

# THEORY OF MACHINES

①

## CHAPTER - 1

## SIMPLE MECHANISMS

Theory of m/c is a branch of science which deals with the study of relative motion between the various parts of a m/c, & the forces which are acting upon them.

It is broadly divided into two parts.

### Kinematics

It deals with the study of relative motion between the various parts of the machine, but the various forces involved in the motion, are not considered.

### Dynamics

It deals with the study of various forces involved in the various parts of the m/c. The forces may be static or dynamic.

## Mechanism & Machine

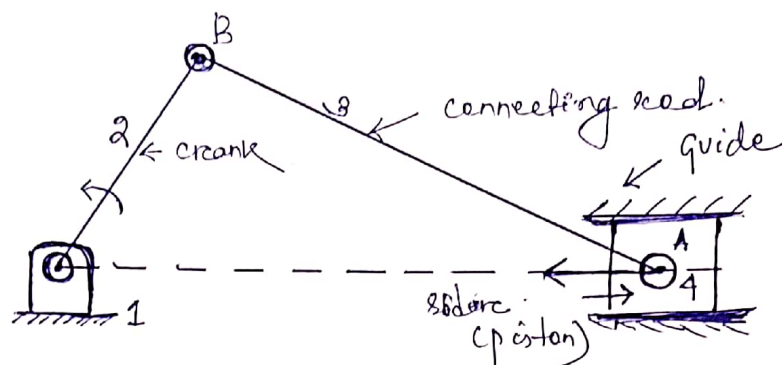
① Mechanism → If a no. of rigid bodies are assembled in such a way that the motion of one causes ~~constraint~~ constrained and predictable motion to the another, is known as a m/c. mechanism.

A mechanism also transmits & modify motion.

② Machine → A machine is a mechanism or combination of mechanisms which not only imparts definite motions to the parts but also transmits & modify the available mechanical energy into some kind of useful energy. This useful energy may be in the shape of some kind of desired work.

sem  
by

for example :- slider crank mechanism.



The above fig. shows a mechanism which is known as slider-crank mechanism. It is a combination of rigid bodies namely crank, connecting rod & slider. They are also so shaped & connected that they move upon each other with definite relative motion.

The slider-crank mechanism will become a m/c become a m/c when it is used in automobile engine by adding valve mechanism etc. In that case it will convert the available energy (force on the piston) into the desired energy (torque on the crank shaft). This torque will move the vehicle.

## Link

A link is defined as a member or a combination of members, connecting other members & having motion relative to them. A slider-crank mechanism consists of following four links.

- i) frame
- ii) crank
- iii) connecting rod
- iv) slider

The slider (4) reciprocates in guide, which is connected to frame. Hence guide also becomes link 1.

Types of link → Rigid link  
Flexible link  
Fluid link



## Kinematic pair

(2)

Two links or element of a machine, when in contact with each other they said to form a pair.

### Types of kinematic pair

1. According to nature of contact.

- Lower pair
- Higher pair

Lower pair — A pair of link having surface or area contact between them known as lower pair. The contact surface of two links are similar.

eg. shaft rotating in bearing  
Nut turning in a screw.

Higher pair — When a pair has a point or line contact between the links, is known as higher pair. The contact surface bet<sup>n</sup> two surface are dissimilar.

eg. cam follower pair  
\*

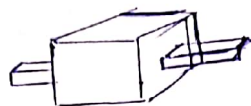
2. Acc. to nature of Mechanical constraint

closed pair — When the elements of a pair of link are held together mechanically, it is known as closed pair. The two elements are geometrically identical & one element envelope the other one. eg.

unclosed pair — When two links of a pair are in contact either due to gravity are called unclosed pair.

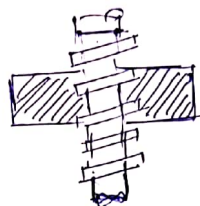
## 3. Acc. to nature of relative motion

Sliding pair -



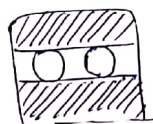
eg. piston & cylinder

Turning pair -



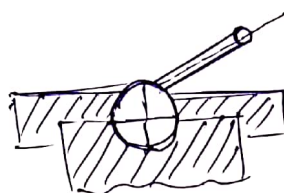
lathe spindle supported in head stock

Rolling pair -



Roller bearing

Spherical pair -



Ball & socket joint attachment of car mirror.

$l \rightarrow$  no. of links  
 $j \rightarrow$  no. of binary joints

## Degree of freedom

It can be defined as the no. of independent motion both translational & rotational a body can have.

$\rightarrow$  Translation motion along  $x, y$  &  $z$  axis.  
 $\rightarrow$  Rotational motion along  $xy, xz$  axis.

$$\boxed{\eta = 3(l-1) - 2j - h}$$

$h \rightarrow$  no. of higher pairs.

It is called as Kutzbach criterion, for a mechanism.

## Kinematic chain

When a kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion, it is called as a kinematic chain.

\* The relation bet<sup>n</sup> no. of link ( $L$ ) & pairs ( $P$ ) is given by

$$L = 2P - 4 \quad \text{--- (1)}$$

\* The relation bet<sup>n</sup> no. of link ( $L$ ) & joints ( $J$ ) is given by

$$J = \frac{3}{2}L - 2 \quad \text{--- (2)}$$

On the equation ① & ② if

③

$LHS > RHS \rightarrow$  then the chain is locked

$LHS < RHS \rightarrow$  then the chain is unconstrained

$LHS = RHS \rightarrow$  " " is constrained

example - 1

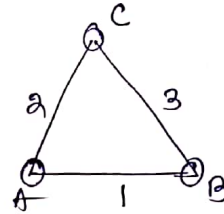
A 3 link chain with 3 Joints.

Hence the chain is locked

$$j = 3$$

$$L = 3$$

$$P = 3$$



①  $L = 2P - 4$

$\Rightarrow 3 = 6 - 4 = 2$

$\Rightarrow 3 > 2 \quad LHS > RHS$

②  $j = \frac{3}{2}L - 2$

$\Rightarrow 3 = \frac{3}{2} \times 3 - 2$

$\Rightarrow 3 > 2.5 \quad LHS > RHS$

Hence proved.

### Inversion Mechanism

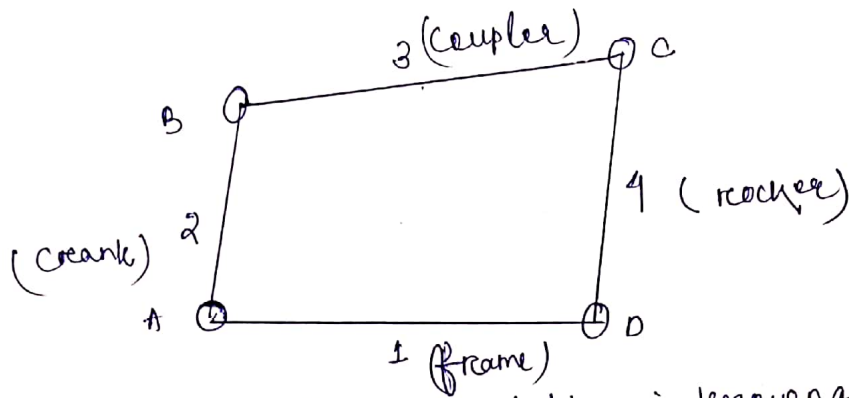
Mechanism is a kinematic chain where one link is fixed. As there are many links in a chain, by fixing one at a time, we can obtain a no. of Mechanism. & This method is known as inversion.

### Four bar chain Mechanism & its inversion.

This is the simplest kinematic chain. It consists of four rigid links which are connected in the form of a quadrilateral by 4 pin joints.

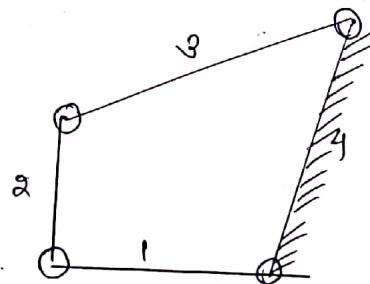
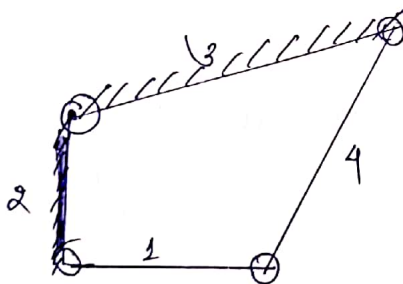
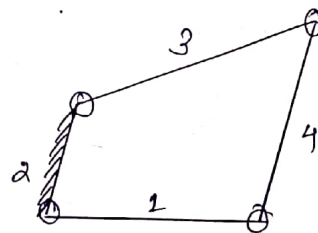
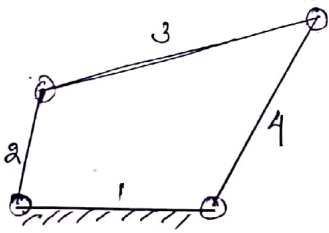
It consists of four turning pairs. Link 1 & Link 2 form first turning pair, link 2 & link 3 form second turning pair. link 3 & 4 and 4 & 1 4th turning pairs respectively.





A link that makes complete revolution is known as crank. The fixed link is known as frame of the mechanism. The link opposite to the fixed link is known as connecting rod. The fourth link is known as lever, or rocker or as another crank (if it rotates). (if oscillates)

If different links of the four bar mechanism are fixed, four diff. mechanism will be obtained.



### Conditions for four bar Mechanism / (Grashof condition)

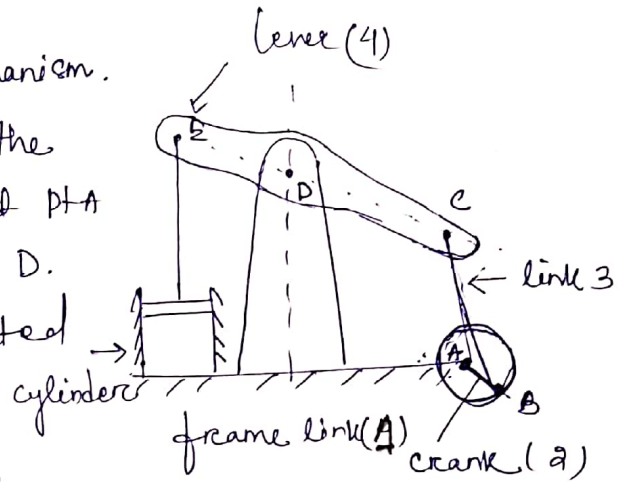
- The length of one link should <sup>not</sup> be greater than the sum of the ~~other~~ other three links.
- One of the shortest link should make a complete revolution relative to other 3 link.
- The sum of the longest & shortest link should not be greater than the sum of the other 2 links.

## Examples of inversions

(1)

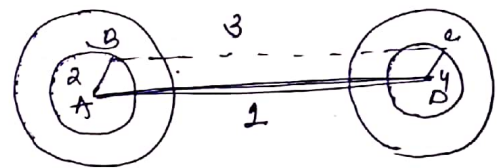
### 1. Beam engine (Crank lever mechanism)

It is also known as crank lever mechanism. which consist of 4 links. As shown in the figure, when the crank rotates about P/A the lever oscillates about fixed centre D. The end 'E' of the lever EDC is connected to a piston rod which reciprocates due to revolution of crank. Thus the rotary motion is converted into reciprocating motion.



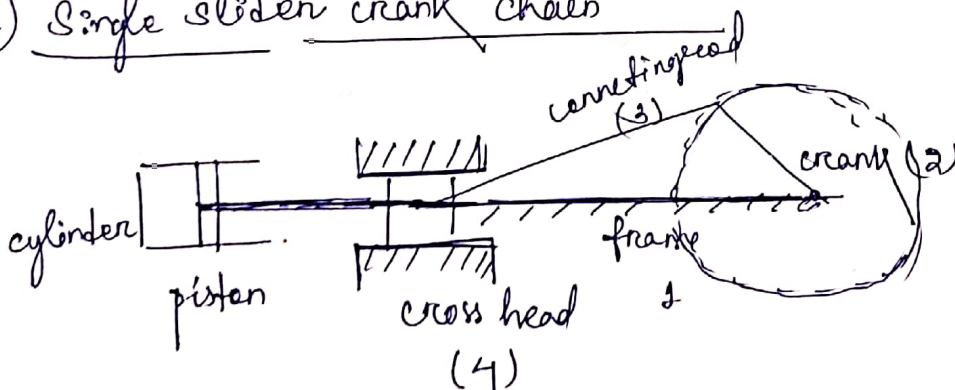
### 2. Coupling rod of locomotive (Double crank mechanism)

Here the links AB & CD are act as cranks & connected to the respective wheels. The link BC act as coupling rod. & AD act as frame or fixed.



— used to transmit rotary motion from one wheel to another wheel.

### 3. Single slider crank chain



— It consist of 4 links having one sliding pair & 3 turning pairs.  
— It converts rotary motion into reciprocating motion & vice versa.

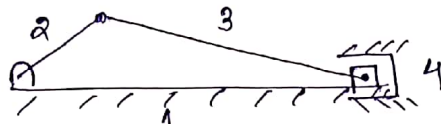
1-2, 2-3, 3-4 → turning pair  
4-1 → sliding pair

## Inversion of single slider crank chain

### 1) 1st inversion

It is obtained when 1 link is fixed & link 2 is the crank & link 4 becomes slider.

Application — reciprocating engine

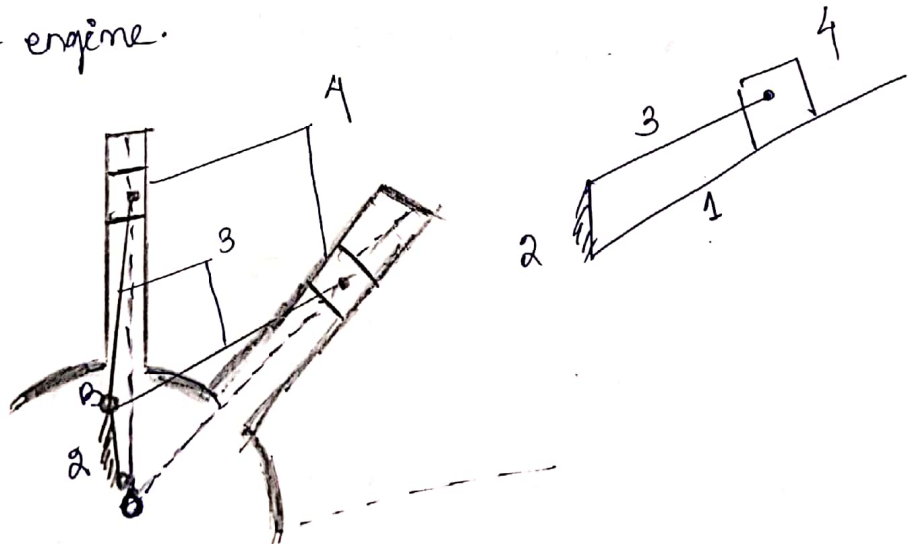


Here link 1 is fixed & 2 is the crank & link 4 is the slider.

### 2) 2nd inversion

It is obtained when link 2 is fixed. When link 2 is fixed instead of link 1, the link 3 along with slider 4 will become the crank. It causes link 1 to rotate about pt. O, with the slider reciprocating.

Application — Rotary engine.



The fig shows the rotary engine mechanism. In this mechanism link 2 is fixed. Link 4 is made as the ~~cylinder~~ piston arrangement. & link 1 made as the cylinder, which reciprocates about link 4. Here instead of one cylinder, seven or nine cylinders are symmetrically placed in regular intervals in the same plane. All the cylinders



rotates about the common fixed center O. The fixed link 2 is common to all cylinders. The only difference between the 1st & 2nd inversion is that, in 1st inversion, the body is fixed & crank is rotating whereas in 2nd inversion, the crank is fixed & body rotates.

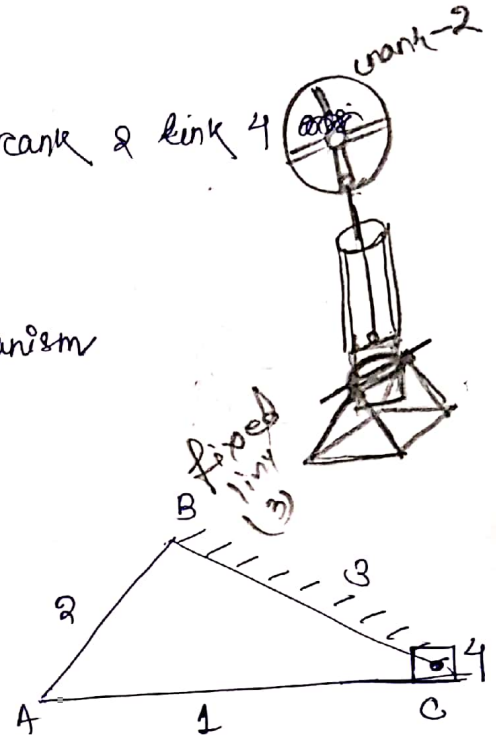
### 3) 3rd inversion

when link 3 is fixed and link 2 act as crank & link 4 oscillates.

Application — oscillating cylinder  
crank & slotted lever mechanism

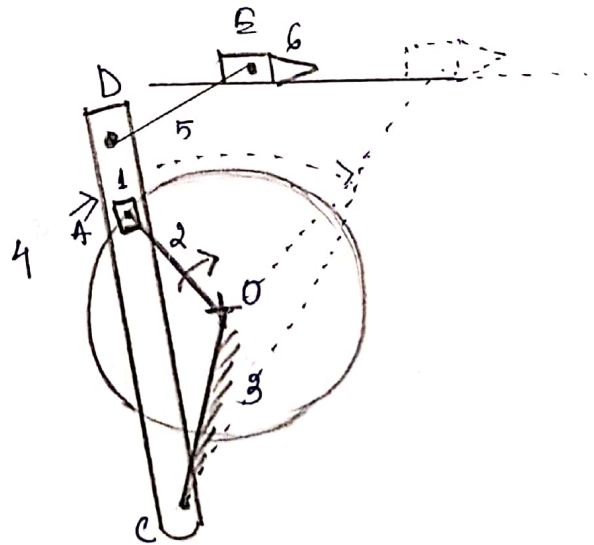
### 8) oscillating cylinder

The piston reciprocates inside the cylinder pivoted to the fixed link 3. when the piston reciprocates the crank rotates about O.



### ii) crank & slotted lever mechanism

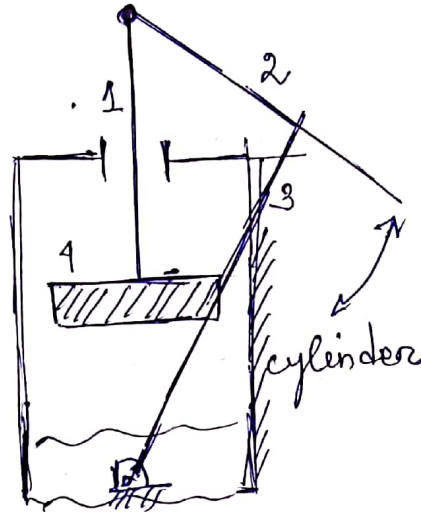
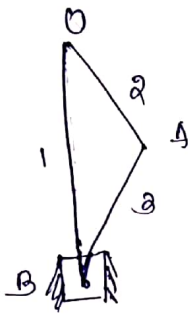
In the fig, link 1 is the slider which slides upon link 4. Link 3 is fixed and link 2 is the crank which rotates counter clockwise about a fixed center O. The link 4 is extended to pt. D which is another link, link, 5. At the end of link 5 link 6 is attached, which is the cutting tool.



### 4) 4th inversion

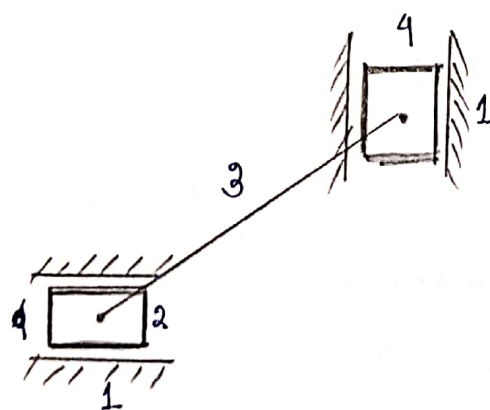
When the 4th link is fixed, the 4th inversion is obtained. Link 3 can oscillate about the fixed pivot B on the link 4. It causes link 2 to oscillate about B & the end O to reciprocate along the axis of the fixed link 4.

Application - Hand pump.



### 3) Double slider crank chain

A four bar chain having 2 turning pairs & 2 sliding pairs such that two pairs of same kind are adjacent is called double slider crank chain.



link 1-2 } sliding pair  
4-1 }

link 2-3 } turning pair  
3-4 }

### i) 1st inversion

It is obtained when link 1 is fixed & 2-3, 3-4 are turning pairs and 1-2 & 4-1 are sliding pairs.

## Application - Elliptical Trammel

(6)

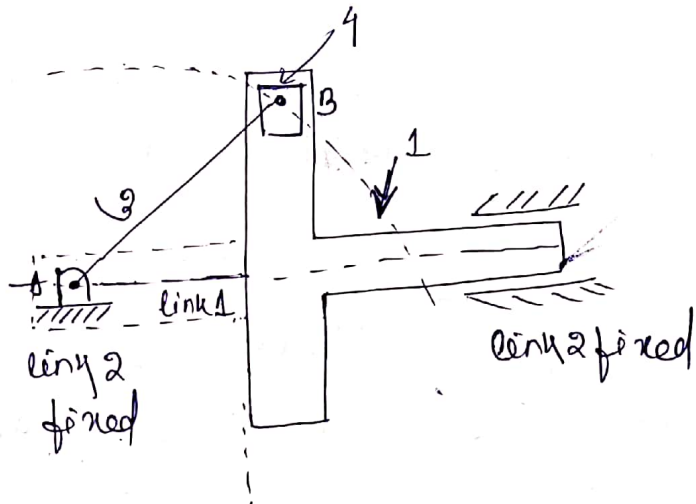
It is an instrument used to draw ellipse.

### i) 2nd inversion

It is obtained when any slide block in the 1st inversion is fixed.

Application: Scotch yoke mechanism.

Here link 2 is fixed and 'B' is a point on link 3 rotates about pt A. and link 1 reciprocates in the horizontal direction. The link 1 reciprocates about fixed link 2.



### iii) 3rd inversion

When link 3 is fixed it is called as the 3rd inversion.

Application - Oldham's coupling.

It is used for connecting 2 parallel shaft whose axes are at a small distance apart. The shafts are coupled such that if one shaft rotates, the other shaft also rotates with the same speed.



### 1) completely constrained motion

→ when the motion between the two elements of a pair is in a definite direction irrespective of the direction of the force applied.

→ It may be linear or rotary.

Eg. piston & cylinder of the steam engine.

Here the motion of the piston is limited to definite direction.

### 2) incompletely constrained motion

When motion between two elements of a pair is possible in more than one direction & depends on the direction of force applied.

Eg. Shaft in circular hole.

(It can slide as well as rotate)

### 3) Successfully constrained motion

When the motion between two elements of a pair is possible in more than one direction but is made to have motion only in one direction by using some external means.

Eg. Shaft in a foot step bearing, the vertical motion is restricted in upward direction by applying load, apart from rotary motion.

## FRICTION

When a solid body slides over a stationary solid body, a force is exerted at the surface of contact by the stationary body on the moving body. This force is called as friction.

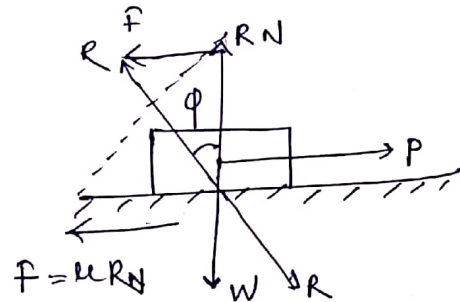
### Types

- static friction → it is the friction, experienced by a body, when at rest.
- Dynamic friction → experienced by a body, when it is in motion.

### coefficient of friction ( $\mu$ )

$$\mu = \frac{\text{Limiting friction}}{\text{Normal reaction}} = \frac{F}{R_N}$$

$$\Rightarrow \boxed{F = \mu R_N} \quad \text{--- ①}$$



### angle of friction ( $\phi$ )

$$\tan \phi = \frac{F}{R_N} = \frac{\mu R_N}{R_N}$$

$$\boxed{\tan \phi = \mu} \quad \text{--- ②}$$

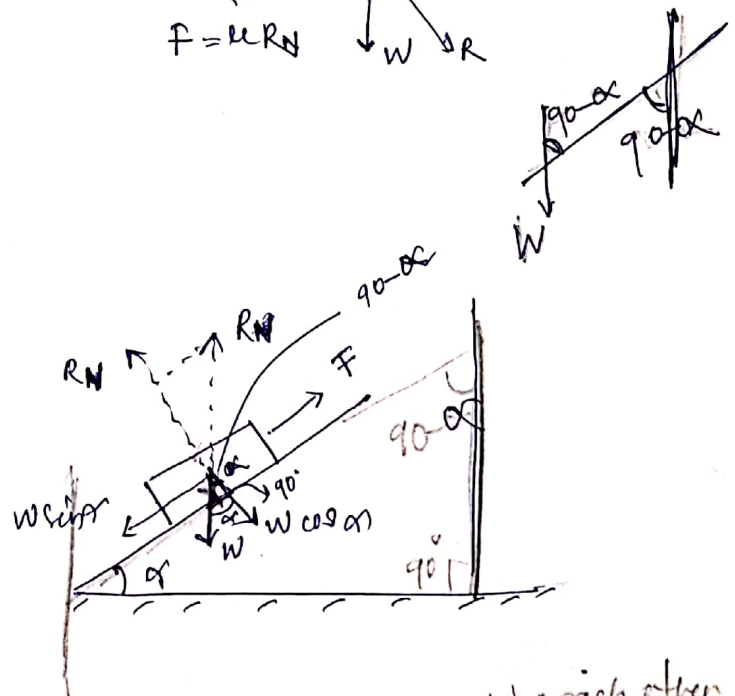
### Angle of repose ( $\alpha$ )

$$\begin{aligned} W \sin \alpha &= F \\ &= \mu R_N \\ &= \mu W \cos \alpha \end{aligned}$$

$$\Rightarrow \mu = \tan \alpha$$

$$\Rightarrow \boxed{\tan \phi = \tan \alpha} \quad \text{--- ③}$$

$$\boxed{\alpha = \phi}$$



|| to each other  
alternate interior angle.  
corresponding angle.

## Screw Friction

Screws, bolts, studs, nuts etc are widely used in various m/c and structures for temporary fastening. These fastenings have screw threads, which is made of cutting a continuous helical groove on a cylindrical surface. If the cuts are outside it is called as external threads & if the cuts are internal it is called as internal threads. Screw threads are 2 types. Square thread.

& V. thread.

### Terms related to threads

Helix - It is the curve traced by a particle, while ~~describing~~ ~~a circular path~~ moving along a screw thread.

pitch - It is the distance from a point of a screw to a corresponding point on the next thread measured parallel to the axis of the screw.

lead - It is the distance which a screw thread advances axially in one turn. (P)

Helix angle - Slope of thread with the horizontal.

$$\tan \alpha = \frac{\text{Lead}}{\text{circumference of screw}} = \frac{P}{\pi d} \quad \text{--- (4)}$$

## Screw Jack

It is a device used for lifting of heavy loads, with very small effort. It ~~consist~~ consist of a nut, screw & a handle fitted to the head of the screw. The nut also forms the body of the Jack.

$W$  = Weight placed on the screw head.

$P$  = Effort applied at the end of handle.

$L$  = Length of the handle.



(2)

$p$  = pitch of the screw

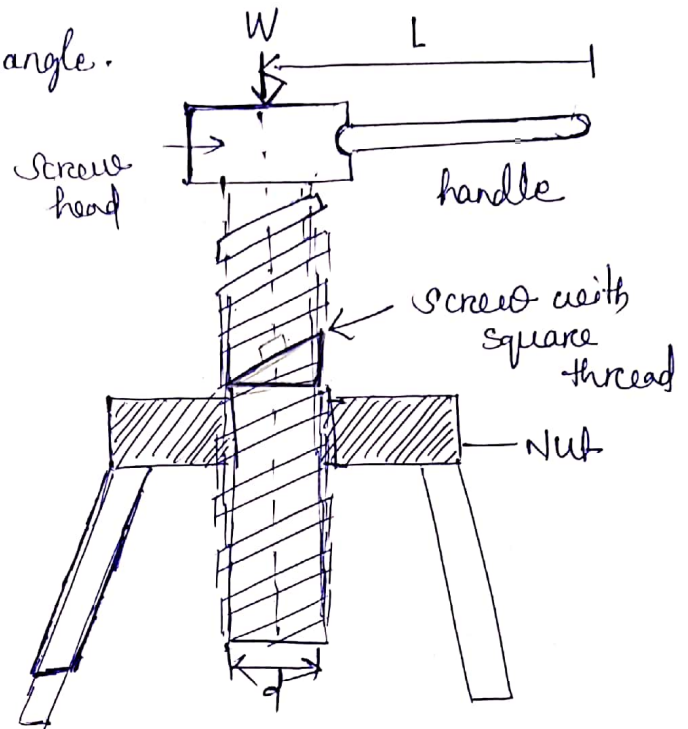
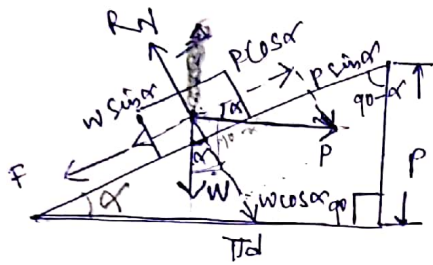
$d$  = mean dia of the screw

$\alpha$  = angle of the screw or helix angle.

$\phi$  = angle of friction

$\mu$  = co-efficient of friction.

Torque required to lift the load



Since the load is being lifted, therefore force of friction will act downwards.

from the above fig

$$p \cos \alpha = W \sin \alpha + F$$

$$\Rightarrow p \cos \alpha = W \sin \alpha + \mu R_N \quad \text{--- (5)}$$

( $\because F = \mu R_N$ )  
(Resolution along horizontal)

$$R_N = p \sin \alpha + W \cos \alpha \quad \text{--- (6)}$$

putting  $R_N$  in eqn (5)

$$p \cos \alpha = W \sin \alpha + \mu (p \sin \alpha + W \cos \alpha)$$

$$\Rightarrow p \cos \alpha = W \sin \alpha + \mu p \sin \alpha + \mu W \cos \alpha$$

$$\Rightarrow p \cos \alpha - \mu p \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

$$\Rightarrow p (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$

$$\Rightarrow p = W \left[ \frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha} \right]$$

$$\Rightarrow p = W \left[ \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha} \right]$$

$$= W \left[ \frac{\sin \alpha + \frac{\sin \phi}{\cos \phi} \cos \alpha}{\cos \alpha - \frac{\sin \phi}{\cos \phi} \sin \alpha} \right]$$

putting  $\mu = \tan \phi$

$$\Rightarrow P = W \times \frac{\sin \alpha \cdot \cos \phi + \sin \phi \cdot \cos \alpha}{\cos \alpha \cdot \cos \phi - \sin \alpha \sin \phi}$$

$$\Rightarrow P = W \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)}$$

$$\Rightarrow \boxed{P = W \tan (\alpha + \phi)} \text{ — (7)}$$

Torque required to overcome friction between screw & nut

$$T = P \times \frac{d}{2}$$

$$\boxed{T = W \tan (\alpha + \phi) \frac{d}{2}} \text{ — (8)}$$

without friction

$$\boxed{P = W \tan \alpha}$$

$$\star \text{ Speed of screw (N)} = \frac{\text{Speed of nut}}{\text{Pitch of screw}}$$

Torque required to lower the load by a screw jack.

$$\boxed{T = W \tan (\phi - \alpha) \frac{d}{2}} \text{ — (9)}$$

## Bearing

A bearing is a m/c element that constraints relative motion to only the desired motion and reduces friction between moving parts. For example rotary bearings hold rotating components such as shafts & axles with in mechanical system.

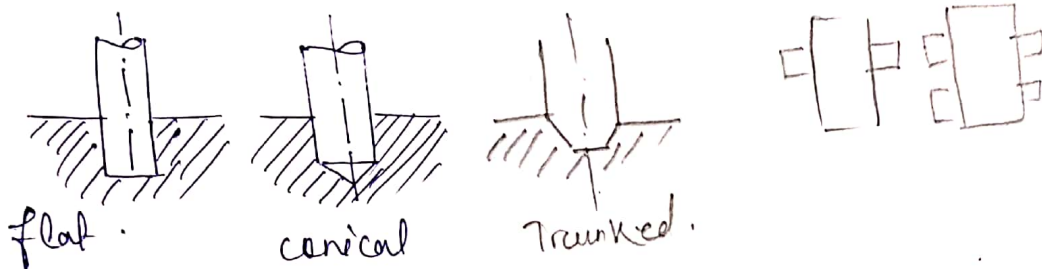
Types — ball bearing

Roller bearing

plain bearing

needle bearing

- ③ The rotating shaft are subjected to axial thrust. These shafts can be kept in correct axial position if bearing surfaces are provided. The bearing surface which is placed at the end of a shaft are known as pivots.
- A pivot may be of flat or conical surface.



For describing friction in bearing, two assumptions has to be taken:

UPT ① The pressure is uniformly distributed throughout the bearing surface.

UPT ② The wear is uniform throughout the bearing surface.

### Flat pivot bearing / Foot step bearing

It is also known as footstep bearing.

$W \rightarrow$  load transmitted over bearing surface due to shaft

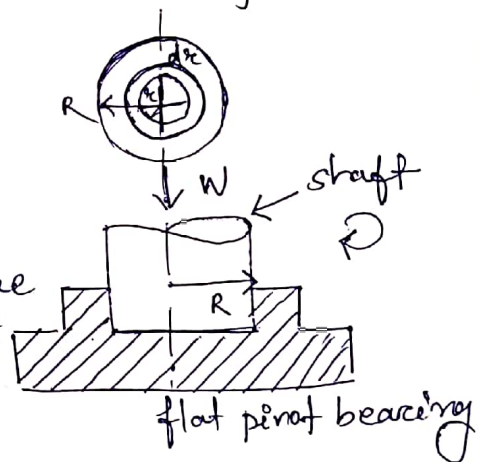
$R \rightarrow$  Radius of bearing surface.

$P \rightarrow$  pressure (intensity of) per unit area.

$\mu \rightarrow$  coefficient of friction.

$T \rightarrow$  Total frictional torque.

Consider a small ring having radius  $r$  & thickness  $dr$ .



Area of  $dA = 2\pi r \cdot dr$ . Area of bearing surface for ring bearing surface  $= 2\pi \times R \cdot dr$   
 $= \pi d \cdot dr$

## i) uniform pressure

Total pressure  $p = \frac{W}{\pi R^2}$

$$A = 2\pi r \cdot dr$$

Load transmitted at the ~~area~~ ring  $\delta W = p \times A$   
 $= p \times 2\pi r \cdot dr$

Frictional resistance to sliding acting at the ring

$$F_{fr} = \mu \cdot \delta W = \mu p 2\pi r \cdot dr$$

$$F_r = 2\pi \mu p r \cdot dr$$

Frictional torque  $T_r = F_r \times r$

$$T_r = 2\pi \mu p r^2 dr$$

Total torque.  $T = \int_0^R T_r = \int_0^R 2\pi \mu p r^2 dr$

$$\Rightarrow T = 2\pi \mu p \int_0^R r^2 dr$$

$$= 2\pi \mu p \left[ \frac{r^3}{3} \right]_0^R = 2\pi \mu p \frac{R^3}{3}$$

$$\Rightarrow T = \frac{2}{3} \pi \mu \times \frac{W}{\pi R^2} \times \frac{R^3}{3} \quad \left( \because p = \frac{W}{\pi R^2} \right)$$

$$\Rightarrow \boxed{T = \frac{2}{3} \times \mu \times W \times R} \quad \text{--- (10)}$$

power lost in friction  $P = T \times \omega$

$$\Rightarrow \boxed{P = \frac{2\pi N T}{60}}$$

## ii) uniform wear

The rate of wear depends upon the intensity of pressure.  
 & the velocity of the rubbing surface ( $v$ ). This rubbing velocity increases with the distance from the axis of bearing (i.e. radius)



$$P \cdot r = C$$

( $C \rightarrow \text{constant}$ )

(4)

$$\Rightarrow P = \frac{C}{r}$$

$$\delta W = p \times 2\pi r \times dr$$

$$= \frac{C}{r} \times 2\pi r \times dr$$

$$= 2\pi C \cdot dr$$

Total load transmitted to the bearing

$$W = \int_0^R \delta W$$

$$= 2\pi C r \Big|_0^R$$

$$W = 2\pi C R$$

Total torque

$$T_r = 2\pi r p r^2 dr \quad (F_r \times r_r)$$

$$= 2\pi r \frac{C}{r} r^2 dr$$

$$= 2\pi C \cdot r \cdot dr$$

$$\text{Total } T = \int_0^R T_r = \int_0^R 2\pi C r \cdot dr = 2\pi C \int_0^R r \cdot dr = 2\pi C \frac{r^2}{2} \Big|_0^R$$

$$\Rightarrow T = \pi C \frac{R^2}{2}$$

$$\Rightarrow T = \pi C \frac{R^2}{2} \quad \Rightarrow T = \pi \mu C R^2 \quad \text{--- (1)}$$

$$\Rightarrow T = \frac{1}{2} \mu W R$$

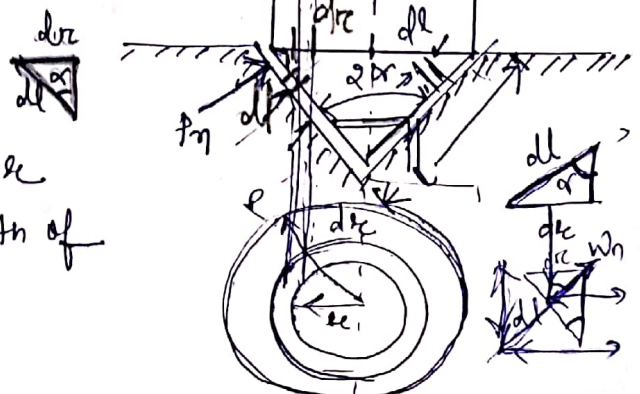
### CONICAL PIVOT BEARING

$P_m \rightarrow$  intensity of pressure normal to cone.

$\alpha \rightarrow$  semi angle of cone.

$R \rightarrow$  radius of shaft.

Consider a small ring having radius  $r$  & thickness  $dr$  & length  $dl$  in the length of cone.



$dl = \frac{dr}{\cos \theta}$  (Load acting on the circular ring, normal to the conical surface)

$A = 2\pi r \cdot dl$  (Area of the small element)  
 $= 2\pi r \cdot dr \cdot \sec \theta$

i) uniform pressure

$\delta W_n = p_n \times A$  (normal load)

$\Rightarrow \delta W_n = p_n \times 2\pi r \cdot dr \cdot \sec \theta$

Vertical load will be

$\delta W_v = \delta W_n \cdot \sin \theta$

$= p_n \times 2\pi r \cdot dr \cdot \sec \theta \cdot \sin \theta$

$\delta W_v = p_n \times 2\pi r \cdot dr$  ( $\because \sec \theta \sin \theta = 1$ )

total vertical load.

$W = \int_0^R p_n \times 2\pi r \cdot dr$

$= p_n 2\pi \int_0^R \frac{r^2}{2}$

$= 2\pi p_n \times \frac{R^2}{2}$

$W = \pi p_n R^2$

$\Rightarrow p_n = \frac{W}{\pi R^2}$

Frictional ~~normal~~ forces acting tangentially

$F_r = \mu \times \delta W_n$

$= \mu \times p_n \times 2\pi r \cdot dr \cdot \sec \theta$

$= 2\pi \mu p_n \cdot \sec \theta \cdot r \cdot dr$

Torque  $T_r = F_r \times r$

$= 2\pi \mu p_n \sec \theta r^2 dr$

Integrating  $T_r$

$T = \int 2\pi \mu p_n \sec \theta r^2 dr$

$T = 2\pi \mu p_n \sec \theta \frac{R^3}{3}$

$T = \frac{2}{3} \times \pi R^3 \mu p_n \sec \theta$

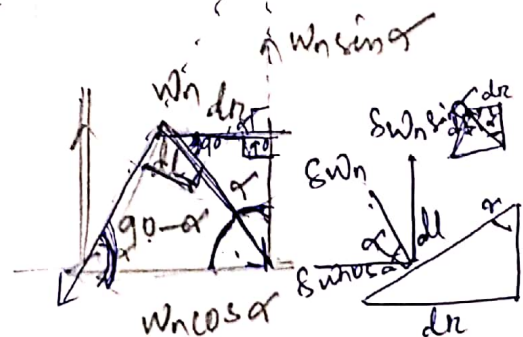
putting  $p_n = \frac{W}{\pi R^2}$

$\Rightarrow T = \frac{2}{3} \times \pi R^3 \times \mu \times \frac{W}{\pi R^2} \sec \theta$

$T = \frac{2}{3} \times \mu W R \sec \theta$

$T = \frac{2}{3} \times \mu W l$  — (12)

( $R \sec \theta = l$ )



Uniform wear

⑤

As we derived previously

$$p_r \times r = C$$

$$\delta W = p_r \times 2\pi r \cdot dr \quad (A = 2\pi r \cdot dr)$$

$$W = \int_0^R 2\pi C \cdot dr$$

$$W = 2\pi C \cdot R$$

$$\Rightarrow C = \frac{W}{2\pi R}$$

Frictional torque acting on the screw.

$$T_r = 2\pi r \mu p_r \cos \alpha \cdot r \cdot dr$$
$$= 2\pi \mu \times \frac{C}{r} \times \cos \alpha \cdot r^2 dr$$

$$T_r = 2\pi \mu C \cos \alpha \cdot r \cdot dr$$

$$\oint T = \int_0^R T_r = \int_0^R 2\pi \mu C \cdot \cos \alpha \cdot r \cdot dr$$
$$= 2\pi \mu C \cdot \cos \alpha \cdot \left[ \frac{r^2}{2} \right]_0^R$$
$$= 2\pi \mu C \cdot \cos \alpha \cdot \frac{R^2}{2}$$

$$T = \pi \mu C \cdot \cos \alpha \cdot R^2$$

$$\Rightarrow T = \frac{1}{2} \mu \cdot W \cdot R \cos \alpha$$

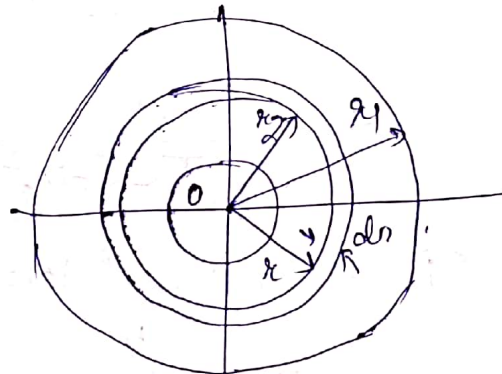
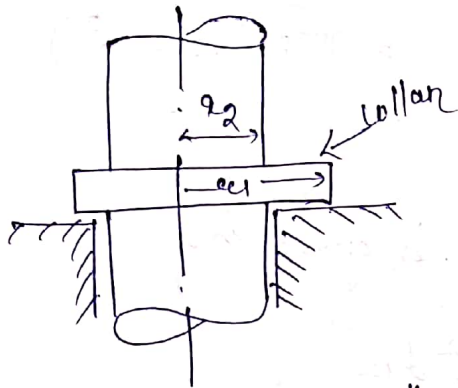
$$\Rightarrow \boxed{T = \frac{1}{2} \mu W \times l} \quad \text{--- (13)}$$

(putting  $C = \frac{W}{2\pi R}$ )



## Flat collar bearing

These are also called as thrust-bearing. The bearing surface provided at any position along the shaft to carry axial thrust known as collar.



$r_1 \rightarrow$  ext. radius of collar  
 $r_2 \rightarrow$  internal " "  
 $p \rightarrow$  Intensity of press.

$w \rightarrow$  Axial load  
 $\mu \rightarrow$  co-eff.  
 $T \rightarrow$  Total frictional load.

Consider a circular ring of thickness  $dr$  at a radius  $r$ .

$$\text{Area of ring} = 2\pi r \cdot dr$$

$$\text{Load on the ring} = p \times 2\pi r \cdot dr$$

$$\begin{aligned} \text{Frictional torque on ring} &= \mu \times \text{Load} \\ &= \mu p 2\pi r \cdot dr \end{aligned}$$

$$\begin{aligned} \text{Torque on ring } dT &= \mu \times p 2\pi r \cdot dr \times r \\ &= 2\pi \mu p r^2 \cdot dr \end{aligned}$$

$$\begin{aligned} \text{Total Frictional torque } T &= \int_{r_2}^{r_1} dT \\ &= \int_{r_2}^{r_1} 2\pi \mu p r^2 \cdot dr \end{aligned}$$

i) uniform pressure

$$p = c$$

$$\begin{aligned} \text{Total load transmitted on the bearing} &= \int_{r_2}^{r_1} \text{load on the ring} \end{aligned}$$

$$W = \int_{r_2}^{r_1} p \cdot 2\pi r \cdot dr$$

$$= p \times 2\pi \int_{r_2}^{r_1} r \cdot dr$$

$$= 2\pi p \left[ \frac{r^2}{2} \right]_{r_2}^{r_1} = 2\pi p \left[ \frac{r_1^2 - r_2^2}{2} \right]$$

$$= \pi p (r_1^2 - r_2^2)$$

$$\Rightarrow p = \frac{W}{\pi(r_1^2 - r_2^2)}$$

Total frictional torque is given by

$$T = \int_{r_2}^{r_1} 2\pi r \mu p r^2 dr$$

$$= 2\pi \mu p \int_{r_2}^{r_1} r^2 dr$$

$$= 2\pi \mu p \left[ \frac{r^3}{3} \right]_{r_2}^{r_1}$$

$$= 2\pi \mu p \left[ \frac{r_1^3 - r_2^3}{3} \right]$$

$$= 2\pi \mu \times \frac{W}{\pi(r_1^2 - r_2^2)} \times \left[ \frac{r_1^3 - r_2^3}{3} \right]$$

$$= \frac{2}{3} \mu W \left[ \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

Uniform wear

$$p \times r = \text{const}$$

$$p = \frac{C}{r}$$

Total load transmitted to the bearing.

$$= \int_{r_2}^{r_1} \text{load on ring}$$

$$= \int_{r_2}^{r_1} p \times 2\pi r \times dr$$

$$W = \int_{r_2}^{r_1} p \times 2\pi r \times dr$$

$$= \int_{r_2}^{r_1} \frac{C}{r} \times 2\pi r \times dr = \int_{r_2}^{r_1} \frac{C}{r} \times 2\pi r \times dr$$

$$= 2\pi C \int_{r_2}^{r_1} dr = 2\pi C \left[ r \right]_{r_2}^{r_1} = 2\pi C (r_1 - r_2)$$

$$C = \frac{W}{2\pi (r_1 - r_2)}$$

Total Frictional torque is given by eqn<sup>n</sup>.

$$T = \int_{r_2}^{r_1} 2\pi \mu p r^2 dr$$

$$= 2\pi \mu \int_{r_2}^{r_1} p r^2 dr = 2\pi \mu \int_{r_2}^{r_1} \frac{C}{r} \times r^2 \times dr$$

$$= 2\pi \mu \int_{r_2}^{r_1} C r \cdot dr = 2\pi \mu C \int_{r_2}^{r_1} r \cdot dr$$

$$= 2\pi \mu C \left[ \frac{r^2}{2} \right]_{r_2}^{r_1} = 2\pi \mu C \left[ \frac{r_1^2 - r_2^2}{2} \right]$$

$$= 2\pi \mu \times \frac{W}{2\pi (r_1 - r_2)} \times \left[ \frac{r_1^2 - r_2^2}{2} \right]$$

$$= \frac{\mu W}{2} (r_1 + r_2)$$



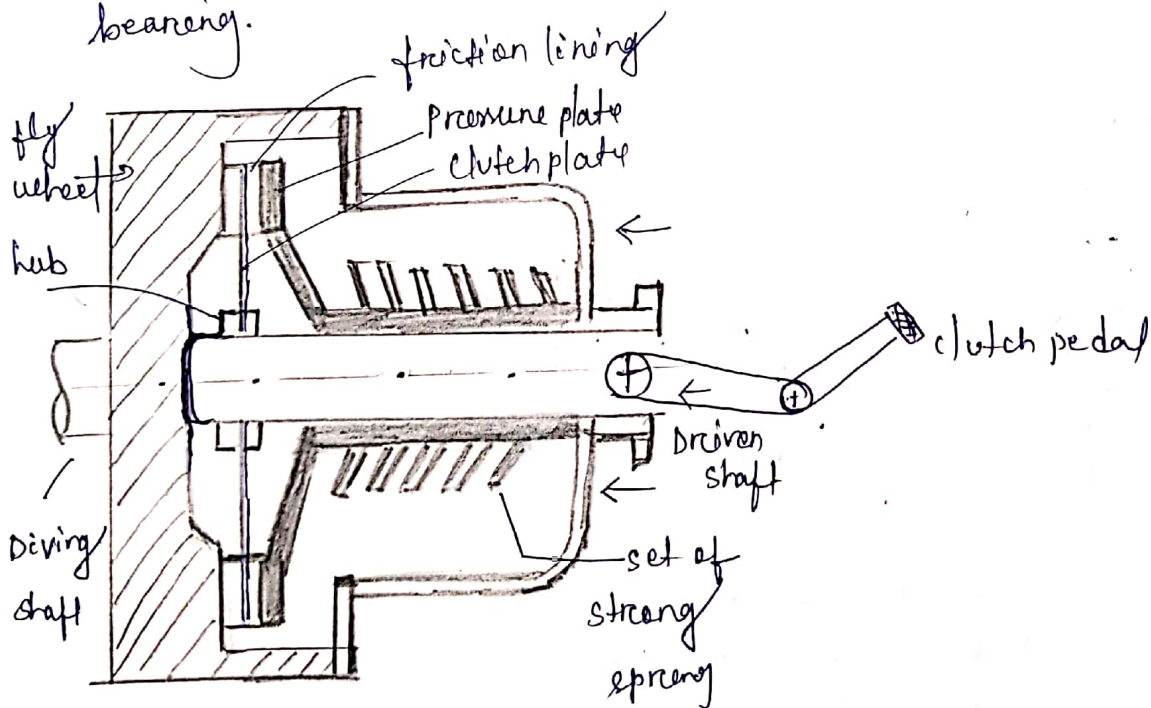
## Friction clutch

⑥

The device used to transmit the rotary motion of one shaft to another or driver shaft to driven shaft is known as friction clutch or "clutch." The engine shaft and the gear box shaft is connected with the help of friction clutch.

### 1. Single plate clutch

A single plate clutch consists of a single clutch plate with friction lining on both sides. This plate is attached to a hub which is free to move axially along the splines of the driver shaft. There is a pressure plate inside the clutch body. This pressure plate pushes the clutch plate towards the flywheel by a set of strong springs. The total clutch body is bolted to the flywheel. The pressure plate & the flywheel rotate with the driving shaft. The movement of the clutch pedal is transferred to the pressure plate through thrust bearing.



Engaged position

When the foot is taken away from the pedal, the set of strong spring will move forward to the pressure plate & a contact is made bet the clutch plate and the flywheel with both side friction lining. Due to the friction lining a tight grip is created between the pressure plate & flywheel, thus the power from the driving shaft is transmitted to the driven shaft. And when the pedal is pressed the both shaft got detached from each other & power trans<sup>n</sup> stops.

### Torque transmitted

Let  $r_1$  = ext. radius of friction lining

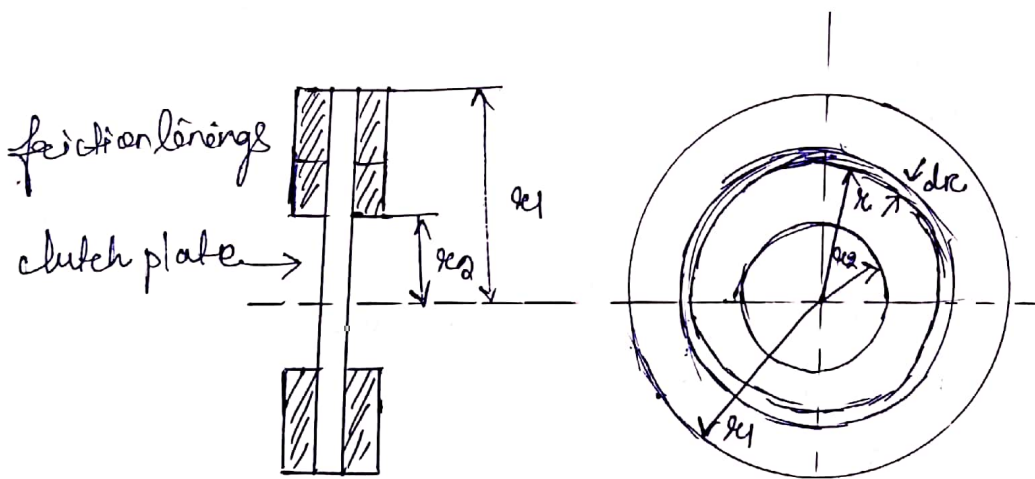
$r_2$  = int. " "

$p$  = intensity of pressure

$w$  = total axial load.

$\mu$  = co. effi. of friction

$T$  = Torque transmitted.



\* For a new clutch the intensity of pressure is approximately ~~high over~~ not uniform over the entire surface where in an old clutch the uniform wear theory is more approximate.

Area of the ring  $dA = 2\pi r \cdot dr$

Axial load in the ring  $dW = p \times 2\pi r \cdot dr$

Radial force "  $dF = \mu \times \text{load in the ring}$   
 $= \mu \times p \times 2\pi r \cdot dr$

Frictional torque  $dT = \text{frictional force} \times \text{radius}$

$$= dF \times r$$

$$= \mu p \times 2\pi r \cdot dr \times r$$

$$= \mu p 2\pi r^2 dr$$

i) For uniform pressure.

$$p = \text{constant}$$

$$= \frac{\text{load}}{A_{\text{clear}}} = \frac{W}{\pi(r_1^2 - r_2^2)} \quad (\text{average pressure})$$

$$T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} \mu p 2\pi r^2 dr$$

$$= 2\pi \mu p \left[ \frac{r^3}{3} \right]_{r_2}^{r_1} = 2\pi \mu p \left[ \frac{r_1^3 - r_2^3}{3} \right]$$

$$= 2\pi \mu \times \frac{W}{\pi(r_1^2 - r_2^2)} \times \left[ \frac{r_1^3 - r_2^3}{3} \right]$$

$$\boxed{T = \frac{2}{3} \mu W \left[ \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]} \quad \text{--- (16)}$$

The above torque for a single single friction plate.

So the total torque on the total clutch plate will be

$$\boxed{T_c = 2T}$$

ii) For uniform wear

$$p \times r = \text{const.}$$

$$p \times r = C$$

$$\Rightarrow p = C/r$$

$$(\because W = 2\pi C (r_1 - r_2))$$

$$\text{We know } dW = 2\pi r dr \times p$$

$$\text{total axial load } W = \int_{r_2}^{r_1} dW = \int_{r_2}^{r_1} 2\pi r \cdot dr \times \frac{C}{r} = 2\pi C [r_1 - r_2]$$

$$\Rightarrow \boxed{C = \frac{W}{2\pi [r_1 - r_2]}}$$



Total frictional torque.

$$T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} \mu \times p \times 2\pi r^2 dr$$

$$= \int_{r_2}^{r_1} \mu \times \frac{C}{r} \times 2\pi r^2 dr = \int_{r_2}^{r_1} \mu C \times 2\pi r \cdot dr$$

$$= 2\pi \mu C \int_{r_2}^{r_1} r \cdot dr = 2\pi \mu C \left[ \frac{r^2}{2} \right]_{r_2}^{r_1} = 2\pi \mu C \left[ \frac{r_1^2 - r_2^2}{2} \right]$$

$$= \mu \times \frac{W}{2\pi(r_1 - r_2)} \times \frac{r_1^2 - r_2^2}{2} \times \pi \quad \left\{ \text{putting value of } C = \frac{W}{2\pi(r_1 - r_2)} \right\}$$

$$= \frac{\mu \times W}{2} (r_1 + r_2)$$

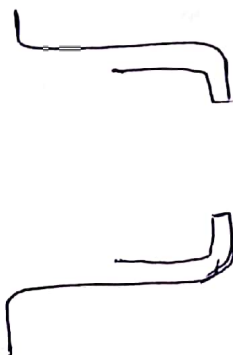
$$T = \frac{\mu \times W (r_1 + r_2)}{2} \quad \text{--- (17)}$$

Total torque on clutch plate  $T_c = 2T$

### Multiplate clutch

The working & construction is same as the single plate clutch except, the friction lining & disc plate. As the name suggest it contains a no. of friction & disc plate.

Diagram.



Torque is same.

$$T = \eta \times \mu \times W \times \frac{2}{3} \left[ \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$T = \frac{\eta \times \mu \times W \times (r_1 + r_2)}{2}$$

$$\eta = \eta_1 + \eta_2 - 1$$

$\eta_1 \rightarrow$  no. of friction plate

$\eta_2 \rightarrow$  " " disc.

## BRAKES

A brake is a mechanical device by means of which artificial frictional resistance is provided or applied to a moving machine, in order to retard or stop the motion of the machine. The energy absorbed by brake is dissipated in the form of heat. The heat is dissipated to the surrounding air.

The capacity of a brake depends upon the following factors.

- The unit pressure bet<sup>n</sup> the braking surface.
- The co-efficient of friction bet<sup>n</sup> the braking surfaces.
- The projected area of the friction surface.
- The ability of ~~brake~~ brake to release heat.

### Material for brakes

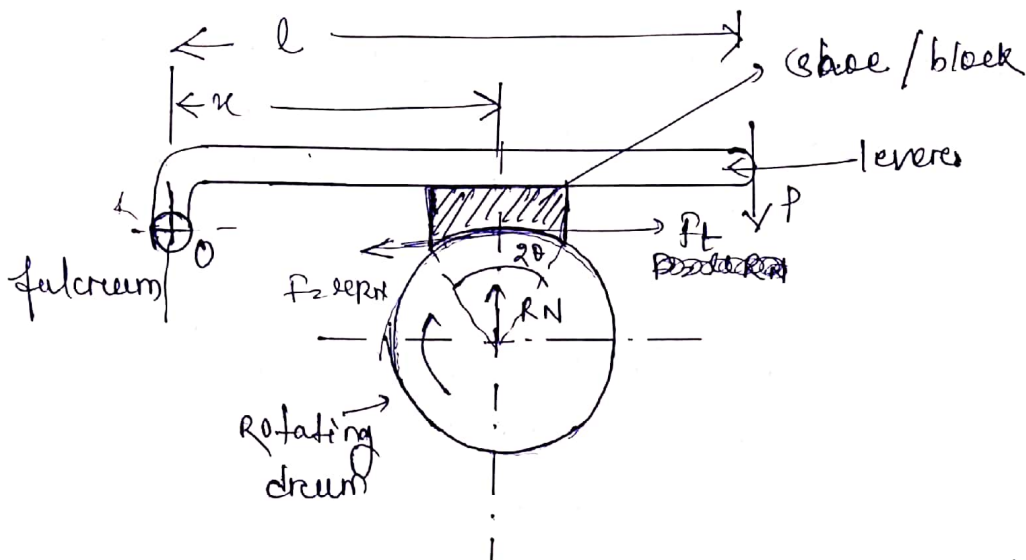
The material used for brake lining should have following characteristic.

- It should have high co-efficient of friction.
- It should have low wear rate.
- It should have high heat resistant.
- It should have enough mechanical strength.
- It should not be effected by moisture & oil.

Some important type of Mechanical brakes are.

- 1) simple shoe / block brake
- 2) Double block brake
- 3) Band brake
- 4) Internal expanding brake etc.

## 1) single block or shoe brake



- This above fig. is the arrangement of simple shoe/block brake. The face of a brake has a special frictioned material which has high value of co-efficient of friction.

- A single block or shoe brake consist of a block which is pressed against a rotating drum. This block is rigidly fixed to the lever. The force is applied at one end of the lever and other end of the lever is pivoted to a fixed fulcrum O. As the force is applied to the lever, the block is pressed against the rotating drum. The friction bet<sup>n</sup> the block & the drum causes a tangential force to act on the drum, which tends to prevent the rotation.

- The block is made up softer material than that of drum, so the block can be replaced on wearing. for eg. ~~tricycle~~ bicycle.

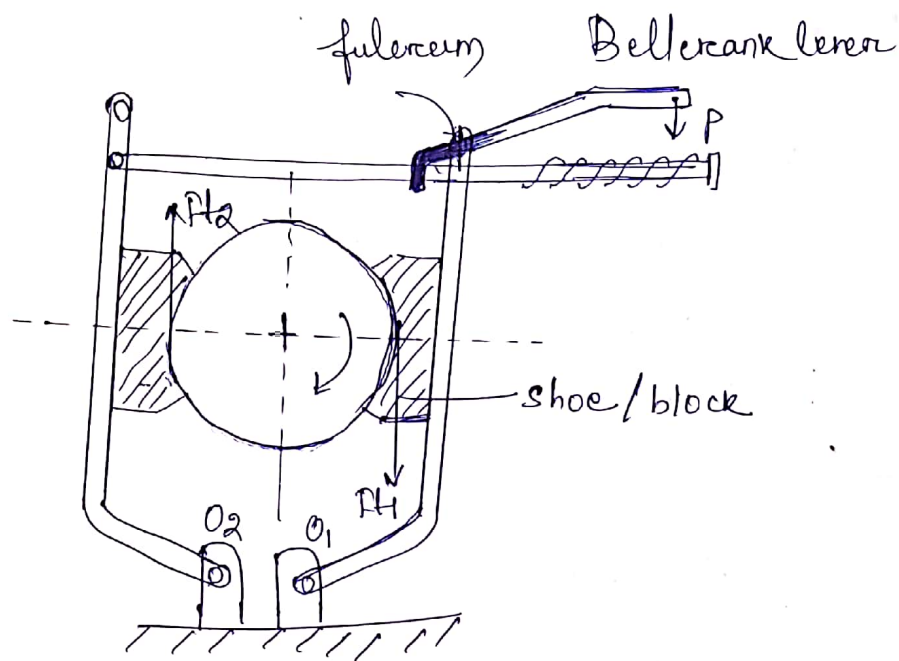
## 2) Double shoe / block brake

- It consist of two block or two shoe applied at opposite ends of a diameter of the wheel. The brake is set by a spring which pulls the upper ends of the brake arm together.

- when a force P is applied to the bell crank lever, the spring is compressed and the brake is released.



This type of brakes often used on electric cranes.

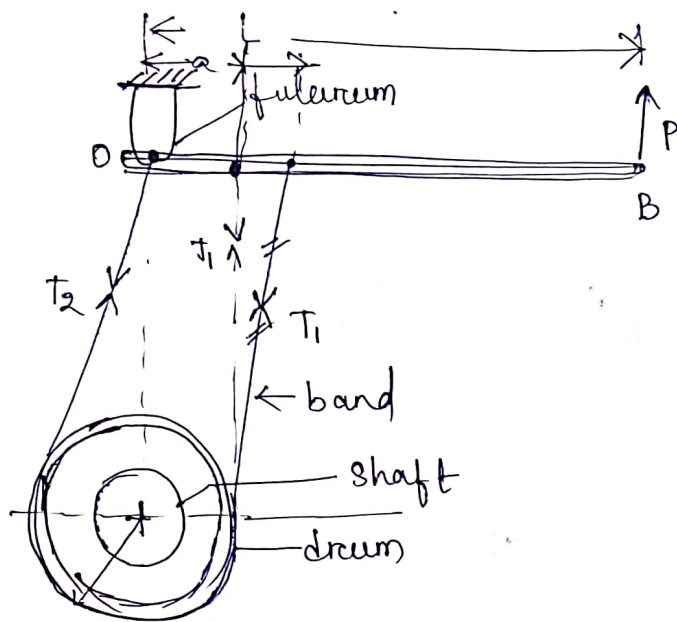


### Simple band brake

A band brake consists of one or more ropes, belt or flexible steel band lined with friction material, which encircle the circumference of rotating drum. The fig. shows a simple band brake in which one end is attached with the fulcrum (fixed pin) of the lever while other end is attached to the lever at distance 'a' from the fulcrum. In order to apply the brake, the band is tightened around the drum & the friction between the band and the drum provides the braking torque.

The force  $P$  is applied at the free end of the lever which turns about the fulcrum  $O$ . This tightens the band on the drum and hence brake is applied. The braking force is provided by the friction between the band & the drum.





$T_1 \rightarrow$  Tension in the tight side

$T_2 \rightarrow$  " " " slack "

$t \rightarrow$  thickness of the band.

$r \rightarrow$  radius of drum

$$r_c = r + t/2$$

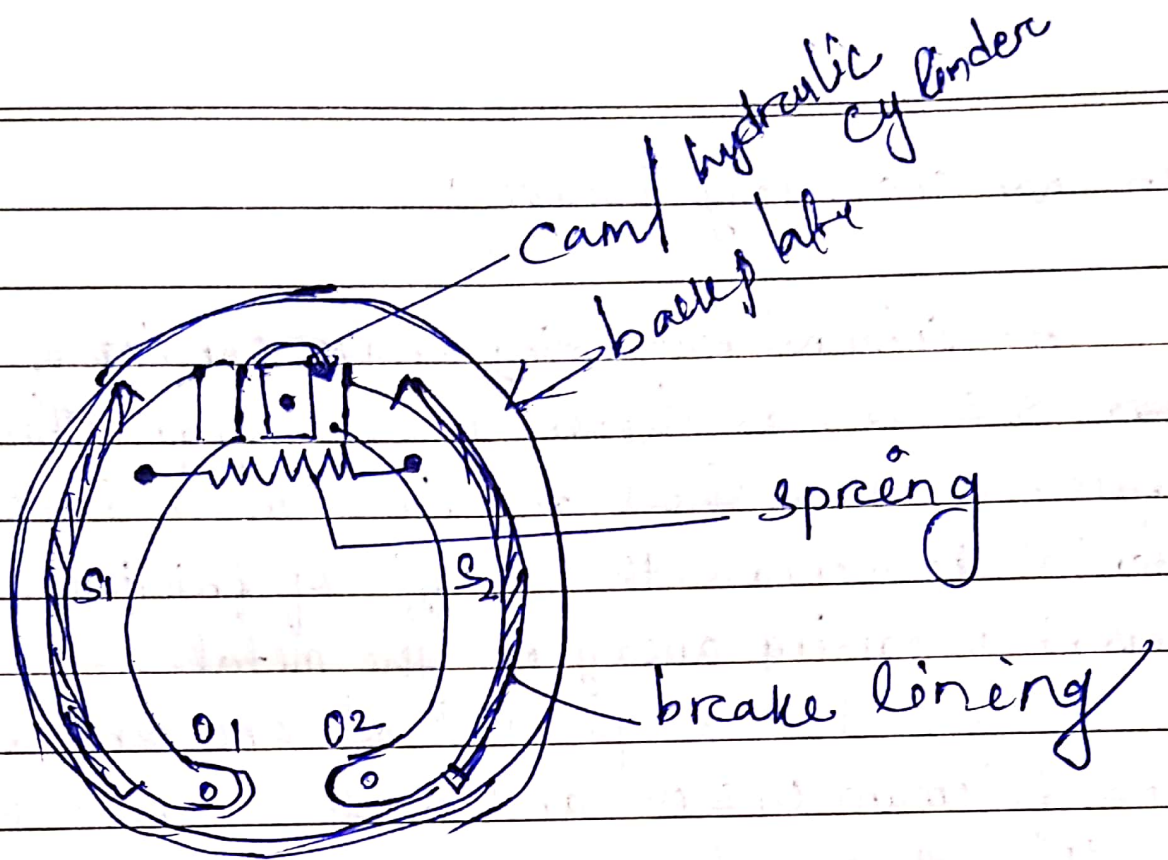
$t \rightarrow$  thickness of band.

Breaking torque on the drum.

$$T_B = (T_1 - T_2) r \quad (\text{neglecting thickness})$$

$$T_B = (T_1 - T_2) r_c$$

$$= (T_1 - T_2) (r + t/2) \quad \text{considering thickness.}$$



## Internal expanding Brake:

An internal expanding brake consist of 2 shoe ~~brake~~  $S_1$  &  $S_2$ , as shown in the fig. The entire surface of the shoes are lined with some friction material to increase the co-effi of friction and to prevent wearing away of the metal.

— Each shoe is pivoted at one end about a fixed fulcrum  $O_1$  &  $O_2$  and made to contact a cam at the other end.

— When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque & hence reduces the speed of the drum. The shoes are normally held in off position by a spring, as shown in the figure. The drum encloses the entire mechanism to keep out dust & moisture. — These are used in motor cars & light weight trucks.



## dynamometers

A dynamometer is a brake but in addition, it has a device to measure the frictional resistance. By measuring the frictional resistance, we may obtain the torque transmitted & hence the power of the engine.

It is broadly divided as 2 categories.

- ① Absorption dynamometer.
- ② Transmission dynamometer.

In absorption dynamometer the entire energy or power produced by the engine is absorbed by the friction resistances of the brake & transformed into heat.

In transmission dynamometer, the energy is not wasted in friction but it is used for doing work. Here the energy or power produced by the engine is transmitted through the dynamometer to some other m/c where the power developed is suitably measured.

### Absorption dynamometer

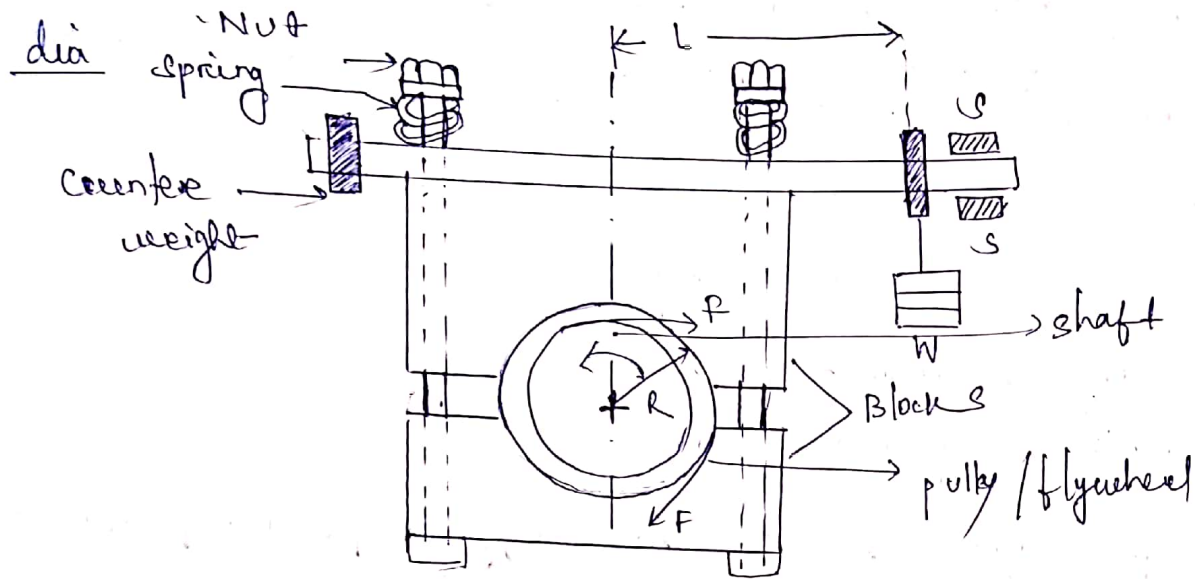
- Pony brake dynamometer.
- Repe brake dynamometer.

### Pony brake dynamometer

A simplest form of an absorption type dynamometer is a pony brake dynamometer. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two bolts & nuts, as shown in the fig. A helical spring is provided between the nut and upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it & carries a weight 'W' at its outer end. A counter weight is placed at the



Other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of the lever.



When the brake is to be put in operation, the long end of the lever is loaded with suitable weights  $W$  & the nuts are tightened until the engine shaft runs at a const. speed & the lever is in horizontal position. Under these conditions, the moment due to the weight  $W$  must balance the moment of the frictional resistance between the blocks & the pulley.

$W$  = weight of the outer end of the lever.

$L$  = horizontal distance of the weight  $W$ , from the centre of the pulley.

$F$  = frictional resistance between the blocks & the pulley in newton.

$R$  = radius of the pulley.

$N$  = speed of the shaft.

$$T = W \times L$$

Work done in one revolution = Torque  $\times$  Angle turned in radian

$$= \cancel{2\pi \times R} T \times 2\pi$$

$$\text{Work done/min} = T \times 2\pi \times N \quad \text{N-m} \quad \left[ \text{B.P} = \frac{T \times 2\pi N}{60} \right] \text{ Watts}$$

## Rope brake Dynamometer

It is another type of absorption type dynamometer, which is commonly used for measuring the brake power of the engine.

It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of the engine. The upper end of the ropes is attached to a spring balance while the lower end of the rope is kept in position by applying a dead weight as shown in the fig. To avoid slipping of rope over the flywheel, wooden blocks are placed at regular intervals around the whole circumference of flywheel.

In operational condition of the brake, the engine is made to run at a const. speed. The frictional torque, due to the rope must be equal to the torque being transmitted by the engine.

$W$  = Dead load

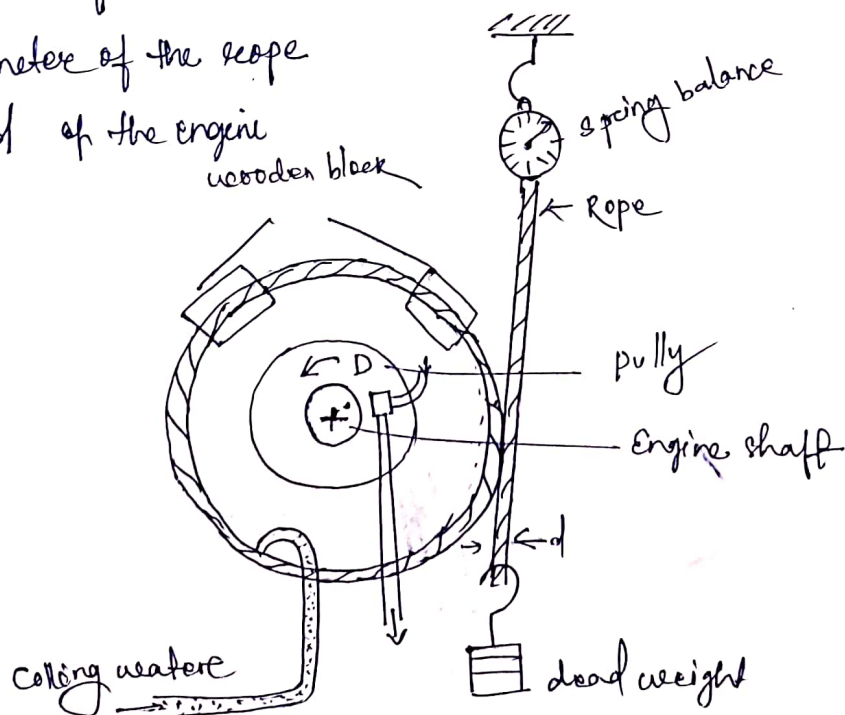
$S$  = spring balance

$D$  = Diameter of the wheel

$d$  = diameter of the rope

$N$  = Speed of the engine

Diagram



$$\text{Net load on the brake} = W - S \quad \text{N}$$

We know the distance moved in one revolution

$$= \pi (D + d) \cdot m$$

$$\text{Work done / revolution} = (W - S) \pi (D + d) \cdot m \quad \text{N-m}$$

$$\& \text{ Work done / min} = (W - S) \pi (D + d) \cdot m \cdot n \quad \text{N-m}$$

$$\text{Brake power} = \frac{\text{Work done / min}}{60}$$

$$\boxed{B.P = \frac{(W - S) \pi (D + d) \cdot m \cdot n}{60} \text{ Watts.}}$$

If dia of rope (d) is neglected. then,

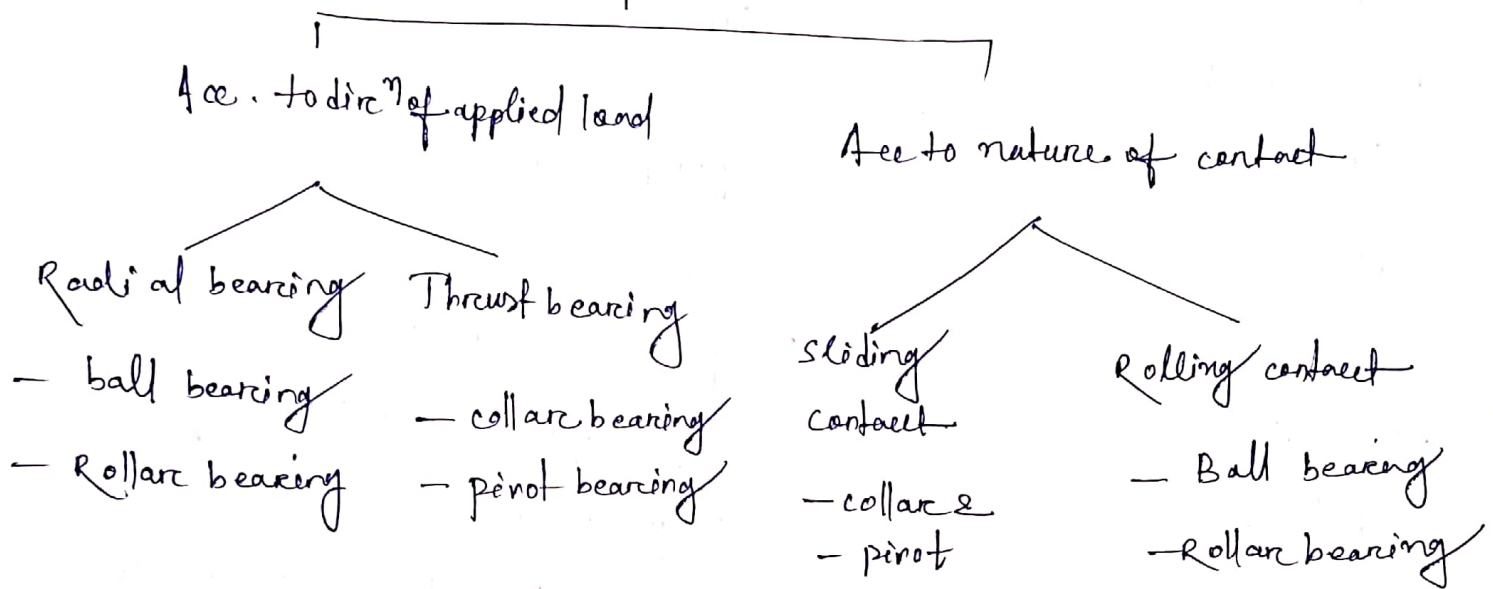
$$\boxed{B.P = \frac{(W - S) \pi (D) \cdot m \cdot n}{60} \text{ watts.}}$$



# Bearing

A bearing is a m/c element that constraints relative motion to only the desired motion, & reduces friction between moving part. It is a device that support load.

## Types of bearing



## Rolling contact bearing

When the action between journal and bearing is of rolling instead of sliding then it is called rolling/contact bearing. It is also called as antifriction bearing as the value of  $\mu$  is very low.

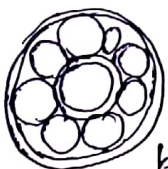
## Advantages

- less friction
- Low cost of maintenance
- easy operation
- comparatively clean.

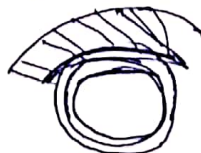
## Limitation

- more initial cost
- Design is complicated
- less resistant to shock

eg. ball bearing & Roller bearing



ball.





## Roller bearing

- very high load capacity
- good speed capability

## Application

- gear box
- Rolling mills
- Electric motors

## Types

- cylindrical
- spherical
- Taper

## Ball bearing

- high speed
- With stand high axial load
- longer life
- Maintenance is less

## Application

- Gearbox
- pump / compressor

## Types

- Deep groove ball bearing
- self aligning ball bearing
- Angular contact ball "

Material for both bearing is same, bronze, babbit, cast iron etc.

## Roller needle bearing

- It is a special type of bearing which is used in automobile components like rock arm, compressor, transmissions.
- compared to ball bearing it have greater surface area in contact & can support greater load.

## Mechanical advantage of screw jack.

Ratio bet<sup>n</sup> load required to raise or lower the load to the effort required.

$$M.A = \frac{W}{P}$$

It is always  $> 1$ .

## Self locking & overhauling screw

$$P \text{ to lower the load} = W \tan(\phi - \alpha)$$

if  $\phi < \alpha$

$P = -ve$

$T = -ve$

(overhauling)

if  $\phi > \alpha$

$P = +ve$

$T = +ve$

(self locking)

\* If  $\phi > \alpha$  then torque req<sup>d</sup> to lower the body will be +ve. This cond<sup>n</sup> is called self locking of screw.

\* If  $\phi < \alpha$ , the torque required to lower the load will be -ve i.e. load will move downwards without application of ~~load~~ torque, this cond<sup>n</sup> is overhauling.

$\eta_{\text{self locking}} \leq 50\%$        $\eta_{\text{overhauling}} > 50\%$

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

## Power Transmission CHAPTER=3

- It can be defined as the transmission of power from one point to another.

- There are various means of power transmission means,  
for eg. belt drive  
Chain drive  
Gear drive etc.

### Belt drive

The belt drive is used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or diff speed.

### Types of drives

Light drives — the belt speed is up to 10 m/s eg. agriculture m/c.

Medium drives — 10 - 22 m/s eg. m/c tools

Heavy drives — more than 22 m/s eg. compressors.

### Types of belts

Flat belts — used in factories & both the pulleys are 8 m apart.

V belts — used in workshops & " " very near to each other

Circular belts — more than 8 m.

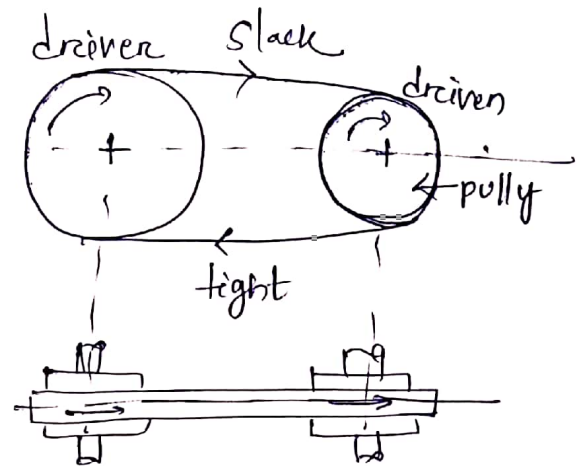
### Materials for belts

- leather
- cotton
- Rubber etc.

# Types Belt drive

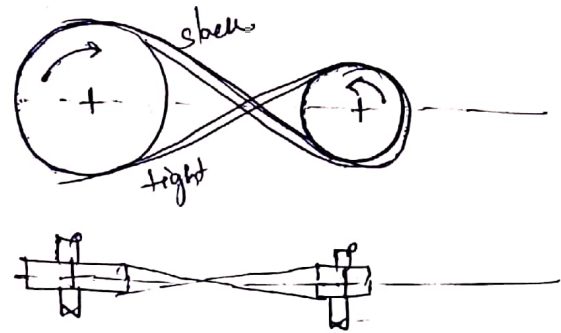
## 1) open belt drive

It is used when shafts are  $\parallel$  & rotating in same direction. Tension in lower side is more than tension in upper side as the driver pulls the belt from one side & deliver it to driven side.



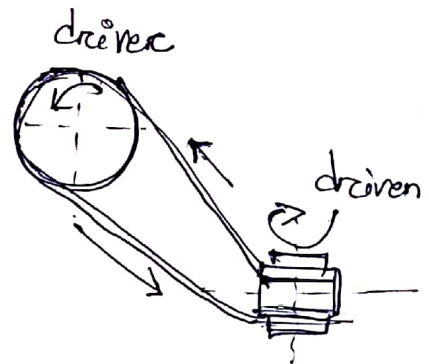
## 2) crossed belt drive

It is used when shafts are  $\parallel$  but moving in opposite direction. At the point where the belt crosses, it rubs against each other so excessive wear & tear occurs. To avoid this the shafts should be placed at a min<sup>m</sup> distance  $2.6 \cdot b$ , where  $b$  is width of belt.



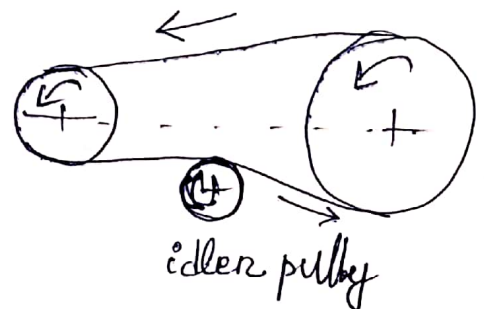
## 3) Quarter turn belt drive

It is also called as right angle belt drive. used when shafts are right angle & rotating in one direction.



## 4) Belt drive with idler pulleys

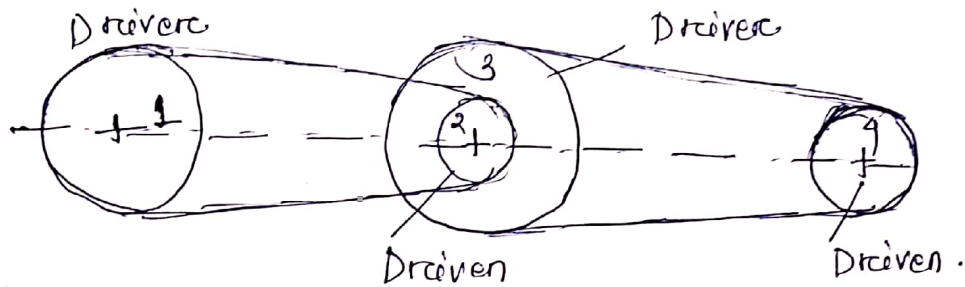
Used when shafts are  $\parallel$  & when open belt drive cannot be used due to small angle of contact on the smaller pulleys. Belt drive with many idler pulleys are used when motion has to be transformed from one shaft to several shafts.





## Compound belt drive

It is used when power is transmitted from one shaft to another through a no. of pulleys.



## Velocity ratio of belt drive

It is the ratio bet<sup>n</sup> velocities of the driver to driven.

Let  $d_1, d_2 \rightarrow$  dia of driver & driven respectively

$N_1, N_2 \rightarrow$  speed in rpm of " "

Length of belt passing over driver in 1 min =  $\pi d_1 N_1$

" " " driven " " =  $\pi d_2 N_2$

As the length is same so.  $\pi d_1 N_1 = \pi d_2 N_2$

$$\Rightarrow \left[ \frac{N_2}{N_1} = \frac{d_1}{d_2} \right] - \text{velocity ratio.}$$

If thickness of belt is considered  $\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$

## Velocity ratio of compound belt drive

Let pulley 1 driving pulley 2.

" 2 & 3 are mounted on same shaft so, 1 & 3 also drives

3 & 4 ultimately drives 4.

Let  $d_1, d_2, d_3, d_4 \rightarrow$  dia of pulley 1, 2, 3 & 4

$N_1, N_2, N_3, N_4 \rightarrow$  speed " " 1, 2, 3 & 4

$$\text{velocity ratio of pulley 1 \& 2} = \frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{--- ①}$$

$$3 \& 4 = \frac{N_4}{N_3} = \frac{d_3}{d_4} \quad \text{--- ②}$$

multiplying (i) & (ii)

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\Rightarrow \boxed{\frac{N_4}{N_1} = \frac{d_1 d_3}{d_2 d_4}} \quad (\because \text{as } N_2 = N_3)$$

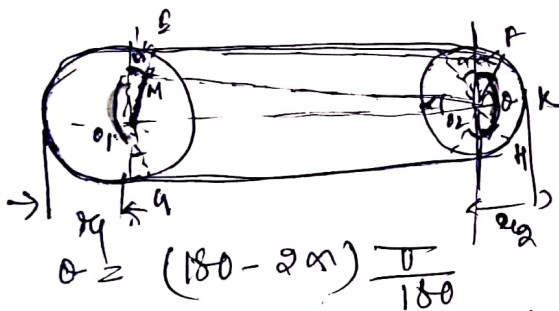
$$\frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{Product of dia of drivers}}{\text{" " " driven}}$$

Angle of contact

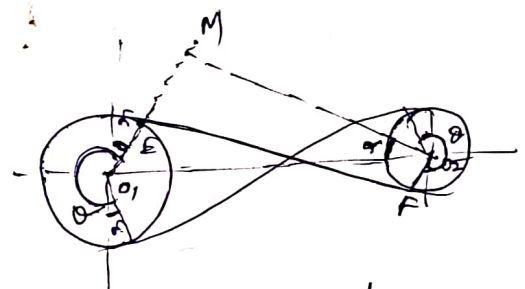
$$\sin \alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - M E}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

open belt drive

crossed belt drive



$\theta = (180 - 2\alpha) \frac{\pi}{180}$   
always taken in the small pulley.



$\theta = (180 + 2\alpha) \frac{\pi}{180}$   
Same on both the pulley.

power transmission by belt

$$\sin \alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + E M}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

As driving pulley pulls the belt from one side and delivers it to the other side, so tension on the tight side is more than tension in slack side.

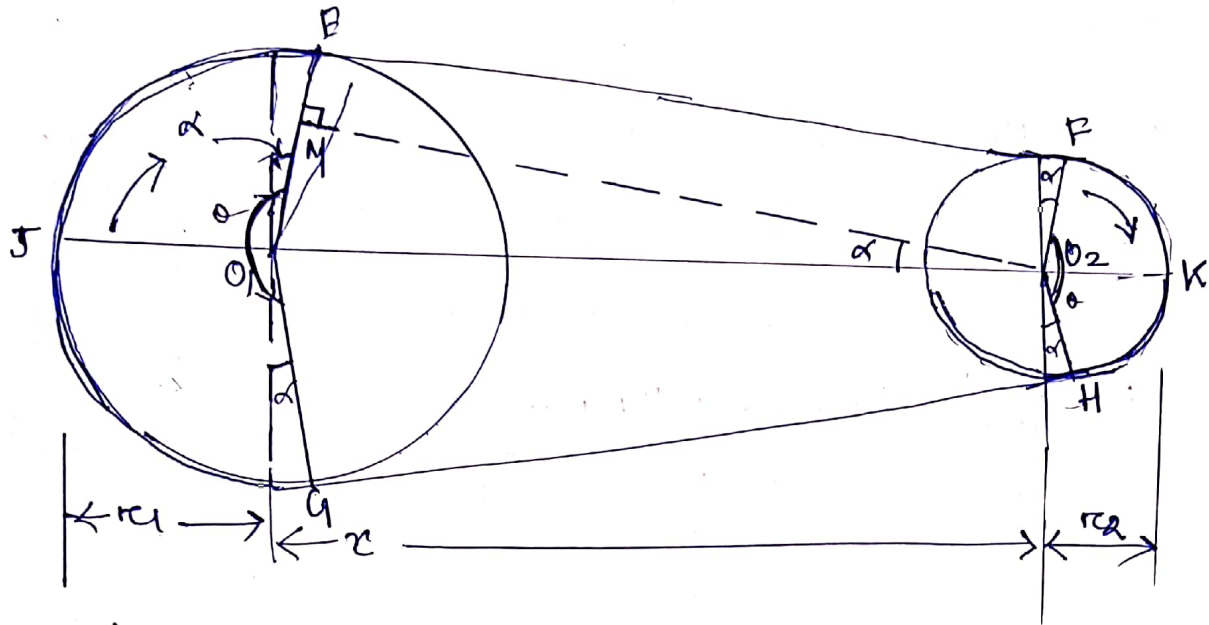
Let  $T_1, T_2 \rightarrow$  Tensions in tight & slack side

$r_1, r_2 \rightarrow$  radii of driver & follower pulley

$v \rightarrow$  velo. of belt in m/s

# Length of an open belt drive.

$$\text{Angle of contact} \\ \theta = (180 - 2\alpha) \frac{\pi}{180}$$



Let  $r_1$  &  $r_2 \rightarrow$  Radii of the larger & smaller pulleys

$x \rightarrow$  Distance between the centres of two pulleys  
(i.e.  $O_1$  &  $O_2$ )

$L \rightarrow$  Total length of the belt.

Let the belt leaves the larger pulley at E & G & the smaller pulley at F & H. Through  $O_2$  draw  $O_2M \parallel$  to  $EP$ . Now  $O_2M$  is  $\perp$  to  $O_1E$ .

Let  $\angle MO_2O_1 = \alpha$

We know that the length of the belt

$$L = \text{Arc GE} + EP + \text{Arc FKH} + HG \\ = 2 (\text{Arc JE} + EP + \text{Arc FK}) \quad \text{--- (1)}$$

From the geometry of the fig, we find that

$$\sin \alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}$$

$\therefore \sin \alpha \rightarrow$  very very small we can write  $\alpha$ .

$$\alpha = \frac{r_1 - r_2}{x} \quad \text{--- (2)}$$

$$\text{Arc JE} = r_1 \left( \frac{\pi}{2} + \alpha \right) \quad \text{--- (3)}$$

$$\text{Arc FK} = r_2 \left( \frac{\pi}{2} - \alpha \right) \quad \text{--- (4)}$$

$$\begin{aligned} EF = MO_2 &= \sqrt{(O_1O_2)^2 - (O_1M)^2} \\ &= \sqrt{x^2 - (r_1 - r_2)^2} = x \sqrt{1 - \left( \frac{r_1 - r_2}{x} \right)^2} \\ &= x \sqrt{1 - \frac{(r_1 - r_2)^2}{x^2}} \end{aligned}$$

By applying binomial theorem.

$$EF = x \left[ 1 - \frac{1}{2} \left( \frac{r_1 - r_2}{x} \right)^2 + \dots \right] = x - \frac{(r_1 - r_2)^2}{2x} \quad \text{--- (5)}$$

putting the values of eqn 3 & 4 & 5 in eqn ①.

$$\begin{aligned} L &= 2 \left[ r_1 \left( \frac{\pi}{2} + \alpha \right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \left( \frac{\pi}{2} - \alpha \right) \right] \\ &= 2 \left[ r_1 \times \frac{\pi}{2} + r_1 \alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2 \alpha \right] \\ &= 2 \left[ \frac{\pi}{2} (r_1 + r_2) + \alpha (r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x} \right] \\ &= \pi (r_1 + r_2) + 2\alpha (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x} \end{aligned}$$

Now putting the value of  $\alpha = \frac{r_1 - r_2}{x}$

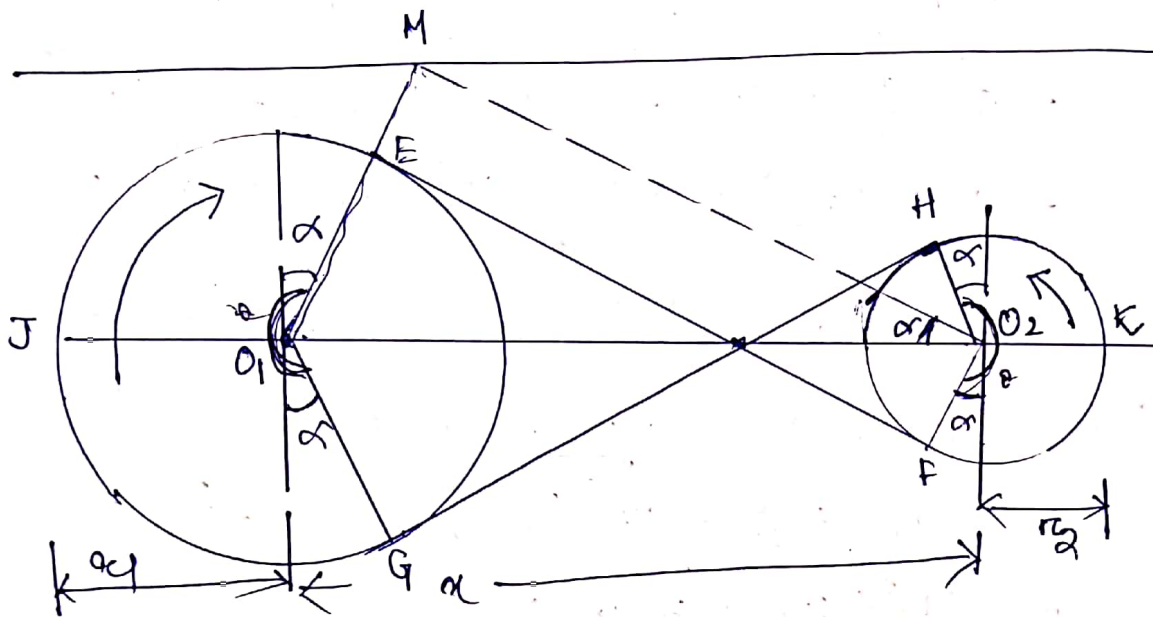
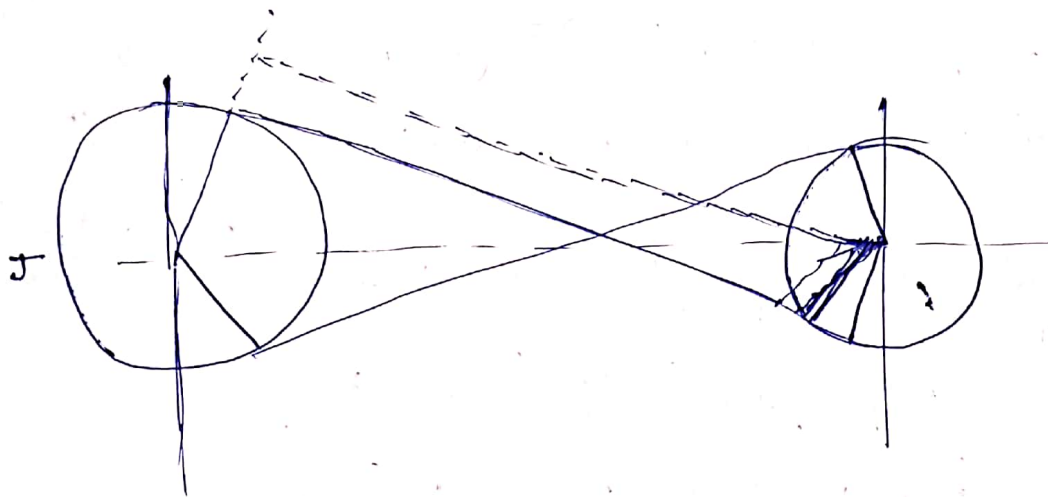
$$\begin{aligned} &= \pi (r_1 + r_2) + 2 \left( \frac{r_1 - r_2}{x} \right) (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x} \\ &= \pi (r_1 + r_2) + 2 \frac{(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x} \end{aligned}$$

$$\boxed{= \pi (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}}$$

In diameter. 
$$= \left[ \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \right]$$



# Length of a cross belt drive



$r_1$  &  $r_2 \rightarrow$  radii of larger & smaller pulley

$x \rightarrow$  distance bet<sup>n</sup> centers  $O_1$  &  $O_2$

$L \rightarrow$  total length of pulley

$$\alpha = (180 + 2\theta) \pi / 180$$

Let belt leaves larger pulley at E & G at smaller pulley at F & H

draw  $O_2M \parallel$  to  $EF$  Now  $O_2M$  is  $\perp$  to  $O_1E$

Let  $\angle O_2O_1 = \alpha$

Now  $L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$

$$= 2(\text{Arc } JE + EF + \text{Arc } FK) \quad \text{--- (1)}$$

$$\sin \alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

Since  $\sin \alpha \approx$  very small  $\approx \alpha. = \frac{r_1 + r_2}{x}$

$$\text{Arcc JE} = r_1 \left( \frac{\pi}{2} + \alpha \right), \text{Arcc DE} = r_2 \left( \frac{\pi}{2} + \alpha \right) \quad \text{--- (3)}$$

$$\text{EF} = r_2 = \sqrt{(OD_2)^2 - (OM)^2} = \sqrt{x^2 - \left( \frac{r_1 + r_2}{2} \right)^2} = x \sqrt{1 - \left( \frac{r_1 + r_2}{2x} \right)^2} \quad \text{--- (2)}$$

Expanding this eqn in binomial form  $\text{EF} = x \left[ 1 - \frac{1}{2} \left( \frac{r_1 + r_2}{2x} \right)^2 + \dots \right]$

$$= x - \frac{(r_1 + r_2)^2}{2x} \quad \text{--- (4)}$$

Now putting value of eqn 2, 3 & 4 in eqn (1)

$$\begin{aligned} L &= 2 \left[ r_1 \left( \frac{\pi}{2} + \alpha \right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \left( \frac{\pi}{2} + \alpha \right) \right] \\ &= 2 \left[ r_1 \frac{\pi}{2} + r_1 \alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \left( \frac{\pi}{2} + \alpha \right) \right] \\ &= 2 \left[ \frac{\pi}{2} (r_1 + r_2) + \alpha (r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x} \right] \\ &= \pi (r_1 + r_2) + 2\alpha (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x} \end{aligned}$$

Now putting  $\alpha = \frac{r_1 + r_2}{2x}$  the eqn will become.

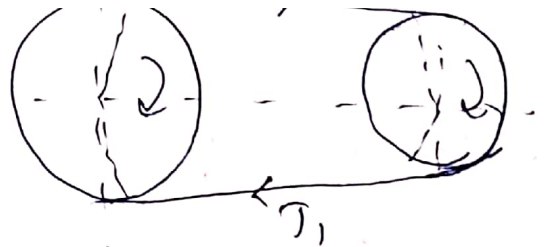
$$\begin{aligned} L &= \pi (r_1 + r_2) + \frac{r_1 + r_2}{x} \times 2 (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x} \\ &= \pi (r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x} \end{aligned}$$

$$\boxed{L = \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}}$$

In terms of pulley diameter  $\boxed{\frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}}$

work done / sec.  $= (T_1 - T_2) V$

power transmitted  $= (T_1 - T_2) V \text{ N/m/s}$



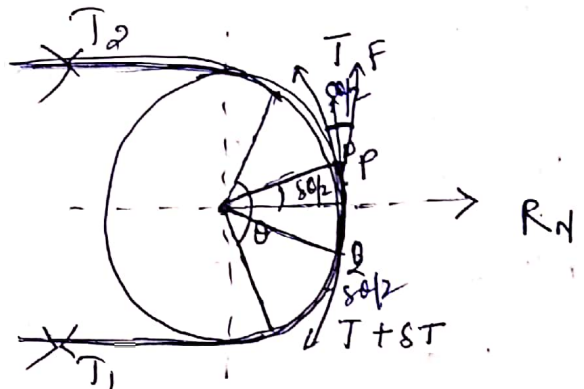
Torque produced on driver pulley  $= (T_1 - T_2) r_1 \checkmark$

" " driven "  $= (T_1 - T_2) r_2 \checkmark$

Ratio of belt tension for flat belt drive :-

Let  $T_1, T_2 \rightarrow$  Tension in the belt on tight side & slack side.

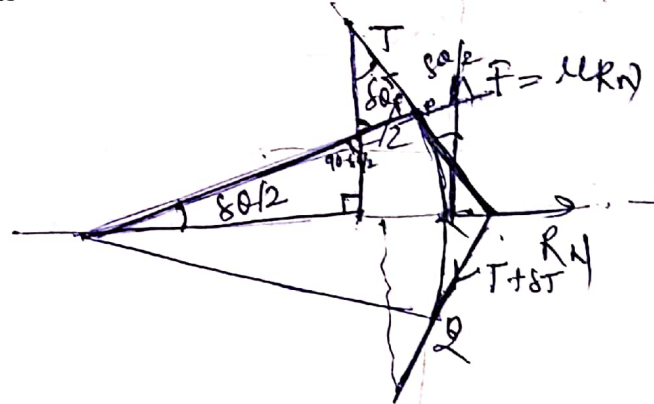
$\theta \rightarrow$  Angle of contact.



consider a small element PQ on the belt which makes an angle  $\delta\theta$  with the center of the pulley.

Forces acting on PQ.

- belt tension (T) at P
- " " (T +  $\delta T$ ) at Q
- Normal reaction  $R_N$
- Frictional force F.



Resolving the forces in horizontal direction.

$$R_N = T \sin\left(\frac{\delta\theta}{2}\right) + (T + \delta T) \sin\frac{\delta\theta}{2}$$

$$\approx T\left(\frac{\delta\theta}{2}\right) + (T + \delta T) \frac{\delta\theta}{2}$$

$\because \sin\frac{\delta\theta}{2} \approx \text{small} \approx \frac{\delta\theta}{2}$

$$= T \frac{\delta \alpha}{2} + T \frac{\delta \alpha}{2} + \frac{\delta T \delta \alpha}{2} \text{ (very small)}$$

$$\boxed{\mu R_N = T \delta \alpha} \quad \text{--- (1)}$$

Resolving vertically.

$$T + T \cos\left(\frac{\delta \alpha}{2}\right) - (T + \delta T) \cos \frac{\delta \alpha}{2} \geq 0 \quad \left\{ \begin{array}{l} \text{As } \delta \alpha \approx \text{small} \\ \cos \frac{\delta \alpha}{2} \approx 1 \end{array} \right.$$

$$\Rightarrow \mu R_N + T - (T + \delta T) \geq 0$$

$$\Rightarrow \boxed{\mu R_N = \delta T} \quad \text{--- (2)}$$

$$\Rightarrow \mu T \delta \alpha = \delta T$$

$$\Rightarrow \int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\alpha} \delta \alpha \quad (\text{integrating over total angle of contact})$$

$$\Rightarrow \ln T \Big|_{T_1}^{T_2} = \mu \alpha$$

$$\Rightarrow \ln(T/T_2) = \mu \alpha =$$

$$\Rightarrow \boxed{T/T_2 = e^{\mu \alpha}} \quad \text{--- (3)}$$

$$\text{or } \boxed{2.3 \log\left(\frac{T_1}{T_2}\right) = \mu \cdot \alpha}$$

### Slip of Belt :-

- The difference in motion between the belt and pulley in driving or driven side is called slip, which occurs due to insufficient frictional grip.

- It reduces the velocity ratios.

- Generally expressed as %.

velo. of driver pulley = velo. of pulley - slip

$$\text{sec.} = \frac{\pi d_1 N_1}{60} - \left[ \left( \frac{\pi d_1 N_1}{60} \right) \cdot \frac{s_1}{100} \right]$$

$$V = \frac{\pi d_1 N_1}{60} \left[ 1 - \frac{s_1}{100} \right]$$

$$\text{velo. of driven pulley/sec} = \frac{\pi d_2 N_2}{60} = \left[ V - V \left( \frac{s_2}{100} \right) \right]$$



$$\Rightarrow \frac{\pi d_2 N_2}{60} = v \left( 1 - \frac{s_2}{100} \right)$$

$$\Rightarrow \frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60} \left( 1 - \frac{s_1}{100} \right) \left( 1 - \frac{s_2}{100} \right)$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2} \left[ 1 - \left( \frac{s_1 + s_2}{100} \right) \right]$$

$$\Rightarrow \boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2} \left[ 1 - \frac{s}{100} \right]} \quad s = \text{total slip.}$$

### Creep in belt drive :-

During working one side of the belt act as tight side & the other side act as slack side. So the elongation or stress produced at different part of belt material are different. This different elongation effects the velocity ratio of belt drive. This phenomenon is called creep.

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \left( \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}} \right)$$

$E \rightarrow$  young's modulus of belt materials

$\sigma_1 \rightarrow$  stress in tight side

$\sigma_2 \rightarrow$  stress in slack side

### creauening of pully

$\rightarrow$  slight convexity of pully rim is called creauening.

$\rightarrow$  it is made to keep the belt in centre of pulley while in motion.

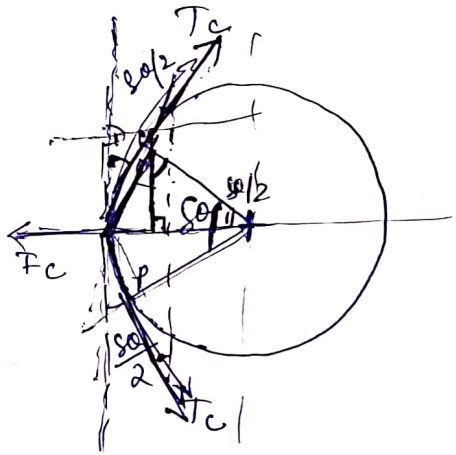
$\rightarrow$  it prevents axial slipping of belt during operation.

$\rightarrow$  The amount of creauening is usually  $\frac{1}{16}$  of the pulley face width.

## Centrifugal tensions ( $T_c$ )

Tension caused by centrifugal force is called centrifugal tension. Its effect is to increase tension on both sides. When the belt is rotating on the pulley it possesses centrifugal force. Due to this force, the belt tends to move away from the pulley.

Consider a small element of the belt making an angle  $\delta\theta$  at the centre of the pulley.



Let  $m \rightarrow$  mass of belt/unit length

$v \rightarrow$  velo. of belt

$r \rightarrow$  radius of pulley.

Centrifugal force produced in belt in small element.

Length of  $PQ = r\delta\theta$

mass " "  $PQ = m r \delta\theta$

$$F_c = m r \delta\theta \frac{v^2}{r} = m \delta\theta v^2$$

For equilibrium, summing the forces.

$$T_c \sin\left(\frac{\delta\theta}{2}\right) + T_c \sin\left(\frac{\delta\theta}{2}\right) = F_c$$

$$\Rightarrow T_c \left(\frac{\delta\theta}{2}\right) + T_c \frac{\delta\theta}{2} = F_c$$

( $\sin \frac{\delta\theta}{2}$  is very small  
 $\approx \frac{\delta\theta}{2}$ )

$$\Rightarrow F_c = T_c \delta\theta$$

$$\Rightarrow m \delta\theta v^2 = T_c \delta\theta$$

$$\Rightarrow \boxed{T_c = mv^2}$$

Note:-

considering centrifugal tension,

$$\text{tension in tight side} = T_H = T_1 + T_c$$

$$\text{" " " slack " } = T_L = T_2 + T_c$$

\* Centrifugal tension has no effect on power transmission.

Max<sup>m</sup> tension in the belt

$\sigma$  = max<sup>m</sup> stress in belt material in N/mm<sup>2</sup>

$b$  = width of belt in mm

$t$  = thickness.

$$\text{Max<sup>m</sup> tension in belt} = \text{max<sup>m}</sup>  $\sigma \times A$$$

$$= \sigma b t$$

$$\text{If } T_c \text{ is considered } T = T_1 + T_c \checkmark$$

$$\text{If } T_c \text{ is not " } T = T_1 \checkmark$$

Condition for maximum power transmission

$$P = (T_1 - T_2) v$$

$$= T_1 \left( 1 - \frac{T_2}{T_1} \right) v$$

$$= T_1 \left( 1 - \frac{1}{e^{\mu \theta}} \right) v$$

$$\Rightarrow P = T_1 k v$$

$$\left( \because k = 1 - \frac{1}{e^{\mu \theta}} \right)$$

$$\text{Max<sup>m</sup> tension} = T = T_1 + T_c$$

$$\Rightarrow T_1 = T - T_c$$

$$\text{or } P = (T - T_c) k v$$

$$\Rightarrow P = (T - mv^2) k v$$

power becomes max<sup>m</sup> when

$$\frac{dP}{dv} = 0$$

$$\Rightarrow \frac{d}{dv} [(T - mv^2) k v] = 0$$

$$\Rightarrow T k - m k 3v^2 = 0$$

$$\Rightarrow \cancel{T k} = \cancel{m k} 3v^2$$

$$\Rightarrow T k = m k 3v^2$$

$$\Rightarrow T = 3mv^2$$

$$\Rightarrow 3T_c = T$$

$$\Rightarrow \boxed{T_c = T/3}$$



## initial tension in belt drive ( $T_0$ )

When belt is wound round the pulleys, its ends are joined & it continuously move over the pulley by frictional grip. So even when the pulley is stationary, the belt is subjected to some tension called initial tension.

$$T_0 = \frac{T_1 + T_2}{2} \quad (T_c \rightarrow \text{neglect}), \quad T_0 = \frac{T_1 + T_2 + 2T_c}{2}$$

$T_c \rightarrow$  considering.

## Length of a belt

$$L = \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{x} + 2x \quad (\text{for open belt drive})$$

$$L = \pi(r_1 + r_2) + \frac{(r_1 + r_2)^2}{x} + 2x \quad (\text{for crossed belt drive})$$

Q

A shaft rotating at 200 rpm drives another shaft at 300 rpm and transmits 6 kW, through a belt. The belt is 100 mm wide and 6 mm thick. The distance between the two shafts is 4 m. The smaller pulley is 0.5 m dia. calculate the stress in the belt if it is.



i) an open belt drive.

ii) cross belt drive.

Take  $\mu = 0.3$

Data

Soln

$$N_1 = 200 \text{ rpm}$$

$$N_2 = 300 \text{ rpm}$$

$$P = 6 \text{ kW}$$

$$b = 100 \text{ mm} \quad d_2 = 0.5$$

$$x = 4 \text{ m}$$

$$\mu = 0.3$$

$$x = 4 \text{ m}$$



$$q \left( \frac{T_1}{T_2} \right) = R$$

$$\Rightarrow \log \frac{T_1}{T_2} = \frac{.3 \times 3.04}{2.3} = .3965$$

$$\Rightarrow \frac{T_1}{T_2} = 2.5 \text{---} \textcircled{2} \text{ (antilog of .3965)}$$

$$T_1 = 2.5 T_2$$

$$T_1 + T_2 = 4677$$

$$2.5 T_2 + T_2 = 4677$$

$$\Rightarrow 3.5 T_2 = 4677$$

$$\Rightarrow T_2 = \frac{4677}{3.5} = 1336 \text{ N}$$

$$T_1 = 2.5 T_2$$

$$= 3341 \text{ N.}$$

$$P = (T_1 - T_2) V$$

$$= (3341 - 1336) \times 21$$

$$P = 42100 \text{ watt}$$

$$P = 42.1 \text{ kW } \underline{\text{Ans.}}$$

Q) Two parallel shaft whose center lines are 4.8 m apart, are connected by open belt drive. The diameter of the larger pulley is 1.5 m & smaller is 1 m. The initial tension in the belt is 3 kN. The mass of the belt is 1.5 kg/m length. The co-efficient of friction between belt & pulley is 0.3. Taking centrifugal tension into account. i) cal. power transmitted when smaller pulley rotates at 400 rpm.

Soln

data given

$$x = 4.8 \text{ m}$$

$$d_1 = 1.5 \text{ m}$$

$$d_2 = 1 \text{ m}$$

$$N_1 = ?$$

$$N_2 = 400 \text{ rpm}$$

$$T_0 = 3 \text{ kN} = 3000 \text{ N}$$

$$\mu = 0.3$$

$$m = 1.5 \text{ kg/m}$$

$$P = (T_1 - T_2) v$$

$$\text{vel. of belt } v = \frac{\pi d_2 N_2}{60} = \frac{\pi \times 1 \times 400}{60} = 21 \text{ m/s}$$

$$T_c = mv^2 = 1.5 \times (21)^2 = 661.5 \text{ N}$$

$$T_0 = \frac{T_1 + T_2 + 2T_c}{2}$$

$$\Rightarrow 3000 = \frac{T_1 + T_2 + 2 \times 661.5}{2}$$

$$\Rightarrow T_1 + T_2 = 3000 \times 2 - 2 \times 661.5$$

$$\Rightarrow T_1 + T_2 = 4677 \text{ N} \quad \text{--- (1)}$$

For open belt drive

$$\alpha = \sin^{-1} \frac{r_1 - r_2}{x}$$

$$= \sin^{-1} 0.0521$$

$$\alpha = 3$$

$$\theta = (180 - 2\alpha) \times \frac{\pi}{180} = (180 - 6) \pi / 180 = 3.04 \text{ rad}$$

$$\frac{N_2}{N_1} = \frac{d_2}{d_1}$$

$$\Rightarrow \frac{200}{300} = \frac{15}{d_1}$$

$$\Rightarrow d_1 = 2.25 \text{ m} \checkmark$$

$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60} = 7.854 \text{ m/s}$$

$$\alpha = 180 - 2\gamma$$

$$\sin \gamma = \frac{r_1 - r_2}{x} \quad (\text{for open belt drive})$$

$$= 1.79$$

$$\alpha = 180 - (2 \times 1.79) = 176.42^\circ$$

$$= 176.42 \times \frac{\pi}{180}$$

$$P = (T_1 - T_2) v$$

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$\Rightarrow T_1 = 2.52 T_2$$

$$P = (T_1 - T_2) v = (2.52 T_2 - T_2) v$$

$$T_2 = 502.6 \text{ N}$$

$$\Rightarrow T_1 = 2.52 \times 502.6 \text{ N}$$

$$= 1266.55 \text{ N}$$

$$T_1 = \sigma b t$$

$$\Rightarrow 1266.55 = \sigma \times 0.01 \times 0.01$$

$$\Rightarrow \sigma = \frac{1266.55}{0.001} = 1266550 \text{ N/m}^2$$

Q Find the power transmitted by a belt running over a pulley of 600 mm diameter at 200 rpm. The co-efficient of belt and the pulley is 0.25. angle of lap  $160^\circ$  and max<sup>m</sup> tension in the belt is 2500 N.

$d = 600 \text{ mm} = 0.6 \text{ m}$      $\mu = 0.25$   
 $N = 200 \text{ rpm}$      $\theta = 160^\circ = 160 \times \pi / 180$   
 $T_1 = 2500 \text{ N}$

we know the velo. of the belt  $v = \frac{\pi d N}{60} = \frac{\pi \times 0.6 \times 200}{60} = 6.284 \text{ m/s}$

$T_2$  = tension in the slack side ~~side~~ of the belt.

$$2.3 \log \left( \frac{T_1}{T_2} \right) = \mu \cdot \theta = 0.25 \times 2.793 = 0.6982$$

$$\log \left( \frac{T_1}{T_2} \right) = \frac{0.6982}{2.3} = 0.3036$$

$$\frac{T_1}{T_2} = 2.01$$

$$\Rightarrow T_2 = \frac{T_1}{2.01} = \frac{2500}{2.01} = 1244 \text{ N}$$

we know that power transmitted by the belt

$$P = (T_1 - T_2) v = (2500 - 1244) 6.284 = 7890 \text{ W}$$

$$\text{or } 7.89 \text{ kW} \quad \underline{\text{Ans}}$$

Q) An engine running at 150 rpm, drives a line shaft by means of a belt. The engine pulley is 750 mm dia & pulley on line shaft is 450 mm. A 900 mm dia pulley on the line shaft drives a 150 mm dia pulley keyed to a dynamo shaft. find the speed of the dynamo shaft, when there is i) no slip ii) with slip is 2% at each drive.

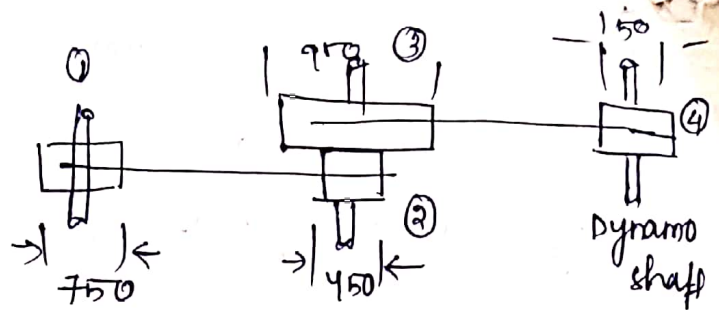


Soln

i) with no slip  $\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$

$\Rightarrow \frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}$

$\Rightarrow N_4 = 1500 \text{ rpm} \checkmark$



ii) with slip

$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \left(1 - \frac{s_1}{100}\right) \left(1 - \frac{s_2}{100}\right)$

$\Rightarrow N_4 = 150 \times \frac{750 \times 900}{450 \times 150} \left(1 - \frac{2}{100}\right) \left(1 - \frac{2}{100}\right)$

$\Rightarrow N_4 = 1440 \text{ rpm}$

Q. The power is transmitted from a pulley 1m dia running at 200 rpm to a pulley 2.25m by belt. Find the speed lost by the driven pulley as a result of creep. If stress of tight side 1.4 MPa & slack side is 0.5 MPa.  $E = 100 \text{ MPa}$

Soln

$d_1 = 1 \text{ m}$

$\sigma_1 = 1.4 \text{ MPa} = 1.4 \times 10^6 \text{ N/m}^2$

$d_2 = 2.25 \text{ m}$

$\sigma_2 = 0.5 \text{ MPa} = 0.5 \times 10^6 \text{ N/m}^2$

$N_1 = 200$

$E = 100 \text{ MPa} = 100 \times 10^6 \text{ N/m}^2$

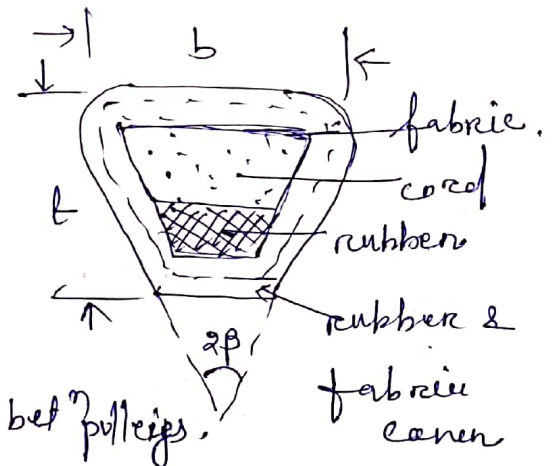
$\frac{N_2}{N_1} = \frac{d_1}{d_2} \Rightarrow N_2 = 200 \times \frac{1}{2.25} = 88.9 \text{ rpm}$  with out

$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E \times \sqrt{\sigma_2}}{E \times \sqrt{\sigma_1}} = 88.7 \text{ rpm}$

Speed lost.  $= 88.9 - 88.7 = 0.2 \text{ rpm} \checkmark$

## V belt drive

- It is used in factories and workshops where large amount of power has to be transmitted and the two pulleys are very near to each other.
- V belts are made of fabric and cords embedded in rubber and covered with fabric & rubber.
- They are trapezoidal in shape, & the angle included is  $30^{\circ}$ - $40^{\circ}$ .
- Hence the rim of the pulley is grooved where it runs.
- To increase the power of several V belts should be operated side by side.



### Advantages

- More compact struct. due to small distance bet<sup>n</sup> pulleys.
- slip is negligible.
- provide longer life i.e. 3 to 5 yrs.
- can be easily installed & removed.
- operation is quiet.
- power transmission is more
- It can be operated horizontal, vertical or inclined.

### Dis advantage

- It can be used, where the center distance is large.
- construction of pulleys for V belt is very complicated.

## Ratio of belt tension in v-belts

$$\boxed{\frac{T_1}{T_2} = e^{\mu \cos \alpha \times \theta}}$$

## Rope Drive

- It is used where large power has to be transmitted between two pulleys.
- Frictional grip of rope drive is more than v-belt.
- A no. of separate drives can be taken from one driving pulley.
- Generally 2 types of ropes are used.
  - i) fibre rope (when pulleys are 60m apart)
  - ii) wire rope ( " " 150m " )

Materials : — hemp, manila, cotton — dia ranges bet 38 to 50mm

- These are strong & more durable.
- These are used in elevators, mines hoists, cranes, suspension bridges etc.

- Here  $\left[ \frac{T_1}{T_2} = e^{\mu \cos \theta} \right]$  Advantages - give smooth, steady & ser
- D's
- high mechanical efficiency
  - less affected by outdoor conditions.

### Chain drive

- made up of rigid links hinged together to provide necessary flexibility for wrapping around the wheels.
- wheels have protruding teeth & fit into the corresponding recesses in the links of the chain.
- So wheel & chain are constrained together to move without slipping & ensures perfect velocity ratio.
- It is used for motion transmission when two shafts are apart by short distance.

### Advantages

- No slip.
- occupy less space
- give high transmission
- Can transmit motion to several shafts by one chain only.

### Disadvantage

- production cost is high
- need accurate mounting



# Gears

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A gear is a rotating element having <sup>cut</sup> teeth & mesh with another toothed part to transmit torque. When 2 or more gear work in sequence they are called gear train or transmission.

When 2 gear meshed, one is bigger than another then a mechanical advantage is produced.

$$\text{mechanical advantage} = \frac{\text{load}}{\text{effort}}$$

smaller one is pinion & larger one is driver.

Material for gear.

Numerous nonferrous alloys, cast iron, powder metallurgy & plastic are used. However steel are most commonly used bcoz. of their high strength & low cost.

Classification of gear

- 1) spur
- 2) helical
- 3) double helical / herringbone
- 4) internal
- 5) rack & pinion
- 6) straight bevel gear.
- 7) spiral bevel gear.
- 8) hypoid bevel gear
- 9) worm gear
- 10) spiral gear.

<u>Type</u>	<u>Features</u>	<u>Application</u>
spur	11 shafting high speed & load highest effi	to all type of gear trains & a wide range of velocity ratio.
Helical	11 shafting very high speed & load effi. less than spur.	Applicable to high speed load.
crossed helical.	Skewed shafting, pt. contact. low speed, low load high sliding	for low velo. ratio light loads. any single skew shaft
internal spur.	11 shafts high speed high load	Internal drives requiring high speeds & high load. offer low sliding, long life good for high capacity
Bevel.	intersecting shafts high speed high load	high velo. ratio & right angle meshes.
worm	Right angle skew shaft high velo. ratio high speed & loads. low efficiency	high velo. ratio. Angular meshes high loads.

## Advantages of gear drive.

- It transmits exact velo. ratio.
- It may be used to transmit large power.
- It has high efficiency.
- It has reliable service.
- It has compact layout.

## Disadvantage

- The manufacture of gear requires special tool & equipments.
- Error in cutting teeth may cause vibration.

## Classification.

### 1) Acc. to position of axis of shaft-

- parallel — spur
- intersecting — bevel
- non intersecting — screw bevel.

### 2) Acc. to peripheral velo.

- low — velo.  $< 3 \text{ m/s}$
- Medium —  $3 \text{ to } 15 \text{ m/s}$
- high —  $> 15 \text{ m/s}$

### 3) Acc. to type of gearing.

- External — Rack & pinion
- internal

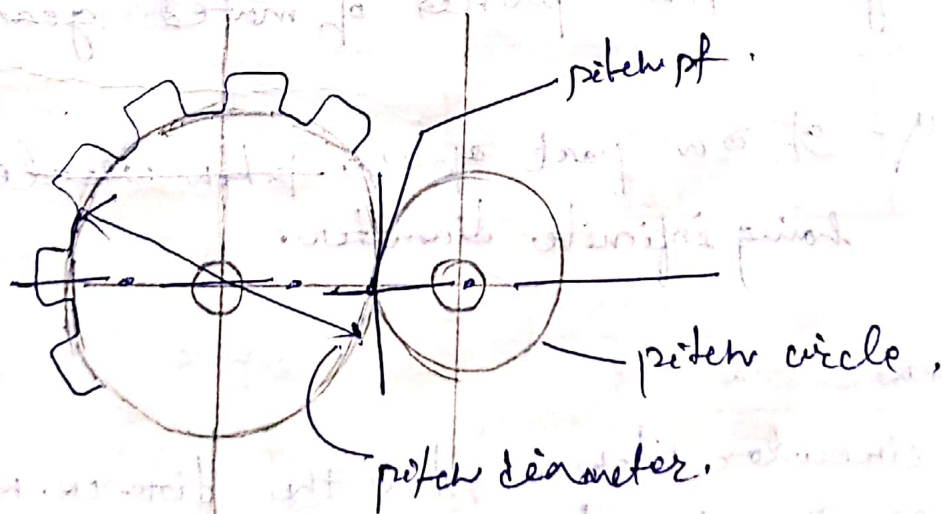
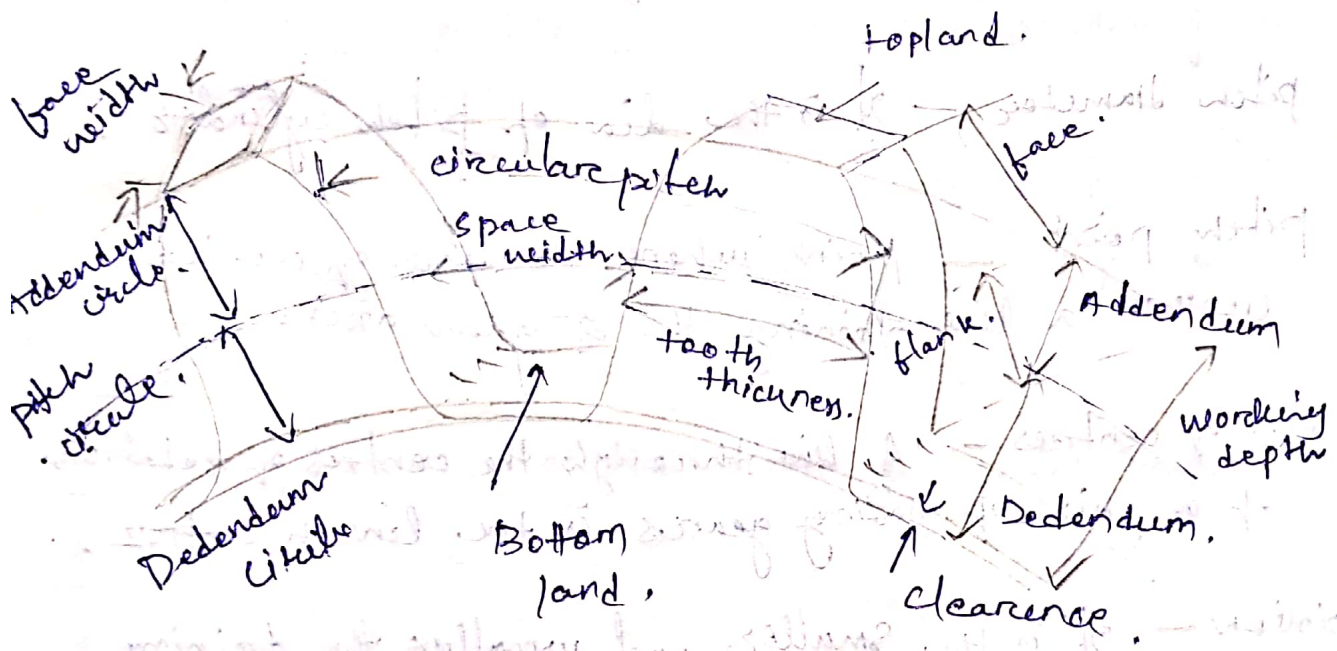


Acc to. position of teeth

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- Straight — Spur.
- inclined — helical
- curved — spiral,

## Gear terminology





pitch cylinder -

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It is a pair of gears in mesh are the imaginary friction cylinder which by pure rolling together, transmit the same motion as pair of gear.

pitch circle - A predetermined diametrical position on the gear where the circular tooth thickness pressure angle - helix angle are defined.

pitch diameter - It is the dia of pitch cylinder.

pitch point - point where the line of action crosses a line joining the 2 gear axes.

line of centres - A line through the centres of rotation of a pair of mating gears is the line of centres.

pinion - It is the smaller and usually the driving gear of a pair of mated gears.

Rack - It is a part of the pitch circle gear wheel having infinite diameter.

pitch -

circular pitch - It is the distance measured along the circumference of the pitch circle from a point on one tooth to the corresponding point on the adjacent tooth.

$$P = \frac{\pi d}{T}$$

$P \rightarrow$  circular pitch  
 $d \rightarrow$  pitch diameter.  
 $T \rightarrow$  no. of teeth.

Diametrical pitch - It is the no. of teeth per unit length of the pitch circle dia in inches.

$$P = \frac{\pi}{d}$$

Gear ratio (G)

It is the ratio of no. of teeth on the gear to that on the pinion.

$$G = \frac{T}{t}$$

$T \rightarrow$  no. of teeth on gear.  
 $t \rightarrow$  no. of " " pinion.

Velocity ratio (VR)

The VR is defined as the ratio of angular velo. of follower to the angular velo. of driving gear.

Subscript 1 - driver  
 2 - follower.

$$VR = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

$$\begin{aligned}
 V &= \frac{2\pi r \times \pi}{60} \\
 &= \frac{2\pi N \times \frac{d}{2}}{60}
 \end{aligned}$$

$$\omega = 2\pi N$$

$$\frac{d_1}{d_2} = \frac{N_2}{N_1}$$

$$P = \frac{\pi d_1}{T_1} = \frac{\pi d_2}{T_2}$$

$$\frac{d_1}{d_2} = \frac{T_1}{T_2}$$

module

$$m = \frac{\text{Pitch dia} \times \frac{2\pi}{60}}{\text{no. of teeth}}$$



Addendum circle - It is the circle passing through the tips of teeth.

Addendum - It is the radial height of a tooth above the pitch circle. Its standard value is 1 module.

Dedendum circle - passing through roots of the teeth.

Dedendum - It is the radial depth of a root below the pitch circle.

Its standard value is  $1.157m$ .

Clearance - Radial difference bet<sup>n</sup> addendum & dedendum of a tooth.

$$C = 0.157m$$

addendum circle dia  $= d + 2m$

dedendum " "  $= d - 2 \times 1.157m$

module -  $m = \frac{D}{T}$

$D \rightarrow$  pitch circle dia  $1.157m - m$

$T \rightarrow$  no. of teeth.  $= 1.157m$

Full depth of teeth.

It is the total radial depth of the tooth space.

$$\text{full depth} = \text{Addendum} + \text{Dedendum}$$

Working depth of teeth

It is max<sup>m</sup> depth to which a tooth penetrates into the tooth space of mating gear.

$$WD = \text{Sum of Addendum of 2 gears.}$$

# Gear Drive

A gear is a rotating m/c part having teeth in its periphery. When gears are used for power transmission it is called as gear drive. In case of belt drive and rope drive sometimes slipping occurs which reduces the velo. ratio. But in case of gear drive no. such slipping occurs, so exact velocity ratio can be obtained. It is also called as positive drive.

## Advantages

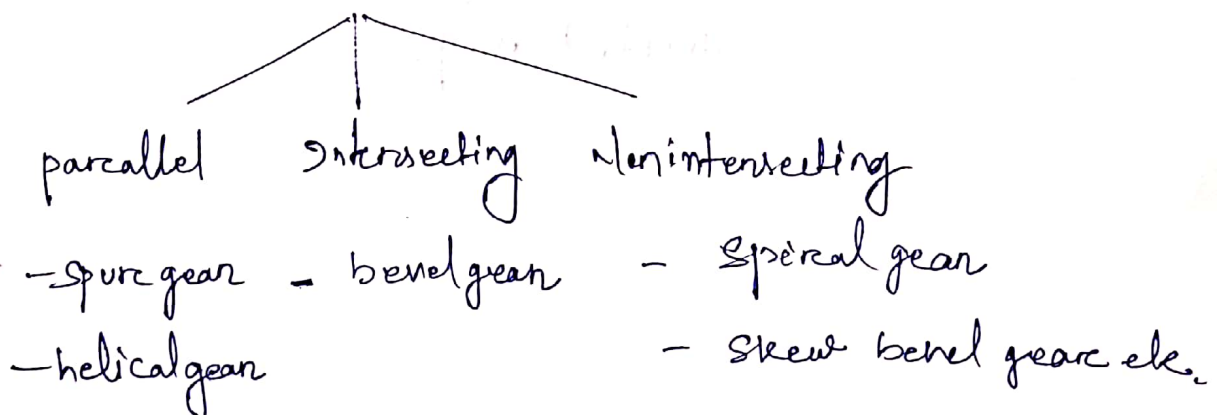
- It transmits exact velo. ratio
- It has high efficiency
- It has reliable service
- It has compact layout.

## Dis

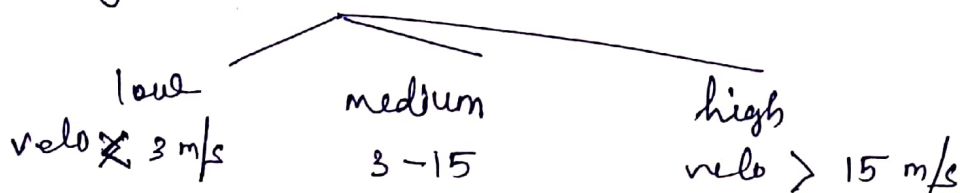
- The manufacturing of gears require special tools.

## Classification

### According to position of axis



### According to velocity of gears





## Gear Train

When two or more gears are mesh with each other to transmit power from one shaft to another, such combination is called as gear train.

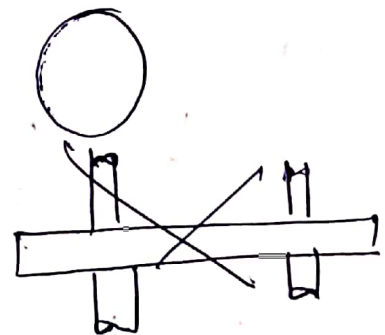
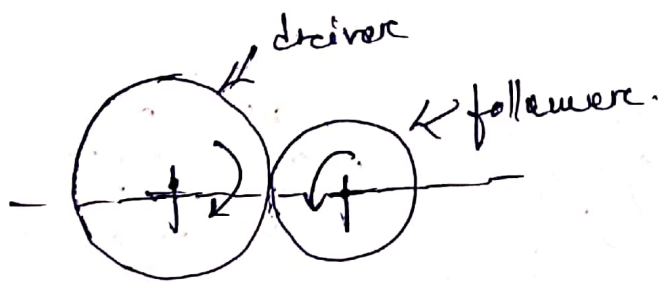
### Types of gear train

Depending upon the arrangement.

- Simple gear train
- Compound "
- Reverted "
- Epicyclic "

#### 1) Simple gear train.

When there is only one gear on each shaft, is known as simple gear train. The gear which drives is called as driver & the gear driven is called as follower. The motion of both the gears are opposite to each other.



$N_1 \rightarrow$  speed of driver

$N_2 \rightarrow$  " " follower

$T_1 \rightarrow$  no. of teeth in driver

$T_2 \rightarrow$  " " in follower.

$$V.R = \frac{\omega_1}{\omega_2}$$

$$= \frac{2\pi N_1}{2\pi N_2} = \frac{N_1}{N_2}$$

$$V = \omega \times r$$

$$= \frac{2\pi N_1}{60} \times r$$

$$= \frac{2\pi N_1 \times \frac{d}{2}}{60} = \frac{\pi d N_1}{60}$$

$$\frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60} \Rightarrow \frac{d_1}{d_2} = \frac{N_2}{N_1}$$

- The speed ratio / velocity ratio is given by

$$\boxed{\frac{N_1}{N_2} = \frac{T_2}{T_1}}$$

- The train value is the reciprocal of speed ratio,

$$\frac{1}{\text{Speed ratio}}$$

$$\text{or } \boxed{\frac{N_2}{N_1} = \frac{T_1}{T_2}}$$

$$P = \frac{2\pi d}{T} \quad \frac{\pi d_1}{T_1} = \frac{\pi d_2}{T_2}$$

$$\Rightarrow \frac{d_1}{d_2} = \frac{T_2}{T_1} \Rightarrow \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

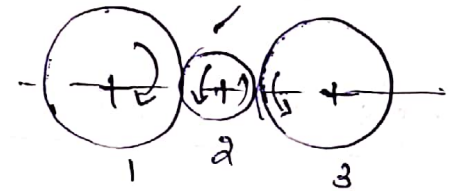
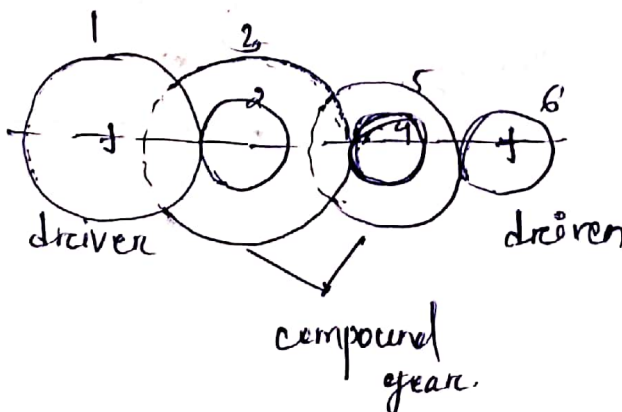
When there are 3 gears.

$$\text{Speed ratio. } \boxed{\frac{N_1}{N_3} = \frac{T_3}{T_1}}$$

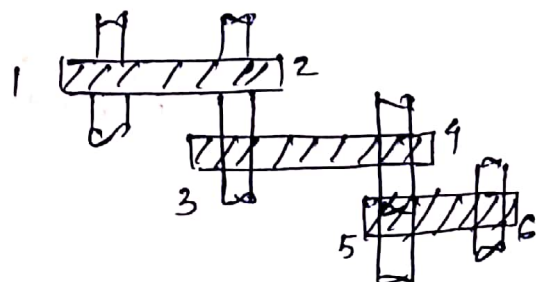
$$V.R = \frac{\omega_2}{\omega_1} = \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1}$$

2) compound gear train

When there are more than one gear on a shaft, it is called compound gear train.



provided where centre distance is larger in the meshing gear. It is also called as intermediate gears



$N_1, N_2, N_3, N_4, N_5, N_6 \rightarrow$  rpm of gear 1, 2, 3, 4, 5 & 6

$T_1, T_2, T_3, T_4, T_5, T_6 \rightarrow$  Teeth " " " " "

Since gear 1 is meshed with gear 2.

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Similarly 3 & 4  $\frac{N_3}{N_4} = \frac{T_4}{T_3}$

" 5 & 6  $\frac{N_5}{N_6} = \frac{T_6}{T_5}$

Combining the 3 equations, the speed ratio of compound gear train will be.

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

Since  $N_2 = N_3$  &  $N_4 = N_5$

$$\boxed{\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}}$$

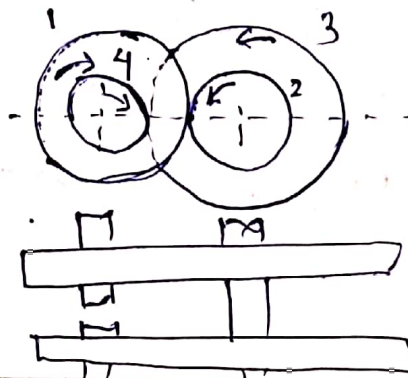
### 3) Reverted gear train

When the axis of 1st gear is co-axial with the last gear, it is known as reverted gear train.

$T_1 \rightarrow$  no. of teeth on gear 1

$\phi_2 \rightarrow$  pitch circle of gear 1

$N_1 \rightarrow$  speed of "



$T_2, T_3, T_4 \rightarrow$  Teeth on 1, 2, 3

$r_1, r_2, r_3 \rightarrow$  pitch circle radius of 1, 2, & 3

$N_1, N_2, N_3 \rightarrow$  rpm of 1 & 2 & 3

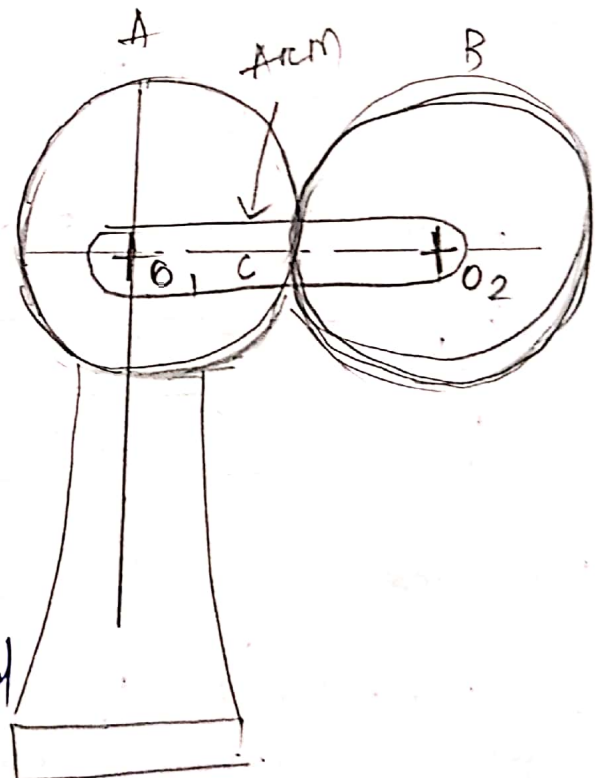
$$\text{Speed ratio} = \left[ \frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3} \right]$$

where  $r_1 + r_2 = r_3 + r_4$

&  $T_1 + T_2 = T_3 + T_4$

#### 4) Epicyclic gear train

A simple epicyclic gear train is shown in the fig. where gear A and the arm C has a common axis at  $O_1$  about which they can rotate. The B gear is meshed with gear A and rotate about  $O_2$ . when the gear is fixed and the arm C is rotating, then it will carry the gear B also. and both the arm & gear B rotate ~~about~~ around the gear A. Such motion is called as epicyclic gear trains.



The speed ratio will be.

$$\left[ \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B} \right]$$



## <CHAPTER - 04> Governers

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- The function of a governor is to maintain the speed of an engine within specified limits whenever there is variation of load.
- If the load on the shaft increases, the engine speed decreases, thus the supply of fuel is increased by opening the throttle valve. & vice versa.
- The throttle valve is operated by the governor through a mechanism for the purpose.

### Types of governors

1) centrifugal

2) inertia

#### Centrifugal governor

pendulum type  
(Watt governor)

loaded type

gravity / dead weight

spring controlled

Porter

Pronell

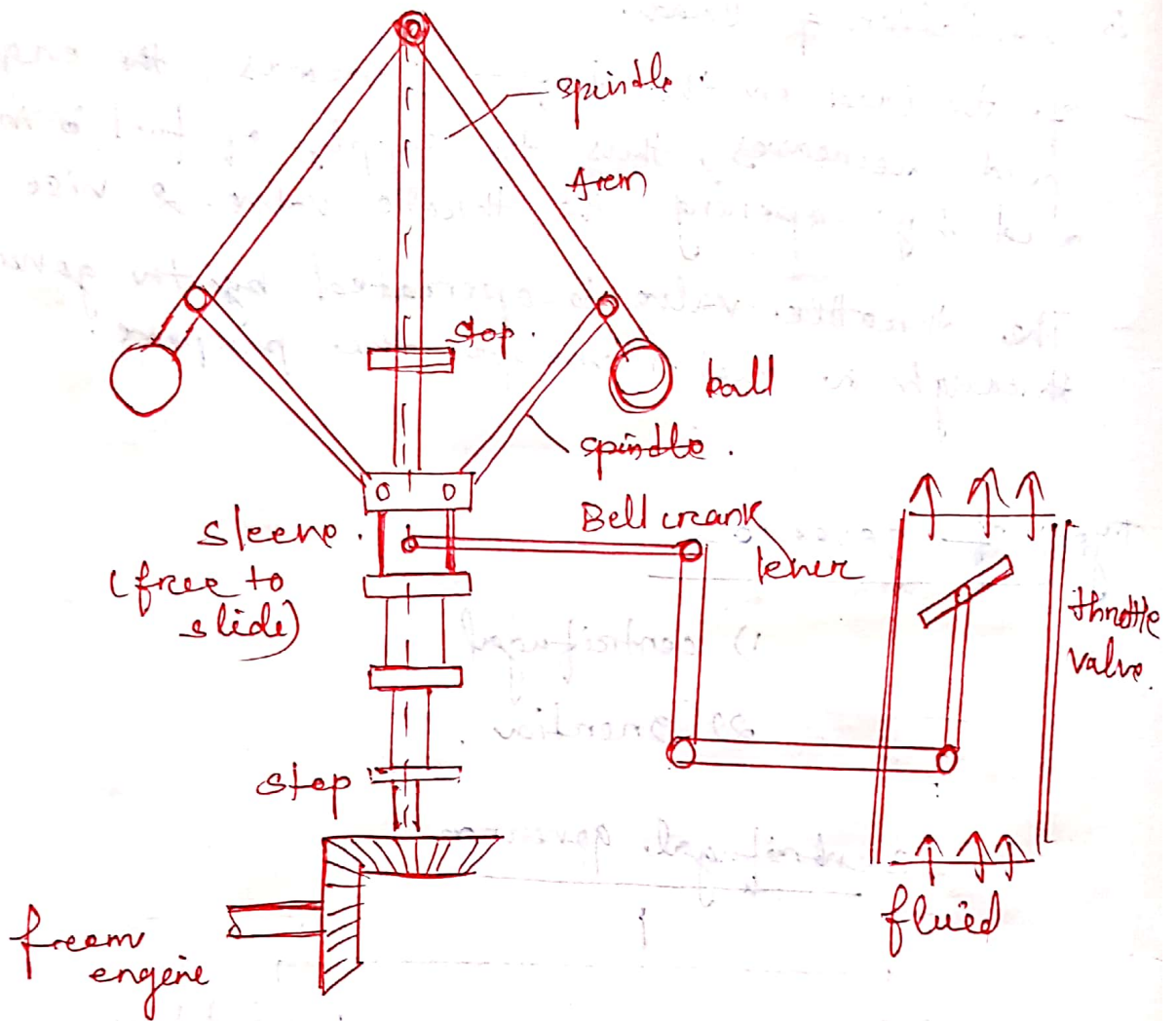
Hartnell

Hartung

Wilsen Hartnell

Pickering

# Centrifugal Governor



This is the more common type. Its action depends on the change of speed. It has a pair of masses, known as governor balls, which rotate with a spindle.

- This spindle is driven by an engine through bevel gears.

- The action of the governor depends upon the centrifugal effects produced by the masses of 2 balls.

with increases in speed, the balls tend to rotate 93 at or greater speed radius from the axis.

- This causes the sleeve to slide up on the spindle & this movement on the sleeve is communicated to the throttle valve through the bell crank lever.
- This closes the throttle valve to the req. extent.
- when speed decreases the vice versa will happen

### Inertia Governor

- In this type the position of the balls are affected by the forces set up by an angular accel<sup>n</sup> or deceleration of the given spindle in addition to centrifugal forces on the balls. using suitable linkages & springs, the change in pos<sup>n</sup> of the balls is made to open or close the valve.

Thus whereas as the balls are operated by the actual change of engine speed in case of centrifugal governors, it is the rate of speed in case of inertia governor.

Therefore, the response of inertia governor is faster than that of centrifugal type.



## Terms used in governor

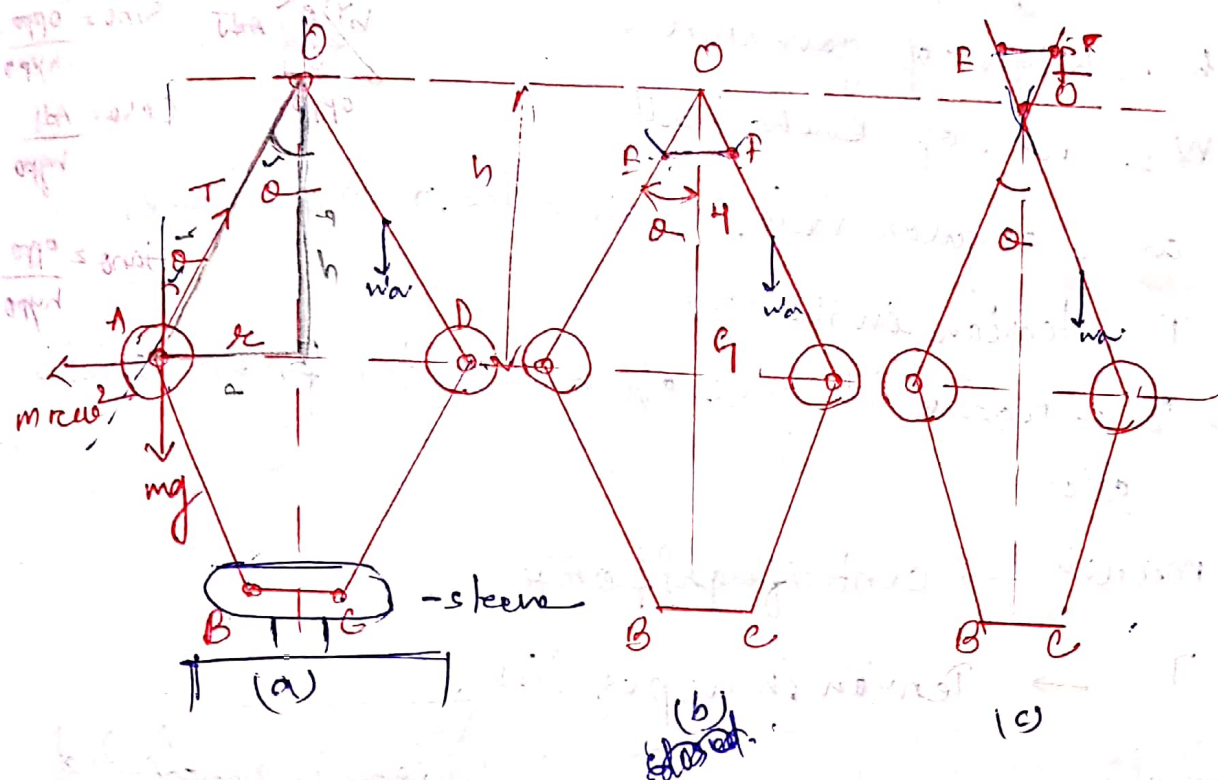
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- 1) Height of governor (h)  $\rightarrow$  It is the vertical distance from the centre of the ball path point where the axis of the arm intersects on the spindle axis.  $\downarrow$
- 2) Equilibrium speed  $\rightarrow$  It is the speed at which the governor balls, arm etc are in complete equilibrium & the sleeve does not tend to move upward & down ward.
- 3) Mean equilibrium speed  $\rightarrow$  It is the speed at the mean position of the balls or the sleeve.
- 4) Sleeve lift  $\rightarrow$  It is the vertical distance which the sleeve travels due to change in equilibrium.

## Watt governor

- The simplest form of a centrifugal governor is a watt governor.
- In this, a pair of balls is attached to the spindle with the help of links.
- The fig shows 3 forms of a watt governor.





In fig (a) the upper links are pinned at O.  
 In " (b) the " " " connected by a horizontal link & the governor is known as open arm type.

In fig (c) the upper links cross the spindle & are connected by a horizontal link & the governor is said to be crossed arm.

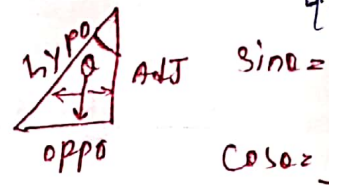
→ In the above 3 cases the lower link is fixed to the sleeve free to move on vertical spindle.

a) Neglecting the weight of arms.

PTD  

$$= 1(0.02 - (-0.02)) = 0.04$$

Let  $m =$  mass of each ball  
 $h =$  height of governor.  
 $W =$  wt. of each ball



$\omega =$  angular velo. of the balls, arm & sleeve.

$T =$  tension in the arm

$r =$  radial distance of ball centre from spin axis

$m\omega^2 r \rightarrow$  centrifugal force

$T \rightarrow$  Tension in upper link.

If sleeve is massless & also friction is neglected the lower link will be tension free.

The equilibrium of the mass provides

$$T \cos \alpha = mg$$

$$T \sin \alpha = m\omega^2 r$$

$$\tan \alpha = \frac{r\omega^2}{g} \Rightarrow \frac{r}{h} = \frac{r\omega^2}{g}$$

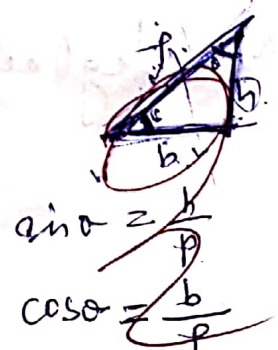
$$\Rightarrow h = \frac{g}{\omega^2}$$

$$\Rightarrow h = \frac{g}{\left(\frac{2\pi N}{60}\right)^2} = \left(\frac{60}{2\pi}\right)^2 \times \frac{9.81}{N^2}$$

$$= \frac{895}{N^2} \text{ m}$$

$$h = \frac{895000}{N^2} \text{ mm}$$

$$= \frac{895}{N^2} \text{ m for } N = 200$$



Q. A simple watt governor rotates at 75 rpm. Cal. its vertical height & the change if the speed increases to 80 rpm. Also cal. the height at 75 rpm if the wt. of ball is 20N & arm 5N.

Soln

$$N_1 = 75 \text{ rpm}$$

$$N_2 = 80 \text{ rpm}$$

with out considering wt of arm.

$$h_1 = \frac{895}{N_1^2} \text{ m}$$

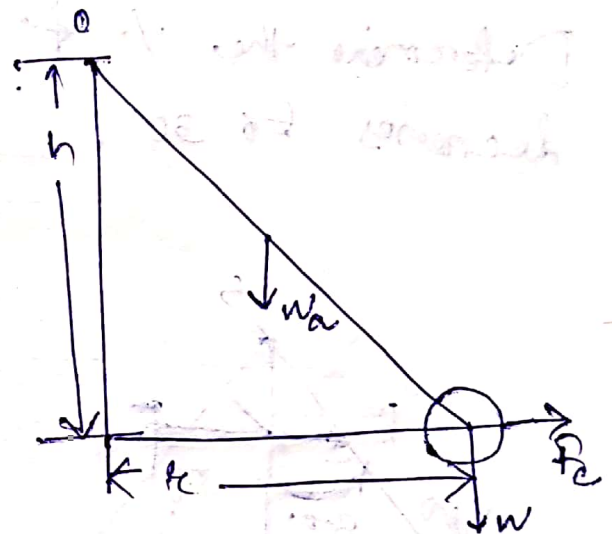
$$= 0.159 \text{ m}$$

$$h_2 = \frac{895}{N_2^2}$$

$$= 0.139 \text{ m} \approx 0.14 \text{ m}$$

$$\text{change in height} = h_1 - h_2$$

$$= 0.02 \text{ m} = 0.019 \text{ m}$$



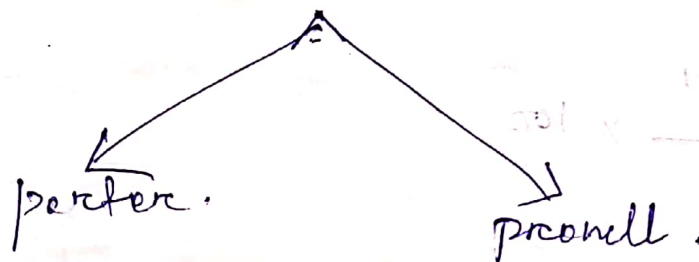
1.65  
F.7



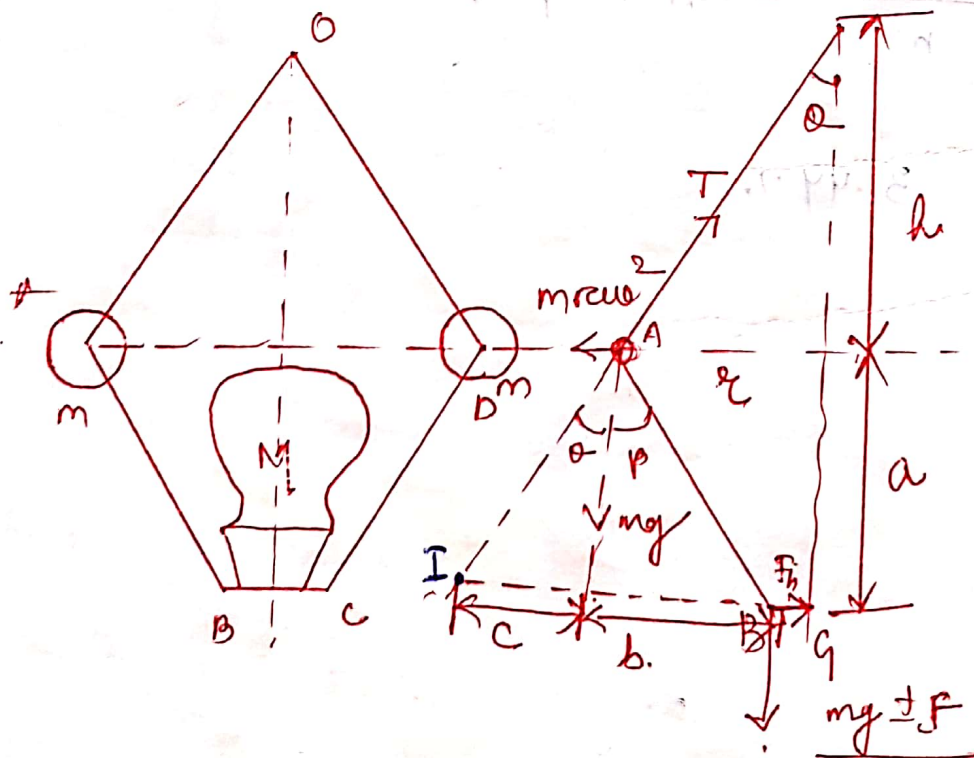
## gravity loaded governor

(192)

- In the gravity loaded governor, a central lead is attached to the sleeve, which slides on the spindle.
- There is a force of friction between the loaded sleeve & spindle.
- The frictional force acts downward when the sleeve moves up & acts upwards when the sleeve moves down. Thus the height of governor increases & decreases from normal value.



### Porter governor.





$M$  = mass of the sleeve

$m$  = mass of each ball

$f$  = force of friction at the sleeve.

(103)

→ When the sleeve moves up, frictional force acts downwards & the downward force acting on the sleeve is  $(Mg + f)$

→ When sleeve moves down,  $f$  acts upwards. The force on sleeve is  $(Mg - f)$ .

— Net force acting on the sleeve is  $(Mg \pm f)$

$h$  → height of governor

$r$  → distance of each ball from axis of rotation.

considering the equilibrium of left hand half of the governor & taking moment about I.

$$m r \omega^2 \cdot a = m g \times c + \left( \frac{Mg \pm f}{2} \right) (c + b)$$

$$\Rightarrow m r \omega^2 = m g \frac{c}{a} + \frac{Mg \pm f}{2} \left( \frac{c}{a} + \frac{b}{a} \right)$$

$$= m g \cdot \tan \alpha + \frac{Mg \pm f}{2} (\tan \alpha + \tan \beta)$$

$$= \tan \alpha \left[ m g + \frac{Mg \pm f}{2} (1 + K) \right] \quad \left| \frac{\tan \beta}{\tan \alpha} = K \right.$$

$$= \frac{r}{h} \left[ m g + \frac{Mg \pm f}{2} (1 + K) \right]$$

$$\Rightarrow \omega^2 = \frac{1}{mh} \left[ \frac{2mg + (Mg \pm f)(1 + K)}{2} \right]$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h} \left[ \frac{(2mg + (Mg \pm f)(1+K))}{2mg} \right]$$

$$\Rightarrow N^2 = \frac{895}{h} \left[ \frac{(2mg + (Mg \pm f)(1+K))}{2mg} \right]$$

$$g = 9.81 \text{ m/s}^2$$

This eqn<sup>n</sup> would provide 2 values for  $N$  for the same height.

$$\text{if } K = 1 \quad N^2 = \frac{895}{h} \left[ \frac{(mg + (Mg \pm f))}{mg} \right]$$

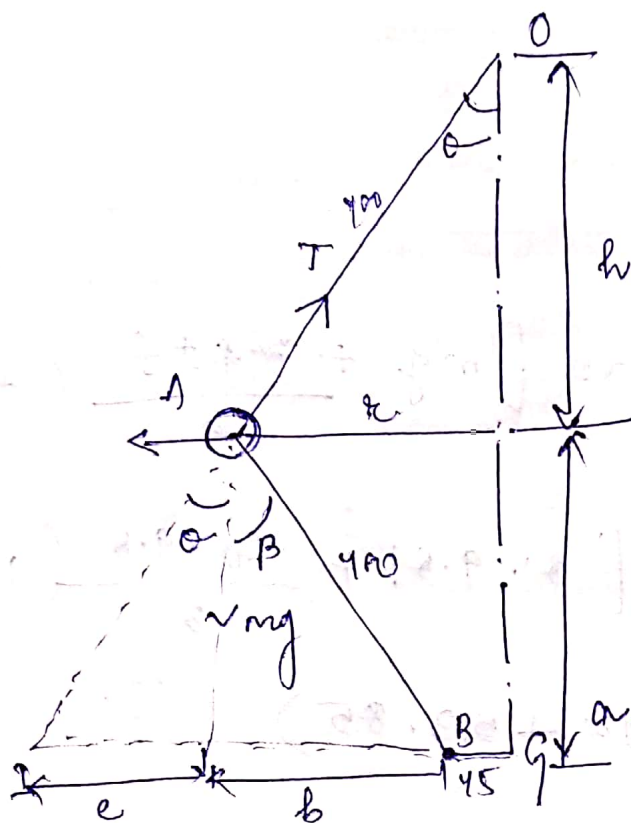
$$\text{if } f = 0 \quad N^2 = \frac{895}{h} \left[ \frac{(2m + M(1+K))}{2m} \right]$$

$$\text{if } K \neq f = 0 \quad N^2 = \frac{895}{h} \left[ \frac{(M+m)}{m} \right]$$

- Q2 In a porter governor, each of the four arms is 400 mm long. The upper arms are pivoted on the axis of the sleeve whereas the lower arms are attached to the sleeve, at a distance of 45 mm from the axis of rotation. Each ball has a mass of 8 kg & lead on the sleeve is 60 kg.
- what will be the equilibrium speed for 2 extreme radii of 250 mm & 300 mm of rotation of the governor balls.

Soln

1.5



when  $i) r_c = 250$ .

$$\tan \alpha = \frac{r_c}{h} = \frac{r_c}{\sqrt{(400)^2 - r_c^2}}$$

$$= \frac{250}{\sqrt{(400)^2 - (250)^2}} = 0.8$$

$$\mu = \frac{\tan \beta}{\tan \alpha} = \frac{b/a}{\tan \alpha}$$

$$b = 250 - 45$$

$$= 205 \text{ mm}$$

$$a = \sqrt{(400)^2 - (b)^2}$$

$$= \sqrt{(400)^2 - (205)^2}$$

$$= 343.4 \text{ mm}$$

3  
over  
come



$$\mu = \frac{205/343.9}{0.8}$$

$$= 0.764$$

$$m r \omega^2 = m g \tan \alpha \left[ m g + \frac{m g \pm f}{2} (1 \pm k) \right]$$

$$\Rightarrow 8 \times 1.25 \times \omega^2 = 0.8 \left[ 8 \times 9.81 + \frac{60 \times 9.8}{2} (1 + 7.1) \right]$$

$$\Rightarrow 2 \omega^2 = 0.8 (78.48 + 513.85)$$

$$\omega^2 = 237$$

$$\omega = \frac{2\pi N}{60} = 15.39$$

$$f = 20$$

$$N = 147 \text{ rpm}$$

(ii) for when  $r = 300 \text{ mm}$

& range of speed. an

12.1 rpm



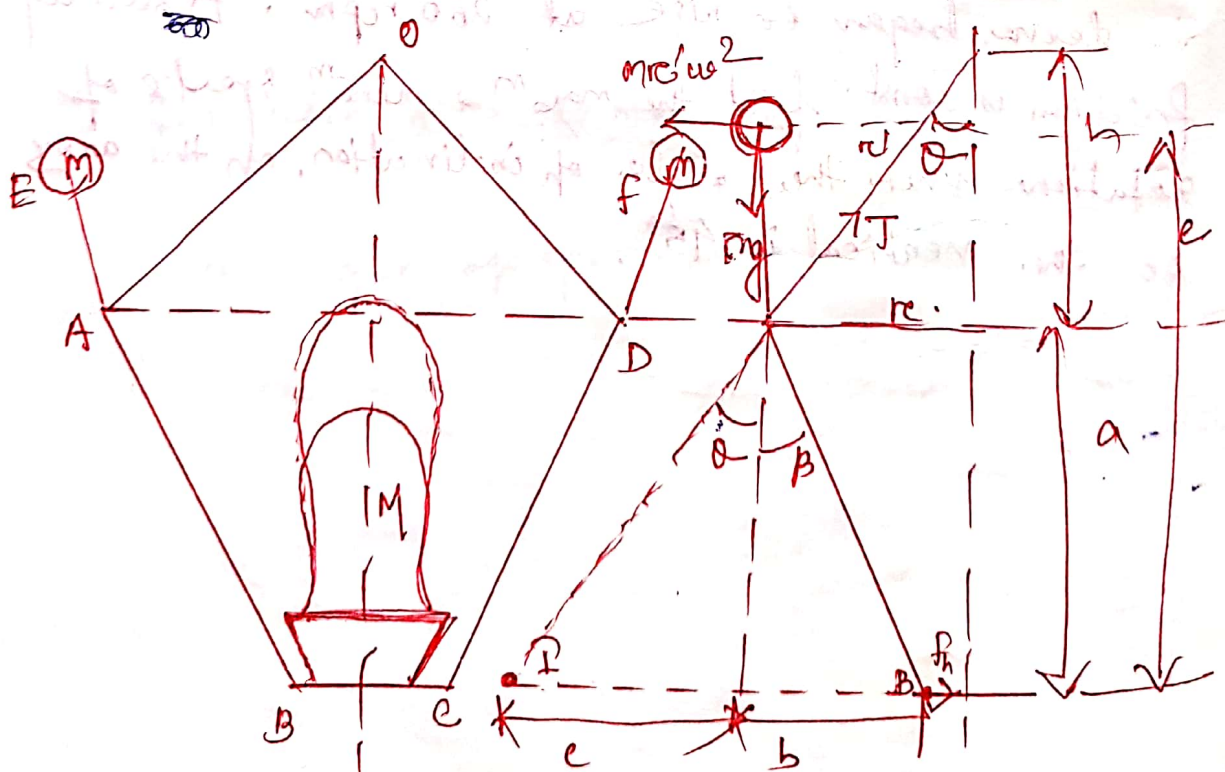
## Pronell governor.

109

A porter governor is known as a pronell governor if the two balls are fixed on the upward extensions of the lower links which are in form of bent links BAE & CDE.

Considering the equilibrium of link,  
the forces acting upon BAE

- The wt. of balls  $mg$ .
- Centrifugal force  $mrc\omega^2$
- tension in the link AD.
- horizontal reaction of the sleeve.
- the wt. of the sleeve & friction  $\frac{1}{2}(Mg \pm f)$



$$mre \omega^2 = \frac{a}{e} \times \frac{1}{h} \left[ \frac{2mg + (Mg \pm f)(1+k)}{2} \right]$$

$$\omega^2 = \frac{a}{e} \times \frac{1}{m \pi \cdot h} \left[ \frac{2mg \mp (Mg \pm f)(1+k)}{2} \right]$$

$$= \frac{a}{e} \times \frac{g}{h} \times \frac{1}{h} \left[ \frac{2mg \mp (Mg \pm f)(1+k)}{2} \right]$$

$$= \frac{a}{e} \times \frac{g}{h} \left[ \frac{2mg + (Mg \pm f)(1+k)}{2mg} \right]$$

(divide g & multiply g)

$$\omega^2 = \frac{a}{e} \times \frac{g}{h} \left[ \frac{2mg + (Mg \pm f)(1+k)}{2mg} \right] \quad g = 9.81$$

$$\Rightarrow N^2 = \frac{895}{h} \times \frac{a}{e} \left[ \frac{2mg + (Mg \pm f)(1+k)}{2mg} \right]$$

$$\text{if } k=1 \quad N^2 = \frac{895}{h} \times \frac{a}{e} \left[ \frac{mg + (Mg \pm f)}{mg} \right]$$

$$\text{if } f=0 \quad N^2 = \frac{895}{h} \times \frac{a}{e} \left[ \frac{2m + M(1+k)}{2m} \right]$$

$$\text{if } k=1, f=0 \quad N^2 = \frac{895}{h} \times \frac{a}{e} \left( \frac{m+M}{m} \right)$$

9 Each arm of a proell governor is 240 mm long & each rotating ball has mass 3 kg. The central load acting on the sleeve is 30 kg. The pivots of all the arms are 30 mm from the axis of rotation. The vertical height of the governor is 190 mm. The extension links of the lower arms are vertical & the governor speed is 180 rpm, when the sleeve is in mid position. Determine the length of the extension links & the tension in the upper arms.

Soln

$$m = 3 \text{ kg}$$

$$h = 190 \text{ mm}$$

$$N^2 = \frac{895}{h} \cdot \frac{a}{e} \left( \frac{m + M}{m} \right)$$

$$180^2 = \frac{895}{190} \cdot \frac{19}{e} \left( \frac{3 + 30}{3} \right)$$

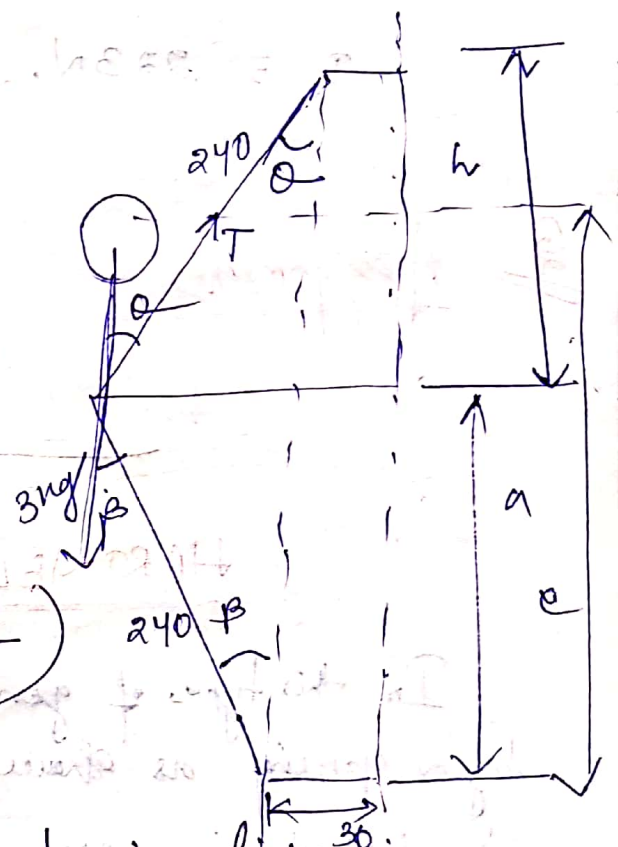
$$e = 0.304 \text{ m}$$

Therefore, length of the extension links

$$e - a = 304 - 190$$

$$= 114 \text{ mm}$$

Let  $T$  be the tension in upper arm.





considering the vertical components of the forces on the lower link,

$$T \cos \alpha = mg + \frac{Mg}{2}$$

$$\cos \alpha = \frac{0.19}{.24} = .792$$

$$T \times .792 = 3 \times 9.81 + \frac{30 \times 9.81}{2}$$

$$T = 223 \text{ N.}$$

## 2 Assignments

### HARTNELL GOVERNER

In this type of governor, the balls are controlled by a spring as shown in fig. Initially, the spring is fitted in compression, so that a force is applied to the sleeve. Two bell cranks levers each carrying a mass at end & a roller or the other, are pivoted to a pair of arms which rotate with the spindle. The rollers fit into a groove in the sleeve.



$W_1 = \text{wt of ball}$   
 $W = \text{wt of sleeve}$

$r_1, r_2 = \text{Max}^m \text{ \& \; min}^m \text{ radii of rotation of the ball.}$

$\omega_1, \omega_2 = \text{ " " angular speed of rotation.}$

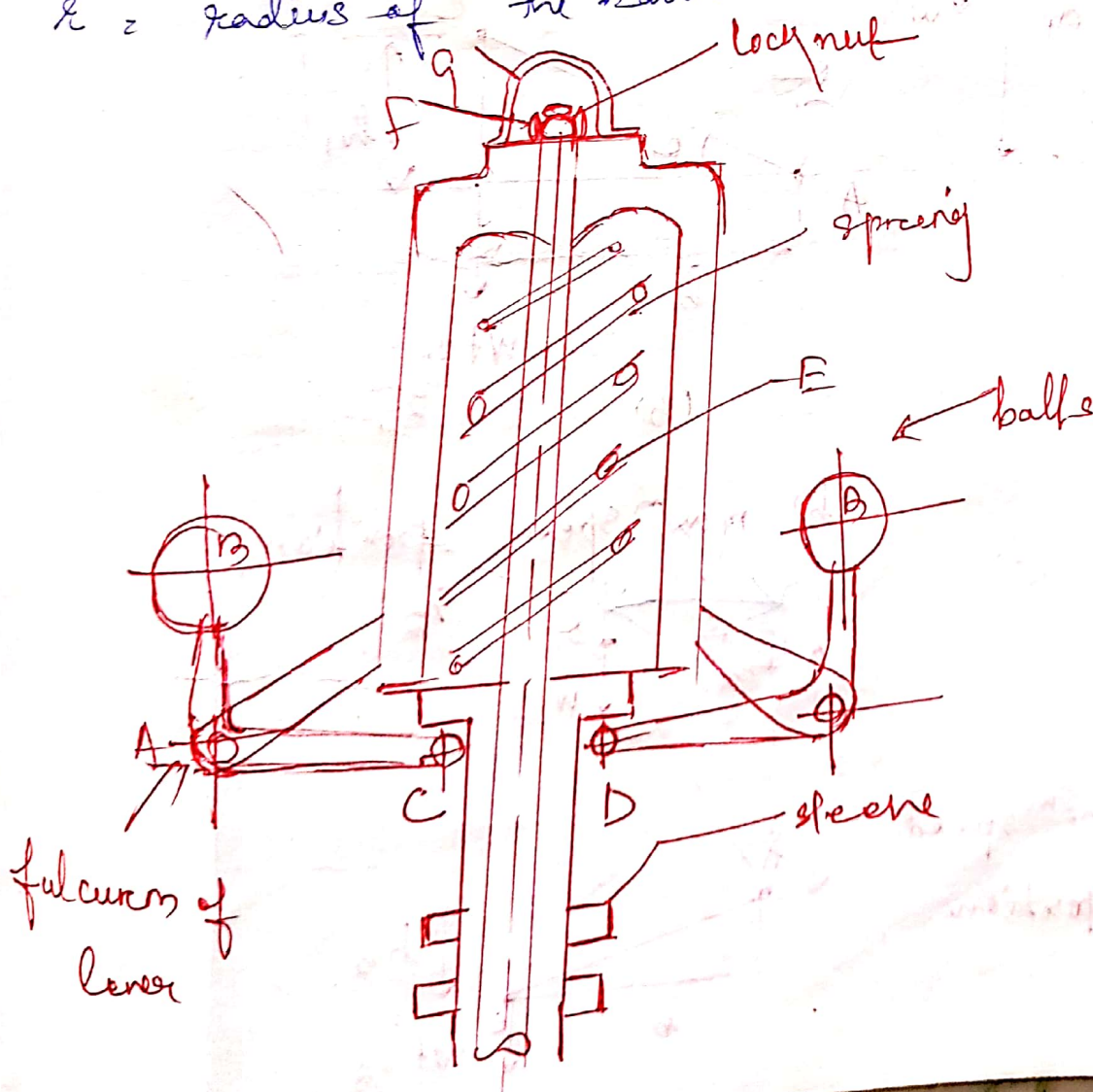
$S_1, S_2 = \text{Max}^m \text{ \& \; min}^m \text{ spring force.}$

$F_{c1}, F_{c2} = \text{centrifugal forces at speed } \omega_1 \text{ \& \; } \omega_2 \text{ respectively.}$

$k = \text{stiffness of the spring}$

$a, b = \text{vertical \& \; horizontal length of arm of ball crank lever.}$

$r = \text{radius of the ball.}$



## isochronism

- when a governor shows an equilibrium speed for all positions of the sleeve or the balls is known as isochronous governor.

- However this is not practical due to friction in the sleeve.

for eg. porter governor. (neglecting friction).

$$h_1 = \frac{g}{\omega_1^2} \left( 1 + \frac{M}{m} \right) \quad h_2 = \frac{g}{\omega_2^2} \left( 1 + \frac{M}{m} \right)$$

for isochronism  $\omega_1 = \omega_2$  thus  $h_1 = h_2$  ✓

## Stability

- Stability and the sensitivity are two opposite characteristics.

- A governor is said to be stable if it brings the speed of engine to the req. value & there is not much hunting.

## Effort of governor

- The mean force acting on the sleeve to raise or lower it for a given change of speed is called effort of governor.

- At const. speed, the governor is in equilibrium & the resultant force acting on sleeve is zero.

- If the force acting at the sleeve changes gradually from zero to a value  $E$  for an increased speed of governor, the mean force or the effort is

$$\frac{0+E}{2} = E/2.$$

## CHAPTER - 05 BALANCING OF ROTATING MASSES

→ Balancing is defined as the process of designing or modifying a m/c in which unbalance force is minimum. If in high speed engines the rotating & reciprocating masses are not balanced properly then dynamic forces will induce, which increases the stress in m/c members.

- 1) - Balancing of single rotating mass
- 2) - Balancing of several rotating masses in same plane.
- 3) - Balancing of " " " " diff. plane.

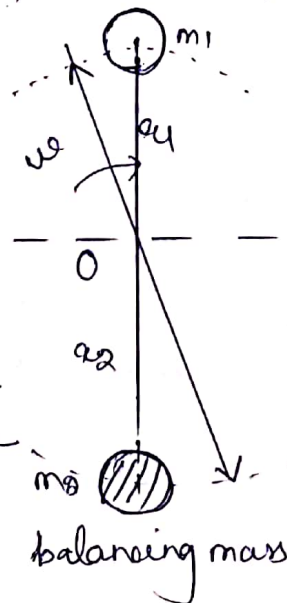
### 1) Balancing of single rotating mass.

A single rotating mass can be balanced in 2 diff. ways.

- i) The balance mass may be rotating in the same plane.
- ii) The balance mass is not rotating in same plane.

ii) The balance mass may be rotating in same plane! —

The fig shows. a disturbing mass  $m_1$  attached to a shaft which is rotating with a angular velo.  $\omega$  rad/s.



The process of providing the 2nd mass in order to counteract the effect of centrifugal force of 1st mass is called balancing of rotating masses.

If certain mass is attached with a rotating shaft, it will exert centrifugal force, which may bend the shaft & produce vibration. So to avoid this, another mass is attached to the opposite side of the shaft to balance the effect of previous mass.



The axis of shaft is  $\perp$  to plane of paper & passing through:

$r_1 \rightarrow$  is the distance <sup>c.g.</sup> of mass  $m_1$  from pt. O.

$m_2 \rightarrow$  balancing mass.

$r_2 \rightarrow$  the distance of <sup>c.g.</sup> of mass  $m_2$  from axis of rotation.

The centrifugal force acting radially outward on mass  $m_1$ ,

$$F_c = m_1 \omega^2 r_1 \text{ ————— (1)}$$

This centrifugal force induces a bending moment on the shaft & to counteract this moment a balance mass  $m_2$  is attached to the shaft in same plane. ~~having~~

centrifugal force due to  $m_2$ .

$$F_c = m_2 \omega^2 r_2 \text{ ————— (2)}$$

equating both eqn.  $m_1 r_1 \omega^2 = m_2 r_2 \omega^2$

$$\Rightarrow \boxed{m_1 r_1 = m_2 r_2} \text{ ————— (3)}$$

This eqn shows that the product of mass & radius is const.

ii) The balancing mass is not rotating in the same plane.

If balance mass is not in the same plane of rotation as disturbing mass, then we can't get accurate result by using a single ~~one~~ balancing mass.

The fig shows the mass ( $m_1$ ) & mass ( $m_2$ ) are in a diff. plane. The centrifugal force induced by these 2 masses have same magnitude but the directions of these 2 forces are opposite. Hence they will form a couple. Hence to balance this couple a second balancing mass will be used so to balance the system.



$$m_2 r_2 \cdot d = m_1 r_1 \cdot d_1$$

$$\Rightarrow m_2 r_2 = \frac{m_1 r_1 \times d_1}{d} \quad \text{--- (2)}$$

moment about P.

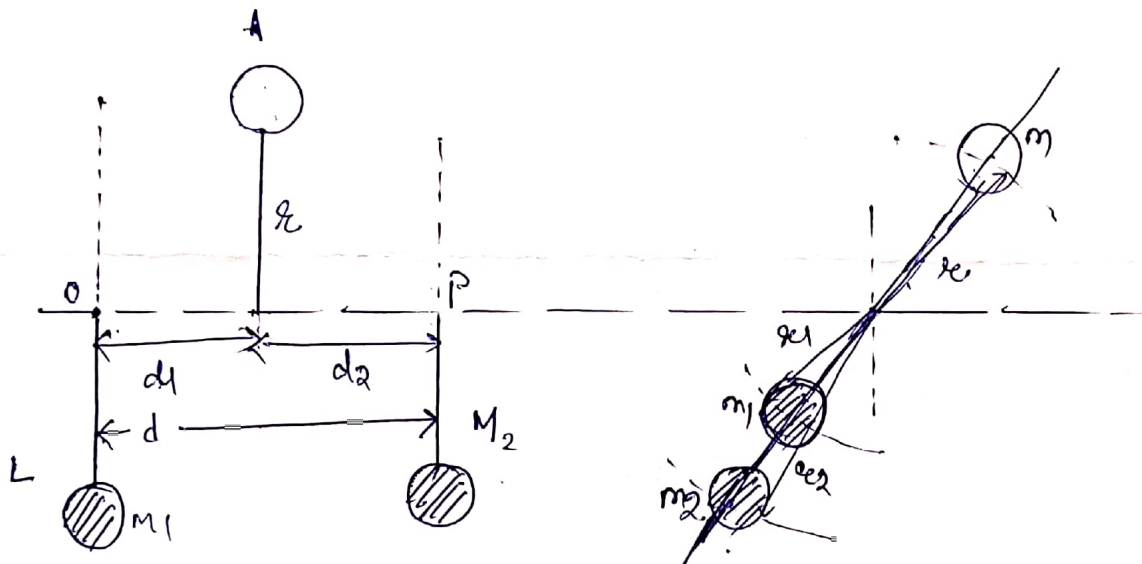
$$F_{c1} \times d = F_{c2} \times d_2$$

$$\Rightarrow (m_1 r_1 \omega^2) d = (m_2 r_2 \omega^2) d_2$$

$$\Rightarrow m_1 r_1 d = m_2 r_2 d_2$$

$$\Rightarrow m_1 r_1 = \frac{m_2 r_2 d_2}{d} \quad \text{--- (3)}$$

6) The masses are either side of the distributing mass in different plane



$$F_c = m r \omega^2$$

$$F_{c1} + F_{c2} = F_c$$

$$F_{c1} = m_1 r_1 \omega^2$$

$$F_{c2} = m_2 r_2 \omega^2$$

$$\Rightarrow m_1 \times r_1 \times \omega^2 + m_2 \omega^2 \times r_2 = m r \omega^2$$

$$\Rightarrow m_1 r_1 + m_2 r_2 = m r \quad \text{--- (1)}$$

Taking moments of the forces about pt. O.

$$F_{c1} \times d_1 = F_{c2} \times d_2$$

$$\Rightarrow m_1 r_1 \omega^2 \times d_1 = m_2 r_2 \omega^2 \times d_2$$

$$\Rightarrow m_2 r_2 = \frac{m_1 r_1 d_1}{d_2} \quad \text{--- (2)}$$

$$\Rightarrow \boxed{m_2 r_2 \times d_2 = m_1 r_1 d_1}$$

taking moment about P

$$\boxed{m_1 r_1 d = m_2 r_2 d_2}$$

$$m_2 r_2 \cdot d = m_1 r_1 \cdot d_1$$

$$\Rightarrow m_2 r_2 = \frac{m_1 r_1 \times d_1}{d} \quad \text{--- (2)}$$

moment about P.

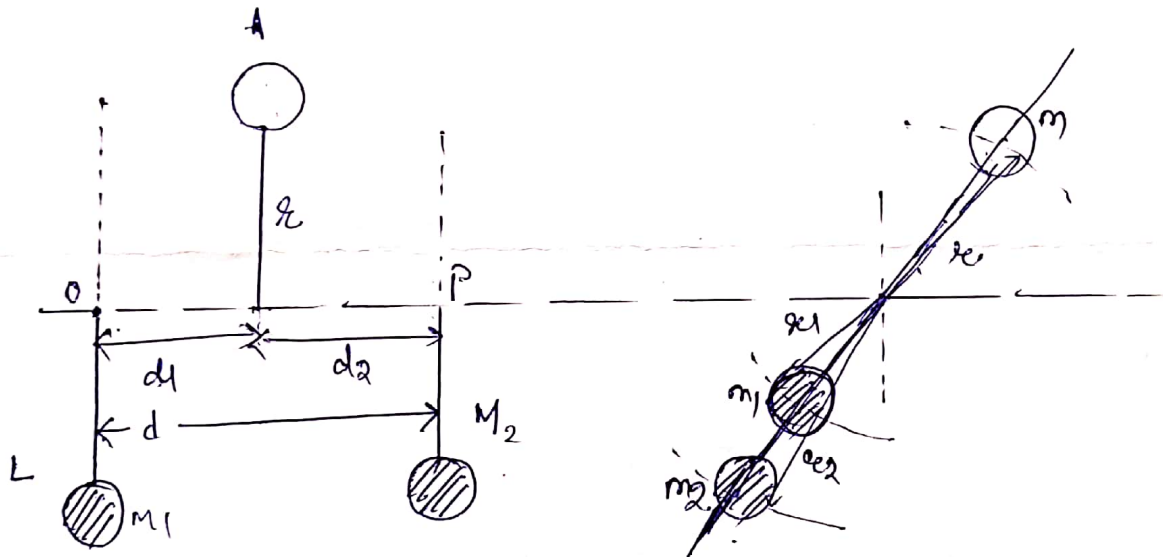
$$F_{c1} \times d = F_c \times d_2$$

$$\Rightarrow (m_1 r_1 \omega^2) d = (m r \omega^2) d_2$$

$$\Rightarrow m_1 r_1 d = m r d_2$$

$$\Rightarrow m_1 r_1 = \frac{m r d_2}{d} \quad \text{--- (3)}$$

6) The masses are either side of the distributing mass in different plane



$$F_c = m r \omega^2$$

$$F_{c1} + F_{c2} = F_c$$

$$F_{c1} = m_1 r_1 \omega^2$$

$$F_{c2} = m_2 r_2 \omega^2$$

$$\Rightarrow m_1 r_1 \omega^2 + m_2 r_2 \omega^2 = m r \omega^2$$

$$\Rightarrow m_1 r_1 + m_2 r_2 = m r \quad \text{--- (1)}$$

Taking moments of the forces about pt. O.

$$F_c \times d_1 = F_{c2} \times d$$

$$\Rightarrow m r \omega^2 \times d_1 = m_2 r_2 \omega^2 \times d$$

$$\Rightarrow m_2 r_2 = \frac{m r d_1}{d} \quad \text{--- (2)}$$

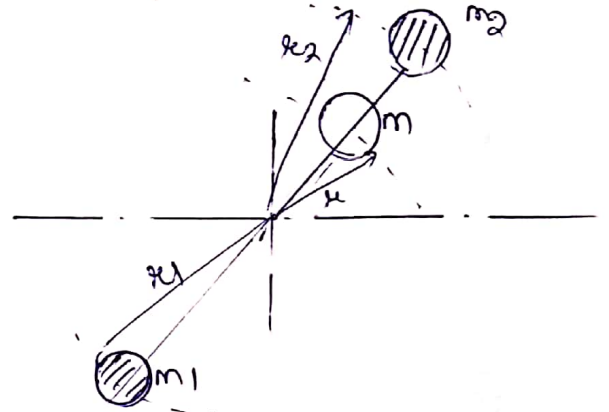
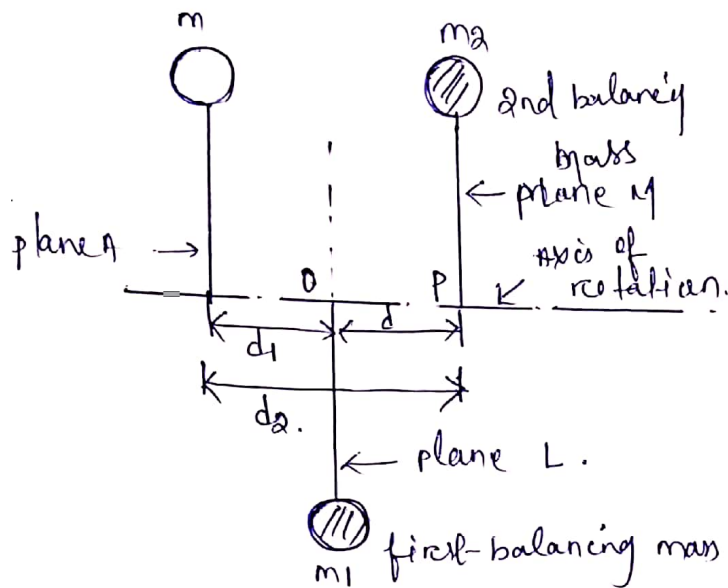
$$\Rightarrow \boxed{m_2 r_2 \times d = m r d_1}$$

taking moment about P

$$\boxed{m_1 r_1 d = m r d_2}$$

& this is possible if the line of action of 3 centrifugal forces are parallel & the sum of their moments about any pt is 0.

a) The masses at same side of the plane.



$$F_c = m r \omega^2$$

$$F_{c1} = m_1 r_1 \omega^2$$

$$F_{c2} = m_2 r_2 \omega^2$$

$$F_c + F_{c2} = F_{c1}$$

$$\Rightarrow m r \omega^2 + m_2 r_2 \omega^2 = m_1 r_1 \omega^2$$

$$\Rightarrow m r + m_2 r_2 = m_1 r_1 \quad \text{--- (1)}$$

For another condition the sum of moments about any pt. should be 0.

Let pt. O is the pt. of intersection of plane L with axis of rotation, & P is the pt. " " " " " " " " " " " "

moment about O, we get.

$$F_{c2} \times d = F_c \times d_1$$

$$\Rightarrow (m_2 r_2 \omega^2) \times d = (m r \omega^2) d_1$$

$$\boxed{m_2 r_2 d = m r d_1}$$



Similarly taking moment about P.

$$F_2 \times d_2 = F_1 \times d$$

$$\Rightarrow (m_2 \omega^2) d_2 = (m_1 \omega^2) \times d$$

$$\Rightarrow \boxed{m_2 d_2 = \frac{m_1 d}{\omega^2}} \quad \text{--- (3)}$$

### Problem

Q1 A disturbing mass 600 kg is attached to a shaft. The shaft is rotating at a uniform angular velo.  $\omega$  rad/s. & the distance of the C.G. of the disturbing mass from the axis of rotation is 270 mm. The disturbing mass is to be balanced by 2 masses in 2 diff planes. The distance of the C.G. of the balancing masses from the axis of rotation is 450 mm each. The distance bet<sup>n</sup> the two planes of the balancing mass is 1.5 m & distance bet<sup>n</sup> the plane of disturbing mass & one of the planes of the balancing masses is 300 mm.

Determine - 1) the distance bet<sup>n</sup> the plane of disturbing mass & the plane of other balancing masses.

ii) Magnitude of balancing masses such as.

a) The plane of balancing are on the same side of the plane of disturbing masses.

b) The planes of the balancing masses are on either side of the plane of disturbing mass.

Sol<sup>n</sup>

$$m = 600 \text{ kg.}$$

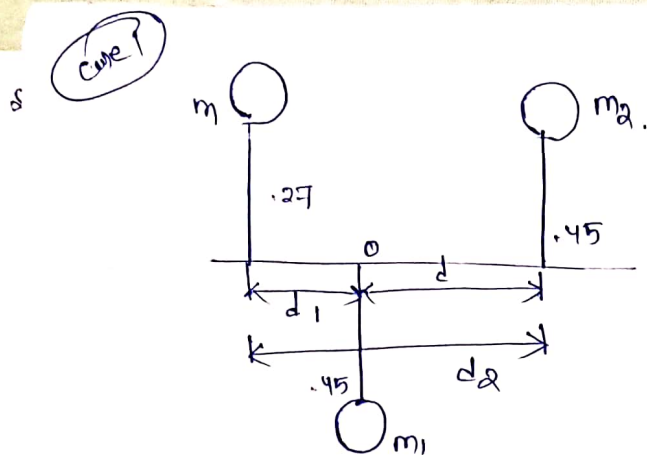
distance of C.G. of mass  $m$  from axis of rotation  $r = 270 \text{ mm} = 0.27 \text{ m}$

" " balancing mass from axis of rotation  $r_1 = r_2 = 450 = 0.45 \text{ m}$

distance bet<sup>n</sup> the plane of disturbing mass & the plane of 1st

balancing mass = ~~0.3 m~~ 0.3 m

distance bet<sup>n</sup> 2 planes of balancing mass  $d = 1.5 \text{ m}$ .



$$m \times r_1 + m_2 r_2 = m_1 r_1$$

$$\Rightarrow 600 \times 0.27 + m_2 (0.45) = m_1 \times 0.45$$

Again resultant moment

$$m_2 r_2 \times d = m \times r_1 d_1$$

$$m_2 r_2 = \frac{m r_1 d_1}{d}$$

$$\Rightarrow m_2 \times 0.45 = \frac{600 \times 0.27 \times 0.03}{1.5}$$

$$\Rightarrow m_2 = 72 \text{ kg}$$

$$\text{New } m_1 = \frac{600 \times 0.27 + 72 \times 0.45}{0.45} = 432 \text{ kg}$$

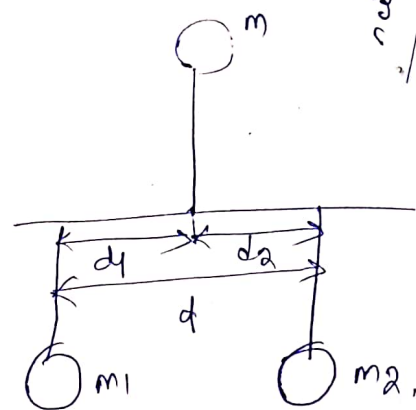
To find  $d_2$ .

$$m_1 r_1 d = m_2 r_2 d_2$$

$$d_2 = \frac{m_1 r_1 d}{m r_2}$$

$$= \frac{432 \times 0.45 \times 1.5}{600 \times 0.27} = 1.8 \text{ m} \quad \underline{\text{Ans}}$$

Core 2



$$m_1 r_1 + m_2 r_2 = m \times r_2$$

$$\Rightarrow m_1 \cdot 0.45 + m_2 \cdot 0.45 = 600 \times 0.27$$

The resultant moment to be zero at pt O.

$$m r_1 d_1 = m_2 r_2 d_2$$

$$\Rightarrow m_2 = 72 \text{ kg}$$

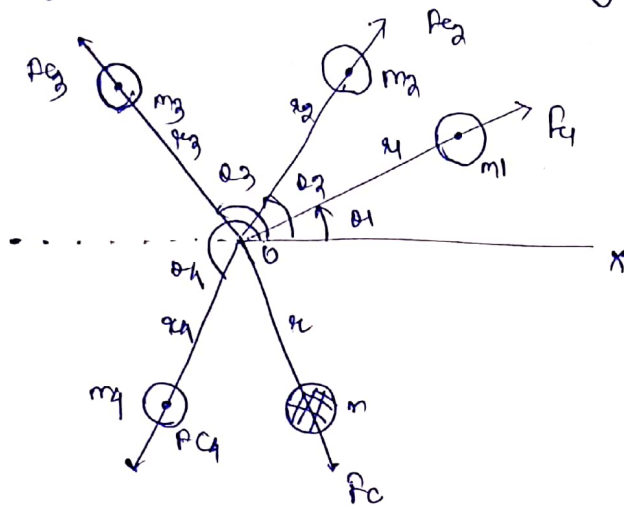
$$m_1 = 288 \text{ kg}$$

Value of  $d_2$ .

$$m \times r_1 \times d_1 = m_1 r_2 d_2$$

$$\Rightarrow d_2 = 1.2 \text{ m} \quad \underline{\text{Ans}}$$

## Balancing of Several masses rotating in the same plane



The above fig shows a no. of masses  $m_1, m_2, m_3$  &  $m_4$  which are rigidly attached to a shaft which is revolving at  $\omega$  rad/s.

The radius of rotation of masses  $r_1, r_2, r_3$  &  $r_4$  & the angle subtended by them with the horizontal axis.  $\theta_1, \theta_2, \theta_3$  &  $\theta_4$ .

The centrifugal forces experienced by them  $P_1, P_2, P_3$  &  $P_4$  & these forces are proportional to corresponding product of mass &

1. Analytical method. radius  $P_1 \propto m_1 r_1$   $P_2 \propto m_2 r_2$   $P_3 \propto m_3 r_3$   $P_4 \propto m_4 r_4$

Resolving each centrifugal forces horizontally & vertically.

$$\Sigma H = P_1 \cos \theta_1 + P_2 \cos \theta_2 + P_3 \cos \theta_3 + P_4 \cos \theta_4$$

$$\Sigma H = m_1 r_1 \omega^2 \cos \theta_1 + m_2 r_2 \omega^2 \cos \theta_2 + m_3 r_3 \omega^2 \cos \theta_3 + m_4 r_4 \omega^2 \cos \theta_4$$

$$\Sigma H = m_1 r_1 \cos \theta_1 + m_2 r_2 \cos \theta_2 + m_3 r_3 \cos \theta_3 + m_4 r_4 \cos \theta_4$$

Similarly.  $\Sigma V$

$$\Sigma V = m_1 r_1 \sin \theta_1 + m_2 r_2 \sin \theta_2 + m_3 r_3 \sin \theta_3 + m_4 r_4 \sin \theta_4$$

$$\text{Resultant centrifugal force } P_c = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

Position of the balancing mass

$$\tan \alpha = \frac{\Sigma V}{\Sigma H}$$

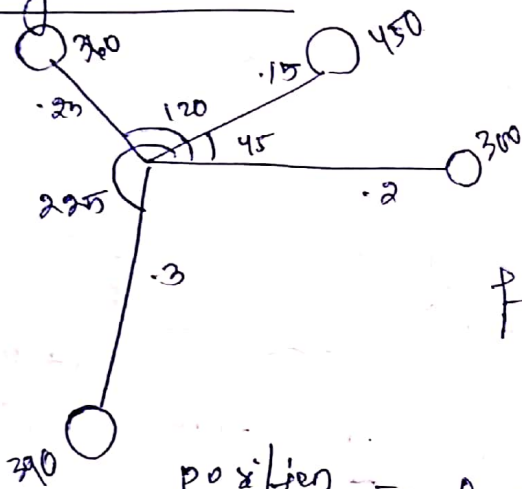
Since the Resultant force & centrifugal force by same magi but oppo. in direction. Hence dir<sup>n</sup> of bal. mass (180°)



Q A shaft is rotating at a uniform angular speed. Four masses  $m_1, \dots, m_4 = 500, 450, 360$  &  $390$  kg are attached to the shaft. All the masses are rotating in a same plane. The radius of rotations are  $200, 150, 250, 300$  mm respectively. Angle are  $0^\circ, 45^\circ, 120^\circ, 225^\circ$ . Find.  $\omega = 1 \text{ rad/s}$ .

- i) find the resultant force. ~~The radius of rotation of balancing mass~~  
 ii) position of the balancing mass if its radius is  $0.2 \text{ m}$ .

Analytical meth



$$m_1 r_1 \cos \theta = 60 + 47.72 + -45 + -87.73 = -25.01$$

$$\Sigma H = 32.45 \text{ kgm}$$

$$\Sigma V = 12.65$$

$$\text{Force} = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= 34.82 \text{ kgm}$$

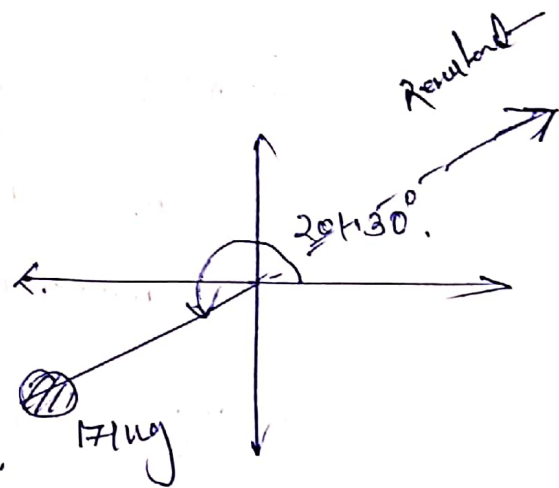
$$\text{position} = \alpha = \tan^{-1} \frac{\Sigma V}{\Sigma H}$$

iii) magnitude of balancing mass

$$\text{Resultant force } F = m r$$

$$\Rightarrow F = m \times 0.2$$

$$\Rightarrow m = 174.1 \text{ kg}$$

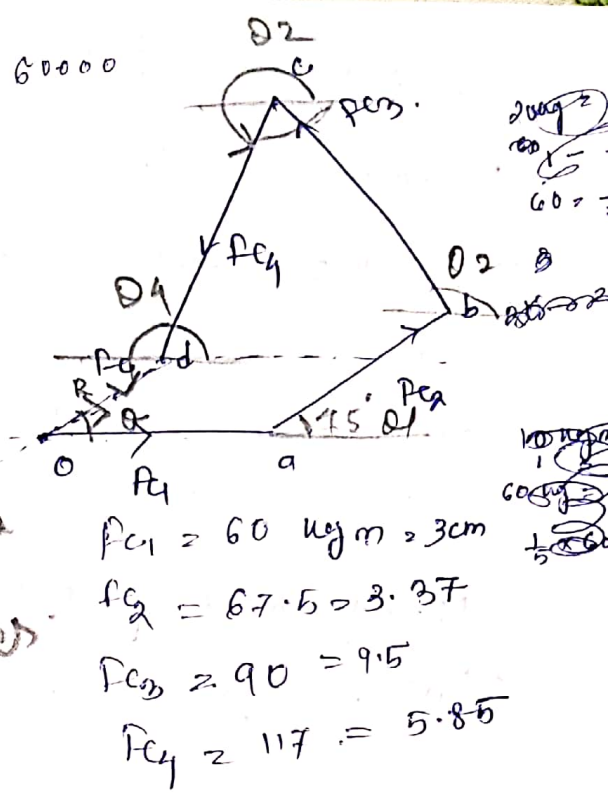
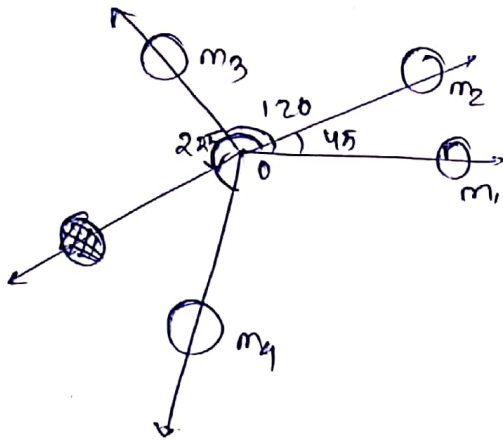


$$\alpha = 21.30^\circ$$

ii) Direction of balancing mass.  $= 180 + \alpha$   
 $= 180 + 21.30$   
 $= 201.30$

# graphical Methode.

1) draw the space dia



Od → Resultant of all the masses.

$$P_{c1} = 60 \text{ kg m} = 3 \text{ cm}$$

$$P_{c2} = 67.5 = 3.37$$

$$P_{c3} = 90 = 9.5$$

$$P_{c4} = 117 = 5.85$$

ii) Cal. the centrifugal force.

$$P_{c1}, P_{c2}, P_{c3}, P_{c4}$$

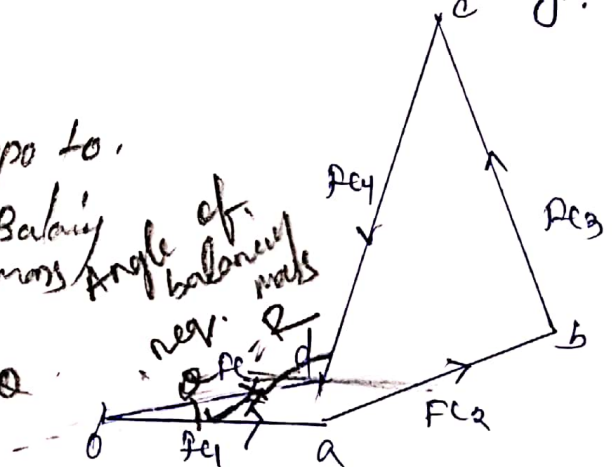
$$P_c = m \times r = 300 \times 200 = 600 \text{ kg m}$$

Pr) take suitable scale.

1) Take any pt from O & draw Oa =  $P_{c1} = m_1 r_1$  parallel to  $P_{c1}$ . From a draw ab || to  $P_{c2}$ . From b, draw bc || to  $P_{c3}$ . From c draw cd || to  $P_{c4}$ . The closing side will represent the resultant force. measure od = 34.8 kg.

balancing mass will act oppo to.

Resultant force. R = Balancing mass  
angle of balancing mass = 180 + 0



$$P_{c1} = 20.3 \text{ cm} = 20.3 \times 3$$

$$OD = 34.8$$

# Balancing of several masses rotating in different plane

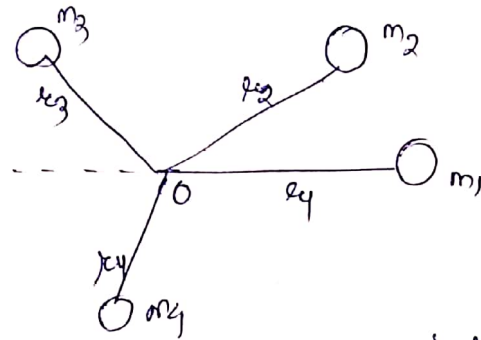
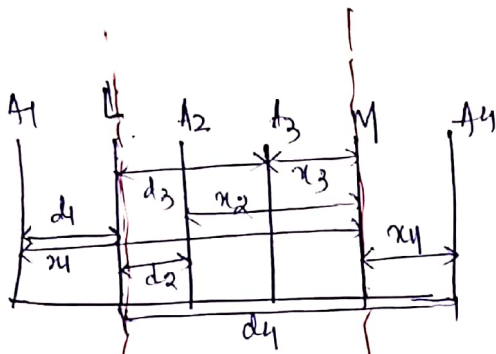


Fig shows several masses ( $m_1, m_2, m_3$  &  $m_4$ ) are rotating in 4 diff plane ( $A_1, A_2, A_3$  &  $A_4$ ) & radii of rotation ( $r_1, r_2, r_3, r_4$ ) respectively

Suppose the balancing masses are in plane L & M, so it is req. to find the masses & their angular position.

plane	mass	radius	centrifugal force $\div \omega^2$	Distance from		balancing force $\div \omega^2$	
				L	M	L	M
$A_1$	$m_1$	$r_1$	$m_1 r_1$	$d_1$	$r_1$	$-\frac{m_1 r_1 d_1}{d}$	$\frac{m_1 r_1 d_1}{d}$
$A_2$	$m_2$	$r_2$	$m_2 r_2$	$d_2$	$r_2$	$-\frac{m_2 r_2 d_2}{d}$	$-\frac{m_2 r_2 d_2}{d}$
$A_3$	$m_3$	$r_3$	$m_3 r_3$	$d_3$	$r_3$	$-\frac{m_3 r_3 d_3}{d}$	$-\frac{m_3 r_3 d_3}{d}$
$A_4$	$m_4$	$r_4$	$m_4 r_4$	$d_4$	$r_4$	$+\frac{m_4 r_4 d_4}{d}$	$+\frac{m_4 r_4 d_4}{d}$

Disturbing mass  $m_1$  is in plane  $A_1$ .

The plane  $A_1$  is left to " L & M.

The balancing force of plane L will be opposite to the centrifugal force on ~~on~~ plane  $A_1$  & M.

If we assume the force on  $A_1$  is the, then conversely the forces on L will be -ve & M will be the.

As follows.



The sign convention is as follows

1. Let 1st disturbing mass is  $M_1$  & it is placed left to both  $L$  &  $M$  so the balancing forces on plane  $L$  must be opposite to the centrifugal force on outer 2. masses i.e. plane  $M$  &  $M$ . So if we assume centrifugal force on  $M_1$  is the 1<sup>st</sup> then balancing force  $L$  will be -ve & balancing force on plane  $M$  is the
2. for mass  $m_2$ , it is bet<sup>n</sup>  $L$  &  $M$ .  $P_c$  at  $m_2$  should be opposite to balancing forces on  $L$  &  $M$ . As we consider centrifugal force as the 1<sup>st</sup> then hence balancing forces on  $L$  &  $M$  will be -ve.
3. If  $m_3$  is also bet<sup>n</sup>  $L$  &  $M$  of 0 condition will be applied,
4.  $m_4$  it is place right to  $L$  &  $M$  & same procedure will be followed.

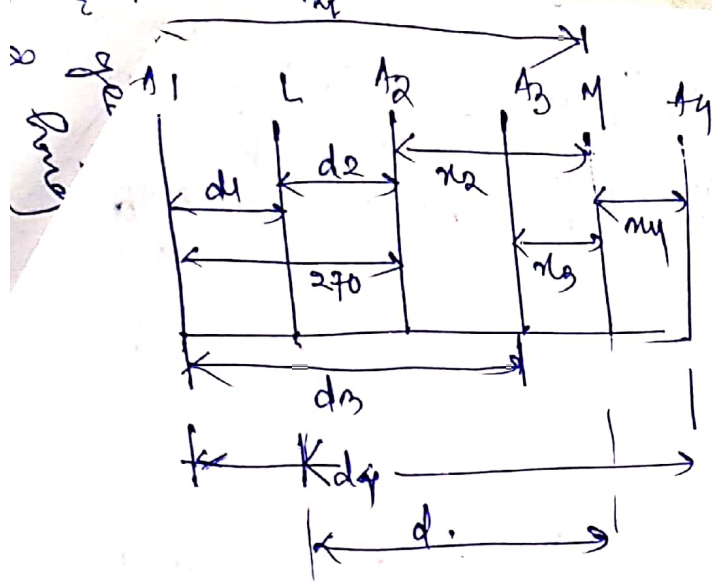
Q. cal. the min<sup>m</sup> speed, max<sup>m</sup> speed & range of speed of porter governor, which has equal arms each 200mm long & pivoted on the axis of rotation. The mass of each ball is 4 kg & the central mass on the sleeve is 20 kg. The radius of rotation of the ball is 100mm when the governor begins to lift & 130mm when the governor is at max<sup>m</sup> speed.

Q. four masses  $m_1 = 20$   $r_1 = 270 = 2.7m$   
 $m_2 = 28m$   $r_2 = 210$   
 $m_3 = 24m$   $r_3 = 8m$   
 $m_4 = 260$   $r_4 = 360$

distance bet<sup>n</sup> L & M = 500 = d.  
 $H \& L = 120 = d_1$   
 $A_1 \& 11 = r_1 = 100mm$

radius of ball & mass 72 mm.

And  $m_L \& m'_m \& r_L \& 0m$ .



distance bet<sup>n</sup> A1 & A2

$$= 270 \text{ mm}$$

$$A1 \text{ \& } A3 = 420 \text{ mm}$$

$$A1 \text{ \& } A4 = 720 \text{ mm}$$

Soln

$$d_1 = 120 \text{ mm}$$

$$d_2 = 270 - 120 = 150 \text{ mm}$$

$$d_3 = 420 - 120 = 300 \text{ mm}$$

$$d_4 = 570 + 150 \\ = 600 \text{ mm}$$

$$x_1 = 620 \text{ mm}$$

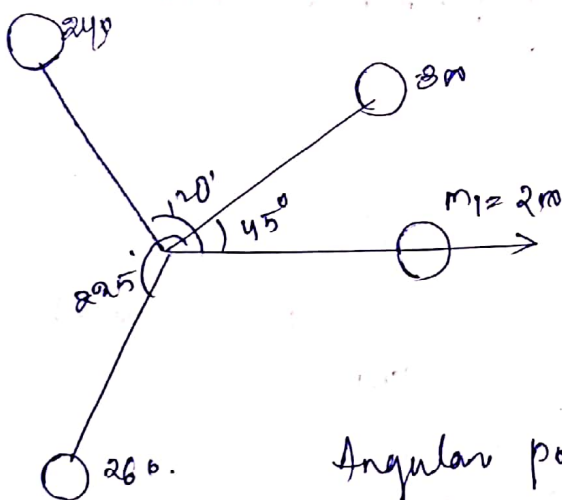
$$x_2 = 620 - 270 = 350$$

$$x_3 = 570 - 300 = 270$$

$$x_4 = 100 \text{ mm}$$

$$d = 500 \text{ mm}$$

$$A1 \text{ \& } A4 = 720 \text{ mm} = 72 \text{ cm}$$



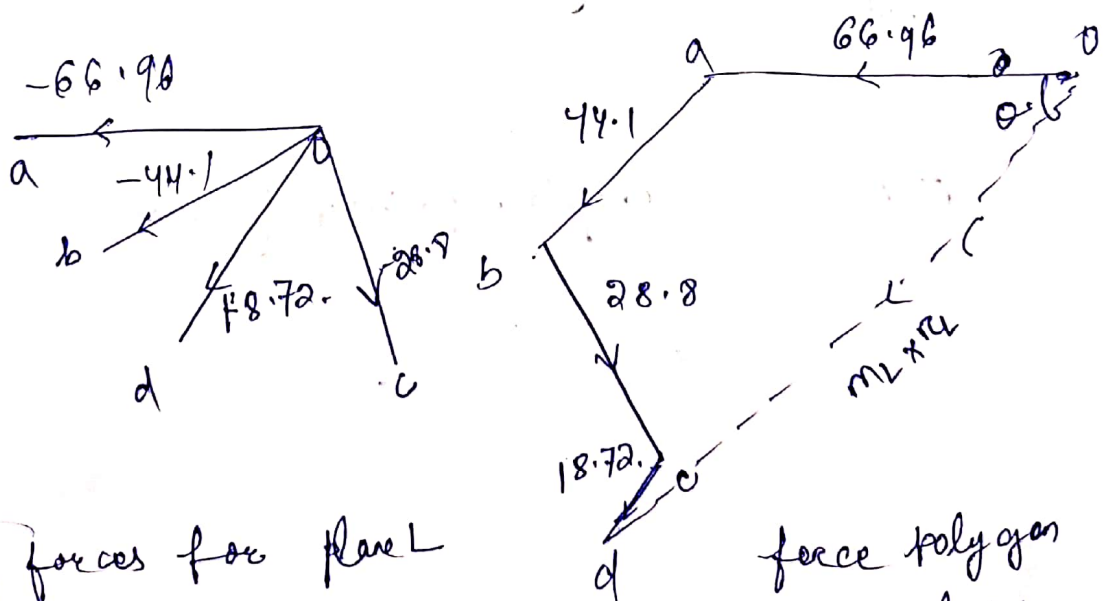
Angular positions of masses.



planes. A	mass (m) (in kg)	radius r (in m)	centrifugal force = $\frac{m r \omega^2}{\omega^2}$	Dist. from		Balancing force =	
				plane L	plane M	plane L $\frac{m \times r \times \omega^2}{\omega^2}$	plane M $\frac{m \times r \times \omega^2}{\omega^2}$
A <sub>1</sub>	200	.27	54 kgm	.12	.62	-66.96	12.96
A <sub>2</sub>	300	.21	63	.15	.35	-44.1	-19.9
A <sub>3</sub>	240	.30	72	.30	.20	-28.8	-43.2
A <sub>4</sub>	266	.36	93.6	.60	.10	18.72	-112.32

Resultant force for plane L.

If the balancing force due to disturbing mass is the 1st column 7., then the direction of balancing force is taken in same direction or if -ve taken in opposite direction.



balancing forces for plane L

force polygon for plane L

suitable scale draw the force polygon for plane L.

$m_L$  = balancing mass for plane L.

$r_L$  = Radius of balancing mass in plane L = .72m

66.96 →

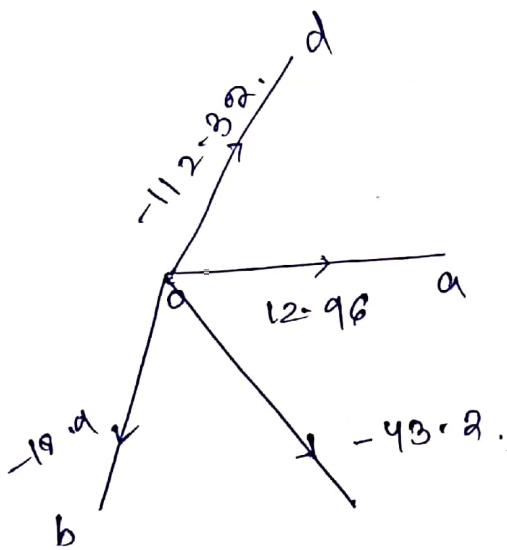
measure  $Od$ ,  $Od = m_L \times r_L$

$$\Rightarrow m_L \times r_L = Od$$

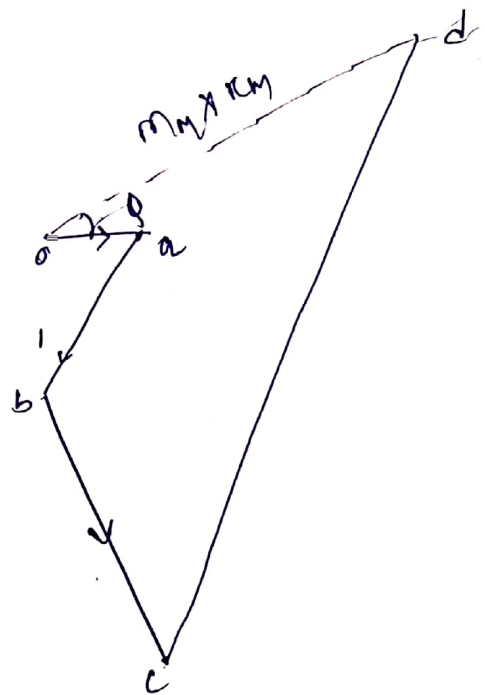
$$\Rightarrow m_L = \frac{Od}{r_L} = ( )$$

measure angle  $\alpha \rightarrow$  from the polygon.

Resultant force for plane M.



balancing forces for plane M.



force polygon for plane M.

Now measure  $Od = m_M \times r_M$

$$\Rightarrow m_M = \frac{Od}{r_M} = ( )$$

measure angle  $\phi$ .

(Longitudinal & Transverse)

Vibrations are due to internal elastic forces within the body. When an elastic body like shaft & springs, which are fixed at one end & displaced at the other end from its equilibrium by the application of an external force. The body starts to move to & fro or up & down, then the body is said to be vibrating. & is said to be in vibration.

Types of vibration

- 1) free or natural vibration
- 2) forced vibration.
- 3) Damped vibration.

1) Free vibration - A vibration in which no. ext force acts on the body, after giving an initial displacement & the motion is maintained by internal elastic forces (when friction & other resistances are neglected) is known as free or natural vibration.  
\* The frequency of free vibration is called natural frequency.

2) Forced vibration - A vibration in which ext. force is applied on the body & the vibrations have the same freq. as the applied force is known as forced vibration.

3) Damped vibration - A vibration in which there is a reduction in amplitude over every cycle of vibration is known as damped vibration. After removing the external force, the energy stored by the body is gradually dissipated in overcoming the internal & external resistance of the body so that the amplitude goes on decreasing.



## Important Definitions for vibrating motion

1. period of vibration or time period - It is the time taken for the motion to repeat itself. It is measured in seconds.
2. cycle - It is the motion completed during one time period.
3. Frequency - No. of cycles completed in one sec. SI unit is Hertz.  
i.e. 1 cycle / 1 sec.
4. Resonance - when freq. of external force is same as that of the natural freq. of the system, resonance takes place. Resonance results in large amplitude of vibration & this may be dangerous.

## Free vibration

In free vibration there is no. ext. force is there. It is divided into 3 types

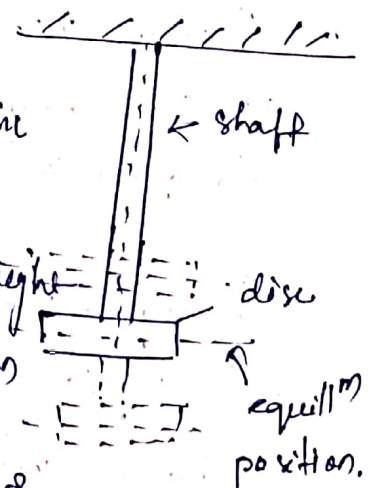
1) Longitudinal

2) Transverse

3) Torsional

### 1) Longitudinal

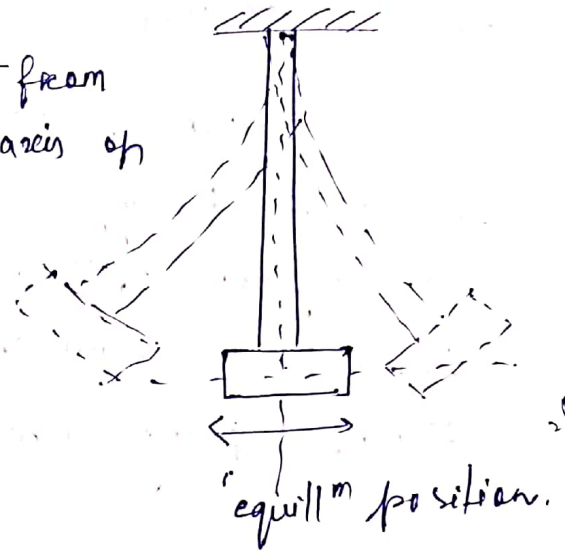
The shaft is assumed to be weightless. The disc is moving up & down as shown in the fig. All the particles of the disc will vibrate along straight path to the axis of the shaft. This type of vibration is known as longitudinal vibration.



In longitudinal vibration the shaft elongates & shortens alternately resulting in compressive & tensile stress in shaft.

## Transverse Vibration

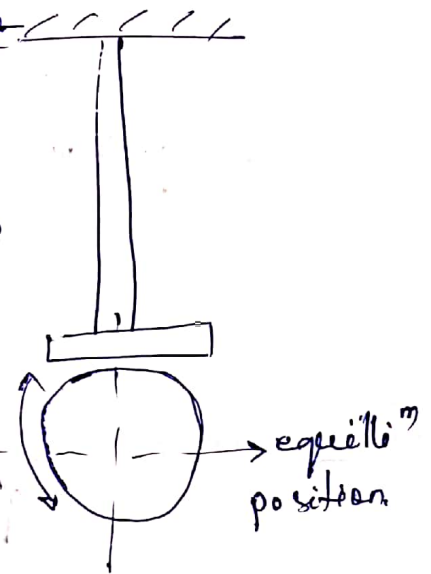
The disc may be given initial displacement from equilibrium position by external force  $\perp$  to the axis of the shaft i.e. horizontally (left or right). This type of vibration is known as transverse vibration. In transverse vibration, the shaft is alternately bent & straight resulting in bending stress in shaft.



## Torsional vibration

The disc may be given initial angular displacement from the equilibrium position by external torque. i.e. it is twisted in a circular manner and then released. All the particles of the disc will vibrate along a circular arc whose centre lies on the axis of shaft.

In torsional vibration, the shaft is alternatively twisted & untwisted resulting in torsional vibration.



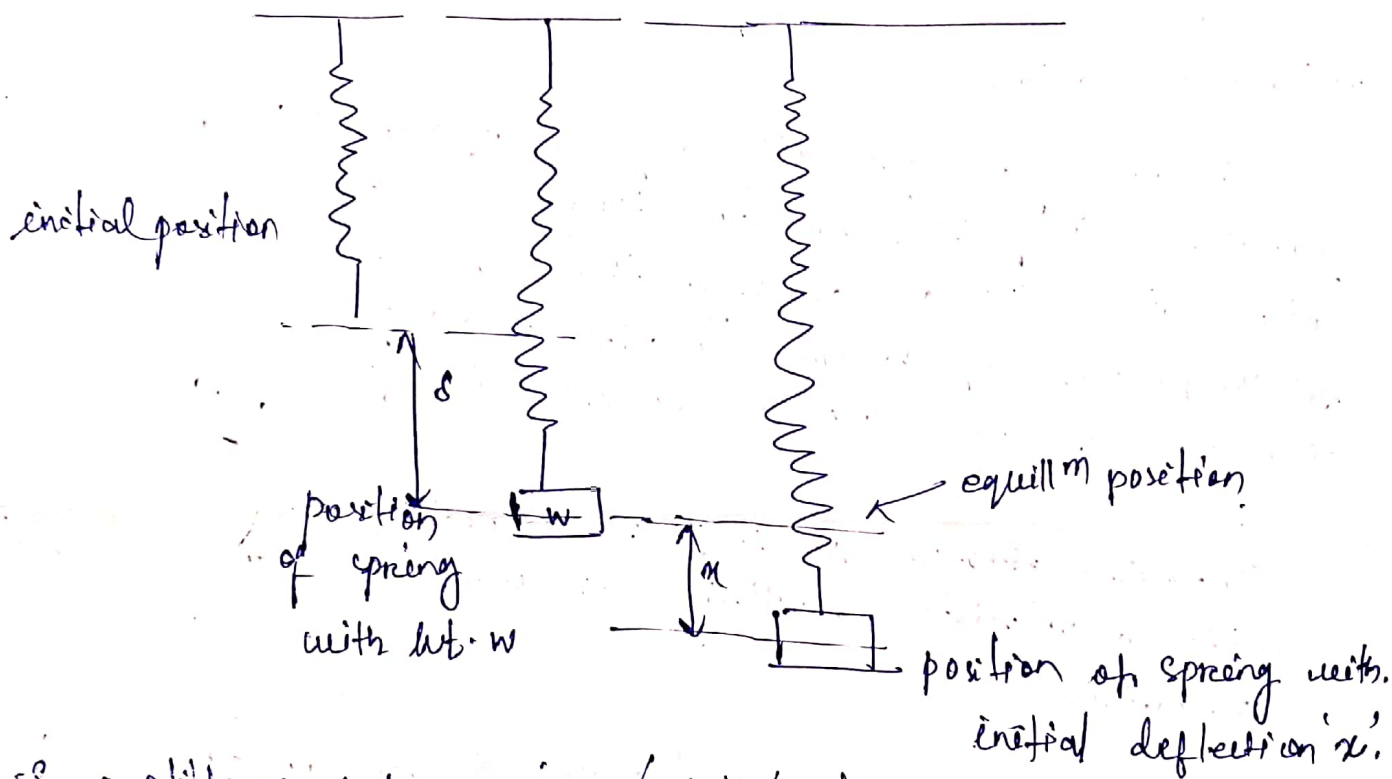
## Methods of finding the natural frequency of free longitudinal vibration

- Equilibrium
- Energy
- Rayleigh's method.

## Equilibrium Method -

the equi

Consider a spring of negligible mass suspended vertically from a rigid support. The other end of spring is free. If weight  $w$  is suspended from the free end, the spring is stretched by a distance  $s$  as shown in fig. The distance  $s$  is known as static deflection of spring under wt.  $w$ .



$s \rightarrow$  stiffness of the spring. / spring constant.  
i.e. the force req. to produce unit deflection.

$m \rightarrow$  mass of wt. suspended  $w = mg$ .

for equilibrium position, wt. suspended = upward force of spring.

$$mg = s \times s$$

Now the wt.  $w$  given an initial displacement  $x$  & then released as shown in fig.



the weight  $w$  is moving downwards & is at a distance of  $x$  from equilibrium position, the net force acting on the weight.

= Downward force - upward force.

$$= W - S(s+x)$$

$$= mg - Ss - Sx$$

$$= Ss - Ss - Sx.$$

$$\boxed{\text{Spring force} = -Sx: \text{--- (1)}} \quad \text{Spring force} = Sx(\uparrow)$$

The net force is equal to the product of mass & accel<sup>n</sup>.

\* Another method

Net force =  $m \times a$  ( $\downarrow$ ) <sup>ve</sup> <sub>inertial</sub>  $\rightarrow$  to D'Alembert's principle  
 internal force =  $m \times \frac{d^2x}{dt^2}$  (2)  
 (Spring force =  $Sx(\uparrow)$ )  
 $\rightarrow$  sum of internal & external force is zero in any dir<sup>n</sup>.

$$\boxed{\text{Inertia force} = m\ddot{x}(\uparrow)}$$

$$\text{or } \frac{md^2x}{dt^2} = -Sx \quad \text{--- (3)}$$

$$\Rightarrow m \frac{d^2x}{dt^2} + Sx = 0. \quad \text{--- (4)}$$

$$\Rightarrow \left[ \frac{d^2x}{dt^2} + \left(\frac{S}{m}\right)x = 0. \right] \quad \text{--- (5)}$$

$$m\ddot{x} + Sx = 0$$

$$\Rightarrow \ddot{x} + \frac{S}{m}x = 0$$

$$\Rightarrow \ddot{x} + \omega_n^2 x = 0$$

$$\omega_n^2 = \frac{S}{m}$$

$$\omega_n = \sqrt{S/m}$$

This eq<sup>n</sup> is similar <sup>to</sup> fundamental eq<sup>n</sup> of simple harmonic.

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0.} \quad \text{--- (6)}$$

from eq<sup>n</sup> (5) & (6)

$$\omega^2 = \frac{S}{m}$$

$$\Rightarrow \omega = \sqrt{S/m}$$

where  $\omega_n$  is known as the natural angular vel, & denoted by  $\omega_n$

$$\omega_n = \sqrt{s/m} \quad \text{--- (5)}$$

$$\rightarrow \text{New the time period } t = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{s/m}} = 2\pi\sqrt{m/s} \quad \text{--- (6)}$$

$$\rightarrow \text{Natural freq } f_n = \frac{1}{t}$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{s}{m}} \quad \text{--- (7)}$$

The natural freq. can also be expressed by static deflection  $\delta$ , by substituting the value of  $s$  in eqn (7)

$$m \cdot g = s \cdot \delta$$

$$\Rightarrow s = \frac{mg}{\delta}$$

$$f_n = \frac{1}{2\pi} \sqrt{\left(\frac{mg}{\delta}\right) \times \frac{1}{m}}$$

$$f_n = \frac{1}{2\pi} \sqrt{g/\delta} \quad \text{--- (8)}$$

$$f_n = \frac{0.4985}{\sqrt{\delta}} \text{ Hz} \quad \checkmark$$

$$E = \frac{\text{Stress}}{\text{strain}} = \frac{\frac{W}{A}}{\frac{\delta}{L}}$$

$$E = \frac{W}{A} \times \frac{L}{\delta}$$

$$\Rightarrow \delta = \frac{W \times L}{EA}$$

$$\Rightarrow \delta = \frac{W \times L}{EA}$$

## Energy Methode : Energy Methode

In free vibrations, the total energy of the system means the sum of P.E & K.E remains const. P.E is taken w.r.t to ~~some~~ certain datum position. Datum position is taken as equilibrium position in case of vibration.

$$K.E + P.E = \text{const.}$$

$$\therefore \frac{d}{dt} [K.E + P.E] = 0. \quad \text{--- (1)}$$

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m \left( \frac{dx}{dt} \right)^2 = \frac{1}{2}m\dot{x}^2$$

P.E = mass force  $\times$  displacement from equilibrium position.

$$= \left( \frac{0 + s \cdot x}{2} \right) \times x$$
$$= \frac{s x^2}{2}$$

$$\left\{ \begin{aligned} \frac{d}{dt} \left[ \frac{1}{2}m\dot{x}^2 + \frac{s x^2}{2} \right] &= 0 \\ \Rightarrow \frac{1}{2}m \cdot 2\dot{x} \cdot \ddot{x} + \frac{1}{2}s \cdot 2x \cdot \dot{x} &= 0 \\ \Rightarrow m\dot{x} + s x &= 0 \end{aligned} \right.$$

Now eqn (1) becomes.  $\Rightarrow \frac{d}{dt} \left[ \frac{1}{2}m \left( \frac{dx}{dt} \right)^2 + \frac{s x^2}{2} \right] = 0$

$$\Rightarrow \frac{1}{2}m \times 2 \cdot \frac{dx}{dt} \times \frac{d^2x}{dt^2} + \frac{1}{2} \times s \times 2x \times \frac{dx}{dt} = 0$$

$$\Rightarrow 2 \cdot \frac{dx}{dt} \left[ \frac{1}{2}m \cdot \frac{d^2x}{dt^2} + \frac{1}{2}s x \right] = 0$$

$$\Rightarrow \boxed{m \cdot \frac{d^2x}{dt^2} + s x = 0}$$

This eqn is same as. eqn is same as eqn no. 2(a).

Hence in this case also the time period & natural freq. will be same as given in equilibrium method.



## Method of Natural freq. of free transverse vibration.

consider a cantilever shaft with one end is fixed & other end is free. Let it is of negligible mass. A wt.  $w$  is attached at the free end, due to wt  $w$  the shaft get deflected by  $\delta$ .



The distance  $\delta \rightarrow$  static deflection under the wt.  $w$ .

$S \rightarrow$  stiffness of the shaft i.e force req. to produce unit deflection in the shaft

$S \cdot \delta \rightarrow$  total force req. to produce  $\delta$  deflection.

Again  $\delta$  is produced due to wt.  $w$

$$\text{So } w = S \cdot \delta \Rightarrow mg = S \cdot \delta \quad \text{--- (1)}$$

the net force acting on the body, when the body is at a distance  $x$  below the equilibrium position.

$\therefore$  Wt. of body - upward resistive force

$$\therefore W - S(x + \delta)$$

$$\therefore mg - S \cdot x - S \cdot \delta$$

$$\therefore -Sx$$

again net force  $\therefore m \cdot a$ .

$$-Sx = m \cdot \frac{d^2x}{dt^2}$$

$$\Rightarrow \frac{m \cdot d^2x}{dt^2} + Sx = 0$$

$$\text{or } \frac{d^2x}{dt^2} + \left(\frac{S}{m}\right)x = 0$$

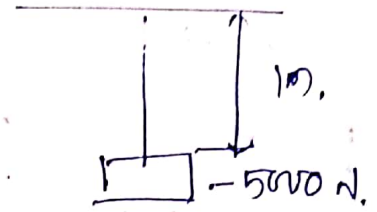
Hence the time period & natural freq. of transverse vibration will be same as that of longitudinal vibrations.

$$T = 2\pi \sqrt{m/S}$$

$$f = \frac{1}{2\pi} \sqrt{S/m} \quad \text{or} \quad \frac{1}{2\pi} \sqrt{g/\delta}$$

for a cantilever shaft loaded at free end - the static deflection  $\delta = \frac{WL^3}{3EI}$

A vertical shaft of 100 mm dia & 1 m in length has its upper end fixed at top. Lower end carries a ~~torque~~ weight of 5000 N. The  $E = 2 \times 10^5 \text{ N/mm}^2$ . Neg. ult. of shaft, determine  $f_n$  for longitudinal & transverse vibrations.



Sol<sup>n</sup>

$$d = 100 \text{ mm}$$

$$L = 1 \text{ m}$$

$$W = 5000$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$= 2 \times 10^{11} \text{ N/m}^2$$

i)

$$f_n = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}}$$

$$\delta = \frac{WL}{AE}$$

$$= \frac{5000 \times 1}{\pi (1)^2 \times 2 \times 10^{11}}$$

$$= 3.183 \times 10^{-6} \text{ m}$$

$$f_n = 279.5 \text{ Hz}$$

ii)  $f_n = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}}$

$$\delta = \frac{WL^3}{8ED}$$

$$= \frac{5000 \times 1^3}{8 \times 2 \times 10^{11} \times \frac{\pi}{64} (1)^4}$$

$$= 1.7 \times 10^{-3} \text{ m}$$

$$f_n = 12 \text{ Hz} \quad \underline{\underline{\text{Ans}}}$$



## Causes of vibration

- 1) unbalanced reciprocating m/c pairs.
- 2) unbalanced rotating m/c pairs.
- 3) Incorrect alignment of the transmission elements like coupling etc.
- 4) use of simple spur gears for power transmission.
- 5) worn out teeth of the gears for power transmission.
- 6) Loose transmission belts & chains.
- 7) Loose fastening of the moving parts.
- 8) vibration due to improper isolation of the m/c.
- 9) Due to more material contact eg. base plates on the foundations for pedestal bearing.
- 10) Non-rigid m/c foundations due to lack of compact soil below which causes misalignment of m/c components.

## Remedies

Vibrations can not be eliminated but can be reduced by,

- partial balancing of reciprocating masses.
- Balancing of unbalanced rotating masses.
- Using helical gears instead of spur gears.
- proper tightening & locking of parts.
- correcting the misalignments of rotating components & checking it time to time.
- Timely replacement of worn-out moving parts, slides & bearings with excessive clearance.
- providing vibration pads in m/c foundations.
- making strong foundations.

# Tool Materials

## **Syllabus:**

- 1.1 Composition of various tool materials**
- 1.2 Physical properties & uses of such tool materials.**

## **Manufacturing technology:**

It is defined as a field of study focused on process techniques or equipments, cost reduction, increased efficiency, enhanced reliability, security safety and anti-pollution measures are its objects.

## **Tool Material:**

The characteristics of the ideal cutting tool material are-

- (a) Hot hardness**
- (b) Wear resistance**
- (c) Toughness**
- (d) Cost and easiness in fabrication**

### **Hot hardness:**

The material must remain harder than the work material at elevated operating temperatures.

### **Wear resistances:**

The material must withstand excessive wear even through the relative hardness of the tool-work materials changes.

### **Toughness:**

The term toughness actually implies a combination of strength and ductility. The material must have sufficient toughness to withstand shocks and vibrations and to prevent breakage.

### **Cost and easiness in fabrication:**

The cost and easiness of fabrication should have within reasonable limits.

### **State the composition of various tool material**

The cutting tool materials are:-

1. Carbon steels
2. Medium alloy steel
3. High speed steels
4. Stellites
5. Cemented carbides
6. Ceramics
7. Diamonds
8. Abrasives
9. Cubic boron nitride(CBN)



### **Composition of carbon steels:**

Carbon steels contain carbon in amounts ranging from 0.008 to 1.5%

### **Composition of medium alloy steel:**

The high carbon medium alloy steels have a carbon content akin to plain carbon steels, but in addition there is, say up to 5% alloy content consisting there of tungsten, molybdenum, chromium and vanadium.

### **Composition of high speed steel:**

High speed steel is the general purpose metal for low and medium cutting speed owing to its superior three type of high speed steel

- 1. High tungsten**
- 2. High molybdenum**
- 3. High cobalt**

Actually these three named modify as following

1. 18-4-1 high speed steel (T-series)
2. Molybdenum high speed steel (M-series)
3. Cobalt high speed steel

### **Composition of 18-4-1 high speed steel (T-series)-**

This steel containing 18% tungsten, 4% cr & 1% vanadium, is considered to be one of the best of all purpose of tool steel.

### **Composition of molybdenum high speed steel (M-series):**

This steel containing 6% molybdenum, 6% w, 4% cr % & 2% vanadium.

### **Composition of cobalt high speed steel:**

This is sometimes called super high speed steel. Cobalt is added from 2 to 15% in increase of this steel contains 20% tungsten 4% cr, 2% v & 12% cobalt.

### **Composition of satellites:**

Satellites is the trade name of a nonferrous cost alloy cobalt, chromium and tungsten. The ranges of elements in these alloys is 40 to 48%, 30 to 35% Cr & 12 to 19% tungsten.

### **Composition of cemented carbides:**

A typical analysis of a carbide suitable for steel machining is 82% tungsten carbide, 10% titanium carbide and 8% cobalt.

### **Composition of ceramics:**

The latest development in the metal cutting tools use Al oxide generally referred to as ceramics

Tools are made by composing aluminium oxide powder in a mould at about  $280 \text{ kg/cm}^2$  or more.

### **Composition of diamonds:**

The diamonds are used for cutting tools are industrial diamonds, which are naturally occurring diamonds.

### **Composition of abrasive:**

Abrasive grains in various forms, loose, bonded into wheels and extended in papers and stony and extended in papers and cloths find wide application in industry. They are mainly used for grinding harder materials and where a superior finish is desired on hardened or unhardened materials.

### **Composition of Cubic Boron Nitride (CBN):**

This material consisting atoms of boron and nitrogen is considered as the hardest tool material available.

- 1. Carbon steel**
- 2. Medium alloy steel**
- 3. High speed steel**
- 4. Cast alloy satellites**
- 5. Cemented carbide tool material**
- 6. oxide or ceramic tool material**
- 7. diamonds**
- 8. abrasives**
- 9. cubic boron nitride(CBN)**

### **Carbon steel:**

#### **Properties**

- I. low hot hardness**
- II. poor hardenability**
- III. can be withstand cutting temperature  $200^{\circ}\text{C}$**
- IV. carbon tool steel are harder than many hss**

**uses:** It can be used most economically under these condition.

- (a) The carbon steels are used for making certain taps and drills.
- (b) For making wood working tools

### **Medium alloy steel:**

#### **Properties**

- i) Better hardenability.
- ii) Higher wear resistance.
- iii) Higher hardness.

#### **Uses**

- i) Used for making drills
- ii) Used for making taps, etc
- iii) It can cut effectively up to temperature  $250$  to  $300^{\circ}\text{C}$ .



## **High speed steel (HSS):**

### **Properties**

- i) **High hot hardness**
- ii) **Cutting tools retain the cutting ability upto 600<sup>0</sup> c.**
- iii) **High wear resistance.**
- iv) **The hardenability is good.**

### **Uses**

- i) **Drills**
- ii) **Broaches**
- iii) **Milling cutters**
- iv) **Lathe cutting tools**
- v) **Taps,etc.**

## **Cast alloy satellite:**

### **Properties:**

- i) **Material is not so hard at room temperature**
- ii) **Hardness above 1000<sup>0</sup>F is greater then high speed steels.**
- iii) **Hat hardness is higher then H.S.S at higher temprature.**
- iv) **This material is very brittle**

### **Uses**

These material are used extensively in some non metal cutting application such as rubbers, plastics.

## **Cemented Carbide:**

### **Properties:**

- i) **High hardness**
- ii) **High heat resistance**
- iii) **High wear resistance**

### **Uses:**

These tool materials are used for machining cast iron, alloy steels.

### **Oxides ceramic tool material:**

#### **Properties:**

- I) The ceramic has extremely high compressive strength. It is quietly brittle.
- II) Heat conductivity is very low. So generally no coolant is required while machining
- III) The ceramic tools can retain strength and hardness upto 1200<sup>0</sup>c.

#### **Uses:**

These tool materials are used for turning boring, etc operations at high speed.

### **Diamonds are cutting tools:**

#### **Properties:**

- i) It has a low co-efficient of friction
- ii) Hardness of the diamond is incompressible.

#### **Uses:**

Diamonds are suitable for cutting very hard material such as glass, plastics, ceramics.

### **Abrasive:**

#### **Uses:**

For most grinding operations there are two kinds of abrasives in general use namely aluminium oxide and silicon carbide. The aluminium oxide abrasive are used for grinding all high tensile materials, where as Silicon Carbide abrasives are more stable for low tensile materials.

# Chapter-2

## Cutting Tools

### Syllabus:

- 2.1 Cutting action of various hand tools such as Chisel, hack saw blade, dies and reamer**
- 2.3 Turning tool geometry and purpose of tool angle**
- 2.5 Machining process parameters (Speed, feed and depth of cut)**
- 2.6 Coolants and lubricants in machining and purpose**

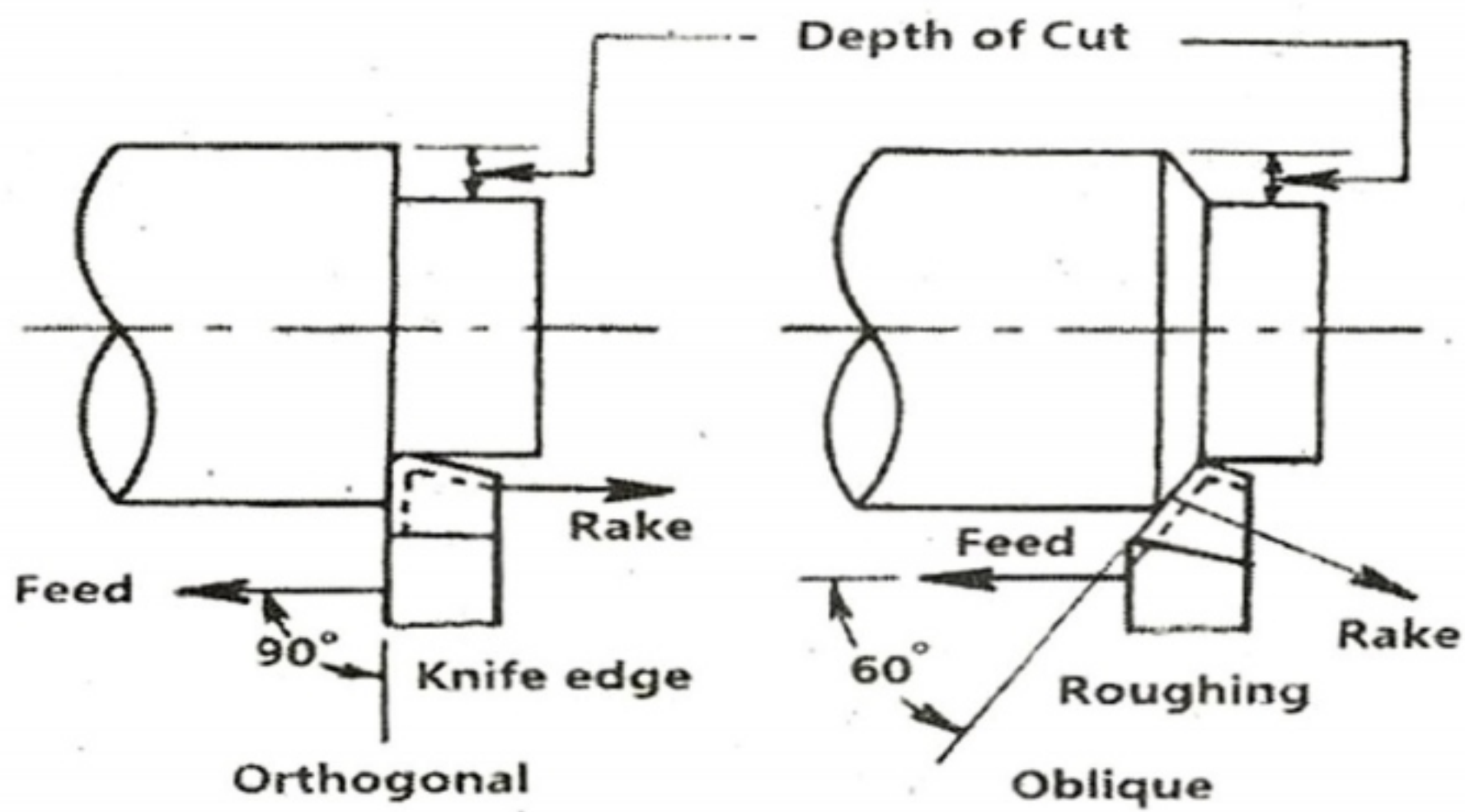
### Cutting tools:

In machining a cutting tool or cutter is any tool which is used to remove the material from the W/P by means of shear difference

Cutting tool must be made of a material harder than the material which is to be cut and the tool must be to withstand the heat generated in the metal cutting process

The angle of cutting facer is also important, also the tool must have a specific geometry and clearance angles designed so that the cutting edge can contact the W/P surface .





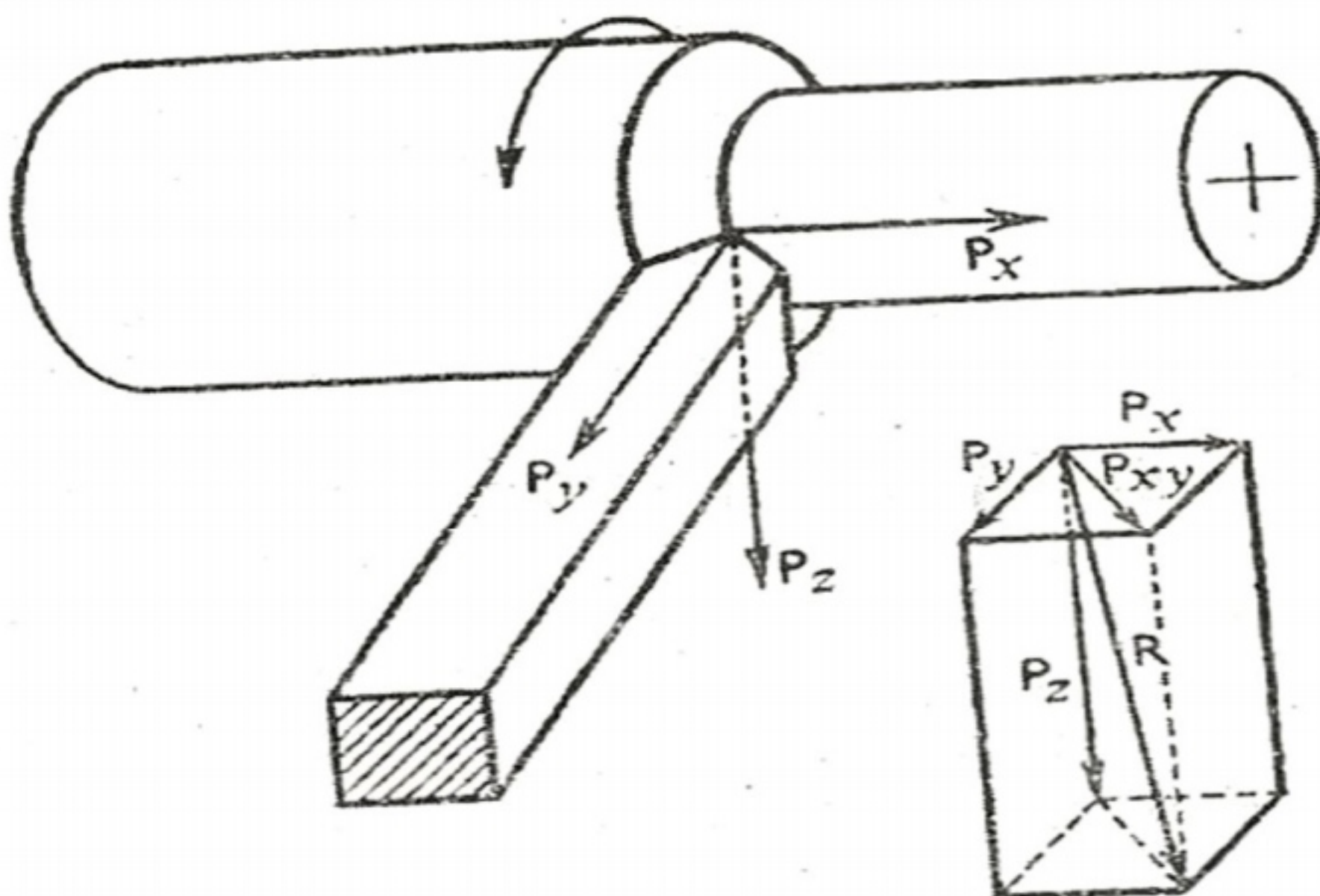
**Figure Orthogonal and oblique cutting**

### Single point cutting tool

This type of cutting tools have only one cutting edge. These used for wide application of lathe, shaper planner, slitter, boring M/C

### Multi point cutting tools

This type cutting tools have more than cutting edge. These are employed for wide application in twist drills, Reamers, tapes, milling cutters etc.



**Figure Cutting forces in conventional turning process**



## Cutting action of hand tools

### Chisel:

A chisel is a hand cutting tools which is shaped cutting edge of blade on its end, forcarving, cutting a hard material such as wood, stone, metal by hand with the help of mechanical power.

In used the chisel are forced in to the material to linear relative motion.

The driving forced into the material may be manually applied by using a hammer.

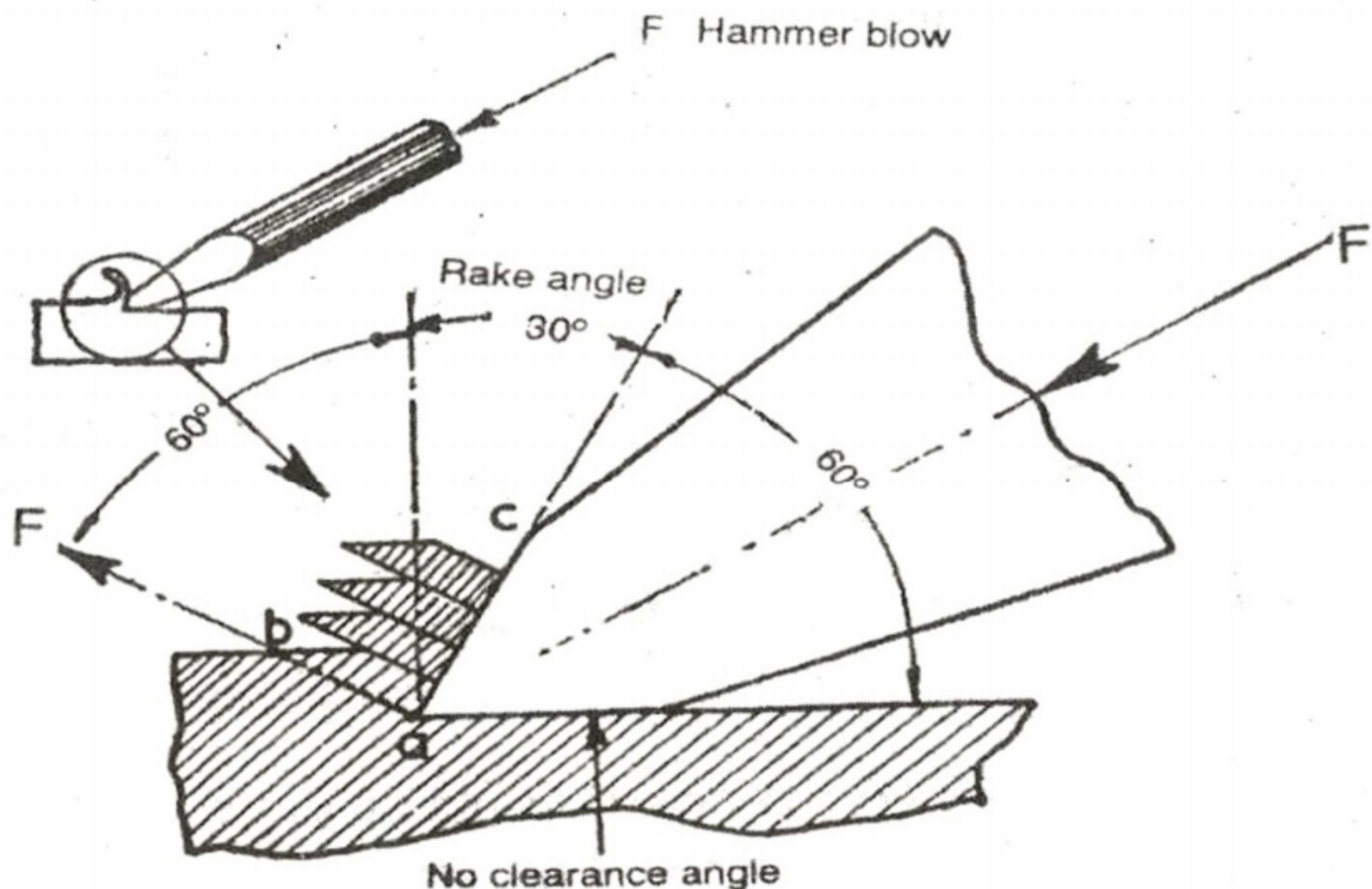
In industrial use, a hydraulic ram or falling weight drives the chisel into the material to be cut .

Chisel is employed to use in wood work, metal working etc.

In wood & stone working used for carving, cutting, shaving, shaping, trimming.

In metal working process chisel use divided into two categories:

### Cold chisel:



Figure

Shearing action of a cold chisel

It is made of from tempered steel.

Use for cutting cold metal.

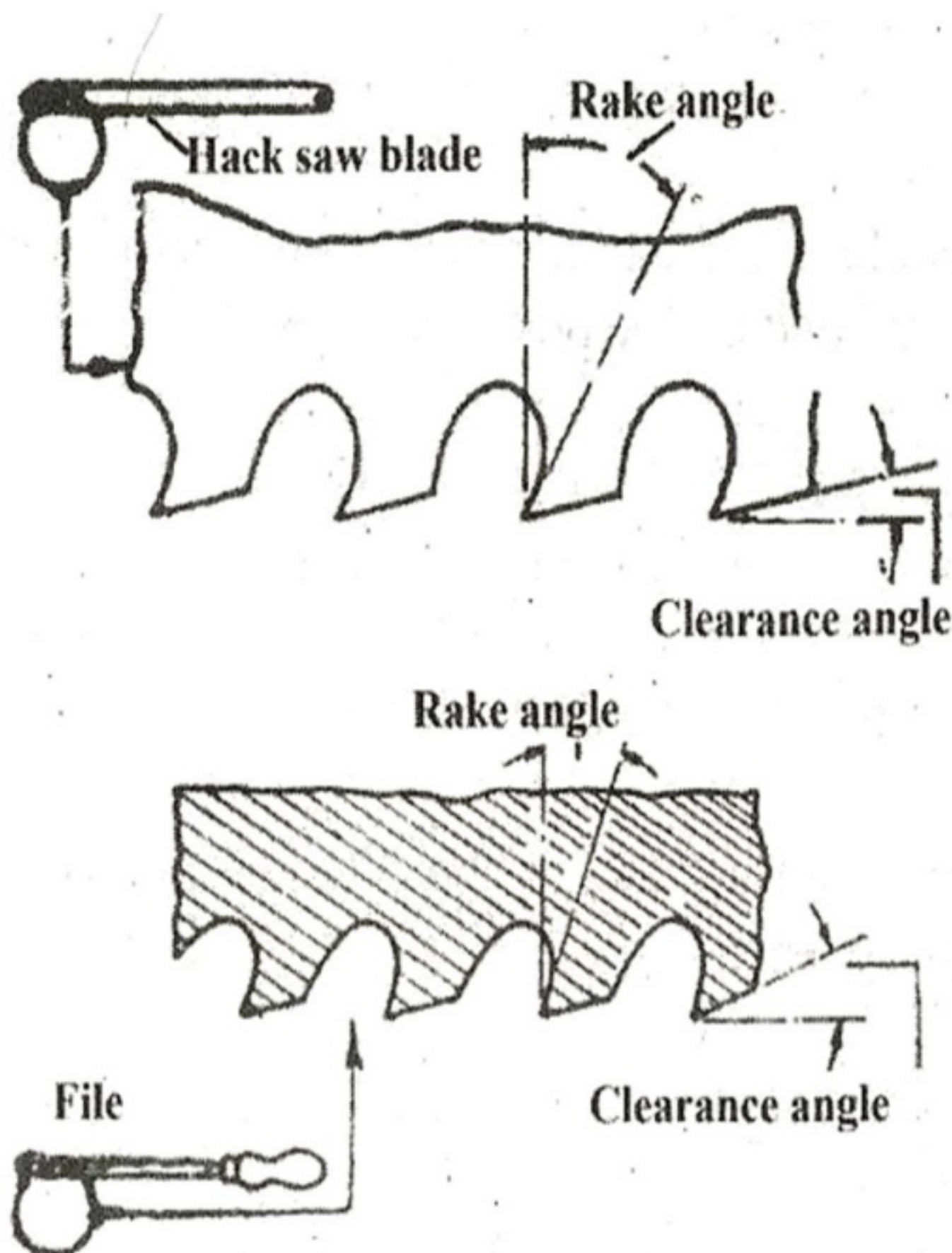
Used to remove waste metal in the situation where a smooth finish is not necessary or when other tools such as file, hacksaws cannot be used .

### **Hot chisel:**

A hot chisel is used to cut metaln that has been heated in a force to sustain the metal.

Used to smooth the metals.

### **Hacksaw blade**



**Figure Rake and clearance angles on hack saw blade and file**



When attached to a C-shaped frame which holds a blade under tension.

The frames may be adjustable to accommodate blades of different sizes.

Blades are available in standardized lengths, usually 10<sup>11</sup> or 12<sup>11</sup> for a standard hand hack-saw.

The pitch of the teeth can be anywhere from 14 to 32 per inch for a hand blade & for large power hack saw blade there are 3 tpi

As hack-saw teeth are so small, they are set in a wave set.

As the blades are normally quite brittle, so proper care should be taken to prevent fracture of the blade.

Blades are made of carbon steel or low alloy steel.

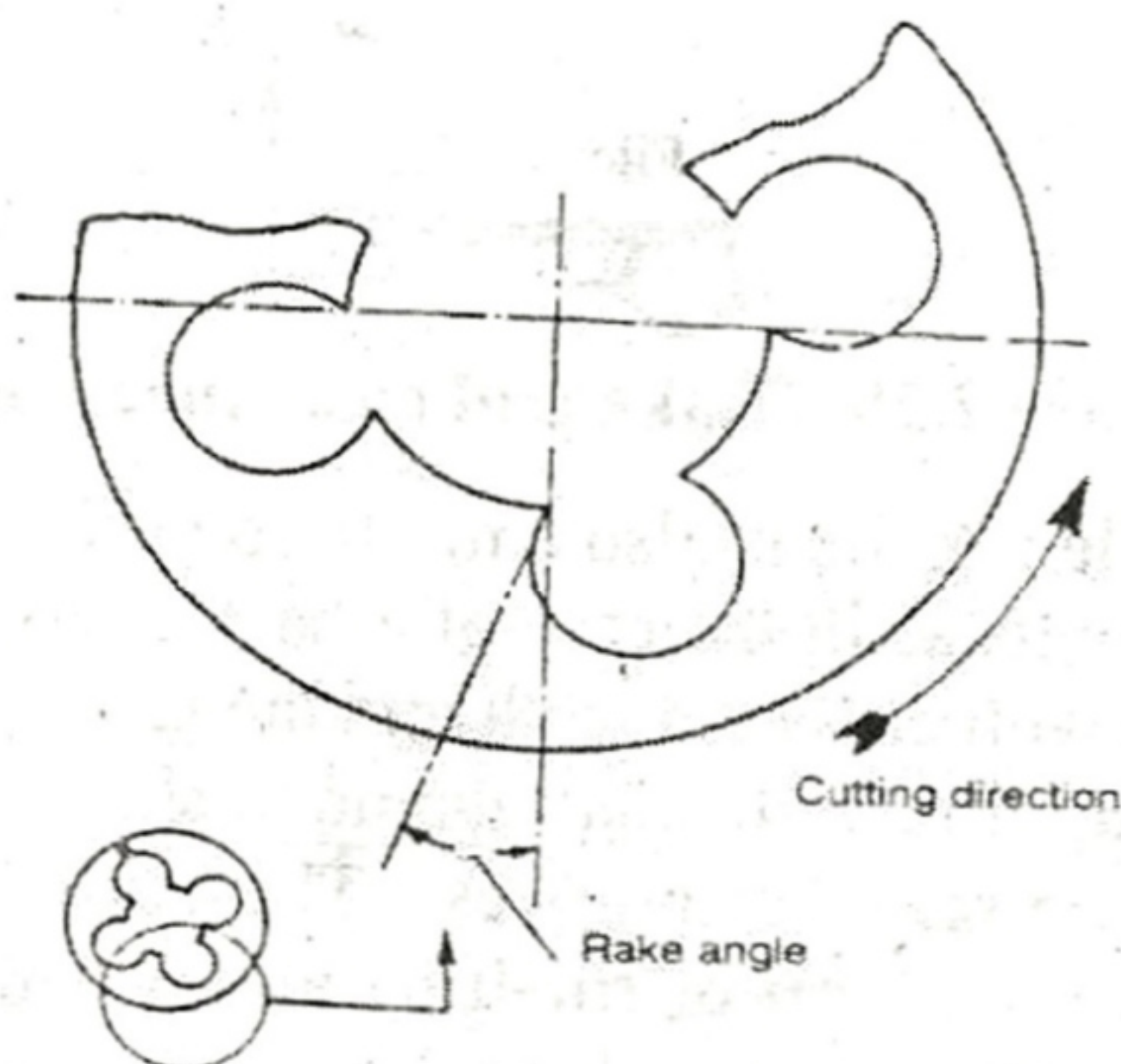
But for several decades now, hack-saw blades have used HSS for their teeth, giving great improved cutting & tooth life.

On hack-saw the blade can be mounted with the teeth facing toward or away from the handle

Resulting and cutting action on either pushes or pull stroke.

In normal use, cutting vertically downwards with work held in a bench, vice, the saw blade Should be set to be face forward.

### Die:



**Figure**

**Rake angle on a die**

Die cutting is the process of using die to shear work of low strength material such as rubber, timber, cloth, plastic, sheet metal etc.

Die cutting can be done on either flat bed or by rotary process .

Rotary die cutting is die cutting using a cylindrical die or a rotary processes .

Dies are used to cut the external thread or the rod or pipe end .

Dies are made of high carbon steel or HSS .

The process of cutting external thread by dies is called dieing .

Shaping is also known as die cutting, is a process which cuts stock without formation of chips or the off during or melting.

The die cutting action can be controlled by electric, hydraulic, pressurized or manual surfaces.

### **Reamer:**

- It is a multiple edge cutting tools.
- The process of enlarging the hole is called reaming.
  - There are many different types reamer and there may be designed for used as a hand tool or in a M/C tool such as milling M/C or drill press.
  - A typical reamer consists of a set of parallel straight or helical cutting edge along the length of a cylindrical body
  - Each cutting edge is grounded at a slight angle and with slight under cut below the cutting edge
  - This may be used to remove small amount of material.
  - Reamers are made of high Carbon or Plain Carbon

Steel

Reamers are of two types

- Hard Reamers
- Machine Reamers

## **Machining Process Parameters:**

Factors affecting tool life:

The life of a tool is affected by many factors such as: cutting speed, feed, depth of cut, chip thickness tool geometry, material of cutting fluid, and rigidity of the machine

### **Cutting Speed:**

The cutting speed can be defined as the relative surface speed between the tool and the job or the amount of length that will pass the cutting edge of the tool per unit of time.

It may be defined as the speed which the cutting edge pass over the material. It is expressed in meters per min (mpm).

### **Feed:**

It is defined as the relation by small movement per cycle of the cutting tool, relative to the workpiece in a direction which is usually to the cutting speed direction.



It is the distance the tool advances into or along the work piece. Each time the tool point passes a certain position in its travel over the surface. It is expressed as mm/tooth.

### **Depth of cut:**

The depth of cut is the thickness of the layer of metal removed in one cut or pass, measured in a direction  $\perp$  to the machined surface.

It is the vertical distance the tool advances into the work piece during one revolution of job it is expressed in mm.

### **Selection of cutting speed, feed & depth of cut:**

- Hard and strong materials require a lower cutting speed, soft & ductile material require higher cutting speeds.
- For light finishing cut – fine feed & higher speed roughing cut – low feed & lower cutting speed.
- Large depth of cut – roughing operation
- Small depth of cut – finishing operation
- Cemented carbide, ceramics, satellite &
- Hss – high cutting speed tool
- Alloy or carbon steel tools – lower cutting speed.

### **Coolants & lubricants:**

Cutting fluid sometimes referred to as lubricants or coolants are liquids and gases applied to the tool and work piece to assist in the cutting operations.

### **Purpose of cutting fluid:**

- To cool the tool
- To cool the work piece
- To lubricate and reduce friction
- To improve surface finish
- To protect the finished surface from corrosion
- To cause chips break up into small parts
- To wash the chips away from the tool

### **Properties of cutting fluids:**

1. High heat absorption for readily absorbing heat developed.
2. Good lubricating qualities to produce low-coefficient of friction.
3. High flash point so as to eliminate the hazard of fire
4. Stability so as not to oxidize in the air
5. Neutral so as not to react chemically
6. Colorless so as not to produce any bad smell even when heated.
7. Harmless to the bearings.
8. Harmless to the skin of the operators
9. Non-corrosive to the work or the machine
10. Transparency so that the cutting action of the tool may be observed.
11. Low viscosity to permit free flow of the liquid

1. Type of operation
2. The rate of metal removal
3. Material of the work piece
4. Material of the tool
5. Surface finish requirements
6. Cost of cutting fluid.

### **Type of cutting fluids:**

#### **Water:**

Pure water is the best cutting fluid available because of its highest heat carrying capacity. But water corrodes the material very quickly so water containing alkali, salt or water-soluble additive but little or no oil or soap are some times used as coolant.

#### **Soluble oils:**

These are emulsions composed of around 80% or more water, soap & mineral oil. The soap acts as an emulsifying agent which breaks the oil into minute particles to disperse them throughout the water. The water increases the cooling effect and the oil provides the lubricating properties.

#### **Straight oils:**

The straight oils may be

- a) Straight mineral oils, kerosene, low-viscosity petroleum fraction such as mineral seal, or higher viscosity mineral oils
- b) Straight fixed or fatty oils consisting animal, vegetable or synthetic equivalent, lard oil etc.
- c) They have both cooling and lubricating properties



## **Mixed oils:**

This is a combination of strength mineral and strength fatty oil. This makes oil excellent lubricant and coolant for atmospheric screw-machine work.

Chemical additive oil: straight oil or mixed oil when mixed- up with sulphur or chlorine is known as chemical additive oil. Sulphur and chlorine are used to increase both lubricating and cooling qualities. These oils used for machining tough, sticky low carbon steels.

Chemical compounds: these compounds consist mainly of a rust inhibitor, such as sodium nitrate, mixed with a high percentage of water.

Solid lubricants: stick waxes. And bar soaps are sometimes used as lubricants.

Metal cutting and cutting tools : in the metal working industry the various working processes fall into groups.

Not-cutting shaping – forging, pressing, drawing

Cutting shaping – turning, drilling, milling.

## **Cutting tools**

A cutting tool may be used either for cutting a part or for removing chips.

Cutting tools are mainly divided into two groups.

- Single point cutting tool
- Multipoint cutting tool

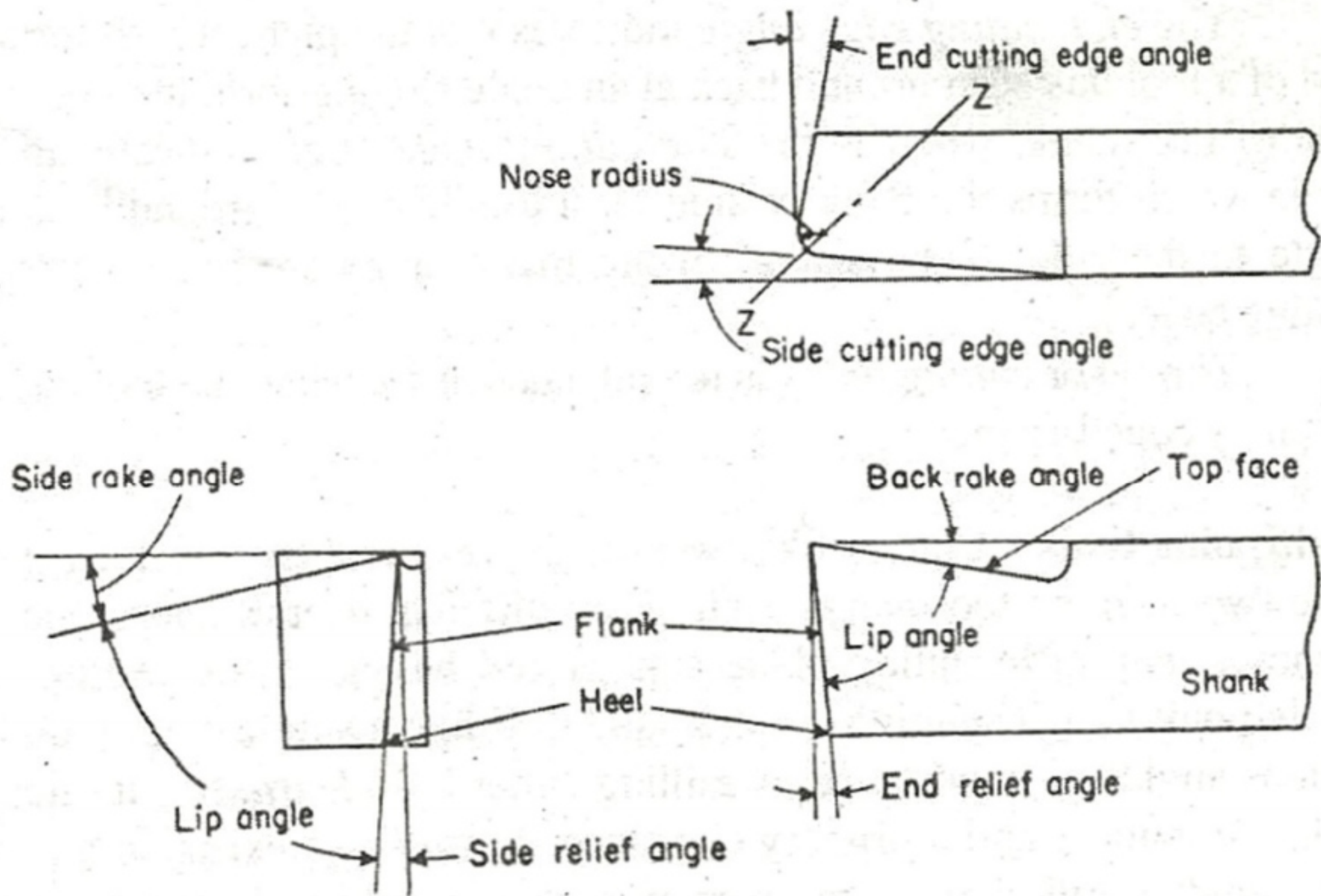
A single point cutting tool consists of a sharpened cutting part called its point.

Ex: lathes, slotting machines

Multipoint cutting tools have arrangement of two or more single point tools as a unit.

Ex – milling cutting, broaching tool, twist drill.

It means schematic naming of the various parts and angles of a cutting tool.



Figure

Tool nomenclature and tool angles

### **Shank:**

It is the main body of the tool.

### **Flank:**

The surface or surfaces below and adjacent to the cutting edge is called flank of the tool.

### **Heel:**

It is the intersection of the flank and the base of the tool.

### **Nose:**

It is the point where the side cutting edge and end cutting edge intersection.

## **Cutting edge .**

It is the edge on the face of the tool which removes the material from the work piece. The total cutting edge consists of major edge (major), end cutting edge (minor) and the nose.

## **Face:**

The surface against which the chip slides upward.

## **Base:**

It is the underside of the shank.

## **Rake:**

It is the slope of the top away from the cutting edge. Larger the rake angle, the cutting force and power reduce.

Designation of cutting tools: there are two systems to designate the tool shape

### **1. American standards association system (ASA)**

Or

American national standards institute (ANSI)

### **2. Orthogonal rake system. (ORS)**

The various tool angles are:

#### **1. Side cutting edge angle (Cs): (Lead angle)**

The angle between the side cutting edge and side of the tool shank

#### **2. End cutting edge angle (Ce):**

This is the angle between the end cutting edge and a line normal to the tool shank



## **Side relief angle:**

It is the angle between the portion of the side flank immediately below the side cutting edge and a line perpendicular to the base of the tool measured at right angle to the side flank.

## **End relief angle**

It is the angle between the portion of the end flank immediately below the side cutting edge and a line perpendicular to the base of the tool measured at right angle to the end flank.

## **Back rake angle:**

It is the angle between the face of the tool and a line parallel to the base of the tool and measured in a plane perpendicular to the side of the cutting edge. The angle is +ve – If side cutting edge slopes downwards from the point towards the shank.

-ve – if the slope of the side cutting edge is reverse.

## **Side rake angle :**

It is the angle between the tool face and a line parallel to the base of the tool and measured in a plane perpendicular to the base and side cutting edge. This angle gives slope of the face of the tool from the cutting edge.

The angle is – ve – if the slope is towards the cutting edge

+ve - If the slope is away from the cutting edge

# Chapter-3

## Lathe Machine

### Syllabus:

#### **3.0 Lathe Machine**

**3.1 Construction and working of lathe Major components of a lathe and their function**

**Operations carried out in a lathe (Turning, thread cutting, taper turning, internal machining, parting off, facing, knurling)**

**Safety measures during machining**

#### **3.2 Capstan lathe**

**Difference with respect to engine lathe**

**Major components and their function**

**Define multiple tool holders**

#### **3.3 Turret Lathe**

**Difference with respect to capstan lathe**

**Major components and their function**

**3.6 Draw the tooling lay out for preparation of a hexagonal bolt & bush**

## **Lathe machine:**

The lathe machine is the one of the oldest machine tools and came the early tree lathe which was a device for rotating and machining a piece of lathe between two adjacent trees. a rope would round the work with its one end attached to a flexible branch of trees and end is pulled by a man to rotate the job hard tools are used them.

### **Function of lathe machine:**

The main function of lathe machine is to remove metal from a piece of work to give it the required shape and size. the work is held securely and rigidly on the machine and then turn against the cutting tools which is remove metal from the work in the forms of chips.

### **Types of lathe:**

- **speed lathe:**
  - **Wood working**
  - **Centering**
  - **Polishing**
  - **Spinning**
- **Engine lathe:**
  - **belt drive**
  - **Individual motor drive**
  - **Gear head lathe**
- **Bench lathe:**
- **Tool room lathe**
- **Caps & turret lathe**
- **Special purpose:**
  - **Wheel lathe**
  - **Gap bed lathe**
  - **Turret lathe**
  - **Duplicating lathe**
- **Automatic lathe:**



### **The speed lathe:**

- It is the simplest of all types of lathe.
- It consists of a bed, headstock, a tailstock and a tool post mounted on an adjusted slide.
- There is no feed box, lead screw or conventional type carriage.
- The tool is mounted on the adjustable slide and is fed into work purely by hand control.
- Spindle speed is very high.(range from 1200 to 3600 rpm).
- This is used for work requiring, spinning, centering, polishing.

### **The engine lathe or centre lathe:**

- The engine lathe in the early days were driven by steam engines
- It consists of bed, head stock, and tail stock.
- More robust head stock and contains mechanism for driving the spindle at multiple speeds.
- Belt driving lathe – receives power from an overhead live shaft
- Individual motor drive- receiving power from an individual motor
- Gear head lathe –gets power from a constant speed motor.

### **The bench lathe:**

- The small lathe mounted on a bench
- It consists of all the parts but small in size.
- It is used for small & precision work.

### **The tool room lathe:**

- It is similar to engine lathe.
- It has spindle speeds ranging from very low to high upto 2500 rpm.
- It consists of chuck, taper turning attachment, thread chasing dial, steady rest, coolant etc.
- This is used for precision work on tools, dies, gauges & for accuracy works

multiple tools are fitted

- Several operations can be done on a work piece without resetting of work or tools & a no. of identical parts can be produced in minimum time.

### **Special purpose lathe:**

- They are used for special purposes.
- The Wheel lathe – for finishing journals & turning the thread on rail road car and locomotive wheels gap bed lathe- to swing extra large diameter pieces.
- T-lathe – for machining of motor for jet engines the axis of the lathe bed is right angles to the axis of the head stock spindle.
- Duplicating lathe – for duplicating the shape of a flat or round template on the work piece

### **Automatic lathe:**

- These are high speed, heavy duty, mass production lathes with complete automatic control.
- After the tools are set and the machine is started it performs automatically all the operations to finish the job.
- The changing of tools, speeds and feeds are done automatically.
- After the job is complete, the machine will continue to repeat the cycles producing identical parts.



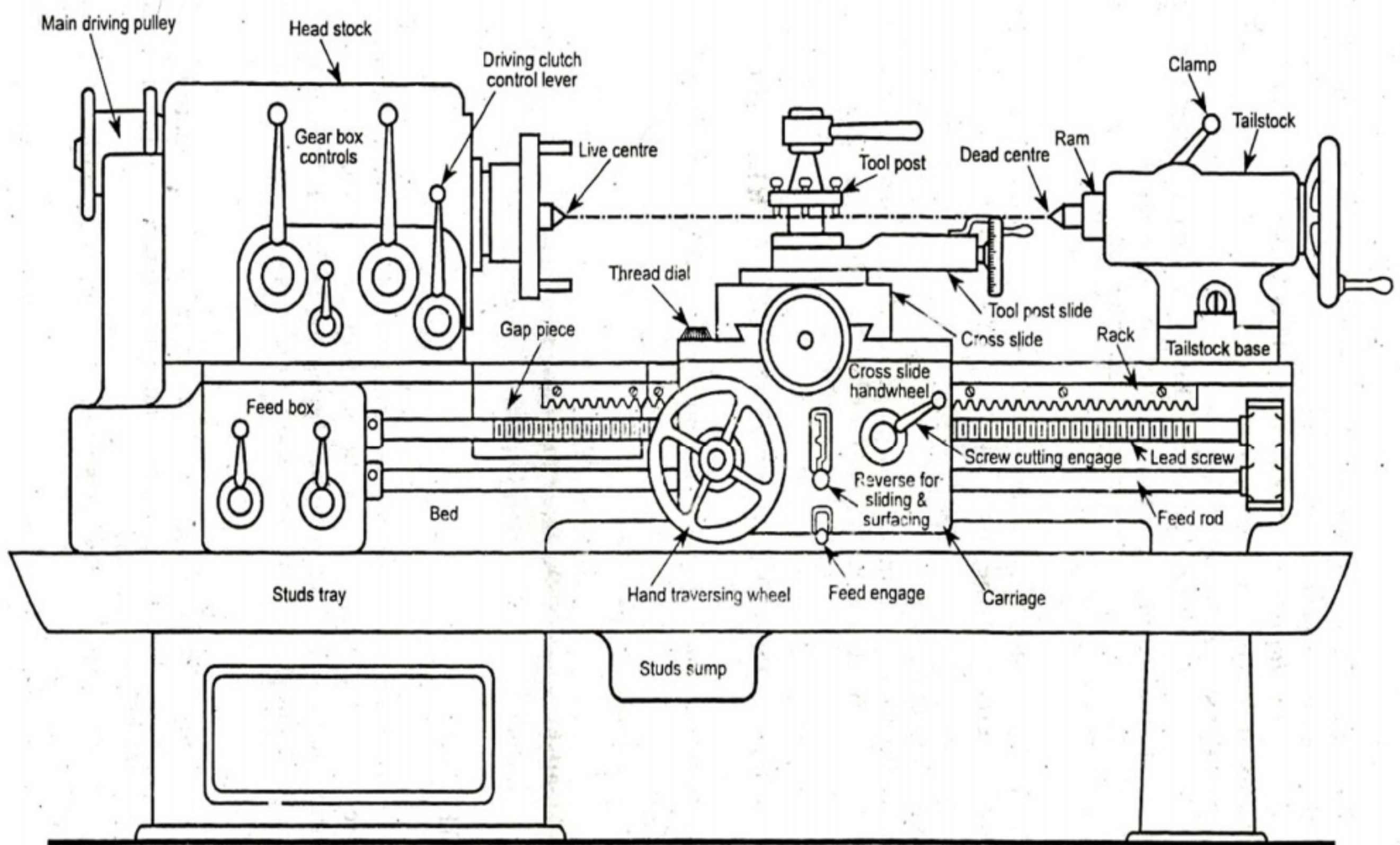


Figure 3.2 Lathe parts

1. Bed
2. Head stock
3. Tail stock
4. Carriage
5. Feed mechanism
6. Screw cutting mechanism

## BED:

The bed is the base or foundation of the lathe. It is made of cast iron. It is a massive and rigid casting made in one piece to resist deflection and vibrations. It supports the head stock, tail stock and carriage. On the top of the bed, there are two sets of slides or guide ways. The outer ways for carriage and inner ways for the tail stock. The guide ways may be flat and inverted –v having included angle of  $90^0$



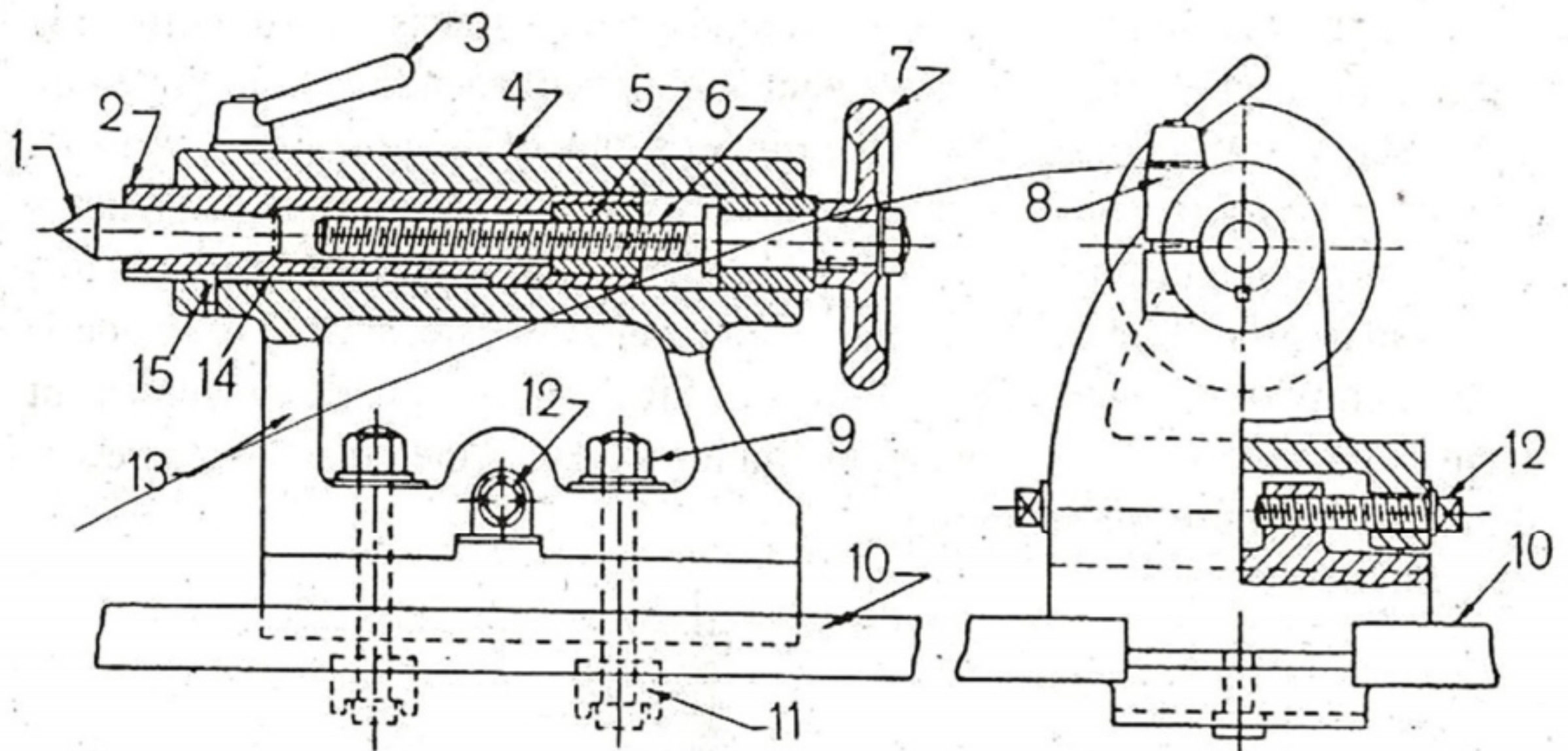
## **Head stock:**

The head stock assembly is permanently fastened to the left hand end of the lathe. It provides mechanical means of rotating the work at multiple speeds.

The spindle of head stock is made of carbon or nickel chrome steel. It protrudes from gear box and contains means for fast ending work holding derives like chuck, face plate, dog plats, live centred spindle nose is turned so that face plate or chuck can be mounted on it hold and rotate the work piece. Hollow spindle is tapered at the nose to receive the live centre.

## **Tail stock or loose head stock:**

The tail stock is located at on the inner ways at the right hand end of the bed and it is a non-rotating part which slides and can be clamped to it. Any position to accommodate different lengths of work pieces.



**Figure Tailstock**

1. Dead centre, 2. Spindle, 3. Spindle clamp, 4. Barrel, 5. Bush, 6. Square threaded screw, 7. Hand wheel, 8. Split lug, 9. Tailstock clamping bolt, 10. Lathe bedways, 11. Clamping plate, 12. Setover screw, 13. Body, 14. Keyway, 15. Key.

It has two uses:

1. Supports the other end of the work when it is machined between two centres.
2. To hold a tool for per forming operations such as drilling, reaming, tapping etc.

## **CARRIAGE:**

In between the head stock and tail stock is the carriage. It is movable on the bed ways and its purpose is to hold the cutting tool and to impart to it either longitudinal or cross feed. It consists of the following parts:

**1. Saddle:**

It is an H-shaped casting that fits over the outer ways of the bed. It carries the cross-slide and tool post.

**2. Cross slide:**

It is mounted on saddle. It provides cutting tool motion which is to the centre like of the lathe. This is known as cross slide can be moved by means of feed screw, which is controlled by a small hand wheel or by power feed.

---

**3. Compound rest:**

It is mounted on the top of the cross slide. It supports the tool post. It has a graduated base and can be swivelled around a vertical axis. It can be swivelled around a vertical axis. It can be moved by means of a screw which is controlled by a small hand wheel and graduated dial & not by power feed.

**4. Tool post:**

It is mounted on the top of the compound rest to hold the tool. The tool post can be moved on the compound rest and can be clamped in any position. It can be rotated also to hold the cutter in desired angle. 1. single tool post 2. Four bolt 3. Open side 4. Four way



## **Lathe operation:**

Lathe operations are performed by following methods.

Operations which are performed in a lathe either by holding the work piece between centers or by a chuck:

- 1. Straight turning**
  - 2. Shoulder turning**
  - 3. Chamfering**
  - 4. Thread cutting**
  - 5. Facing**
  - 6. Knurling**
  - 7. Filling**
  - 8. Taper turning**
- 
- 9. Eccentric turning**
  - 10. Polishing**
  - 11. Grooving**
  - 12. Spinning**
  - 13. Spring winding**
  - 14. Forming**

, using a torved gauge the do is given by compound slide and the to read is finished in the usual manner.

### **Safety measures during machining:**

Some safety precautions should be needed while working on lathe.

- Before operating the machine ,one should fully understand its operations controls and how to stop it .
- All gears and gear ends of the lathe should be properly guarded .
- Safety goggles are preferred to avoid damage to eyes by flying chips.
- Avoid wearing rings, bracelet or watch.
- Machine should not be left running and opperater should be alert during a job.
- Before starting a lathe spindle by power , spindle should be rotated by one revolution by hand to make it sure that no fouling is there.
- Safe distance from revolving chuck should be maintained.
- Tools and instruments should not be placed over lathe bed.
- Sliding parts of the lathe should be cleaned and lubricated periodically.
- Chips should never be removed by hand. It can be removed by brush.
- Before starting the machine, the work should be clamped properly.
- Before moving the carriage, the carriage clamping screw should be unlocked.
- On hearing unusual noise, machine should be stopped immediately and should not be operated till the fault is clear.

## Capstan & Turret lathes:

A capstan or a turret lathe is a production lathe used to manufacture any number of identical pieces in the minimum time. The main feature is the six sided block mounted on one end of the bed replacing the normal tailstock six tools can be mounted at on cross slide two tool posts are mounted, one in the front and the other in the rear. Each one can hold four tools .Thus the total carrying capacity is a maximum 14 tools

### Difference between CAPSTAN &TURRET and an ENGINE LATHE

CAPSTAN &TURRET	ENGINE LATHE
1. The head stock possesses wider range of speeds and in heavier in construction it require 15 hp power to drive the spindle.	1. It requires 3hp to drive the spindle.
2. The tool post mounted on the cross slide is a four way & a rear tool post is mounted on the rear side which also holds 4 tools.	2. In engine lathe one tool can be mounted at one time for different operation.
3. In turret lathe, the tail stock is replaced by a turret which is a hexagonal block which contains 6 tools on each face.	3. It can accommodate one tool of limited size.
4. The feed movement of each tool set on square or hexagonal turret is regularity by stops & feed strips.	4. The feed movement is given by hand.
5. Combination cuts can be taken by mounting two or more tools on the same face of the turret.	5. Combination cuts can not be done.
6. The labour cost is less.	6. Labour cost is more.



7. The threads are cut by dieheads & taps.	7. The threads are cut by lead screws Centre lathe is suitable for odd jobs having different shapes & sizes.
8. Turret lathes are suitable for producing large no. Of identical pieces.	8. The threads are cut by lead screws.

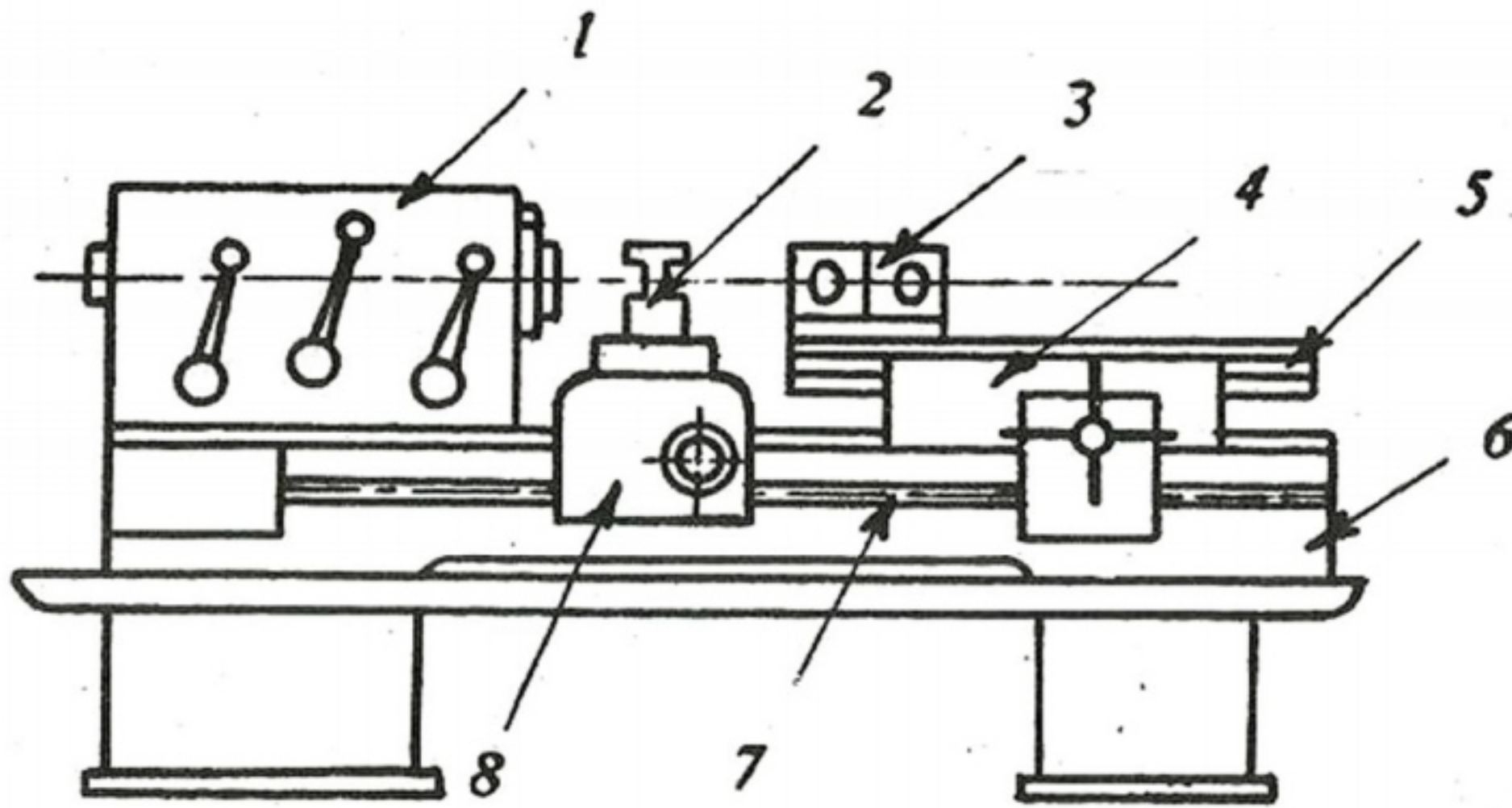
### **Difference between capstan & turret lathe:**

Capstan lathe	Turret lathe
<ol style="list-style-type: none"> <li>1. Its turret head is mounted in slide, which moves on the guide ways produced on the saddle.</li> <li>2. For feeding the tool to the work, the saddle is fixed at convenient distance from the work.</li> <li>3. It is suitable for smaller size &amp; lighter jobs. It is not suitable for heavy cutting condition.</li> <li>4. It is suitable to work for smaller bar upto 60 mm dia.</li> <li>5. The turret head may be hexagonal or circular.</li> <li>6. It is smaller in size compared to turret lathe.</li> <li>7. The tool traverse is faster and offer less fatigue to the hands of the operator.</li> </ol>	<ol style="list-style-type: none"> <li>1. Its turret head is mounted directly on the saddle.</li> <li>2. For feeding the tool to the work, the entire saddle unit is moved.</li> <li>3. It is suitable for long and heavy work and severe cutting condition.</li> <li>4. It is used to work for large size bar upto 200mm dia.</li> <li>5. Turret head is hexagonal.</li> <li>6. It is large in size as compared to capstan lathe.</li> <li>7. The tool feeding is relatively slower and provide more fatigue to operator hands.</li> </ol>

## Parts of capstan and turret lathe:

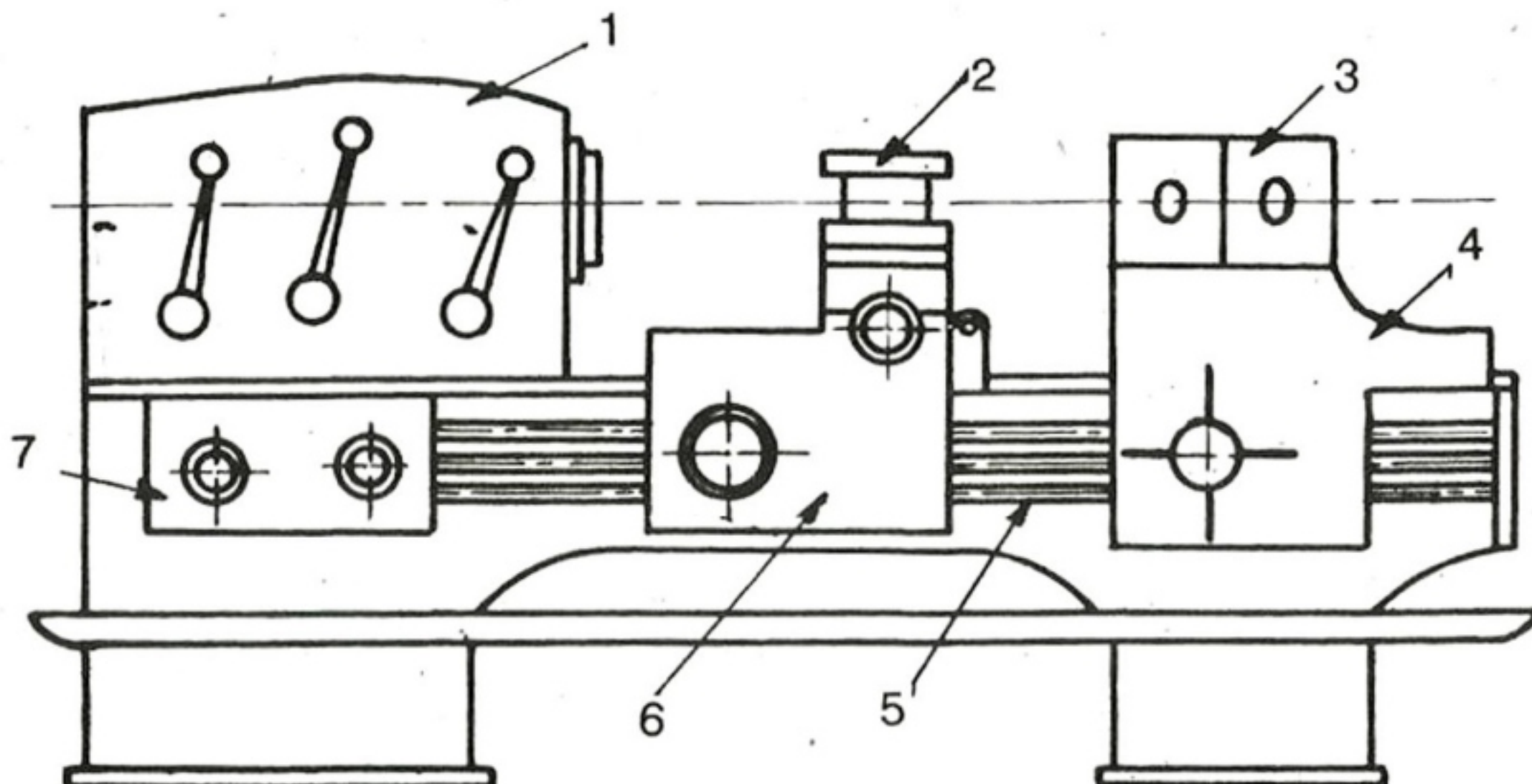
### Bed:

The bed is longer box like casing provided with accurate guide ways on which the carriage and turret slid are mounted. It is designed to ensure strength, rigidity and permanency of alignment under heavy duty services.



**Figure 4.3. Capstan lathe parts**

1. Headstock, 2. Cross-slide toolpost, 3. Hexagonal turret, 4. Saddle for auxiliary slide, 5. Auxiliary slide, 6. Lathe bed, 7. Feed rod, 8. Saddle for cross-slide.



**Figure 4.4 Turret lathe parts**

1. Headstock, 2. Cross-slide toolpost, 3. Hexagonal turret, 4. Turret saddle, 5. Feed rod, 6. Saddle for cross-slide.



## **Headstock:**

It is similar to engine lathe in construction . It is larger & heavier in construction and wider range of speeds speed may range from 30 to 2000rpm two types of headstocks .

- a) Electric head - variable speed motor is mounted .
- b) All geared heads - wider range of speeds

The spindle is hollow and bar stock can be fed through a collet chuck .

## **Gross slide & saddle:**

There are two types of slides used in turret lathe

- Conventional type
- Slide hunk type

The conventional type of carriage bridge the gap between the front and rear bed ways

The slide hunk type carriage is generally fitted with heavy duty turret lathe. Large diameter of work pieces can be swing over bed. The longitudinal movement of each tool may be regulated by using stop bars or shafts set against the stop fitted on the bed and carriage. The stops are set so the each tool will feed into the work to desired length the stop bars are indexed by hand

## **The turret saddle and auxiliary slide :-**

The turret saddle bridges the gap between two bed ways. The hexagonal turret is mounted on the auxiliary slide. In turret lathe, the turret is directly mounted on the top of the saddle.

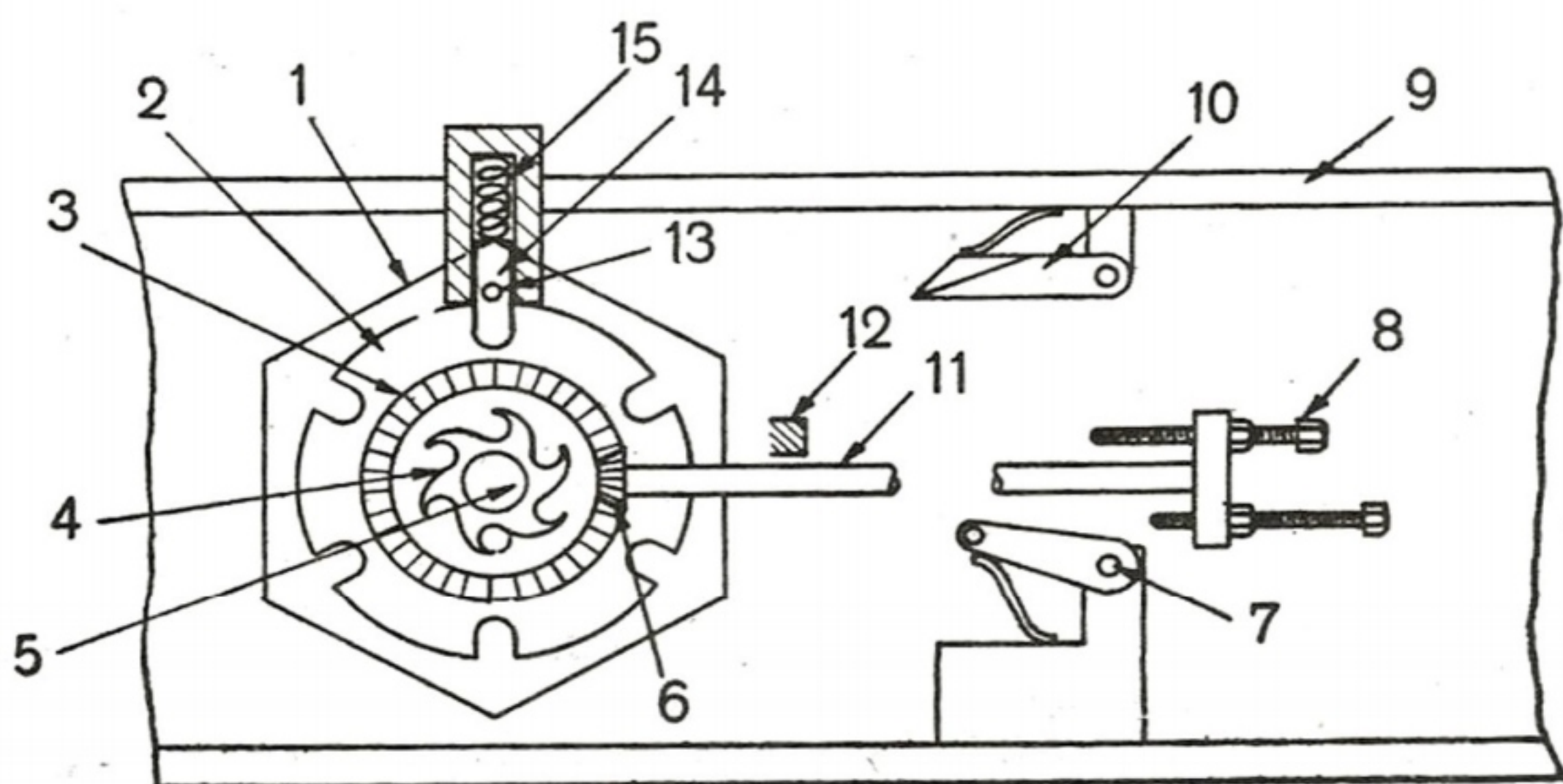
The turret is a hexagonal shaped tool holder intended for six or more tools. The centre line of each hole coincides with the axis of the lathe. Six stop bars are mounted on the saddle which restricts the movement of each tool mounted on each face of turret to be fed predetermined amount. After one operation the turret is brought backward from the spindle nose the turret indexes automatically.



## Capstan and turret lathe mechanism:-

### Turret head indexing mechanism:-

This is an inverted plan of turret assembly. The turret is mounted on the spindle. The index plate, the, bevel gear and an indexing ratchet are keyed to the spindle. The plunger fitted within the housing and mounted on the saddle locks the index plate by spring pressure and prevents any rotary movement of the turret as the tool feeds into the work.



**Figure 4.5 Turret indexing mechanism**

1. Hexagonal turret, 2. Index plate, 3. Beveled gear, 4. Indexing ratchet, 5. Turret spindle, 6. Beveled pinion, 7. Indexing pawl, 8. Screw stop rods, 9. Lathe bed, 10. Plunger actuating cam, 11. Pinion shaft, 12. Stop, 13. Plunger pin, 14. Plunger, 15. Plunger spring.

A pin is fitted on the plunger projects out of the housing. An actuating cam and the indexing pawl are attached to the lathe bed at desired positions. Both the cam and the pawl are spring loaded. As the turret reaches the backward position, the attaching cam lifts the plunger out of the groove in the index plate due to the riding of the pin on the bevelled surface of the cam and thus unlocks the index plate.

The spring loaded pawl which by this time engages with a groove of ratchet plate, causes the ratchet to rotate as the turret head moves backward.



When the index plate or turret rotates through one sixth of revolution , the pin and the plunger drops out of the cam and the plunger locks the index plate at the next groove.

The turret is thus indexed by one sixth of revolutions and again backed into the next position automatically. The turret holds the next tool is now fed forward and the pawl is released from the ratchet plate by the spring pressure.

The bevel opinion meshes with the bevel gear mounted on the turret spindle. The extension of the pinion shaft carries a plate holding six adjustable stop rods. As the turret rotates through one sixth of the revolution , the bevel gear causes the plate to rotate.

The ratio of the teeth between the pinion and the gear are so chosen that when the tool mounted on the face of the turret is indexed to bring it to the cutting position, the particular stop rod for controlling the longitudinal travelling of the tool is aligned with the stop.

The setting of the stop rods for limiting the feed of each operation may be adjusted by unscrewing the lock nuts and rotating the stop rods on the plate. Thus, six stop rods may be adjusted for controlling the longitudinal travel of tools mounted on six faces of the turret.

### **Bar feeding mechanism:-**

On the capstan and turret lathes , some arrangements is need to be feed the bar stock through the collet or chuck after each finished work piece is parted off. Bar may be fed by hand also but has a safety measure one has to stop the machines first for every feeding of bar. It also wastes lot of time.

# Chapter-4

## Shaper

### Syllabus:

- 4.1 Potential application areas of a shaper machine**
- 4.2 Major components and their function**
- 4.3 Explain the automatic table feed mechanism**
- 4.4 Explain the construction & working of tool head**
- 4.5 Explain the quick return mechanism through sketch**
- 4.6 State the specification of a shaping machine.**

### Shaper:-

The shaper is a reciprocating type of machine tool intended to produce flat surfaces. The surface may be horizontal, vertical or inclined

### Working principle:-

The job is fixed rigidly in a suitable vice or directly clamped on the machine table. The tool is held in the tool post mounted on the ram of the machine. This ram reciprocates to and fro, and in doing so, makes the tool to cut the material in the forward stroke. No cutting takes place during the return stroke of the ram. It is called idle stroke. The job is given an intended feed, in a direction normal to the line of action of the cutting tool.



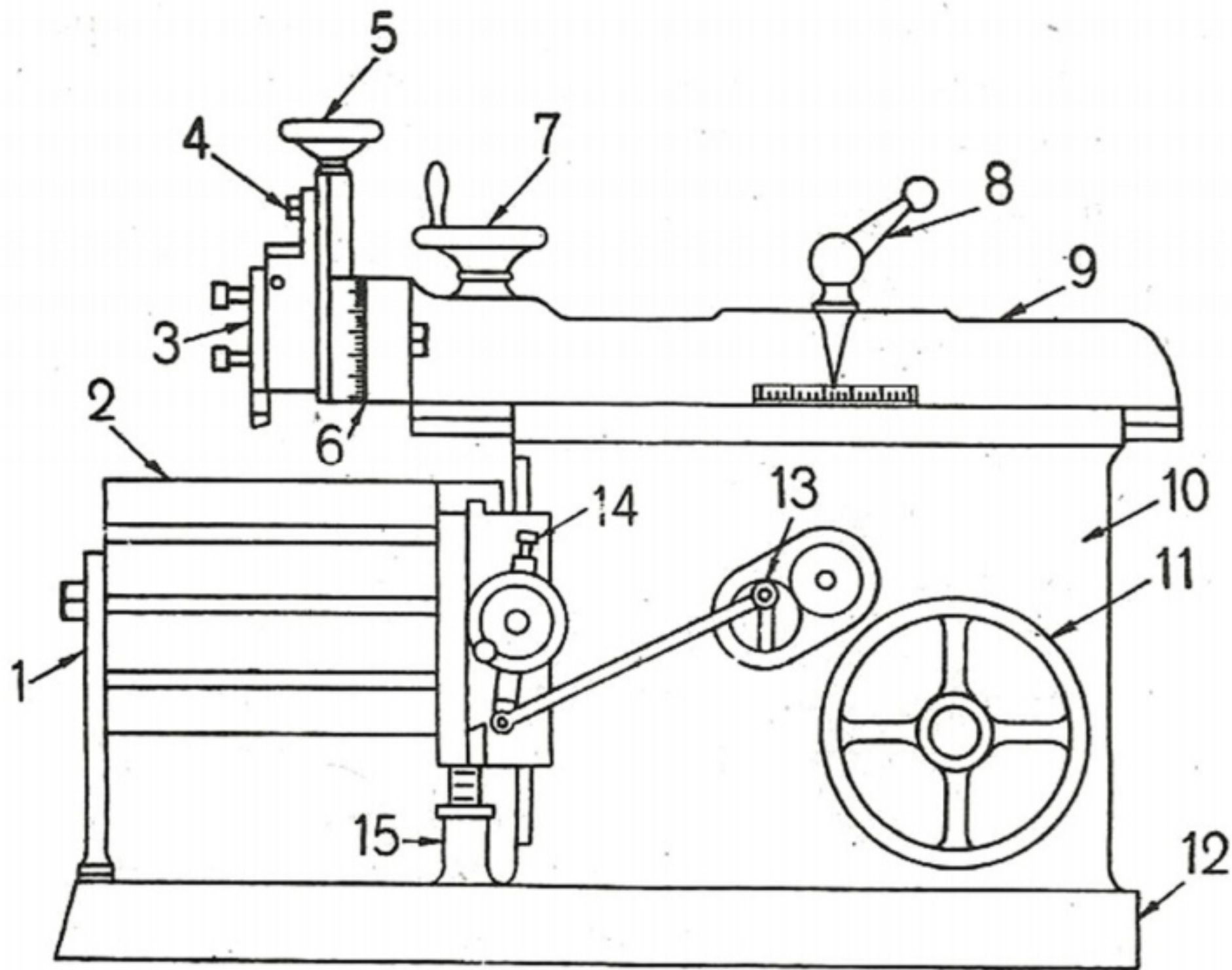
## **Types of shapers:-**

1. According to the type of mechanism used for giving reciprocating motion to the ram.
  - a. Crank type
  - b. Geared type
  - c. Hydraulic type
2. According to the position and travel of ram .
  - a. Horizontal type
  - b. Vertical type
  - c. Travelling head type
3. According to the type of design of the table
  - a. Standard shaper
  - b. Universal shaper
4. According to the type of cutting stroke
  - a. Push type
  - b. Draw type

## **Specification of shaper:-**

1. Maximum length of stroke(175-900mm)
2. Maximum horizontal travel of table
3. Maximum vertical travel of table
4. Maximum distance from table to ram
5. Tool box, vertical adjustment
6. Length and width of the table
7. Numbers and range speeds available
8. Numbers and range feeds available
9. Horse power and speed of driving motor
10. Weight of the machine and floor space required

## DIFFERENT PARTS OF A SHAPER:-



**Figure 7.1 Parts of a standard shaper**

1. Table support, 2. Table, 3. Clapper box, 4. Apron clamping bolts, 5. Downfeed hand wheel, 6. Swivel base degree graduations, 7. Position of stroke adjustment handwheel, 8. Ram block locking handle, 9. Ram, 10. Column, 11. Driving pulley, 12. Base, 13. Feed disc, 14. Pawl mechanism, 15. Elevating screw.

### BASE:-

The base provides stability for the shaper as it supports all other equipments present as well as absorb the forces coming due to the cutting . it is made of cast iron and have a necessary arrangements of bolts so that it can be bolted to the factory floor.

### Column(housing):-

The column of the shaper is a hollow casting and is mounted on the base. It houses the ram driving mechanism. For the ram and table, on the top of the column, necessary guide ways are provided for the linear movement of the ram and the front vertical face is for cross rail.



### **Cross rail:-**

The cross rail is mounted on the front vertical guideways of the column. It has two parallel guide ways on its top vertical plane that are perpendicular to the ram axis. The table may be raised or lowered to accommodate different sizes of job by rotating elevating screw.

### **Saddle:-**

The saddle is mounted on the cross rail which holds the table . on its top crosswise movement of the table is powered by rotating cross feed screw.

### **Table:-**

The work table of a shaper is fastened to the front of the column. The table is provided T-slots for mounting the work pieces. The table can be moved up and down and crosswise by cross rail and saddle. Jobs can be held by vice.

### **Ram:-**

The ram is a reciprocating member of the shaper. It is semicylindrical in form and heavily ribbed inside to make more rigid. It slides on the guideways of the column. A single point cutting tool is fastened in the tool post.

### **Tool head:-**

The single point cutting tool is held in the tool post. The tool head holds the tool provides vertical and angular feed movement and allows the tool to have an automatic relief during return stroke. The vertical side of the tool head has a swivel base which is held on a circular seat on the ram.

The swivel base is graduated in degrees so that the vertical slide may be set perpendicular to the work surface at any desired angle. By rotating the down feed screw handle, the vertical slide carrying the tool executes down feed or angular feed movement while machining vertical or angular surface.

The amount of feed or depth of cut may be adjusted by a micrometer dial on the top of the down feed screw. Apron consisting of clapper box,



clapper block and tool post is clamped upon the vertical slide by a screw. By releasing the clamping screw, the apron may be swivelled upon the apron swivel pin either towards left or towards right w.r.t the vertical slide. This arrangement is necessary to provide relief to the tool while making vertical or angular cuts.

The two vertical walls on the apron called clapper box houses the clapper block which is connected to it by means of a hinge pin. The tool post is mounted upon the clapper block.

On the forward cutting stroke the clapper block fits securely to the clapper box to make a rigid tool support on the return stroke a slight frictional drag of the tool on the work lifts the block out of the clapper box a sufficient amount preventing the tool cutting edge from dragging and consequent wear. The work surface is also prevented from any damage due to dragging.

### **Shaper mechanism:-**

The metal is removed in the forward cutting stroke, while the return stroke no metal is removed during this period.

To reduce the total machining time it is necessary to reduce time taken by the return stroke. The shaper mechanism should be so designed that it can allow the ram holding the tool to move at comparatively slower speed during the forward cutting stroke and during the return stroke the ram move faster rate to reduce the idle return time. The mechanism is called quick return mechanism.

- 1. Crank and slotted mechanism**
- 2. Whitworth quick return mechanism**
- 3. Hydraulic shaper mechanism**

### **Crank and slotted link mechanism:-**

The motion or power is transmitted to the bull gear through a pinion which receives its motion from an individual motor or overhead line shaft through speed control mechanism. Speed can be changed by shifting gears.

A radial slide is bolted to the centre of the bull gear, carries a sliding block into which the crank pin is fitted. rotation of the bull gear causes the

clapper block and tool post is clamped upon the vertical slide by a screw. By releasing the clamping screw, the apron may be swivelled upon the apron swivel pin either towards left or towards right w.r.t the vertical slide. This arrangement is necessary to provide relief to the tool while making vertical or angular cuts.

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### **Crank and slotted link mechanism:-**

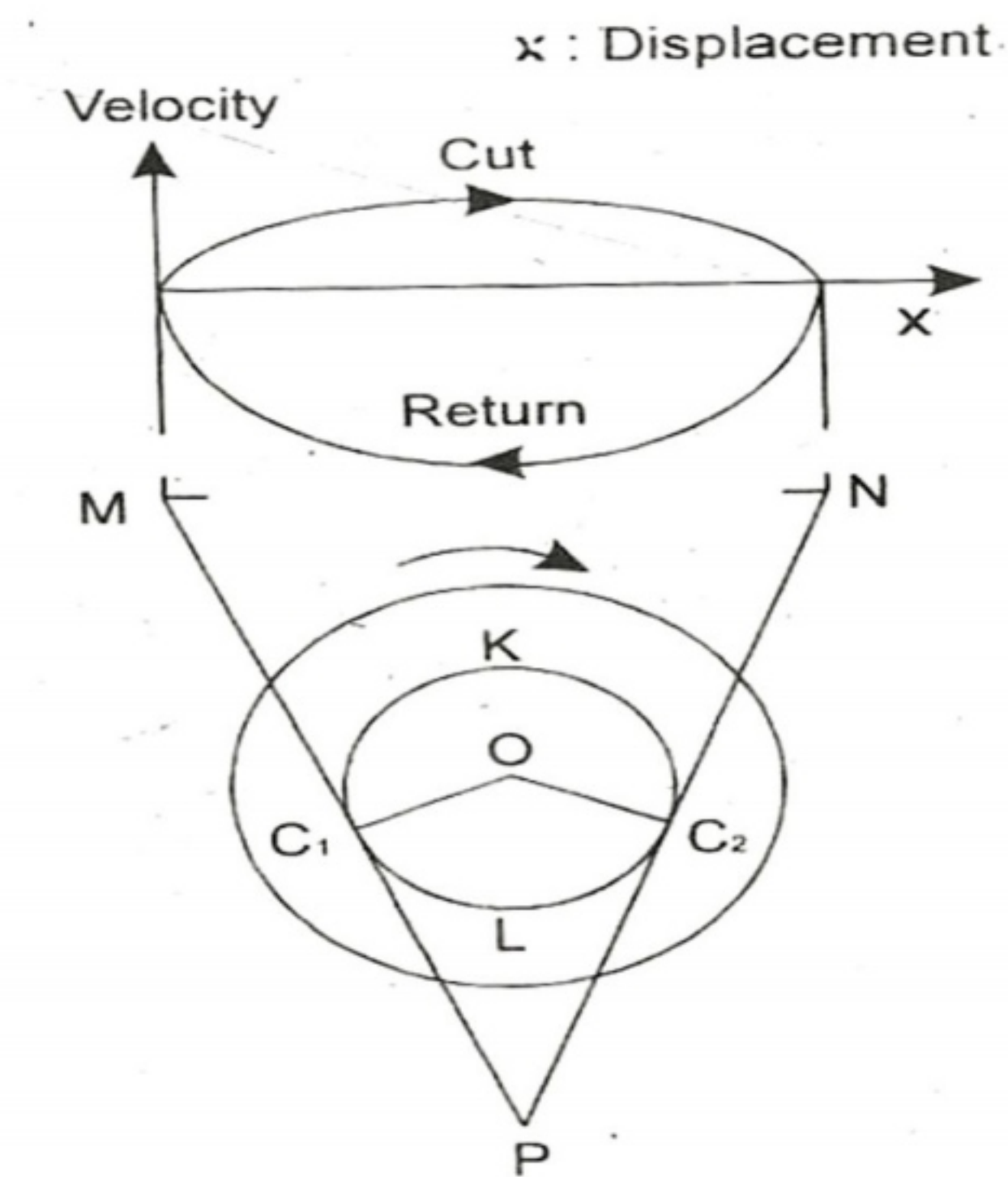
The motion or power is transmitted to the bull gear through a pinion which receives its motion from an individual motor or overhead line shaft through speed control mechanism. Speed can be changed by shifting gears.

A radial slide is bolted to the centre of the bull gear, carries a sliding block into which the crank pin is fitted. rotation of the bull gear causes the



crank pin to rotate at a uniform speed. Sliding block which is invented on the crank pin is fitted within the slotted link. The slotted link is pivoted at its bottom end attached to the frame of the column. The upper end is forked and connected to the ram block by a pin.

As bull gear rotates causes the crank pin to rotate, the sliding block fastened to the crank pin will rotate on the crank pin circle, and at same time will move up and down in the slot giving a rocking movement which is communicated to the ram. The rotary motion of the bull gear converted to reciprocating movement of the ram.



**Figure 7.4 Principle of quick return mechanism**

When the link is in the position PM, ram will be at the extreme backward of its stroke.

When at PN – extreme forward position

PM & PN are tangent to the crank pin circle.

$C_1 \hat{K} C_2 \Rightarrow$  forward cutting stroke

$C_2 \hat{L} C_1 \Rightarrow$  return stroke



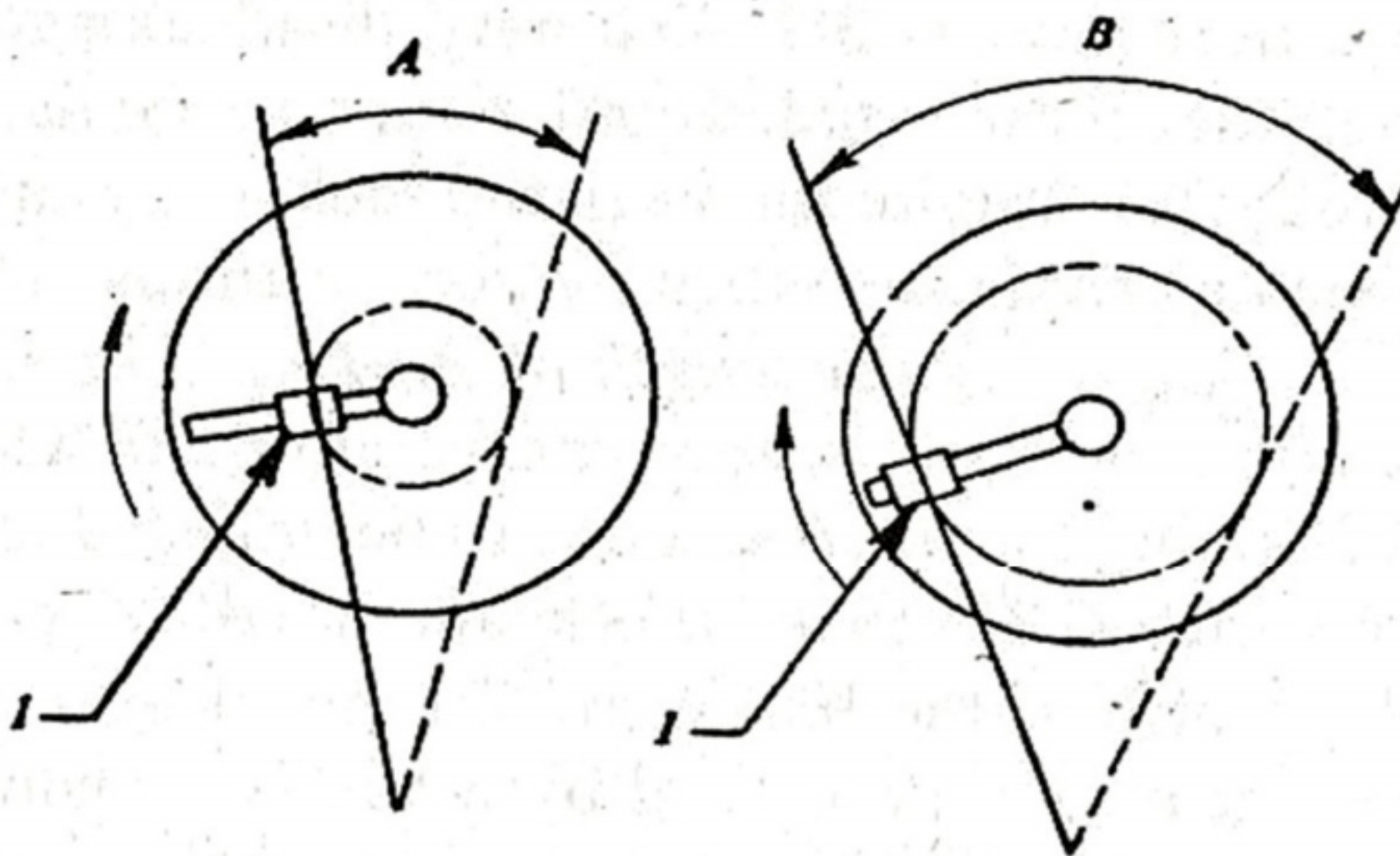
It is clear that the angle made by the forward stroke > return stroke. The angular velocity is constant  $\Rightarrow$  rotated by speed metre . crank rotates at uniform speed.

$$\frac{\text{time taken in cutting stroke}}{\text{time taken in return stroke}} = \frac{C_1 \hat{k} C_2}{C_2 \hat{L} c_1} = \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha}$$

Generally varies 2:1 and practical 3:1

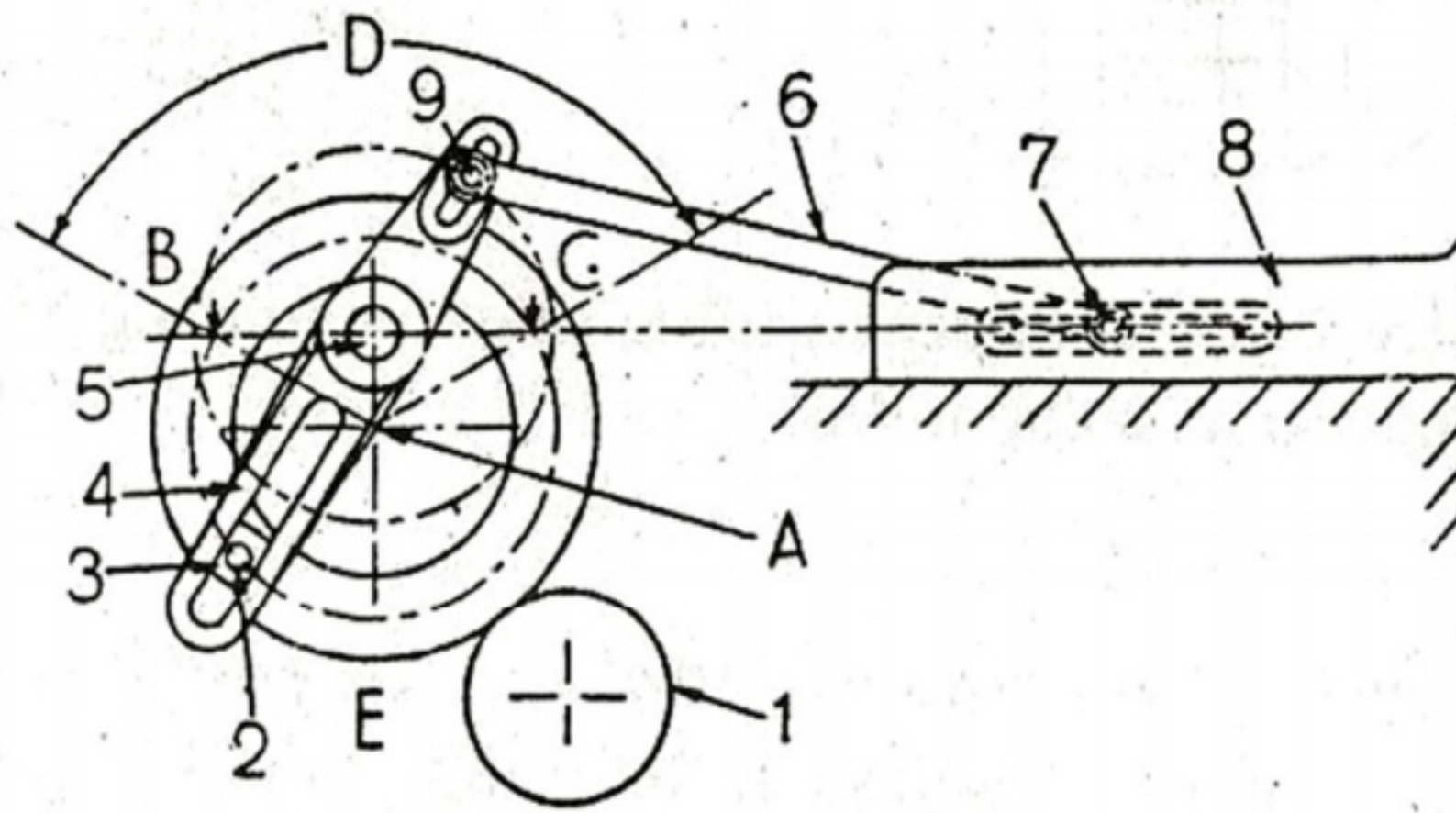
### Whitworth quick return mechanism:-

A bull gear is mounted on a large fixed pin a upon which it is free to rotate. The crank plate is pivoted eccentrically upon the fixed pin at 5. fitted on the face of the bull gear is the crank pin 2 on top of which sliding block is mounted. Sliding block fits into the slot provided on the crank plate. ,a connecting rod connects the crank plate by a pin and the ram by a pin.



**Figure**                      **Stroke length adjustment**  
1. Position of crankpin, A. Short stroke length, B.  
Long stroke length.





**Figure Whitworth quick return mechanism**

1. Driving pinion, 2. Crank pin, 3. Sliding block, 4. Crank plate, 5. Pivot for crank plate, 6. Connecting rod, 7. Connecting pin for ram, 8. Ram, 9. Pin, A. Fixed pin.

When the bull gear will rotate at constant speed the crank pin with the sliding block will rotate on a crank pin with the sliding block will rotate on a crank circle of radius  $A_2$  and the sliding block will cause the crank plate to rotate about the point 5 with a variable angular velocity.

Pin fitted on the other end of the crank plate will rotate in a circle and the rotary motion of the pin will be converted into reciprocating movement in the ram. Similar to crank and connecting rod mechanism. The axis of reciprocating of the ram passes through the pin and normal to the line  $A_3$ .

When the pin cutting stroke is at position C  $\Rightarrow$  extreme backward position

When the pin cutting stroke is at position B  $\Rightarrow$  forward position

$\widehat{CEB}$  = cutting stroke (backward to forward)

$\widehat{BDC}$  = return stroke (forward to backward)

As angular velocity of the crank pin is uniform for the time taken i.e.,

$$\frac{\text{time taken in cutting stroke}}{\text{time taken in return stroke}} = \frac{\widehat{CEB}}{\widehat{BDC}} = \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha}$$

# Chapter-5

## Planning Machine

### Syllabus:

- 5.1 Application area of a planar and its difference with respect to shaper**
- 5.2 Major components and their functions**
- 5.3 The table drive mechanism**
- 5.4 Working of tool and tool support**
- 5.5 Clamping of work through sketch.**

The planner like a shaper is a machine tool to produce plane and flat surfaces by a single point cutting tool

### **Difference between shaper & planner**

The work is stationary; the tool along with ram reciprocates against the work

The work which is supported on the table reciprocates and the tool is stationary



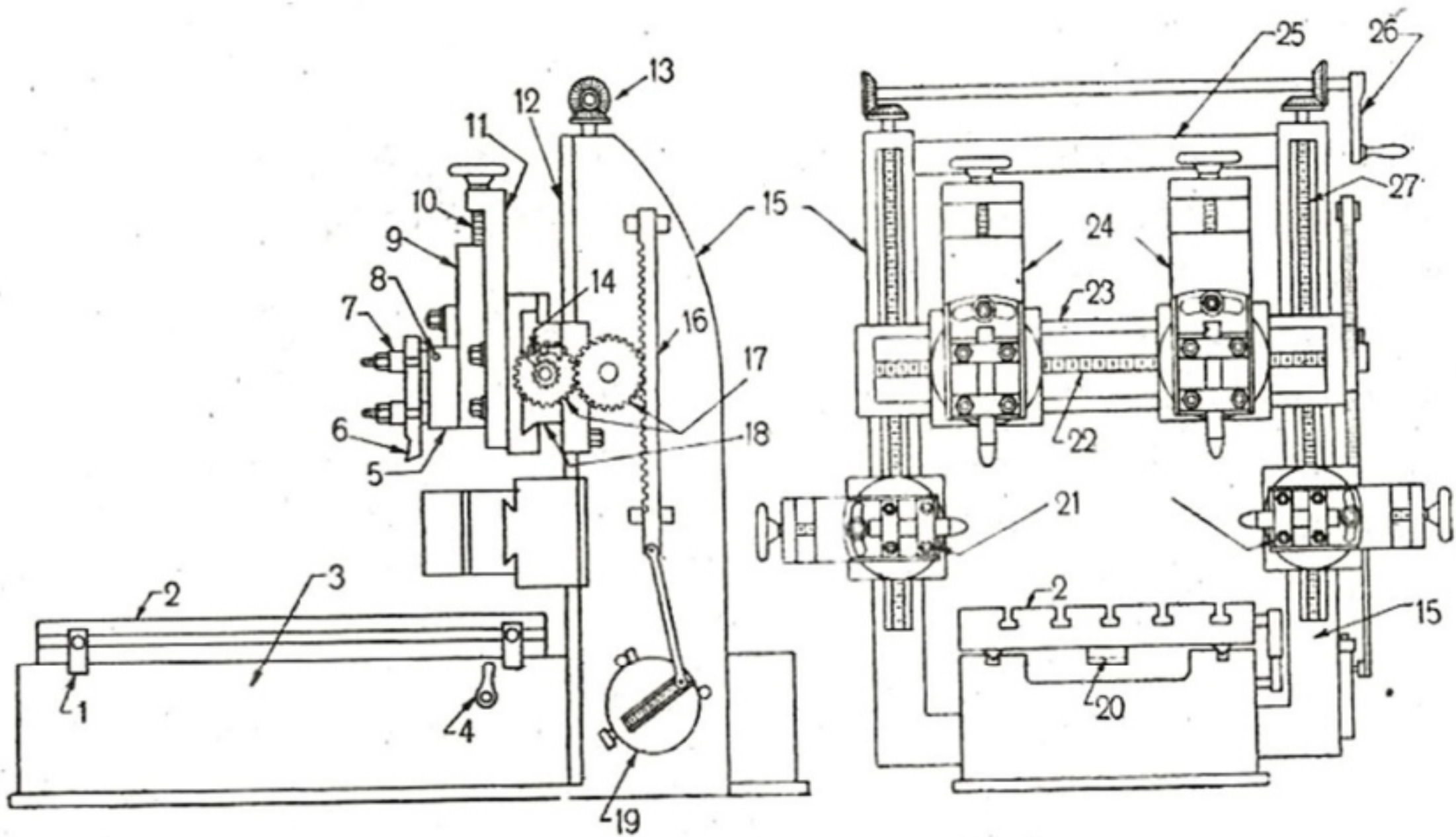
## **Principal parts of a planner:**

### **BED:**

- The bed of a planner is a box like casting cross ribs.
- It is a very large in size and heavy in weight and it supports the column and all other moving parts of the machine.
- The guides ways are provided on the bed for the movement of the table.
- The hollow space within the box like structure of the bed home the driving mechanism for the table.

### **Table:**

- The table supports the job and reciprocates along the ways of the bed.
- The planner table is heavy rectangular casting and is made of cast iron.
- The top face of the table is accurately finished in order to hold the job correctly.
- The top face of the table is 'T' slotted for clamping the job and job holding devices.
- At each end of the table or hollow space is left for collecting chips.
- A groove is cut on the side of the table for clamping planner revising dugs at different position.



**Figure 8.1 Standard double housing planer**

1. Trip dog, 2. Table, 3. Bed, 4. Reversing lever, 5. Clapper box, 6. Tool, 7. Tool Post, 8. Hinge pin, 9. Vertical slide, 10. Downfeed screw, 11. Slide, 12. Guideways on column face, 13. Feed screw for elevating crossrail, 14. Pawl, 15. Column or housing, 16. Rack, 17. Feed gears, 18. Saddle, 19. Feed disc, 20. Table rack, 21. Slide toolhead, 22. Feed screw, 23. Crossrail, 24. Vertical toolheads, 25. Crossmember, 26. Crossrail, elevating handle, 27. Cross elevating screw.

### **Column:**

- It is a rigid vertical box like structure.
- The front face of each housing is accurately machined to provide guide ways on which the crossrail may be slipped up and down for accommodating the different suitable heights job.
- The housing is enclosed with vertical elevating screw and cross feed screws for tool heads and counter balanced the weight of the cross rail.

### **CROSS RAIL:**

- It is a rigid box like casting connecting the two housings.
- The cross rail can be raised or lowered on the face of the housing and can be clamped at any desired position.
- The cross rail when clamped should remain parallel to the top surface of the table.



- The two elevating screws in two the housing are rotated by an equal horizontal any in position.
- The two tool heads are mounted upon the cross rail.
- The cross rail has screws for vertical and cross flow of the tool heads and a screw for elevating the rail.

### **TOOL HEAD:**

- The tool head of the planer is similar to that of a shaper in construction and operation.

### **CLAMPING OF JOB:**

- There are three important points to be noted while clamping the job on the planer table.
- The work should be connected rigidly to the table so that it may not be shifted out of its position while cutting progresses.
- Proper clamping should be done all round the job.
- The job should be so held that the surface planed should remain in proper position with other surface.
- The job may be located on the planning machine table by the following methods.
- By standard clamping devices.
- By special fixtures.
- The standard clamping devices are t-bolts, stops, planer jacks, heavy duty vises, angle plates & planner centres etc.

### **PLANER TOOLS:**

- The cutting tools which are used in planer are single point cutting tool which are used in lathe and shaper.
- Planer tools may be solid for get type or bit type.
- The bit may be brazed, welded or mechanically held on a m.s bar.
- As a planer tool has to take up heavy cuts, the tools are made heavier and larger in cross section.



# Chapter-6

## Milling Machine

### Syllabus:

- 6.1 Types of milling machine and operations performed by them**
- 6.2 Explain work holding attachment**
- 6.3 Construction & working of simple dividing head, universal dividing head**
- 6.4 Procedure of simple and compound indexing**
- 6.7 Illustration of different indexing methods**

### Milling m/c:

A milling m/c is a type of metal cutting device which remove metal with a fast rotating multi-touch cutter.

As this m/c yield high production of different varieties of jobs, in choice for production m/c, comes after the lathe.

Generally smaller jobs are employed for machining in milling m/c. If larger jobs are handled, then the m/c will perhaps be slower.

Because of using multi tooth cutters & various forms of cutters, a milling m/c can be economically employed for generating varieties of surfaces quite speedily.

### **Types of milling m/c:**

Milling m/c are available in various designs covering a wide range of work & capacities. The choice for a particular m/c depends on the nature & size of the work to be undertaken.

- a) column & knee type. Milling m/c.
  - a. Hand milling m/c
  - b. Plain milling m/c
  - c. Universal milling m/c
  - d. Omniversal milling m/c
  - e. Vertical milling m/c
- b) manufacturing/ fixed bed type. Milling m/c
- c) planer type milling m/c
- d) special type milling m/c.
  - a. Rotary table machine.
  - b. Drum milling m/c.

### **Milling attachment:**

It is a rotary table type work holding device bolted on the table. It provides a rotary motion to the w/p in addition to longitudinal, cross & vertical motions.

### **Dividing head/indexing head:**

Indexing is an operation of dividing a periphery of a cylindrical w/p into equal no. of divisions by the help of index crank & index plate.

Indexing is accomplished by using a special attachment known as dividing head/indexing head.

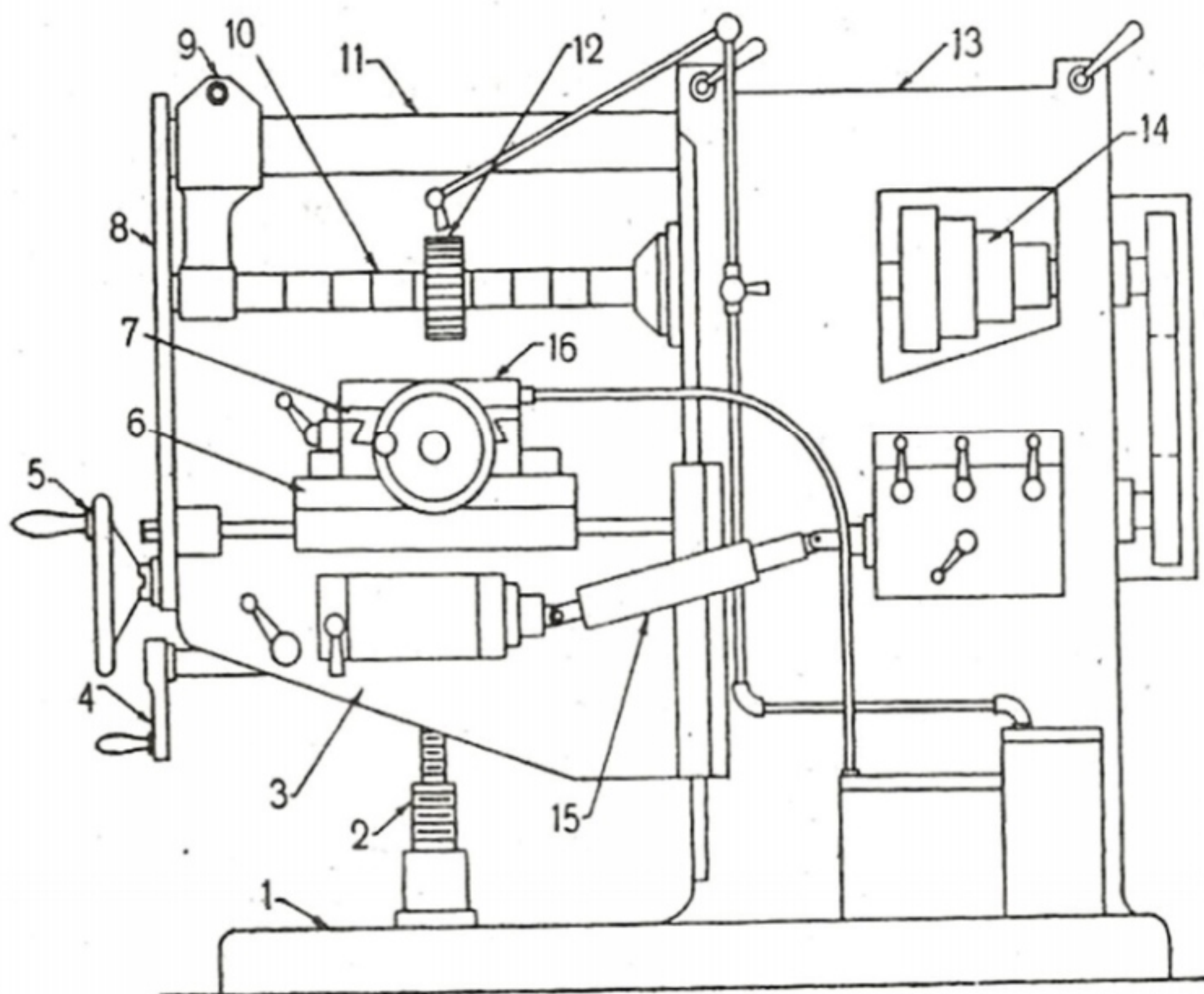
Dividing heads are 3 types

- 1. Plain/simple dividing head:**
- 2. Universal dividing head**



### Plain/ simple dividing head:

- It has a spindle which carries job holding devices such as three jaw chuck, face plate with centre carrier
- A worm wheel is rigidly fixed on spindle. While an indexing crank is mounted on the worm shaft such that the rotation of index crank finally results in the rotation of the spindle.
- In a plane dividing head, its spindle rotates only around horizontal axis.
- The index plate remains fixed & does not rotate only around horizontal axis.
- The index plate remains fixed & does not rotate while performing simple indexing operation.



**Figure 11.1 Column and knee type milling machine**

1. Base, 2. Elevating screw, 3. Knee, 4. Knee elevating handle, 5. Crossfeed handle, 6. Saddle, Table, 8. Front brace, 9. Arbor support, 10. Cone pulley, 15. Telescopic feed shaft.



The amount of the spindle relative to the worm depends on the ratio between the relations of worm & the worm wheel.

The most common ratio is 40:1, which means 40 revolutions of index crank or worm will move the worm wheel or spindle through one complete revolution.

### **Universal dividing head:**

- It is the most commonly used type of attachment on the milling m/c.
- It is used for the following purposes.
- Setting the w/p in horizontal, vertical & inclined positions relative to the milling m/c table.
- Turning the work periodically through a given angle performing indexing of the work.
- Imparting a continuous rotary motion to the w/p for milling helical grooves.
- Dividing head spindle is connected with the table feed screw through a gear train attachment to impart a continuous rotary motion to the w/p for helical milling.
- Working mechanism of an universal dividing head is the crank which is rigidly fixed at one end of the work shaft, while the bevel gear runs free on the worm shaft.
- The index plate is blotted with gear & can be locked against the rotation of lock pin.

### **Optical dividing head:**

It is used for high precision angular indexing of the job with respect to the cutter.

For reading the angle, an optical system is built into the dividing head.

### **Spindle:**

Spindle is located in the upper part of the column.

It gets driving power from motor to transmit it to the arbor.

The front end of the spindle just projects from the column face & is provided with a tapered hole to accommodate various cutting tools & arbors.

### **Arbour:**

The arbour is connected with a spindle through a bolt, which serves as an extension of the m/c spindle on which milling cutters are safely mounted & rotated

It is taper shank for proper alignment with the spindle having tapered hole at its nose.

### **Work holding devices:**

Work holding device used on a milling m/c includes the following: Vices which may be plain vice, swivel vice & tool maker's universal vice.

- **Angle plates.**
- **v-blocks**
- **special fixtures.**

### **Work holding attachment:**

Special attachment are used on milling m/c for performing different typical operations. Common attachments include:

- a. vertical milling attachment: used for facilitating the horizontal spindle milling m/c to do facing on horizontal surface or for making grooves.
- b. universal milling attachment: similar to vertical attachment with added features for swivelling the spindle about two mutually perpendicular axis.

### **slitting attachment :**

used for converting rotational motion of the horizontal spindle m/c into up & down vertical reciprocating motions. The slotting head is fitted to the spindle & is used for cutting slats. The attachment can be swivelled to any angular positions.

### **high speed milling attachment:**

it is a gearing system used for increasing the spindle speed by four to six times. The attachment is bolted to the face of the column.

### **rack milling attachment:**

It is bolted to the face of the column. Used for cutting rake teeth.

### **Differential Indexing :**

Available no. Of index plates with different hole circles sometimes limit the range of plate indexing. In such case differential indexing is found useful. In this process, the indexing plate rotates itself in relation to the crank during the process of indexing. For making necessary calculation to find the change gears to be placed between the spindle and the power shaft use the following relation

Where  $N$  = no. Of divisions to be indexed

$n$  is a no. Of slightly or less than  $N$ . After simplification, the above relation  $n$  will give the gear ratio between the gears to be placed on the spindle (driver) and the own shaft (driver).

Gears may be arranged in a simple train or in a compound train as the case may be.

When  $(n - N)$  is positive, the index plate must rotate in the direction in which the crank is rotated.

If  $(n - N)$  is negative the index plate rotates in the opposite direction to that of the crank.

### **Dividing head attachment:**

A dividing head attachment is also a special work holding device which is bolted on the machine plate. The work may be mounted on a chuck fitted on the dividing head spindle or may be supported between live or dead centre.

The dead centre is mounted on a foot stock as in a lathe tail stock that is bolted on the machine table after correctly aligning its spindle axis with the dividing head's spindle



### **universal dividing head:**

The attachment is similar to the vertical attachment but it has an added arrangement for swivelling the spindle about two mutually perpendicular axes . This feature of the attachment permits the cutting spindle axis to swivel at practically any angle and machine any compound angle surface of the work. The attachment is supported by the over-ram and operates at either the same speed or at higher speed than the speed of machine.

# Chapter-7

## Slotter

### Syllabus:

- 7.1 Major components and their function**
- 7.2 Construction and working of slotter machine**
- 7.3 Tools used in slotter**

### SLOTTER

It is used for cutting grooves, key ways and slots of various shapes for making regular and irregular surfaces both internal and external.

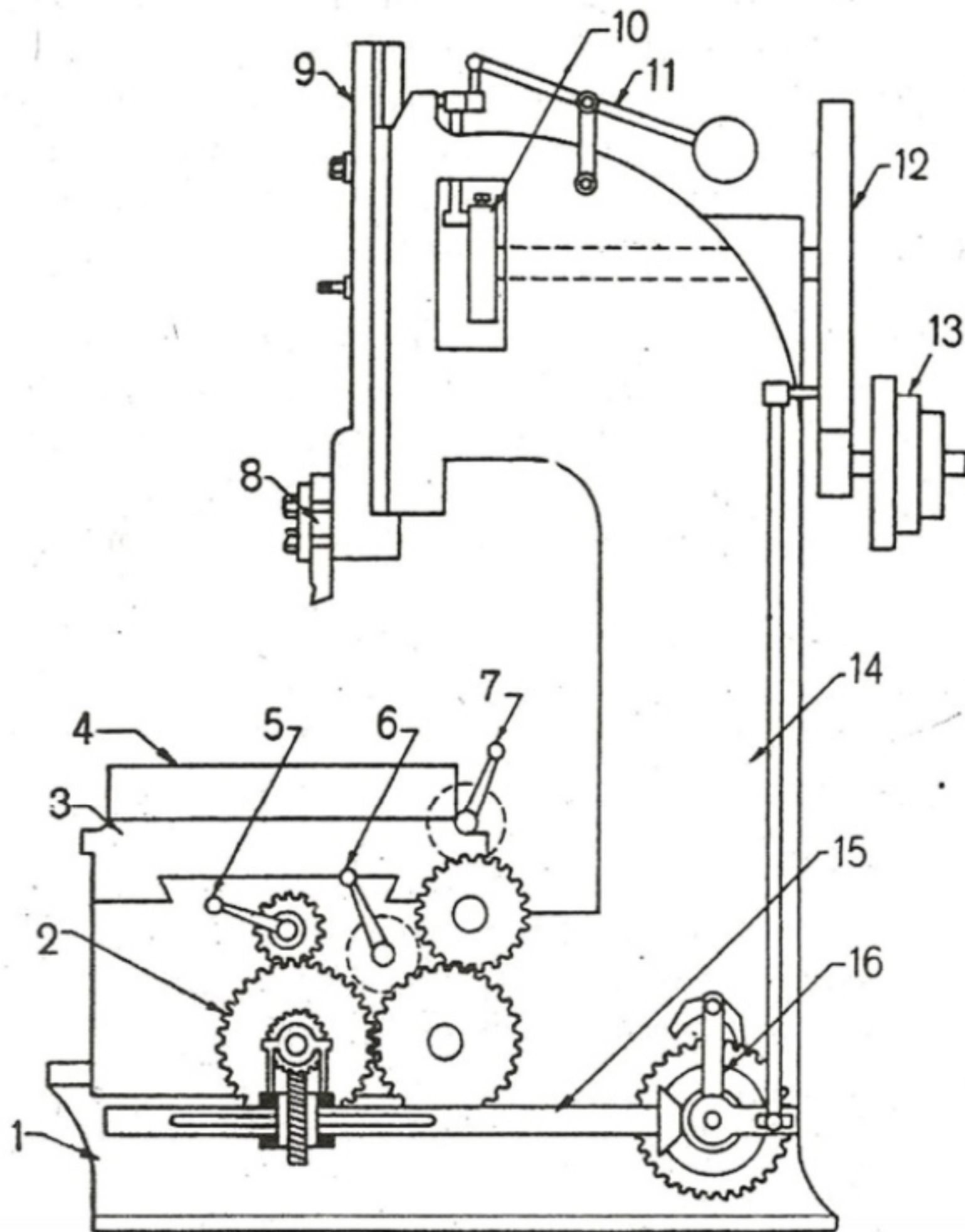
Difference between slotter and shaper:

<b>SLOTTER</b>	<b>SHAPER</b>
The ram holds the tool and reciprocates in a vertical axis	The ram holds the tool and reciprocates in a horizontal axis.

## **SLOTING MACHINE PARTS:**

### **Base/bed:**

The base of the machine is rigidly built to withstand all cutting forces and entire load of the machine.



**Figure 9.1 Slotting machine**

1. Base, 2. Feed gear, 3. Cross-slide, 4. Table, 5. Crossfeed handle, 6. Longitudinal feed handle, 7. Circular feed handle, 8. Tool, 9. Ram, 10. Crank disc, 11. Lever for counterbalance weight, 12. Bull gear, 13. Cone pulley, 14. Column, 15. Feed shaft, 16. Pawl actuating crank.



### **COLUMN:**

- The column is the vertical member which is cast integral with the base.
- Driving mechanism and feeding mechanism are inside the column.
- The front vertical face of column is accurately finished for providing ways on which the ram reciprocates.

### **SADDLE:**

- The saddle is mounted upon the guideways and may be moved towards or away from the column either by power/ manual control to supply longitudinal feed to the work.
- The top surface of the saddle is accurately finished to provide guideways for the cross slide.
- These guideways are perpendicular to the guideways on the base.

### **CROSS SLIDE:**

- The cross slide is mounted upon the guideways of the saddle and may be moved parallel to the face of the column. The movement of the slide may be controlled either by hand or power to supply cross feed.

### **ROTATING TABLE:**

- The rotating table is a circular table which is mounted on the top of the cross slide. The table may be rotated by rotating a worm which meshes with a worm gear connected to the under side of the table.
- In some machines the table is graduated in degrees that enables the table to be rotated for indexing.
- T-slots are cut on the top face of the table for holding the work by different clamping devices.
- The rotary table enables a circular or contoured surface to be generated on the work piece.

### **RAM & TOOLHEAD ASSEMBLY:**

- The ram is the reciprocating member of the machine mounted on the guide ways of the column.
- It supports the tool at its bottom end on toolhead.

## **RAM DRIVE MECHANISM:**

- A slotter removes metal during downward cutting stroke only whereas during upward return stroke no metal is removed.
- To reduce the idler time quick return mechanism is incorporated in the machine.

## **FEED MECHANISM:**

- The feed is given by the table.
- A slotting machine table has 3 types of feed movements.

## **LONGITUDINAL:**

- If the table is fed perpendicular to the column towards or away from the face the feed movement is termed as longitudinal.

**CROSS:** If the table is fed parallel to the face of the column the feed movement is termed as cross.

**CIRCULAR:** If the table is rotated on a vertical axis the feed movement is circular.

## **SLOTTER TOOLS:**

$\alpha$  = top rake angle

$\beta$  = flank clearance angle

- In a slotter the pr. Acts along the length of the tool.
- In slotter tool the angles are provided for better cutting.
- The nose of the tool projects slightly beyond the shank to provide clearance.
- The slotter tools are robust in cross section and usually of forged type.
- Bit type tools are fitted in heavy duty tool holders.
- Key way cutting tools are thinner at the cutting edges.
- Round nose tools are used for machining circular/ contoured surfaces.
- Square nosed tools are used for machining flat surfaces.

# Chapter-8

## Grinding

### Syllabus:

**8.1 Significance of grinding operations**  
**8.2 Manufacturing of grinding wheels**  
**8.3 Criteria for selecting of grinding wheels**  
**8.4 Specification of grinding wheels with example**  
**Working of**  
**Cylindrical Grinder**□  
**Surface Grinder**□  
**Centre less Grinder**□

### Grinding M/C:

- Grinding is the process of removing material from workpiece. The removing of material from the W/P is either to bring its dia metre within very close tolerance (0.02mm) or to give a fine finish on the work surface.
- The grinding machine supports and rotates the grinding wheel for smooth operation. Grinding machine is used for roughing and finishing flat, cylindrical and conical surface.



- The grinding wheel is made of fine grains of abrasive materials. The grains during the rotation of the wheels remove very small chips.
- As the selection of chips removed is very small and high cutting speed are involved, the grinding operation results into a very good finish on the work surface and high accuracy in work dimensions.

### **Types of Grinding:**

Often used to grind casting and weldments using portable grinders or pedestal grinder

### **Fine Grinding :**

- It is applied for finishing of those materials which are too hard to be machined by other methods of metal cutting.
- It is also used for producing surface on the job to attain higher dimensional accuracy and finish.

### **BED/BASE**

- It is a heavy cast iron construction and supports other parts of the slotting machine such as column, ram and its driving mechanism, table etc.
- The top of the base is accurately finished to provide guide ways for mounting of the saddle. The cross-slide guide ways are perpendicular to the column face.

### **Column**

- It is a vertical structure cast integral with the base.
- It houses the mechanism for driving ram and feeding mechanism.
- The front vertical face of the column carries guide ways for ram to reciprocate upon it.

### **Saddle:**

- It is mounted upon guide ways and can be moved towards or away from the column.
- The saddle carries guide ways for cross-slide.
- The feed is given by manual or power.

### **Cross-slide:**

- It is mounted upon guide ways made at the top of the saddle and can be moved parallel to the front face of the column.
- Feed is given manual or by power.
- Table
- It is a circular rotary table mounted on the top of the cross-slide
- A circular feed handle for the table is provided
- Rotation of the table is effected by hand or by power
- The table carries T-slots to help mounting of jobs on the table
- Ram and tool head
- They reciprocates up and down on the guide ways made on the front face on the column.
- The ram carries a tool head at its bottom end.
- In some mechanism special tool head is provided to release the tool during its return stroke
- A quick return mechanism is given with the machine which enables the return or ideal stroke to be completed faster than the cutting stroke

### **Methods of grinding:**

According to type surface to be ground main kinds of grinding methods are as follow:

External cylindrical grinding: produces a straight or tapered surface on a w/p when it is rotated about its own axis between centres as it passes lengthwise across the face of a revolving grinding wheel.

Internal cylindrical grinding: it produces internal cylindrical holes & tapers. The work is chucked & rotated on its axis, while the grinding wheel rotates against the work.

### **Surface grinding:**

It produces flat surfaces & the work may be ground either by periphery or by end face of the grinding wheel.

### **Face grinding:**

It is a method of grinding vertical flat surface & the wheel spindle may be vertical or horizontal.

### **Form grinding:**

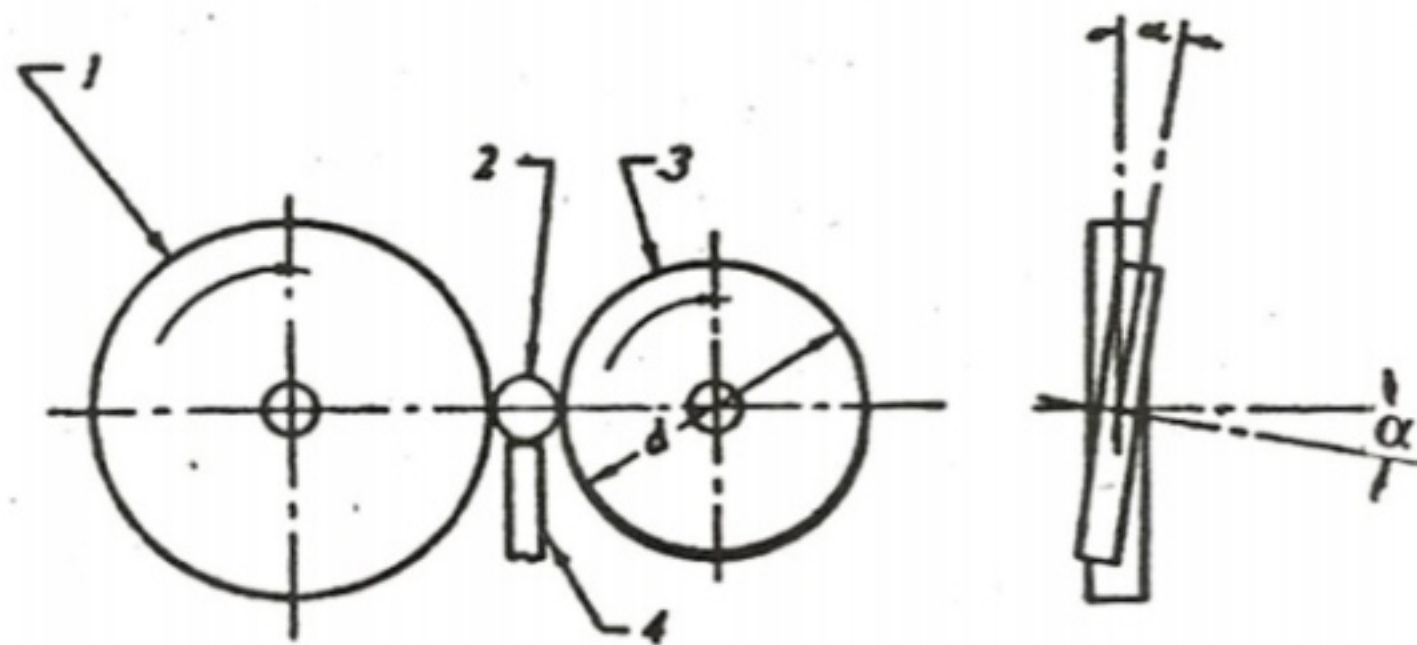
It is done by specially shaped grinding wheels to grind formed surfaces as gear teeth, theards, shaft, dovetails etc.

### **Set wheel grinding:**

It is a method of grinding short w/p without changing the grinding wheel.

### **Centre less grinding:**

It is a method of grinding external & internal cylindrical surfaces in which the work is supported among a regulating wheel, a grinding wheel & a work rest blade.



**Figure External centreless grinding**  
1. Grinding wheel, 2. Work, 3. Regulating wheel, 4. Work-rest.

### **Off-hand grinding:**

It is rough grinding method in which work is held in hand and pressed

Against the rotating grinding wheel. For example grinding a chisel on pedestal grinder



### **Types of grinding machine:**

Grinding machine may be broadly classified as follows

- 1. Surface grinder**
- 2. Cylindrical grinder**
- 3. Center-less grinder**
- 4. Portable grinder**
- 5. Pedestal grinder**
- 6. Cutter and tool grinder**
- 7. Internal grinder**
- 8. Flexible shaft grinder**

#### **Surface grinder :**

- a. It is used for grinding flat surfaces
- b. Surface grinding effective for removing hard spots and sets from the work surface
- c. Surface grinding machine differs according the shape of grinding wheel and motion given to the work table during working.
- d. Some common type surface grinding machine are prescribed in the following

### **Horizontal spindle surface grinding machine:**

It is used for the circumference of a straight grinding wheel and are able to handle a wide range of work with super finish and externally fine limits of accuracy

### **Vertical spindle flat grinding machine:**

These are strongly built machine. They yield more output with cup type wheel rather than using of straight wheels.

### **Disc grinding machine:**

These are used for rough semi precision grinding rapid removal of metal is done by this type of machine.

# Chapter-9

## Internal Machining operations

### Syllabus:

**Classification of drilling machines**

**9.1 Working of**

**Bench drilling machine** □

**Pillar drilling machine** □

**Radial drilling machine** □

**9.2 Boring**

**Basic Principle of Boring** □

**Different between Boring and □ drilling**

**9.3 Broaching**

**Types of Broaching (pull type, □ push type)**

**Advantages of Broaching and □ applications**



## Introduction

The drilling machine is one of the most important machine tools in a workshop. As regards its importance it is second only to the lathe. Although it was primarily designed to originate a hole, it can perform a number of similar operations. In a drilling machine holes may be drilled quickly and at a low cost. The hole is generated by the rotating edge of a cutting tool known as the drill which exerts large force on the work clamped on the table. As the machine tool exerts vertical pressure to originate a hole it is loosely called a “drill press”.

Holes were drilled by the Egyptians in 1200 B.C. about 3000 years ago by bow drills. The bow drill is the mother of present day metal cutting drilling machine.

## TYPES OF DRILLING MACHINE

Drilling machines are made in many different types and sizes, each designed to handle a class of work or specific job to the best advantage. The different types of drilling machines are :

1. Portable drilling machine.
2. Sensitive drilling machine.
  - (a) Bench mounting,
  - (b) Floor mounting
3. Upright drilling machine.
  - (a) Round column section,
  - (b) Box column section
4. Radial drilling machine.
  - (a) Plain
  - (b) Semiuniversal
  - (c) Universal
5. Gang drilling machine.
6. Multiple spindle drilling machine.
7. Automatic drilling machine.
8. Deep hole drilling machine.
  - (a) Vertical
  - (b) Horizontal

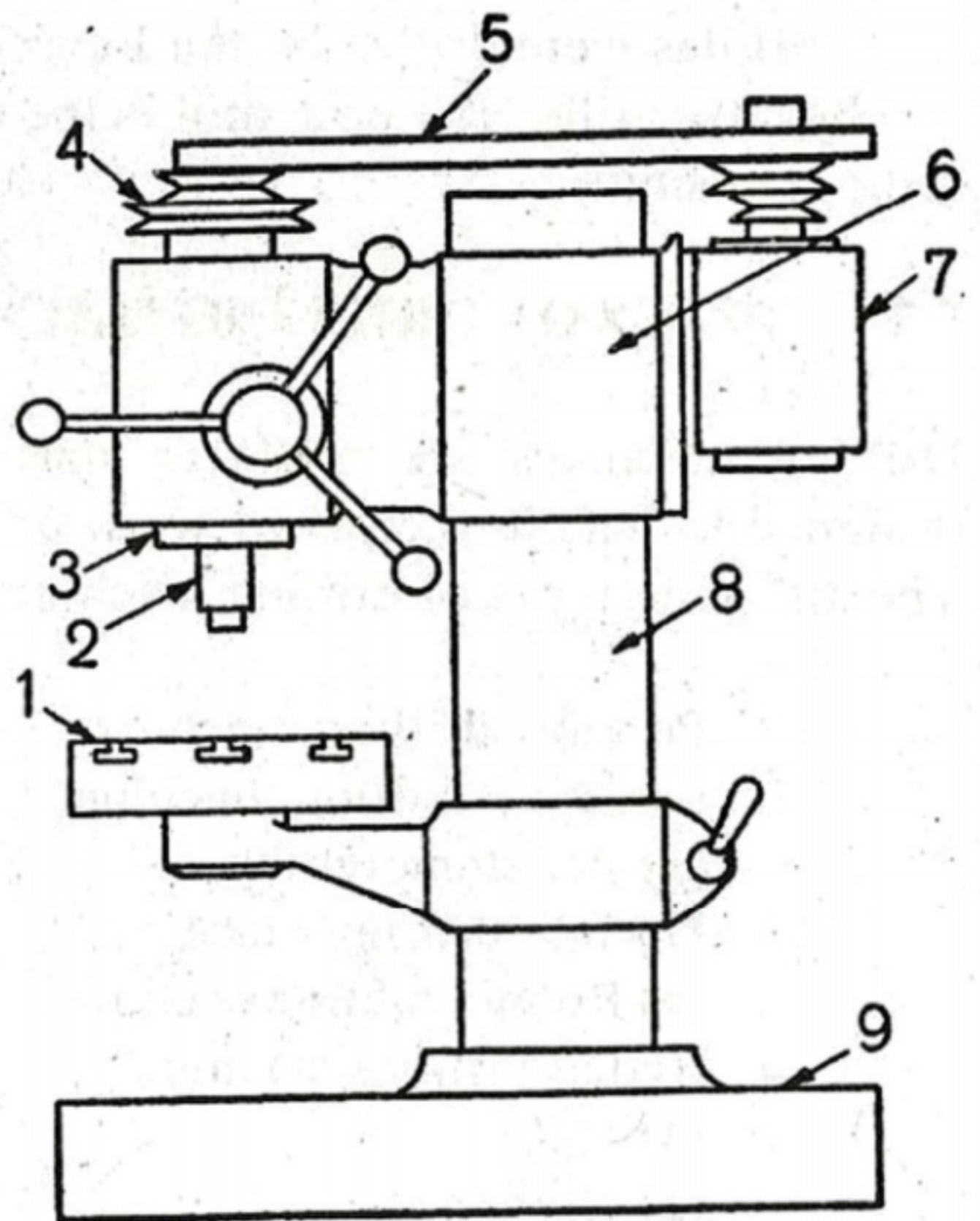


## PORTABLE DRILLING MACHINE

As the name implies this type of drilling machine can be operated with ease any where in the workshop and is used for drilling holes in workpieces in any position which can not be drilled in a standard drilling machine. Some of the portable machines are operated by hand power, but most of the machines are driven by individual motor. The entire drilling mechanism including the motor is compact and small in size. The motor is usually of universal type which may be driven by both A.C. and D.C. The maximum size of the drill that it can accommodate is not more than 12 to 18 mm. The machine is operated at high speed as smaller size drills are only used. Some of the portable machines are driven by pneumatic power.

## SENSITIVE DRILLING MACHINE

The sensitive drilling machine is a small machine designed for drilling a small holes at high speed in light jobs, The base of the machine may be mounted on a bench or on the floor. It consists of a vertical column, a horizontal table, a head supporting the motor and driving mechanism, and a vertical spindle for driving and rotating the drill. There is no arrangement for any automatic feed of the drill spindle. The drill is fed into the work by purely hand control. High speed and hand feed are necessary for drilling small holes. High speeds are necessary to attain required cutting speed by small diameter drill. Hand feed permits the operator to feel or sense the progress of the drill into the work, so that if the drill becomes worn out or jams on any account, the pressure on the drill may be released immediately to prevent it from



**Figure Sensitive drilling machine**  
1. Table, 2. Vertical drill spindle, 3. Sleeve, 4. Cone pulley, 5. V-belt, 6. Head, 7. Driving motor, 8. Vertical column, 9. Base.

released immediately to prevent it from



breaking. As the operator senses the cutting action, at any instant, it is called sensitive drilling machine. Sensitive drilling machines are capable of rotating drills of diameter from 1.5 to 15.5 mm. Super sensitive drilling machines are designed to drill holes as small as 0.35 mm in diameter and the machine is rotated at a high speed of 20,000 r.p.m. or above. Fig.5.1 illustrates a sensitive drilling machine.

## UPRIGHT DRILLING MACHINE

The upright drilling machine is designed for handling medium sized workpieces. In construction the machine is very similar to a sensitive drilling machine for having a vertical column mounted upon the base. But this is larger and heavier than a sensitive drill and is supplied with power feed arrangement. In an upright drilling machine a large number of spindle speeds and feeds may be available for drilling different types of work. The table of the machine also have different types of adjustments. There are two general classes of upright drilling machine:

1. Round column section or pillar drilling machine.
2. Box column section.

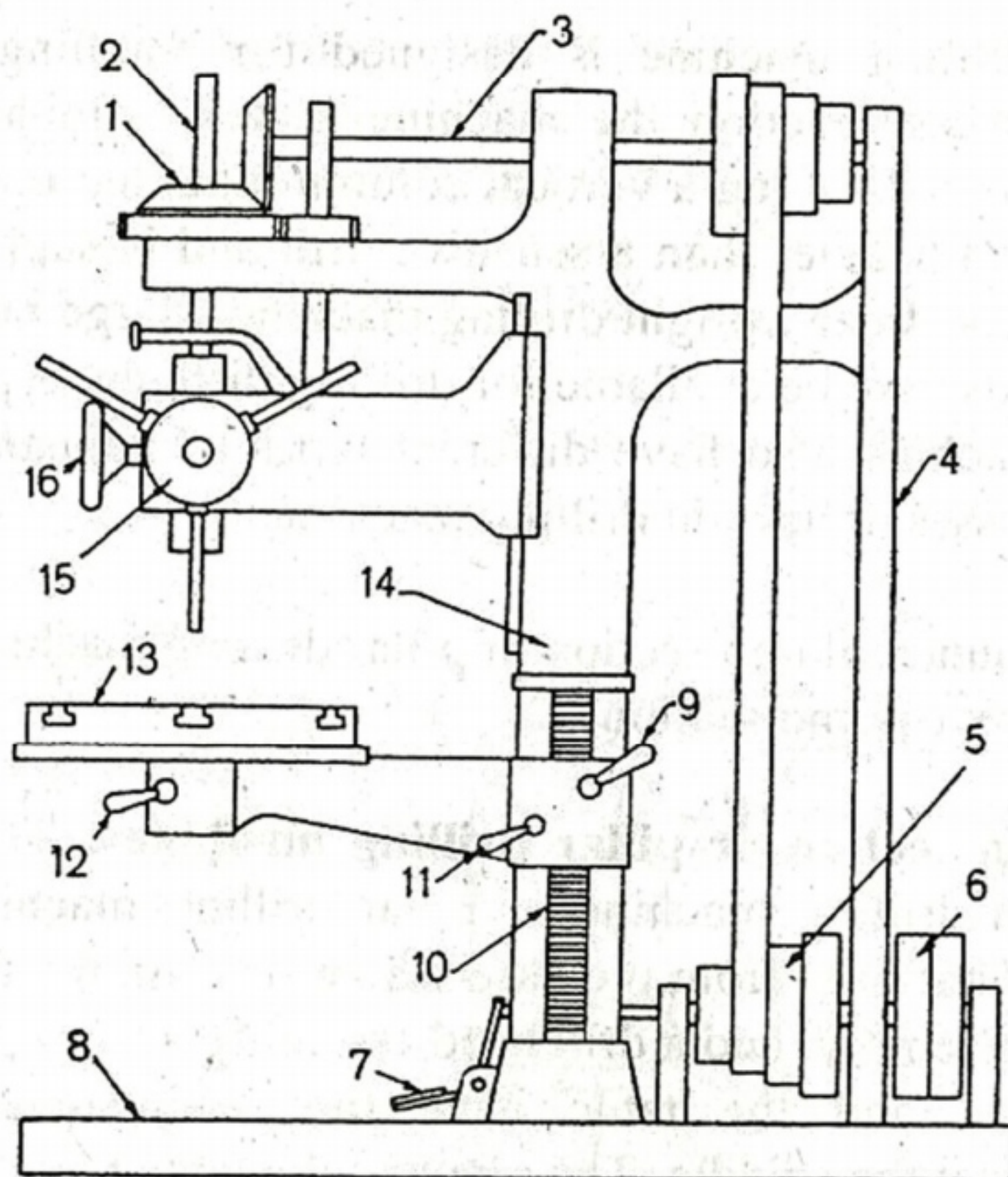
**Round column section or pillar drilling machine :** The round column section upright drilling machine or pillar drilling machine consists of a round column that rises from the base which rests on the floor, an arm and a round table assembly, and a drill head assembly.

The arm and the table have three adjustments for locating workpieces under the spindle. The arm and the table may be moved up and down on the column for accommodating workpieces of different heights. The table and the arm may be moved in an arc upto  $180^\circ$  around the column and may be clamped at any position. This permits setting of the work below the spindle. Moreover, heavy and odd-size work may be supported directly on the base of the machine and drilled after the arm is swung out of the way. The table may be rotated  $360^\circ$  about its own centre independent of the position of the arm for locating workpieces under the spindle.

The construction of the machine being not very rigid and the table being supported on a horizontal arm, this is particularly intended for lighter work. The maximum size of holes that the machine can drill is not more than 50 mm.



**Box column section upright drilling machine :** The upright drilling machine with box column section has the square table fitted on the slides at the front face of the machine column. Heavy box column gives the machine strength and rigidity. The table is raised or lowered by an elevating screw that gives additional support to the table. These special features permit the machine to work with heavier workpieces, and holes more than 50 mm in diameter can be drilled by it.



**Figure Upright pillar drilling machine**

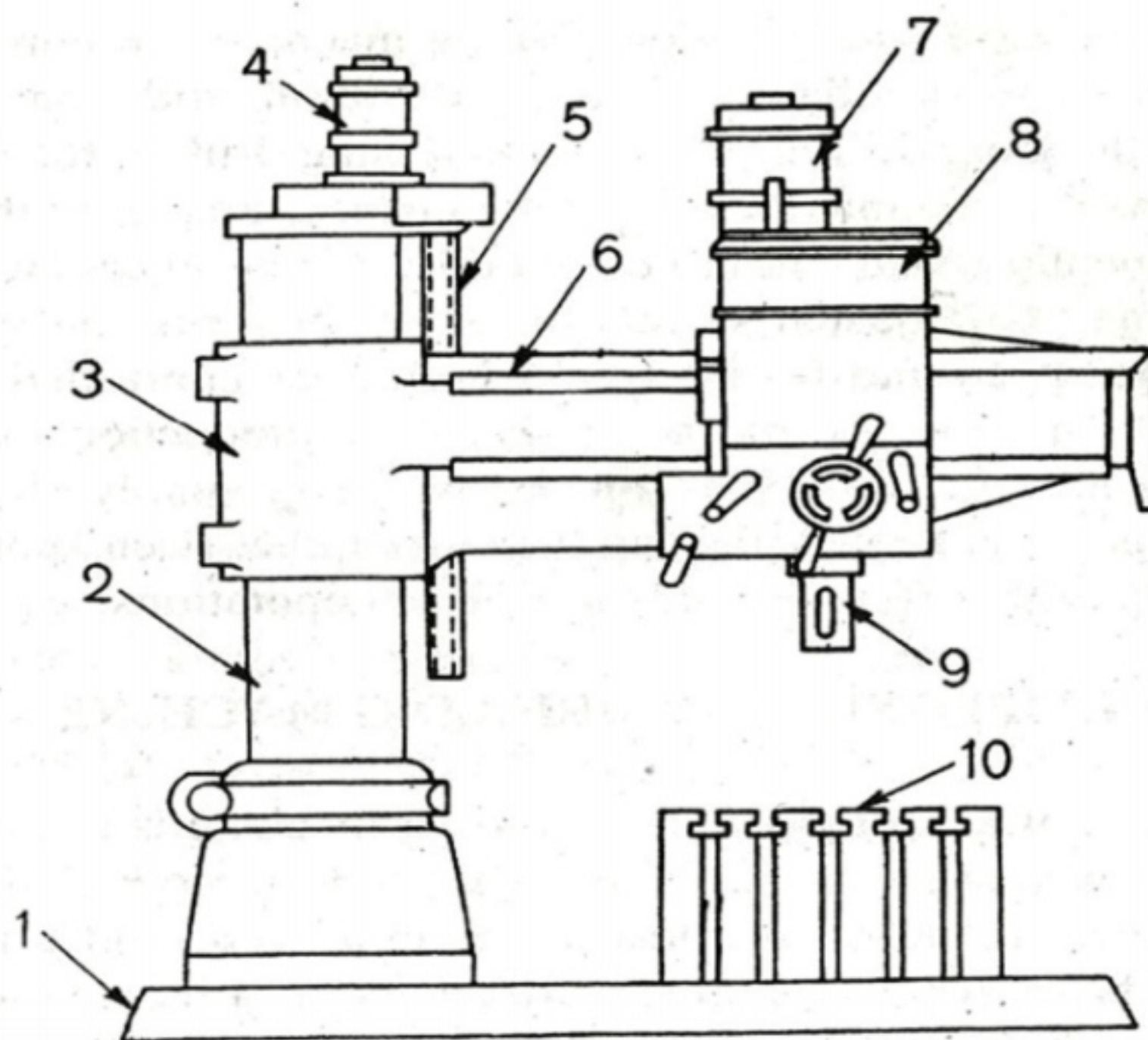
1. Bevel gear drive to spindle, 2. Spindle, 3. Overhead shaft, 4. Back stay, 5. Counter shaft cone pulley, 6. Fast and loose pulley, 7. Table elevating handle, 8. Foot pedal, 9. Base, 10. Rack on column, 11. Table elevating clamp handle, 12. Table clamp, 13. Table, 14. Column, 15. Handwheel for quick hand feed, 16. Handwheel for sensitive hand feed.

## **RADIAL DRILLING MACHINE**

The radial drilling machine is intended for drilling medium to large and heavy workpieces. The machine consists of a heavy, round, vertical column mounted on a large base. The column supports a radial arm which can be raised and lowered to accommodate workpieces of different



heights. The arm may be swung around to any position over the work bed. The drill head containing mechanism for rotating and feeding the drill is mounted on a radial arm and can be moved horizontally on the guide-ways and clamped at any desired position. These three movements in a radial drilling machine when combined together permit the drill to be located at any desired point on a large workpiece for drilling the hole. When several holes are drilled on a large workpiece, the position of the arm and the drill head is altered so that the drill spindle may be moved from one position to the other after drilling the hole without altering the setting of the work. This versatility of the machine allows it to work on large workpieces. The work may be mounted on the table or when the work is very large it may be placed on the floor or in a pit. Fig.5.3 illustrates a radial drilling machine.



**Figure Radial drilling machine**

1. Base, 2. column, 3. Radial arm, 4. Motor for elevating the arm, 5. Elevating screw, 6. Guide ways, 7. Motor for driving the drill spindle, 8. Drill head, 9. Drill spindle, 10. Table

**Plain radial drilling machine :** In a plain radial drilling machine provisions are made for vertical adjustment of the arm, horizontal movement of the drill head along the arm, and circular movement of the arm in horizontal plane about the vertical column.

**Semiuniversal machine :** In a semiuniversal machine, in addition to the above three movements, the drill head can be swung about a horizontal axis perpendicular to the arm. This fourth movement of the drill head permits drilling hole at an angle to the horizontal plane other than the normal position.

**Universal machine :** In a universal machine, in addition to the above four movements, the arm holding the drill head may be rotated on a horizontal axis. All these five movements in a universal machine enables it to drill on a workpiece at any angle.



The boring machine is one of the most versatile machine tools used to bore holes in large and heavy parts such as engine frames, steam engine cylinders, machine housings, etc. which are practically impossible to hold and rotate in an engine lathe or a drilling machine. Boring machines have, therefore, been developed primarily to do this. In addition to its primary purpose of boring the range of speeds and feeds provided to the various traversing components allow drilling, milling and facing to be performed with equal facility. By the fitting of simple attachments, the use of the machine can be extended still further to include screw cutting, turning, planetary grinding, or gear cutting.

## **TYPES OF BORING MACHINES**

The boring machines may be classified under the four headings :

1. Horizontal boring machine.
  - (a) Table type.
  - (b) Floor type.
  - (c) Planer type.
  - (d) Multiple head type.
2. Vertical boring machine.
  - (a) Vertical turret lathe.
  - (b) Standard vertical boring machine.
3. Precision boring machine.
4. Jig boring machine.
  - (a) Vertical milling machine type.
  - (b) Planer type.

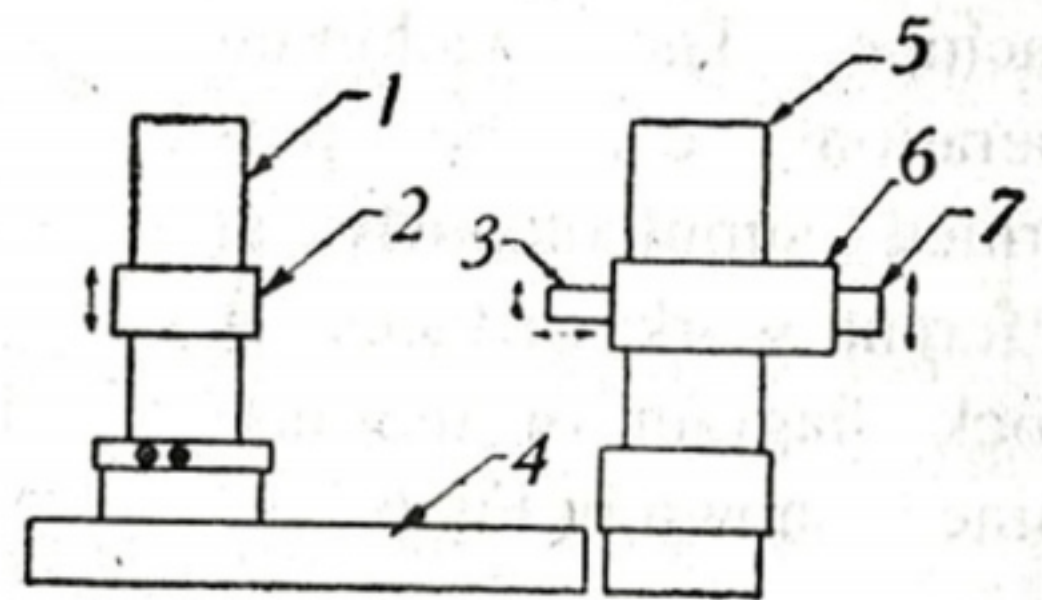
## **HORIZONTAL BORING MACHINE**

In a horizontal boring machine, the work is supported on a table which is stationary and the tool revolves in a horizontal axis. A horizontal boring



motion. The machine essentially consists of a bed, headstock supporting column, end supporting column, headstock, saddle and table, and boring bar. The table, saddle and headstock may be adjusted by leadscrews using micrometer dials. This type of machine is suitable for general purpose work where other operations, in addition to boring, are required to be performed. A block diagram of a table type machine is shown in Fig.6.2.

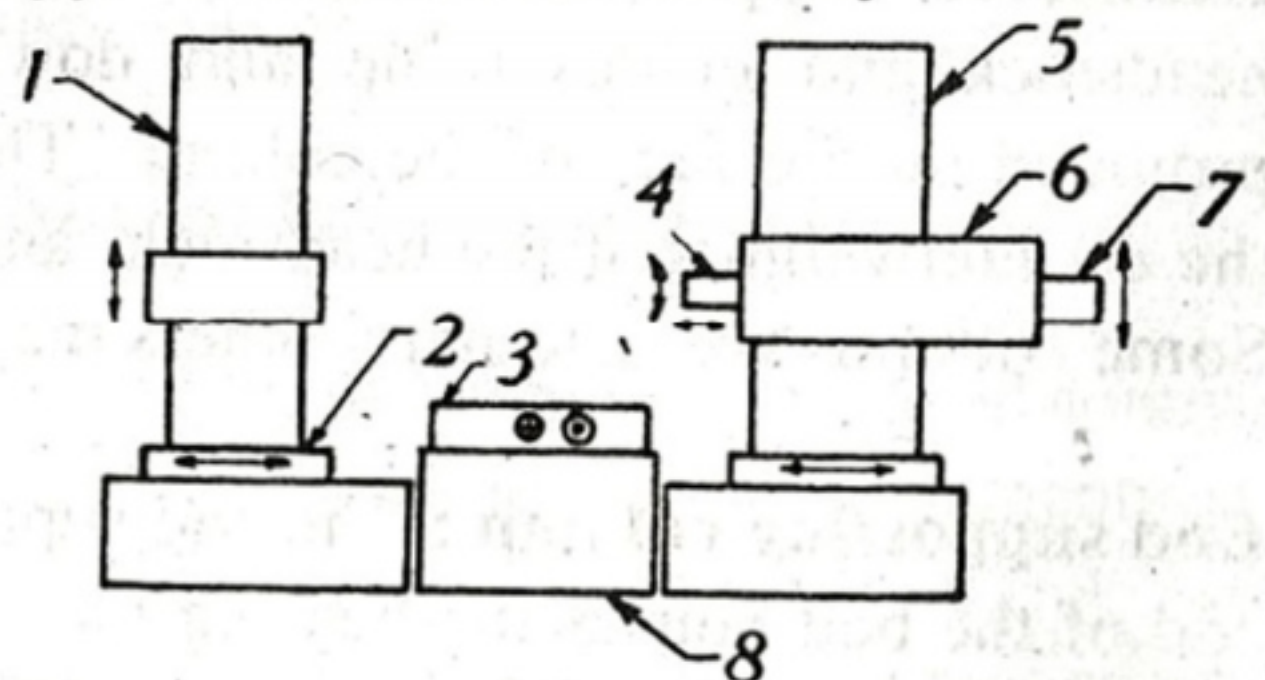
**Floor type horizontal boring machine :** The floor type horizontal boring machine having no table uses a stationary floor-plate on which T-slots are provided to hold the work. The headstock supporting column and the end supporting column and the end supporting column are mounted on the runways which are placed at right angles to the spindle axis. Thus any crosswise adjustment or cross-feed movement is provided by the spindle itself and not by the work. This is so designed for holding very large and heavy workpieces which are difficult to be mounted and adjusted on a table. A block diagram of a floor-type machine is shown in Fig.6.3.



**Figure Floor type horizontal boring machine**

1. End supporting column, 2. Column base, 3. Spindle, 4. Floor plate, 5. Headstock, 7. Motor.

**Planer type horizontal boring machine :** The planer type horizontal boring machine resembles the table type but table slides directly on the bed instead of on a saddle and reciprocates at right angles to the spindle similar to a planer. The end supporting column and headstock supporting column may be adjusted towards or away from the table for accommodating different widths of work. This type of machine is suitable for supporting a long work. A block diagram of a planer type machine shown in Fig.6.4.

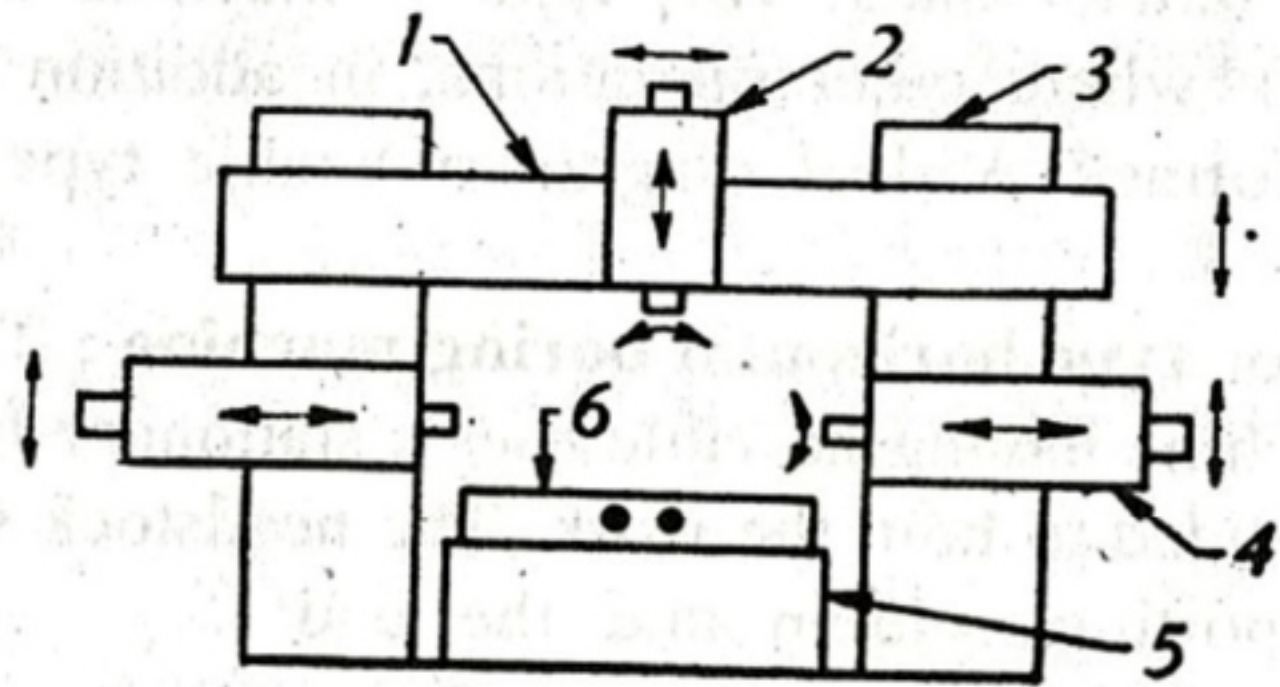


**Figure Planer type horizontal boring machine**

1. End supporting column, 2. Column base, 3. Table, 4. Spindle 5. Headstock supporting column, 6. Headstock, 7. Motor, 8. Bed.



**Multiple head type horizontal boring machine :** The machine resembles a double housing planer or a plano-miller. The table is supported on a long bed on which it reciprocates. There are two vertical columns at two sides of the bed, nearly at the middle of the bed. The two columns are bridged by a crossrail. The machine may have two, three or four headstocks. This type of machine may be used both as a horizontal and vertical machine. The machining operations can be performed simultaneously at different work surfaces. A block diagram of the machine is shown in Fig.6.5.



**Figure Multiple head type horizontal boring machine**

1. Crossrail, 2,4. Headstocks, 3. Column, 5. Bed, 6. Table.

## PART OF A HORIZONTAL BORING MACHINE

The different parts of a horizontal; boring machine are illustrates in Fig.

**Bed :** The bed is that part of the machine which is fitted on the floor of the shop and has a box like casting. The bed supports the columns, tables and other parts of the machine.

**Headstock supporting column :** The column provides support to the headstock and guides it up and down accurately by the guide ways provided on the face of the column. The column which is hollow houses the counterweights of the headstock, and is heavily ribbed to add rigidity. Some columns are stationary, others may be made to slide along the bed.

**End supporting column :** The end supporting column situated at the other end of the bed houses the bearing block for supporting a long boring bar. The column may be adjusted on the sideways of the bed towards or away from the spindle for supporting different lengths of boring bars or it may be moved at right angles to the spindle as in the case of a floor type machine.

**Headstock :** The headstock mounted on the column supports, drives, and feeds the tool. The spindle revolves within a quill. The spindle provides



The teeth of a gear or splint may be broached altogether or one or a few at a time. A comparatively simple broach can be made to cut one or a few tooth spaces. After one pass, the gear blank is indexed, and more of its teeth are cut. Successive passes are made until all the teeth are finished.

## **ADVANTAGES AND LIMITATIONS OF BROACHING**

Broaching has been adopted for mass production work because of the following outstanding features and advantages :

1. Rate of production is very high. With properly applied broaches, fixtures, and machines, more pieces can be turned out per hour by broaching than by any other means,
2. Little skill is required to perform a broaching operation. In most cases the operator merely loads and unloads the workpiece.
3. High accuracy and a high class of surface finish is possible. A tolerance of  $\pm 0.0075$  mm and a surface finish of about 0.8 microns (1 micron = 0.001mm) can be easily obtained in broaching.
4. Both roughing and finishing cuts are completed in one pass of the tool.
5. The process can be used for either internal or external surface finishing.
6. Any form that can be reproduced on a broaching can be machined.
7. Cutting fluid may be readily applied where it is most effective because a broach tends to draw the fluid into the cut.

Certain reasons, however, limit the application of the broaching process. They are :

1. High tool cost. A broach usually does only one job and is expensive to make and sharpen.
2. Very large workpieces cannot be broached.
3. The surfaces to be broached cannot have an obstruction.
4. Broaching cannot be used for the removal of a large amount of stock.
5. Parts to be broached must be capable of being rigidly supported and must be able to withstand the forces that set up during cutting.



# Chapter-10

## Surface finish, lapping

### Syllabus:

#### 10.1 Definition of Surface finish

Define super finishing □

#### 10.2 Description of lapping & explain their specific cutting.

### Introduction:

In a manufacturing plant, a product may be shaped, turned, milled or drilled, and left in that condition as being satisfactory for use. However, if a better finish is desired, for looks, for accuracy, for wearing qualities, or for any other reasons, one of the micro finished that include lapping, honing, super finishing, polishing, buffing, may be employed. In some cases other operations are done only to get durable finishes.



## **LAPPING:**

Lapping is an abrading process that is used to produce geometrically true surfaces, correct minor surface imperfection, improve dimensional accuracy, or provide a very close fit between two contact surfaces. Very thin layers of metal (0.005 to 0.01 mm) are removed in lapping and it is therefore, evident that it is unable to correct substantial error in the form and size of surface, it is however a low efficiency process is used only when specified accuracy and surface finish cannot be obtained by other method. Abrasive powders (floors such as emery, corundum iron oxide, chromium oxide mixed with oil is mixed with oil or special paste with some carrier are used in lapping.

Most lapping is done by means of lapping shoes or quills called laps that are rubbed against the work. The face of a lap becomes 'charged' with abrasive particle. Charging a lap means to embed the abrasive grains into surface. Laps may be made of almost of any material soft enough to receive and retain the abrasive grains. They are made of soft iron, brass, copper, lead or soft steel.

The method of charging a lap depends upon the shape of lap. When the lap is once charged it should be used without applying more abrasive until it ceases to cut. Lap may be operated by hand or machine, the motion being rotary or reciprocating.

Cylindrical work may be lapped by rotating the work in the lathe or drill press and reciprocating the lap over the work in an ever-changing path. Small flat surfaces may be lapped by holding the work against a rotating disc, or the work may be moved by hand in an

irregular path over a stationary face plate lap. In equalising lapping the work and lap mutually improve each other surfaces as they slide on each other.

There are three important types of lapping machines. The vertical axis lapping machine laps flat or round surfaces between two oppose laps on vertical spindle. The centre-less lapping machine is designed for continuous production of round parts such as piston pins, bearing races and cups, valve tappets and shafts.

The centre less lapping machine on the same principle as centre less grinding. The abrasive belt lapping machine lapps bearings and cam surfaces by means of abrasive coated clothes.



## Fluid Mechanics

### Chapter - I Properties of fluid

Any substance which flows is called as fluid. For example, oil, water, gas. Mechanics deals with the rest and motion of any substance. So fluid mechanics is the study of either rest or motion of any fluid. In this subject we will deal with only liquids. We know that liquid possesses many properties out of which some are described below.

#### Properties:

##### Density or specific mass or Mass Density:

It is defined as the ratio of mass with volume at standard pressure and temperature.

Mathematically,  $\text{Density} = \frac{\text{mass}}{\text{Volume}}$

It is denoted by 'S'. Both S.I. and M.K.S. units are  $\text{kg/m}^3$ . The density of water is  $1000 \text{ kg/m}^3$ .

##### Specific weight or weight density:

It is defined as the ratio of weight with volume at standard pressure and temperature. It is denoted by 'w'.

Mathematically,  $\text{specific weight} = \frac{\text{Weight}}{\text{Volume}}$

The S.I. unit is  $\text{N/m}^3$ , while M.K.S. unit is  $\text{kg/m}^3$ .

The specific weight of water is  $9800 \text{ N/m}^3$ .

##### Specific Gravity:

It is defined as the ratio of the specific weight of any liquid with the specific weight of a standard substance at the standard temperature. For liquids, the pure water at  $4^\circ\text{C}$  is taken as the standard substance.

Mathematically,  $\text{Specific Gravity} = \frac{\text{Specific weight of the liquid (any liquid)}}{\text{Specific weight of pure water at } 4^\circ\text{C}}$

It is denoted by 'S'. It is unitless.

Mathematically,  $S_{\text{liquid}} = \frac{w_{\text{liquid}}}{w_{\text{water}}}$



The specific gravity of water is 1, while for Hg (Mercury) it is 13.6

### Specific volume:-

It is defined as the ratio between volume with mass. Mathematically,  $\text{specific volume} = \frac{\text{volume}}{\text{mass}}$

It is the reciprocal of density.

### Viscosity:-

It is defined as the property which resists the movement of one layer of liquid on the other layer (adjacent). It is also the resistance offered against the flow of the liquid.

### Newton's law of viscosity:-

It states that the shear stress on a layer of fluid is directly proportional to the rate of shear strain. Mathematically,  $\tau \propto \frac{v}{y}$

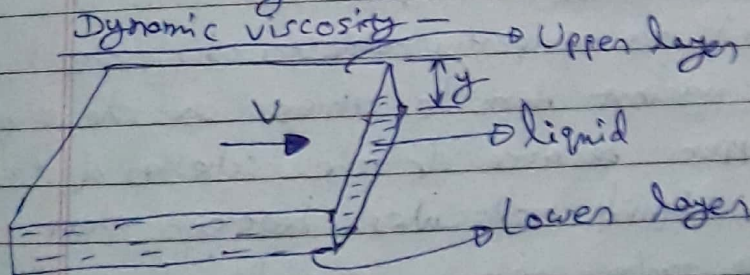
where;  $\tau$  = Shear stress,

$v$  = velocity of fluid or liquid

$y$  = thickness of the liquid layer

$\frac{v}{y}$  = Rate of shear strain or velocity gradient

### Dynamic viscosity:-



$\therefore \tau = (\mu) \left( \frac{v}{y} \right)$  where;  $\mu$  = proportionality constant or coefficient of viscosity or coefficient of dynamic viscosity or absolute viscosity

In S.I. system, the unit of ' $\mu$ ' is poise

Also unit of ' $\mu$ ' is  $\text{N-s/m}^2$  (S.I.)

1 Poise =  $0.1 \text{ N-s/m}^2$

Centipoise (cP) is the smaller unit used.

1 centipoise =  $\frac{1}{100}$  of Poise

(P-2)

$$\mu = \tau \left( \frac{y}{v} \right)$$



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" $\mu$ " is called as the dynamic viscosity.

### Kinematic viscosity -

It is the ratio of the absolute viscosity with the density of the liquid. It is denoted by " $\nu$ " and is the modified form of viscosity.

Mathematically,  $\nu = \frac{\mu}{\rho}$

where;  $\mu$  = absolute viscosity,  $\rho$  = density

In S.I., the unit is stoke and also  $\frac{m^2}{s}$ .

1 stoke =  $10^{-4} m^2/s$

The smaller unit is centistoke and

1 centistoke =  $\frac{1}{100}$  of stoke.

### Compressibility:-

It is the property due to which the volume changes with change in pressure. But with change in pressure, the change in volume of water is very small and can be neglected. So, for practical purposes, the water is considered to be incompressible.

### Surface Tension:-

It is the property due to which the liquid resists against tensile stress at the surface. It happens because of the cohesion of the surface liquid molecules between them.

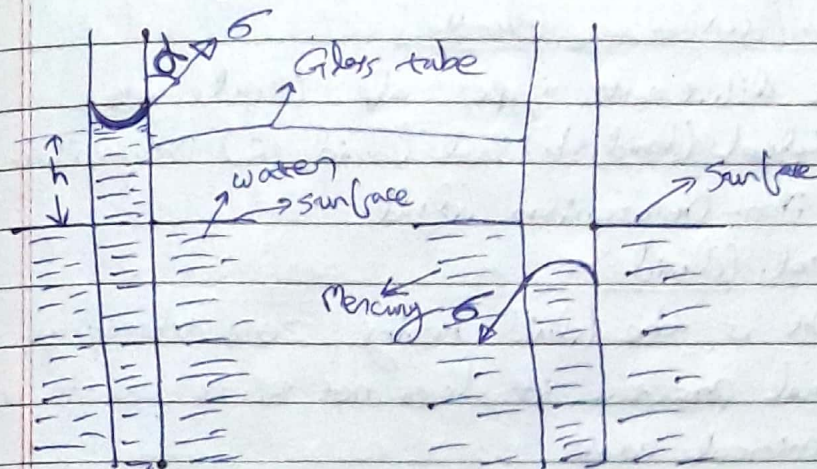
Due to surface tension, the liquid surface tries to reduce its surface as small as possible.

Due to this, the rain water drops falling on the earth are of spherical shape. The water surface inside a small glass tube dipped in water becomes concave due to it. It also makes the mercury surface convex when mercury is taken in the glass tube instead of water. It is also used to produce the spherical lead shots.



It is denoted by ' $\sigma$ ' and S.I. unit is N/mm.

Capillary:-



When we dip any small diameter glass tube in water, then water rises in the tube having upward concave surface. Here the adhesion between the glass tube and water molecules is more than the cohesion between the water molecules. While taking mercury in the glass tube in place of water, the mercury falls in the tube having downward convex surface. Here the adhesion between glass and mercury molecules is less than the cohesion between mercury molecules. Thus there is either rise or fall of liquid inside the glass tube. This rise or fall is called capillary rise or capillary fall. This property of liquid is called as capillarity. We know that,

Downward weight of water column in the glass tube  
= Vertical component of force of surface tension  
or,  $(wh) \times \frac{\pi}{4} d^2 = (\sigma)(\pi d)(\cos \theta)$

where,  $w$  = specific weight of water,

$h$  = height of capillary rise of water,

$d$  = Diameter of the glass tube

$\theta$  = angle of contact of water with tube surface

$\sigma$  = Force of surface tension/unit length of tube



$$or, h = \frac{4G \cos \alpha}{\omega d}$$

### Classification of fluids:

The different types of fluids are;  
 (a) Ideal fluid, (b) Real fluid, (c) Newtonian fluid,  
 (d) Non-Newtonian fluid.

#### Ideal fluid:-

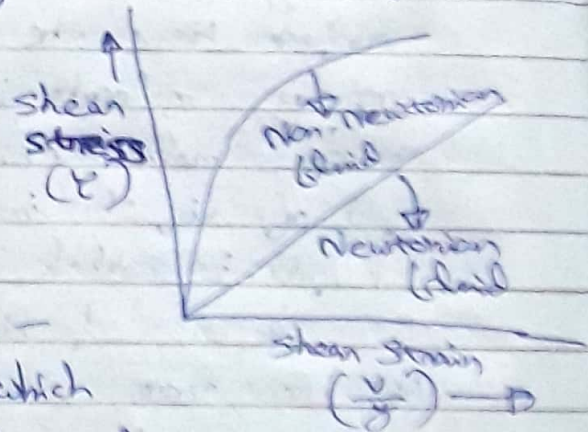
It is the fluid having zero viscosity. In actual practice it does not exist. So it is theoretical case.

#### Real fluid:-

It is the fluid which possesses some viscosity. Practically all fluids are real fluids.

#### Newtonian fluid:-

It is the fluid which obeys the Newton's law of viscosity. It is the fluid having no change in viscosity with the rate of deformation. It is represented by the straight line as shown in the side graph.



#### Non-Newtonian fluid:-

It is the fluid, which does not obey the Newton's law of viscosity.

Here, the viscosity value changes with the rate of deformation or shear strain. In the graph, the Non-Newtonian fluid is shown by a curve.



### Problem - 1

The weight of  $2.5 \text{ m}^3$  of a liquid was found to be  $18.75 \text{ kN}$ . Calculate the specific weight and density of the liquid.

Solution

Volume,  $V = 2.5 \text{ m}^3$ ; weight,  $W = 18.75 \text{ kN} = 18750 \text{ N}$   
Specific weight of liquid,  $w = \frac{W}{V} = \frac{18750}{2.5} = 7500 \frac{\text{N}}{\text{m}^3}$

Mass of liquid,  $m = \frac{W}{\text{Acceleration due to gravity } (g)}$   
 $= \frac{18750}{9.81} = 1.91 \times 10^3 \text{ kg}$

Density of liquid,  $\rho = \frac{m}{V} = \frac{1.91 \times 10^3}{2.5} = 764 \text{ kg/m}^3$

### Problem - 2

A container having volume of  $4 \text{ m}^3$  contains oil having weight equal to  $30.2 \text{ kN}$ . Calculate the specific gravity of the oil.

Solution

Volume,  $V = 4 \text{ m}^3$ ; Weight,  $W = 30.2 \text{ kN} = 30200 \text{ N}$   
Specific weight of oil,  $w_{\text{oil}} = \frac{W}{V} = \frac{30200}{4} = 7550 \frac{\text{N}}{\text{m}^3}$

Specific gravity of oil,  $S_{\text{oil}} = \frac{w_{\text{oil}}}{w_{\text{water}}} = \frac{7550}{9810} = 0.77$

### Problem - 3

The space between two parallel square plates each of side  $0.8 \text{ m}$  is filled with an oil of specific gravity  $0.8$ . If the space between the plates is  $12.5 \text{ mm}$  and the upper plate moves with velocity of  $1.25 \text{ m/s}$  needs force equal to  $51.2 \text{ N}$ , then calculate both dynamic and kinematic viscosities of oil.

Solution

Side of each square  $= 0.8 \text{ m}$ ;  $S_{\text{oil}} = 0.8$ ;

(P-6)



Distance between plates  $y = 12.5 \text{ mm} = 0.0125 \text{ m}$ ;  $V = 1.25 \text{ m/s}$   
 Force  $= 51.2 \text{ N}$

Area of square plate  $= \text{side} \times \text{side} = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Shear stress,  $\tau = \frac{\text{Force}}{\text{Area}} = \frac{51.2}{0.64} = 80 \text{ N/m}^2$

Again,  $\tau = (\mu) \left( \frac{V}{y} \right)$

Dynamic viscosity  
 or,  $\mu = \frac{(\tau)(y)}{V} = \frac{(80)(0.0125)}{1.25}$

or,  $\mu = 0.8 \frac{\text{Ns}}{\text{m}^2} = 8 \text{ P}$ , where P = poise

Density of oil,  $\rho_{\text{oil}} = \rho_{\text{oil}} \times \rho_{\text{water}}$   
 $= 0.8 \times 1000 = 800 \text{ kg/m}^3$

Kinematic viscosity,  $\nu = \frac{\mu}{\rho_{\text{oil}}} = \frac{0.8}{800}$

or,  $\nu = 10^{-3} \text{ m}^2/\text{s} = 10 \text{ stokes}$

#### Problem - 4

Find out the capillary effect for a glass tube of 4mm diameter when immersed in water and in mercury. The surface tension for water and mercury in contact with air are  $0.0735 \text{ N/m}$  and  $0.51 \text{ N/m}$  respectively. The contact angle for water and mercury are  $0^\circ$  and  $130^\circ$  respectively.

Solution

Diameter of tube,  $d = 4 \text{ mm} = 0.004 \text{ m}$ ;

$\sigma_w = 0.0735 \text{ N/m}$ ;  $\sigma_{\text{Hg}} = 0.51 \text{ N/m}$ ;  $\theta_w = 0^\circ$ ;  $\theta_{\text{Hg}} = 130^\circ$

Specific weight of water,  $w_w = 9800 \text{ N/m}^3$

Capillary rise of water,  $h_w = \frac{4 \sigma_w \cos \theta_w}{(w_w)(d)}$

or,  $h_w = \frac{4 \times 0.0735 \times \cos 0^\circ}{9800 \times 0.004} = 0.0075 \text{ m}$   
 $= 7.5 \text{ mm}$

Capillary fall of mercury,  $h_{\text{Hg}} = \frac{4 \times \sigma_{\text{Hg}} \times \cos \theta_{\text{Hg}}}{(w_{\text{Hg}})(d)}$

$= \frac{4 \times 0.51 \times \cos 130^\circ}{132800 \times 0.004} = -0.00247 \text{ m}$   
 $= -2.47 \text{ mm}$

(P-7)



### Question Bank:

- (1) Define about fluid mechanics.
- (2) What do you mean by mass density?
- (3) Define specific weight with unit (S.I.).
- (4) Define about specific gravity.
- (5) What is specific volume of liquid?
- (6) What do you understand by viscosity?
- (7) Write down the law of viscosity (Newton's).
- (8) Define about Dynamic viscosity.
- (9) Define Kinematic viscosity.
- (10) Define about compressibility.
- (11) Write down about surface tension in one sentence.
- (12) In a single sentence write down about capillarity.
- (13) What do you mean by ideal fluid?
- (14) What is real fluid?
- (15) Define about Newtonian fluid.
- (16) Define Non-Newtonian fluid.

### Long Questions -

- (17) Write short notes about, density, specific weight and specific gravity of liquid.
- (18) Write briefly about both dynamic and kinematic viscosity.
- (19) Explain briefly about the surface tension and capillarity of liquid.
- (20) Classify the different types of fluids and write brief notes about them.



## Chapter - 2

### Fluid Pressure and its measurement

#### Fluid Pressure:-

Whenever any fluid is kept in a container, then it exerts force at all points of all sides of the vessel. This force acting per unit area is called as fluid pressure. It is measured in Pascal (Pa) and in  $N/m^2$  (SI), in  $kgf/cm^2$  (MKS) and also in bar.

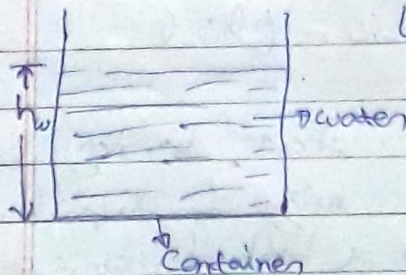
#### Intensity of Pressure:-

It is the ratio of the force of fluid with the area on which it is acting. It is denoted by 'P'. Mathematically,  $P = \frac{F}{A}$  where, F = Force acting and A = Area.

The intensity of pressure always acts at right angles to the surface on which fluid supplies force. It is also called as pressure. It is measured in bar, pascal,  $N/m^2$  and  $kgf/cm^2$ .

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

#### Pressure Head:-



Let the container as shown in figure contains water. This water will produce pressure on all sides and bottom of the container.

$w_w$  = specific weight of water

$h_w$  = height of water in the container

A = Area of base of the container

The water will exert pressure on the base of the container due to its weight (of the water).

Hence, pressure,  $P = \frac{\text{Weight of water}}{\text{Area of container base}}$

$$\text{or, } P = \frac{w \times h \times A}{A}$$

$$\text{or, } \boxed{P = w h_w} \quad \text{or, } \boxed{h_w = \frac{P}{w}}$$

(P-9)



Thus we know that the intensity of pressure is proportional to the height of water above the base. So the pressure can be expressed in terms of depth of water and this height of water is called as pressure head. So,  $h_w$  = Pressure head of water. Both MKS and S.I. units are mtr.  
Pascal's Law:-

It states that the intensity of pressure at any point in a fluid (which is at rest), is same in all the directions.

Atmospheric Pressure:-

It is the pressure of the atmosphere or atmospheric air on the earth surface. As air is compressible, hence its density varies with height, temperature and humidity. So, it cannot be calculated like the liquids. But it can be measured by finding the liquid column height which it can support. At the sea level, the pressure exerted by  $1\text{m}^2$  cross-sectional area air column is 103 kN. So atmospheric pressure at sea level is 103 kN or 103 kPa or 10.3 mtr. of water column or 760 mm. of mercury column.

Gauge Pressure:-

The pressure which is measured by using any pressure measuring instrument by taking the atmospheric pressure as datum is called as gauge pressure. Generally the atmospheric pressure is marked as zero on the gauge scale. This gauge pressure is of two types like (a) positive gauge pressure and (b) negative gauge pressure.

Positive gauge Pressure:-

The value of it is positive and it is

(P-10)

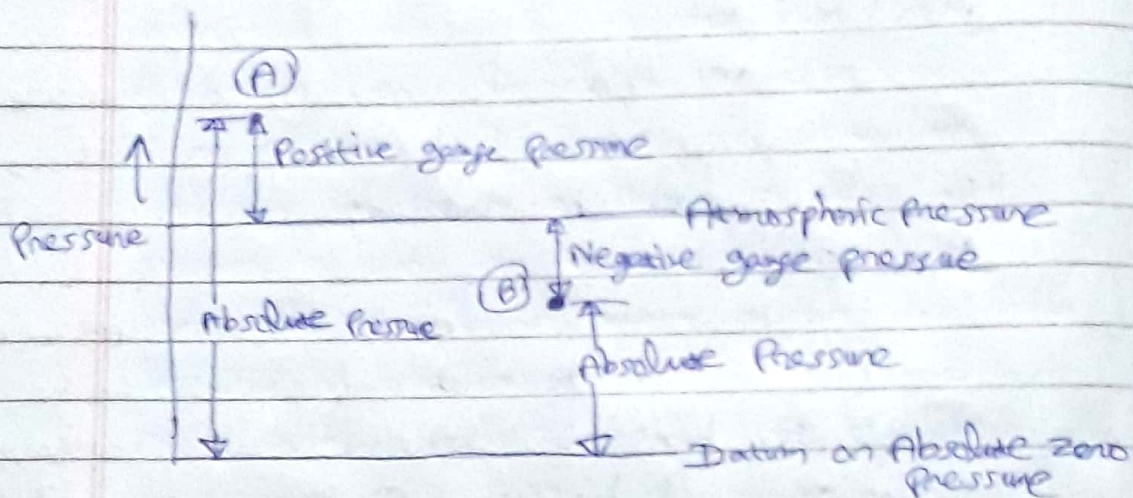


above the atmospheric pressure.

Negative Gauge Pressure or Vacuum Pressure —

It has negative value and measured below the atmospheric pressure. It is either having unit of  $N/m^2$  or  $kgf/m^2$  or  $mm$ .

Absolute Pressure —



Absolute pressure is the pressure equal to the algebraic sum of atmospheric pressure and gauge pressure. Let 'A' is a point. Then pressure at 'A' from Absolute zero pressure is the absolute pressure at A. Absolute pressure at A = Gauge pressure at A (Positive gauge pressure) + Atmospheric pressure. But Absolute pressure at B = Atmospheric pressure - Gauge pressure at B (negative gauge pressure). So, absolute pressure can also be defined as the total pressure measured from the absolute zero pressure.

Mathematically,  $P_{abs} = P_{atm} + P_{gauge}$

where,  $P_{abs}$  = Absolute Pressure

$P_{atm}$  = Atmospheric Pressure

$P_{gauge}$  = Gauge Pressure

Hence,  $P_{abs} = P_{atm} + P_{gauge}$  (for positive gauge pressure)

and  $P_{abs} = P_{atm} - P_{gauge}$  (for vacuum gauge pressure)

(P-11)



## Pressure Measurement:-

It can be measured in 2 ways.

- ① By balancing one liquid column whose pressure is to be measured with another liquid column. These are called as tube gauges.
- ② By balancing the liquid column with spring force on dead weight. These are called as mechanical gauges.

### Tube gauges —

These are of two types.

- ① Piezometer
- ② Manometer

### Piezometer —

It is a L-shaped glass tube used to measure positive gauge pressure only.

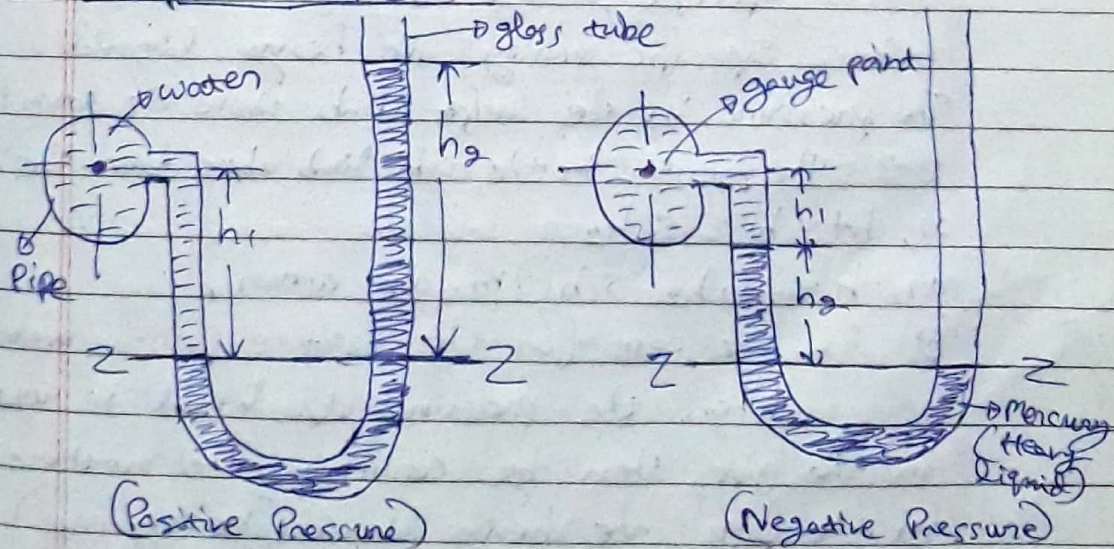
### Manometer —

It is the improved version of piezometer and can measure both positive and negative gauge pressures.

The different types are;

Simple manometer, Differential manometer.

### Simple manometer —



The simple manometer is a U-shaped bent glass tube attached to the gauge point for measuring both positive and negative gauge pressure.



and also high pressure. It is also called as U-tube manometer. In the figure the manometer is connected to a pipe containing light liquid (water here) at one end and other end is free to atmosphere. Another heavy liquid (Mercury here) is taken in the U-tube. When connected, the high pressure light liquid will force the heavy liquid in the left limb of U-tube to move downward. Due to this the heavy liquid will rise in the right limb. The datum line or common surface is the line where both heavy and light liquid will meet in the left limb. The Z-Z is the datum line.

$h_1$  = height of light liquid above the datum line in the left limb in m

$h_2$  = height of the heavy liquid above the datum line in the right limb in m

$h$  = Pressure of water in the pipe in terms of water head in m

$s_1$  = Specific gravity of light liquid

$s_2$  = Specific gravity of heavy liquid

The pressure in the left limb above the datum line

Z-Z = The pressure in the right limb above datum line Z-Z

$$\text{or, } h + s_1 h_1 = s_2 h_2$$

$$\text{or, } h = (s_2 h_2 - s_1 h_1) \text{ m. of water.}$$

It is the case for the positive pressure measurement

When the pressure of liquid is negative in the pipe, then by connecting the manometer in the pipe, the heavy liquid (mercury here) will rise in the left limb. The heavy liquid will fall in the right limb. The datum line may be taken as the meeting line of both



Light and heavy liquid in the left limb or the height of the heavy liquid in the right limb. Here second case is taken as 2-2 line due to simple in calculation.

$h_1$  = height of light liquid in the left limb above the meeting line of light and heavy liquid

$h_2$  = height of heavy liquid in the left limb above the datum line 2-2

So, Pressure in the left limb above datum line (2-2)

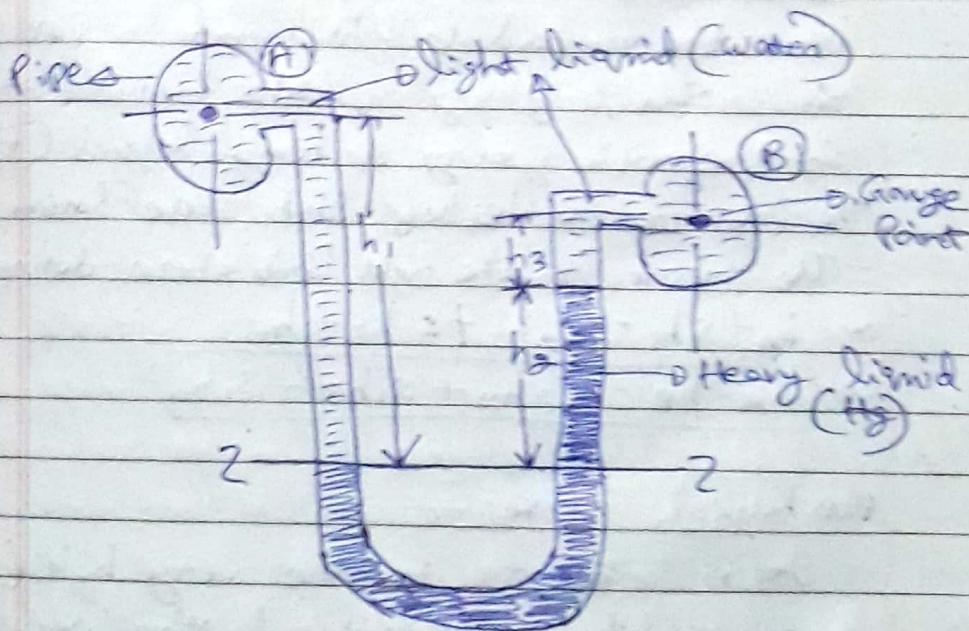
= Pressure in the right limb above datum line (2-2)

or,  $h_1 \rho_1 + \rho_2 h_2 = 0$

or,  $h = -\rho_1 h_1 - \rho_2 h_2 = -(\rho_1 h_1 + \rho_2 h_2)$  m of water

Thus it measures the negative gauge pressure

Differential manometer —



It is used to measure pressure difference between any two points or between different pipes. The manometer containing heavy liquid (Hg here) is connected with two different pipes having different gauge points named as A and B by means of its two ends. The pipes contain light liquid (water here), where

(P-14)



the pressure of liquid at A is more than the pressure of liquid at B. So, the high pressure liquid at A will force the heavy liquid to move downwards, while there will be corresponding rise of heavy liquid in the right limb. The meeting line of light and heavy liquid in the left limb is taken as datum line Z-Z here.

$h_1$  = Height of light liquid in left limb above Z-Z

$h_2$  = Difference in height of heavy liquid in both left and right limb or reading of differential manometer or manometer reading

$h_3$  = height of liquid (light) in the right limb above Z-Z

$h_A$  = Pressure head of liquid (light) in Pipe A

$h_B$  = Pressure head of light liquid in Pipe B

$S_1$  = Specific gravity of light liquid (water)

$S_2$  = Specific gravity of heavy liquid (Hg)

The pressure in the left limb above datum Z-Z

= The pressure in the right limb above datum Z-Z

$$\text{or, } h_A + S_1 h_1 = S_2 h_2 + S_1 h_3 + h_B$$

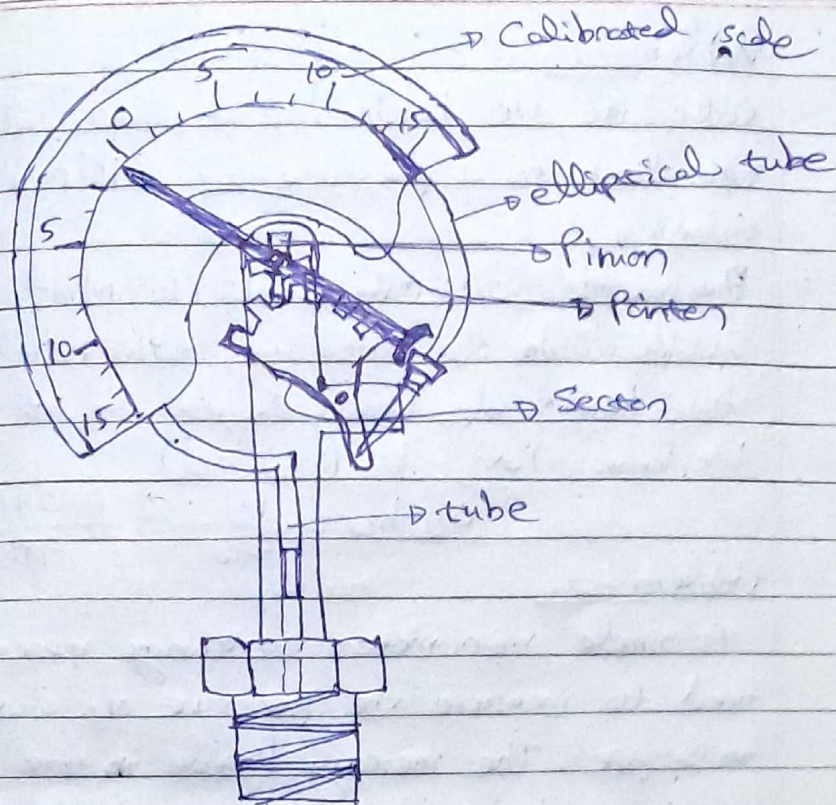
$$\text{or, } h_A - h_B = S_2 h_2 + S_1 h_3 - S_1 h_1$$

### Mechanical Gauges —

It is used to measure very high fluid pressures like in boilers and other high pressure equipments (Compressor). The Bourdon's tube pressure gauge is one of the commonly used type. Bourdon's tube pressure gauge —

It is used to measure both positive and negative gauge pressure. It consists of an elliptical tube which is called as Bourdon tube.





(Bourdon Tube)

The elliptical tube is encased in a circular cover. The elliptical tube is connected with a tube containing fluid under pressure at the bottom side. So the fluid under pressure enters the elliptical tube and the elliptical tube tries to straighten itself. As it is inside circular cover, so the tube becomes circular. Due to this elastic deformation the sector attached with one end of Bourdon's tube moves. Then the pinion attached with sector also moves. So finally the pointer attached with the pinion rotates over a calibrated scale and shows directly the pressure reading. Thus we can measure the pressure of the fluid directly and get the numerical value of it. This type is most commonly used.



### Problem - 1

Calculate the height of a water column equivalent to a pressure of 0.15 MPa.

Solution

$$\text{Pressure, } p = 0.15 \text{ MPa} = 0.15 \times 10^6 \text{ N/m}^2$$

$$\text{Specific weight of water, } w_w = 9810 \text{ N/m}^3$$

$h_w$  = height of water column in m.

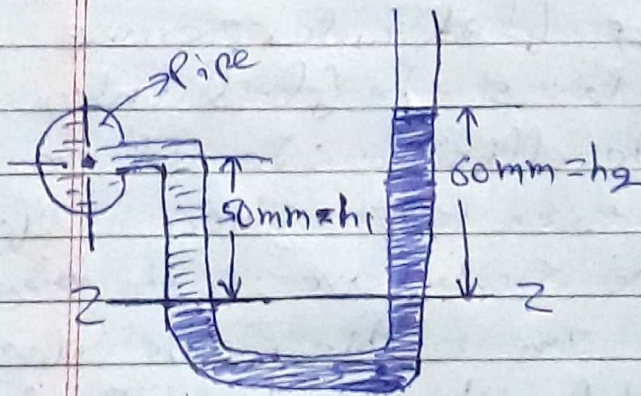
$$\text{We know that; } p = (w_w)(h_w)$$

$$\text{or, } h_w = \frac{p}{w_w} = \frac{0.15 \times 10^6}{9810} = 15.3 \text{ m}$$

### Problem - 2

A simple manometer containing mercury is used to measure the pressure of water flowing in a pipe. The mercury level in the open tube is 60 mm higher than that on the left tube. If the water height in the left tube is 50 mm, then calculate the pressure in the pipe in terms of water head.

Solution



The height of water in the left tube,  $h_1 = 50 \text{ mm}$ .  
The height of mercury in the right tube above the datum line Z-Z,  $h_2 = 60 \text{ mm}$ .

$h$  = Pressure in the pipe in terms of water head.

Pressure in the left tube above datum Z-Z

= Pressure in the right tube above datum Z-Z

$$\text{or, } h + s_1 h_1 = s_2 h_2 \quad ; \text{ where, } s_1 = \text{specific gravity of water, and } s_2 = \text{specific gravity of Hg.}$$

$$\text{or, } h = h_2 s_2 - s_1 h_1 = 60 \times 13.6 - 1 \times 50 = 766 \text{ mm. of water.}$$

### Problem - 3

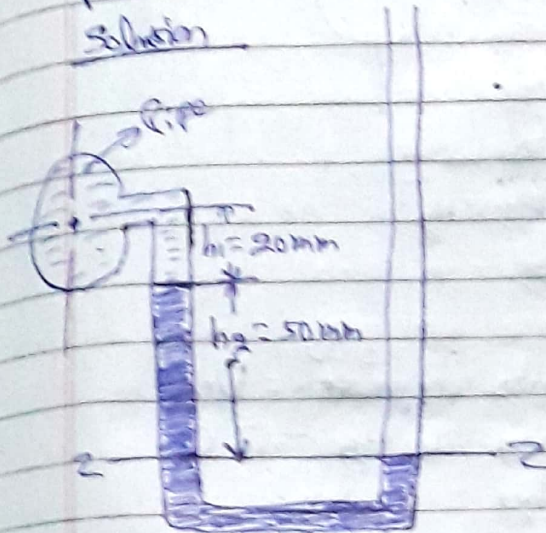
A simple manometer having mercury was used to

(P-11)



Find the negative pressure in the pipe containing water. The mercury level in the left side above datum line is 50 mm and water level in the left side from the mercury meniscus line upto the pipe centre line is 20 mm. Calculate the negative pressure below the atmosphere in the pipe.

Solution



$h_1$  = height of water from the upper level of Hg upto the centre line of pipe = 20 mm (in left side)  
 $h_2$  = height of Hg from Datum line 2-2 upto the meniscus line

of water and mercury in the left tube = 50 mm  
 $h$  = water pressure in the tube in terms of head of water.

Pressure in the left tube above datum 2-2 = Pressure in the right tube above datum 2-2  
 or,  $h_1 s_1 + h_2 s_2 = 0$

where,  $s_1$  = specific gravity of water = 1

and  $s_2$  = specific gravity of mercury = 13.6

$$\text{or, } h = -h_1 s_1 - h_2 s_2 = -20 \times 1 - 50 \times 13.6 = -700 \text{ mm}$$

$$\text{or, } h = -7 \text{ m. of water}$$

Gage pressure in the pipe,  $P = \rho_w \times h$

where,  $\rho_w$  = specific weight of water

$$\text{or, } P = 9810 \times (-7) = -68670 \text{ N/m}^2 = -68670 \text{ Pa} \\ = 68670 \text{ Pa (Vacuum)}$$

Problem-4

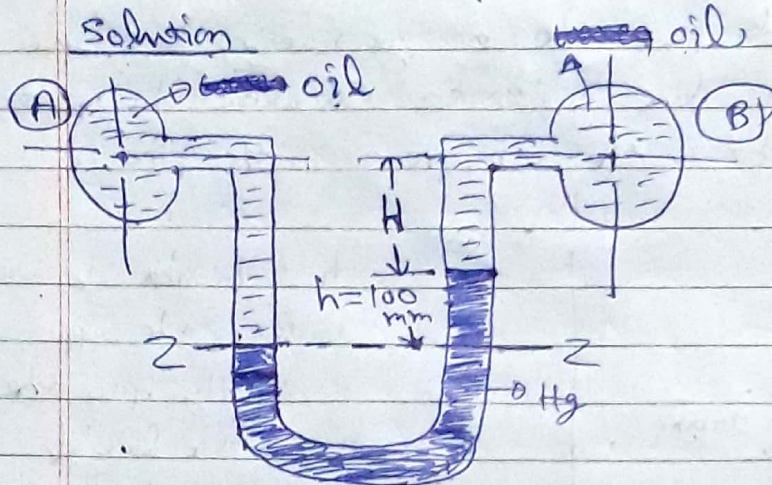
A differential manometer connected at two points A and B at the same level in a pipe

(P-15)



containing an oil of specific gravity 0.8, shows a difference in mercury level as 100 mm. Find out the difference in pressures at the two points.

Solution



$S_1$  = Specific gravity of <sup>oil</sup> ~~mercury~~ = 0.8

$S_2$  = Specific gravity of Hg = 13.6

$h$  = manometer reading = 100 mm

$H$  = Height between upper level of mercury in right tube and pipe centre named as B.

Pressure head in the left limb above datum Z-Z

= Pressure head in the right limb above Z-Z

$$\text{or, } h_A + S_1 H + S_1 h = h_B + S_2 h + S_1 H$$

where;  $h_A$  = pressure head in pipe having point A,  
 $h_B$  = pressure head in pipe having point B

$$\text{or, } h_A + 0.8 \times 100 = h_B + 13.6 \times 100$$

$$\text{or, } h_A - h_B = 13.6 \times 100 - 0.8 \times 100 = 1280 \text{ mm. of } \text{<sup>oil</sup> ~~mercury~~}$$

$\therefore$  Pressure difference,  $P_A - P_B = w_o (h_A - h_B)$

where,  $w_o$  = specific weight of oil

$$= w_w \times S_1 = 9810 \times 0.8 = 7848 \text{ N/m}^3$$

where,  $w_w$  = specific wt. of water

$$\text{or, } P_A - P_B = 7848 \left( \frac{1280}{1000} \right)$$

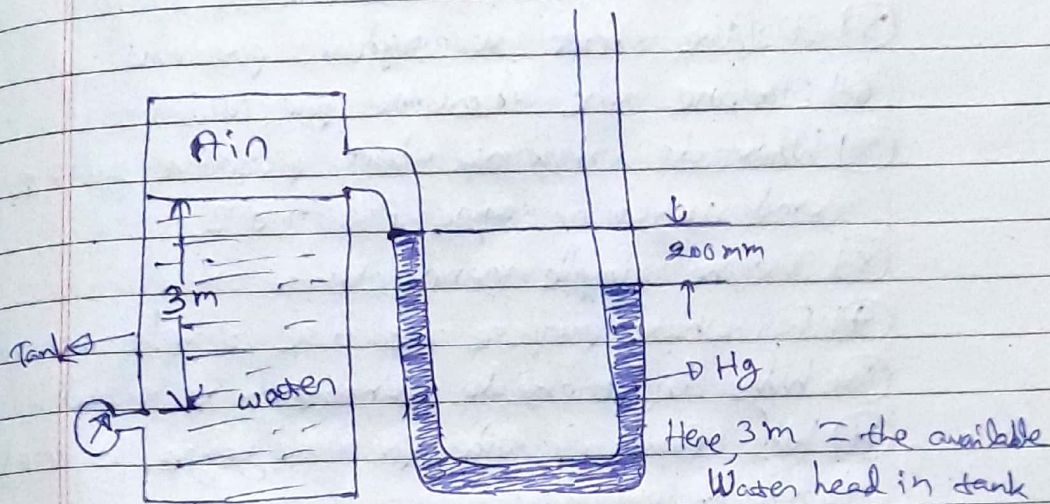
$$= 7848 (1.28)$$

$$= 10045.44 \text{ N/m}^2$$



### Problem - 5

A closed tank fitted with a gauge and a manometer contains water as shown in figure. Find the gauge reading if the manometer, containing mercury shows a reading of 200 mm.



$$\text{Manometer reading} = 200 \text{ mm} = 0.2 \text{ m}$$

As air is connected to the manometer, so the air pressure will be equal to the manometer reading. As the right side of manometer shows a lower level than the left side, so the air pressure is negative.

$$\begin{aligned} \text{Pressure of air (in terms of water head)} &= -0.2 \times 13.6 \\ &= -2.72 \text{ m (specific gravity of mercury = 13.6)} \end{aligned}$$

$$\begin{aligned} \text{Gauge reading} &= \text{water head in tank} + \text{Pressure of air} \\ &= 3 \text{ m} + (-2.72) = 0.28 \text{ m of water} \end{aligned}$$

$$\begin{aligned} \therefore \text{Gauge reading (in terms of pressure intensity)} &= \text{Specific wt. of water} \times \text{water head} \\ &= 9810 \times 0.28 \\ &= 2750 \text{ N/m}^2 \\ &= 2750 \text{ Pa} \end{aligned}$$

$$(\because \text{Pressure intensity, } p = w \times h)$$



## Question Bank (Short questions) :-

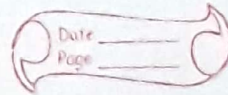
- ① Define fluid pressure and write its SI unit.
- ② What is intensity of pressure?
- ③ What do you mean by pressure head and write down the unit?
- ④ State the "Pascal's law".
- ⑤ Define short atmospheric pressure.
- ⑥ Define the absolute pressure.
- ⑦ What do you know about positive gauge and negative gauge pressure?
- ⑧ Define simple manometer.
- ⑨ For what purpose we use simple manometer?
- ⑩ Why differential manometer is used?
- ⑪ Why Bourdon's tube pressure gauge is used?

## Long Questions

- ⑫ Write briefly about fluid pressure, intensity of pressure and pressure head of any fluid.
- ⑬ Write a short note about atmospheric pressure, positive gauge pressure and negative gauge pressure.
- ⑭ Explain about the absolute pressure of any fluid in detail.
- ⑮ Explain about simple manometer, draw the line diagram and derive mathematical derivation to find out both positive and negative gauge pressure heads by using it.
- ⑯ Explain the differential manometer with proper diagram and also derive the mathematical derivation for the pressure difference.
- ⑰ Explain in detail about the Bourdon's tube pressure gauge with proper diagram.



## Chapter-3 Hydrostatics



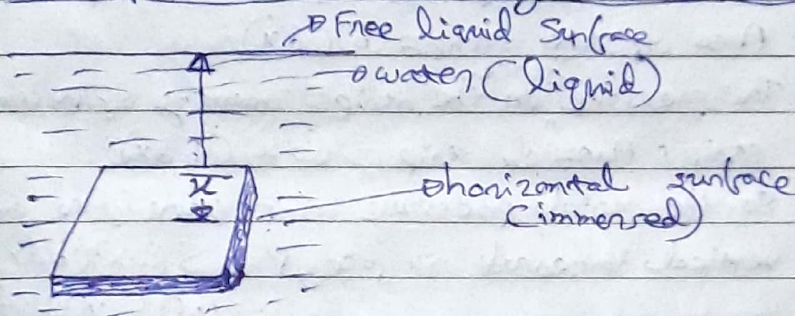
### Hydrostatic Pressure:-

It is the pressure exerted by the liquid at rest on any body which is immersed in it. The direction of this pressure is always at right angles with the immersed surface.

### Total Pressure:-

It is defined as the total pressure (force) exerted by the liquid on the immersed surface. The immersed surface may be either in horizontal or vertical position.

### Total pressure on horizontally immersed surface:-



The plane horizontal surface is immersed in water as shown in the figure.

$w$  = specific weight of the liquid (water)

$A$  = Top area of the immersed surface

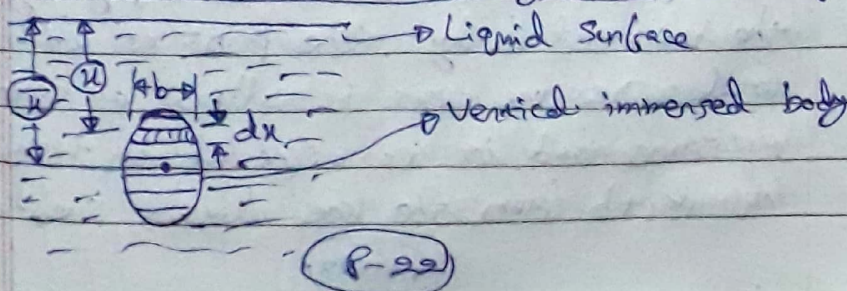
$x$  = Depth of the horizontal surface (from liquid surface)

Total pressure on the surface,  $P$  = weight of the liquid (water) above the horizontally immersed surface  
= specific wt. of the liquid  $\times$  Area of surface  $\times$  liquid Depth

$$\text{Or, } P = wAx$$

The S.I. unit is N.

### Total Pressure on vertically immersed surface:-





The plane surface is vertically immersed in the liquid and it is divided into a number of small parallel strips.

Let us take;  $w$  = specific weight of the liquid

$A$  = Total area of the immersed surface

$\bar{u}$  = Depth of Centre of Gravity of the immersed surface from liquid surface

Now take a small strip having width ' $b$ '; thickness as ' $dx$ ' and at a depth of ' $x$ ' from the free liquid surface.

The intensity of pressure on the strip (small) =  $(w)(x)$

Area of the strip =  $(b)(dx)$

The pressure <sup>force</sup> on the strip = Intensity of pressure on the strip  $\times$  Area of strip =  $(wx)(b dx)$ .

So, the total pressure or pressure force on the vertical immersed surface,  $P = \int (wx)(b dx)$   
 $= (w) \int (x)(b dx)$

But,  $\int (x)(b dx)$  = the moment of the total surface area of immersed body about the liquid level  
 $= (A)(\bar{u})$

So,  $\boxed{P = (w)A(\bar{u})}$

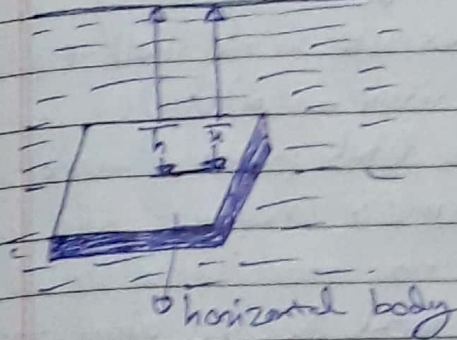
Centre of Pressure —

The intensity of pressure increases when the depth of the liquid increases, hence the pressure is more over the lower part of the immersed body than the upper part. Hence the resultant force (Total pressure) will act at some point below the centre of gravity of the immersed body. The point at which the total pressure acts is called as centre of pressure and it is expressed in terms of its depth from the free liquid surface.



## Centre of Pressure for horizontally immersed body

→ Free liquid surface



The horizontal body (rectangular) is immersed in the liquid.

$w$  = specific weight of the liquid

$A$  = Area of the immersed surface

$\bar{u}$  = Depth of C.G. of immersed surface from free surface

$\bar{h}$  = Depth of centre of pressure from free surface

Total Pressure,  $P$  = specific weight  $\times$  Volume of liquid

$$\text{or, } P = w \times (A) (\bar{u}) = (w) (A) (\bar{h})$$

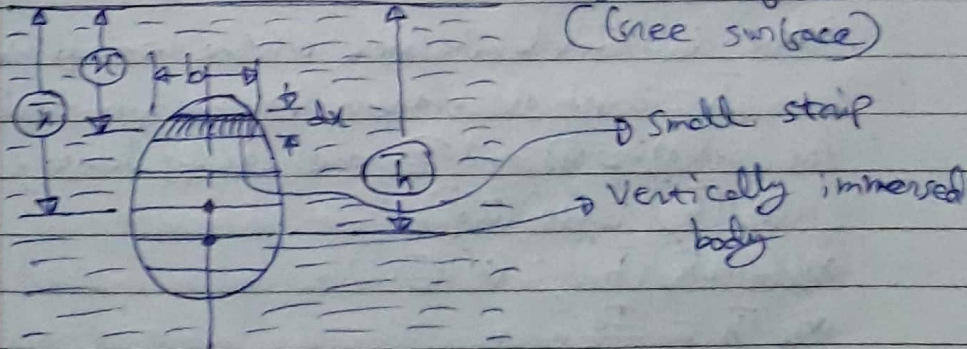
here;  $\bar{u} = \bar{h}$

$$\text{So, } \boxed{\bar{h} = \frac{P}{wA}}$$

## Centre of pressure of vertically immersed surface

→ Liquid surface

(free surface)



→ small strip

→ Vertically immersed body

The plane surface shown above is vertically immersed in liquid and it is divided into a number of small parallel strips.

Let us take,  $w$  = specific weight of the liquid

$A$  = Area of the vertical immersed surface

$\bar{u}$  = Depth of centre of gravity of the vertical immersed surface from free surface

Now consider a small strip having breadth 'b'



thickness ' $dx$ ' and having a depth of ' $x$ ' from the free liquid surface.

The intensity of pressure on the small strip =  $(w)(x)$

Area of the small strip =  $(b)(dx)$

The pressure force or total pressure on the small strip

= Intensity of pressure  $\times$  Area of strip

$$= (w)(x)(b dx)$$

The moment of this total pressure about the liquid surface =  $(wx)(b dx)(x) = (wx^2)(b dx)$

The sum of moments of all the total pressures about the liquid surface,  $M = \int (wx^2)(b dx)$

$$= (w) \int (x^2)(b dx)$$

But,  $\int (x^2)(b dx) = I_0$  (Moment of Inertia of the vertical immersed surface about the liquid surface or the second moment of area)

$$\text{Hence, } M = (w)(I_0) \quad \text{--- (1)}$$

Again, the sum of the moments of the pressure =  $(P)(\bar{h})$  --- (2)

where,  $P$  = Total Pressure on the vertical immersed surface

$\bar{h}$  = Depth of centre of pressure from liquid surface

As equations (1) and (2) are equal; hence,

$$(P)(\bar{h}) = (w)(I_0)$$

$$\text{or, } (wA\bar{x})(\bar{h}) = (w)(I_0)$$

$$\text{or, } \bar{h} = \frac{I_0}{A\bar{x}}$$

Again from the Parallel Axis Theorem;

$$I_0 = I_G + Ah^2$$

where,  $I_G$  = Moment of Inertia of the vertical immersed surface about the horizontal axis through its centre of gravity and

$h$  = Distance between the liquid surface and centre of gravity of the vertical immersed surface =  $\bar{x}$



Date \_\_\_\_\_  
Page \_\_\_\_\_

$$\text{So, } \bar{h} = \frac{I_G + A h^2}{A(\bar{x})} = \frac{I_G}{A(\bar{x})} + \frac{A(\bar{x})^2}{A(\bar{x})}$$

$$\text{or, } \boxed{\bar{h} = \frac{I_G}{A(\bar{x})} + (\bar{x})}$$

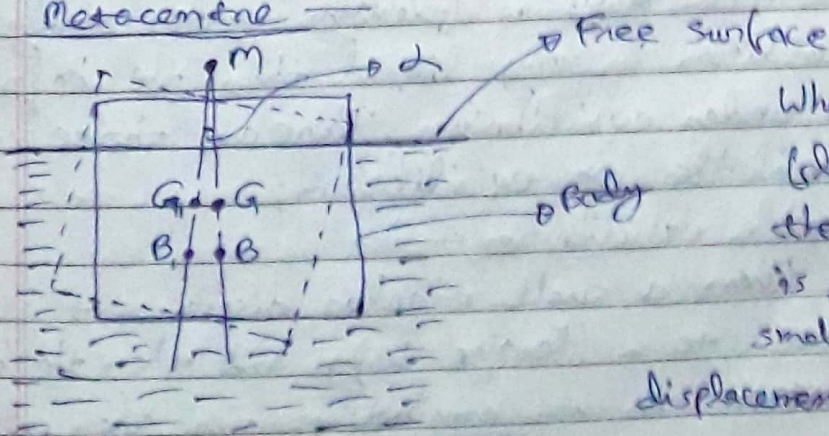
### Archimedes' Principle —

It states that when a body is immersed fully or partly in a fluid, then it is buoyed up by a force equal to the weight of the fluid (liquid) displaced by the body. So, when a body is immersed, then the resultant force acting on it is equal to the difference between the downward force due to gravity and upward force of fluid at the bottom of the body.

### Buoyancy —

It is the property of the fluid to lift a submerged body in the upward direction due to the upward force of the fluid. The upward force is called as force of buoyancy. It is equal to the weight of the fluid displaced by the submerged body. When the force of buoyancy is more than the body weight (submerged), then the body will float. When the buoyant force is less than the body weight, then the body will sink down.

### Metacentre —



When a body floats in the liquid is given small angular displacement, then



it starts oscillating about some point and that point is called as metacentre. It is mentioned in the figure as point 'M'. It is also the point of intersection of the line passing through original centre of Buoyancy (B) and centre of gravity of the body and the vertical line through the new centre of Buoyancy (B<sub>1</sub>).

#### Metacentric Height -

It is the distance between the centre of gravity of a floating body and the metacentre of the body. Here,  $GM$  = Metacentric Height as shown in the figure.

#### Floatation -

Whenever a body is placed in a liquid, then two forces like gravitational force and upthrust of liquid acts on the body. Both the mentioned forces are opposite in direction to each other. When the gravitational force is less than the upthrust, then the body will float. But if the gravitational force is more than the upthrust of liquid, then the body will sink down in the liquid.

#### Conditions of equilibrium (Floating Body):

When a body is floating under steady condition in any liquid then the body is under equilibrium. There are three conditions available for equilibrium;

#### (a) Stable equilibrium -

When a small angular displacement is given to a floating body and then the body returns to its original position, then only

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the body is under stable equilibrium.

(b) Unstable equilibrium —

When a small angular displacement is given and the body instead of returning to original position moves further away, then the body is under unstable equilibrium.

(c) Neutral equilibrium —

When a small angular displacement is given and the body achieves a new position of rest then the body is under neutral equilibrium.

Problem-1

A tank  $3\text{m} \times 4\text{m}$  contains  $1.2\text{m}$  deep oil of specific gravity  $0.8$ . Calculate the intensity of pressure at the base of the tank and total pressure on the base of the tank.

Solution

Given data,

size of the tank,  $A = 3\text{m} \times 4\text{m} = 12\text{m}^2$ ;

Depth of oil,  $\bar{x} = 1.2\text{m}$ ;

Specific gravity of oil,  $S_o = 0.8$ ;

Specific weight of oil,  $w_o = S_o \times w_w$

$$= 0.8 \times 9810$$

$$= 7850 \text{ N/m}^3$$

where,  $w_w$  = specific weight of water

$$= 9810 \text{ N/m}^3$$

Intensity of pressure at the base of the tank,

$$p = w_o \times h$$

$$= 7850 \times 1.2 = 9420 \text{ N/m}^2$$

where,  $h$  = depth of oil.

Total pressure on the base of the tank,

$$P = w_o A \bar{x}$$

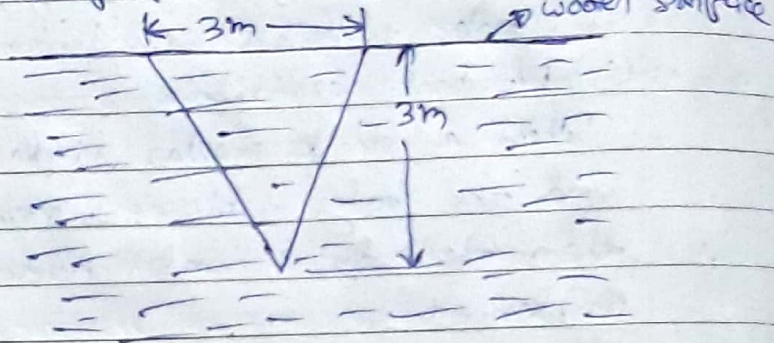
$$= 7850 \times 12 \times 1.2 = 113040 \text{ N}$$

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### Problem - 2

An isosceles triangular plate of base 3m and altitude 3m is immersed vertically in water as shown in the figure. Calculate the total pressure and centre of pressure of the plate.



### Solution

Given data; Width of the base,  $b = 3\text{m}$ ;

Altitude of the triangle,  $h = 3\text{m}$

Surface area of the triangular plate,

$$A = \frac{bh}{2} = \frac{3 \times 3}{2} = 4.5 \text{ m}^2$$

Depth of centre of gravity of the triangular plate from the water surface,  $\bar{x} = \frac{3}{3} = 1\text{m}$ .

Total pressure on the plate,  $P = w_o A \bar{x}$

$$= (9810)(4.5)(1) = 44100 \text{ N}$$

where;  $w_o$  = specific weight of water

Moment of Inertia of the triangular plate about its centre of gravity and parallel to the water surface (free surface),

$$I_G = \frac{bh^3}{36} = \frac{3(3)^3}{36} = 2.25 \text{ m}^4$$

The depth of centre of pressure from the

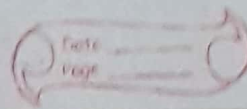
$$\text{water surface, } \bar{h} = \frac{I_G}{A \bar{x}} + \bar{x}$$

$$= \frac{2.25}{(4.5)(1)} + 1$$
$$= 1.5 \text{ m}$$

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## Question Bank



### Short questions;

- (1) Define about hydrostatic pressure.
- (2) Define total pressure of liquid on immersed body.
- (3) Define the term centre of pressure on immersed body in the liquid.
- (4) Write down the mathematical equation for total pressure on a horizontal immersed body.
- (5) Write down the mathematical formula for total pressure on a vertical immersed body.
- (6) Define Archimede's principle.
- (7) Define about the term buoyancy.
- (8) What do you mean by metacentre?
- (9) What is metacentric height?
- (10) What do you understand by flotation?
- (11) <sup>long question</sup> Derive the mathematical equation for finding the total pressure on a horizontally immersed surface.
- (12) Derive the mathematical expression for finding out the total pressure on a vertical immersed surface in liquid.
- (13) Derive the mathematical equation to find out the centre of pressure for a horizontal immersed surface in any liquid.
- (14) Find out the mathematical equation by deriving for the centre of pressure when a vertical body is immersed in liquid.
- (15) Write short notes about the buoyancy and flotation.



## Chapter-4

### Kinematics of Flow



#### Types of Fluid Flow:-

There are many types or ways in which the fluid particles may flow and they are as follows

##### ① Uniform flow -

When the velocities of different fluid particles are equal at all sections of any conduit through which the particles flow, then the type of flow is called as uniform flow.

##### ② Non-uniform flow -

If the velocities of different fluid particles are different at all sections of any conduit, then the type of flow is called as non-uniform flow.

##### ③ Steady flow -

When the quantity of fluid flowing per time period is constant, then the type of flow is called as steady flow.

##### ④ Unsteady flow -

When the quantity of fluid flowing per time period is not constant, then the type of flow is called as unsteady flow.

##### ⑤ Streamline flow -

When each fluid particle in a fluid flow has a particular path and the paths of different particles do not cross with each other, then the flow is called as streamline flow.

##### ⑥ Turbulent flow -

When each fluid particle does not possess a particular path and the paths of different particles cross with each other, then the flow is called as turbulent flow.

##### ⑦ Rotational flow -

When the fluid particles while flowing also





rotate about their own axis, then the flow is called as rotational flow.

### ② Irrotational flow -

If the fluid particles at the time of flowing, do not rotate about their own axis, then the flow is called as irrotational flow.

### ③ Compressible flow -

If the density of the fluid changes during flow, then it is called as compressible flow.

### ④ Incompressible flow -

If the density of the fluid does not change during the flow, then it is called as incompressible flow.

### ⑤ One-dimensional flow -

When the streamlines of flowing particles are represented by a straight line and as straight line has only one dimension, then the flow is called as one-dimensional flow.

### ⑥ Two-dimensional flow -

When the streamlines of flowing particles are represented by a curve and as curve possesses two mutually perpendicular directions, then the flow is called as two-dimensional flow.

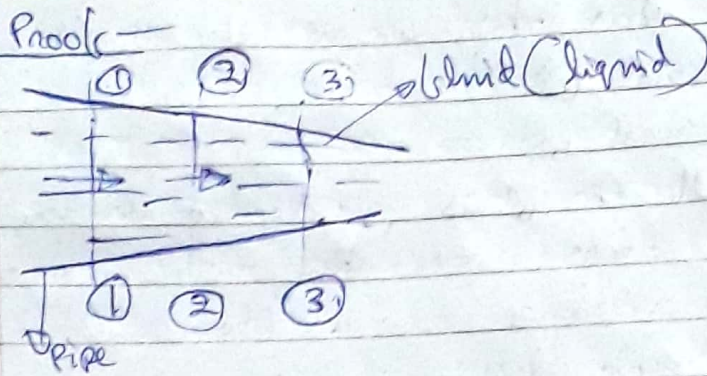
### ⑦ Three-dimensional flow -

When the streamlines of flowing particles are represented in space and as space has three mutually perpendicular directions, then the flow is called as three-dimensional flow.

### Continuity Equation -

It states that when any incompressible fluid is flowing continuously through a conduit, then the quantity of fluid passing per unit period is same at all the sections of the conduit.





Let us take a tapered pipe as shown in above, through which liquid is flowing.

Let us assume;  $a_1$  = area of cross-section of pipe at 1-1,

$V_1$  = velocity of liquid at section 1-1,

$a_2$  = area of cross-section of pipe at section 2-2,

$V_2$  = velocity of liquid at section 2-2,

$a_3$  = area of cross-section of pipe at 3-3,

$V_3$  = Velocity of liquid at section 3-3

Total quantity of liquid passing at section 1-1,

$$Q_1 = (a_1)(V_1)$$

Total quantity of liquid passing at section 2-2,

$$Q_2 = (a_2)(V_2)$$

Total quantity of liquid passing at section 3-3,

$$Q_3 = (a_3)(V_3)$$

According to the law of conservation of matter, the total quantity of liquid flowing through all the sections must be same.

$$\text{So, } Q_1 = Q_2 = Q_3$$

$$\text{or, } a_1 V_1 = a_2 V_2 = a_3 V_3$$

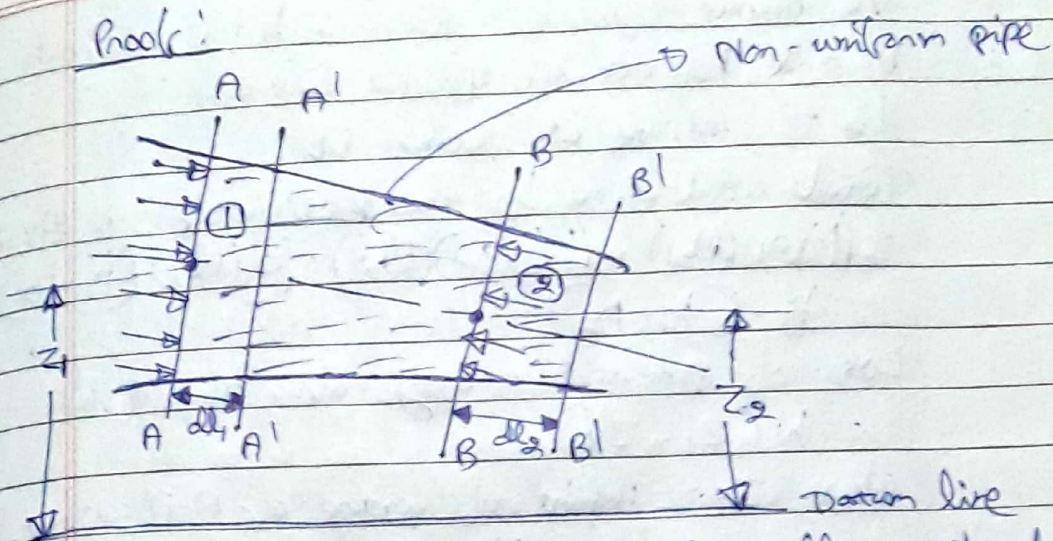
$$\text{or, } Q_1 = a_1 V_1 = Q_2 = a_2 V_2 = Q_3 = a_3 V_3$$

Bernoulli's Equation:—

It states that when a perfect, incompressible liquid is flowing continuously in a conduit, then the total energy possessed by liquid particles remains same at all sections of the conduit.



Proof:



The perfect, incompressible liquid is flowing through the tapered pipe as shown in the figure both continuously and fully. The AA and BB are two sections of the pipe. The liquid available between sections AA and BB is assumed to move to sections A'A' and B'B' through very small distances of  $dl_1$  and  $dl_2$ . Thus the liquid (equivalent) between AA and A'A' is moved to the section between BB and B'B', while liquid between A'A' and BB remains unaffected. Suppose,  $W$  = weight of the liquid between AA and A'A' or,  $W = w a_1 dl_1 = w a_2 dl_2$

where;  $a_1$  = area of cross-section of pipe at AA  
 $a_2$  = area of cross-section of pipe at BB  
 $w$  = specific weight of liquid flowing  
 or,  $a_1 dl_1 = a_2 dl_2 = \frac{W}{w}$   
 or,  $a_1 dl_1 = a_2 dl_2$

Now, the work done by pressure for moving the liquid from section AA to section A'A'  
 = Force  $\times$  distance =  $(P_1)(a_1)(dl_1)$

where,  $P_1$  = pressure of liquid at AA

Work done by pressure for moving the liquid from section BB to section B'B'  
 =  $(-P_2)(a_2)(dl_2)$



The minus sign is given with  $p_2$  as both  $p_1$  and  $p_2$  act in opposite directions.

$p_2$  = pressure at section BB'

$$\begin{aligned} \text{Total work done by the pressure of the liquid} \\ = (p_1)(a_1)(\delta l_1) - (p_2)(a_2)(\delta l_2) = (a_1 \delta l_1)(p_1 - p_2) \\ = \frac{W}{g}(p_1 - p_2) \end{aligned}$$

$$\begin{aligned} \text{Loss of potential energy of the liquid} \\ = W(z_1 - z_2) \end{aligned}$$

where,  $z_1$  = Height of centre of A-A section from the datum line

$z_2$  = Height of centre of B-B from datum

$$\begin{aligned} \text{Gain in kinetic energy} = W \left( \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \right) \\ = \frac{W}{2g}(v_2^2 - v_1^2) \end{aligned}$$

where,  $v_1$  = velocity of liquid at section A-A

$v_2$  = velocity of liquid at section B-B

$g$  = acceleration due to gravity

Again; Work done by pressure of liquid + Loss of Potential energy of liquid = Gain in kinetic energy of the liquid

$$\text{or, } \frac{W}{g}(p_1 - p_2) + W(z_1 - z_2) = \frac{W}{2g}(v_2^2 - v_1^2)$$

$$\text{or, } \left[ \frac{p_1}{\rho} + z_1 + \frac{v_1^2}{2g} \right] = \left[ \frac{p_2}{\rho} + z_2 + \frac{v_2^2}{2g} \right]$$

Limitations of Bernoulli's equation:-

(1) In Bernoulli's theorem, it is assumed that the velocity of every liquid particle at any cross-section of the pipe is uniform. But the velocity of liquid particles at the centre of the pipe is maximum and reduces towards the pipe wall because of friction. So, actually mean velocity is taken.

(2) It is also assumed that except the gravitational force, there is no other external



force acting on the liquid. But actually external force like pipe friction is available.

(3) The other assumption is that there is no loss of energy from the liquid particles. But in actual practice, some energy losses occur.

(4) When the liquid is flowing in a curved path, then we have to consider the energy due to centrifugal force.

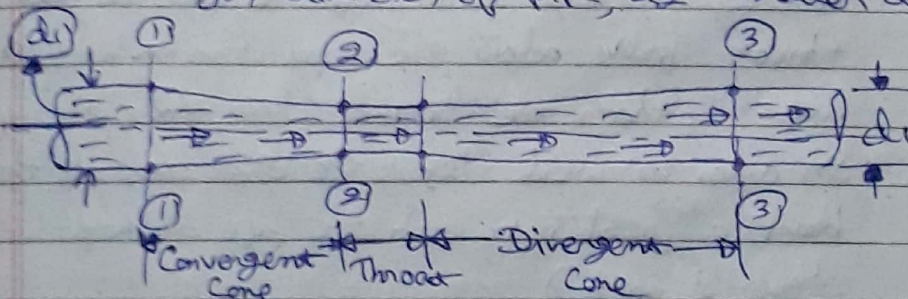
(5) It is also assumed that the liquid is perfect. But in actual practice, there is no perfect liquid which means that all liquids are real.

### Applications of Bernoulli's theorem:-

This theorem is generally applied in hydraulics and applied hydraulics. The other practical applications are;

#### (a) Venturimeter -

$d_1$  = larger diameter of pipe,  $d_2$  = smaller diameter



The venturimeter is the apparatus used to find out the discharge ( $Q$ ) of a liquid flowing in a pipe by applying Bernoulli's equation. It has three main parts like; (i) Convergent part, (ii) Throat, (iii) Divergent Part.

#### Convergent part -

It is the part of the pipe which gradually reduces from pipe diameter ( $d_1$ ) to a smaller diameter of pipe ( $d_2$ ). It is also called as the inlet of the venturimeter. The slope



of the convergent cone generally varies from 1 in 4 to 1 in 5.

Throat -

It is the part having the smaller diameter of the pipe ( $d_2$ ).

Divergent part -

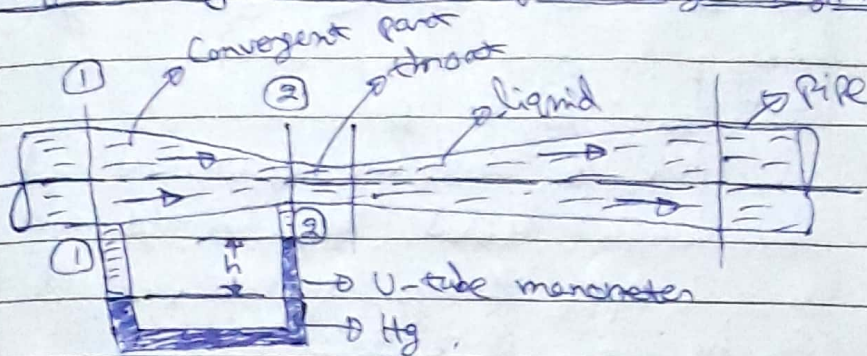
It is the part having gradual increase in diameter from lower diameter ( $d_2$ ) upto the higher diameter ( $d_1$ ). It is also called as the outlet part. The length of this part is 3 to 4 times more than convergent part.

As the liquid gets accelerated while flowing from convergent part to throat (section 1 to section 2), so velocity at the throat becomes higher than velocity of liquid at section 1, so the pressure at the throat is reduced as compared to the convergent part. If the pressure head at throat falls below the separation head (2.5 m of water), then the liquid flow tends to separate. In order to avoid this, the diameter of throat bears a fixed ratio with the pipe diameter (generally  $\frac{d_2}{d_1} = \frac{1}{3}$  to  $\frac{1}{2}$ ).

The liquid when flows through the divergent part, deceleration occurs and hence the pressure at the outlet of divergent part becomes more. When the increase in pressure becomes quick, then liquid particles break away from the wall because of boundary layer effect. To avoid this and also to minimise the losses due to friction, the divergent part length is made more than the length of the convergent part.



## Equation for calculation of discharge through Venturimeter



The liquid is flowing in the venturimeter as shown in the figure and an U-tube manometer is connected between the convergent part and throat. The U-tube manometer contains heavy liquid mercury and shows the manometer reading of  $h$ .

$P_1$  = pressure at section 1

$V_1$  = velocity of liquid at section 1

$Z_1$  = Datum head at section 1

$a_1$  = area of the venturimeter at section 1

$P_2$  = pressure at section 2

$V_2$  = velocity of liquid at section 2

$Z_2$  = Datum head at section 2

$a_2$  = area of the venturimeter at section 2

Applying Bernoulli's equation, we get;

$$\frac{P_1}{w} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{w} + Z_2 + \frac{V_2^2}{2g}$$

where,  $w$  = specific weight of the liquid

As the pipe is horizontal, so both datum heads are equal. Hence,  $Z_1 = Z_2$ .

$$\text{So, } \frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$\text{or, } \frac{P_1}{w} - \frac{P_2}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \text{--- (1)}$$

As per continuity equation,

$$a_1 V_1 = a_2 V_2 \text{ or, } V_1 = \frac{a_2 V_2}{a_1}$$

Putting the value of  $V_1$  in equation (1);

$$\frac{P_1}{w} - \frac{P_2}{w} = \frac{V_2^2}{2g} - \frac{(a_2)^2}{(a_1)^2} \left( \frac{V_2^2}{2g} \right)$$

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$$\text{or, } \frac{P_1}{\omega} - \frac{P_2}{\omega} = \frac{V_2^2}{2g} \left( 1 - \frac{a_2^2}{a_1^2} \right) = \frac{V_2^2}{2g} \left( \frac{a_1^2 - a_2^2}{a_1^2} \right)$$

$$\text{Let } \frac{P_1}{\omega} - \frac{P_2}{\omega} = h_2$$

where,  $h_2$  = Difference of pressure head of liquid flowing in the venturimeter between convergent part and throat

$$\text{So, } h_2 = \frac{V_2^2}{2g} \left( \frac{a_1^2 - a_2^2}{a_1^2} \right)$$

$$\text{or, } V_2^2 = (2g)(h_2) \left( \frac{a_1^2}{a_1^2 - a_2^2} \right)$$

$$\text{or, } V_2 = \sqrt{2g(h_2)} \times \frac{a_1}{\sqrt{a_1^2 - a_2^2}}$$

Discharge through the venturimeter,

$$Q = C_v \times a_2 V_2$$

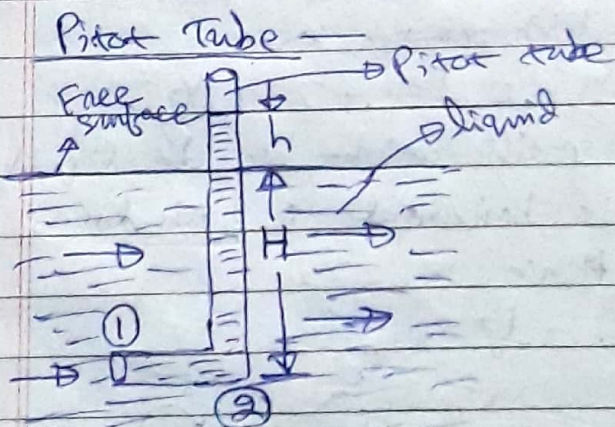
$$\text{or, } Q = C_v \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

where,  $C_v$  = Coefficient of Venturimeter

$$\text{Also, } h_2 = h \left( \frac{S_{Hg} - S_L}{S_L} \right)$$

where,  $S_{Hg}$  = Specific gravity of Hg

$S_L$  = Specific gravity of liquid



It is used to find out the velocity of liquid flowing at any required point in any stream. It is a glass tube bent with an angle of  $90^\circ$ . The lower end of



tube forces the direction of flow and liquid rises in the tube because of the pressure of liquid (flowing).

$h$  = height of liquid in the tube above free surface of liquid

$H$  = Depth of the tube inside the flowing liquid

$v$  = velocity of the flowing liquid

Applying Bernoulli's equation between section (1) and (2)

$$\frac{P_1}{\rho} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + Z_2 + \frac{V_2^2}{2g}$$

For section (1) and (2);  $Z_1 = Z_2$

$$\text{So, } \frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g}$$

where,  $Z_1$  = Datum head at section 1;

$Z_2$  = Datum head at section 2;

$P_1$  = Pressure at section 1;

$P_2$  = Pressure at section 2;

$\rho$  = specific wt. of the liquid

$$\text{But, } \frac{P_1}{\rho} = H \text{ and } \frac{P_2}{\rho} = H + h$$

$$\text{Hence, } H + \frac{V_1^2}{2g} = H + h + 0$$

because,  $V_2 = 0$  as the velocity of liquid in the tube becomes zero at section 2 and then liquid rises,

$V_1$  = velocity of liquid at section 1 =  $v$

$V_2$  = velocity of liquid at section 2

$$\text{So, } H + \frac{v^2}{2g} = H + h$$

$$\text{or, } \frac{v^2}{2g} = h$$

$$\text{or, } v^2 = 2gh$$

$$\text{or, } v = \sqrt{2gh}$$

Problem-1

Water is flowing through a pipe of 100mm diameter with an average velocity of 10 m/s. Calculate the rate of discharge of water and the velocity of water at the other end of the pipe. The diameter of the

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pipe is gradually changed to 200mm

Solution

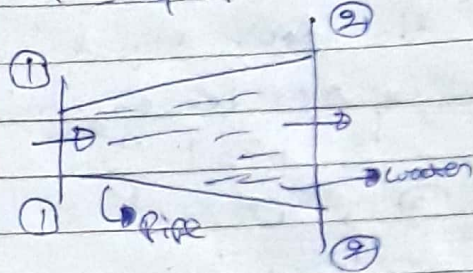
Given data;  $d_1 = 100\text{mm} = 0.1\text{m}$ ,  $V_1 = 10\text{m/s}$ ,  
 $d_2 = 200\text{mm} = 0.2\text{m}$ .

Cross-sectional area of pipe at point 1 (section 1),

$$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (0.1)^2 = 0.007854\text{m}^2$$

Rate of discharge,

$$Q = a_1 V_1 \\ = (0.007854)(10) \\ = 0.07854\text{m}^3/\text{s}$$



Cross-sectional area of pipe at section 2,

$$a_2 = \frac{\pi}{4} (d_2)^2 = \frac{\pi}{4} (0.2)^2 = 0.03142\text{m}^2$$

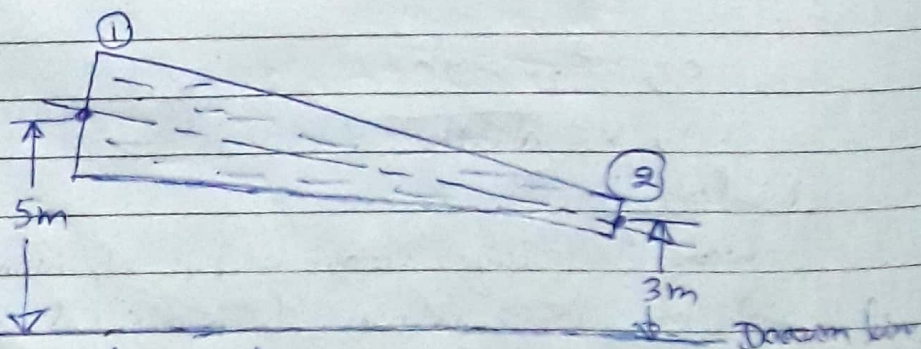
Velocity of water at section 2,  $V_2 = \frac{Q}{a_2}$

$$= \frac{0.07854}{0.03142} = 2.5\text{m/s}$$

Problem-2

The diameter of a pipe changes from 200mm at a section 5m above datum to 50mm at a section 3m above datum. The pressure of water at 1st section is  $500\text{KN/m}^2$ , the velocity of flow at the same section is  $1\text{m/s}$ , calculate the pressure intensity at the second section.

Solution



Given data;  $d_1 = 200\text{mm} = 0.2\text{m}$ ,  
 $z_1 = 5\text{m}$ ,  $d_2 = 50\text{mm} = 0.05\text{m}$ ,

(P-41)



$$Z_2 = 3\text{m}, P_1 = 500 \text{ kN/m}^2, V_1 = 1\text{m/s}$$

$P_2$  = pressure at section 2,  $V_2$  = velocity at section 2

$$\text{Area of pipe at section 1, } a_1 = \frac{\pi}{4} (d_1)^2 = \frac{\pi}{4} (0.2)^2 \\ = 0.03142 \text{ m}^2$$

$$\text{Area of pipe at section 2, } a_2 = \frac{\pi}{4} (d_2)^2 = \frac{\pi}{4} (0.05)^2 \\ = 0.001964 \text{ m}^2$$

Applying continuity equation;  $a_1 V_1 = a_2 V_2$

$$\text{or, } V_2 = \frac{a_1 V_1}{a_2} = \frac{(0.03142)(1)}{0.001964} = 16 \text{ m/s}$$

Applying Bernoulli's equation;

$$\frac{P_1}{\rho} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + Z_2 + \frac{V_2^2}{2g}$$

$$\text{or, } \frac{500}{9.81} + 5 + \frac{(1)^2}{2 \times 9.81} = \frac{P_2}{9.81} + 3 + \frac{(16)^2}{2 \times 9.81}$$

$$\text{or, } 51 + 5 + 0.05 = \frac{P_2}{9.81} + 3 + 13.05$$

$$\text{or, } \frac{P_2}{9.81} = 40 \quad \text{or, } P_2 = 40 \times 9.81 = 392.4 \text{ kN/m}^2$$

### Problem-3

A venturimeter with a 150 mm diameter at inlet and 100 mm at throat is laid with its axis horizontal and is used for measuring the flow of oil having specific gravity 0.9. The oil-mercury differential manometer shows a gauge difference of 200 mm. The coefficient of venturimeter is assumed to be 0.98. Calculate the discharge of oil.

### Solution

Given data;  $d_1 = 150\text{mm} = 0.15\text{m}$ ;  $d_2 = 100\text{mm} = 0.1\text{m}$ ;

Specific gravity of the oil,  $S_o = 0.9$ ;

Gauge difference,  $h = 200\text{mm} = 0.2\text{m}$  of Hg;

Coefficient of Venturimeter,  $C_v = 0.98$

$$\text{Area at the inlet, } a_1 = \frac{\pi}{4} (d_1)^2 \\ = \frac{\pi}{4} (0.15)^2 \\ = 0.01767 \text{ m}^2$$

$$\text{Area at the throat, } a_2 = \frac{\pi}{4} (d_2)^2 = \frac{\pi}{4} (0.1)^2 \\ = 0.007854 \text{ m}^2$$

(P-42)



Difference of pressure head of oil flowing through the venturimeter between section (1) and section (2),  $h_x = h \left( \frac{S_{Hg} - S_o}{S_o} \right)$

$$= 2(0.2) \left( \frac{13.6 - 0.9}{0.9} \right) \\ = 2.82 \text{ m of oil}$$

Discharge through the venturimeter,

$$Q = \frac{(C_v)(a_1 a_2)}{\sqrt{a_1^2 - a_2^2}} (\sqrt{2gh}) \\ = \frac{(0.98)(0.01767)(0.007854)}{\sqrt{(0.01767)^2 - (0.007854)^2}} \left( \sqrt{2 \times 9.81 \times 2.82} \right)$$

$$\therefore Q = 0.0639 \text{ m}^3/\text{s}$$

#### Problem-4

A pitot tube was inserted in a pipe to measure the velocity of water in it, if the water rises in the tube is 200 mm, find the velocity of water.

#### Solution

$$\text{Given data; } h = 200 \text{ mm} \\ = 0.2 \text{ m}$$

Velocity of water in the pipe,

$$V = \sqrt{2gh}$$

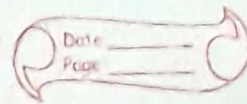
$$\text{where, } g = \text{acceleration due to gravity} \\ = 9.8 \text{ m/sec}^2$$

$$\therefore V = \sqrt{2 \times 9.81 \times 0.2}$$

$$\therefore V = 1.98 \text{ m/s}$$



# Question Bank



## Short Questions

- (1) Define Uniform fluid flow.
- (2) Define about Non-uniform fluid flow.
- (3) What is steady fluid flow?
- (4) What do you mean by unsteady fluid flow?
- (5) What is streamline flow?
- (6) Define about turbulent fluid flow.
- (7) What do you understand by rotational flow?
- (8) What is irrotational flow?
- (9) Define about compressible flow.
- (10) Define incompressible fluid flow.
- (11) What is one-dimensional flow?
- (12) Define about the two-dimensional flow.
- (13) What is three-dimensional flow?
- (14) State Continuity equation for one-dimensional flow.
- (15) State Bernoulli's equation.
- (16) Why Venturimeter is used?
- (17) For what purpose pitot tube is used?
- (18) Long Questions  
State and prove the continuity equation for one-dimensional fluid flow.
- (19) State and prove the Bernoulli's equation for fluid flow.
- (20) Write in brief the limitations of Bernoulli's equation.
- (21) Write a short note about Venturimeter.
- (22) Derive the equation to find out the discharge of flowing fluid through venturimeter.
- (23) Explain in brief about Pitot tube.



## Chapter-5

### Orifices, Notches, Weirs

#### Orifice:-

Orifice is defined as an opening in a vessel through which liquid comes out and the liquid level on the upstream side is above the top of the orifice. It is used to measure the discharge of liquid. It may be provided in the vertical side or along the base.

#### Types of Orifice:-

① According to size;

(a) Small orifice, (b) Large orifice

② According to shape

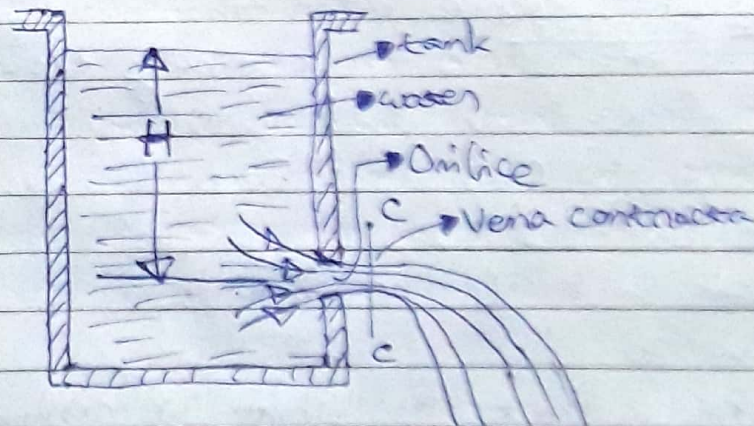
(a) Circular orifice, (b) Rectangular orifice,

(c) Triangular orifice

③ According to the type of discharge

(a) Fully submerged orifice, (b) Partially submerged orifice

#### Flow through Orifice:-



The tank as shown above contains water and fitted with an orifice at one side. The water flows out through the orifice. The water particles above the orifice take a turn in order to pass through the orifice. By making this turn the water particles lose some of the energies possessed by



them. So the water jet (water coming out of orifice) becomes contracted after leaving the orifice. The maximum contraction of water jet occurs at a particular section on the downstream side of the orifice. At this section the water particles become parallel with each other. This section is called as vena contracta and in ~~the~~ the figure it is shown by section C-C. The position of this section is generally at a distance of  $\frac{d}{2}$  (from the orifice in the downstream side ( $d$  - diameter of the orifice)). After vena contracta the water particles are attracted by the gravitational pull (force of the earth), so the cross-section of water jet increases after vena contracta and the water jet moves in the downward direction.

Velocity of the water jet (theoretical),  $V = \sqrt{2gH}$   
 where,  $H$  = Head of water available in the upstream side of the orifice in the tank

So, theoretical Discharge,  $Q_{th} = (a) \sqrt{2gH}$

where,  $a$  = area of cross-section of the orifice

Hydraulic coefficients :-

The orifice has four types of hydraulic coefficients

① Coefficient of Contraction —

It is defined as the ratio of area of water jet at the vena-contracta with the area of orifice.

It is denoted by  $C_c$ . Mathematically,

$$C_c = \frac{\text{Area of jet (water) at vena-contracta}}{\text{Area of the orifice}}$$

The average value of  $C_c$  is 0.64. The value of  $C_c$  depends on the head available, size and shape of orifice.

② Coefficient of velocity —

It is defined as the actual velocity of the



water jet at the vena-contracta with the theoretical velocity of water jet, It is denoted by  $C_v$ . Mathematically,

$$C_v = \frac{\text{Actual jet velocity at vena contracta}}{\text{Theoretical jet velocity}}$$

The average value is 0.97. It depends upon the different shapes of orifice edges and also on the orifice size.

### (3) Coefficient of discharge —

It is defined as the ratio between the actual discharge through the orifice with the theoretical discharge through the orifice. It is denoted by  $C_d$ . Mathematically,

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}}$$

The average value of it varies from 0.6 to 0.64. It also depends upon shape and size of the orifice.

### (4) Coefficient of resistance —

It is defined as the ratio between loss of head in the orifice with the available head of water at the orifice exit. It is denoted by  $C_r$ . Mathematically,

$$C_r = \frac{\text{head loss in the orifice}}{\text{Head of water available at orifice exit}}$$

As the value is very less, hence the value of it is neglected while solving problems.

### Relation between Hydraulic coefficients —

$$\begin{aligned} C_d &= \frac{\text{Actual discharge of water jet}}{\text{Theoretical discharge of water jet}} \\ &= \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}} \end{aligned}$$

(P-41)

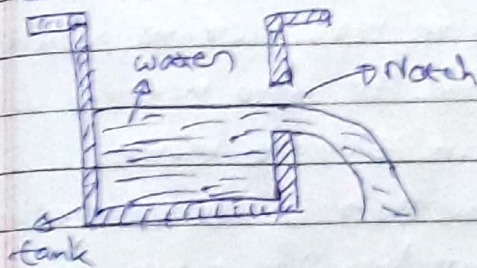


$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual area}}{\text{Theoretical area}}$$

$$= C_v \times C_c$$

$$\text{or, } C_d = (C_v)(C_c)$$

Notch:-

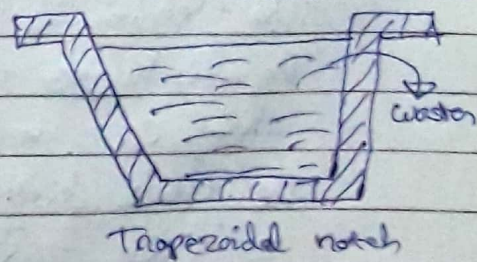
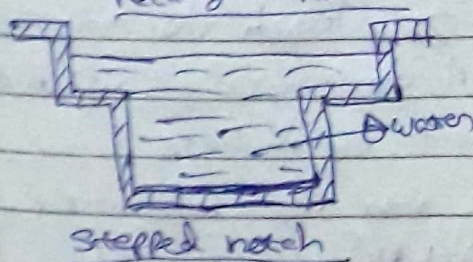
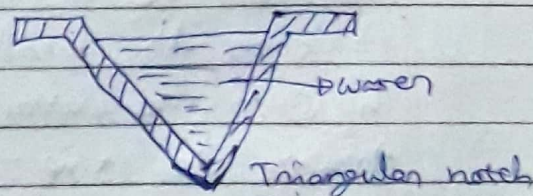
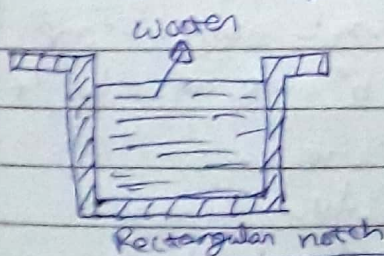


The notch is defined as the opening provided in one side of the tank when the upstream liquid level is below the top edge of the opening. So, notch only has working sides and bottom edge. The notch is used to measure the discharge of liquid and generally made of metal. The bottom edge of the notch is called as crest or sill. The sheet of water flowing over the notch is called as vein or nappe.

Classification of notch:-

The different types are;

- (a) rectangular notch, (b) Triangular notch,
- (c) Stepped notch, (d) Trapezoidal notch





The rectangular notch is of having rectangular shape. It gives more accurate results for high discharges.

The triangular notch is having the shape of a triangle. It is also called V-notch. The triangular notch gives more accurate results for low discharge and it also measures wide range of flows in accurate manner.

The stepped notch is the combination of many rectangular notches. It is used for higher discharges.

The trapezoidal notch is the combination of one rectangular notch and two triangular notches. So it combines both the advantages of rectangular and triangular types.

Discharge over Rectangular Notch:-



The above shown rectangular notch is fitted on the side of the tank. The water is flowing over the rectangular notch.

Let us take a horizontal water strip having thickness ' $dh$ ', breadth ' $b$ ' and at a height of ' $h$ ' from the water surface.

Area of the strip =  $(b)(dh)$

The theoretical velocity through the strip is  $\sqrt{2gh}$ .

The discharge through the strip,  $dQ = (C_d)(b)(dh)\sqrt{2gh}$

where,  $C_d$  = Coefficient of discharge

The total discharge over the whole notch,  $Q$   
 $= \int_0^H dQ = \int_0^H (C_d)(b)(dh)\sqrt{2gh}$   
 (p-48)

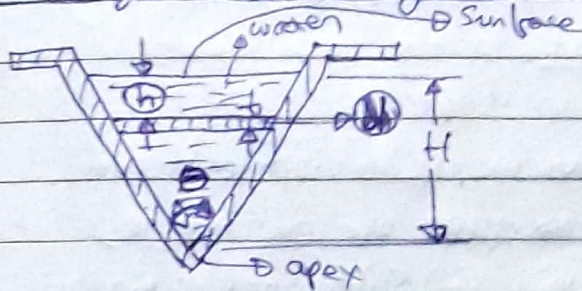


$H$  = height of water above the bottom of the notch on the upstream side

$$\text{or, } Q = (C_d)(b)(\sqrt{2g}) \int_0^H (\sqrt{h}) dh = (C_d)(b)(\sqrt{2g}) \left[ \frac{h^{3/2}}{(3/2)} \right]_0^H$$

$$\text{or, } Q = \frac{3}{2} (C_d)(b)(\sqrt{2g}) (H)^{3/2}$$

Discharge over Triangular notch :-



The above figure shows a triangular notch over which water is flowing.

$H$  = height of the water above the apex of the notch

$\theta$  = angle of the notch

$C_d$  = coefficient of discharge

Let us consider a small water strip having thickness 'dh' and at a height of 'h' from water surface.

The width of the notch at the water surface =  $2H \tan(\frac{\theta}{2})$

Area of the small strip =  $[2(H-h) \tan(\frac{\theta}{2})] (dh)$

Theoretical velocity of water through the strip =  $\sqrt{2gh}$

The discharge over the small strip,  $dQ$

$$= (C_d)(2)(H-h) \left( \tan \frac{\theta}{2} \right) (dh) \sqrt{2gh}$$

The discharge over the full triangular notch,

$$Q = \int_0^H (C_d)(2)(H-h) \left( \tan \frac{\theta}{2} \right) (dh) \sqrt{2gh}$$

$$= \tan \left( \frac{\theta}{2} \right) (2 C_d (\sqrt{2g})) \int_0^H (H-h) (\sqrt{h}) (dh)$$

$$= (2)(C_d)(\sqrt{2g}) \left( \tan \frac{\theta}{2} \right) \int_0^H \left[ H h^{1/2} - h^{3/2} \right] dh$$

$$= (2)(C_d)(\sqrt{2g}) \left( \tan \frac{\theta}{2} \right) \left[ \frac{H(h)^{3/2}}{(3/2)} - \frac{h^{5/2}}{5/2} \right]_0^H$$

$$\text{or, } Q = \left( \frac{8}{15} \right) (C_d)(\sqrt{2g}) \left( \tan \frac{\theta}{2} \right) (H)^{5/2}$$



## Weir:-

It is a structure made to reserve or contain river water, over which the water flows. It is similar to notch but bigger in size. It is made of concrete or masonry.

### Classifications:-

① According to shape;

(a) Rectangular weir, (b) Cipolatti weir

② According to type of discharge;

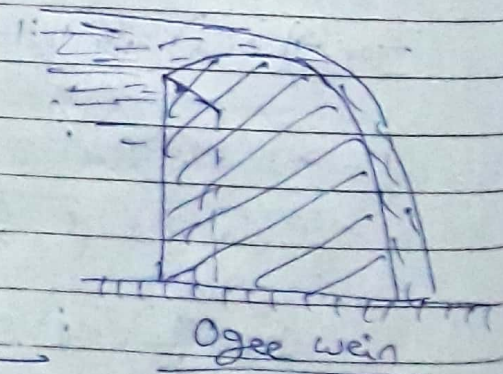
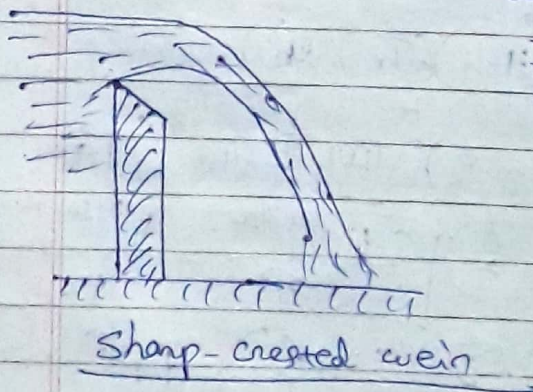
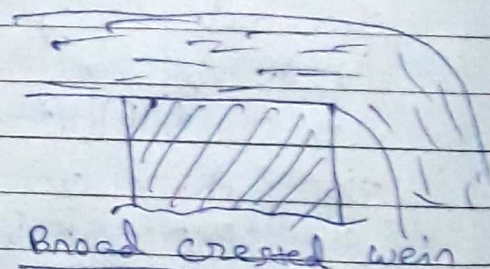
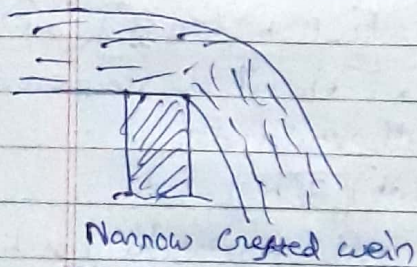
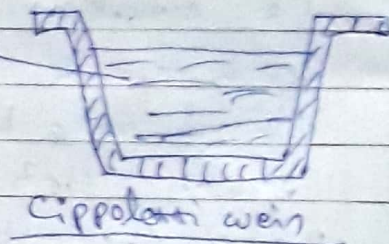
(a) Ordinary weir, (b) Submerged weir

③ According to crest width

(a) Narrow-crested weir, (b) Broad-crested weir

④ According to the nature of crest

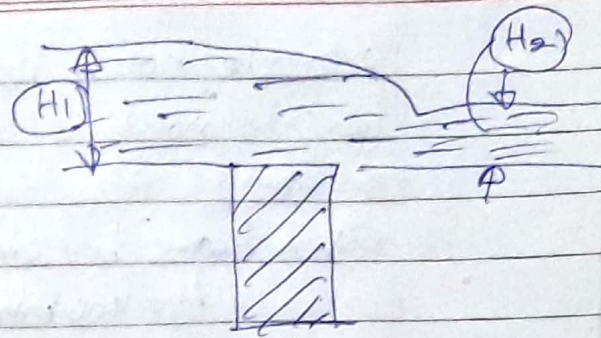
(a) Sharp-crested weir, (b) Ogee weir







Ordinary weir



Submerged weir

Problem - 1

A 50 mm diameter orifice is discharging water under a head of 9m. Calculate the actual discharge through the orifice and actual velocity of the jet at vena contracta. Given,  $C_d = 0.625$  and  $C_v = 0.98$ . Also find out  $C_c$ .

Solution

Given data;  $d = 50 \text{ mm} = 0.05 \text{ m}$ ,

$H = 9 \text{ m}$ ,

$C_d = 0.625$ ,  $C_v = 0.98$

The cross-sectional area of the orifice,  $a = \frac{\pi}{4}(d)^2$   
 $= \frac{\pi}{4}(0.05)^2 = 0.002 \text{ m}^2$

The theoretical discharge through the orifice,

$$Q_{th} = (a)(\sqrt{2gh}) = (0.002)(\sqrt{2 \times 9.81 \times 9})$$

$$= 0.0376 \text{ m}^3/\text{s}$$

Actual discharge through the orifice,  $Q_{ac} = (C_d)(Q_{th})$

$$= (0.625)(0.0376) = 0.0235 \text{ m}^3/\text{s}$$

Theoretical velocity of the jet,  $V_{th} = \sqrt{2gh}$

$$= \sqrt{2 \times 9.81 \times 9} = 13.3 \text{ m/s}$$

Actual velocity of the jet,  $V_{ac} = (C_v)(V_{th})$

$$= (0.98)(13.3) = 13 \text{ m/s}$$

We know that,  $C_d = (C_c)(C_v)$

$$\therefore C_c = \frac{C_d}{C_v} = \frac{0.625}{0.98} = 0.6377$$

Problem - 2

A rectangular notch of 0.5 m wide has a constant head of 400 mm. Find the discharge

(P-52)



over the notch, if the coefficient of discharge for the notch is 0.62.

Solution

Given data;  $b = 0.5 \text{ m}$ ;

$$H = 400 \text{ mm} = 0.4 \text{ m}, C_d = 0.62$$

The discharge over the rectangular notch,

$$\begin{aligned} Q &= \left(\frac{2}{3}\right) (C_d) (b) (\sqrt{2g}) (H)^{3/2} \\ &= \left(\frac{2}{3}\right) (0.62) (0.5) (\sqrt{2 \times 9.81}) (0.4)^{3/2} \\ &= 0.231 \text{ m}^3/\text{s} \end{aligned}$$

Problem - 3

A right angled V-notch was used to measure the discharge. The depth of water at V-notch is 200 mm. Calculate the discharge over the notch.

Assume; coefficient of discharge = 0.62

Solution

Given data;  $\theta = 90^\circ$ ,  $H = 200 \text{ mm} = 0.2 \text{ m}$ ,  $C_d = 0.62$

Discharge over the triangular notch,

$$\begin{aligned} Q &= \left(\frac{8}{15}\right) (C_d) (\sqrt{2g}) \left(\tan \frac{\theta}{2}\right) (H)^{5/2} \\ &= \left(\frac{8}{15}\right) (0.62) (\sqrt{2 \times 9.81}) \times \left(\tan \frac{90}{2}\right) (0.2)^{5/2} \\ &= 0.026 \text{ m}^3/\text{s} \end{aligned}$$

Problem - 4

A rectangular weir of 4.5 m long has a 300 mm head of water. Determine the discharge over the weir if coefficient of discharge is 0.6

Solution

Given data;  $L = 4.5 \text{ m}$ ,  $H = 300 \text{ mm} = 0.3 \text{ m}$ ,  $C_d = 0.6$

The discharge over the weir,

$$\begin{aligned} Q &= \left(\frac{2}{3}\right) (C_d) (L) (\sqrt{2g}) (H)^{3/2} \\ &= \left(\frac{2}{3}\right) (0.6) (4.5) (\sqrt{2 \times 9.81}) (0.3)^{3/2} \\ &= 1.31 \text{ m}^3/\text{s} \end{aligned}$$

(P-53)



# Question Bank



## Short Questions

- (1) Define about orifice.
- (2) What do you mean by vena-contracta?
- (3) Define about the coefficient of contraction.
- (4) What is the coefficient of velocity?
- (5) What do you understand by coefficient of discharge?
- (6) Define the term coefficient of resistance.
- (7) Define about the notch.
- (8) Why rectangular notch is preferably used?
- (9) For what advantages we use triangular notch?
- (10) Define about the weir.
- (11) What is nappe of notch?
- (12) What do you mean by crest of notch?
- (13) Write down the materials used for making weir.

## Long Questions

- (14) Write a short note about the flow through orifice.
- (15) Derive the mathematical relation between the three hydraulic coefficients of orifice.
- (16) Derive the mathematical equation for the discharge of liquid over the rectangular notch.
- (17) Derive the mathematical equation to find out the discharge of liquid over the triangular notch.
- (18) Briefly classify the different types of weirs.



## Chapter - 6 Flow through Pipe



Pipe:-

The pipe is defined as the closed conduit generally having circular cross-section and carries liquid or fluid. A pipe when running full is said to be pipe flow under pressure.

Loss of energies in Pipes:-

Whenever any liquid or fluid flows through the pipe, then it experiences resistance to its flow. Due to the resistance, the velocity and head of liquid are reduced. The losses are of generally two types, like (a) major loss and (b) minor loss.

Major loss -

The major loss is because of the frictional resistance of the pipe. The frictional resistance generally depends on the inside surface roughness of the pipe. With increase in roughness the frictional resistance also increases. This friction is also called as fluid friction.

Mr. Fraude conducted many experiments on fluid friction and found that:

- (a) The frictional resistance changes approximately with the square of the velocity.
- (b) The frictional resistance also changes according to the nature of the surface.

Minor losses -

There are energy losses at the entrance of the pipe. There are also energy losses because of the liquid velocity at the pipe exit. In actual cases, the minor losses are neglected such that the mathematical equations and

(P-55)



calculations become simple. Also the minor losses are small when compared with major losses.

Darcy's formula for frictional head loss in pipe:-

The mathematical equation is;

$$h_f = \frac{4(f')(l)(v^2)}{(w)(d)}$$

where,  $h_f$  = Loss of head because of friction

$f'$  = Frictional resistance per unit area per unit velocity

$l$  = length of the pipe

$v$  = velocity of liquid in the pipe

$w$  = specific weight of the liquid

$d$  = diameter of the pipe

Again;  $f' = \frac{(f)(w)}{(2g)}$

where,  $f$  = Darcy's coefficient or friction coefficient

Again,  $f = 0.005 \left(1 + \frac{1}{12d}\right)$  for smooth and new pipe

$f = 0.01 \left(1 + \frac{1}{12d}\right)$  for rough and old pipe

So,  $h_f = 4 \times \frac{(f)(w)}{(2g)} \times \frac{(l)(v^2)}{wd} = \frac{4flv^2}{2gd}$

Again;  $Q = \frac{\pi}{4} d^2 \times v$  or,  $v = \frac{4Q}{\pi d^2}$

So;  $h_f = \frac{4fl}{2gd} \times \frac{16Q^2}{\pi^2 d^4} = \frac{32flQ^2}{\pi^2 g d^5}$

where,  $Q$  = discharge of liquid

Chezy's formula for head loss:-

The mathematical equation is;

$$v = C \sqrt{m i}$$

where,  $v$  = velocity of liquid in the pipe

$C$  = Chezy's constant

$$C = \sqrt{\frac{w}{f'}}$$



where,  $w$  = specific weight of the liquid  
 $f'$  = Frictional resistance per unit area  
 per unit velocity

Again,  $m = \frac{A}{P}$

where,  $m$  = hydraulic mean depth

$A$  = Area of the pipe

$P$  = Wetted perimeter =  $\pi d$

$d$  = diameter of the pipe

Again,  $i = \frac{h_f}{L}$

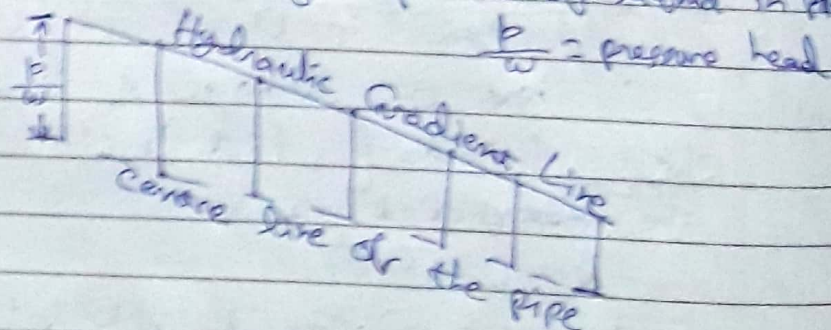
where,  $i$  = loss of head per unit length

$h_f$  = Loss of head due to friction

$L$  = length of the pipe

Hydraulic gradient line:-

When the pressure heads of the liquid flowing through the pipe are plotted as the vertical ordinates on the centre line of the pipe, then the line joining the tops of all the vertical ordinates is called as hydraulic gradient line. In short it is called as H.G.L. From this line we are able to know how the pressure head changes along the length of the pipe and also gives us idea about the flow pattern of liquid in pipe.



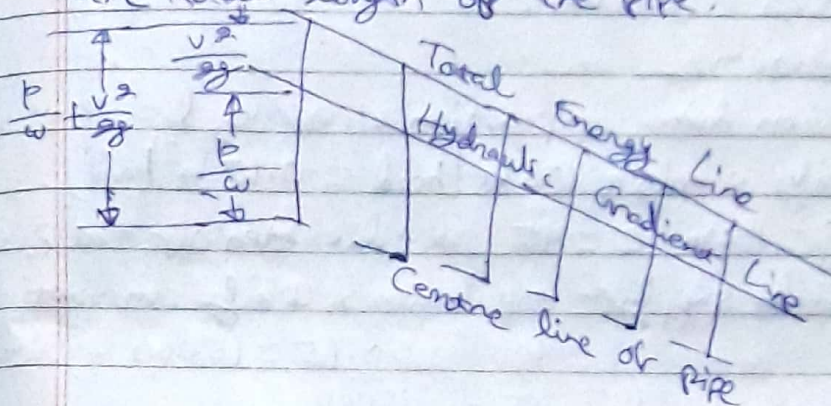
Total Energy Line:-

When the sum of pressure heads and velocity heads of the liquid are plotted

(P-57)



as vertical ordinates on the centre line of the pipe, then the line joining the tops of all the vertical ordinates is called a Total Energy Line. In short it is also called as T.E.L. This line maintains constant velocity heads above the H.G.L. and lies above it. It also provides information about total energy along the total length of the pipe.



### Problem - 1

Find the loss of head, due to friction, in a pipe of 500 mm diameter and 1.5 km long. The velocity of water in the pipe is 1 m/s. Take coefficient of friction as 0.005.

### Solution

Given Data;  $d = 500 \text{ mm} = 0.5 \text{ m}$ ,  
 $L = 1.5 \text{ km} = 1500 \text{ m}$ ,  
 $V = 1 \text{ m/s}$ ,  
 $f = 0.005$

$$1000d = 1000 \times 0.5 = 500 \text{ m}$$

As, length of pipe 1500 m is more than 500 m, so it is a long pipe and hence all the minor losses are neglected. The only loss is frictional loss.

According to Darcy's equation, head loss due to friction;

$$h_f = \frac{4fLV^2}{2gd} = \frac{4(0.005)(1500)(1)^2}{2 \times 9.81 \times 0.5} = 3.01 \text{ m}$$



### Problem - 2

A town having a population of 100000 is to be supplied with water from a reservoir at 5 km. distance. The one-half of the daily supply of 150 lt/head are delivered within 8 hours. Find the size of the pipe to furnish the supply, if the head available is 12m. Take  $C=45$  in the Chezy's formula.

#### Solution

Given data: population = 100000,  $L = 5 \text{ km} = 5000 \text{ m}$   
Daily supply = 150 lt/head =  $0.15 \text{ m}^3/\text{head}$ ,  
 $h_f = 12 \text{ m}$ ,  $C = 45$ ,  $d$  = diameter of the pipe.  
Total supply = population  $\times$  Daily consumption  
 $= 100000 \times 0.15 = 15000 \text{ m}^3/\text{day}$ .

$$\text{Maximum flow, } Q = \frac{15000}{24 \times 3600} \\ = 0.26 \text{ m}^3/\text{sec}.$$

Hydraulic mean depth of pipe;

$$m = \frac{d}{4}$$

Loss of head per unit length,

$$i = \frac{h_f}{L} = \frac{12}{5000} = 0.0024$$

velocity of water,  $V = C \sqrt{mi}$

$$= 45 \sqrt{\frac{d}{4} \times 0.0024} \\ = 45 \sqrt{0.0006d}$$

Discharge through the pipe,  $Q = a \times V$

where,  $a$  = area of pipe =  $\frac{\pi}{4} d^2$

$$\text{or, } Q = \frac{\pi}{4} d^2 \times V$$

$$\text{or, } 0.26 = \frac{\pi}{4} (d)^2 \times 45 \sqrt{0.0006d}$$

$$\text{or, } 0.0676 = \left(\frac{\pi^2}{16}\right) \times d^4 \times 2025 \times 0.0006d$$

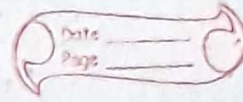
(squaring both sides)

$$\text{or, } 0.0676 = 0.7495 d^5 \text{ or, } d = 0.618 \text{ m.}$$

(P-59)



## Question Bank



### Short questions

- (1) Define about the pipe.
- (2) Define about the frictional resistance in pipe flow.
- (3) Write down the findings of Mr. Froude.
- (4) Why minor losses are not considered as compared with the major losses when any fluid is flowing through the pipe.
- (5) Write down the mathematical formula for the Darcy's in case of fluid flow in pipe.
- (6) Write the mathematical equations for friction coefficient in both new and old pipes when liquid is flowing through it.
- (7) What is Chezy's constant?
- (8) What do you mean by hydraulic mean depth?
- (9) Write the mathematical equation for head loss per unit length.
- (10) Define about H.G.L.

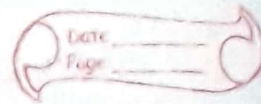
### Long Questions

- (11) Write down in details about the energy losses in pipes when fluid is flowing through it.
- (12) Write both Darcy's and Chezy's formula and elaborate the different terms in the formula.
- (13) Write short note about Hydraulic Gradient line and Total Energy Line for pipe flow.



## chapter - 7

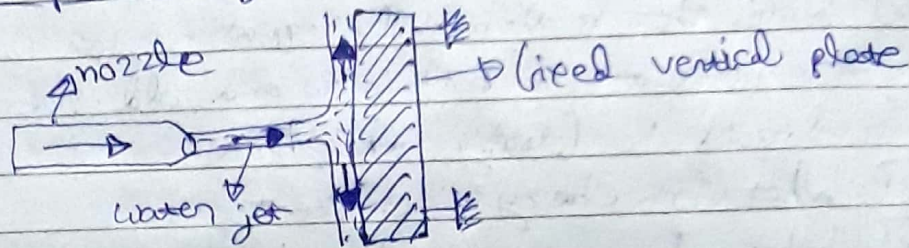
### Impact of Jets



#### Impact of jet:-

The liquid jet when strikes on a fixed plate, then it produces force on the plate. According to Newton's 2nd law of motion, the force produced is equal to the rate of change of momentum of the water jet. When the plate is not fixed, then the plate moves in the direction of the liquid jet due to the force.

#### Impact of jet on a vertical fixed plate:-



A liquid jet is making impact on the fixed vertical plate.

$V$  = velocity of liquid (water) jet.

$a$  = cross-sectional area of the water jet.

Mass of water flowing/sec. =  $\frac{w a V}{g}$  kg

where,  $w$  = specific weight of the water,

$g$  = ~~acceleration~~ acceleration due to gravity.

The velocity of jet after impact with the vertical plate is zero in the direction of water jet or in the horizontal direction.

The force exerted by the water jet on the plate,

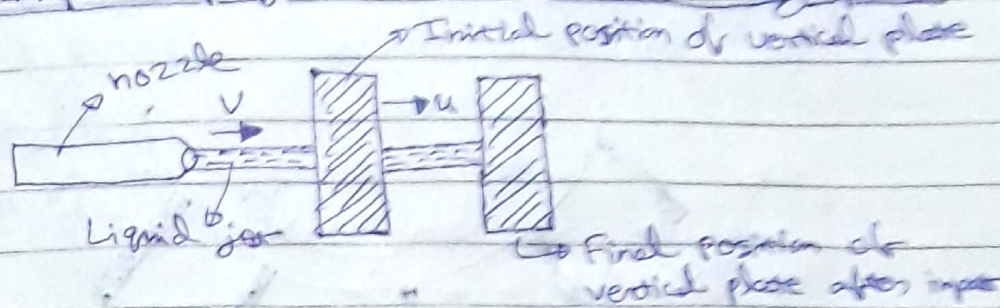
$F$  = mass of water flowing/sec  $\times$  Velocity change

$$= \left( \frac{w a V}{g} \right) (V - 0)$$

$$\therefore F = \frac{w a V^2}{g}$$



## Impact of jet on a vertical moving plate :-



The liquid jet is making impact on the vertical plate. After impact the vertical plate moves in the direction of liquid jet.

$v$  = velocity of liquid jet

$a$  = cross-sectional area of the liquid jet

$u$  = velocity of the vertical plate after impact of the liquid jet

$v-u$  = Relative velocity of the liquid jet with respect to the vertical plate

Hence, initially the jet of liquid is moving with the relative velocity of  $(v-u)$ . After impact the relative velocity becomes zero in the direction of impact.

The Force exerted by the liquid jet on the plate,

$$F = \text{mass of water flowing per sec.} \times \text{velocity change} \\ = \left[ \frac{wa(v-u)}{g} \right] [(v-u) - 0]$$

where,  $w$  = specific weight of the liquid,

$g$  = acceleration due to gravity

$$\text{or, } F = \frac{wa(v-u)^2}{g}$$

The work done by the liquid jet on the plate/sec.

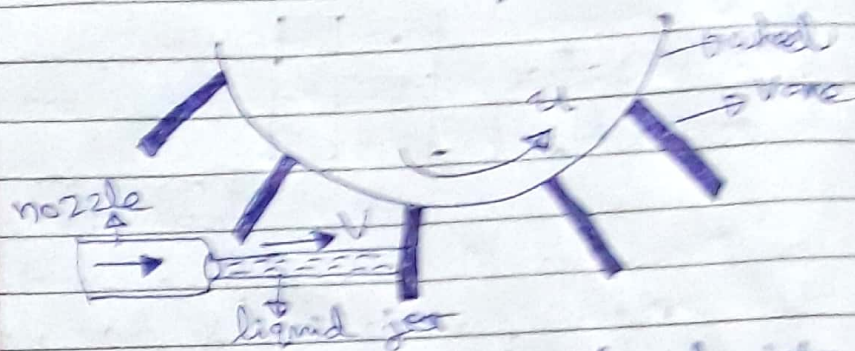
$W$  = Force exerted by the liquid jet  $\times$  the distance travelled by the plate/sec.

$$= \left[ \frac{wa(v-u)^2}{g} \right] (u)$$

$$\text{or, } W = \frac{(wa)(u)(v-u)^2}{g}$$



## Impact of jet on a series of vanes:-



In actual practice, the liquid jet continuously becomes unable to give impact on the moving vertical flat plates (one plate). So, practically the liquid jet impacts on a series of vanes on plates which are mounted on the circumference of a wheel. As shown in the above figure, the liquid jet strikes on the vane with a velocity ' $V$ ' and due to it the vanes move with a velocity ' $u$ '.

$V$  = Velocity of liquid jet

$a$  = cross-sectional area of liquid jet

$u$  = Velocity of the series of vanes after impact

Force exerted by the jet on the vanes,

$$F = \text{Mass of liquid flowing/sec} \times \text{Velocity change} \\ = \left( \frac{w a V}{g} \right) (V - u)$$

where,  $w$  = specific wt. of the liquid

$$\text{or, } F = \frac{w a V (V - u)}{g}$$

Work done by the jet/sec,

$W$  = Force exerted by the jet  $\times$  Distance moved by the vane/sec

$$= \frac{w a V (V - u)}{g} \times u$$



Energy of the liquid jet = Kinetic energy of jet  
 $= \frac{1}{2} \times \text{mass of jet} \times \text{velocity of jet}$   
 $= \frac{1}{2} \left( \frac{w a v}{g} \right) (v)^2$

The efficiency,  $\eta = \frac{\text{Work done by the jet}}{\text{Energy of the jet}}$   
 $= \frac{\left( \frac{w a v}{g} \right) (v-u)(u)}{\left( \frac{1}{2} \right) \left( \frac{w a v}{g} \right) (v^2)}$   
 or,  $\eta = \frac{2(v-u)u}{v^2}$

Maximum efficiency —

The efficiency to be maximum;

$$\frac{d\eta}{du} = 0$$

$$\text{or, } \frac{d}{du} \left[ \frac{2(v-u)u}{v^2} \right] = 0$$

$$\text{or, } \frac{d}{du} \left( \frac{2vu - 2u^2}{v^2} \right) = 0$$

$$\text{or, } \left( \frac{1}{v^2} \right) \left( \frac{d}{du} \right) (2vu - 2u^2) = 0$$

$$\text{or, } \left( \frac{d}{du} \right) [2vu - 2u^2] = 0$$

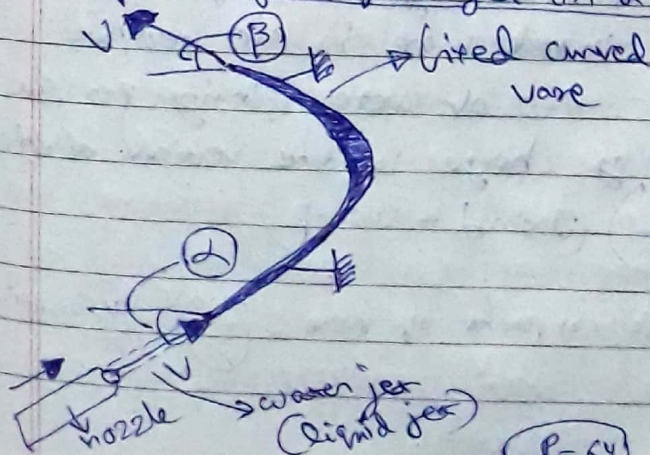
$$\text{or, } 2v - 4u = 0$$

$$\text{or, } 2v = 4u$$

$$\text{or, } v = \frac{4u}{2} = 2u$$

So, the condition is;  $v = 2u$  (for maximum efficiency)

Impact of liquid jet on a fixed curved vane:



The liquid jet is entering and leaving the fixed curved vane in the



tangential direction. The liquid jet produces force on the fixed curved vane. This force has two components out of which one component is acting in the horizontal direction and other one is acting in the vertical direction.

$V$  = Velocity of the liquid jet

$a$  = cross-section area of the liquid jet

$\alpha$  = Inlet angle of the liquid jet

$\beta$  = outlet angle of the liquid jet

The force exerted by the jet on the curved fixed vane in the direction normal to the vane or in the horizontal direction

= mass of liquid flowing/sec  $\times$  change in velocity in the horizontal direction

$$= \left( \frac{w a V}{g} \right) [V \cos \alpha - (-V \cos \beta)]$$

$$= \left( \frac{w a V}{g} \right) (V \cos \alpha + V \cos \beta)$$

where,  $w$  = specific weight of the liquid,  
 $g$  = acceleration due to gravity.

$V \cos \beta$  is negative as it acts in opposite direction that of  $V \cos \alpha$ .

→  $V \cos \alpha$  and ←  $V \cos \beta$

The force exerted by the liquid jet on the fixed curved vane in the vertical direction = mass of water flowing per sec.

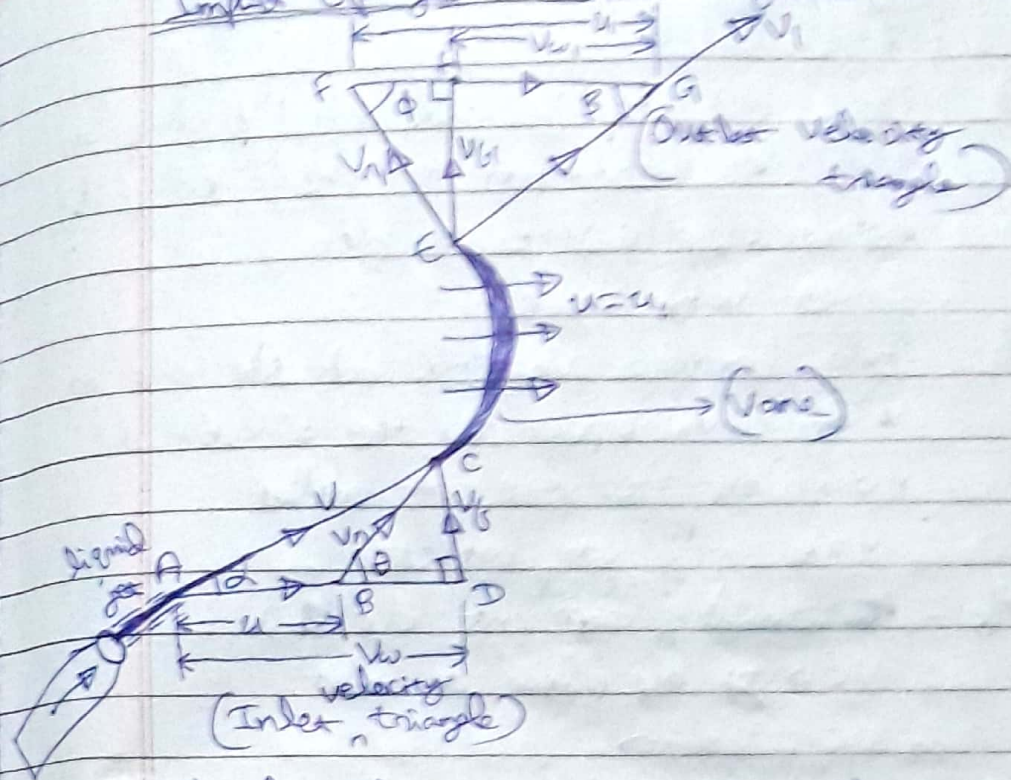
$\times$  velocity change in the vertical direction

$$= \left( \frac{w a V}{g} \right) (V \sin \alpha - V \sin \beta)$$

$$\text{or, Vertical component of force} = \left( \frac{w a V}{g} \right) (V \sin \alpha - V \sin \beta)$$



## Impact of jet on a moving curved vane:-



The liquid jet is entering the moving curved vane tangentially and also leaving the vane tangentially.

$V$  = velocity of liquid jet entering the vane = AC

$V_1$  = velocity of liquid jet leaving the vane = EG

$u$  = velocity of vane at inlet = AB

$u_1$  = velocity of vane at outlet = FG

$V_n$  = relative velocity of the liquid jet with the vane at the entry = BC

$V_{n1}$  = relative velocity of the liquid jet with the vane at the outlet = EF

$V_w$  = Horizontal component of the velocity ' $V$ '  
= Velocity of whirl at inlet  
= AD =  $V \cos \alpha$

$\alpha$  = Liquid jet entry angle with the vane with respect to the horizontal axis

$\beta$  = Liquid jet outlet angle with the vane with respect to the horizontal axis

$V_{w1}$  = Horizontal component of the velocity ' $V_1$ '



= velocity of whirl at the outlet

$$= H\alpha = V_1 \cos \beta$$

$\theta$  = Angle between  $V_1$  and with the vane at the inlet with respect to the direction of motion of the vane at inlet

= Inlet vane angle

$\phi$  = Angle between  $V_2$  and with the vane at outlet with respect to the direction of motion of the vane at outlet

= Vane angle at outlet

$V_f$  = ~~Horizontal~~ <sup>Vertical</sup> component of  $V$

= Velocity of flow at inlet

$$= DC = V \sin \alpha$$

$a$  = cross-sectional area of the liquid jet

$V_{f2}$  = Vertical component of  $V_2$

= velocity of flow at the outlet

$$= EH = V_2 \sin \beta$$

The liquid jet must enter and leave the curved vane in tangential manner in order to reduce <sup>impact of jet and</sup> friction. The inner surface of the curved vane must also

be very smooth in order to reduce friction. The ABCD in the inlet velocity triangle.

First of all, it is to be drawn as per given data. Then the vane is drawn in such a manner that the both  $V_1$  and  $V_{f1}$

will be tangent to the vane. Then the outlet velocity triangle (EFHG) is drawn. Both the triangles represent mainly

the different velocity components of the jet and hence so named. As both the impact load and friction losses are



very less, so we can assume the following for both the velocity triangles.

$$The u = u_1$$

$$and V_n = V_{n1}$$

The force exerted by the liquid jet in the direction of motion of the vane or in the horizontal direction;

$$\begin{aligned} F_x &= \text{Mass of water (liquid) flowing/sec.} \times \text{the velocity change (Horizontal components)} \\ &= \text{mass of liquid jet} \times \text{change in velocity of whirl} \\ &= \left( \frac{w a V}{g} \right) (V_w - V_{w1}) \end{aligned}$$

where,  $w$  = specific weight of the liquid

$g$  = acceleration due to gravity

Work done by the liquid jet in the direction of motion of the vane or in the horizontal direction/sec.

= Force in the horizontal direction  $\times$  Distance covered by the vane in the horizontal direction per second

$$= \left( \frac{w a V}{g} \right) (V_w - V_{w1}) (u)$$

Energy of the jet of liquid

= Kinetic energy of the jet of liquid

=  $\frac{1}{2} \times \text{mass of jet of liquid} \times (\text{velocity of liquid})^2$

$$= \left( \frac{1}{2} \right) \left( \frac{w a V}{g} \right) (V)^2$$

Efficiency,  $\eta = \frac{\text{Work done by the liquid jet}}{\text{Energy of the liquid jet}}$

$$\text{or, } \eta = \frac{\left( \frac{w a V}{g} \right) (V_w - V_{w1}) (u)}{\left( \frac{1}{2} \right) \left( \frac{w a V}{g} \right) (V)^2} = \frac{2 (V_w - V_{w1}) (u)}{(V)^2}$$

(P-68)



### Problem - 1

A jet of water of 50 mm diameter is discharging water under a constant head of 70 m. Calculate the force exerted by the water jet on a fixed plate. Assume,  $C_v = 0.9$

#### Solution

Given data;  $d = 50 \text{ mm} = 0.05 \text{ m}$ ,

$H = 70 \text{ m}$ ,

$C_v = 0.9$

Area of cross-section of the jet,

$$a = \frac{\pi}{4} (d)^2 = \frac{\pi}{4} (0.05)^2 = 0.001964 \text{ m}^2$$

Velocity of the jet,

$$V = C_v \sqrt{2gH} = (0.9) \sqrt{2 \times 9.81 \times 70} = 33.4 \text{ m/s}$$

Force exerted by the jet on the plate,

$$F = \frac{\rho a V^2}{g} = \frac{9810 \times 0.001964 \times (33.4)^2}{9.81}$$

$$= 2190 \text{ N}$$

### Problem - 2

A water jet having 50 mm diameter moving with a velocity of 26 m/s is impinging normally on a plate. Calculate the pressure on the plate when it is moving with a velocity of 10 m/s in the direction of the jet. Also find out the work done per second by the jet.

#### Solution

Given data,  $d = 50 \text{ mm} = 0.05 \text{ m}$ ,

$V = 26 \text{ m/s}$ ,

$u = 10 \text{ m/s}$ .

Pressure on the plate when it is moving

$$= \frac{\rho a (V-u)^2}{g} = \frac{9810 \times \frac{\pi}{4} (0.05)^2 (26-10)^2}{9.81}$$

(P-69)



where,  $a = \text{area of the jet} = \frac{\pi}{4} (d)^2 = \frac{\pi}{4} (0.05)^2$   
 or, Pressure = 503 N (again,  $w = \text{specific weight of water} = 9810 \text{ N/m}^3$ )

Work done per second by the jet  
 = Pressure force  $\times$  (distance/sec.)  
 =  $(503)(10) = 5030 \text{ N-m/sec.}$

### Problem - 3

A jet of water 50 mm in diameter, moving with a velocity of 15 m/s, impinge on a series of vanes moving with a velocity of 6 m/s. Calculate the force exerted by the jet, work done per second by the jet and efficiency of the jet.

### Solution

Given,  $d = 50 \text{ mm} = 0.05 \text{ m}$ ,  
 $V = 15 \text{ m/s}$ ,  
 $u = 6 \text{ m/s}$ .

Cross-section area of the jet,  $a = \frac{\pi}{4} (d)^2$   
 $= \frac{\pi}{4} (0.05)^2 = 0.001964 \text{ m}^2$

Force exerted by the jet,

$$F = \frac{(w a V)(V - u)}{g}$$

$$= \frac{(9810)(0.001964)(15)(15 - 6)}{9.81} = 265 \text{ N}$$

where,  $w = \text{specific weight of water} = 9810 \text{ N/m}^3$

Work done per second by the jet/sec.

= Force  $\times$  Distance moved/sec.

$$= 265 \times 6 = 1590 \frac{\text{N-m}}{\text{sec}}$$

Efficiency of the jet;

$$\eta = \frac{2(V - u)u}{V^2} = \frac{2(15 - 6)(6)}{(15)^2}$$

$$\text{or, } \eta = 0.48 = 48\%$$



#### Problem - 4

A jet of water 40 mm diameter enters a fixed curved vane with a velocity of 50 m/s at an angle of  $20^\circ$  to the horizontal. Find the normal and tangential forces exerted by the jet, if it leaves the vane at an angle of  $15^\circ$  to the horizontal.

Solution

Given data,  $d = 40 \text{ mm} = 0.04 \text{ m}$ ,

$$V = 50 \text{ m/s},$$

$$\alpha = 20^\circ$$

$$\beta = 15^\circ$$

Area of cross-section of the jet,

$$a = \frac{\pi}{4} (d)^2 = \frac{\pi}{4} (0.04)^2 = 0.001257 \text{ m}^2$$

Normal force exerted by the jet,

$$F_n = \left( \frac{w a V}{g} \right) (V \cos \alpha + V \cos \beta)$$

where,  $w$  = specific weight of water =  $9810 \text{ N/m}^3$

$$\therefore F_n = \frac{9810 \times (0.001257) (50)}{(9.81)} (50 \cos 20^\circ + 50 \cos 15^\circ)$$

$$= (1000) (0.063) (50 + 0.9397 + 50 \times 0.9659) \\ = 6000 \text{ N}$$

Tangential force exerted by the jet,

$$F_t = \left( \frac{w a V}{g} \right) (V \sin \alpha - V \sin \beta)$$

$$= \frac{9810 \times (0.001257) (50)}{(9.81)} (50 \sin 20^\circ - 50 \sin 15^\circ)$$

$$= (63) (50 \times 0.3420 - 50 \times 0.2598)$$

$$= 262 \text{ N}$$

#### Problem - 5

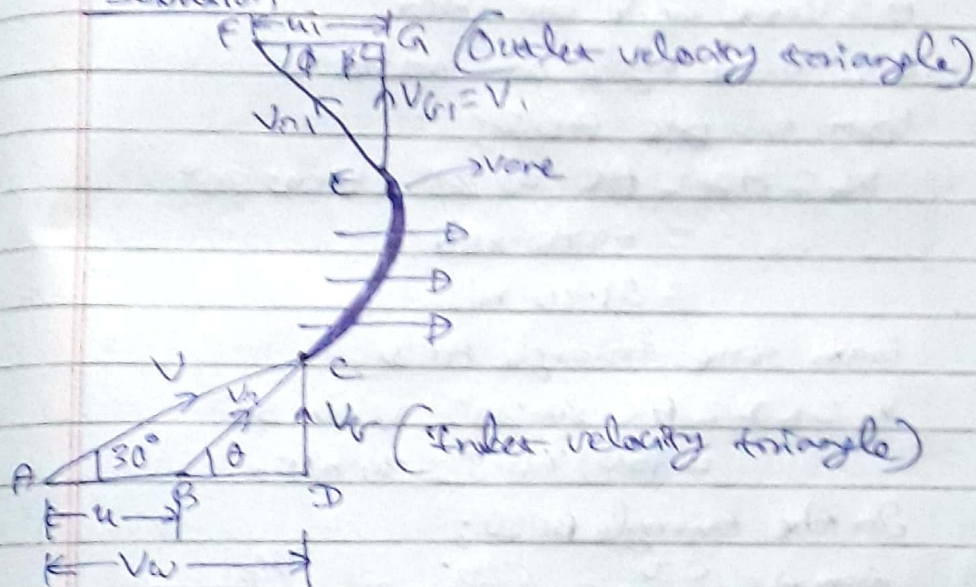
A jet of water having a velocity of

(P-71)



60 m/s impinges on a curved vane and vane is moving at 25 m/s. The water jet enters the vane at  $30^\circ$  with the direction of jet. The water leaves the blade normally to the motion of the vanes. Draw both inlet and outlet velocity triangles and find the vane angles for no shock at entry and exit. The relative velocity at the outlet to be 0.85 of the relative velocity at the inlet.

Solution



Given data;  $V = 60 \text{ m/s}$ ,  $u = 25 \text{ m/s}$ ,  $\alpha = 30^\circ$ ,  $V_{r2} = 0.85 V_{r1}$

- Draw a horizontal line and cut it as AB equal to 25 m/s as per any suitable scale. So  $AB = u$ .
- Draw a line at an angle of  $30^\circ$  at point A and cut it equal to 60 m/s as per the same scale as AC. So,  $AC = V$ .
- Then join B with C and  $BC = V_r$ .
- Now draw a perpendicular from the point C on the AB line extended such that it cut at point D. So,  $CD = V_f$  and  $AD = V_w$ .
- Then draw a curved vane such that  $V_r$  is tangential at the inlet tip of vane.



(f) Now draw a line EF tangential at the point of outlet 'E' of the vane by taking  $EF = 0.85(BC)$  by the same scale taken.  $EF = V_{n1} \approx 0.85 V_n$ .

(g) Draw a horizontal line from point 'F' towards right and draw a perpendicular on it from point 'E' to meet both at point 'G'. Hence the outlet velocity triangle is completed.  $FG = u_1$ ,  $EG = V_{G1}$  and  $\angle FGE = \angle B = 90^\circ$ .

$\theta$  = Vane angle at inlet,

$\phi$  = Vane angle at outlet,

From triangle ACD;

$$\begin{aligned} V_w \approx AD &= AC \cos 30^\circ = V \cos 30^\circ \\ &= 60 \times 0.866 \\ &= 51.96 \text{ m/s.} \end{aligned}$$

From same triangle ACD;

$$\begin{aligned} V_G = CD &= AC \sin 30^\circ = V \sin 30^\circ \\ &= (60) \times 0.5 = 30 \text{ m/s.} \end{aligned}$$

In the triangle BCD;

$$\tan \theta = \frac{CD}{BD} = \frac{CD}{AD - AB} = \frac{V_G}{V_w - u}$$

$$\text{or, } \tan \theta = \frac{30}{51.96 - 25} = 1.1128$$

$$\text{or, } \theta = 48.1^\circ$$

In the same triangle BCD;

$$V_n \approx BC \approx \frac{CD}{\sin \theta} = \frac{V_G}{\sin 48.1^\circ} = \frac{30}{0.7447}$$

$$\text{or, } V_n = 40.31 \text{ m/s.}$$

In the outlet triangle EFG;

$$V_{n1} = EF = 0.85(BC) = 0.85(V_n) = (0.85)(40.31) = 34.26 \text{ m/s}$$

In the triangle EFG;

$$\cos \phi = \frac{FG}{EF} = \frac{u_1}{V_{n1}} = \frac{25}{34.26} = 0.7297$$

$$\text{or, } \phi = 43.1^\circ$$

(P-73)



## Question Bank



### Short questions

- (1) Define about impact of jet of liquid.
- (2) Write down the mathematical equation for the force exerted by liquid jet on the fixed vertical plate.
- (3) Write down the mathematical equation of work done per second by the liquid jet on the moving vertical plate.
- (4) What is the condition for the maximum efficiency of impact of jet on series of vanes.
- (5) Write down the mathematical equation for the horizontal force exerted by the liquid jet on the fixed curved vane.
- (6) Write the mathematical equation for the efficiency of jet when liquid jet impinges on the moving curved vane.

### Long Questions

- (7) Derive the equation for the force exerted by the liquid jet on a moving vertical plate.
- (8) Derive the formula for efficiency of the liquid jet when the liquid jet impacts on a series of vanes.
- (9) Derive the equation for the force exerted in the vertical direction when liquid jet enters the fixed curved vane.
- (10) Draw both inlet and outlet velocity triangles and derive the equation for work done when liquid jet impinges on a moving curved vane.



# Chapter-1

## Performance of I.C engine

Engine →

It is a device which transform one form of energy into another form.

Internal combustion engine →

It is a machine which converts chemical energy of fuel into mechanical energy.

Fuel is burnt in the combustion chamber, which releases its chemical energy in the form of heat, & it converts mechanical energy with the help of reciprocating piston & crank mechanism. Two types of reciprocating IC engines are used. 1) Otto cycle engine  
2) Diesel " "

Performance of Internal combustion engines →

Engine is selected for a particular application on the basis of its power o/p & rated speed. Some performance parameters are

1. Indicated power
2. Brake " "
3. Frictional " "
4. Fuel consumption
5. Air " "
6. Brake thermal efficiency
7. Indicated " "
8. Mechanical " "
9. Volumetric " "
10. Air-fuel ratio

1) Indicated power (IP) →

→ It is defined as the rate of work done on the piston by burning of charge inside the cylinder.

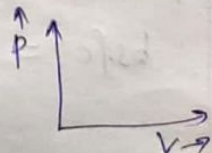
→ It is obtained from the indicated diagram obtained from the engine.

→ It is the gross power produced by the engine.

IP = Indicated mean effective pressure × Swept volume rate

$$= \frac{P \cdot L \cdot A \cdot n \cdot k}{60} \text{ (kW)}$$

where  $n = \text{No. of cyls}$





where  $P$  = Indicated mean effective pressure (in  $\text{kg/cm}^2$ )  
 $L$  = stroke length (in m)  
 $A$  = cross-sectional area of cylinder bore  
 $= \frac{\pi}{4} d^2$   
 $n = N$  (for 2-stroke engine)  $K \rightarrow$  no. of cylinders  
 $= \frac{N}{2}$  (for 4-stroke "  
 $n \rightarrow$  No. of working strokes per minute, when engine has a speed of  $N$  rpm.

### 2) Brake power (BP) $\rightarrow$

It is the net power available at the engine shaft for external use. It is measured by brake or dynamometer.

$BP = \text{brake load} \times \text{velocity of brake drum}$

$$= F \times \frac{2\pi RN}{60}$$

$$P = F \times v$$

$$= \frac{2\pi N(FR)}{60} = \frac{2\pi NT}{60} \text{ kW}$$

where  $F$  = braking force in N

$R$  = effective radius of the brake drum

$= \frac{\text{Dia of brake drum} + \text{Dia of rope}}{2}$

$T$  = Torque acting  $= F \times R$

$N$  = speed of the engine in rpm

### 3) Friction power (FP) $\rightarrow$

It is the part of the indicated power which is used to overcome the frictional effects within the engine.

$$FP = IP - BP$$

### 4) Specific fuel consumption (sfc) $\rightarrow$

It is defined as the ratio of the mass of fuel consumed per hour per unit power o/p (BP).

It is of 2 types  $\rightarrow$  bsfc  
 $\rightarrow$  isfc

Bsfc (Brake Specific fuel Consumption)  $\rightarrow$

The specific fuel consumption based on the indicated power is called bsfc.

$$bsfc = \frac{\dot{m}_f (\text{kg/h})}{BP (\text{kW})}$$

$\text{kg/kW-h}$



ISFC (Indicated Specific fuel Consumption)  
It is the SFC on based on the indicated power.  

$$ISFC = \frac{\dot{m}_f \text{ (kg/h)}}{IP \text{ (kW)}} \quad \text{kg/kWh}$$

### 5) Air-Fuel Ratio (A/F) →

It is defined as the ratio between the mass of the air & mass of the fuel supplied to the engine.

$$A/F = \frac{\dot{m}_a}{\dot{m}_f}$$

Theoretically correct (stoichiometric) A/F is 15.

A/F is 12 to 14 for petrol engine.

A/F is 20 to 60 " diesel "

### Efficiencies of IC engines: →

#### i) Brake thermal efficiency ( $\eta_{bth}$ ) →

It is defined as the ratio between brake power to the heat supply rate.

$$\eta_{bth} = \frac{BP}{\text{energy supply rate}} = \frac{BP}{\dot{m}_f \times CV}$$

where  $\dot{m}_f$  = mass flow rate of fuel (kg/s)  
 $CV$  = calorific value of fuel (kJ/kg)

#### ii) Indicated thermal efficiency ( $\eta_{ith}$ ) →

It is defined as the ratio between indicated power to input fuel energy.

$$\eta_{ith} = \frac{IP}{\dot{m}_f \times CV}$$

#### iii) Mechanical Efficiency ( $\eta_{mech}$ ) →

It is defined as the ratio between brake power to indicated power.

$$\eta_{mech} = \frac{BP}{IP}$$

$$\text{or } \eta_{mech} = \frac{\eta_{bth}}{\eta_{ith}}$$

$$\text{or } \eta_{mech} = \frac{\overset{\text{brake}}{B_{mep}}}{\underset{\text{Indicated}}{I_{mep}}}$$



iv) **Relative efficiency** →  
It is the ratio between actual thermal efficiency to air standard efficiency of the engine. It is also called as efficiency ratio.

$$\eta_{\text{relative}} = \frac{\eta_{\text{Brake thermal}}}{\eta_{\text{air standard}}}$$

It varies from 75% to 95%.

v) **Volumetric efficiency** → ( $\eta_{\text{vol}}$ )

It is defined as the ratio of mass or volume of actual charge induced into the cylinder to the mass or volume of the charge ~~can~~ displaced by the system.

$$\eta_{\text{vol}} = \frac{V_{\text{act}}}{V_{\text{swept}}}$$

vi) **Calorific Value of fuel (CV)** →

It is defined as the amount of heat energy liberated by complete combustion of unit quantity of fuel. The quantity of heat energy liberated by combustion at const. pressure & under normal condition  
Unit → KJ/kg

vii) **Mean effective pressure** →

It is the theoretical parameter used to measure the performance of an engine. It is the average pressure in the cylinder.

It is a hypothetical average pressure, which if acted on the piston during the entire power stroke, will produce the same power o/p as produced during the actual cycle.

$$P_{\text{mep}} = \frac{W_{\text{net}}}{V_{\text{swept/displacement}}}$$

\* It is used to compare the o/p of similar engines of different sizes.



Q-1 A 4 cylinder, 4S petrol engine develops indicated power of 14.7 kW at 1000 rpm. The mean effective pressure is 5.5 bar. Calculate the bore of stroke of the engine, if the stroke is 1.5 times the bore.

Sol<sup>n</sup>

Given  $K = 4$

$$IP = 14.7 \text{ kW}$$

$$P_m = 5.5 \text{ bar} = 550 \text{ kPa}$$

$$L = 1.5 d$$

$$\eta = \frac{N}{2}$$

$$N = 1000 \text{ rpm}$$

$$IP = \frac{P_m L A n K}{60} \Rightarrow 14.7 = \frac{550 \times \frac{1.5d}{60} \times \left(\frac{\pi}{4} d^2\right) \times \frac{1000}{2} \times 4}{60}$$

$$\Rightarrow d^3 = 6.806 \times 10^{-4} \text{ m}^3 \Rightarrow d = 0.08796 \text{ m} = 87.96 \text{ mm}$$

$$\text{So, } L = 1.5d = 131.94 \text{ mm}$$

Q-2 The following results were obtained from a test on a single cylinder, 4S diesel engine. Dia of cylinder is 30 cm, stroke of the piston is 45 cm, indicated mean eff. P is 540 kPa & engine speed is 240 rpm; Calculate the IP of the engine.

Sol<sup>n</sup>

Given  $K = 1$

$$L = 45 \text{ cm} = 0.45 \text{ m}$$

$$d = 30 \text{ cm} = 0.3 \text{ m}$$

$$N = 240 \text{ rpm}$$

$$P_m = 540 \text{ kPa}$$

$$\eta = \frac{N}{2} = 120 \text{ working stroke/minute}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} 0.3^2 = 0.07068 \text{ m}^2$$

$$IP = \frac{P_m L A n K}{60} = \frac{540 \times 0.45 \times 0.07068 \times 120 \times 1}{60} = 34.353 \text{ kW}$$



Q-3 In a single cylinder 4 stroke diesel engine indicated mean effective pressure = 755 KPa, cylinder dia = 10 cm, Piston stroke = 15 cm, engine speed = 480 rpm. Brake wheel dia = 62.5 cm, net load on brake wheel = 170 N. Find IP, BP,  $\eta_{mech}$ .

Sol<sup>n</sup> Given data  $P_m = 755 \text{ KPa}$   
 $d = 10 \text{ cm}$   
 $L = 15 \text{ cm}$   
 $N = 480 \text{ rpm}$

$$D = 62.5 \text{ cm} = 0.625 \text{ m}$$

$$F = 170 \text{ N}$$

$$T = F \times r = 170 \times \frac{0.625}{2} = 53.125 \text{ N-m}$$

$$B.P = \frac{2\pi NT}{60,000} = \frac{2\pi \times 480 \times 53.125}{60,000} = 2.67 \text{ kW}$$

$$I.P = \frac{P_m L A N K}{60,000} = \frac{755 \times 0.15 \times \frac{\pi}{4} \times 0.1^2 \times \frac{480}{2} \times 1}{60} = 3.54 \text{ kW}$$

$$\eta_{mech} = \frac{BP}{IP} = \frac{2.67}{3.54} = 0.75 = 75\%$$

Q-4 In a 4 cylinder 2S petrol engine 30 kW power is generated at 2500 rpm. The  $P_m$  on each piston is 8 bar &  $\eta_{mech} = 80\%$ . Calculate the dia & stroke of each cylinder if the stroke to bore ratio is 1.5. Also calculate the fuel consumption of the engine if the  $\eta_{bth} = 28\%$ . The CV of fuel is 43900 KJ/kg.

Sol<sup>n</sup> Given  $K = 4$   $\eta = N =$

$$BP = 30 \text{ kW}$$

$$N = 2500 \text{ rpm}$$

$$P_m = 8 \text{ bar} = 800 \text{ KPa}$$

$$\eta_{mech} = 0.8$$

$$d = ?$$

$$L = ?$$

$$\frac{L}{d} = 1.5 \Rightarrow L = 1.5d$$

$$sfc = ?$$

$$\eta_{bth} = 0.28$$

$$CV = 43900 \text{ KJ/kg}$$

$$\eta_{mech} = \frac{BP}{IP}$$

$$\Rightarrow 0.8 = \frac{30}{IP} \Rightarrow IP = 37.5 \text{ kW}$$

$$\eta_{bth} = \frac{BP}{\dot{m}_f \times CV}$$

$$\Rightarrow 0.28 = \frac{30}{\dot{m}_f \times 43900} \Rightarrow \dot{m}_f = 2.44 \times 10^{-3} = 8.78 \text{ kg/h}$$

$$IP = \frac{P_m L A N K}{60} = \frac{800 \times \frac{1.5d}{2} \times \frac{\pi}{4} d^2 \times 2500 \times 4}{60} = 37.5$$

$$\Rightarrow d = 0.062 \text{ m} = 62 \text{ mm}$$



$$L = 1.5d = 93 \text{ mm}$$

$$\text{sfc for BP} = \frac{\dot{m}_f}{BP} = \frac{2.44 \times 10^{-3}}{30} = 8.13 \times 10^{-5} \text{ kg/kW-h}$$

$$\text{sfc for IP} = \frac{\dot{m}_f}{IP} = \frac{2.44 \times 10^{-3}}{37.50} = 6.50 \times 10^{-5} \text{ kg/kW-h}$$

$$\text{Bsfc} = \frac{\dot{m}_f \text{ (kg/h)}}{BP \text{ (kW)}} = \frac{8.78}{30} = 0.293 \text{ kg/kW-h}$$

Q-5 The following results found from a petrol engine test.  
 IP = 30 kW, BP = 26 kW, engine speed = 1000 rpm.  
 Bsfc = 0.35 kg/kW-h, CV of fuel = 43900 kJ/kg  
 calculate a) Indicated thermal efficiency

b) brake

c) Mech.

$$\text{bsfc} = 0.35 \text{ kg/kW-h}$$

Sol<sup>n</sup> Given IP = 30 kW  
 BP = 26 kW  
 N = 1000 rpm

a)  $\eta_{ith} = ?$

b)  $\eta_{bth} = ?$

c)  $\eta_{mech} = ?$

$$\text{fuel consumption rate} = \dot{m}_f = \text{bsfc} \times BP$$

$$= 0.35 \times 26 = 9.1 \text{ kg/h}$$

$$= 2.53 \times 10^{-3} \text{ kg/s}$$

a)  $\eta_{ith} = \frac{IP}{\dot{m}_f \times CV}$

$$= \frac{30}{2.53 \times 10^{-3} \times 43900} = 0.27 = 27\%$$

b)  $\eta_{bth} = \frac{BP}{\dot{m}_f \times CV} = \frac{26}{2.53 \times 10^{-3} \times 43900} = 0.234 = 23.4\%$

c)  $\eta_{mech} = \frac{BP}{IP} = \frac{26}{30} = 0.867 = 86.7\%$

Q-6 The mechanical efficiency of a single cylinder, 4S engine is 80%. The friction power is estimated to be 26 kW. calculate the indicated power & brake power developed by the engine.

Sol<sup>n</sup> Given data  $\eta_{mech} = 80\% = 0.8$  IP = ?

$$F_p = 26 \text{ kW}$$

$$BP = ?$$

$$\eta_{mech} = \frac{BP}{IP} \Rightarrow 0.8 = \frac{BP}{IP} \Rightarrow BP = 0.8 IP$$

$$FP = IP - BP \Rightarrow 26 = IP - 0.8 IP \Rightarrow IP = \frac{26}{0.2} = 130 \text{ kW}$$

$$BP = IP - FP = 130 - 26 = 104 \text{ kW}$$



Q7 A 2 stroke, diesel engine develops a brake power of 420 kW. The engine consumes 195 kg/h of fuel and air-fuel ratio is 22:1. CV of fuel is 42000 kJ/kg. If 76 kW of power is required to overcome the frictional losses, calculate

- $\eta_{mech}$
- air consumption
- $\eta_{bth}$  (brake thermal efficiency)

sol<sup>n</sup> Given data  $\eta = N$   
 $BP = 420 \text{ kW}$   
 $\dot{m}_f = 195 \text{ kg/h}$   
 $A/F = 22:1$   
 $CV = 42000 \text{ kJ/kg}$   
 $FP = 76 \text{ kW}$

a)  $IP = BP + FP = 420 + 76 = 496 \text{ kW}$

$$\eta_{mech} = \frac{BP}{IP} = \frac{420}{496} = 0.8467 = 84.67\%$$

b) fuel consumption rate

$$\dot{m}_a = \dot{m}_f \times \frac{A}{F} = 195 \text{ kg/h} \times \frac{22}{1} = 4290 \text{ kg/h}$$

$$= 71.5 \text{ kg/min}$$

c)  $\eta_{bth} = \frac{BP}{\dot{m}_f \times CV}$

$$= \frac{420}{\frac{195}{3600} \times 42000 \text{ kJ/kg}} = 0.1846 = 18.46\%$$

Q8 Calculate the brake mean effective pressure of a 4 cylinder, 2S engine of 100 mm bore, 125 mm stroke, when it develops a torque of 490 N-m.

sol<sup>n</sup> Given data  $P_{meq} = ?$

$$K = 4$$

$$\eta = N$$

$$d = 100 \text{ mm} = 0.1 \text{ m}$$

$$L = 125 \text{ mm} = 0.125 \text{ m}$$

$$T = 490 \text{ N-m}$$

$$BP = \frac{2\pi NT}{60,000} = \frac{P L A n K}{60} = \frac{P \times 0.125 \times \frac{\pi}{4} \times 0.1^2 \times N \times 4}{9 \times 60}$$

$$\Rightarrow P = 784 \text{ kPa} = 7.84 \text{ bar}$$



Q A single cylinder, CI engine with a brake thermal efficiency of 30% uses diesel oil having a CV of 42000 kJ/kg. If its  $\eta_{mech} = 80\%$ , calculate  
 a) BSFC    b) ISFC    c)  $\eta_{ith}$

Sol<sup>n</sup>

Given  $\eta_{bth} = 0.3$      $\eta_{mech} = 0.8$   
 $CV = 42000 \text{ kJ/kg}$

a)  $\eta_{bth} = \frac{BP}{\dot{m}_f \times CV}$

$\Rightarrow 0.3 = \frac{BP}{\dot{m}_f \times 42000} \Rightarrow \frac{BP}{\dot{m}_f} = \frac{0.3 \times 42000}{1} \text{ kJ/kg}$

$bsfc = \frac{sfc}{BP} = \frac{\dot{m}_f \times 3600}{BP} = \frac{1}{0.3 \times 42000} \times 3600$   
 $= 0.286 \text{ kg/kWh}$

b)  $\eta_{mech} = \frac{ISFC}{bsfc}$

$\Rightarrow 0.8 = \frac{ISFC}{0.286} \Rightarrow ISFC = 0.8 \times 0.286 = 0.229 \text{ kg/kWh}$

c)  $\eta_{mech} = \frac{BP}{IP} = \frac{\eta_{bth}}{\eta_{ith}}$

$\Rightarrow \eta_{ith} = \frac{\eta_{bth}}{\eta_{mech}} = \frac{0.3}{0.8} = 0.375 = 37.5\%$

Q For a single cylinder, 2-stroke engine

$P_{mep} = 550 \text{ kPa}$

cylinder dia = 21 cm

piston stroke = 28 cm

engine speed = 360 rpm

brake torque = 628 N-m

fuel consumption = 8.16 kg/h

CV of fuel = 42700 kJ/kg

calculate a)  $\eta_{mech}$  b)  $\eta_{ith}$  c)  $\eta_{bth}$  d) BSFC in kg/kWh

Sol<sup>n</sup>

Given data

$P = 550 \text{ kPa}$

$d = 21 \text{ cm} = 0.21 \text{ m}$

$L = 28 \text{ cm} = 0.28 \text{ m}$

$N = 360 \text{ rpm}$

$T = 628 \text{ N-m}$

$\dot{m}_f = 8.16 \text{ kg/h}$

$CV = 42700 \text{ kJ/kg}$

$K = 1$



$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.21^2 = 0.0346 \text{ m}^2$$

$$IP = \frac{PLANK}{60}$$

$$= \frac{550 \times 0.28 \times 0.0346 \times 360 \times 1}{60} = 32.0 \text{ kW}$$

$$BP = \frac{2\pi NT}{60,000} = \frac{2\pi \times 360 \times 628}{60,000} = 23.675 \text{ kW}$$

$$a) \eta_{mech} = \frac{BP}{IP} = \frac{23.675}{32} = 0.7397 = 74\%$$

$$b) \eta_{th} = \frac{IP}{\dot{m}_f CV} = \frac{32 \text{ kW}}{\left(\frac{8.16}{3600} \text{ kg/s}\right) \times (42700 \text{ kJ/kg})}$$

$$= 0.330 = 33\%$$

$$c) \eta_{bth} = \frac{BP}{\dot{m}_f \times CV}$$

$$= \frac{23.675 \text{ kW}}{\left(\frac{8.16}{3600} \text{ kg/s}\right) \times (42700 \text{ kJ/kg})}$$

$$= 0.244 = 24.4\%$$

$$d) bsfc = \frac{\dot{m}_f (\text{kg/h})}{BP (\text{kW})} = \frac{8.16 \text{ kg/h}}{23.675 \text{ kW}} = 0.3446 \text{ kg/kW}$$

Q A single cylinder 4s diesel engine works on the following data.

cylinder bore = 15 cm

stroke = 25 cm

speed = 250 rpm

Area of Indicator diagram = 6 cm<sup>2</sup>

Length " = 7.9 cm

spring const = 7.5 bar/cm

bsfc = 0.24 kg/kWh

CV = 42000 kJ/kg

dia of brake wheel = 70 cm

Rope dia = 3.5 cm

brake load = 40 kg

= 392.4 N

Calculate a) B.P b)  $P_{net}$  c) IP d)  $\eta_{mech}$  e)  $\eta_{th}$

Soln

Given data

$$n = \frac{N}{2}$$

$$k = 1$$

$$d = 15 \text{ cm}$$

$$L = 25 \text{ cm}$$

$$N = 250 \text{ rpm}$$

$$A_i = 6 \text{ cm}^2$$

$$l = 7.9 \text{ cm}$$

$$Sp. const = 7.5 \text{ bar/cm}$$

$$bsfc = 0.24 \text{ kg/kWh}$$

$$CV = 42000 \text{ kJ/kg}$$

$$D = 70 \text{ cm}$$

$$W = 392.4 \text{ N}$$

$$d_r = 3.5 \text{ cm}$$



$$\text{Effective brake radius} = r_{\text{brake}} = \frac{D_{\text{brake}} + d_{\text{rope}}}{2} = \frac{70 + 3.5}{2} \\ = 36.75 \text{ cm} = 0.3675 \text{ m}$$

$$T = \text{Load} \times \text{eff. brake radius} \\ = W_{\text{brake}} \times r_{\text{brake}} = 392.4 \times 0.3675 = 144.2 \text{ N-m}$$

$$a) \text{ BP} = \frac{2\pi NT}{60} = \frac{2\pi \times 250 \times 144.2}{60} = 3775 \text{ W}$$

$$b) P_{\text{me}} = \frac{\text{Area of Indicator diagram}}{\text{Length of " "}} \times \text{spring const.} \\ = \frac{6 \text{ cm}^2}{9 \text{ cm}} \times 7.5 \text{ bar/cm} \\ = 5 \text{ bar} = 500 \text{ kPa}$$

$$c) \text{ IP} = \frac{PLANk}{60} = \frac{500 \times 0.25 \times \pi \times 0.15^2 \times 250 \times 1}{4 \times 60 \times 2} \\ = 4.6 \text{ kW}$$

$$d) \eta_{\text{mech}} = \frac{\text{BP}}{\text{IP}} = \frac{3.775}{4.6} = 0.820 = 82\%$$

$$e) \text{ bsfc} = \frac{\dot{m}_f}{\text{BP}} \Rightarrow 0.24 = \frac{\dot{m}_f}{3.775} \Rightarrow \dot{m}_f = 0.906 \text{ kg/h} \\ = 2.516 \times 10^{-4} \text{ kg/s}$$

$$\eta_{\text{ith}} = \frac{\text{IP}}{\dot{m}_f \times \text{CV}} \\ = \frac{4.6}{2.516 \times 10^{-4} \times 42000} = 0.435 = 43.5\%$$

Q A single cylinder CI engine with a brake thermal efficiency 80%. Diesel fuel has CV = 42000 kJ/kg.  $\eta_{\text{mech}} = 80\%$ . calculate bsfc, isfc &  $\eta_{\text{ith}}$ .

Sol

$$K = 1 \quad \text{bsfc} = ? \\ \eta_{\text{bth}} = 0.8 \quad \text{isfc} = ? \\ \text{CV} = 42000 \text{ kJ/kg} \quad \eta_{\text{ith}} = ? \\ \eta_{\text{mech}} = 0.8$$

$$\eta_{\text{bth}} = \frac{\text{BP}}{\dot{m}_f \times \text{CV}} \Rightarrow \frac{\dot{m}_f}{\text{BP}} = \frac{1}{\eta_{\text{bth}} \times \text{CV}} = \frac{1}{0.8 \times 42000} \text{ kg/kWh} \\ = \frac{3600}{0.8 \times 42000} \text{ kg/kWh} \\ \eta_{\text{mech}} = \frac{\text{isfc}}{\text{bsfc}}$$

$$\eta_{\text{mech}} = \frac{\eta_{\text{bth}}}{\eta_{\text{ith}}} \Rightarrow 0.8 = \frac{0.8}{\eta_{\text{ith}}} \Rightarrow \eta_{\text{ith}} = \frac{0.8}{0.8} = 0.375 = 37.5\%$$

$$\text{bsfc} = \frac{\dot{m}_f}{\text{BP}} = \frac{3600}{0.8 \times 42000}$$



Now  $\eta_{mech} = \frac{18fc}{b8fc}$

$\Rightarrow 0.8 = \frac{18fc}{\frac{2600}{0.8 \times 42000}}$

$\Rightarrow 18fc = 0.228 \text{ kg/kw-h}$

Q In a petrol engine indicated power = 30 kW, BP = 26 kW,  $N = 1000 \text{ rpm}$ .  $sfc = 0.35 \text{ kg/kw-h}$ ,  $CV = 43000 \text{ kJ/kg}$ . Calculate indicated thermal  $\eta$ ,  $\eta_{bth}$ ,  $\eta_{mech}$ .

Sol<sup>n</sup>

Given IP = 30 kW

BP = 26 kW

$N = 1000 \text{ rpm}$

$sfc = 0.35 \text{ kg/kw-h}$   
 $= \frac{0.35}{3600} \text{ kg/whrs}$

$CV = 43000 \text{ kJ/kg}$

$\eta_{ith} = ?$

$\eta_{bth} = ?$

$\eta_{mech} = ?$

$\eta_{mech} = \frac{BP}{IP} = \frac{26}{30} = 86\%$

$sfc = \frac{\dot{m}_f}{BP}$

$\Rightarrow \frac{0.35}{3600} = \frac{\dot{m}_f}{30}$

$\Rightarrow \dot{m}_f =$

$sfc = \frac{\dot{m}_f}{IP}$

$\Rightarrow 0.35 = \frac{\dot{m}_f}{30}$

$\Rightarrow \dot{m}_f =$

$\eta_{ith} = \frac{IP}{\dot{m}_f \times CV}$

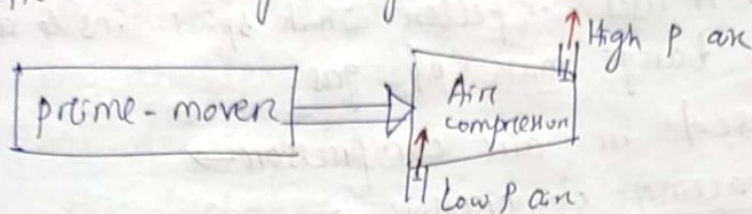
$\eta_{bth} = \frac{BP}{\dot{m}_f \times CV}$



## Chapter-2

# Reciprocating Air Compressor

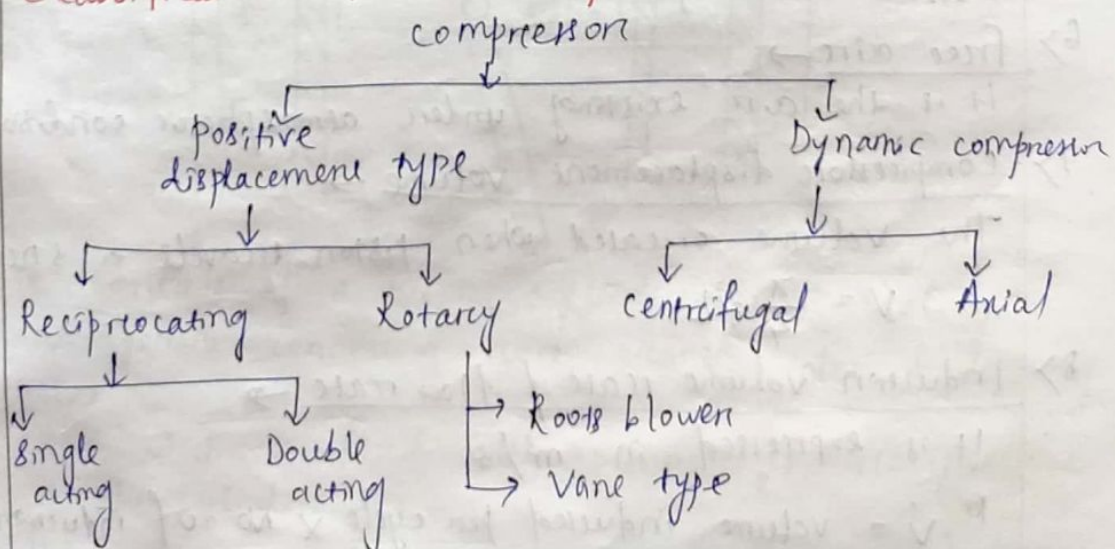
Air compressor is a machine which takes atmospheric air with the help of mechanical energy & delivers it at high pressure. It is also called as air pump. It increases pressure of air by decreasing its specific volume by using some mechanical means.



### Use of compressed air →

1. In air refrigeration & cooling of large buildings.
2. In driving pneumatic tools in shops like drills, riveters, screw drivers etc.
3. In blast furnaces.
4. In spray painting & spraying fuel in diesel engines.
5. Operating of air brakes in buses, trucks & trains.
6. In inflating automobile & aircraft tyres.
7. In supercharging of internal combustion engines.
8. In driving air motors in mines, where electric motor & IC engines cannot be used due to fire risks as inflammable gases are present there.
9. In operation of lifts, hoists, cranes, pumps.
10. In automobile suspension system.
11. cleaning purposes.

### Classification of air compressor →





- Reciprocating compressor is used to produce high pressure gas. It uses displacement of piston in the cylinder for compression. It handles low mass of gas & a high pressure ratio.
- Rotary compressors are used for low & medium pressures. It uses impeller which spins inside a housing. It handles large mass of gas.

### Terms used in air compressor →

#### 1) Single acting compressor →

Here suction, compression & delivery of gas takes place only on one side of the piston during a cycle of one revolution of crank shaft.

#### 2) Double acting compressor →

Here suction, compression & delivery of gas takes place on both sides of the piston & 2 cycles takes place during one rev<sup>n</sup> of crankshaft.

#### 3) Single stage compressor →

Here compression of gas to final delivery pressure is done in one cylinder only.

#### 4) Multi stage compression →

Here compression of gas is done in more than one cylinder in series.

#### 5) Pressure ratio →

Ratio between absolute discharge pressure to absolute suction pressure.

#### 6) Free air →

It is the air existing under atmospheric condition.

#### 7) Compressor displacement volume →

The volume created when piston travels a stroke.

$$V = \frac{\pi}{4} d^2 L$$

#### 8) Induction volume rate / flow rate →

It is expressed in m<sup>3</sup>/s.

$\dot{V} = \text{volume induced per cycle} \times \text{No. of inductions per revolution} \times \text{No. of revolutions per sec.}$



$$\dot{V} = \frac{\pi}{4} d^2 L \frac{N}{60} \quad (\text{for single acting compression})$$

$$\dot{V} = \frac{\pi}{4} d^2 L \left( \frac{2N}{60} \right) \quad (\text{" double " " " "})$$

9) Capacity of a compressor →

It is the actual quantity of air delivered per unit time at atmospheric cond<sup>n</sup>.

10) Free Air delivery (FAD) →

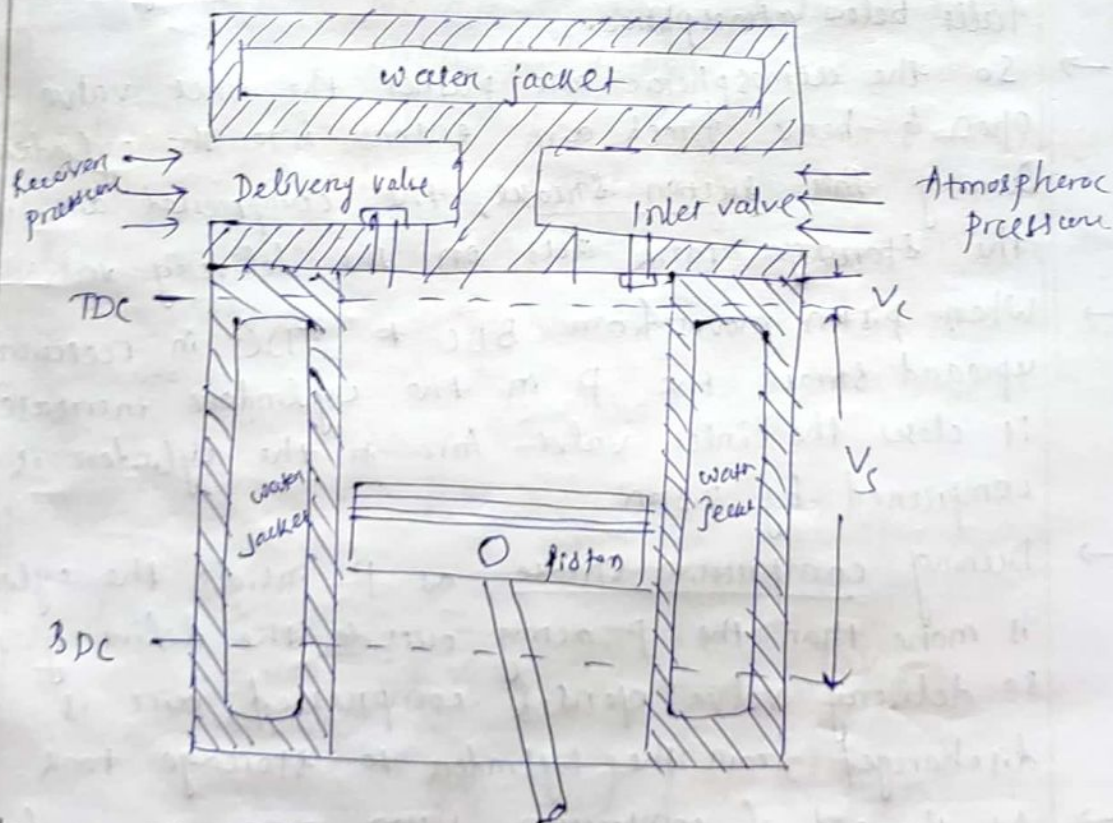
It is the discharge volume of the compressor corresponding to ambient conditions.

11) Piston speed →

It is the linear speed of the piston measured in m/min.

$$V_{\text{piston}} = 2LN$$

Construction of air compressor →



It consists of a piston, cylinder with cooling arrangement, connecting rod, crank, inlet & delivery valves.

Piston, which is fitted with piston rings, reciprocates in the cylinder.

The prime mover (an engine or electric motor) drives

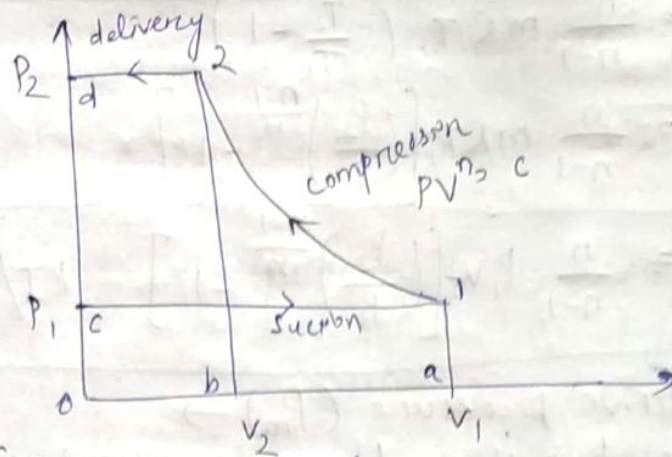


- the crank shaft. The crank shaft rotates & converts rotary motion into reciprocating motion of piston with the help of connecting rod.
- The cylinder head consists of spring loaded inlet & delivery valves, which are operated by small pressure difference across them.
  - The piston rings seal the gap between the piston & cylinder wall.
  - The cylinder is surrounded by a water jacket or metallic fin for proper cooling of air during compression.

### Working →

- When piston moves from TDC to BDC in downward stroke, any residual compressed air left in the cylinder from previous cycle expands first.
- When piston moves further,  $P$  inside the cylinder falls below atmospheric.
- So the atmospheric air pushes the inlet valve to open & hence fresh air enters into the cylinder. During this suction stroke, the compressed air in the storage tank acts on the delivery valve.
- When piston moves from BDC to TDC in return/upward stroke, the  $P$  in the cylinder increases & it closes the inlet valve. Air in the cylinder is compressed by piston.
- During compression stroke, as  $P$  inside the cylinder is more than the  $P$  acting outside the delivery valve, so delivery valve opens & compressed air is discharged from the cylinder to storage tank.
- At the end of compression, piston again moves from TDC to BDC & the cycle repeats.





[ P-V diagram of single stage compressor without clearance ]

- Process c-1 → Suction stroke → Inlet valve opens & air enters the compressor at const P, (P<sub>1</sub>)
- " 1-2 → Polytropic compression of air from P<sub>1</sub> to P<sub>2</sub>.
- " 2-d → Discharge of compressed air through delivery valve at P (P<sub>2</sub>).
- " d-c → No air in the cylinder & return of piston for suction stroke.

Indicated work done for single acting reciprocating compressor without considering clearance →

Net workdone = area <sup>behind</sup> under the curve on P-V diagram

$$W_{in} = \text{Area } c-1-2-d-c$$

$$= \text{Area } 2-d-o-b-2 + \text{Area } 1-2-b-a-1$$

$$- \text{Area } 1-c-o-a-1$$

$$= P_2 V_2 + \left( \int_1^2 P dV \right) - P_1 V_1$$

for compression process

$$P V^n = c$$

$$\therefore W = - \int P dV = \frac{P_2 V_2 - P_1 V_1}{n-1}$$

$$= P_2 V_2 + \frac{P_2 V_2 - P_1 V_1}{n-1} - P_1 V_1$$

$$= (P_2 V_2 - P_1 V_1) \left( \frac{1}{n-1} + 1 \right)$$

$$= (P_2 V_2 - P_1 V_1) \left( \frac{1+n-1}{n-1} \right)$$

$$= \frac{n}{n-1} (P_2 V_2 - P_1 V_1) \text{ KJ/cycle}$$

from characteristic gas eqn  $PV = mRT$

$$\therefore W_{in} = \frac{n}{n-1} mR (T_2 - T_1)$$



$$\Rightarrow W_{in} = \frac{n}{n-1} m R T_1 \left( \frac{T_2}{T_1} - 1 \right)$$

$$\Rightarrow W_{in} = \frac{n}{n-1} m R T_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$\Rightarrow W_{in} = \frac{n}{n-1} P_1 V_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{ kJ/cycle}$$

mean effective pressure ( $P_m$ )  $\rightarrow$

It is the hypothetical avg. pressure, which if acted on the piston during the entire compression stroke will require the same power i/p as required during actual cycle.

Net work input in a cycle  $= W_{in} = P_m \times (\text{swept volume})$

$$\Rightarrow W_{in} = P_m \times V_s$$

$$\Rightarrow P_m = \frac{W_{net}}{V_s}$$

\* Again  $P_m = \frac{\text{Area of indicator diagram (mm}^2\text{)}}{\text{Length of " " (mm)}} \times \text{spring constant (kPa/mm)}$

Power &  $\eta_{mech} : \rightarrow$

1) IP  $\rightarrow$  The workdone on air per unit time is called indicated power input to the compressor.

Power required by an air compressor, running at  $N$  rpm is  $= IP = \text{work i/p per cycle} \times \text{No. of cycles per unit of time}$

$$\Rightarrow IP = \frac{W_{in} N k}{60} \text{ (kW)}$$

from indicated diagram,  $IP = P_m \times V_s$

$$= \frac{P_m L A N k}{60} \text{ (kW)}$$

$k \Rightarrow$  No. of suction per revolution of crank shaft  
 $= 1$  for single acting reciprocating compressor  
 $= 2$  " double " " "

2) BP  $\rightarrow$  The actual power (shaft power) i/p to the compressor is more than IP as some work is required to overcome the irreversibilities & mechanical frictional effects.

$$BP = IP + F.P.$$



$$3) \quad \eta_{\text{mech}} = \frac{IP}{BP} \quad (\text{for compression})$$

$$\text{BHP} = \text{motor power / engine power} = \frac{\text{shaft power (or BP)}}{\eta_{\text{mech of motor \& drive}}}$$

Ex A single stage reciprocating air compressor takes in 1.4 kg of air per minute at 1 bar & 17°C & delivers it at 6 bar. Assuming compression process follows the law  $PV^{1.35} = C$ , calculate IP input to compressor.

Sol<sup>n</sup> Given data  $\dot{m}_a = 1.4 \text{ kg/min}$

$$P_1 = 1 \text{ bar}$$

$$P_2 = 6 \text{ bar}$$

$$T_1 = 17^\circ\text{C} = 290 \text{ K}$$

$$n = 1.35$$

$$PV^{1.35} = C$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow T_2 = 290 \left(\frac{6}{1}\right)^{\frac{1.35-1}{1.35}} = 461.46 \text{ K}$$

$$W_{\text{in}} = \frac{n}{n-1} \dot{m}_a R (T_2 - T_1)$$

$$= \frac{1.35}{1.35-1} (1.4 \text{ kg/min}) (0.287 \text{ kJ/kg}\cdot\text{K}) (461.46 - 290)$$

$$= 265.72 \text{ kJ/min}$$

$$IP = \frac{W_{\text{in}}}{60} = \frac{265.72 \text{ kJ/min}}{60} = 4.43 \text{ kW}$$

Ex 2) A single acting, single cylinder reciprocating air compressor has a cylinder dia of 200 mm & a stroke of 300 mm. Air enters the cylinder at 1 bar, 27°C. It is then compressed polytropically to 8 bar acc<sup>n</sup> to  $PV^{1.3} = C$ . If the speed of the compressor is 250 rpm, calculate the mass of air compressed per minute & power required in kW for driving the compression.

Sol<sup>n</sup> Given data  $d = 200 \text{ mm} = 0.2 \text{ m}$

$$L = 300 \text{ mm} = 0.3 \text{ m}$$

$$P_1 = 1 \text{ bar} = 100 \text{ kPa}$$

$$P_2 = 8 \text{ bar}$$

$$N = 250 \text{ rpm}$$

$$n = 1.3$$

$$T_1 = 27^\circ\text{C} = 300 \text{ K}$$

$$V_2 = V_1 = \frac{\pi}{4} d^2 L = \frac{\pi}{4} \times 0.2^2 \times 0.3 = 9.424 \times 10^{-3} \text{ m}^3$$



$$P_1 V_1 = m_a R T_1$$

$$\Rightarrow m_a = \frac{P_1 V_1}{R T_1} = \frac{100 \text{ kPa} \times 9.424 \times 10^{-3} \text{ m}^3}{0.287 \text{ kJ/kg} \cdot \text{K} \times 300 \text{ K}} = 0.0109 \text{ kg/s}$$

$$\dot{m}_a = \text{mass of air} \times \text{no. of suction/min}$$

$$= 0.0109 \times 250 = 2.74 \text{ kg/min}$$

$$\text{Again } \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}}$$

$$\Rightarrow T_2 = 300 \times \left( \frac{8}{1} \right)^{\frac{1.3-1}{1.3}} = 484.73 \text{ K}$$

$$W_{in} = \frac{n}{n-1} \dot{m}_a R (T_2 - T_1)$$

$$= \frac{1.3}{1.3-1} \times 2.74 \text{ kg/min} \times 0.287 \text{ kJ/kg} \cdot \text{K} (484.73 - 300)$$

$$= 629.56 \text{ kJ/min}$$

$$= 10.49 \text{ kW}$$

Q-3 A single acting, single cylinder reciprocating air compressor is compressing 20 kg/min of air from 110 kPa, 30°C to 600 kPa & delivers it to a receiver. Law of compression is  $PV^{1.25} = C$ .  $\eta_{\text{mech}} = 80\%$ . Find the power I/P to compressor, neglecting losses due to clearance, leakage & cooling.

Sol<sup>n</sup> Given data  $\dot{m}_a = 20 \text{ kg/min}$   $PV^{1.25} = C$

$$P_1 = 110 \text{ kPa}$$

$$T_1 = 30^\circ\text{C} = 303 \text{ K}$$

$$P_2 = 600 \text{ kPa}$$

$$\eta_{\text{mech}} = 0.8$$

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \Rightarrow \frac{T_2}{303} = \left( \frac{600}{110} \right)^{\frac{0.25}{1.25}} \Rightarrow T_2 = 425.4 \text{ K}$$

$$\text{IP input to compressor} = \dot{W} = \frac{n}{n-1} \dot{m}_a R (T_2 - T_1)$$

$$= \frac{1.25}{0.25} \left( \frac{20}{60} \text{ kg/s} \right) \times 0.287 \text{ kJ/kg} \cdot \text{K} (425.4 - 303)$$

$$= 58.55 \text{ kW}$$

The motor (brake) power

$$\text{BP} = \frac{\dot{W}}{\eta_{\text{mech}}} = \frac{58.55}{0.8} = 73.18 \text{ kW}$$

Q-4

A single cylinder, double acting reciprocating air compressor receives air at 1 bar,  $17^\circ\text{C}$ , compresses it to 6 bar according to the law  $PV^{1.25} = C$ . The cylinder dia is 300 mm. The avg. piston speed is 150 m/min at 100 rpm. Calculate the power required in kW for driving the compressor. Neglect clearance.

Sol<sup>n</sup>

Given data  $P_1 = 1 \text{ bar} = 100 \text{ kPa}$   $PV^{1.25} = C$   $N = 100 \text{ rpm}$   
 $T_1 = 17^\circ\text{C} = 290 \text{ K}$   $d = 300 \text{ mm} = 0.3 \text{ m}$   $K = 2$   
 $P_2 = 6 \text{ bar}$   $V_{\text{piston}} = 150 \text{ m/min}$   $\eta = 1.25$

$$V_{\text{piston}} = 2LN$$

$$\Rightarrow L = \frac{V_{\text{piston}}}{2N} = \frac{150 \text{ m/min}}{2 \times 100 \text{ rpm}} = 0.75 \text{ m}$$

$$V_s = V_1 = \frac{\pi}{4} d^2 L = \frac{\pi}{4} 0.3^2 \times 0.75 = 0.053 \text{ m}^3$$

Indicated work /p to compressor

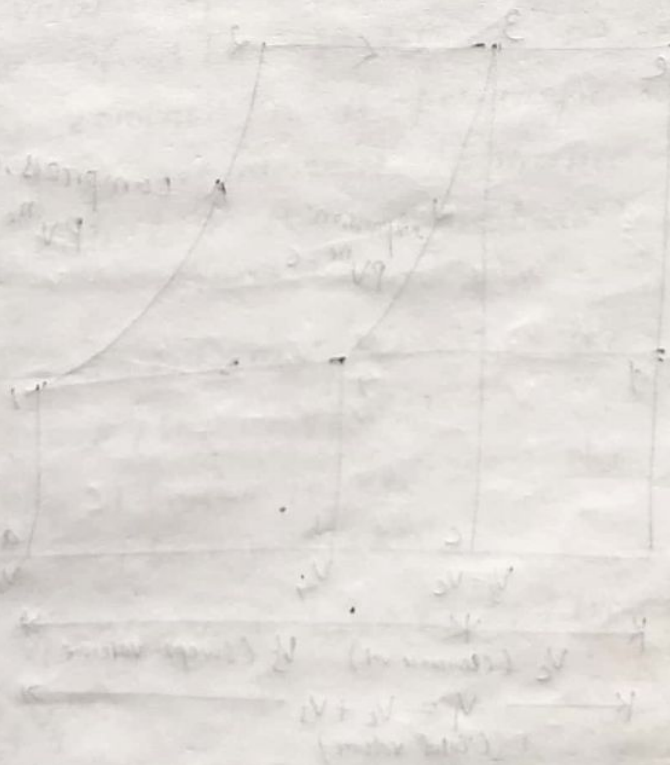
$$W_m = \frac{n}{n-1} P_1 V_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.25}{0.25} 100 \times 0.053 \left[ \left( \frac{6}{1} \right)^{\frac{0.25}{1.25}} - 1 \right]$$

$$= 11.42 \text{ kJ/cycle}$$

For double acting reciprocating compressor,

$$IP = \frac{W_m NK}{60} (\text{kW}) = \frac{11.42 \times 100 \times 2}{60} = 38.1 \text{ kW}$$





## Clearance volume in a compressor →

It is the space left in the cylinder when piston reached TDC. It is provided:

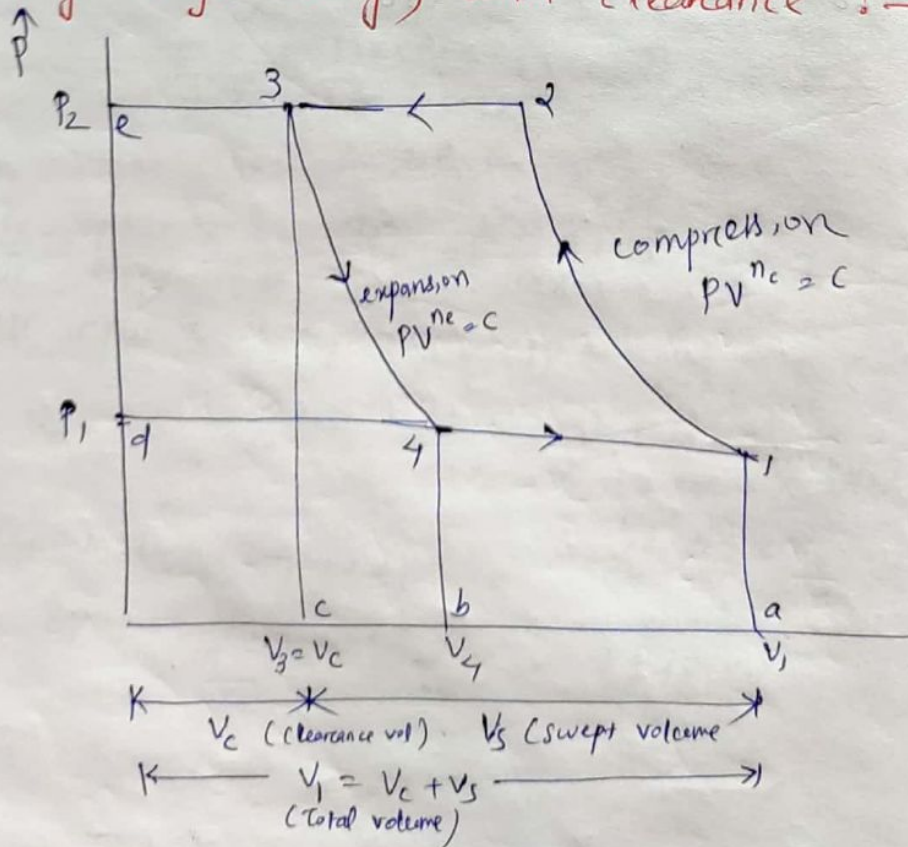
- to avoid the piston striking the cylinder head &
- to accommodate valve's position inside the cylinder.
- \* Compressor should have smallest clearance volume, as compressed air left here, first re-expands in the cylinder during suction, which reduces suction capacity.
- \* clearance ratio =  $\frac{\text{clearance volume}}{\text{swept volume}}$

It varies from 2 to 10 %.

## Effect of clearance volume →

1. The volume of air taken in per stroke is less than the swept volume, thus the volumetric efficiency decreases.
2. More power i/p is required to drive the compressor for same pressure ratio, due to increase in volume to be handled.
3. The max<sup>m</sup> compression pressure is controlled by clearance volume.

Indicated work input to the compressor (single stage single acting) with clearance: →





Here after delivery of compressed air, the air remaining in the clearance volume at  $P_2$  expands, when the piston proceeds for the next suction stroke. As soon as the  $P_1$  reaches at state 4, the induction of fresh charge starts & continues up to state 1.

Indicated work done = Area of 1-2-3-4-1 on the  $P$ - $V$  diagram  
 $\Rightarrow W_{in} = (\text{Area of 1-2-e-d}) - (\text{Area of 3-e-d-4})$

$$\text{Area of 1-2-e-d} = W_{comp} = \frac{n_c}{n_c - 1} P_1 V_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right]$$

$$\text{Area of 3-e-d-4} = W_{exp} = \frac{n_e}{n_e - 1} P_3 V_3 \left[ \left( \frac{P_2}{P_3} \right)^{\frac{n_e - 1}{n_e}} - 1 \right]$$

$$\text{Now } P_4 = P_1 \text{ \& } P_3 = P_2$$

$$\text{So, } W_{in} = \frac{n_c}{n_c - 1} P_1 V_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right] - \left\{ \frac{n_e}{n_e - 1} P_1 V_4 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n_e - 1}{n_e}} - 1 \right] \right\}$$

$$\text{If } n_c = n_e = n$$

$$W_{in} = \frac{n}{n - 1} P_1 (V_1 - V_4) \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n - 1}{n}} - 1 \right]$$

Q A single stage, single acting reciprocating air compressor has a displacement volume of 14 Ltr & a clearance volume of 0.7 Ltr. It receives the air at a pressure of 1 bar & delivers it at a pressure of 7 bar. The compression is polytropic with an index of 1.3 & re-expansion is isentropic with an index of 1.4. Calculate the net indicated work of a cycle.

Sol<sup>n</sup> Given  $V_s = 14 \text{ Ltr} = 14 \times 10^{-3} \text{ m}^3$   
 $V_c = 0.7 \text{ Ltr} = 0.7 \times 10^{-3} \text{ m}^3$   
 $P_1 = 1 \text{ bar} = 100 \text{ kPa}$   
 $P_2 = 7 \text{ bar} = 700 \text{ kPa}$

$$n_c = 1.3$$

$$n_e = 1.4$$

$$W_{in} = ?$$

$$\text{Total volume} = V_1 = V_s + V_c = 14 + 0.7 = 14.7 \text{ Ltr} = 14.7 \times 10^{-3} \text{ m}^3 = 0.0147 \text{ m}^3$$

$$P_3 V_3^{n_e} = P_4 V_4^{n_e}$$

$$\Rightarrow \left( \frac{V_4}{V_3} \right)^{n_e} = \frac{P_3}{P_4} = \frac{P_2}{P_1} \Rightarrow \frac{V_4}{V_3} = \left( \frac{P_2}{P_1} \right)^{\frac{1}{n_e}} \Rightarrow V_4 = V_3 \left( \frac{P_2}{P_1} \right)^{\frac{1}{n_e}}$$



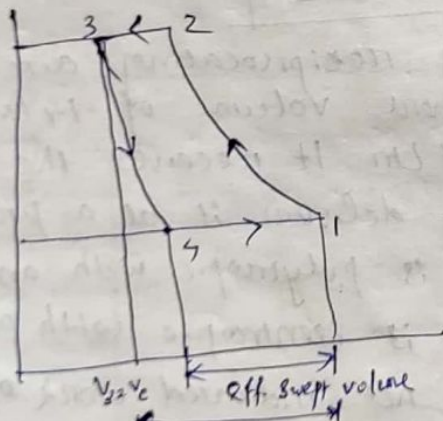
$$\Rightarrow V_4 = V_c \left( \frac{P_2}{P_1} \right)^{\frac{1}{n_c}} = 0.7 \times 10^{-3} \left( \frac{7}{1} \right)^{\frac{1}{1.4}} = 0.00281 \text{ m}^3$$

$$\begin{aligned} W_{in} &= \frac{n_c}{n_c - 1} \left[ P_1 V_1 \left( \frac{P_2}{P_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right] - \frac{n_c}{n_c - 1} P_1 V_4 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right] \\ &= \frac{1.3}{1.3 - 1} \left[ 100 \times 0.0147 \left( \frac{7}{1} \right)^{\frac{0.3}{1.3}} - 1 \right] - \left\{ \frac{1.3}{1.3 - 1} 100 \times 0.00281 \left[ \left( \frac{7}{1} \right)^{\frac{0.3}{1.3}} - 1 \right] \right\} \\ &= 3.61 - 0.731 \\ &= 2.88 \text{ KJ/cycle.} \end{aligned}$$

### \* Volumetric efficiency $\rightarrow$

It is the ratio between actual volume of air sucked into the compressor (measured at atmospheric  $P$  &  $T$ ) to the piston displacement volume.

$$\begin{aligned} \eta_{vol} &= \frac{\text{Actual mass sucked}}{\text{mass corresponding to swept volume at atmospheric } P \text{ \& } T} \\ &= \frac{\text{effective swept volume}}{\text{piston displacement volume}} \end{aligned}$$



$$V_3 = V_1 - V_3 \text{ piston displacement volume}$$

$$\begin{aligned} \eta_{vol} &= \frac{V_1 - V_4}{V_1 - V_c} = \frac{V_3 + V_c - V_4}{V_3 + V_c - V_c} = 1 + \frac{V_c}{V_3} - \frac{V_4}{V_3} \\ &= 1 + C - C \left( \frac{V_4}{V_3} \right) \text{ where } C = \frac{V_c}{V_3} \end{aligned}$$

$$\text{Again } \frac{V_4}{V_3} = \left( \frac{P_3}{P_4} \right)^{\frac{1}{n_c}} = \left( \frac{P_2}{P_1} \right)^{\frac{1}{n_c}}$$

$$\Rightarrow \eta_{vol} = 1 + C - C \left( \frac{P_2}{P_1} \right)^{\frac{1}{n_c}}$$

$$\text{If } n_c = n_e = n, \left( \frac{P_2}{P_1} \right)^{\frac{1}{n_c}} = \left( \frac{P_2}{P_1} \right)^{\frac{1}{n_e}} = \frac{V_1}{V_2}$$



$$\Rightarrow \boxed{\eta_{vol} = 1 + c - c \left( \frac{V_1}{V_2} \right)}$$

Q-1 A single stage, single acting reciprocating air compressor receives air at 1.013 bar, 27°C & delivers it at 9.5 bar. The compressor has a bore = 250 mm & stroke = 300 mm & it runs at 200 rpm. mass flow rate of air is 200 kg/h. Calculate  $\eta_{vol}$ .

Sol:

$$P_1 = 1.013 \text{ bar}$$

$$T_1 = 27^\circ\text{C}$$

$$P_2 = 9.5 \text{ bar}$$

$$d = 250 \text{ mm} = 0.25 \text{ m} \quad \dot{m}_a = 200 \text{ kg/h}$$

$$L = 300 \text{ mm} = 0.3 \text{ m}$$

$$N = 200 \text{ rpm}$$

$$V_1 = \frac{\pi}{4} d^2 L = \frac{\pi}{4} (0.25)^2 \times 0.3 = 0.0147 \text{ m}^3$$

$$m_a = \frac{P_1 V_1}{RT_1} = \frac{1.013 \times 0.0147}{0.287 \times 300} = 0.0173 \text{ kg/cycle}$$

$$\text{mass flow rate per hour} = \dot{m}_a = \text{mass per cycle} \times \text{No. of turns/rev} \times \text{No. of rev}^\circ/\text{hour}$$

$$= 0.0173 \times 1 \times (200 \times 60)$$

$$\eta_{vol} = \frac{200}{207.6} = 0.963 = 96.3\%$$

### Limitations of single stage compression →

Pressure ratio for a single stage reciprocating air compressor is limited to 7.

Increase in pressure ratio causes the following effects

- i) Greater expansion of clearance air in the cylinder. Hence decreases effective suction volume, which decreases fresh air induction.
- ii) With high delivery  $P$ , delivery  $T$  increases. It increases specific volume of air in the cylinder, so more compression work is needed.
- iii) For high  $P$  ratio, the cylinder size should be large, strong & heavy working parts of compressor are needed. It will increase balancing problems & high torque fluctuation will need heavier flywheel.



## Multi-stage compression: $\rightarrow$

The compression of air in two or more cylinders in series is called multi-stage compression.

The limitations of single stage compression can be reduced by using multi stage compression.

- \* Compressor needs min work input with isothermal compression. But the delivery temp  $T_2$  increases with pressure ratio & the  $\eta_{vol}$  decreases as pressure ratio increases.

Air cooling between stages provides the means to achieve appreciable reduction in compression work & maintaining the air temp within safe operating limits.

## Advantages of Multi-stage compression $\rightarrow$

1. The gas can be compressed to a sufficiently high pressure.
2. Cooling of air is more efficient with intercoolers & cylinder wall surface.
3. By multistaging, the pressure ratio of each stage is lowered.
4. Low pressure ratio in a cylinder improves  $\eta_{vol}$ .
5. The size of the two cylinders (i.e. High pressure & Low pressure) may be adjusted to suit the volume & pressure of air.
6. It reduces leakage loss considerably.
7. It gives more uniform torque, so smaller size flywheel is required.
8. It reduces cost of compressor.
9. It provides effective lubrication due to lower temp range.

Q A single stage, single acting reciprocating air compressor has a bore of 200mm & a stroke of 300mm. It receives air at 1 bar & 20°C & delivers it at 5.5 bar. If compression follows  $pV^{1.3} = c$  &  $V_c = 5\%$  of  $V_s$ . Find i)  $P_{mex}$  ii) power required to drive the compressor, if it runs at 500 rpm.



Sol<sup>n</sup>

Given data

$$d = 200 \text{ mm} = 0.2 \text{ m}$$

$$L = 300 \text{ mm} = 0.3 \text{ m}$$

$$P_1 = 1 \text{ bar}$$

$$P_2 = 5.5 \text{ bar}$$

$$T_1 = 28^\circ\text{C} = 293 \text{ K}$$

$$pV^{1.3} = C$$

$$V_c = 5\% \text{ of } V_s$$

$$N = 500 \text{ rpm}$$

$$V_s = \frac{\pi}{4} d^2 L = \frac{\pi}{4} 0.2^2 \times 0.3 = 0.00942 \text{ m}^3$$

$$V_c = 5\% \text{ of } V_s = 0.05 \times 0.00942 = 0.00047 \text{ m}^3$$

$$\text{Now, } V_1 = V_c + V_s$$

$$= 0.00942 + 0.00047 = 0.00989 \text{ m}^3$$

$$\text{Again } V_4 = V_c \left( \frac{P_2}{P_1} \right)^{\frac{1}{n}} = 0.00047 \left( \frac{5.5}{1} \right)^{\frac{1}{1.3}} = 0.00174 \text{ m}^3$$

$$\text{effective swept volume} = V_1 - V_4 = 0.00989 - 0.00174 = 0.00815 \text{ m}^3$$

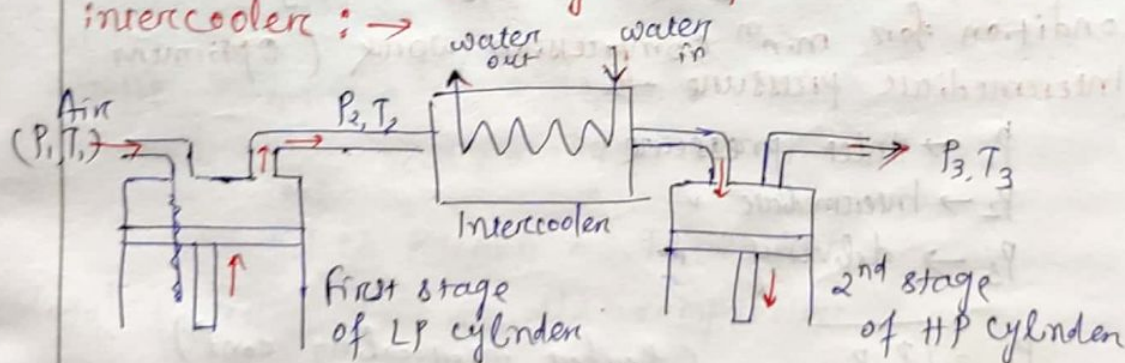
$$W_m = \frac{n}{n-1} P_1 (V_1 - V_4) \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times 1 \times 10^5 \times 0.00815 \left[ \left( \frac{5.5}{1} \right)^{\frac{0.3}{1.3}} - 1 \right]$$

$$= 1702 \text{ N-m}$$

$$P_{\text{mep}} = \frac{W_m}{V_s} = \frac{1702}{0.00942} = 180700 \text{ N/m}^2$$

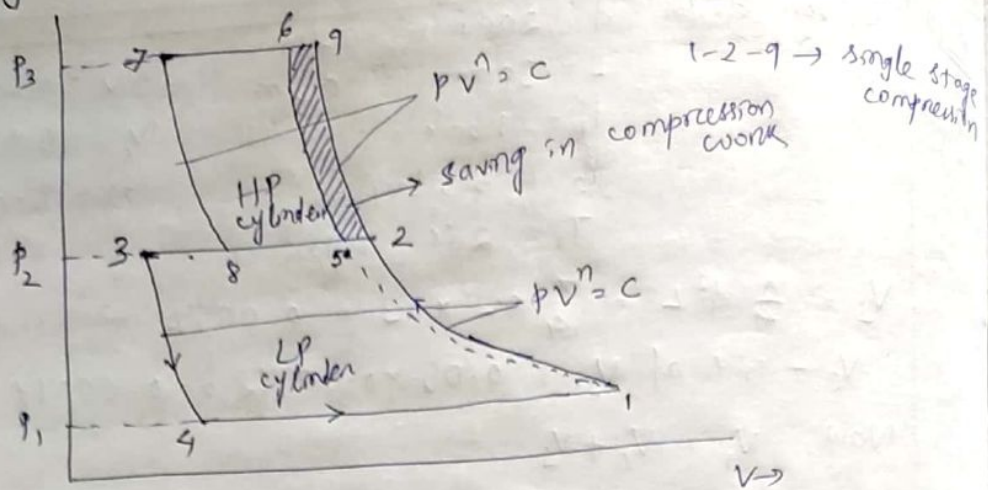
Work done in Multi stage compression with intercoolers: →



- Here air at  $P_1$  &  $T_1$  is first drawn into 1<sup>st</sup> stage or Low pressure (LP) cylinder.
- It is partially compressed to some intermediate pressure  $P_2$  & temp<sup>n</sup>  $T_2$  & then discharged to intercooler (cools air to its initial temp<sup>n</sup>  $T_1$ ).
- The cooled air then enters the 2<sup>nd</sup> stage or high pressure (HP) cylinder & is compressed to a



delivery pressure  $P_3$  & temp<sup>n</sup>  $T_3$ .



$$\begin{aligned}
 W_{in} &= W_{LP} + W_{HP} \\
 &= \text{Area of } 1-2-3-4-1 + \text{Area of } 5-6-7-8-5 \\
 &= \frac{n}{n-1} P_1 (V_1 - V_4) \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} P_1 (V_5 - V_8) \left[ \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 1 \right] \\
 &= \frac{n}{n-1} m_a R T_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} m_a R T_1 \left[ \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 1 \right]
 \end{aligned}$$

$$\text{Here } m_a = \frac{P_1 V_1}{R T_1} = \frac{P_1 (V_1 - V_4)}{R T_1} = \frac{P_2 (V_5 - V_8)}{R T_1}$$

& suffix 1  $\rightarrow$  free air cond<sup>n</sup>.

For given mass flow rate  $\dot{m}_a$  (kg/s),

$$IP = \frac{n}{n-1} \dot{m}_a R T_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} + \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 2 \right]$$

condition for min<sup>m</sup> compression work (Optimum Intermediate pressure  $\rightarrow$ )

$P_1 \rightarrow$  Inlet pressure

$P_2 \rightarrow$  Intermediate "

$P_3 \rightarrow$  delivery "

$$P_2 = \sqrt{P_1 P_3} \quad (\text{Perfect intercooling cond<sup>n</sup>})$$

when  $T_3 = T_1$

cond<sup>n</sup>

- $\rightarrow$  Pressure ratio of each stage should be same.
- $\rightarrow$  " " of any stage is the square root of overall pressure ratio, for a 2 stage compression.
- $\rightarrow$  Air after compression in each stage should be cooled to initial temp<sup>n</sup> of air intake.
- $\rightarrow$  work i/p to each stage is same.



stage  
pressure

Q calculate the power required to compress  $25 \text{ m}^3/\text{min}$  atmospheric air at  $101.3 \text{ kPa}$ ,  $20^\circ\text{C}$  to a pressure ratio of 7 in an LP cylinder. Air is then cooled at const. P to  $25^\circ\text{C}$  in an intercooler, before entering HP cylinder, where air is again compressed to a pressure ratio of 6. Assume polytropic compression with  $n = 1.3$  &  $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ .

Sol Given  $\dot{V}_1 = 25 \text{ m}^3/\text{min}$   $T_3 = 25^\circ\text{C} = 298 \text{ K}$   
 $P_1 = 101.3 \text{ kPa}$   $\frac{P_3}{P_2} = 6$   
 $T_1 = 20^\circ\text{C} = 293 \text{ K}$   $R = 0.287 \text{ kJ/kg}\cdot\text{K}$   
 $\frac{P_2}{P_1} = 7$

$$\dot{m}_a = \frac{P_1 \dot{V}_1}{R T_1} = \frac{(101.3 \text{ kPa}) \times (25 \text{ m}^3/\text{min})}{(0.287 \text{ kJ/kg}\cdot\text{K}) \times (293 \text{ K})} = 30.11 \text{ kg/min}$$

Temp<sup>n</sup> of air after 1<sup>st</sup> stage compression

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 293 (7)^{\frac{1.3-1}{1.3}} = 459.08 \text{ K}$$

$$W_{in, LP} = \frac{n}{n-1} \dot{m}_a R (T_2 - T_1)$$

$$= \frac{1.3}{0.3} 30.11 \times 0.287 (459.08 - 293)$$

$$= 6219.24 \text{ kJ/min} = 103.65 \text{ kW}$$

Temp<sup>n</sup> of air after 2<sup>nd</sup> stage compression,

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{n-1}{n}} = 298 (6)^{\frac{1.3-1}{1.3}} = 450.59 \text{ K}$$

$$W_{in, HP} = \frac{n}{n-1} \dot{m}_a R (T_4 - T_3)$$

$$= \frac{1.3}{0.3} 30.11 \times 0.287 (450.59 - 298)$$

$$= 5714 \text{ kJ/min} = 95.23 \text{ kW}$$

Total power input to the compressors

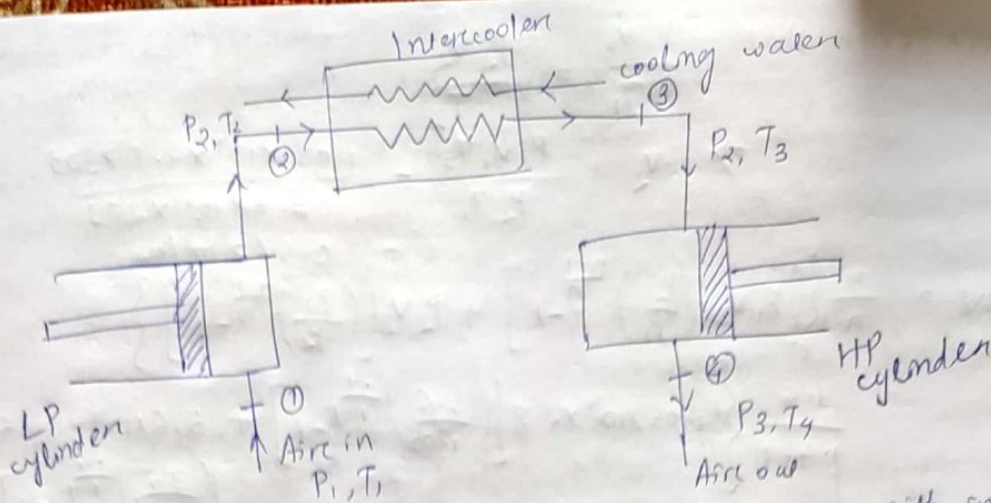
$$IP = IP_{LP} + IP_{HP}$$

$$= 103.65 + 95.23 = 198.88 \text{ kW}$$

Q-2 The LP cylinder of a 2 stage double acting reciprocating air compressor running at 120 rpm has a 50 cm dia & 75 cm stroke. It draws air at a P of 1 bar &  $20^\circ\text{C}$  & compresses it adiabatically to a P of 3 bar. The air is then delivered to the intercooler, where



it is cooled at const. pressure to  $35^\circ\text{C}$  & is then further compressed polytropically ( $n=1.3$ ) to 10 bar in HP cylinder. Determine the power required to drive the compression if  $\eta_{\text{mech}} = 90\%$  &  $\eta_{\text{motor}} = 86\%$ .



2-stage reciprocating air compressor with intercooling

### Work-done

$P_1$  = pressure of air entering LP cylinder  
 $V_1$  = volume of

$P_2$  = pressure of air leaving LP & entering HP cylinder  
 $V_2$  = volume of HP cylinder

$P_3$  = pressure of air leaving the HP cylinder.

i) when intercooling is incomplete (imperfect intercooling i.e.  $T_3 \neq T_1$ )  
 Workdone per cycle in LP cylinder =  $W_1 = \frac{n}{n-1} P_1 V_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$

in HP cylinder =  $W_2 = \frac{n}{n-1} P_2 V_2 \left[ \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 1 \right]$

$$\text{Total } W = W_1 + W_2 = \frac{n}{n-1} P_1 V_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} P_2 V_2 \left[ \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 1 \right]$$

ii) when intercooling is complete (perfect intercooling) i.e.  $T_3 = T_1$

Here  $P_1 V_1 = P_2 V_2$

$$\text{So, } W = \frac{n}{n-1} P_1 V_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} + \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 2 \right]$$

$$= \frac{n}{n-1} m R T_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} + \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 2 \right]$$

Ex Find workdone by a 2 stage reciprocating single acting air compressor to compress  $2.8 \text{ m}^3$  of air per minute at  $1.05 \text{ bar}$  &  $10^\circ\text{C}$  to a final pressure of  $35 \text{ bar}$ . The intermediate receiver cools the air to  $30^\circ\text{C}$  &  $5.6 \text{ bar}$  pressure. for air take  $n = 1.4$

Soln Given  $V_1 = 2.8 \text{ m}^3/\text{min}$   
 $P_1 = 1.05 \text{ bar} = 1.05 \times 10^5 \text{ Pa}$   
 $T_1 = 10^\circ\text{C} = 283 \text{ K}$   
 $P_3 = 35 \text{ bar}$   
 $T_3 = 30^\circ\text{C} = 303 \text{ K}$

$P_2 = 5.6 \text{ bar}$   
 $= 5.6 \times 10^5 \text{ Pa}$   
 $n = 1.4$



we know that  $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_3}$

$$\Rightarrow V_2 = \frac{P_1 V_1 T_3}{P_2 T_1} = \frac{1.05 \times 10^5 \times 2.8 \times 303}{5.6 \times 10^5 \times 283} = 0.562 \text{ m}^3/\text{min}$$

$$W = \frac{n}{n-1} \left\{ P_1 V_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right\} + P_2 V_2 \left\{ \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 1 \right\}$$

$$= \frac{1.4}{1.4-1} \left\{ 1.05 \times 10^5 \times 2.8 \left[ \left( \frac{5.6}{1.05} \right)^{\frac{.4}{1.4}} - 1 \right] + 5.6 \times 10^5 \times 0.562 \left[ \left( \frac{35}{5.6} \right)^{\frac{.4}{1.4}} - 1 \right] \right\}$$

$$= 13.9 \times 10^5 \text{ N-m/min.}$$

\* Power required to drive a two-stage reciprocating air compressor  $= P = \frac{W \times N}{60}$  Wart. No. of working strokes per minute

Q Calculate mm<sup>m</sup> work required to compress 1 kg of air from 1 bar, 27°C to 16 bar in two stages, if  $pV^{1.25} = c$  & the intercooling is perfect.  $R = 287 \text{ J/kg-K}$

Sol

$$m = 1 \text{ kg} \quad P_3 = 16 \text{ bar}$$

$$P_1 = 1 \text{ bar} = 10^5 \text{ Pa} \quad n = 1.25$$

$$T_1 = 27^\circ\text{C} = 300 \text{ K} \quad R = 287 \text{ J/kg-K}$$

For perfect intercooling,  $P_2 = \sqrt{P_1 P_3} = \sqrt{1 \times 16} = 4 \text{ bar}$

work done

$$W = 2 \times \frac{n}{n-1} m R T_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= 275090 \text{ J}$$

Q A two-stage single acting reciprocating air compressor draws in air at 1 bar & 17°C & compresses to 60 bar. After compression in LP cylinder, the air is cooled at const. P of 8 bar & to 37°C. The LP cylinder has dia 150 mm & both cylinders have 200 mm stroke. If compression is  $pV^{1.35} = c$ , find power of the compressor when it runs at 200 rpm. Take  $R = 287 \text{ J/kg-K}$

Soln

Given  $P_1 = 1 \text{ bar} = 10^5 \text{ Pa}$   
 $T_1 = 17^\circ\text{C} = 290 \text{ K}$   
 $P_3 = 60 \text{ bar}$   
 $P_2 = 8 \text{ bar}$   
 $T_3 = 37^\circ\text{C} = 310 \text{ K}$   
 $d = 150 \text{ mm} = 0.15 \text{ m}$   
 $L = 200 \text{ mm} = 0.2 \text{ m}$   
 $n = 1.35$   
 $N = 200 \text{ rpm}$   
 $R = 287 \text{ J/kg-K}$



$V_1$  = Volume of LP cylinder

$$V_1 = \frac{\pi}{4} d^2 L = \frac{\pi}{4} 0.15^2 \times 0.2 = 0.0035 \text{ m}^3$$

$V_2$  = volume of HP cylinder

we know  $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$\Rightarrow V_2 = \frac{P_1 V_1 T_2}{P_2 T_1} = \frac{10^5 \times 0.0035 \times 310}{8 \times 10^5 \times 290} = 0.00047 \text{ m}^3$$

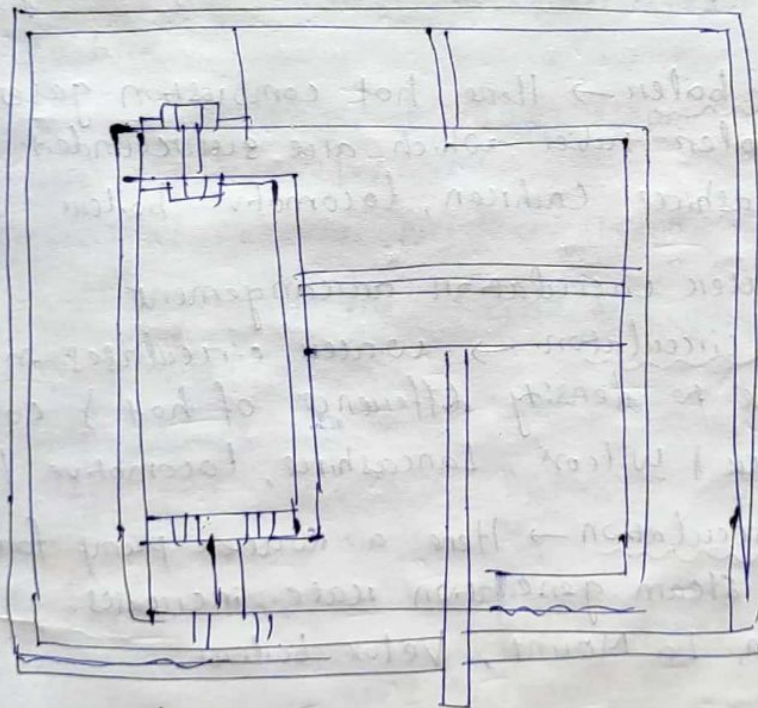
$$W_{in} = \frac{n}{n-1} \left[ P_1 V_1 \left\{ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right\} + P_2 V_2 \left\{ \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 1 \right\} \right]$$

$$= \frac{1.35}{1.35-1} \left[ 10^5 \times 0.0035 \left\{ \left( \frac{8}{1} \right)^{\frac{1.35}{1.35}} - 1 \right\} + 8 \times 10^5 \times 0.00047 \left\{ \left( \frac{60}{8} \right)^{\frac{1.35}{1.35}} - 1 \right\} \right]$$

$$= 3.86 (250 + 258) = 1961 \text{ N-m}$$

$$P = \frac{W_{in} \times N}{60} = \frac{1961 \times 200}{60} = 6540 \text{ W}$$

Double acting reciprocating air compressor  $\rightarrow$



Construction of double acting air compressor is similar to single acting air compressor, except two inlet & two delivery valves on two ends of the cylinder are present to allow air entry & delivery on two sides of the piston.

$\rightarrow$  When piston compresses the air on its one end side, it creates suction on the other side. So suction & compression of air takes place on two sides of the piston simultaneously.



## Chapter-4

### Steam Generator

- A steam boiler or steam generator is a closed vessel in which water is heated, vapourised & converted into steam at a pressure higher than the atmospheric pressure.
- Heat energy required for steam generation is produced by burning fuel in the furnace.
- Steam generated in boiler can be used for producing power or for heating purposes.

#### Classification of boiler →

- 1) Acc<sup>n</sup> to relative passage of water & hot gases
  - a) water tube boiler → Here water flows through a no. of small tubes which are surrounded by hot combustion gases. Ex- Babcock & Wilcox, Stirling, Benson boilers.
  - b) fire tube boiler → Here hot combustion gases pass through boiler tubes which are surrounded by water. Ex- Lancashire, Cochran, locomotive boiler.
- 2) Acc<sup>n</sup> to water circulation arrangement
  - a) Natural circulation → water circulates in the boiler due to density difference of hot & cold water. Ex- Babcock & Wilcox, Lancashire, locomotive boiler.
  - b) forced circulation → Here a water pump forces the water, so steam generation rate increases. Ex- Benson, La Mount, Velox boilers.
- 3) Acc<sup>n</sup> to use
  - a) stationary boiler → Used for power generation in thermal power plants or process steam plants.
  - b) portable boiler → These are mobile boilers used for temporary uses at the sites.
  - c) locomotive boiler → Specially designed boilers. It produces steam to drive railway engines.
  - d) Marine boiler → Used on ships.



- 4) Acc<sup>n</sup> to position of furnace
- Internally fired → Furnace is located inside the shell. Ex- Cochran, Lancashire boiler.
  - Externally fired → Furnace is located outside the boiler shell. Ex- Babcock & Wilcox, Stirling boiler.
- 5) Acc<sup>n</sup> to position of boilers
- Horizontal
  - Vertical
  - Inclined
- 6) Acc<sup>n</sup> to pressure of steam generated
- Low pressure boiler → produces steam at pressure of 15-20 bar. This steam is used for process heating.
  - Medium pressure boiler → It produces steam from 20 bar to 80 bar. This steam is used for power generation & process heating.
  - High pressure boiler → It produces steam at pressure above 80 bar.
    - Sub-critical boiler → produces steam at pressure  $< P_{critical}$  i.e. 221.1 bar.
    - Supercritical " → produces steam "  $> P_{critical}$
- 7) Acc<sup>n</sup> to charge in the furnace
- Pulverised fuel
  - Supercharged
  - Fluidised bed combustion boiler.

### Parts of boiler :-

- Drum/Shell  
It consists of one or more steel plates bent into the cylindrical form & riveted or welded together. Ends of the shell are closed by flat or curved plates called as boiler head.
- Setting  
Also called as foundation & is constructed of bricks. It supports boiler drum & other components. It forms the wall of the furnace, combustion chamber & passage to flue gases.
- Grate  
It is the space located below the furnace & consists of



cast iron bars upon which fuel is burned. The air can pass through the spaces between the bars & can support the combustion process, the ash can fall down through these spaces.

4) Furnace

It is the space above the grate & below the boiler shell. Here combustible gases are burnt & flue gases are generated.

5) Flue gases

It is the hot mixture of products of combustion generated in the furnace.

6) Flue

The hot gas passage in the boiler is called flue. It provides the drain to the hot gases to pass around the boiler.

7) Heating surface

It is the surface of the boiler which is exposed to hot flue gases on one side, water on other side.

8) Stoker

It is a mechanical system for charging of coal to the furnace & keep the firing continued.

9) Water wall

The closely spaced water tubes arranged near the furnace wall form a layer like a wall, called as water wall. The tube surface in the water wall receives the heat by radiation.

10) Water space

Space of the boiler shell occupied by water. Its level can be seen by water level indicator.

11) Steam space

Entire space of boiler shell not occupied by water.

12) Feed water

The water supplied to the boiler is called feed water.

The pump supplying water is called feed pump.

13) Working pressure

P of steam generated in boiler & superheater.

14) Economiser

Feed water supplied by feed pump is heated by waste heat of flue gases before leaving to atmosphere.



through chimney. It  $\uparrow \eta$ .

15) Air preheater

Fresh air going to the furnace is preheated, which increases combustion efficiency. It also uses waste heat of the gases.

16) Superheater

Used to heat saturated steam generated in the boiler to superheated steam. They are located above the furnace &  $\uparrow \eta$  of the system.

17) combustion chamber

It is the space, generally, below the boiler shell used for burning fuel to produce steam from the water contained in the shell.

18) Boiler mountings

These are the fittings which are mounted on the boiler for its proper functioning. Ex- pressure gauge, water level indicator, steam stop valve, feed check valve, blow off cock, boiler safety valve.

Boiler can't function safely without mountings.

19) Boiler accessories

These are integral part of the boiler but not mounted on it. These are auxiliary parts of boiler. Ex- superheater, economiser, air preheater, ESP, feed pump etc.

Characteristics of a good boiler  $\rightarrow$

$\rightarrow$  Should have min<sup>m</sup> steam generation rate with min<sup>m</sup> fuel consumption.

$\rightarrow$  can be started or stopped quickly.

$\rightarrow$  initial cost, running & maintenance cost should be low.

$\rightarrow$  should have control & safety apparatus.

$\rightarrow$  " high rate of heat transfer & better combustion efficiency.

$\rightarrow$  Should occupy less<sup>stn</sup> space.

$\rightarrow$  All parts of boiler should be accessible for cleaning & inspection.

$\rightarrow$  Should obey IBR act



# Comparison between fire tube & water tube boilers

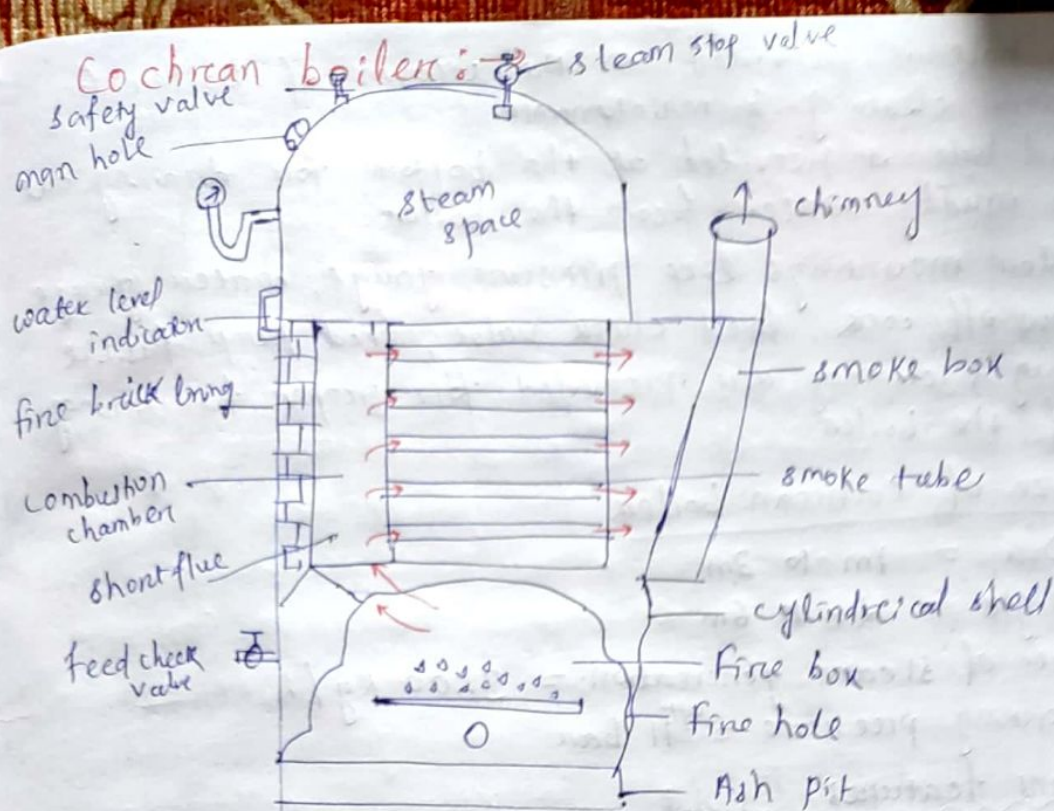
## Fire Tube

- i) Hot flue gases pass through tubes & water surrounds them.
- ii) Operated at low pressure upto 20 bar.
- iii) Rate of steam generation & quality of steam are low. So not suitable for power generation.
- iv) Load fluctuations can not be handled.
- v) Requires more floor area for a given o/p.
- vi) Bulky & difficult to transport.
- vii) Overall  $\eta$  is upto 75%.
- viii) Water does not circulate in a definite dirn.
- ix) Less initial cost but cost per unit is more.
- x) Simple in design, easy to erect & low maintenance cost.
- xi) Less skill required for oper<sup>n</sup>.
- xii) Used in process industry.

## Water Tube

- i) Water passes through tubes & hot flue gases surrounds them.
- ii) upto 250 bar in supercritical boiler.
- iii) . . . . . better. So suitable for power generation.
- iv) . . . . . are easily handled.
- v) . . . less floor area.
- vi) Light in weight. So easy to transport.
- vii) overall  $\eta$  with economiser is upto 90%.
- viii) Dirn<sup>n</sup> of water circulation is well defined.
- ix) Initial cost is high but cost/unit is less.
- x) complex design, difficult to erect & high maintenance cost.
- xi) Skilled operations are required.
- xii) Used in large power plants.





### Construction

- It is a vertical, coal or oil fired, fire tube boiler.
- Here the flue gases from the furnace pass through a number of small tubes surrounded by water.

### Construction

- It consists of a cylindrical shell with a hemi-spherical crown, grating, fire box, chimney & different mountings.
- The grate is placed at the bottom of the hemispherical furnace. Coal is fed into the grate through fire door & the ash formed is collected by ash pit, which is present below the grate & removed manually.

### Working

- The fuel is burnt on grate.
- Now the hot flue gases pass to combustion chamber through short flue, small horizontal smoke tubes & then collected in smoke box, from this it is disposed to atmosphere via chimney.
- Here the heat is transferred to water by radiation through the dome of the fire place & by convection from the walls of the smoke tubes.
- During heating water is vaporised & converted to steam. This steam is collected in the steam space above the water.
- Now this steam is taken for use via main steam stop valve.



- Man hole is provided in the crown of boiler for periodic cleaning & maintenance.
- Mud hole is provided at the bottom for draining out the muddy water from the boiler.
- Boiler mountings like pressure gauge, water gauge, blow off cock, feed check valve, feed pump, fusible plug & chimney are provided for proper functioning of the boiler.

### Size of Cochran boiler

Dia = 1m to 3m

height = 2m to 6m

Rate of steam generation = 3600 kg/h

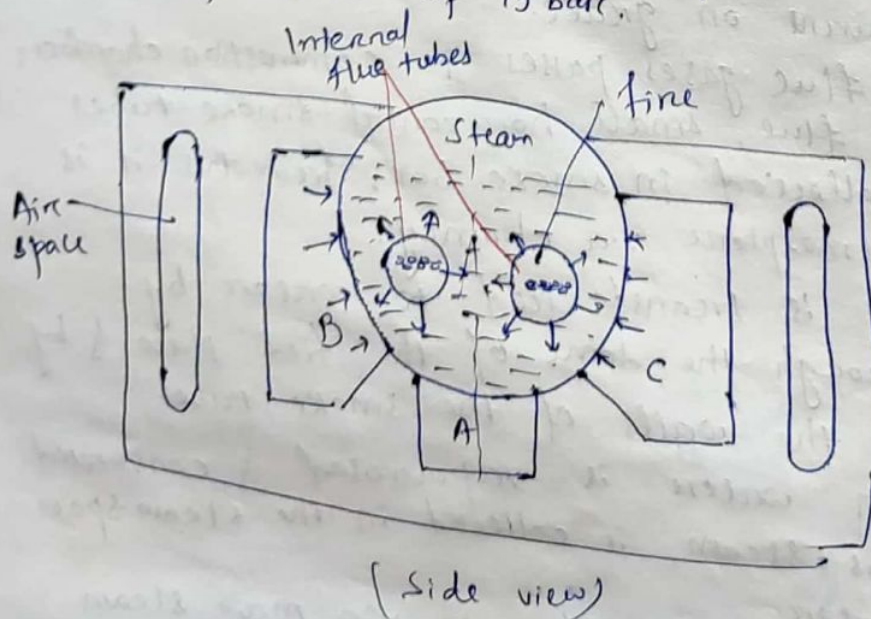
working pressure = <sup>upto</sup> 11 bar

### Salient features

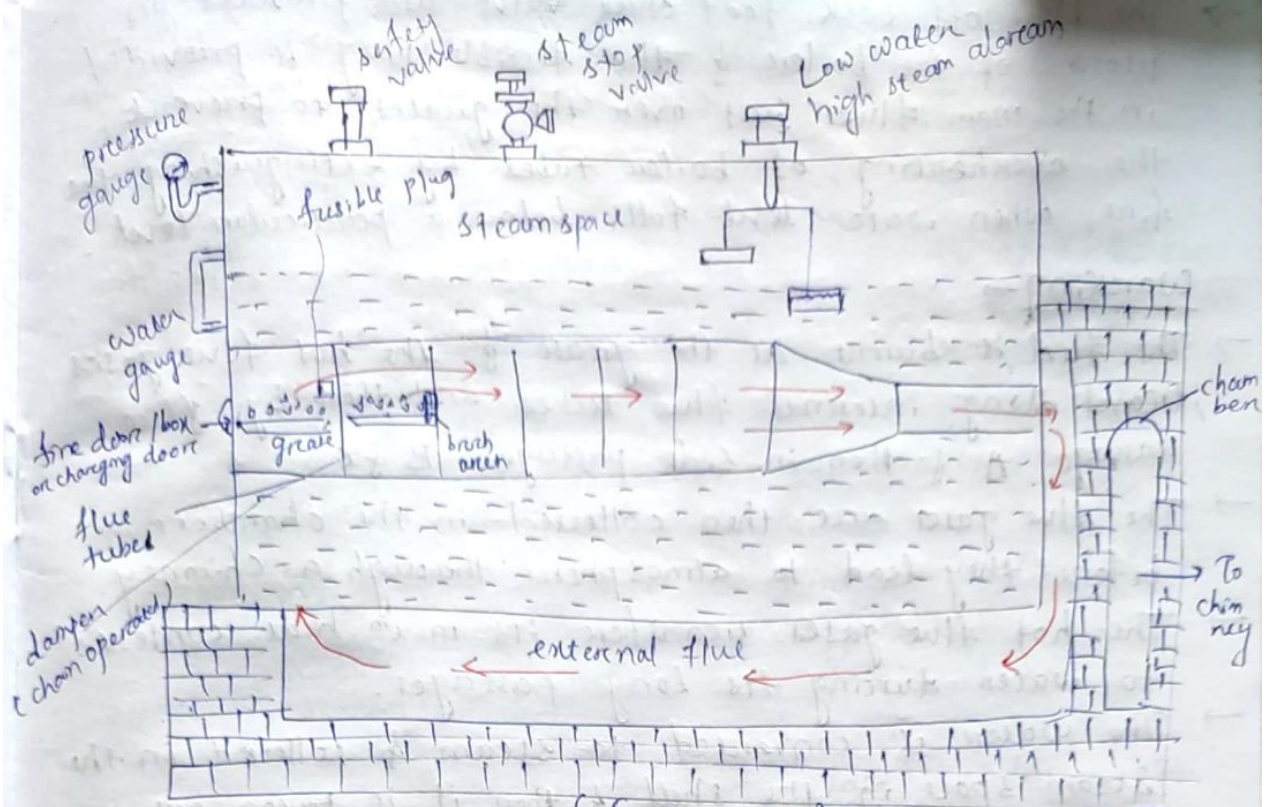
- Spherical crown & spherical shape of fire box are special features as these shapes require least material for a given volume.
- It is very compact & requires less floor area.
- Any type of fuel can be burnt.
- well suited for small industries.
- gives about 70% of  $\eta_{thermal}$  with coal firing.

### Lancashire boiler →

- It is a horizontal, internally fired, fire tube, natural circulation, stationary boiler.
- It is used widely due to capacity of good steam generation.
- It is also used for power generation at moderate steam pressure of 15 bar.







### Construction

(front view)

- It consists of a large shell supported by refractory brick masonry.
- The cylindrical shell is 2-3 m dia & 7-9 m long.
- Two large, horizontal & parallel flue gas tubes pass through shell. (dia 0.4 times dia of shell)
- The fire place is located in front of the flue tubes.
- In brick work, a flue passage A below the boiler shell, & two flue passages B & C at the sides of boiler are formed.
- The flue passages B & C are connected to a chamber & then to the chimney.
- The dampers in the form of sliding doors are located at the end of side flues to control the flow of gases. It regulates rate of combustion and rate of steam generation.
- Boiler is also provided with mountings like pressure gauge, water level indicator, steam stop valve, safety valve, low water & high steam safety valve, man hole etc on the top of the shell.

↓  
It gives an audio signal for low water level & high steam pressure.

\* Brick arch → Deflect the flue gases upward



- The blow off cock, feed check valve are provided in front of the boiler & the fusible plug is provided in the main flue just over the grate to prevent the overheating of boiler tubes by extinguishing the fire, when water level falls below a particular level.

### Working →

- The fuel is burnt at the grate & the hot flue gases travel along internal flue tubes and then by flue passage A & then in side passages B & C.
- The flue gases are then collected in the chamber, before they lead to atmosphere through a chimney.
- The hot flue gases transfer its max<sup>m</sup> heat contents to water during its long passages.
- The water is converted to steam & collected in the steam space in the shell & then it is taken out through steam stop valve for use.

### Special feature

- Its heating surface area per unit volume is large.
- Maintenance is easy.
- can be handled easily fluctuations of load.
- Highly suitable for process industries.

Shell dia → 2m

" height → 8-10 m

fire tube dia → 80 cm - 100 cm (2 nos)



# Babcock & Wilcox boiler: →

→ It is a <sup>horizontal</sup> water tube boiler, straight tube, externally fired, stationary type boiler. <sup>natural circulation</sup> <sup>fired</sup>

## components

welded steel drum

uptake & down take headen

water tubes inclined at about 15° (dia 10cm) <sup>2/3 of boiler shell is filled with water.</sup>

externally fired furnace

mountings & accessories.

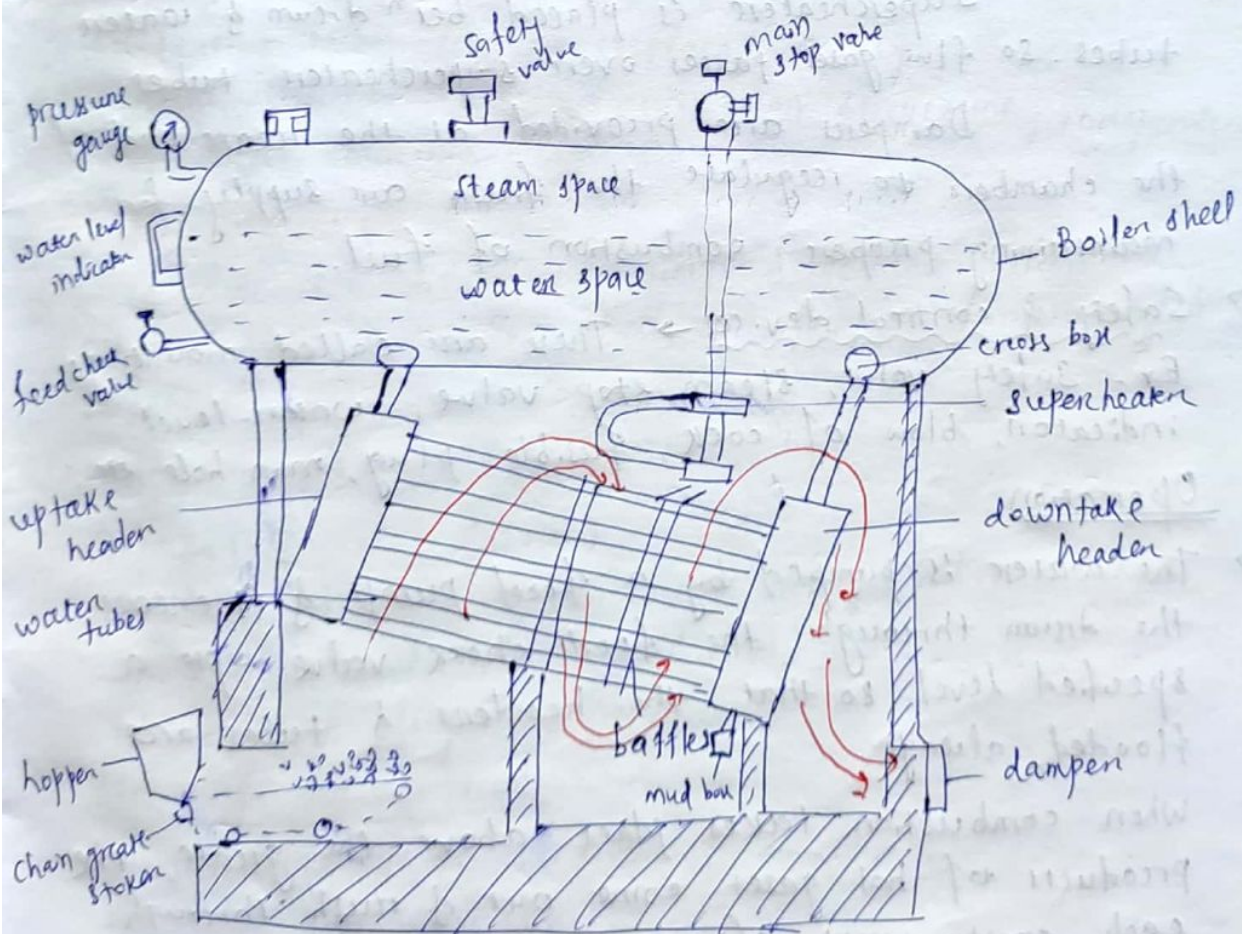
baffles = deflection (deflects flue gas travel to ↑ surface area for heat conduction)

Anti priming pipe → removes <sup>reduces</sup> moisture content of <sup>wet</sup> steam & send it to superheaters.

grate → where fuel is burnt. made of C.I. & have gap for air passage.

mud box is located at bottom of down take headen.

blow off pipe → used to blow the muddy water from mud box.





- Construction
- horizontal boiler shell → It is the main part of boiler. It is supported by a steel structure at certain height & independent of brick works. All safety & control devices are mounted on it.
- bundle of steel tubes → They connect uptake header & downtake header. They are fitted at an angle of  $5^\circ$  to  $15^\circ$  with horizontal to promote water circulation.
- combustion chamber - It is the space above the grate & below the front end of the drum where combustion of fuel takes place. It is enclosed by brickwork & lined with fire bricks from inside. Doors are provided for cleaning, inspection & repairing.
- It is divided into 3 separate chambers by baffles. 1<sup>st</sup> is hottest & the last is having lowest temp of flue gases.

Superheater is placed bet<sup>n</sup> drum & water tubes. So flue gases pass over superheater tubes.

- Dampers are provided at the rear end of the chamber to regulate the fresh air supply for maintaining proper combustion of fuel.

- Safety & control devices → These are called mountings. Ex - safety valve, steam stop valve, water-level indicator, blow off cock, fusible plug, man hole etc.

### Operation

- The water is pumped by a feed pump & it enters the drum through the feed check valve upto a specified level so that the headers & tubes are flooded always.
- When combustion takes place above the grate, the products of hot gases come out & rush through each compartment of combustion chamber.
- When water is inside the tube comes in contact with outside hot flue gases, heat transfer takes place.
- When water is heated inside the tubes, it becomes lighter & rises up in the tube.



- Due to continuous heat supply, some of the water gets vapourised into steam & the mixture of steam & water enters the boiler drum through uptake header.
- The cold water from boiler drum comes down through downtake header & enters the lower end of water tubes for further heating.
- This natural circulation of water remains continuous due to difference in temp<sup>n</sup>.
- The steam generated gets collected in steam space. To remove moisture or water particles, it is further passed over superheater. Now superheated steam is available for use.

### Special feature

- Operating capacity = 20,000 to 40,000 kg/h
- " pressure = 11.5 to 17.5 bar
- draught losses mm<sup>m</sup>.
- defective tubes can be replaced easily.
- entire structure rests on an iron structure, independent of brick structure.

### Boiler Mountings →

These are the different fittings & devices which are mounted on the boiler shell for proper functioning & safety.

#### 1) mountings for safety

- i) safety valve (2 nos)
- ii) High pressure & low water safety valve (1)
- iii) water level Indicator (2 nos)
- iv) Fusible plug (1 nos)

#### 2) Mountings for control

- i) pressure gauge (1 nos)
- ii) Steam stop valve ( " )
- iii) Feed check " ( " )
- iv) Blow off cock ( " )
- v) man hole ( " )
- vi) Mud box ( " )



## Boiler Mountings: →

These are those mechanical appliances which are considered essential for smooth & safe operation of boiler. These are usually mounted on the surface of a boiler.

Example of boiler mountings are

- i) water level indicator
- ii) pressure gauge
- iii) Safety valve
- iv) fusible plug
- v) steam stop valve
- vi) Blow off valve or blow down cock
- vii) feed check valve
- viii) man hole & mud box
- 1) water level indicator / water gauge

It is used to indicate the level of water present in the boiler. If level goes below the fixed mark so that corrective action can be taken to avoid accident.

### 2) Pressure gauge

It is used to indicate the gauge pressure of a fluid. It indicates the gauge pressure of steam within the boiler. Ex- Bourdon tube pressure gauge

### 3) Safety valve

It is a relief valve and it prevents the rising of boiler pressure above its normal working pressure by automatically opening when boiler pressure exceeds the normal working pressure & thus allows excess steam to escape into atmosphere until the pressure comes down to its normal value. It also provides safety to a pipe or vessel containing water at high pressure.



## types of safety valve

- a) Lever type S.V
- b) Dead weight S.V
- c) High steam & low water S.V
- d) Spring-loaded S.V

### 4) Fusible plug

It is used to put off fire in the furnace of the boiler when the water level in the boiler falls below an unsafe level and thus avoids explosion which may take place due to overheating of the tubes and shells.

This plug is usually fitted over the <sup>head</sup> crown of the furnace or over the combustion chamber.

### 5) Steam stop valve

Also called as main stop valve.

Used to control the flow of steam from within the boiler and to stop it completely when required.

### 6) Blow-off valve / blow-down cock / Block-off cock

Its function is to

a) To empty the boiler when necessary for cleaning, repair and inspection.

b) To discharge the mud and sediments carried with the feed water and accumulated at the bottom of the boiler.

→ It is ~~connected~~ fitted to the lowest part of the boiler either directly with the boiler shell or to a pipe connected with the boiler.

→ When blow-off cock is opened, the water which is under the pressure of steam, rushes out with tremendous velocity & thus carry ~~and~~ out the sediments along with it.

### 7) Feed check valve

Used to control the flow of water from the feed pump to the boiler and to prevent the back flow of water from the boiler to the pump when the pump pressure is less than the



boiler pressure on when work.  
→ It is placed at the boiler end of the delivery pipe of the feed pump.

### Boiler accessories: →

These are the auxiliary parts required to increase the overall efficiency of the boiler & to improve the operating conditions.

### Difference bet<sup>n</sup> mountings & accessories

- Boiler mountings is an essential apparatus without which a boiler cannot be operated, but boiler accessories is not an essential part, used only to
- mountings are mounted on the boiler but accessories are not generally mounted on the boiler but installed within or near the boiler
- mountings occupy less floor space but accessory occupy more floor space.

### Example

- i) Economiser
- ii) Feed water heater
- iii) Superheater
- iv) Steam separator
- v) Steam trap
- vi) Feed pump
- vii) Injectors

### Electrostatic precipitator

- viii) Air preheater

### Air preheater →

- It recovers some portion of the waste heat of the flue gases. Air supplied to the combustion chamber is preheated by using heat of waste flue gases.
- These are placed after the economiser & before the gases enter the chimney.
- Preheating of air results the following advantages
  - waste heat from the flue gases is recovered for heating air & reduces the fuel consumption



- by about 1.5% for each  $100^{\circ}\text{C}$  drop in gas temp.
- Inferior grades of coal can be burnt efficiently with preheating air
  - Combustion can be more efficient & an intense flame can be achieved in the furnace so  $\uparrow$  evaporation rate of the boiler.

### Economiser $\rightarrow$

It is a heat exchanger. It extracts the waste heat of the chimney gases to preheat the water before it is fed to the boiler. It ensures economy of fuel. So it is called economiser.

### advantages of preheating of boiler feed water

- $\rightarrow$  Saving of fuel as waste heat of flue gas is utilised to heat feed water.
- $\rightarrow$  Dissolved gases as air &  $\text{CO}_2$  are removed by preheating feed water & reduces corrosion & pitting.
- $\rightarrow$  Less temp. stress in boiler plates as feed water enters the boiler at high temp.
- $\rightarrow$  Circulation of  $\text{H}_2\text{O}$  is well maintained as quick evaporation is possible due to hot feed water.
- $\rightarrow$   $\uparrow$  overall  $\eta$  by  $\downarrow$  fuel consumption.

### Types of economiser

#### 1) Independent type

Installed in chamber upset from the boiler setting. Chamber is situated at passage of the flow of the flue gases from the boiler or boiler to chimney.

#### 2) Integral type.

Part of the boiler heating surface & is installed within the boiler setting.

- $\rightarrow$  Water moves from top to bottom pipe & then to boiler.
- $\rightarrow$  The gases move around the pipes in direction opposite to flow of water & consequently heat transfer through the surface of pipe takes place & water is thereby heated.
- $\rightarrow$  Blow-off cock is provided at back end of vertical pipes to remove sediment deposit in bottom boxes.



- To remove soot of flue gases on pipe of economiser are used which move up & down to keep it clean.
- By pass arrangement enables to isolate or include the economiser in path of flue gases.

### Advantages

- temp. range betn various parts of boiler is reduced which results in reduction of stresses due to unequal expansion.
- evaporative capacity of boiler ↑
- Movement of plant ↑

### Air preheater →

- Used to ↑ the temp. of air before it enters the furnace. It is generally placed after economiser. So flue gases pass through economiser & then the air is preheated.

- It consists of plates or tubes with hot gases on one side & air on other. Preheated air accelerates the combustion & facilitates the burning of coal.

- Degree of pre-heating depends on

- type of fuel
- " " " burning equipment
- Rating at which boiler & furnace are operated.

- Types of air preheater

- Tubular
- plate
- Storage

- After leaving boiler or economiser, flue gas preheats the air & is supplied to furnace. Finally the gases escape to the atmosphere through stack (chimney). Temp. of gases leaving the chimney should be kept as low as possible to minimize losses.

### Superheater →

- Used to increase temp of saturated steam into superheated steam after it comes from the boiler.

### Advantages

- Steam consumption of turbine is reduced.
- Saving in fuel consumption (about 15-20%).



→  $\eta \uparrow$

→ Losses due to condensation in cylinders & steam pipe is reduced.

→ Superheaters are located in path of furnace gases so that heat is recovered by the superheater from the hot gases.

→ Types of Super-heater

1) convective → uses heat of the <sup>flue</sup> gases by convective mode

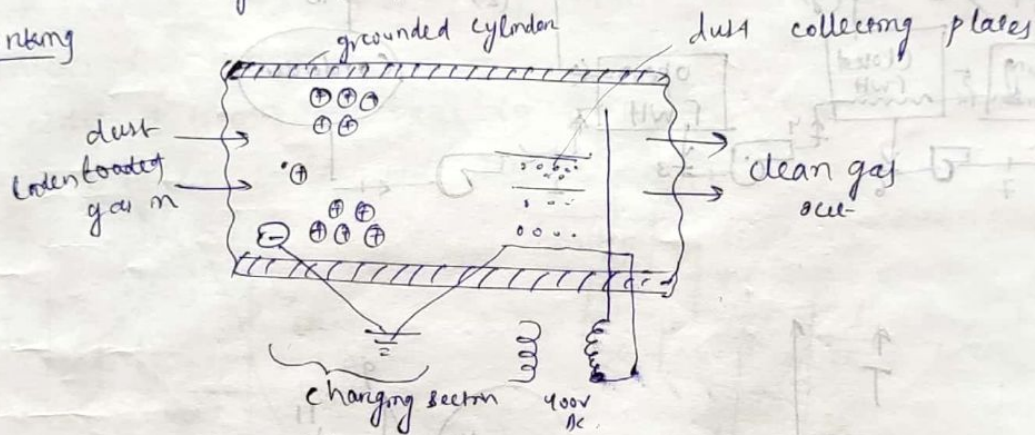
2) Radiant → placed in furnace's wall tubes receive heat from burning fuel through radiation

3) combination type <sup>(high temp. requirement)</sup> → heat is transferred by both mode.

ESP →

→ Used for removal of dust & ash particle carried with exhaust gases of thermal power plants.

Working



When dust laden gas is passed bet<sup>n</sup> oppositely charged conductors it becomes ionised as voltage applied bet<sup>n</sup> the conductors. particles get charged +vely & -vely. The collecting unit collect the dust which has metal plates arranged that are alternatively charged.

advantage

- Remove very small particles
- $\eta$  99.5 %
- ease of operation
- maintenance of charge is easier

Disadvantage

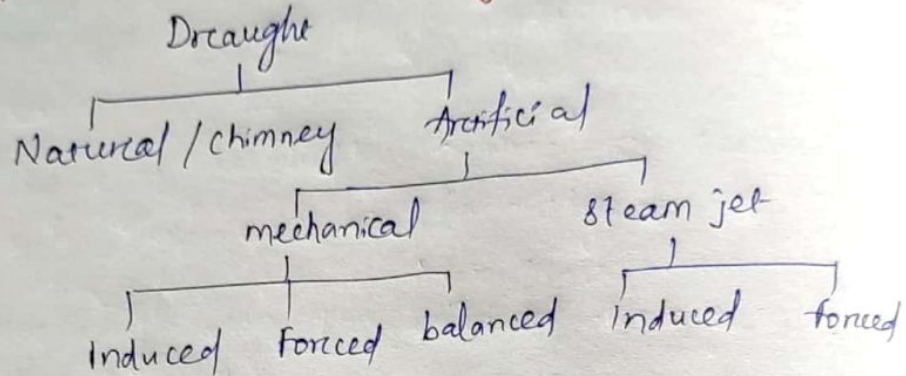
- DC current not available directly
- running cost is high
- space required is large
- velocity of gas is not maintained  $\eta$  is affected



**Boiler Draught**  $\rightarrow$  (caused)  
It is defined as the pressure difference which causes continuous flow of gases inside the boiler.

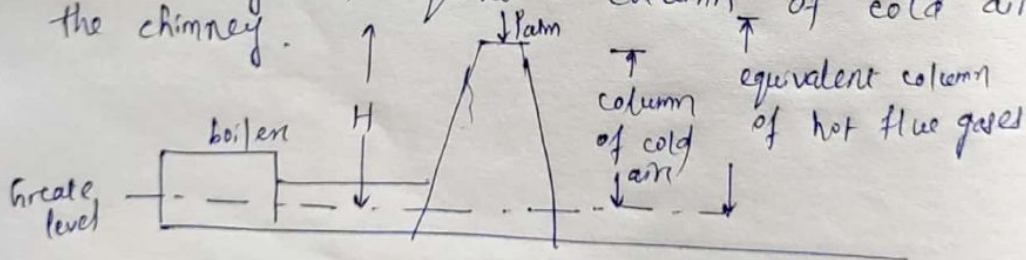
- Function of draught**  $\rightarrow$
- $\rightarrow$  Forces sufficient quantity of air into the furnace for proper combustion of fuel.
  - $\rightarrow$  circulates the hot flue gases through the flue tubes, superheater, economiser, air preheater etc.
  - $\rightarrow$  Discharges the hot flue gases to the atmosphere through the chimney.

**Classification of boiler draught**  $\rightarrow$



**Natural draught**  $\rightarrow$

- $\rightarrow$  It is produced by using chimney.
- $\rightarrow$  chimney is a vertical long cylindrical structure made of brick masonry, RCC or steel.
- $\rightarrow$  The draught produced by chimney is due to density difference bet<sup>n</sup> the column of hot flue gases inside the chimney and equivalent column of cold air outside the chimney.



Pressure at grate level in combustion chamber

$$P_1 = P_{atm} + \rho_g g H$$

Pressure at grate level outside the chimney

$$P_2 = P_{atm} + \rho_a g H$$

$$\rho_g < \rho_a$$

$$\Delta P = P_2 - P_1 = (\rho_a - \rho_g) g H$$

- \* The pressure difference causing the flow of gases is called static draught & is very small, so measured by water manometers.



## Advantages

- easy to construct
- No power is required for producing draught
- Long life of chimney
- No maintenance required

## Disadvantages

- Tall chimney required
- poor  $\eta$ .
- $\eta \downarrow$  with  $\uparrow$  in outside temp.
- No flexibility to create more draught to take peak loads.

## Artificial Draught →

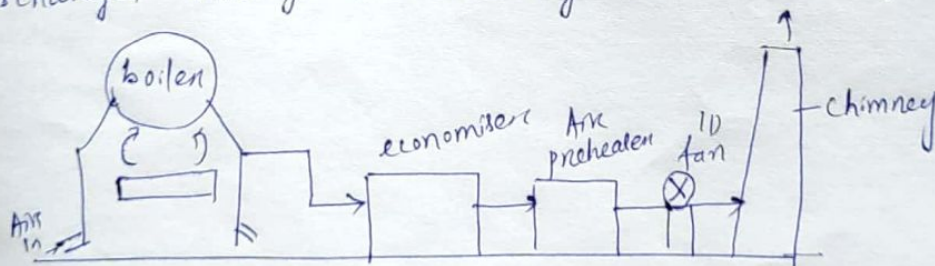
Draught produced by any artificial means.

When artificial draught is produced by fan or blower it is called mechanical draught & that produced by steam jet is called steam jet draught.

## Mechanical draught →

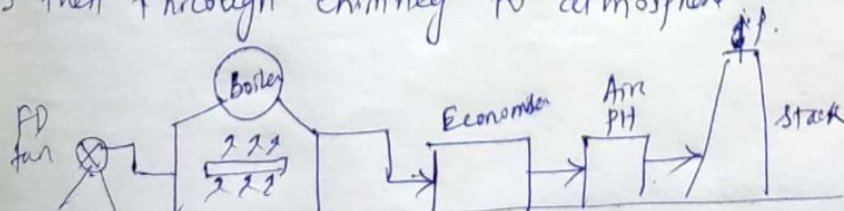
### 1) Induced draught →

Here fan is placed near the base of the chimney. The fan draws the flue gases from the furnace. So the pressure above the fuel bed is reduced below the atmospheric pressure. The fresh air rushes to the furnace & after combustion, the flue gases get discharged through the chimney in the atmosphere.



### 2) Forced draught →

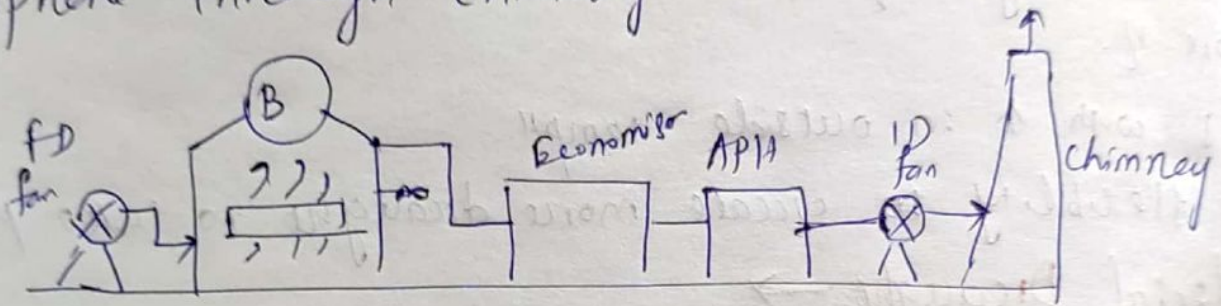
- Here fan or blower is located near or at the base of the boiler grate to force atmospheric air on to the furnace under pressure. This pressure helps in circulation of flue gases through components of boiler & then through chimney to atmosphere.





## Balanced draught →

It is the combination of induced & forced draught. A forced draught fan located near the grate supplies air under pressure through the furnace & an ID fan located near the chimney base, draws in flue gases through the economiser, APH etc & discharges to atmosphere through chimney.



## Advantages of artificial draught over natural draught →

- i) Forced draught has better control on combustion.
- ii) Low grade fuel can be efficiently burnt.
- iii) overall  $\eta$  of a thermal power plant  $\uparrow$  as better heat recovery from the exhaust flue gases in economiser & APH.
- iv) Height of chimney can be reduced.
- v) smoke formation is less.
- vi) Tendency for air leakage in the furnace is less.



# Working Substance Chapter-5

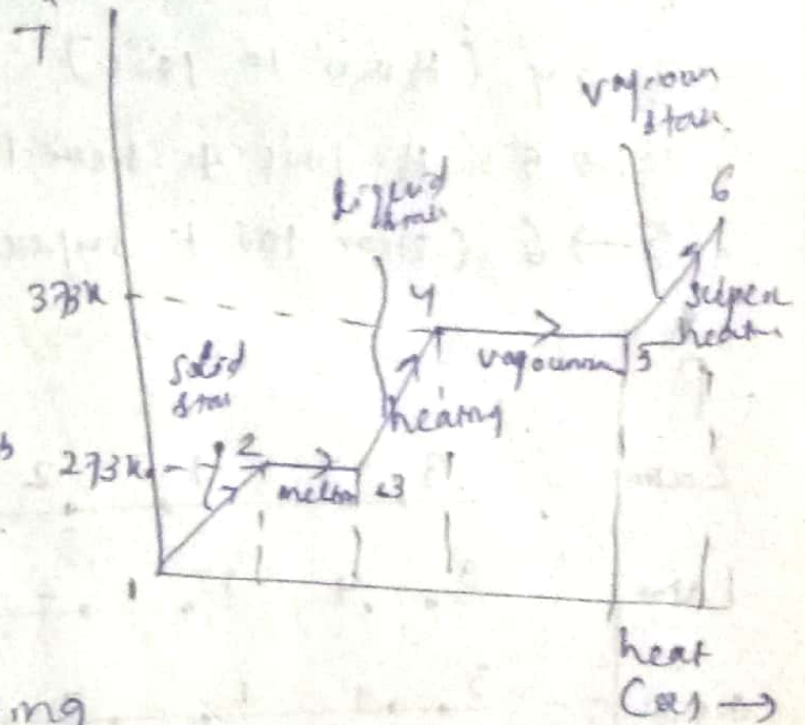
## Pure substance →

It is a substance of fixed chemical composition throughout its mass. It is one component system & exists in one or more phases. Ex-  $O_2$ ,  $N_2$ , air, steam etc.

## Formation / Generation of steam →

Consider 1 kg of ice at pressure  $P$  & at temp  $0^\circ C$  is contained in a vessel. Now it is heated by keeping  $P$  const. The following changes occur,

- i) Temp of ice ↑ gradually till it reaches  $0^\circ C$  at atm  $P$ . (1 → 2)
- ii) By ↑ heat further, ice starts melting by absorbing latent heat of fusion. At const  $P$  &  $T$ . (2 → 3)
- iii) Further adding heat, liquid  $H_2O$  absorbs heat till it reaches its boiling point. (3 → 4)
- iv) Again heating water at  $100^\circ C$  will be converted to steam at  $100^\circ C$  by absorbing latent heat of vapourisation at const  $P$  &  $T$ . (4 → 5)
- v) Further heating called superheating & the steam follows all gas laws. increases its  $T$ .



## Terms used in steam →

- 1) Wet steam → when the steam contains moisture or particles of water. It contains partly vapour & partly liquid.
- 2) Dry saturated steam → when wet steam is further heated, it does not contain any suspended particles of water called as dry saturated steam.
- 3) Superheated steam → when dry saturated steam is further heated at const  $P$  and thus raising its temp, it is called superheated steam. The process is called superheating.

$$\text{degree of superheat} = t^{\circ} - t_s^{\circ} \quad \text{where } t_s^{\circ} = \text{saturation temp.}$$

## Quality or dryness fraction →

Defined as ratio between mass of actual dry steam to the total mass of steam. denoted by  $x$ .

$$\text{Mathematically } x = \frac{m_g}{m_g + m_f} = \frac{m_g}{m} \quad \begin{matrix} m_g = \text{mass of dry saturated vapour} \\ m = \text{mass of mixture} \end{matrix}$$



5) ~~Specific~~<sup>Sensible</sup> heat of water  $\rightarrow$

Sensible heat = mass  $\times$  sp. heat  $\times$  rise in temp

$$= 1 \times 4.2 (1 - 0)$$

6) Enthalpy or total heat of steam  $\rightarrow$

enthalpy = sensible heat + latent heat.

P-V diagram for pure substance  $\rightarrow$

P-V diagram for pure substance  $\rightarrow$

①  $-10^\circ\text{C ice} \rightarrow 0^\circ\text{C ice} \rightarrow 0^\circ\text{C water} \rightarrow 100^\circ\text{C water} \rightarrow 100^\circ\text{C steam} \rightarrow 110^\circ\text{C steam} \leftarrow$

②  $(0^\circ\text{C ice to } 0^\circ\text{C})$

③  $(0^\circ\text{C water to } 100^\circ\text{C})$

④  $(100^\circ\text{C water to } 100^\circ\text{C steam})$

⑤  $(100^\circ\text{C steam to } 110^\circ\text{C})$

⑥  $(110^\circ\text{C steam to } 100^\circ\text{C})$

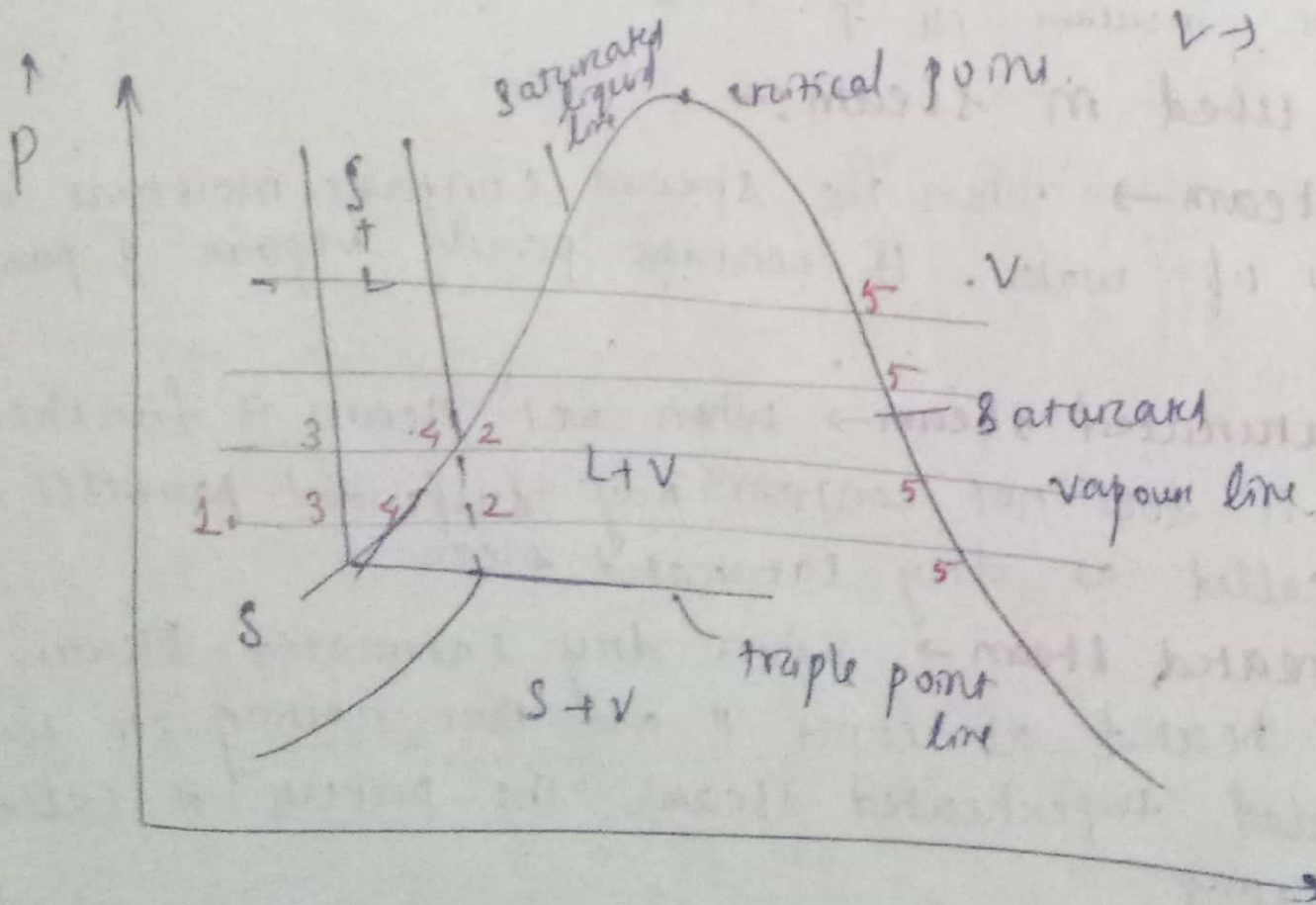
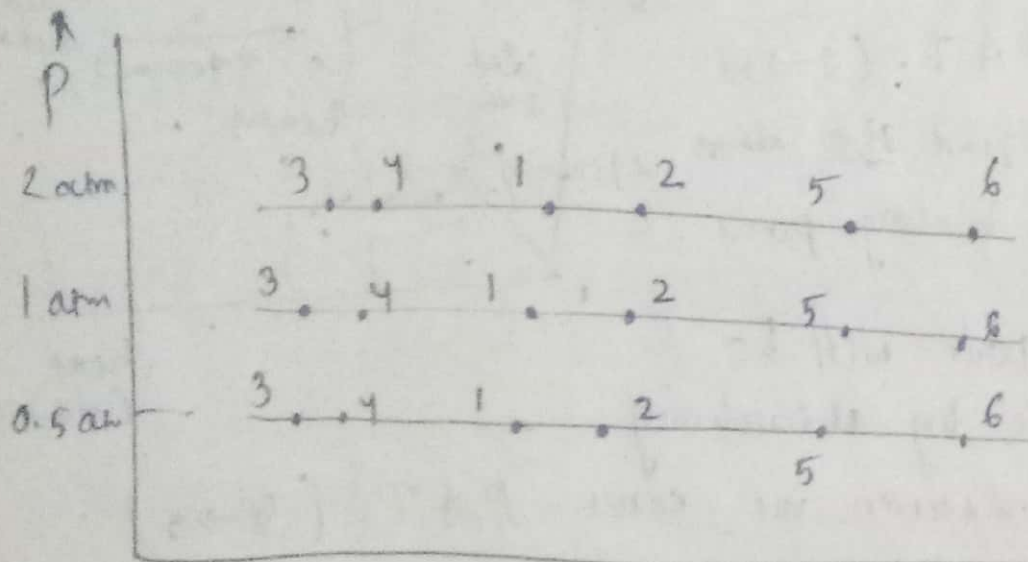
1 → 2 (ice - loc + oc)

2 → 3 (ice  $0^\circ$  to  $H_2O 0^\circ$ )

3 → 4 (H<sub>2</sub>O 0° 100°C)

4 → 5 (H<sub>2</sub>O 100% & steam 100%)

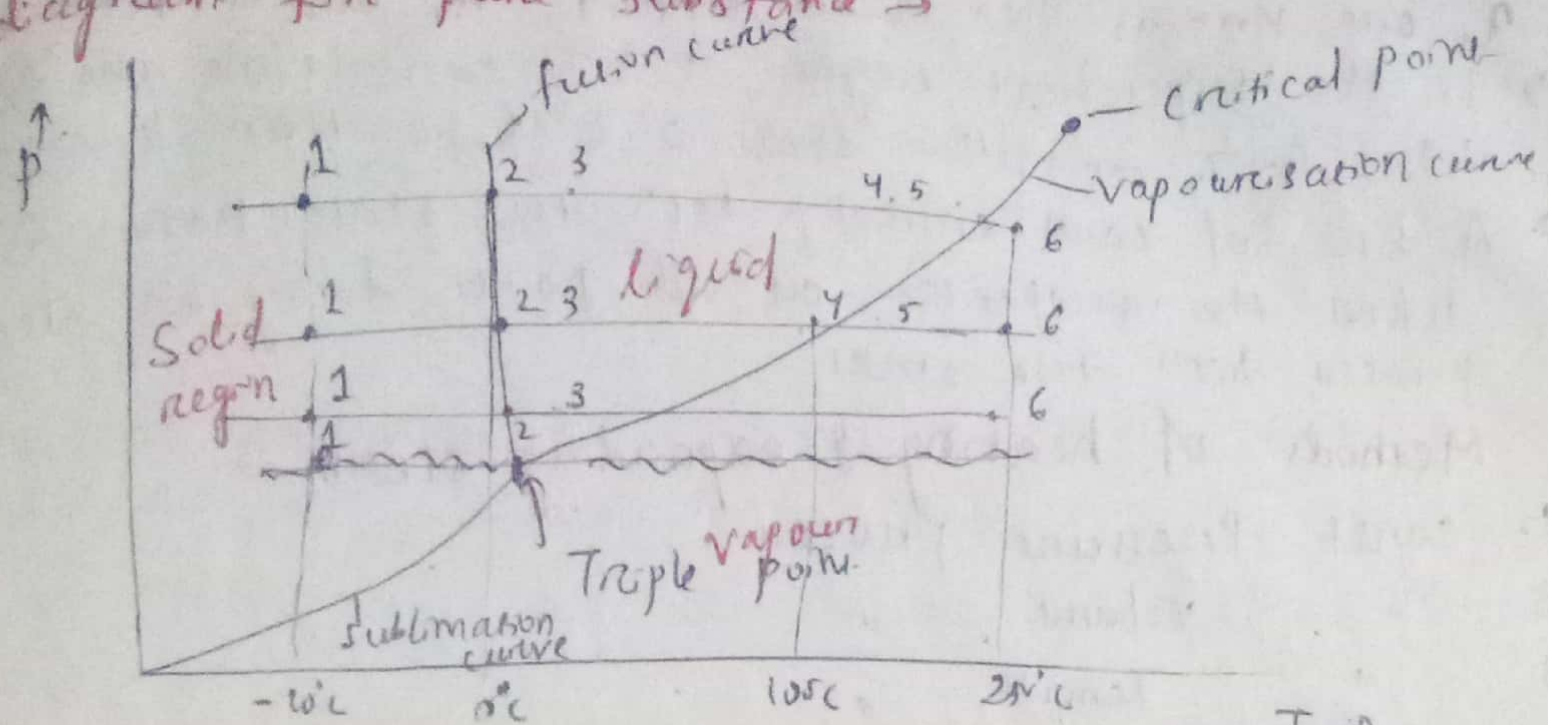
5 → 6 (steam 100% + superheated steam at 100%)



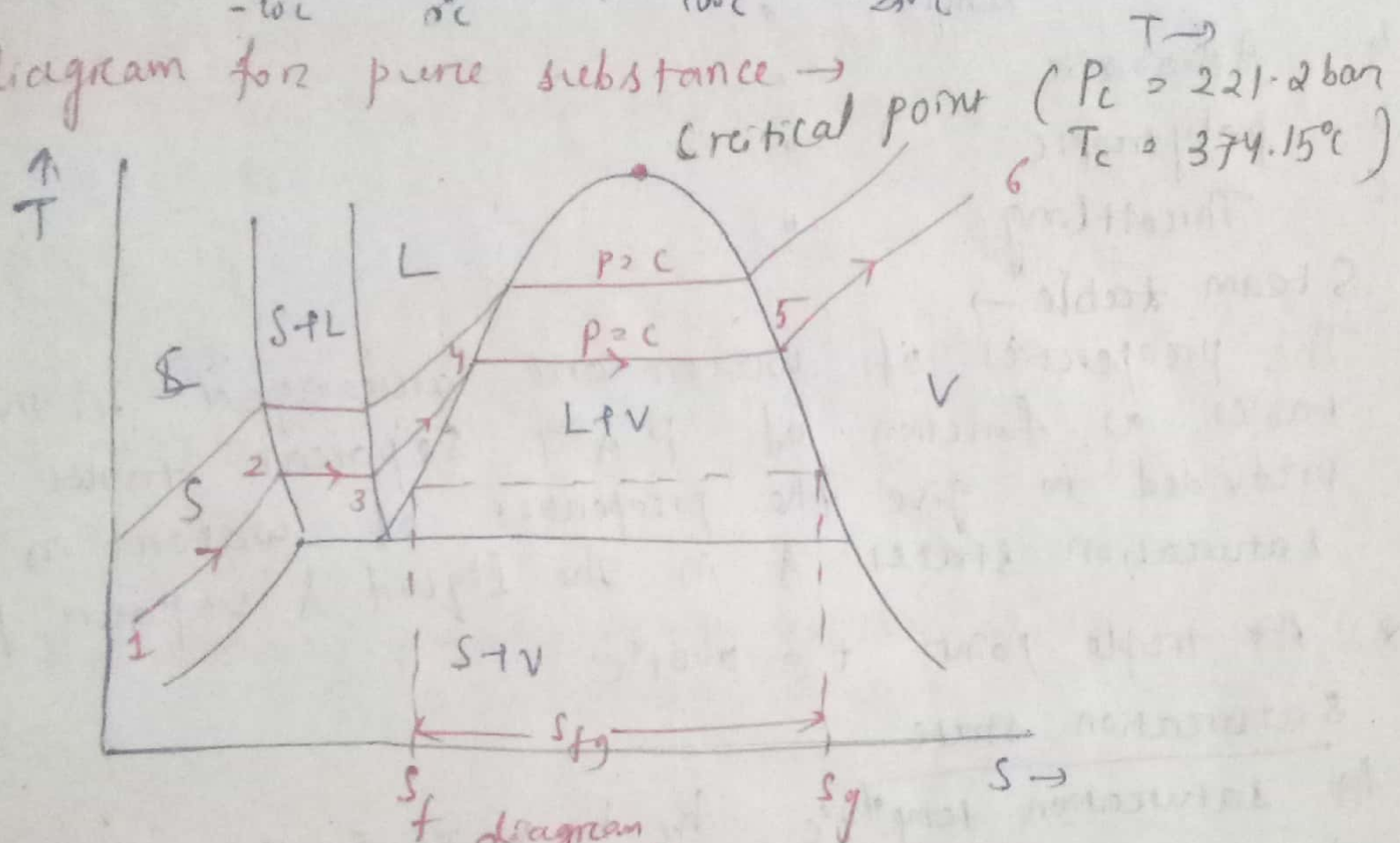
line passing through all saturated solid state  
 " " " " liquid  
 " " " " vapor



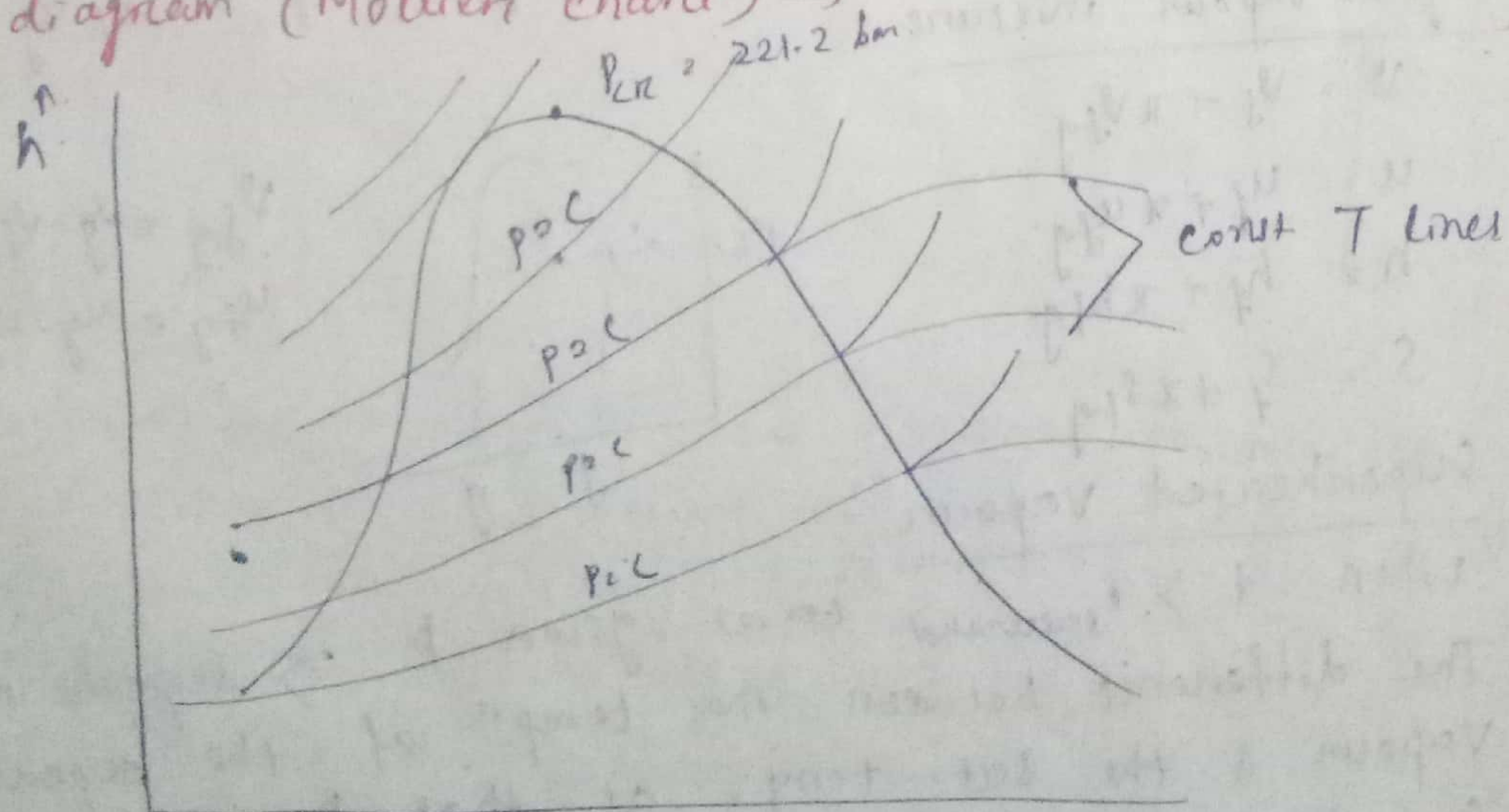
P-T diagram for pure substance →



T-S diagram for pure substance  $\rightarrow$



H-S diagram (Mollier chart)  $\rightarrow$



Here vertical coordinate represents enthalpy  $\overset{S \rightarrow}{H}$  & base represents entropy.

Lines of const  $P$ , are indicated by  $P_1, P_2$  - etc & lines of const  $T$  by  $T_1, T_2$  - etc.

Any two independent properties which appear on the chart are sufficient to define the state. e.g.  $P, T$



- $T_1$  are known. Then from state 1,  $h_1$  can be found out.
- In the superheat region,  $P$  &  $T$  can define the state.  
e.g.  $P_2, T_2$  can define state 2 & from there  $h_2$  can be found.
- A line of const entropy bet<sup>n</sup> two state points 2 & 3 defines the properties at all points during an isentropic process bet<sup>n</sup> two states.

### Methods of heating & expanding steam →

1. const Pressure process
2. " volume "
3. " Temp<sup>n</sup> "
4. Adiabatic
5. polytropic
6. Throttling

### Steam table →

The properties of water are arranged in the steam tables as function of  $P$  &  $T$ . Separate tables are provided to give the properties of water in the saturation states & in the liquid & vapour phase.

\* At triple point  $t = 0.01^\circ\text{C}$

### Saturation state

At saturation temp<sup>n</sup>  $t^\circ\text{C}$ ,  
Liquid-vapour mixture

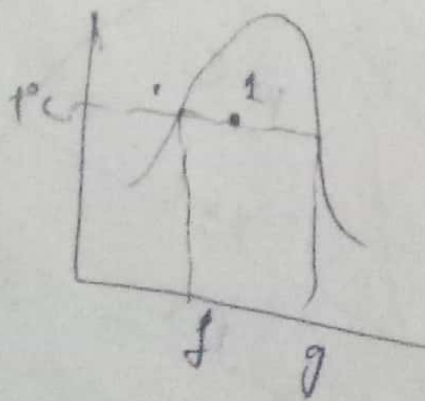
$$v = v_f + x v_{fg}$$

$$u = u_f + x u_{fg}$$

$$h = h_f + x h_{fg}$$

$$S = S_f + x S_{fg}$$

$$h_f, h_g, v_f, v_g, S_f, S_g, u_f, u_g \text{ etc.}$$



$$v_{fg} = v_g - v_f$$

$$u_{fg} = u_g - u_f$$

### Superheated vapour

When  $t > t_{\text{saturation}}$  at given  $P$ .

\* The difference between the temp<sup>n</sup> of the superheated vapour & the sat. temp. at that  $P$  is called "Superheat" or the "degree of superheat".

$$t_1 - t_{\text{sat}} = \text{Superheat}$$

### Compressed or subcooled liquid

When temp. of liquid is less than  $t_{\text{saturation}}$ , at given  $P$  then the liquid is sub-cooled.

\* As its properties varies slightly, so saturation temp. table data



are taken  
 Q-1 Find the saturation temp, change in specific volume & entropy <sup>of steam</sup> during evaporation at 1 MPa.

Sol:  $t_{sat} =$   $s_{fg} =$

Q-2 Find the enthalpy & entropy of steam when  $p$  is 2 MPa & sp. volume is  $0.09 \text{ m}^3/\text{kg}$ .

Sol: At  $p = 2 \text{ MPa}$ ,  $v_f = 0.001177 \text{ m}^3/\text{kg}$  &  $v_g = 0.09963 \text{ m}^3/\text{kg}$   
 $v = v_f + x v_{fg}$

$$\Rightarrow 0.09 = 0.001177 + x(0.09963 - 0.001177)$$

$$\Rightarrow x = 0.904$$

At 2 MPa,  $h_f =$

$$s_g =$$

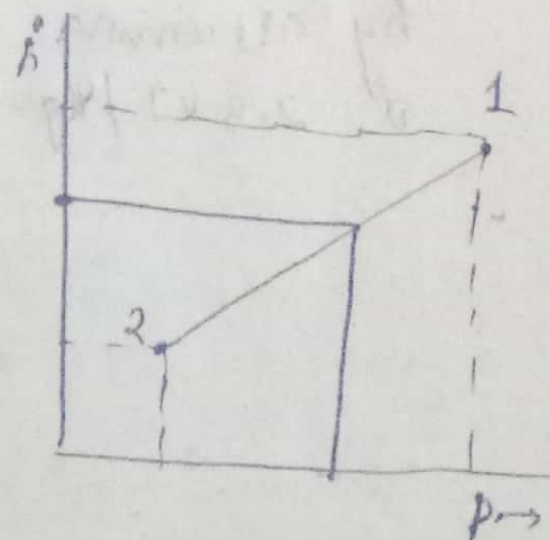
$$s_f =$$

$$s_{fg} =$$

$$h = h_f + x h_{fg} =$$

$$s = s_f + x s_{fg} =$$

Find the enthalpy, entropy & volume of steam at 1.4 MPa,  $380^\circ\text{C}$ .



Determine the quality of heat required to produce 1 kg of steam at a pressure of 6 bar & at a temp. of  $25^\circ\text{C}$ , under the following cond<sup>n</sup>s

1) When steam is wet having  $x = 0.9$

2) " dry saturated

3) " superheated at a constant  $p$  at  $250^\circ\text{C}$ .

assuming mean sp. heat of superheated steam to be  $2.3 \text{ kJ/kg}\cdot\text{K}$

From S.S.T.

At 6 bar,  $25^\circ\text{C}$ ,  $h_f = 670.4 \text{ kJ/kg}$ ,  $h_{fg} = 2085$ ,  $t_{sat} = 158.8^\circ\text{C}$



$$h = h_f + x h_{fg}$$

$$= 670.4 + (0.9 \times 2085) = 2546.9 \text{ kJ}$$

A) Water is at temp.<sup>n</sup> of 25, so heat already in water =  $4.2 \times 25 = 105 \text{ kJ}$

∴ Heat actually required =  $2546.9 - 105 = 2441.9 \text{ kJ}$

i)  $h_g = h_f + h_{fg} = 670.4 + 2085 = 2755.4 \text{ kJ}$

Heat actually required =  $2755.4 - 105 = 2650.4 \text{ kJ}$

ii)  $h_{sup} = h_g + C_p (t_{sup} - t_{sat})$

$$= 2755.4 + 2.3(250 - 158.8) = 2965.16 \text{ kJ}$$

Heat actually required =  $2965.16 - 105 = 2860.16 \text{ kJ}$

Q Determine the quantity of heat required to produce 1 kg of steam at pressure of 6 bar at a temp<sup>n</sup> of 250°C. under the following cases.

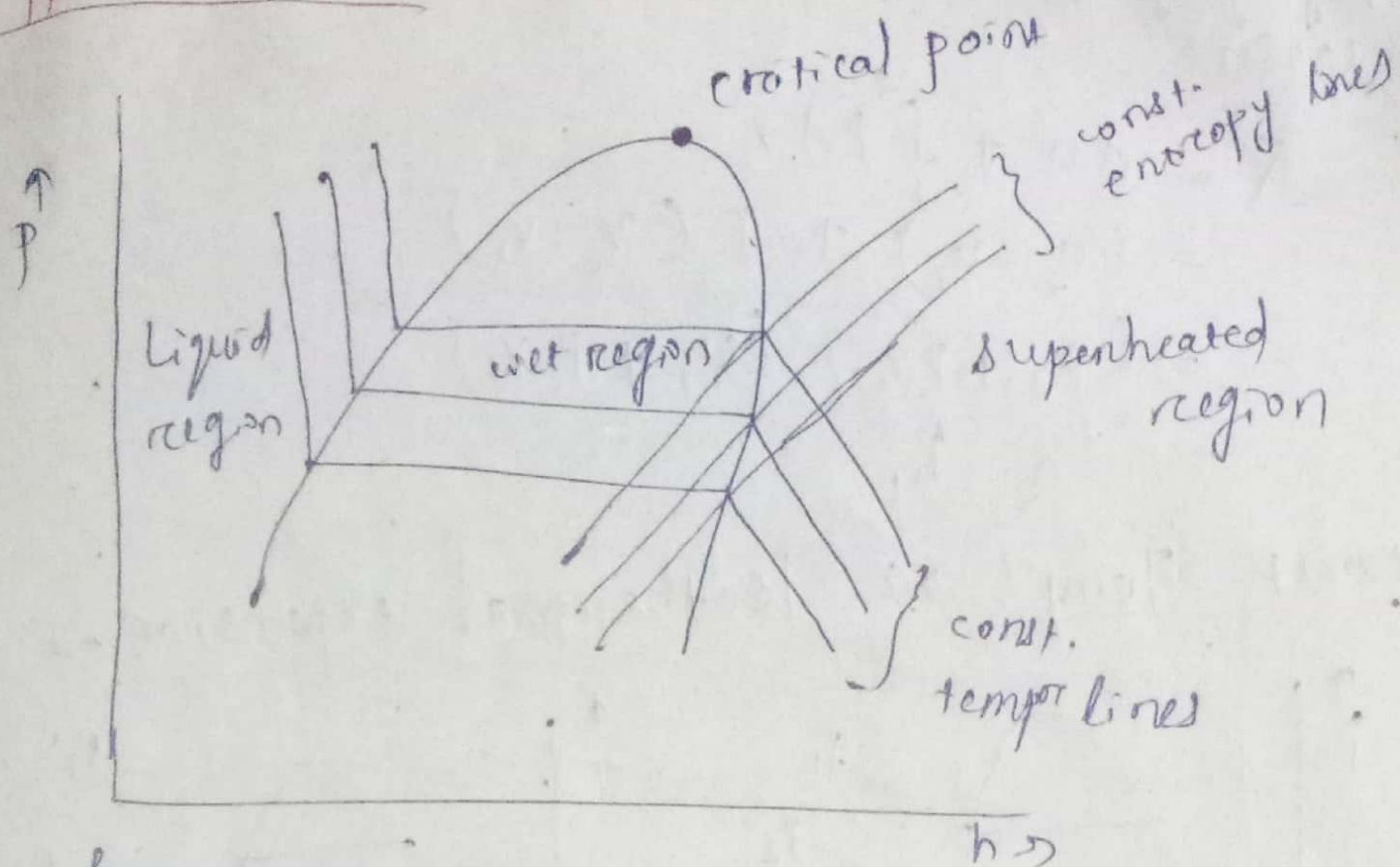
i) steam is wet having 95% quality

ii) steam is dry saturated

iii) steam is superheated at constant pressure at 250°C by assuming mean sp. heat of superheated steam is  $2.3 \text{ kJ/kg-}^\circ\text{C}$ .



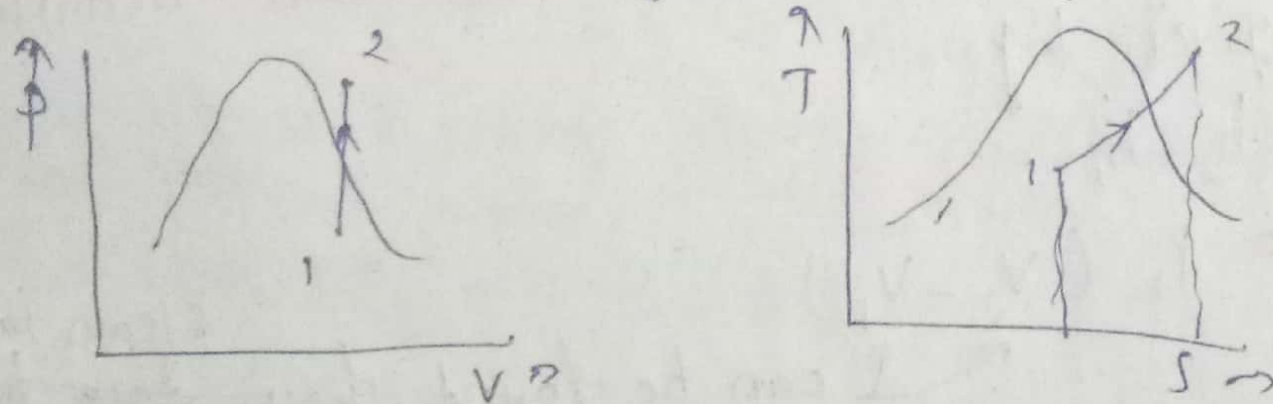
## P-h chart



→ In refrigeration system P-h chart is used  
 → It is useful in locating const. enthalpy lines & const. entropy lines starting from saturated liquid conditions.

## Different flow & non-flow processes of vapour →

1) const. volume heating or cooling →

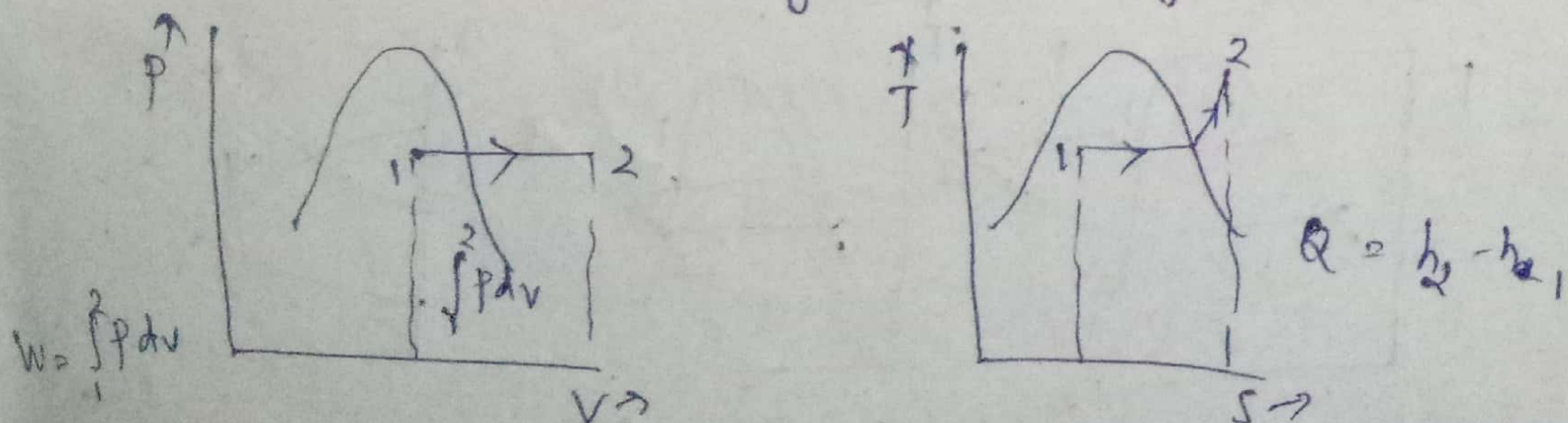


Steam is in wet cond<sup>n</sup> before heating at  $P_1, P_1$  & becomes superheated after heating at  $P_2, P_2$ .

mass of steam remains const during the process.

Data can be extracted from the steam table.

2) const. pressure heating or cooling →



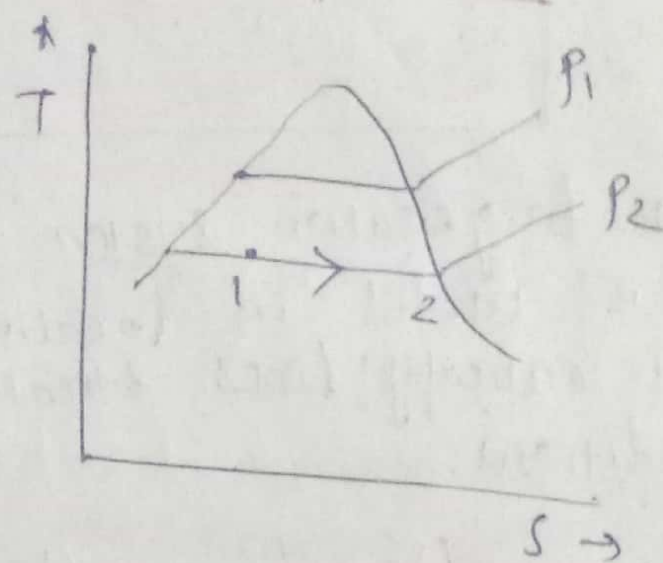
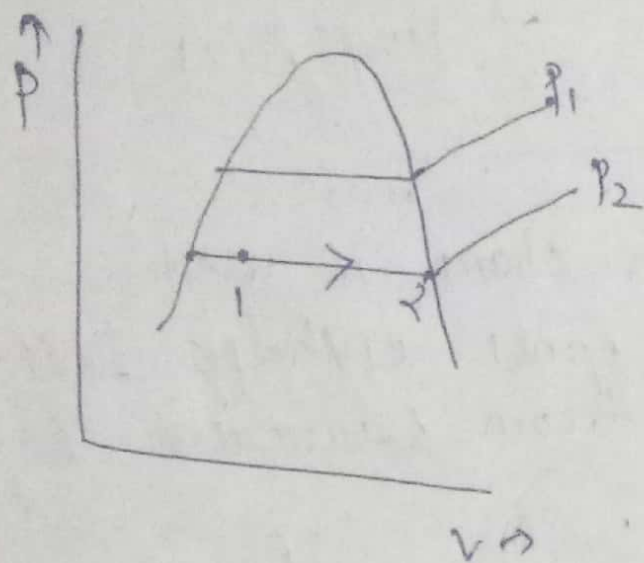
Ex - generation of steam in boiler [const. P heating]



Applying 1<sup>st</sup> law of TD for const. P, non-flow process

$$\begin{aligned}
 Q &= \Delta U + \int_1^2 P dv \\
 &= (u_2 - u_1) + P(v_2 - v_1) \\
 &= (u_2 + Pv_2) - (u_1 + P.v_1) \\
 &= h_2 - h_1
 \end{aligned}$$

3) const Temp<sup>n</sup> or Isothermal expansion →



When wet steam is heated at const. temp<sup>n</sup> till it becomes dry saturated, then ~~the~~ heat transfer Q is given by

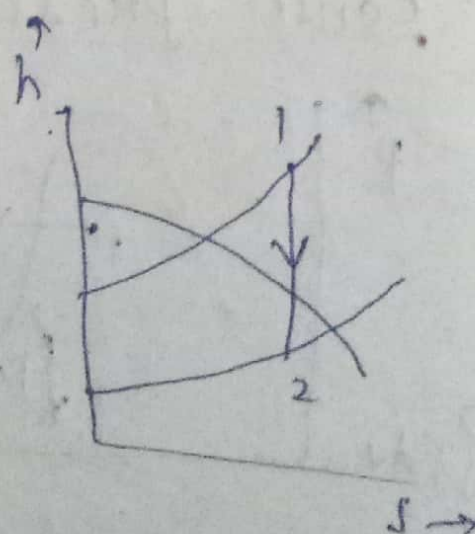
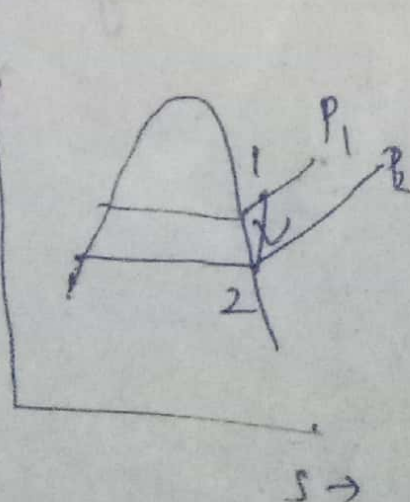
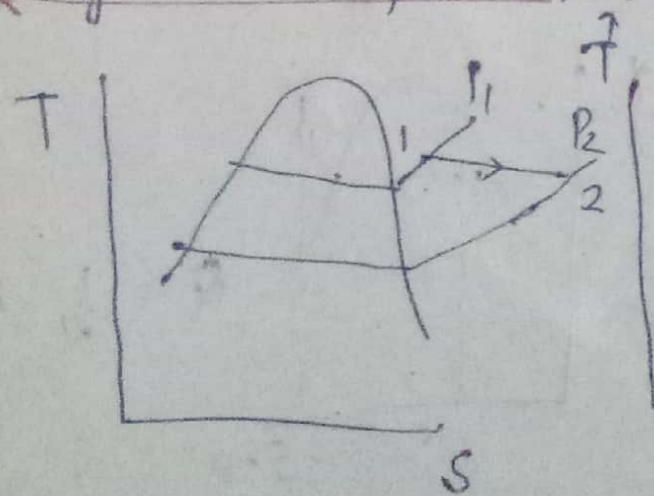
$$Q = h_2 - h_1$$

$$W = P_1 (v_2 - v_1)$$

↖ ↗ can be found from ~~data book~~ <sup>steam table</sup>

\* In wet region const P = const T process. In superheated region steam will behave like a gas & const T process in superheated region become hyperbolic ( $Pv = c$ ).

4) Hyperbolic process →



process  $Pv = c$ . It is also an isothermal



process: any thermodynamic process.

$$W = \int_1^2 P dv$$

$$Q = \Delta u + \int_1^2 P dv$$

$$= (h_2 - h_1) + P_1 v_1 \ln \left( \frac{v_2}{v_1} \right)$$

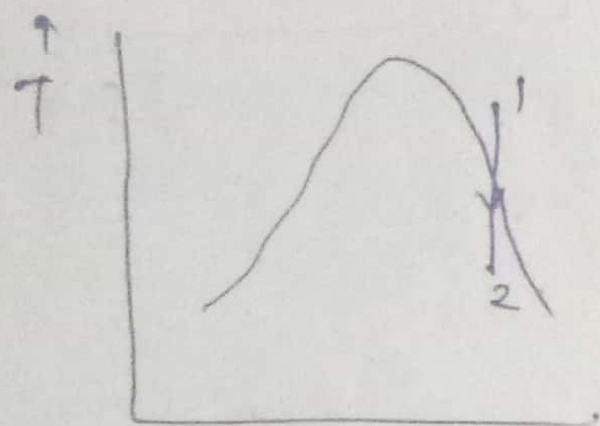
$$= (u_2 - u_1) + P_1 v_1 \ln \left( \frac{v_2}{v_1} \right)$$

$$\therefore P_2 v_2 = P_1 v_1$$

$$= (h_2 - P_2 v_2) - (h_1 - P_1 v_1) + P_1 v_1 \ln \frac{v_2}{v_1}$$

$$= (h_2 - h_1) + P_1 v_1 \ln \frac{v_2}{v_1}$$

5) Reversible adiabatic or Isentropic process



By applying 1<sup>st</sup> law,  $Q = \Delta u + \int_1^2 P dv$

$$\Rightarrow Q = u_2 - u_1 + W$$

If the process is steady flow reversible adiabatic, then as per 1<sup>st</sup> law applied to this process

$$u_1 + P_1 v_1 + Q - W = u_2 + P_2 v_2$$

$$h_1 + 0 - W = h_2$$

$$\Rightarrow \boxed{W = h_1 - h_2}$$

6) Polytropic process →

$$P v^n = C$$

Workdone,  $W = \frac{P_1 v_1 - P_2 v_2}{n-1}$  N-m/kg.

Applying 1<sup>st</sup> law,  $Q = \Delta u + W$

$$h, u + P v$$

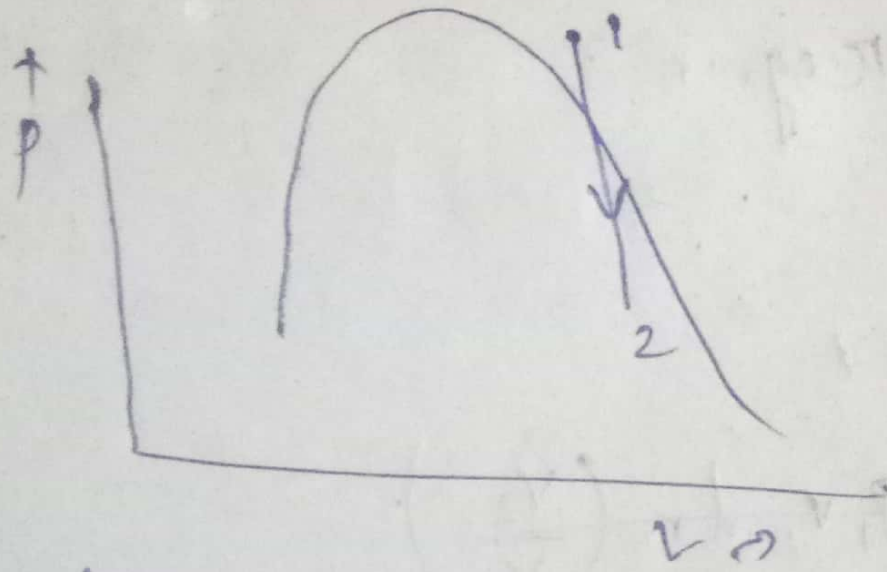
$$= u_2 - u_1 + \frac{P_1 v_1 - P_2 v_2}{n-1}$$

$$= (h_2 - P_2 v_2) - (h_1 - P_1 v_1) + "$$

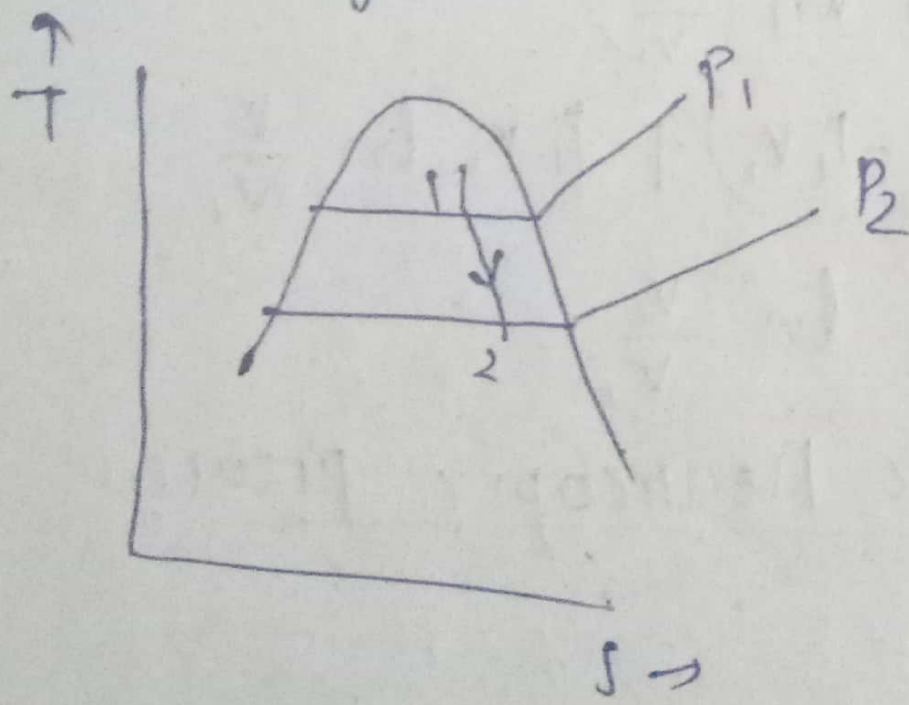
$$= (h_2 - h_1) + (P_1 v_1 - P_2 v_2) \left( 1 + \frac{1}{n-1} \right)$$

$$\left( 1 + \frac{1}{n-1} \right) = \frac{n}{n-1} (P_1 v_1 - P_2 v_2)$$

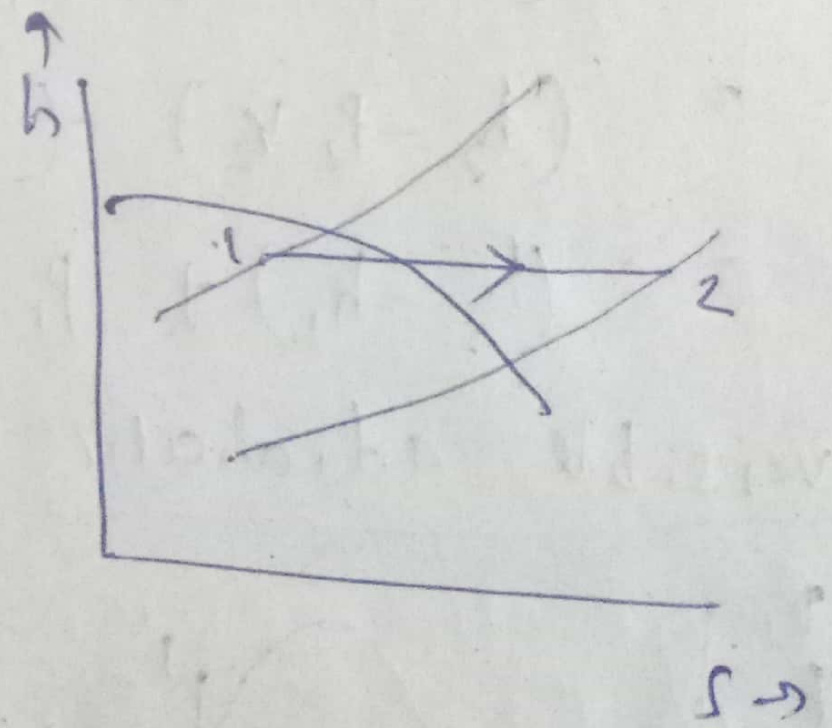




7) Throttling process →



$$h_1 = h_2$$



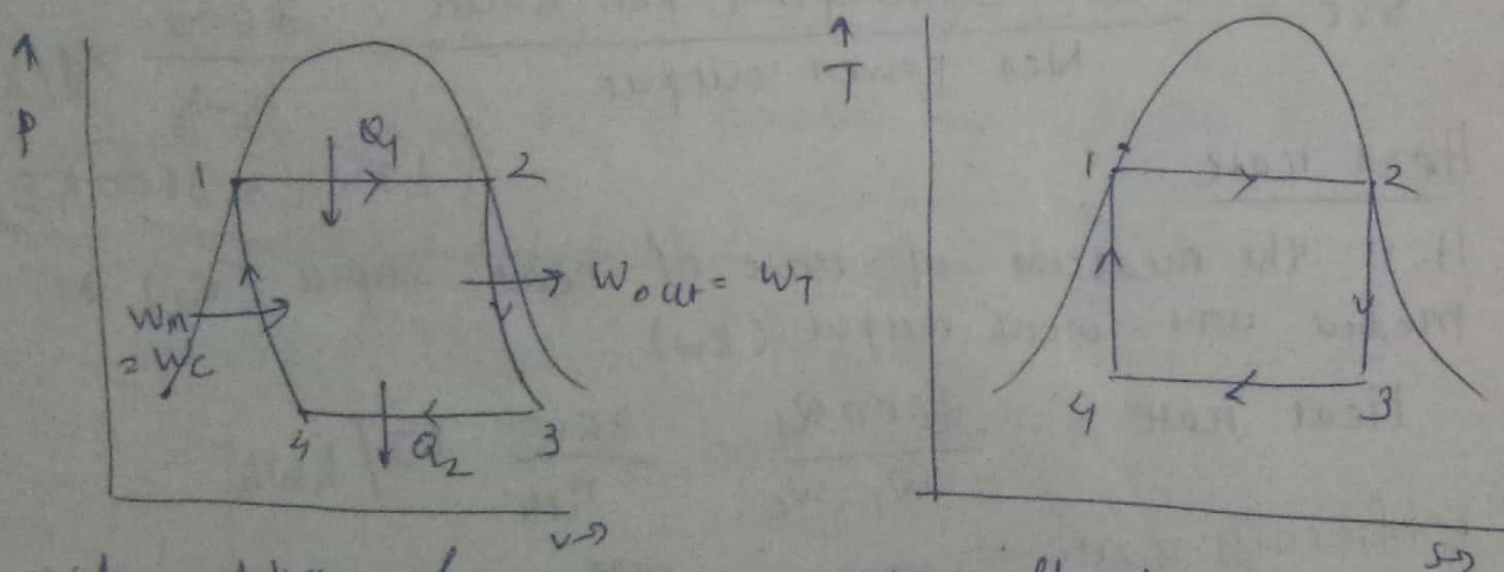
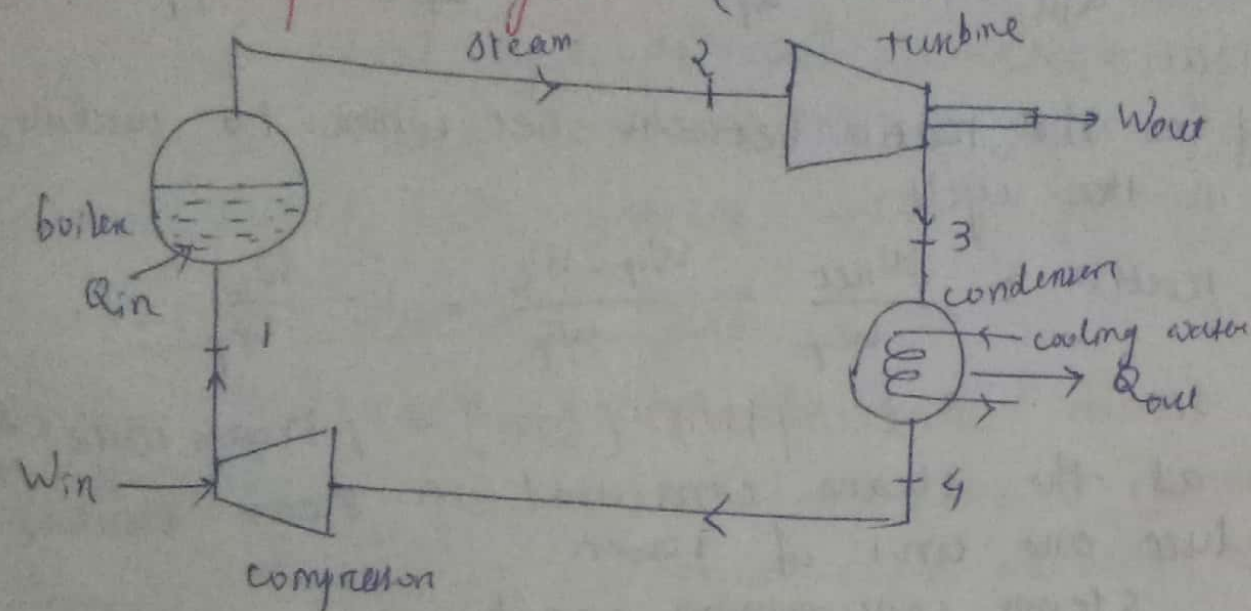


## 1.2 Vapour Power Cycle [steam power plant cycle]

Here working fluid is alternately vapourised & condensed. working fluid is essential to convert heat into work in any thermodynamic process. If steam is used as the working fluid then it is called vapour power cycle.

Steam, mercury, potassium etc can be used as working fluid but steam is widely used due to its low cost, easy availability & high enthalpy of vapourisation.

## 1.3 Carnot Vapour cycle → (with steam as working fluid)



Consider 2 kg of steam as working fluid

Process 1-2 (reversible isothermal heat addition)

Heat water is heated isothermally in a boiler at const. temp. & pressure to convert saturated steam by addition of heat in the boiler.

$$\text{Amount of heat added} = Q_1 = h_2 - h_1 \quad (\text{or}) \quad Q_1 = T_1 (\Delta S) = T_1 (S_2 - S_1)$$

Process 2-3 (reversible isentropic expansion)

Here dry saturated steam is expanded isentropically in a turbine & work is done by the system.

$$W_{out} = h_2 - h_3$$

Process 3-4 (reversible isothermal heat rejection)

Here steam is condensed reversibly & isothermally in a condenser at const. pressure.

$$\text{Heat rejected} = Q_2 = h_3 - h_4$$



process 4-1 (reversible, isentropic compression)

The vapour <sup>wet steam</sup> is compressed isentropically by a compressor to its initial state. Thus the cycle completes.

$$W_c = h_1 - h_4$$

Performance criteria of thermodynamic vapour cycle →

1) Thermal efficiency →

Defined as the ratio between net work done to net heat supplied

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{h_3 - h_4}{h_2 - h_1}$$

2) Work ratio →

Defined as the ratio between net work to turbine work done in the cycle

$$\text{Work ratio} = \frac{W_{net}}{W_T} = \frac{W_T - W_c}{W_T} = 1 - \frac{W_c}{W_T}$$

3) Specific steam consumption (SSC) → / Steam rate / Specific rate of flow of steam  
Defined as the steam consumed in steam turbine to produce one unit of power.

$$SSC = \frac{\text{Steam consumption per hour}}{\text{Net power output}} = \frac{3600}{h_2 - h_3} \text{ kg/kWh} = \frac{3600}{W_{net}}$$

Heat rate

$$(1 \text{ kWh} = 3600 \text{ kJ})$$

It is the measure of rate of heat input ( $Q_1$ ) to produce unit work output (kW)

$$\text{Heat rate} = \frac{3600 Q_1}{W_T - W_c} = \frac{3600}{\eta_{th}} \text{ kJ/kWh}$$

4) Efficiency ratio →

ratio of ~~th~~ thermal efficiency <sup>and</sup> to Rankine efficiency (ideal)

$$E.R. = \frac{\eta_{th}}{\eta_{Rankine}}$$

Limitations of Carnot cycle →

1) Theoretically a Carnot cycle is most efficient. However the following difficulties are associated with it when steam is used as working fluid.

i) It is difficult to compress wet vapour i.e. 2 phase mixture, isentropically i.e. process (4-1)

ii) It is difficult to control the quality of the condensate coming out of the condenser so that state 4 can be exactly obtained

iii) The vapour has large specific volume, so to accommodate this compressor size has to be quite big.



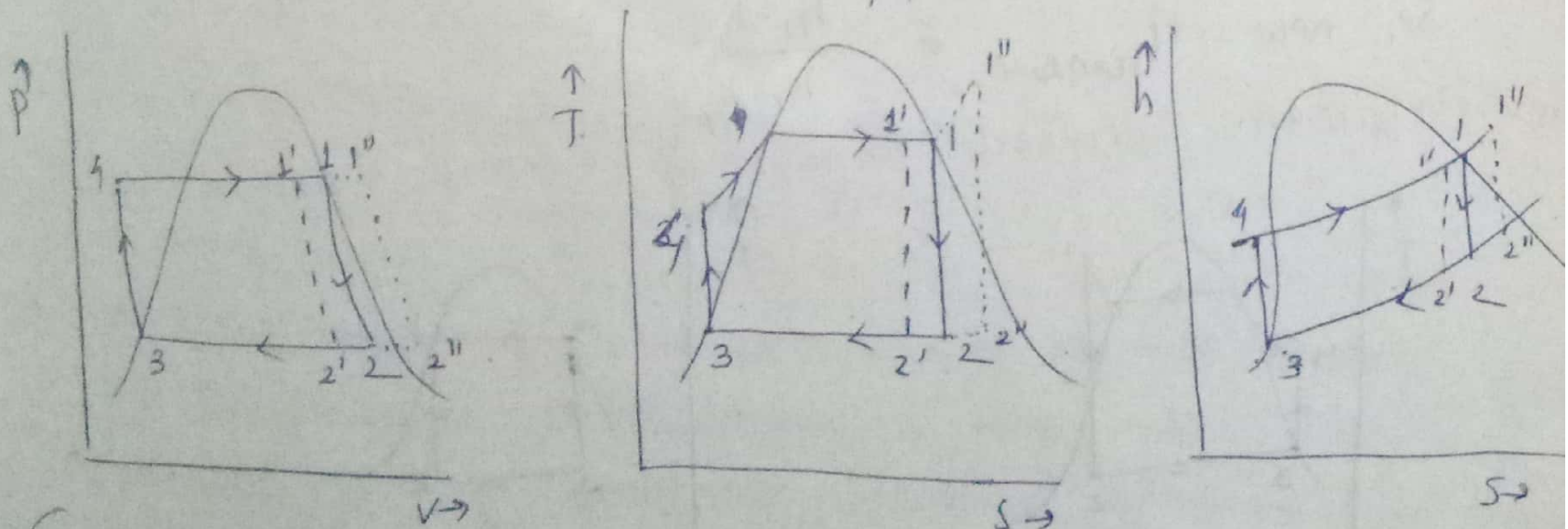
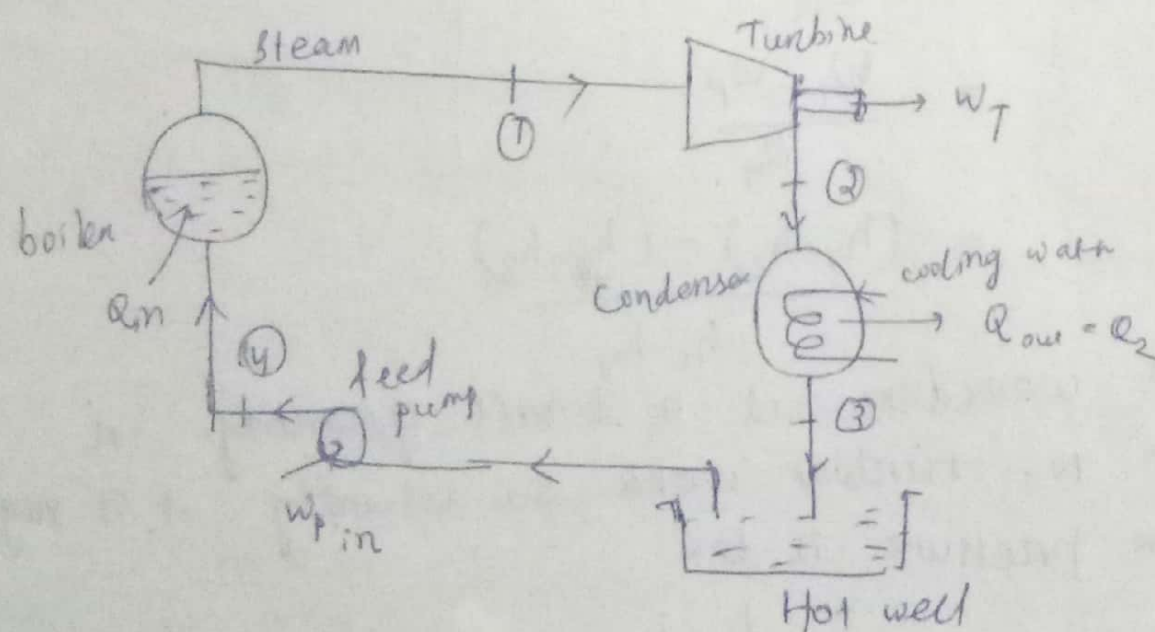
- iv) Carnot cycle has high SSC & low work ratio.  
at isentropic compression of vapour requires more work
- v)  $\eta$  of Carnot cycle is greatly affected by the upper limit temp  $T_1$ . As critical temp of steam is only  $374^\circ\text{C}$ , so cycle has to be operated within two phase region only which limit the max<sup>m</sup>  $\eta$ .
- vi) Difficult to operate <sup>with</sup> superheated steam as superheated steam has to be supplied at const temp instead of const  $P$ .

### 1.4 Rankine cycle $\rightarrow$

It is a modified form of Carnot cycle. It is the simplest or ideal cycle of all reversible vapour power cycles. Here condensation is allowed upto the saturated liquid line unlike Carnot cycle.

#### Elements of Rankine cycle

- 1) Boiler  $\rightarrow$  generates steam at const  $P$ .
- 2) Turbine  $\rightarrow$  steam expands isentropically & work is produced.
- 3) Condenser  $\rightarrow$  heat is rejected from exhaust steam at const  $P$  & converted to <sup>sat</sup> liquid water.
- 4) Pump  $\rightarrow$  Increases  $P$  of water upto boiler  $P$ .



- It consists of the following processes  $\rightarrow$
- 1-2  $\rightarrow$  Dry saturated steam at exit of boiler
  - 1'-2'  $\rightarrow$  wet steam at the exit of boiler
  - 1''-2''  $\rightarrow$  superheated "



Process 1-2 (reversible adiabatic expansion)  
 steam is expanded in the turbine from boiler  $P_1$  to condenser  $P_2$ . workdone is determined by enthalpy

$$\text{Turbine work} = W_T = h_1 - h_2 \quad \text{as } h_1 = W_T + h_2$$

Process 2-3 (reversible const  $P$  heat rejection)  
 Steam from turbine is condensed at const  $P$  &  $T$  to water.

$$Q_{rej} = Q_2 = h_2 - h_3$$

Process 3-4 (reversible adiabatic compression, / pump work)  
 pumping of condensate occurs isentropically from condenser  $P$  to boiler pressure.

$$W_p = h_4 - h_3 = v_f (P_1 - P_2) = v_3 (P_1 - P_2)$$

$$h_3 + W_p = h_4 \Rightarrow W_p = h_4 - h_3 = v_3 (P_1 - P_2)$$

Process 4-1 (const  $P$  heat addition)

heat is added in boiler at const  $P$

$$Q_m = Q_1 = h_1 - h_4 = h_1 - h_3$$

$$\begin{aligned} \text{Now, } \eta_{\text{rankine}} &= \frac{W_{\text{net}}}{Q_{\text{in}}} \\ &= \frac{W_T - W_p}{Q_1} \\ &= \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} \end{aligned}$$

$$dQ = dU + Pdv$$

$$h = u + Pv$$

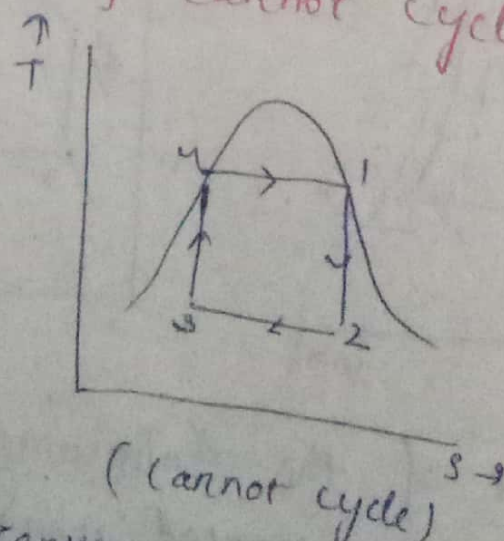
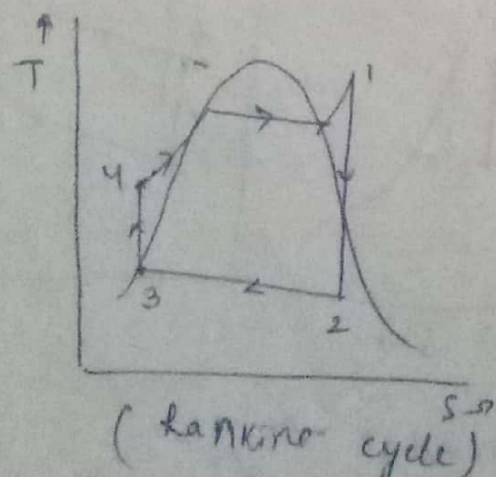
$$\Rightarrow dh = du + Pdv + vdp$$

=

\* The feed pump workdone is a small quantity in comparison to turbine work so usually it is neglected when boiler pressure is low.

$$\text{So, now } \eta_{\text{rankine}} = \frac{h_1 - h_2}{h_1 - h_3}$$

Comparison between Rankine & Carnot cycle  $\rightarrow$



i) Bet<sup>n</sup> same temp<sup>n</sup> limit Rankine cycle provides a higher specific work o/p than a Carnot cycle. Rankine cycle requires smaller SSC.



It requires higher grades of heat transfer in the boiler & condenser.

ii) Here only a part of heat is supplied isothermally at  $T_1$ ,  $\Rightarrow$  its  $\eta$  is lower than Carnot cycle & its  $\eta$  approaches Carnot if the degree of superheat is reduced.

iii) Advantages of using pump to feed liquid to the boiler instead of compressing a wet vapour is obvious because the work of compression is very large as compared to pump work.

Q A steam power plant is supplied with dry saturated steam at a  $P$  of 12 bar & exhausts into a condenser at 0.1 bar. Calculate the  $\eta_{\text{Rankine}}$ .

Sol<sup>n</sup> At  $P = 12 \text{ bar}$

$$h_f = 798.4 \text{ kJ/kg}, \quad h_{fg} = 1984.3$$

$$S_f =$$

$$S_{fg} =$$

$$S_g = 6.519 \text{ kJ/kg-K}$$

At  $P = 0.1 \text{ bar}$

$$h_f = 191.8 \text{ kJ/kg}, \quad h_{fg} = 2393$$

$$S_f = 0.649 \quad S_{fg} = 7.502$$

$$h_1 = h_g|_{12 \text{ bar}} =$$

$$S_1 = S_2$$

$$\Rightarrow 6.519 = S_{f2} + x_2 S_{fg2}$$

$$\Rightarrow 6.519 = 0.649 + x_2 \times 7.502$$

$$\Rightarrow x_2 = 0.783$$

$$h_2 = h_{f2} + x_2 h_{fg2}$$

$$= 191.8 + (0.783 \times 2393) = 2065.5 \text{ kJ/kg}$$

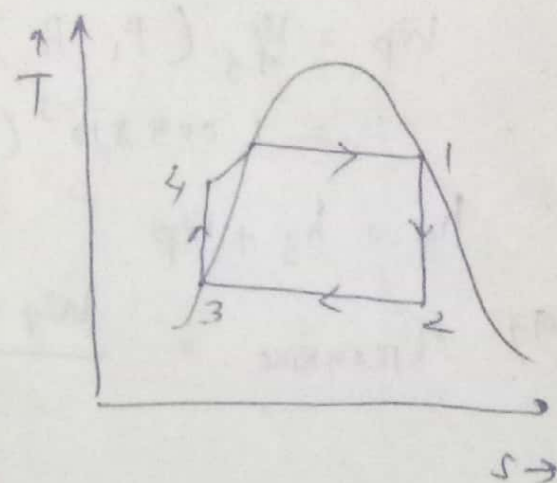
$$h_3 = h_f|_{0.1 \text{ bar}} = 191.8 \text{ kJ}$$

$$W_p = V_{f3} (P_1 - P_2) =$$

$$h_4 = h_3 + W_p$$

$$\eta_{\text{Rankine}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)} =$$

$$27.7\%$$



Q A steam power plant operates on Rankine cycle, has steam at a  $P$  of 40 bar & temp  $400^\circ\text{C}$  & is exhausted into a condenser, where  $P$  of 0.05 bar is maintained. Determine a)  $\eta_{\text{Rankine}}$

b) power developed

c) SSC

d) Heat rejected into the condenser

Take  $m$  of steam = 160 kg/s. e)  $\eta_{\text{Carnot}}$



Sol<sup>n</sup>

At 40 bar & 400°C (Superheated steam table)

$$h = 3215.7 \text{ kJ/kg}$$

$$s = 6.773 \text{ kJ/kg K}$$

At 0.05 bar (Sat<sup>n</sup> steam table)

$$h_f = 137.8$$

$$s_f = 0.476$$

$$h_{fg} = 2423.7$$

$$s_{fg} = 7.919$$

$$\text{So, } h_1 = 3215.7 \text{ kJ/kg}$$

$$\text{Again } s_1 = s_2$$

$$\Rightarrow 6.773 = 0.476 + (x_2 \times 7.919)$$

$$\Rightarrow x_2 = 0.795$$

$$h_2 = h_{f2} + x_2 h_{fg2} = 137.8 + (0.795 \times 2423.7) = 2064.64 \text{ kJ/kg}$$

$$v_3 = 1.005 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$h_3 = 137.8 \text{ kJ/kg}$$

$$W_p = v_{f3} (P_1 - P_2)$$

$$= 1.005 \times 10^{-3} (4000 - 5)$$

$$h_4 = h_3 + W_p$$

$$a) \eta_{\text{Rankine}} = \frac{W_T - W_p}{Q_1} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} = 37\%$$

$$b) \text{ Power developed} = m_s \times W_T = 160 (h_1 - h_2) = 184.256 \text{ MW}$$

$$c) \text{ SSC} = \frac{3600}{h_1 - h_2} = 3.125 \text{ kg/kWh}$$

$$d) Q_2 = m_s (h_2 - h_3) = 308294.4 \text{ kJ/s}$$

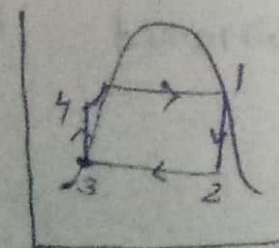
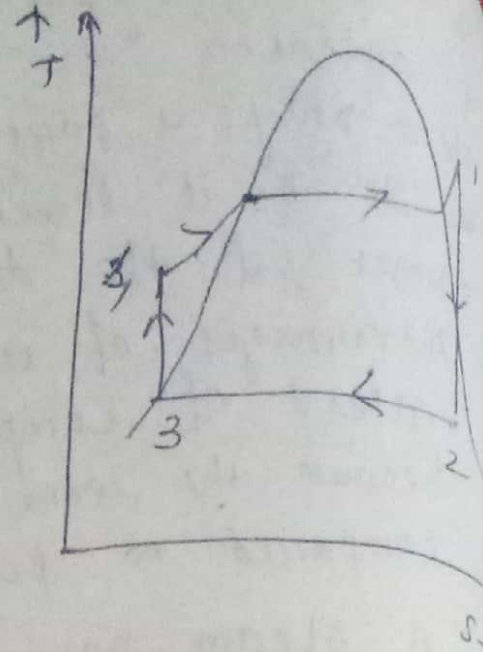
$$e) \eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1} = 1 - \frac{273 + 32.9}{273 + 400} = 54.5\%$$

Ex In a steam power cycle, the steam is supplied at 15 bar & dry saturated. The condenser p is 0.4 bar. Calculate Carnot & Rankine  $\eta$  of the cycle neglecting pump work

$$\text{Sol}^n \quad \eta_{\text{Rankine}} = \frac{h_1 - h_2}{h_1 - h_3}$$

$$\eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1}$$

Steam at 15 bar (sat) enters a steam turbine & comes out to the condenser at 0.4 bar. Calculate  $\eta_m$  of the cycle



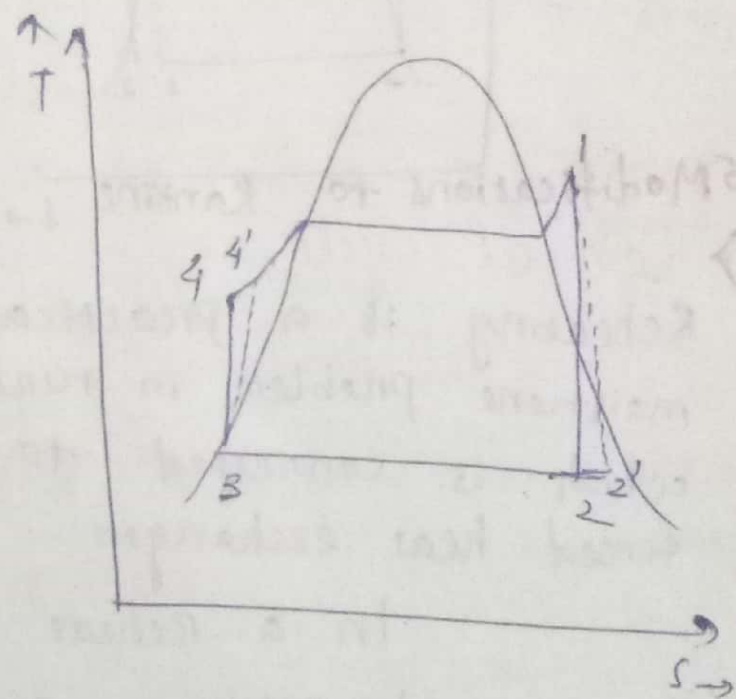
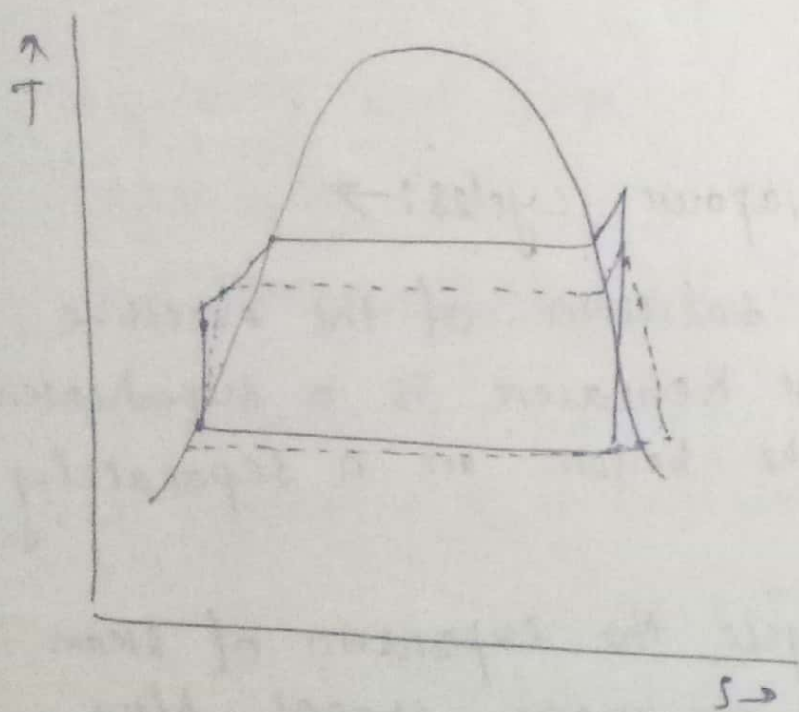


## Actual Vapour Power Cycle: →

- Actual vapour power cycle differs from ideal Rankine cycle due to irreversibilities in various components due to fluid friction & heat loss to the surrounding
- Fluid friction causes pressure drops in boiler, condenser & piping bet<sup>n</sup> various components. So steam leaves at lower  $p$  in boiler &  $p$  at turbine entrance is also lower (because of  $p$  drop).  $p$  drop in condenser is very small.

To compensate for these  $p$  drops, the water must be pumped to a sufficiently higher  $p$  than the ideal cycle. So requires a large pump with high i/p power.

- To compensate the heat losses, more heat requires to be transferred to the steam in the boiler.
- So due to irreversibility pump requires greater work & turbine produces less work.



$$\eta_p = \frac{h_4 - h_3}{h_{4'} - h_3} = \frac{\text{Isentropic work i/p}}{\text{Actual "}}$$

$$\eta_T = \frac{h_1 - h_{2'}}{h_1 - h_2} = \frac{\text{Actual turbine work}}{\text{Isentropic "}}$$

## Improving the efficiency of Rankine cycle →

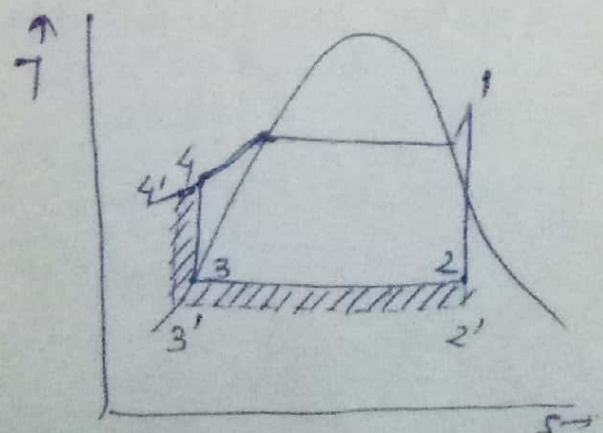
- 1) By lowering the condenser pressure →

hatched area → ↑ of work o/p.

So,  $W_T \uparrow$  ( $1 \rightarrow 2'$ )

&  $Q_1$  also increase ( $4' \rightarrow 1$ ) instead of as  $p \downarrow$  so sat<sup>n</sup> temp ↓.

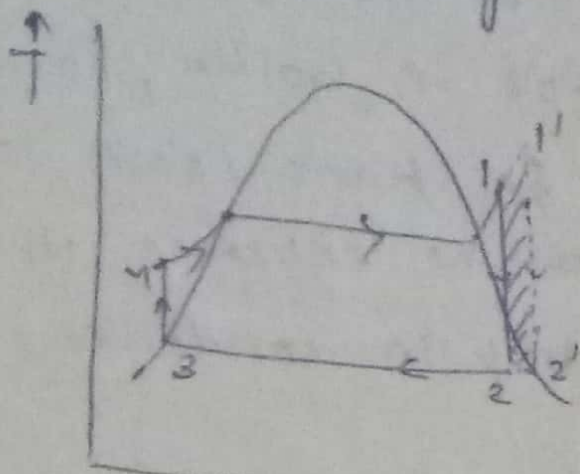
- $p$  is lowered not below the sat<sup>n</sup>  $p$  corresponds to temp. of cooling medium.





2) Superheating the steam to high temp →

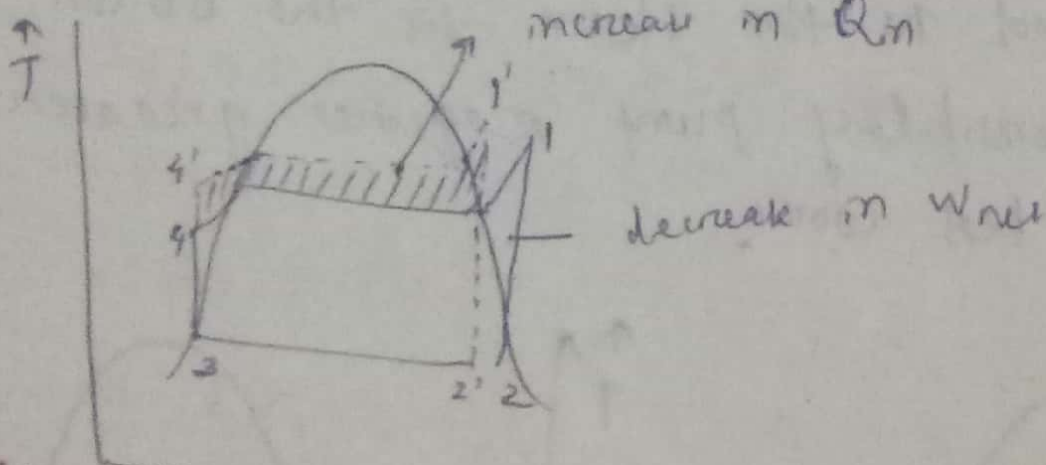
The avg. temp at which heat is transferred to the steam can be increased by superheating the steam to higher temp without increasing the boiler pressure.



work output increases & shown by hatched area  
 ⇒  $\eta \uparrow$

3) By Increasing boiler pressure →

By increasing boiler pressure, it automatically increases the operating temp at which water will boil. So  $\uparrow$  avg. temp at which heat is transferred to the steam & thus  $\eta_{th}$  increases.



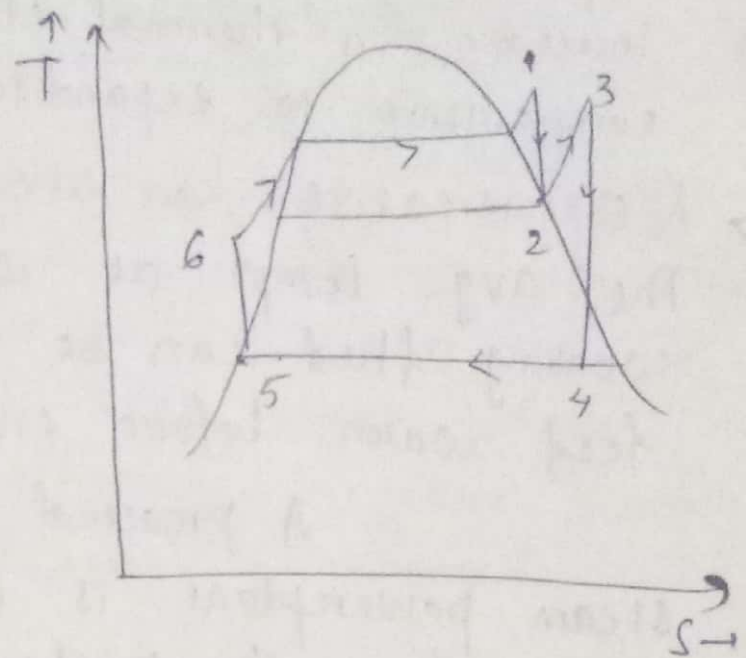
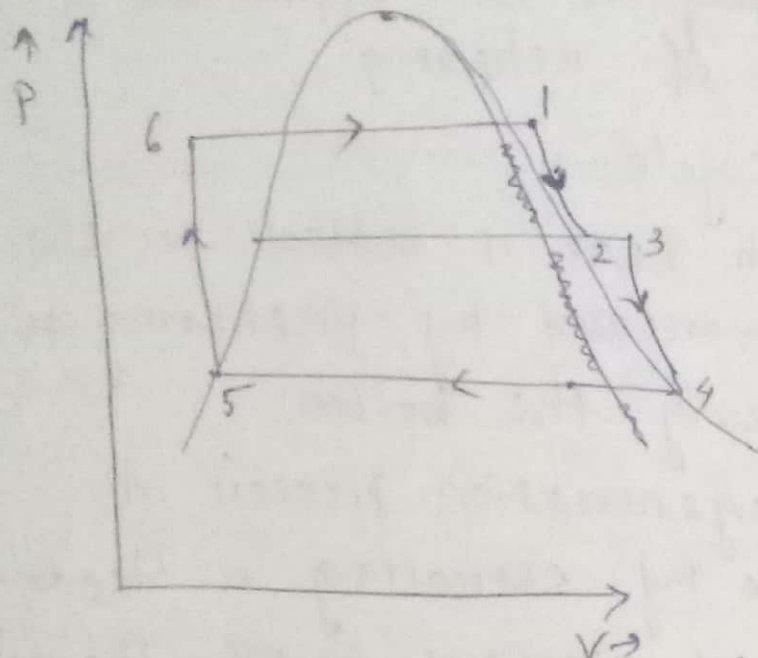
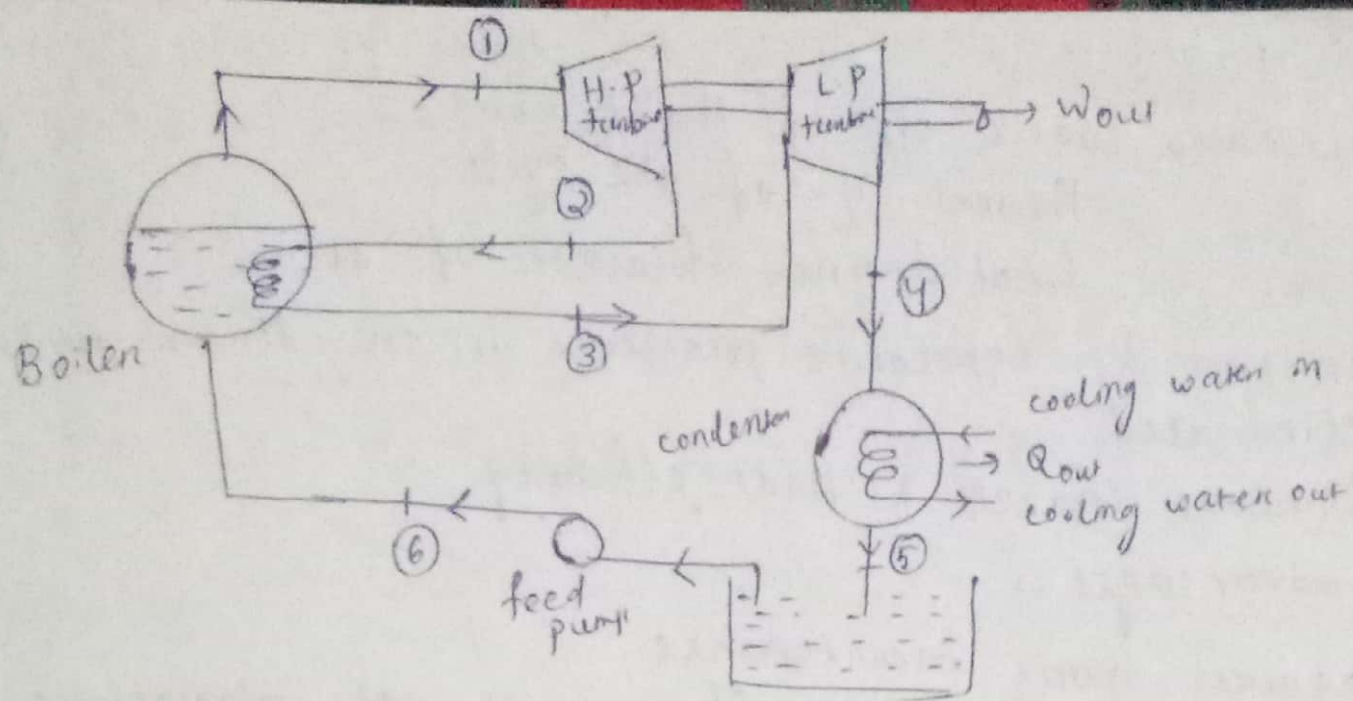
1.5 Modifications to Rankine vapour cycles →

1) Reheat cycle →

Reheating is a practical solution of the excessive moisture problem in turbine. Reheater is a superheater which is connected to the boiler or a separately fired heat exchanger.

In a reheat cycle, the expansion of steam is carried out in two or more stages. After partial expansion to an intermediate pressure in the high pressure turbine, the steam is withdrawn & reheated to the original temp at const. pressure. Steam is then expanded in the low pressure turbine.





Considering unit mass of steam

Turbine work o/p = work done in H.P. turbine + w.D. in low P turbine

$$\Rightarrow W_T = (h_1 - h_2) + (h_3 - h_4)$$

$$\text{Pump work} = W_P = h_6 - h_5$$

$$\text{Heat supplied} = Q_1 = \text{heat supplied in boiler} + \text{heat supplied in superheater} \\ = (h_1 - h_6) + (h_3 - h_2)$$

$$\text{Rankine reheat cycle efficiency} = \eta = \frac{W_{\text{net}}}{Q_1} = \frac{W_T - W_P}{Q_1}$$

$$\Rightarrow \eta = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

\* Avg temp<sup>n</sup> during the reheat process can be increased by increasing the no. of expansion & reheat stages. But more than 2 stages are not practical.

Reheat cycle decreases the moisture content at low P turbine & thus erosion & corrosion in the turbine can be reduced. It also ↑ the o/p of turbine & thus ↑ overall  $\eta$  of the plant.



### Advantages →

- i) Increases work o/p of the turbine
- ii) " thermal  $\eta$  of the cycle
- iii) " final dryness fraction of steam
- iv) erosion & corrosion problems in the steam turbine are eliminated.
- v) Increases nozzle & blade efficiency.

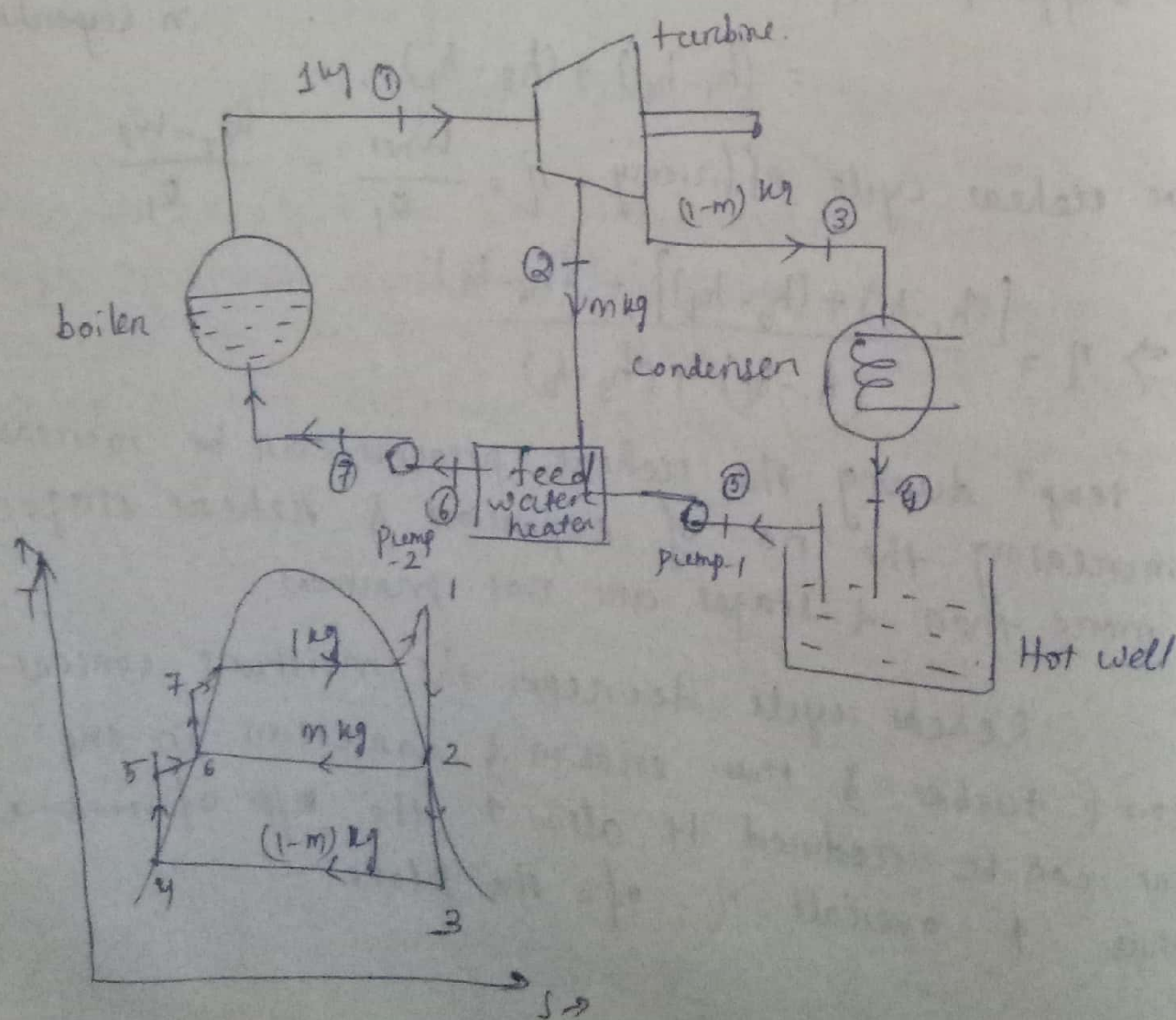
### Disadvantages →

- i) Requires more maintenance
- ii) Increase in thermal efficiency is not appreciable in comparison to expenditure of reheating.

### 2) Regenerative Rankine cycle →

The avg. temp<sup>n</sup> at which heat is added to the working fluid can be increased by preheating the feed water before entering the boiler.

A practical regeneration process in steam powerplant is done by extracting a bleeding steam from the turbine at various points. The rest of the steam expands to the condenser pressure in the turbine. The steam thus extracted is mixed with feed water coming from the condenser. The device in which the feed water is heated by regeneration is called a regenerator or feed water heater.





For unit mass of steam

$$W_T = 1(h_1 - h_2) + (1-m)(h_2 - h_3)$$

pump work  $W_P = W_{P_1} + W_{P_2}$

$$= (1-m)(h_5 - h_4) + 1(h_7 - h_6)$$

$$= (1-m)v_{f_4}(P_5 - P_4) + 1v_{f_6}(P_7 - P_6)$$

$$\eta_{\text{regenerative}} = \frac{W_{\text{net}}}{Q_1} = \frac{W_T - W_P}{Q_1}$$
$$= \frac{(h_1 - h_2) + (1-m)(h_2 - h_3) - (\dots)}{h_1 - h_7}$$

\* Thermal efficiency of the cycle increases as a result of regeneration. This is due to increase in avg. temp<sup>n</sup> at which heat is transferred to the steam in the boiler by raising the temp<sup>n</sup> of feed water before it enters the boiler. The cycle efficiency increases further as the no. of feed water heaters is increased.

Advantages : →

- i) ↑  $\eta_{th}$  as avg. temp of heat addition in boiler ↑.
- ii) Thermal stresses in the boiler are minimised as the temp<sup>n</sup> ranges in the boiler are reduced.
- iii) Heating process in the boiler tends to become reversible.
- iv) Size of the condenser is reduced.
- v) Blade height of the turbine reduces due to reduced amount of steam passing through the low pressure stages.

Disadvantages : →

- i) Greater maintenance is required for the addition of more heaters.
- ii) Plant becomes complicated.
- iii) For a given power a large capacity boiler is required.

\* For one heater case

Heat supplied in the boiler,  $Q_1 = h_1 - h_7$

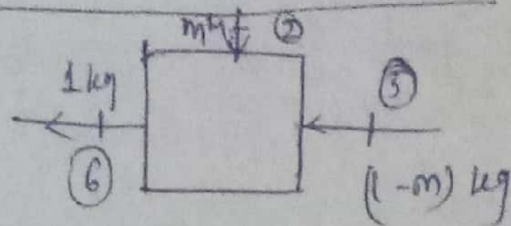
Heat rejected in the condenser,  $Q_2 = (h_3 - h_4)(1-m)$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

- The  $\eta$  of ideal regeneration rankine cycle is equal to the carnot cycle. As it is practically infeasible so actual regeneration cycle is used.



## Energy balance across heaters



Heat & mass balance across heater

For steam entry = For condensate entry

⇒ Heat gained lost by bleed steam = Heat gained by cond.

$$\Rightarrow m(h_2 - h_6) = (1-m)(h_6 - h_5)$$

$$\Rightarrow \frac{1-m}{m} = \frac{h_2 - h_6}{h_6 - h_5}$$

$$\Rightarrow \frac{1}{m} - 1 = \frac{h_2 - h_6}{h_6 - h_5}$$

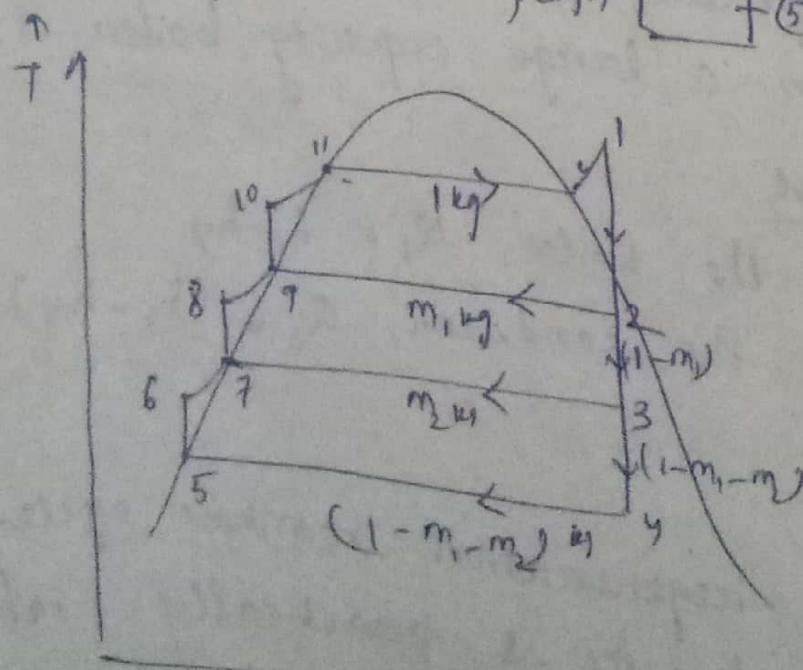
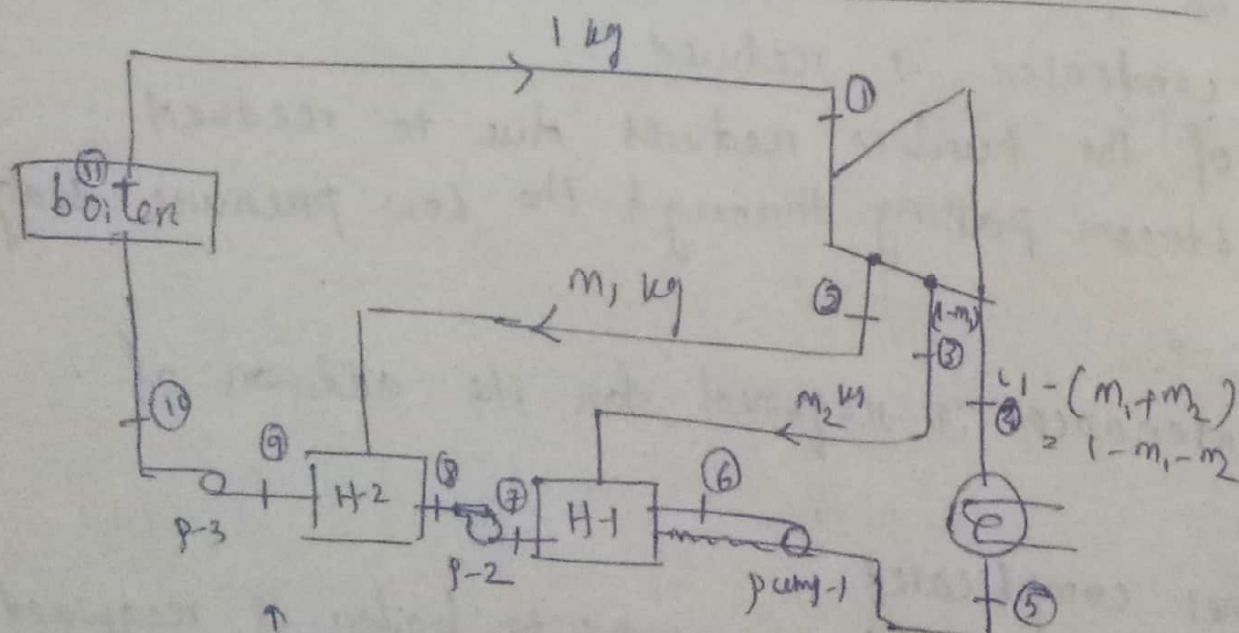
$$\Rightarrow \frac{1}{m} = \frac{h_2 - h_5}{h_6 - h_5}$$

$$\Rightarrow \boxed{m = \frac{h_6 - h_5}{h_2 - h_5}}$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{(h_3 - h_4)(1-m)}{(h_1 - h_2)}$$

$$= 1 - \frac{(h_3 - h_4)(h_2 - h_5)}{(h_1 - h_2)(h_2 - h_5)}$$

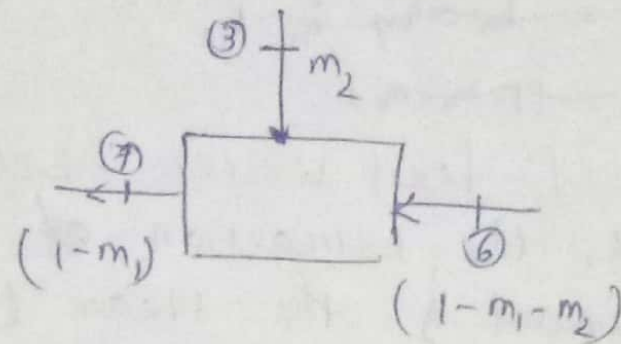
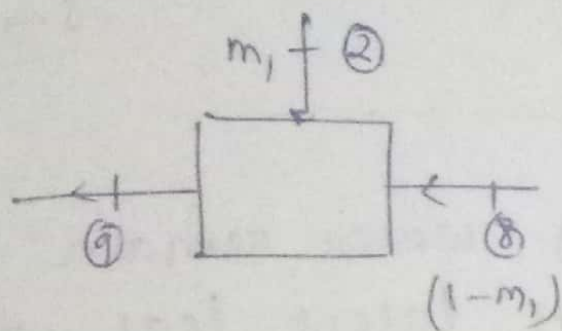
## Actual regeneration with two heaters





- i) Heat rejected in Condenser  $= Q_2 = (1 - m_1 - m_2)(h_4 - h_5)$   
 ii) Heat added in boiler  $= Q_1 = h_1 - h_{10}$   
 iii) Work done by turbine,  $W_T = (h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4)$

$m_1$  &  $m_2$  can be found by



iv) Heat balance for heater across (8)-(9)

$$m_1(h_2 - h_9) = (1 - m_1)(h_9 - h_8)$$

$$\Rightarrow \boxed{m_1 = ?}$$

v) Heat balance for heater across (6)-(7)

$$m_2(h_3 - h_7) = (1 - m_1 - m_2)(h_7 - h_6)$$

$$\Rightarrow \boxed{m_2 = ?}$$

vi) work done by pumps

$$W_{P1} = h_{10} - h_9$$

$$W_{P2} = (h_8 - h_7)(1 - m_1)$$

$$W_{P3} = (1 - m_1 - m_2)(h_6 - h_5)$$

Types of heaters: →

- 1) Open or direct type feed heaters
- 2) closed or indirect " "

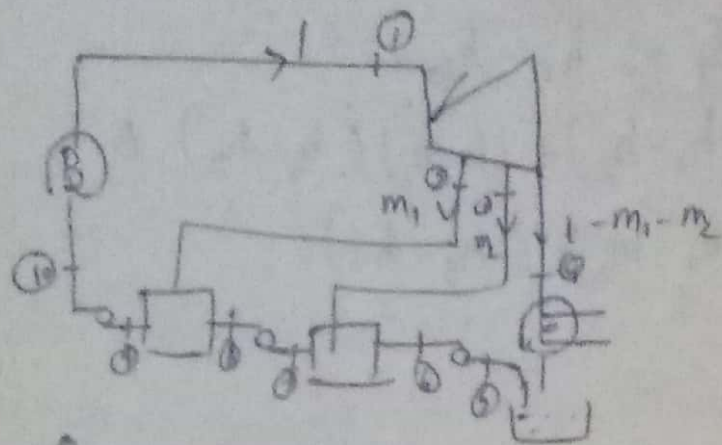
1) Open Feed Water heaters →

→ In an open heater, feed water & extraction steam are mixed together & the temp<sup>n</sup> of feed water reaches the saturation temp<sup>n</sup> corresponding to the extraction steam pressure.

- The latent heat of steam is used to increase the temp<sup>n</sup> of feed water
- Hence the no. of pumps required is one more than the no. of heaters. So power consumption in the pumps is higher
- They are simple & inexpensive & have good heat

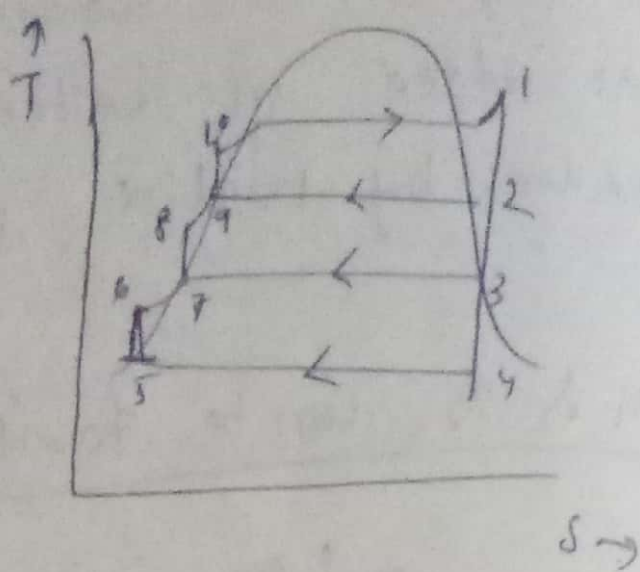


transfer characteristics

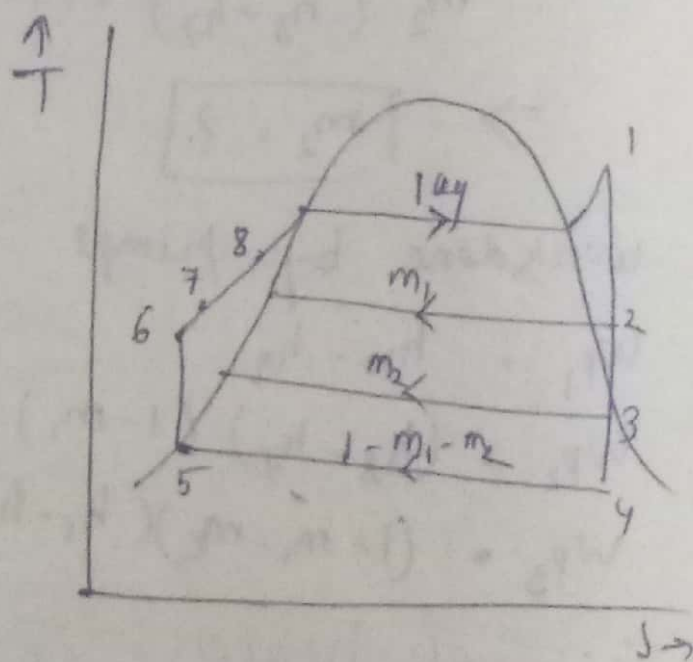
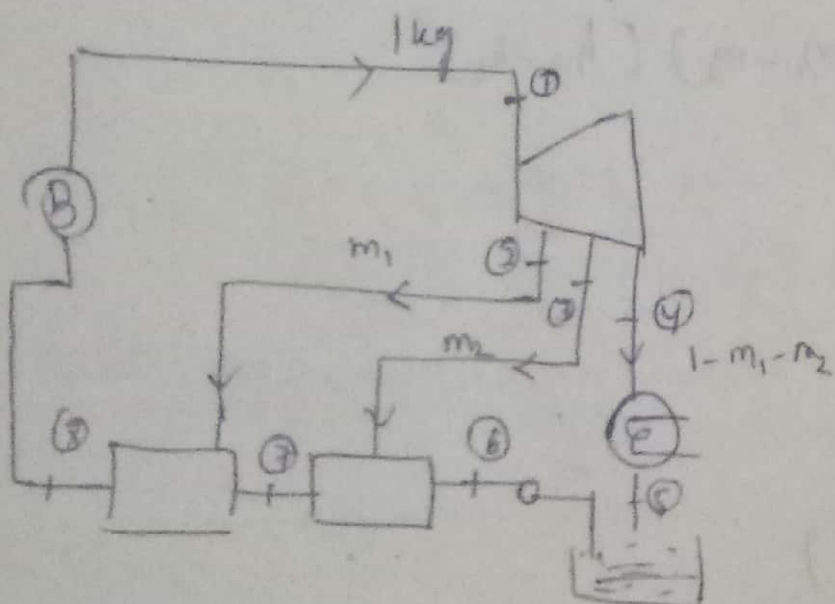


$$h_1 = h_2 = h_3 = h_4 = h_5 = h_6$$

$$h_1 = h_2 = h_3 = h_4 = h_5 = h_6$$



- closed feed water heater →
- Here, the extraction of steam & water remain separated & the steam loses its latent heat which is used for increasing the temp<sup>n</sup> of feed water.
- The condensate of extraction steam is called drip.
- Here only one pump is sufficient for sending the feed water to boiler. So power consumption in pump is less than open feed water system.



\* For Indirect type heater, effectiveness of feed heater is always less than 100% & where as for direct heater it is 100%.

Ex Dry sat steam at 150 bar enters a steam turbine & comes out at 1 bar. Calculate cycle efficiency.

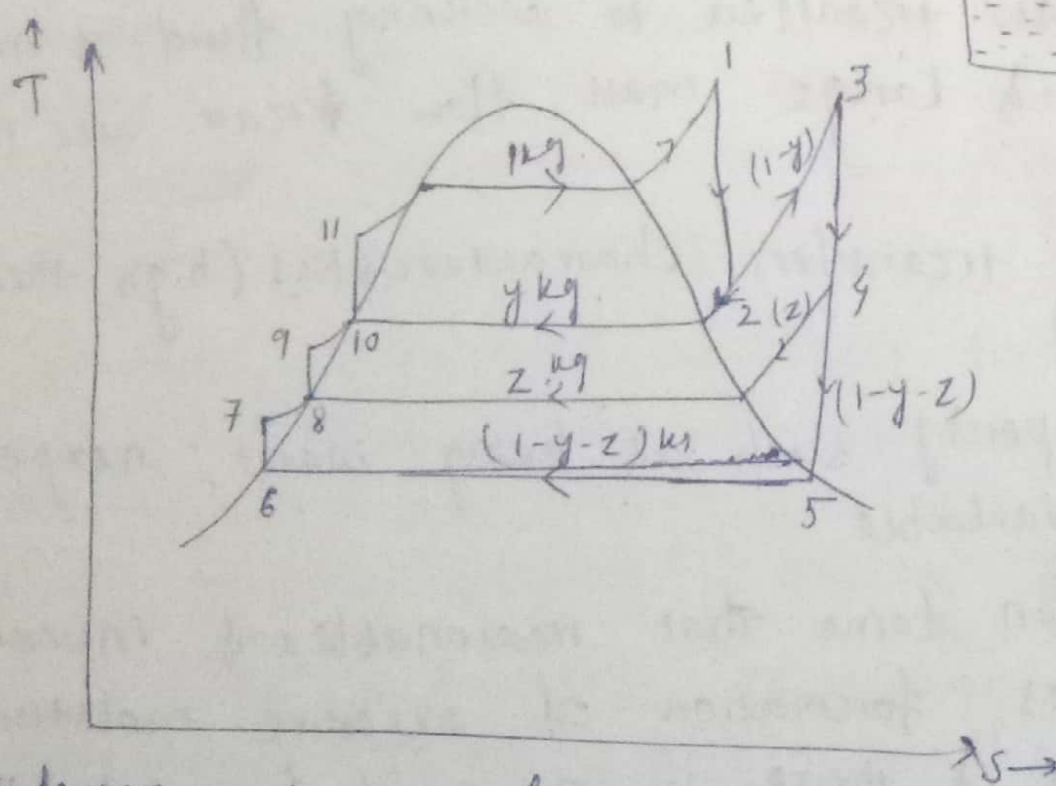
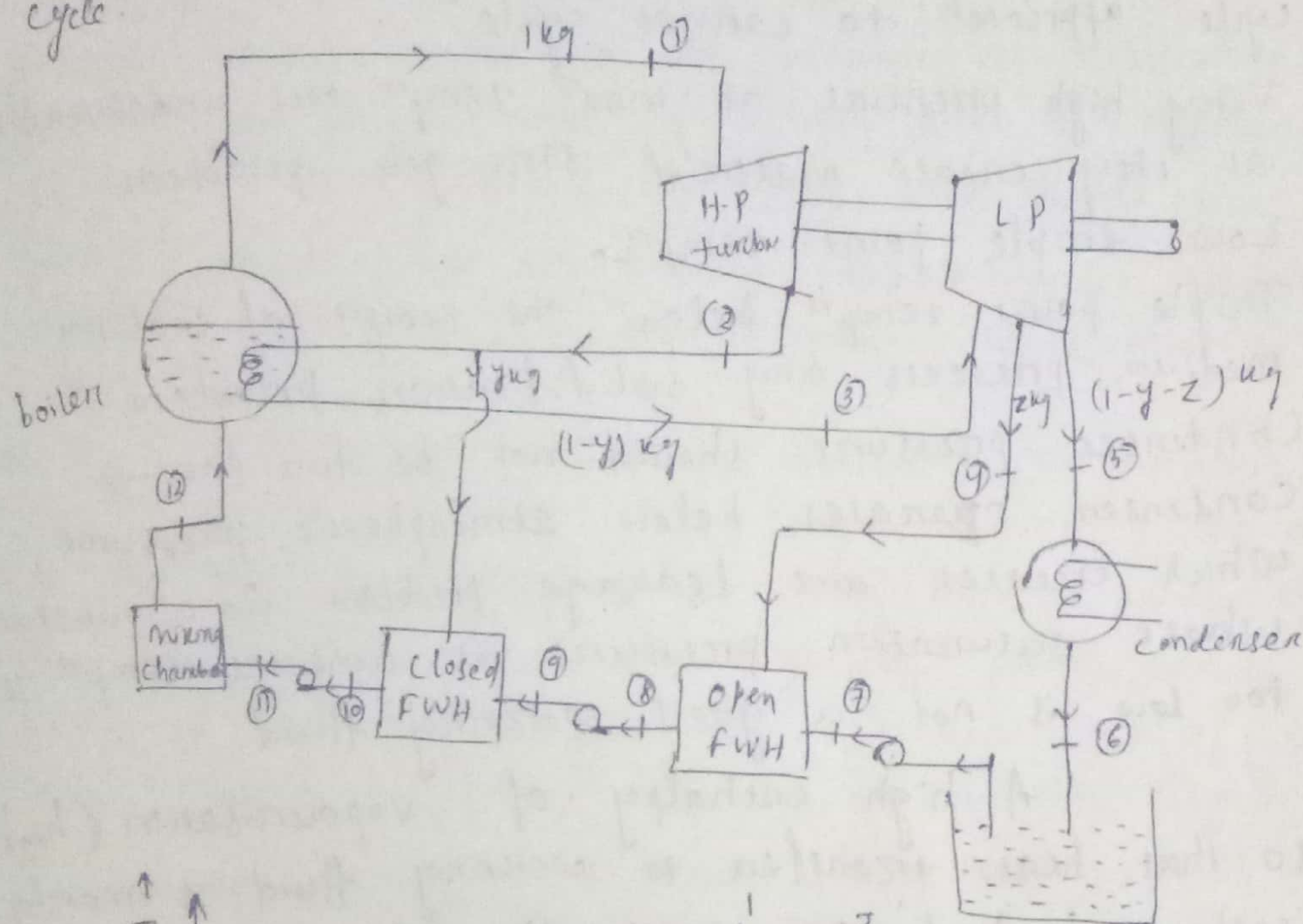
Ex If reheating is done with pressure ratio (0.25) & HP turbine inlet parameters (150 bar, 540°C). Calculate cycle  $\eta$  if reheating is done upto 540°C.

$$\frac{P_2}{P_1} = 0.25$$



### 3) Reheat-Regenerative Rankine cycle: →

- Reheating is done with high  $T$  &  $p$  but gain in thermal  $\eta$  by reheating is rather small
- But regeneration or heating up of feed water by steam extracted from turbine rewards a considerable gain in  $\eta_{th}$
- So modern power plant operates on reheat-regenerative cycle



considering unit mass of steam,

$$Q_{add} = Q_1 = 1(h_1 - h_{11}) + (1-y)(h_3 - h_2)$$

$$Q_{out} = Q_2 = (1-y-z)(h_5 - h_6)$$

$$W_T = W_{T1} + W_{T2}$$

$$= 1(h_1 - h_2) + (1-y)(h_3 - h_4) + (1-y-z)(h_4 - h_5)$$

$$W_P = W_{P1} + W_{P2} + W_{P3}$$

$$= (1-y-z)(h_7 - h_6) + (1-y)(h_9 - h_8) + 1(h_{11} - h_{10})$$



# HEAT TRANSFER

## Chapter 6

Heat transfer can be defined as the transmission of energy from one region to another as a result of temp<sup>n</sup> gradient.

### Modes of heat transfer →

#### 1) conduction →

It is the transfer of heat from one part of a substance to another part of the same substance or from one substance to another in physical contact with it, without appreciable displacement of molecules forming the substance.

In solid, heat is conducted by two mechanisms

- i) By Lattice vibration (faster moving molecules or atoms in the hottest part of a body transfer heat by impacts some of their energy to adjacent molecules).
- ii) By transfer of free electrons (

In gas & liquid, K.E of a molecule is a function of temp<sup>n</sup>. It loses energy by collision.

#### 2) convection →

It is the transfer of heat within a fluid by mixing of one portion of the fluid with another.

It is possible only in fluid medium & is directly linked with the transport of medium itself.

~~It is~~

→ The heat flow depends on the properties of fluid & is independent of the properties of the material of the surface.

→ It is of 2 types

i) Free or natural convection - It occurs when the fluid circulates by virtue of the natural density difference of hot & cold fluids.

The denser portion of fluid moves downward due to greater force of gravity as compared to less dense fluid.

ii) Forced convection - When work is done to blow or pump the fluid.



### 3) Radiation →

It is the transfer of heat through space or matter by means other than conduction & ~~convection~~ <sup>convection</sup>. It occurs due to emission of electromagnetic waves. It requires no medium & occurs mainly in infrared region.

### 4.2 Imp Heat transfer by conduction →

#### 5 Fourier's Law of heat conduction →

It states that "The rate of flow of heat through a simple homogeneous solid is directly proportional to the area of the ~~sur~~ section at right angles to the dir<sup>n</sup> of heat flow, & to temp<sup>n</sup> gradient in the direction of heat flow".

temp<sup>n</sup> gradient → change of temp<sup>n</sup> w.r.t length of the path of heat flow.

mathematically,  $Q \propto A$   
 $Q \propto \frac{dT}{dx}$

$$\text{So, } Q \propto A \frac{dT}{dx}$$

$$\Rightarrow Q = -KA \frac{dT}{dx}$$

where  $Q$  = Rate of heat transfer or amount of heat flowing through a body in unit time.

$A$  = Surface area normal to dir<sup>n</sup> of heat flow.

$dT$  = Temp<sup>n</sup> difference on the two faces of body

$dx$  = Thickness of the body through which heat flows. It is taken along the dir<sup>n</sup> of heat flow.

$K$  = proportionality constant called as thermal conductivity of the body.

∴ sign is used to consider decreasing temp<sup>n</sup> along with the dir<sup>n</sup> of heat flow

$\frac{dT}{dx}$  is always -ve along positive  $x$  dir<sup>n</sup>, so  $Q$  will be +ve.

#### Assumption

1. conduction of heat takes place under steady state cond<sup>n</sup>
2. The heat flow is unidirectional
3. Temp<sup>n</sup> gradient is const & temp<sup>n</sup> profile is linear.



4. There is no internal heat generation  
The material is homogeneous & isotropic.

### Thermal conductivity $\rightarrow$

It is defined as the ability of a substance to conduct heat.

We know  $Q = K A \frac{dT}{dx}$       Unit of  $K = \frac{J}{m^2 K} = J/mK$

$$\Rightarrow K = \frac{Q}{A} \frac{dx}{dT} \quad \text{or} \quad \frac{W}{m \cdot K}$$

When  $Q=1$ ,  $A=1$  &  $\frac{dx}{dT}=1$ , then  $K=1$

$\rightarrow$  So, Thermal conductivity of a material is defined as "The amount of energy conducted through a body of unit area, & unit thickness in unit time when the difference in temp<sup>n</sup> between the faces causing heat flow is unit".

$\rightarrow$  materials with high thermal conductivity are good conductors & with low  $K$  are insulators.

$\rightarrow$  It depends on

- i) material structure
- ii) moisture content
- iii) Density of the material
- iv)  $P$  &  $T$

$\Rightarrow$  Calculate the rate of heat transfer per unit area through a copper plate 45 mm thick, whose one face is maintained at 350°C & the other face at 50°C. Take  $K$  for Cu = 370 W/m°C

Sol<sup>n</sup>  $dT = T_2 - T_1 = 50 - 350 =$

$L = 45 \text{ mm} = 0.045 \text{ m}$

$K_{Cu} = 370 \text{ W/m}^\circ\text{C}$

$$q = \frac{Q}{A}$$

We know  $Q = -KA \frac{dT}{dx}$

$$\begin{aligned} \Rightarrow q &= -370 \times \frac{(50 - 350)}{0.045} \\ &= 2.466 \times 10^6 \text{ W/m}^2 \end{aligned}$$



### Heat flux $\rightarrow$

It is the quantity of heat transfer per unit time per unit area of isothermal substance.

$$q = \frac{Q}{A}$$

The greatest rate of heat flow will be along the line normal to isothermal surface.

$$Q_x = -K_x A \frac{dT}{dx}$$

$$Q_y = -K_y A \frac{dT}{dy}$$

$$Q_z = -K_z A \frac{dT}{dz}$$

A material having  $K_x = K_y = K_z = K$  is called an isotropic material.

### Characteristics of thermal conductivity

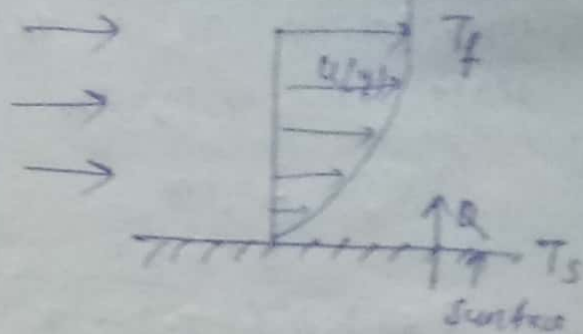
- $\rightarrow$   $K$  of a material is due to flow of free electrons.
- $\rightarrow$   $K$  for pure metal is highest & decreases with impurity.
- $\rightarrow$   $K$  for most metals  $\downarrow$  with  $\uparrow$  in temp<sup>n</sup> (Al & V are exception). In gas  $K \uparrow$  with  $T \uparrow$ .
- $\rightarrow$  In solids & liquids,  $K$  weakly depends on  $P$ , but in gases  $K$  is independent of  $P$ .



## Convective heat transfer →

For a fluid flowing at a mean temp<sup>n</sup> ( $T_f$ ) over a surface at a temp<sup>n</sup>  $T_s$ , Newton proposed the following heat conduction eq<sup>n</sup> called as Newton's law of cooling.

fluid flow



It states that "Heat transfer from a hot body to a cold body is directly proportional to surface area & difference of temp<sup>n</sup> bet<sup>n</sup> the bodies".

$$Q \propto A$$

$$Q \propto T_s - T_f$$

$$\Rightarrow Q = hA(T_s - T_f)$$

Where  $Q$  = Rate of conductive heat transfer

$A$  = Area exposed to "

$T_s$  = Surface temp<sup>n</sup>

$T_f$  = Fluid temp<sup>n</sup>

$h$  = Co-efficient of convective heat transfer

Unit of  $h$

$$h = \frac{Q}{A(T_s - T_f)} = \frac{W}{m^2 K} \text{ or } \frac{W}{m^2 ^\circ C}$$

\* Coeff  $h$  can be defined as "the amount of heat transmitted for a unit temp<sup>n</sup> difference between the fluid & unit area of the surface in unit time".

The value of  $h$  depends on

i) Thermodynamic & transport properties of fluid (e.g. viscosity, density, sp. heat etc)

ii) Nature of fluid flow

iii) Geometry of the surface

iv) Prevailing thermal conduction.

\* At the surface, as viscosity is zero, so heat transfer occurs there by conduction only.

→ It is possible only in fluid medium & directly linked with the transport of medium itself. The effectiveness of heat transfer by convection depends largely upon the mixing motion of the fluid.



Q A flat plate of length 1m & width 0.5m is placed in air at 30°C flowing parallel to it. Calculate convective heat transfer coeff, if the plate is maintained at a temp of 300°C & heat transfer rate is 4.05 kW.

Sol<sup>n</sup> Given  $L = 1\text{m}$   $T_f = 30^\circ\text{C}$   $Q = 4.05\text{ kW}$   
 $b = 0.5\text{m}$   $T_s = 300^\circ\text{C}$   $= 4.05 \times 10^3\text{ W}$

$$Q = hA [T_s - T_f]$$

$$\Rightarrow h = \frac{Q}{A [T_s - T_f]} = \frac{4.05 \times 10^3}{(1 \times 0.5) [300 - 30]} = 30\text{ W/m}^2\text{ }^\circ\text{C}$$

Q A hot plate 1m x 1.5m is maintained at 300°C. Air at 20°C blows over the plate. If the convective heat transfer coefficient is 20 W/m²°C, calculate the rate of heat transfer.

Sol<sup>n</sup>  $A = 1 \times 1.5 = 1.5\text{ m}^2$   
 $T_s = 300^\circ\text{C}$   
 $T_f = 20^\circ\text{C}$   
 $h = 20\text{ W/m}^2\text{ }^\circ\text{C}$   
 $Q = hA (T_s - T_f)$   
 $= 20 \times 1.5 (300 - 20) = 8400\text{ W} = 8.4\text{ kW}$

#### 4.5 Radiation heat transfer :->

- It is defined as "The transfer of energy across a system boundary by means of an electromagnetic mechanism which is caused by temp<sup>n</sup> difference".
- It does not require any medium & occurs most effectively in vacuum.
- Ex- furnaces, combustion chambers, nuclear explosions, & space application; solar energy incident on earth.

Electromagnetic waves → The energy which radiating surface releases is not continuous but in the form of discrete / separate packet or quanta of energy called photons, called EMW. It travels with velocity of light in straight path with same frequency.

When they approach receiving surface, the conversion of wave motion into thermal energy occurs which is partly absorbed, reflected or transmitted through the receiving surface.







When incident radiation also called irradiation ( $G$ ) (defined as total incident radiation on a surface from all dir's per unit time & per unit area of surface) impinges on a surface, a part is reflected back ( $G_r$ ), a part is transmitted ( $G_t$ ) & the rest is absorbed ( $G_a$ ).

By conservation of energy principle

$$G_a + G_r + G_t = G$$

$$\Rightarrow \frac{G_a}{G} + \frac{G_r}{G} + \frac{G_t}{G} = 1$$

$$\Rightarrow \alpha + \rho + \tau = 1$$

Where  $\alpha$  = absorptivity

$\rho$  = reflectivity

$\tau$  = transmittivity

### Black body

- For perfectly absorbing body  $\alpha = 1$ ,  $\rho = 0$ ,  $\tau = 0$
- So it is the body which neither reflects nor transmits any part of the incident radiation but absorbs all of it.
- In real it does not exist

### Opaque body

When no incident radiation is transmitted through the body.

Here  $\tau = 0$

$$\text{so, } \alpha + \rho = 1$$

Ex - Glass, liquid

### White body

If all incident radiations are reflected back.

$$\text{ie. } \rho = 1, \alpha = 0 \text{ \& } \tau = 0$$

### Gray body

The body whose absorptivity of a surface does not vary with temp<sup>n</sup> & wavelength of incident radiation.

$$\alpha = \alpha_\lambda = \text{const.}$$



## concept of black body $\rightarrow$

Black body is an object which absorbs all the radiant energy reaching its surface. So,  $\alpha = 1$  &  $\rho = \tau = 0$ . The concept of it is important but does not exist practically.

### Properties of black body

- $\rightarrow$  Absorbs all incident radiation irrespective of dir<sup>n</sup>.
- $\rightarrow$  Emits max<sup>m</sup> amount of " at all wavelengths at any specified temp<sup>n</sup>.
- $\rightarrow$  It is a diffuse emitter (i.e. radiation emitted by a black body is independent of dir<sup>n</sup>).

### 4.8 Kirchhoff's law relating to spectral emissive power to absorptivity $\rightarrow$

States that "At any temp<sup>n</sup> the ratio of total emissive power  $E$  to the total absorptivity  $\alpha$  is a constant for all substances which are in thermal equm<sup>m</sup> with their environment".

or) States that, "Emissivity of a body is equal to its absorptivity when the body remains in thermal equm<sup>m</sup> with its surroundings.

### ~~Max~~ Planck's law $\rightarrow$

It gives the spectral distribution of radiation intensity of black body & is given by

$$(E_{\lambda})_b = \frac{2\pi c^2 h \lambda^{-5}}{e^{\left(\frac{ch}{\lambda K T}\right)} - 1} \quad (\text{Planck's law})$$

where  $(E_{\lambda})_b$  = monochromatic emissive power of black body

$c$  = velocity of light in vacuum =  $3 \times 10^8$  m/s.

$h$  = Planck's const =  $6.625 \times 10^{-34}$  J.s

$\lambda$  = wavelength,  $\mu\text{m}$

$K$  = Boltzmann const =  $1.3805 \times 10^{-23}$  J/K

$T$  = absolute temp<sup>n</sup>, K

Unit of  $(E_{\lambda})_b$  =  $\text{W/m}^2(\mu\text{m})$



#### 4.6 Theories of radiation: →

Actual mechanism of radiation propagation is not fully understood till date still two theories are in use.

##### 1) Maxwell's theory →

Acc<sup>n</sup> to Maxwell's electromagnetic theory, the energy is transferred from a hot body to cold body in the form of electromagnetic waves. All electromagnetic waves travel with speed of light.

This concept is useful in prediction of the radiation properties of the surfaces & materials.

##### 2) Max-Planck's theory →

Acc<sup>n</sup> to Max Planck's concept, the propagation of thermal radiation takes place in the form of discrete quanta called photons.

Each quanta have energy  $E = h\nu$

where  $h$  = Planck's const

$\nu$  = frequency of photons

This concept is used to predict the magnitude of emitted energy by a body at a given temp<sup>n</sup> under ideal cond<sup>n</sup>.