LECTURE NOTE
ON
STRUCTURAL MECHANICS
FOR
DIPLOMA IN CIVIL ENGINEERING
(3RD SEMESTER STUDENTS)

AS PER SCTE&VT SYLLABUS

PREPARED BY:

Miss Pinky Sahu

PTGF in Civil Engineering

Department of Civil Engineering

Government Polytechnic, Sambalpur (Rengali)

www.gpsambalpur.com

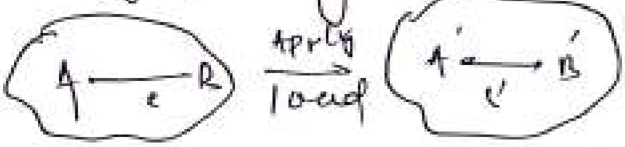
Lec-1

Solid Mechanics

Engineering Mechanics

→ Branch of engineering which ~~study~~ deals with study of rigid body.

→ Rigid body



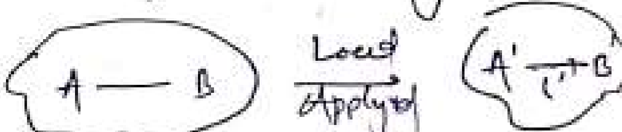
$$\boxed{l = l'}$$

→ Equilibrium Eq^n

Strength of Material

→ Branch of engineering deals with study of deformable body.

→ Deformable body



$$\boxed{l \neq l'}$$

→ Branch of engineering which deals with the deformable body in Equilibrium.

2D

3 equilibrium eq^n.

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_z = 0$

3D

6 equilibrium Eq^n

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma F_z = 0$

$\Sigma M_x = 0$

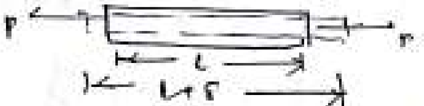
$\Sigma M_y = 0$

$\Sigma M_z = 0$

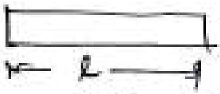
$$\boxed{\text{Moment} = \text{force} \times \text{lever Arm}}$$ **

Deformable Body

Elastic body

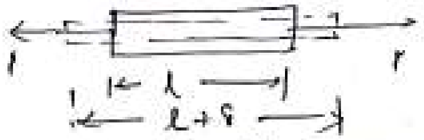


↓ after removal of load



Returns of its original position,

plastic body



↓ after removing of load

There will be some permanent set.

Visco-elastic or visco plastic body

Visco

→ High deformation zone

→ Strain in time ~~deforme~~ dependent.

→ strain is not a ~~st~~time dependent quantity

Elastic Body

lineanearly Elastic body

→ Hook's law is valid

→ Example – mild steel, Fe415, Fe500, Al, Cu, Br

non-lineanealy Elastic body.

→ Hook's law is not valid.

→ Example – Rubber.

* Hook's law! – Within proportionality limit stress is directly proportional to strain.

Note! – ~~stoon~~ strain is an independent quantity and stress is the function which depends on strain

Note! -

Strength of Meterial !- It is the branch of engineering which deals with leanearly elastic ~~Elastic~~ ~~[illegible]~~ deformable body in equlibrium.

Failure of Elastic Body !-

Fracture	Yield or Chalking
Sudden failure	→ Not a sudden failure
Break into piece	→ Give indication in the form of yending (developed of cracks) before getting fail.
no-indication of failure	
~~Dest~~ Huge loss of life and property.	→ No loss of life and property.
observed in high strength material. (Brittle material)(+)	→ low strength material. ↓ → shown in (Ductile material)(+)

2

Ductile Meterial

It is a deformable body.

Large deformation before getting failed

Strength is less compared to other material due to weak bonding. (Less amount of carbon content is present in very)

Compressive strength > tensile strength > shear strength

$$\boxed{\sigma_{uc} > \sigma_{ut} > \tau_u}$$

σ → normal stress

u - ultimate.

c → compression

t - tension.

Continue..

→ A ductile material will fail at a plane of maximum shear stress

→ Material showing greater fracture strain will be more ductile.

Brittle Material **

→ Material which does not elongate more and break into pieces

→ More strength as compare to other material due to greater resistance.

→ $\boxed{\sigma_{cu} > \tau_u > \sigma_{ut}}$ **

→ It is weak in tension.

→ It will fail on a plane at which tensile stress is maximum.

→ Greater the ultimate strain more the brittleness.

→ TYPES OF LOADING :-

1) AXIAL LOADING [Concentrated
Eccentric

2) TRANSVERSE LOADING

3) CONCENTRATED MOMENT

4) TWISTING MOMENT

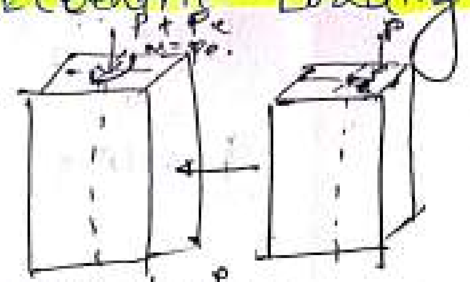
1) AXIAL LOADING.

Concentric Loading

→ It is a normal load acting perpendicular to the cross sectional area.

Apart from 'P' no other loading is generated.

Eccentric Loading



P after transform.

P is direct load

M= P.e. (indirect Moment)

2) TRANSVERSE LOADING :-

The load which is ⊥r to the axil direction.

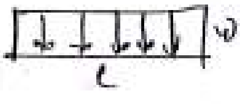
longitudinal direction	Transverse direction
z →	x, y
x →	y, z
y →	x, z

a) Concentrated Point load $\downarrow^{w}$.

b) Uniformly distributed load (UDL) $\underbrace{w^{\,\text{k/m}}}_{l}$

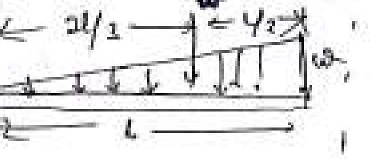
→ load is uniformly distributed over an area.

→ Total load = Area under the diagram

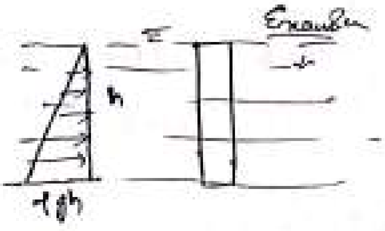


= w·l.

c) Uniformly Varying load (UVL)

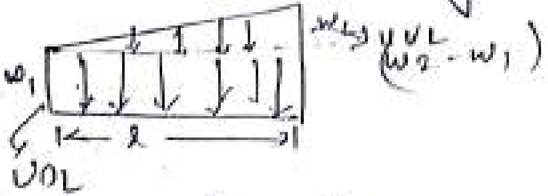


Total load (W) = $\frac{1}{2}$ w.l



continue —

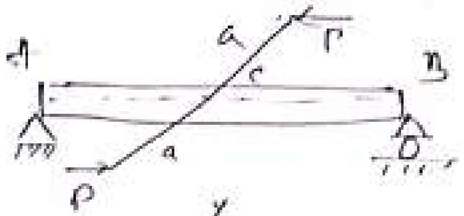
d) Trapizodial loading



e) Rooling load.

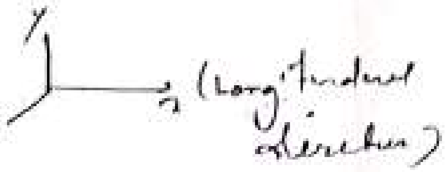
transverse loading

3) CONCENTRATED MOMENT! -



Couple = f × L

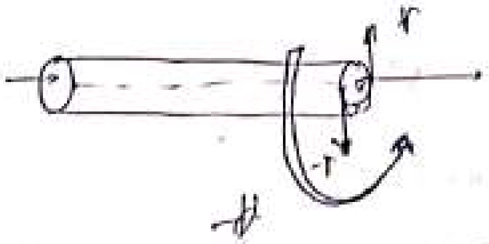
= P × 2a



Axis of rotation → Transverse direction (x)

In above case → The moment can be in the M_y, M_z.

4) TWISTING MOMENT! -



T = r × F

twisting Effect.

- Twisting moment is always along the axis of rotation

Note ⇒ The moment is developed along the axis is twisting and ⊥r to the axis is bending.

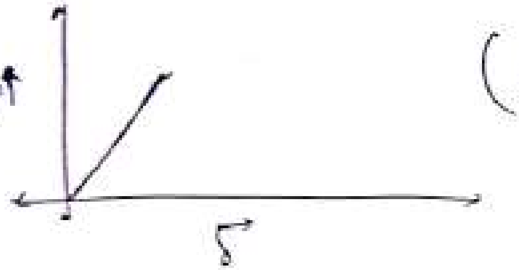
TYPE OF LOAD APPLICATION :-

- GRADUAL LOAD
- SUDDEN LOAD
- IMPACT LOAD

GRADUAL LOAD :-

→ Total load is applyed on the structur gradually

Example- i) Construction of building, Increasing waterlevel in the reservour.



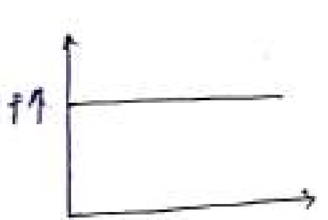
(Load deflection curve)

SUDDEN LOAD :-

→ The total load is applied at an instant. without any energy.

→ This is also reffered as Instantanious loading.

Example:- 1) Train Arriving / stopping in the track
2) Weighing load.



(Load deflection curve)

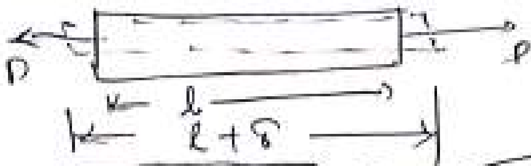
IMPACT LOAD :-

load applied with certain Energy.

Example:- 1) breaking material with hammer, catching ball from height to dead bask, Crashing of car,

STRAIN ENERGY.

→ Internal (Energy) → work done → force × displacement



(External work) done by load P. (Area under P-δ curve) → Get stored in the body in the form of internal energy which is reffered as strain energy.

Note * External work stored in the body i/n the form of internal energy which is refered as strain energy.

* Internal energy stored in deformable body.

→ Forms of Strain Energy :—

i) Toughness, Modulus of toughness

ii) Resilience, Proof resilience, Modulus of resilience

(i) Toughness and Modulus of toughness :—

* Both are developed due to impact loading just before failure.

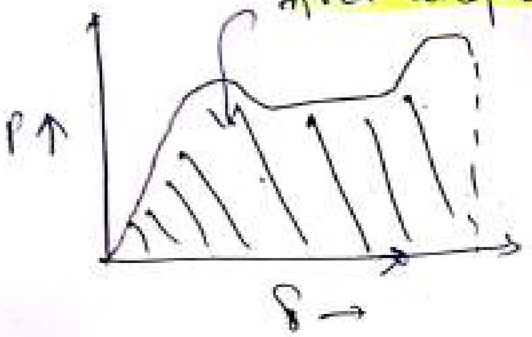
→ Toughness is the strain energy for entaire volume.

→ The strain energy per unit volum of speciman is called Modulus of toughness, its also defined as Toughness per unit volume.

→ Toughness is determined by finding P-δ curve.

→ ~~for~~ find by using Area under stress strain curve.

Area under P-δ curve = Toughness.



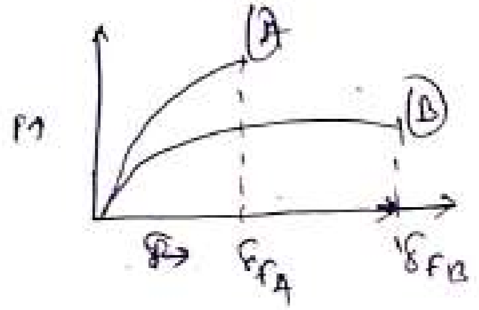
**

~~Modulus~~

$$\boxed{MOT = \frac{\text{Toughness}}{\text{per Unit volume}}}$$

Note:-

Generally Greater the toughness, greater will be ductility.



$$\boxed{\delta_{FB} > \delta_{FA}}$$

B is more ductile than A

Area under curve B > Area of curve of A

$$\boxed{\text{Toughness of B} > \text{toughness of A}}$$

2) Resilience, Proof resilience and modulus of resilience :-

* All due to static load it may be for Gradual load and sudden load. It also a strain Energy.

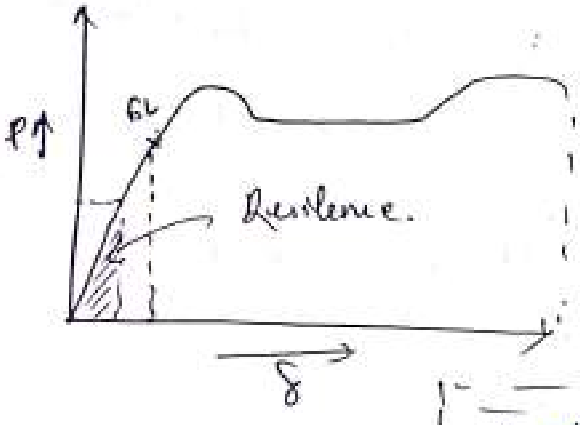
* Resilience:- The strain Energy stored in the body due to static loading within elastic limit. For Entire volume.

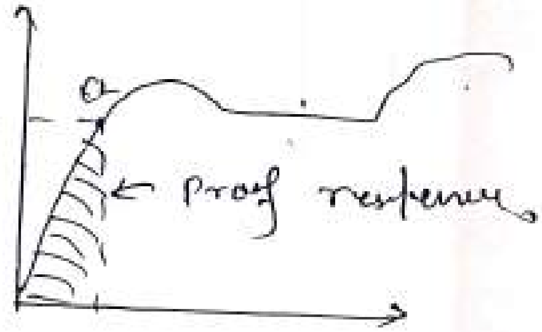
- Proof Resilience :- Strain stored in a body due to static load upto the elastic limit loading for the entire volume.

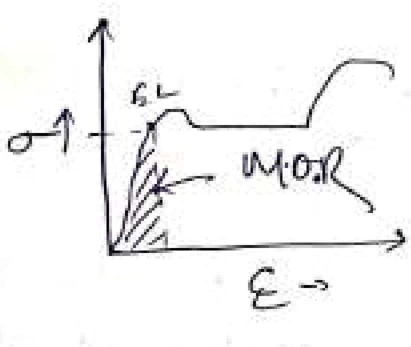
- Modulus of resilience :- Strain energy stored in a body upto the elastic limit per unit volume of the body / Component.

$$\boxed{MOR = \frac{\text{Proof resilience}}{\text{Volume of component}}}$$ **

P-δ curve ~~of~~ of a material.







* Assumptions :-

→ No loss of energy

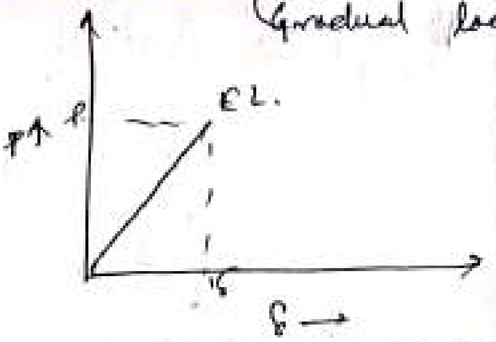
→ P.L, Y.L, E.L. are considered to be nearly same.

→ δ-P curve and σ-ε curve assume to be linear upto elastic limit

→ Strength at elastic limit as yield strength.

* Formulation **[Strain Energy due to Axial loading]** -

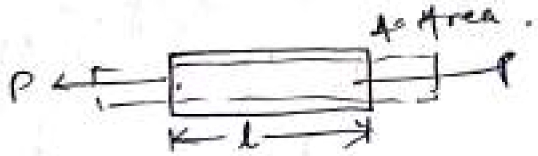
(Gradual loading)



Resilience $\boxed{(U) = \frac{1}{2} P.\delta}$ ---- (I)

Stress $(\sigma) = \frac{\text{load}}{\text{Area}} = \frac{P}{A}$

$\boxed{P = \sigma . A}$ ---- ②



Strain $(\epsilon) = \frac{\text{Change in length}}{\text{original length}}$

$\epsilon = \frac{\delta l}{l}$

$\boxed{\delta = \varepsilon . l}$ ---- ③

~~Stat~~ substitute eqn ② & ③ in I.

$U = \frac{1}{2} \sigma . A . \epsilon . L$

$U = \frac{1}{2} . \sigma . \epsilon . (A . L)$

$\boxed{U = \frac{1}{2} \epsilon . \sigma . V .}$ ---- ④

Hook's law is valid

$\sigma \propto \epsilon$

$\boxed{\sigma = E \varepsilon}$ ---- (5)

↓ youngs modulus

$\boxed{\varepsilon = \frac{\sigma}{E}}$ ---- ⑤

Replace σ in 4

$\boxed{U = \frac{1}{2} E \epsilon^2 \times V}$ ---- ⑥

Replace ⑥ in ④

$\boxed{U = \frac{\sigma^2}{2E} . V}$ ---- ⑦

Resilience

1* Proof Resilience: -

$\boxed{U = \frac{\sigma_y^2}{2E} \times V}$

2 Modulus of resilience (u): -

$u = \frac{U}{V}$

$\boxed{u = \frac{\sigma_y^2}{2E}}$

Note

All ~~the~~ form of eqn Strain energy only due to gradual loading.

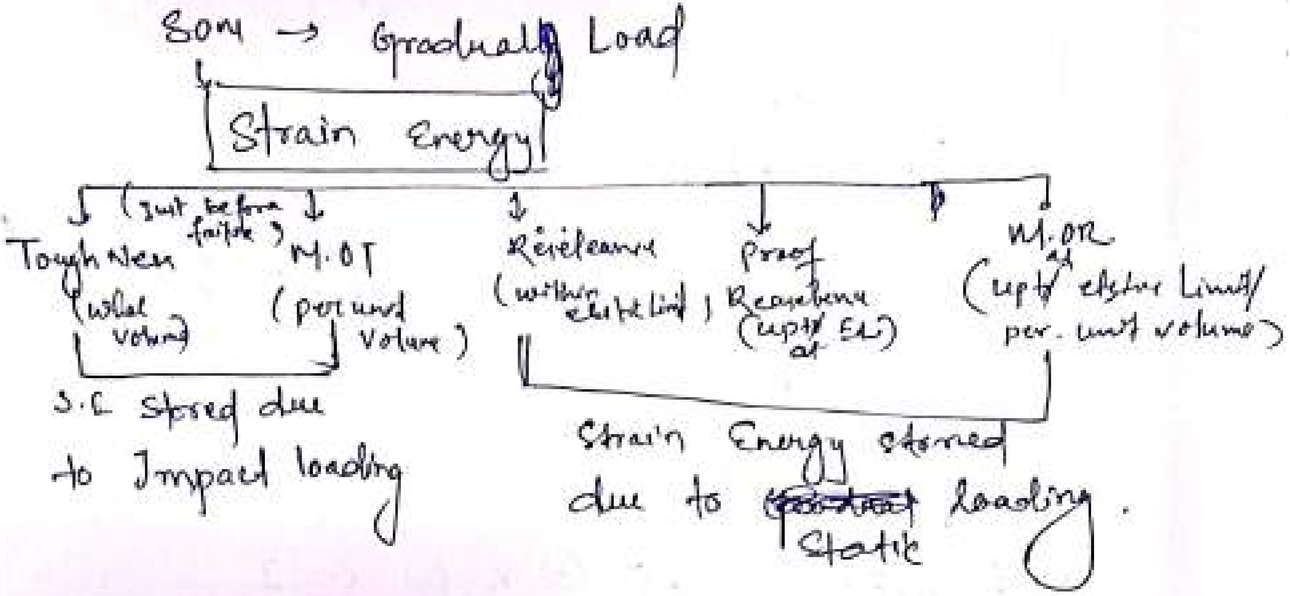
[MOR unit]

$\therefore \boxed{\frac{\text{Joule}}{m^3} , \frac{N\text{-}m}{m^3}}$

$\boxed{\frac{N\text{-}mm}{mm^3}}$

Lec- 4

* flow Chat of Strain Energy.

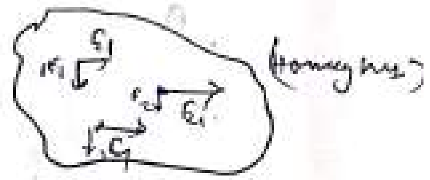


* ASSUMPTION In SOM:- (Important topic)

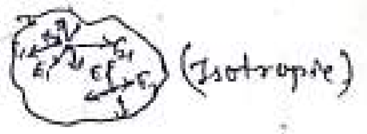
i) Material is deformable and continuous,
↳(No cracks.)

2) Material is homogenous & Isotropic.

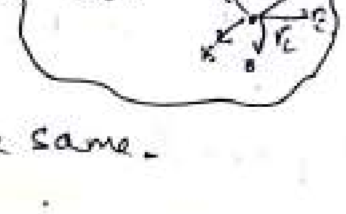
* Homogeneous!- all point within a body. in one direction. elastic property in same



* Isotropic !- At any particular point within the body in all the direction. elastic properties is same.

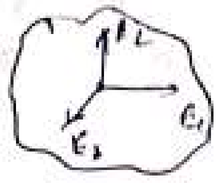


* Homogeneous & Isotropic !- At any part within the body at in any particular direction material elastic properties is found to be same.



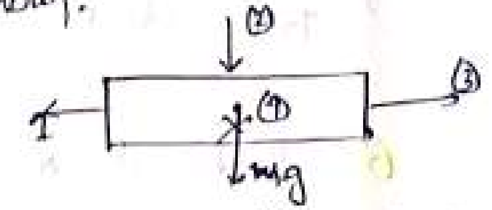
* Anysotropic !- At any point with in the body in all the direction material elastic poi properties is different.

* Orthotropic ! - At any point within the body of elastic property in 3 mutually ⊥r direction is different then the material is orthotropic.

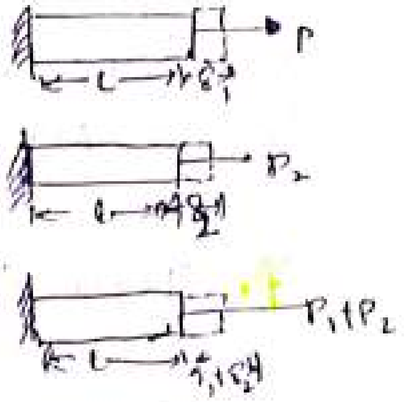


Note :- Orthotropic is a part of Anisotropic.

3. Self weight or dead load of the member are neglected and only surface forces are considered.



4. Super position Principal is valid because Hooks Law is valid.

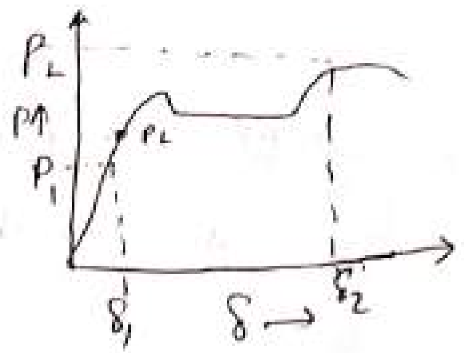


Super position Principle !- $\boxed{\delta = \delta_1 + \delta_2}$

* Effect of load sum of the individual deformation.

* Question IES 2006 !-

I) If $P_1 + P_2$ load is applied, then the deformation is given by.



a) δ_1 b) δ_2

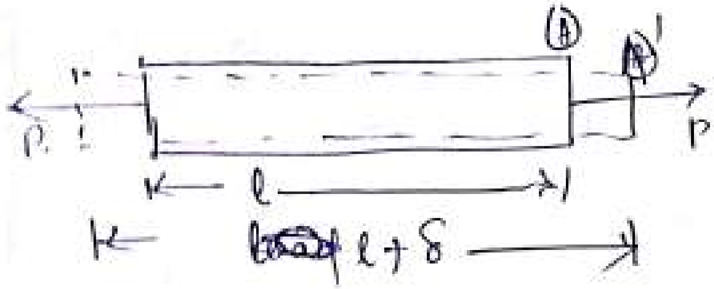
c) $\delta_1 + \delta_2$ d) none of these.

Ans → none of these.

5. Engineering stress and strain are considered when true stress and strain are neglected.

Contd...

Note:

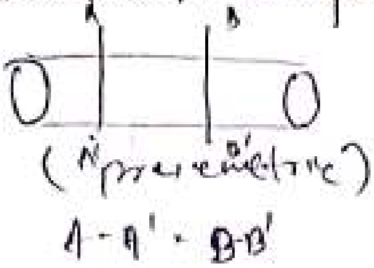


* Engineering Stress = $\frac{\text{load}}{\text{Initial area}} = \frac{P}{A}$

* True Stress = $\frac{\text{Load}}{\text{Instantaneous area}} = \frac{P}{A'}$

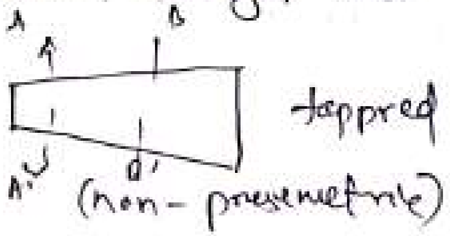
6. Material is Prismatic.

Prismatic :- Uniform cross section throughout the length.



(prismatic)

A-A' = B-B'



tappred bar.

(non-prismatic)

AA' ≠ BB'

7. There is no residual stress is Present / considered

Residual Stress :- Before actual loading ~~so~~ present of a stress is known as Residual stress.

Note:- ~~No~~ Before actual loading not any stress present in the structure.

** Factor of Safty :-

Assumption

homo, Iso,
Eng. stress,
Super position.
→ there assumption are not valid. (in field)

Strength of any material. ≠ Working stress (stress in the field)

$$\boxed{\text{factor of } \cancel{\text{safty}} \text{ safety} = \frac{\text{Strength}}{\text{Working Stress}}}$$

$$\boxed{FOS > 1}$$

Factor of Safety

Ductile Material
↓
yield failure
↓
$\boxed{\text{Yield Strength}}$

$$\boxed{fos = \frac{\text{Yield strength}}{\text{Working stress}}} \text{**}$$

Brittle material
↓
→ fracture
→ $\boxed{\text{Ultimate strength}}$

$$\boxed{FOS = \frac{\text{Ultimate strength}}{\text{working stress}}} \text{**}$$

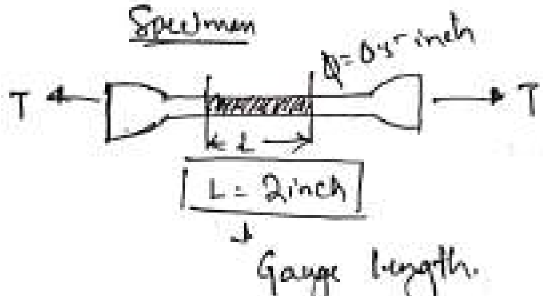
FOS:-

for industry fos → 1.6 to 4

* If $\boxed{fos < 1.6}$
→ Structure is not durable.

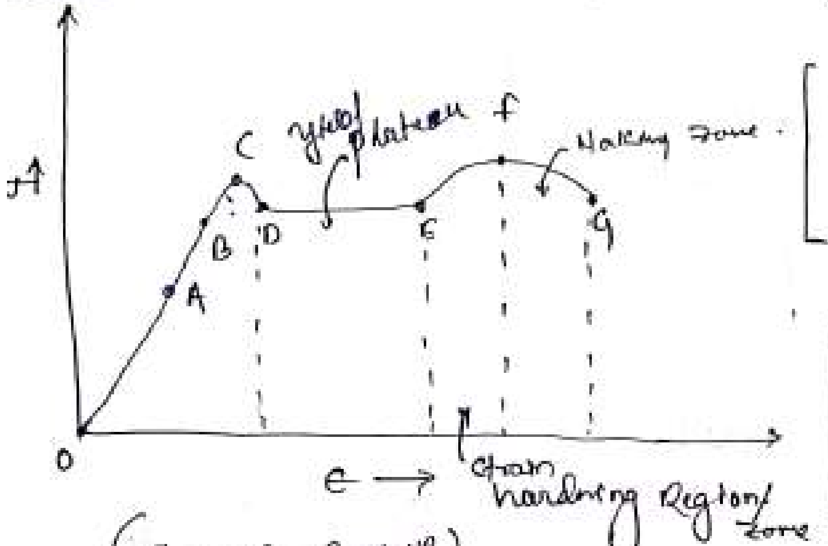
* If $\boxed{Fos > 4}$
→ higher quality material will be used
→ More amount of lower quality

TENSILE TEST ON MILD STEEL !-



* Universal Testing mechine (UTM)
* By American standerd for testing mechine.

Result



(σ – ε curve)

OA → Strength line. $\boxed{\sigma \propto \epsilon}$

A → Proportionality limit $y \propto x^{=1}$

AB → Curved line
Hook's law is not valid

B → Elastic limit

C → Uper Yield point (not constant for any material) (Not considered as yield strength.)

D → lower yield point. → Actual yield starts at this point.
↳ It is a material property
→ Designed yield strength

DE → Yield Plateau !- No stress but continous strain.

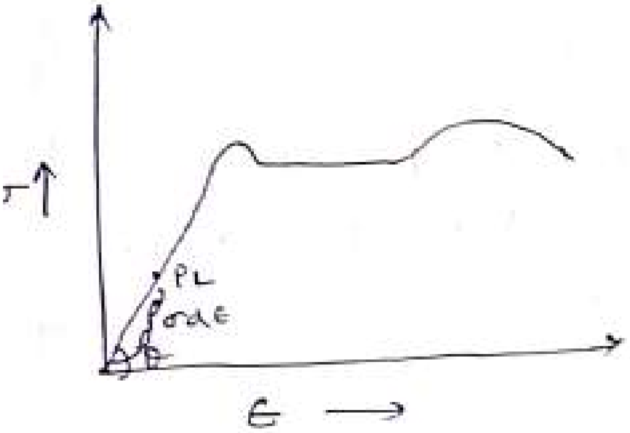
F → Strain hardning Region !- Material become stiff, more stress for smalar strain.

F → Ultimate stress point.

FG → Neking Region ⟶ Strain softning Region

G → Broking Stress point (At this point of failure)

Stress - Strain Curve for different material ! -



$$\theta = \tan^{-1}\left(\frac{\sigma}{\epsilon}\right) = \tan^{-1}(E)$$

** Slope of stress-strain Curve within proportionality limit = Young's modulus of Elasticity

$\downarrow$

Material properties

Signifies elasticity of material.

$E_{MS} = 2\times10^{11}$ N/m^2

$$\boxed{\theta = \tan^{-1}(E) \approx 90^\circ}$$

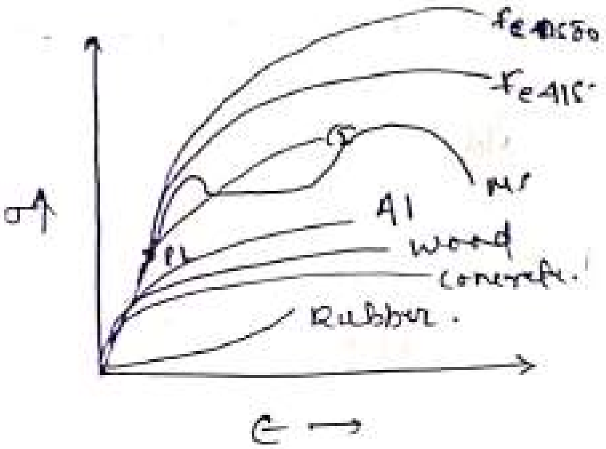
$E_{CI} = \frac{E_{MS}}{2}$

$E_{Al} = \frac{E_{MS}}{3}$

$E_{wood} = \frac{1}{8}$ to $\frac{1}{15}$ E_{MS}

$E_{concrete} = \frac{1}{10}$ to $\frac{1}{20}$ E_{MS}

Relation of young modulus of different Material.



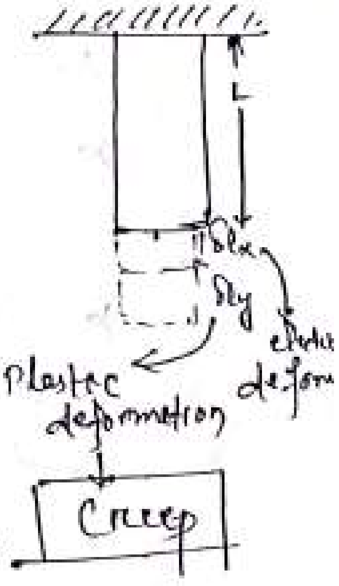
Note :- Steel is more elastic then rubber because it has greater young's modulus of elasticity.

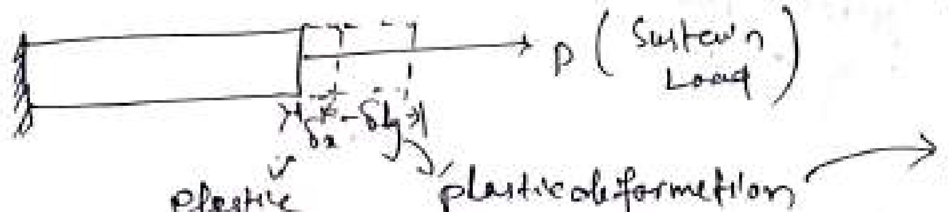
* CREEP :-

→ It is a plastic deformation.

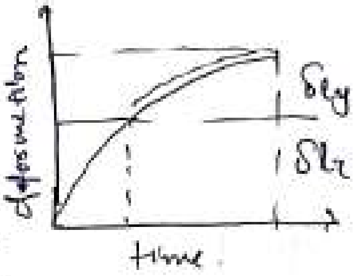
→ Occuring over period of time.

→ Due to dead load or constant Load.





* Factor Affecting Creep ! -

(i) Tamperature (↑)	(ii) Time (↑)	(iii) Magnadude of loading
→ Greater the temp, Greater will be the creep (↑) for Example - Fe - MP → 1539°c If $\frac{MP}{2}$ ↓ → creep tolerable. If $\frac{MP}{2}$ ↑ → Creep intolerable.	→ With time creep (↑) increases but creep Rate (↓) decrease ! 	→ Greater Load Greater will be the creep.

Key $\boxed{\frac{MP}{2} \rightarrow \text{Homologeus temp.}}$ MP ⇒ melting point

Note! - During Creep. The stress will be constant.

acture! 6

Fatigue and endurance limit ! -

* Fatigue ! - failure phenomena due to dynamic loading / cyclic loading or reversal of loading.

* Endurance Limit ! -

→ It is the strength for which material need to be design of do not fail in fatigue.

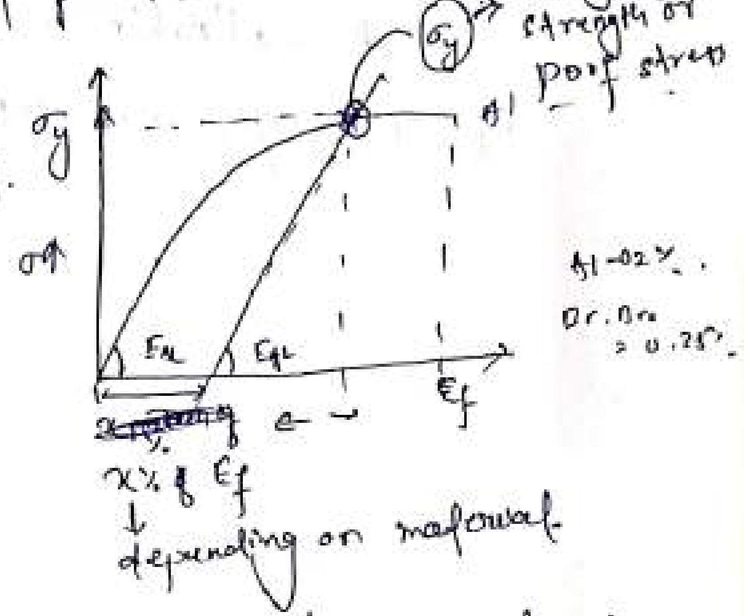
→ The Endurance limit less then yield strength and ultimate strength.

→ whether the material is brittle or ductile, due to cyclic loading material will fail suddenly with out any indication.

PROOF STRESS / Design yield Strength :-

Certain material like Al, Br, Copper. do not show definite yield point. for such material, an ~~opset~~ offset method is adopted to find yield point.

Note
for a certain material ~~of~~ where definite yield stress point is not obtained the stress obtained using the offset method is reffered as proof stress.



Effect of Carbon %age on material properties! -

Carbon %age is increased (↑) :-

- Ductility (↓)
- Toughness (↓)
- * Brittleness (↑)
- * Ultimate strength (↑)
- formability (↓)
- * Hardness (↑)
- * yield strength (↑)
- * wear and tear resistance (↑)
- * shock resistance (↑)
- Malleability (↓)
- Machinability (↓)
- * Resilience (↑)

Hardness :- It is the ability of material to resist the indentation or surface abrasion.

⇒ Brinell hardness test is used for checking hardness of a material

* $$\boxed{\text{Brinell hardness number} = \frac{P}{\frac{\pi D}{2}\left[D - \sqrt{D^2 - d^2}\right]}}$$

Malleability :- It is the properties of a material which permits that the material to be extended in all direction to without rupture.

→ A malleable material posses a high degree of plasticity but not necessarily great strength.

CH-2 SIMPLE, STRESS, & STRAIN (THARMAL STRESS)

(Topic-1) STRESS

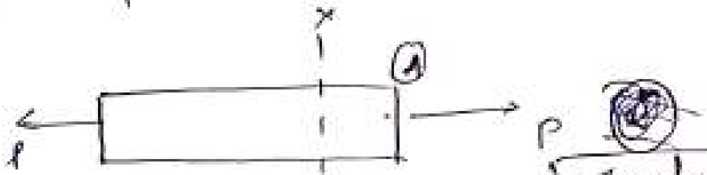
* Stress, Strength and Pressure !-

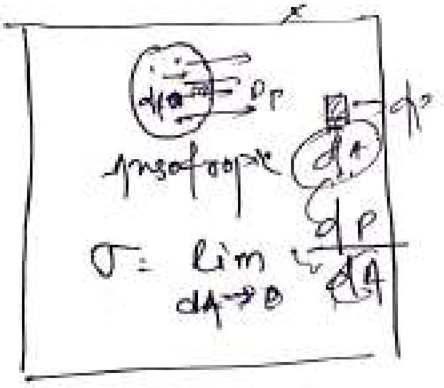
* **Pressure** ! → External normal load per unit area.
  → Thus pressure act both on rigid and deformable body.

* **Strength** : —
  - It is the maximum stress ~~that~~ to resist fracture/ failure under load.

* **Stress** : —

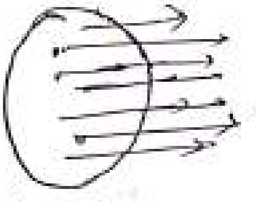
It is the a internal resistance developed in the deformable body due to externally applied load. in order to resist the deformation.





$$\sigma = \lim_{dA \to 0} \frac{dP}{dA}$$

Isotropic & homogeneous



σ_{av} = Numerically to pressure applied.

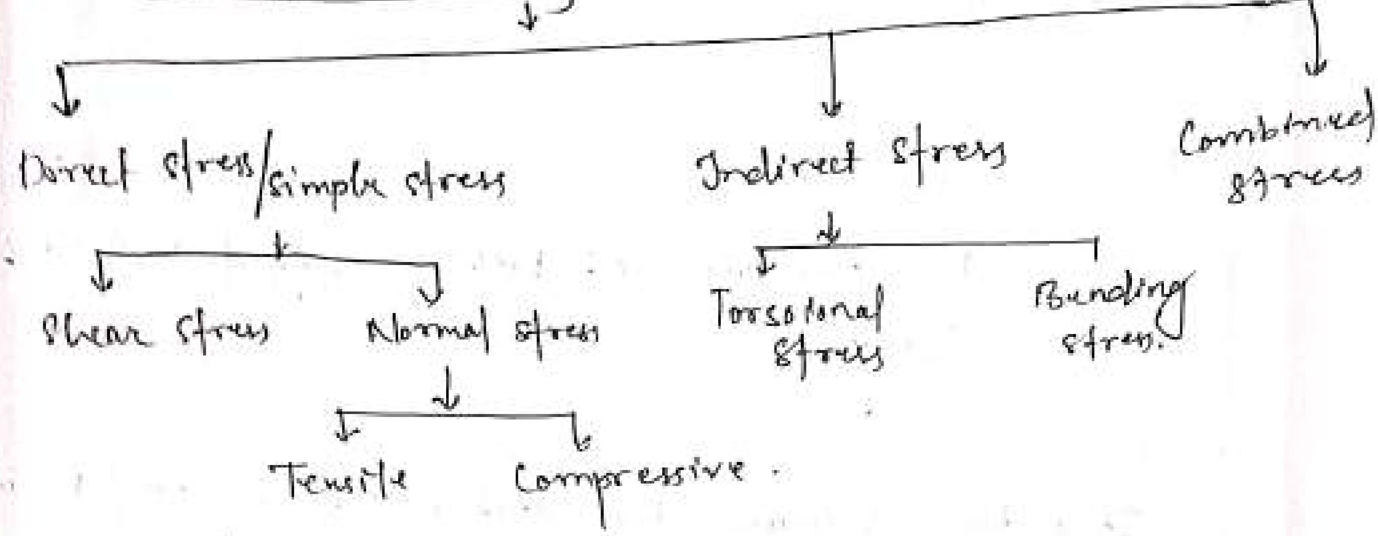
$$\boxed{\sigma_{av} = \frac{P}{A}}$$

Stress !-
Stress dependent on Load & Area.

Strength → is a material properties
→ doesnot depending on Load & Area

Note !- Maximum value of stress before the material fail. is known as strength of material.

Classification Of Stress



- **Shear stress :-**

the stress which is tangential to the cross sectional Area.

- **Normal stress :-** ⊥ʳ to the c/s area.

Tension :-

Compression :-

- **STRESS REPRESENTATION :-**

Scalar	Vector
→ have only magnatude.	→ have both magnatud & one direction.
Ex → distance, speed, length	Ex → Moment, displacement, acceleration. velocity

Tensor Quantity

→ One magnatude, two direction

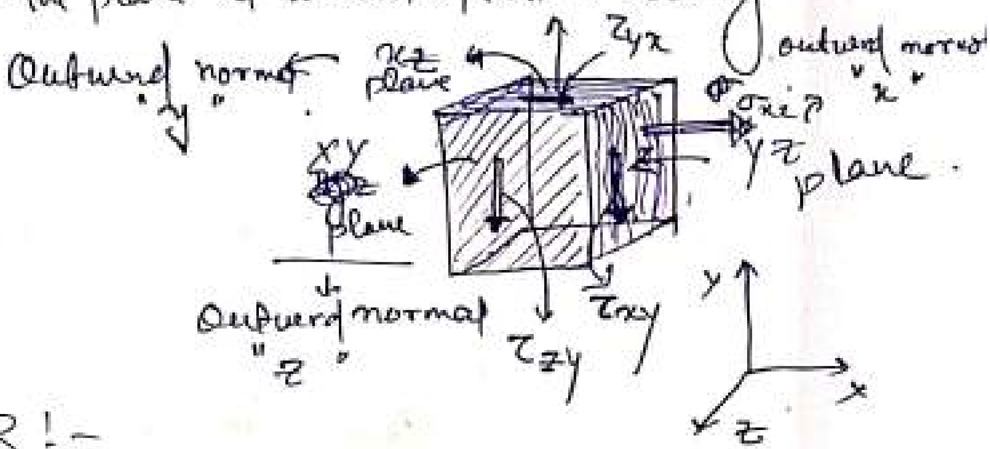
Example - Stress, Strain, product moment of inertia, MOI.

(2nd order quantity)

continue -

xx

σ i:is Direction of the stress.
↳ direction ⊥r to the plane at which stress is acting



7

✡ STRESS TENSOR !-

- ✱ Representation of state of stress at a point is refered as stress tensor.

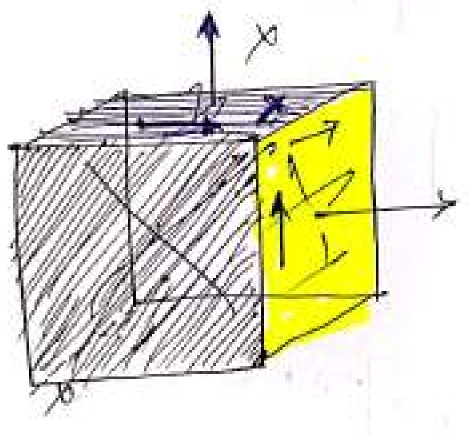
- ✱ For representation are required 3 mutually perpendicular plane. one plane can have 1 normal and 2 shear.

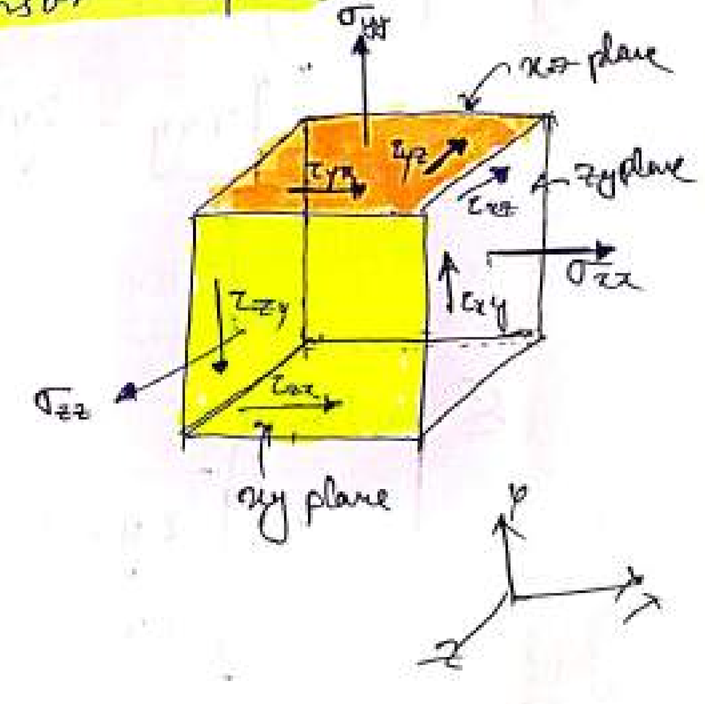
1D
1 plane ⟶ 1 normal & 2 shear.
3 plane ⟶ 3 normal & 6 shear.

9 stress.

xx
2nd order tensor quantity:-
M.O.I, Strain, stress

- ✱ Representation of this 9 stresses in matrix form is reffered as stress tensor matrix.





$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\boxed{\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}}$$

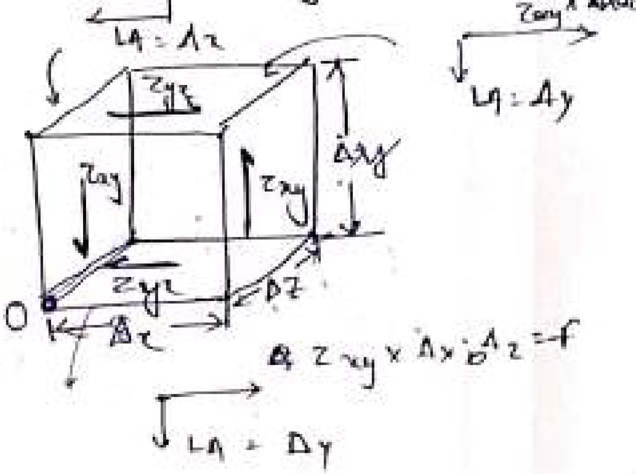
Stress tensor matrix

Equality of Shear Stress !-

$\Sigma M_0 = 0$ (↺ +ve , ↻ −ve)

Moment = F × L.A

= Stress × c/s Area × LA



$\Sigma M_{O_z} = 0$

$\Rightarrow -[\tau_{xy} \times (\Delta z \times \Delta y) \times \Delta x]$

$+ [\tau_{yx} \cdot (\Delta x \times \Delta z) \times \Delta y] = 0$

$\Rightarrow (\Delta x \times \Delta y \times \Delta z)(-\tau_{xy} + \tau_{xy}) = 0$

$\boxed{\tau_{xy} = \tau_{yx}}$ (when plane are perpendicular)

$\boxed{\tau_{xy} \neq \tau_{yx}}$ (when pane are not ⊥r)

Similarly

$$\boxed{\begin{array}{l} \tau_{yz} = \tau_{zy} \\ \tau_{xz} = \tau_{zx} \end{array}}$$

→ from ΣM_x

→ from ΣM_y

Qstn :- Equality of shear stress is based on which principle.

a) Moment of momentum eqn

b) Momentum Eqn

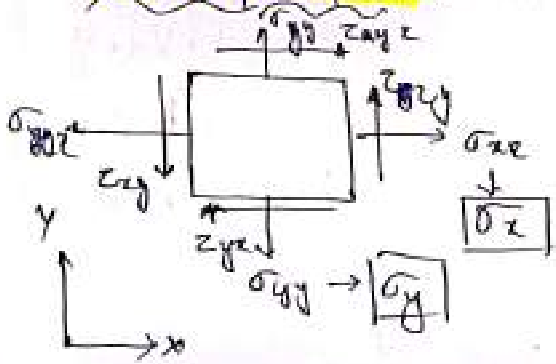
c) Moment equilibrium Eqn

d) None of these.

Ans → Moment equilibrium eqn.

** Biaxial and triaxial Stress :-

* **Biaxial Stress** :-



$$\boxed{a_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}}$$ ***

↳ Represent → 4 element

↳ Define → 3 element.

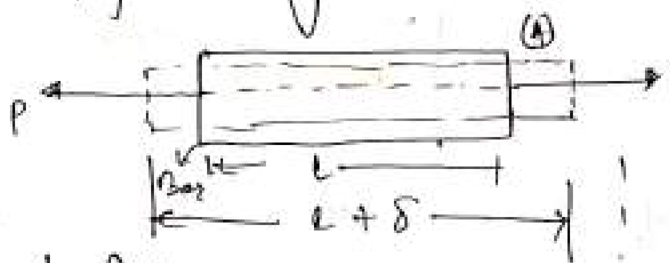
* **Triaxial Stress** :-

$$a_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

Represent ⟶ 9 element

Define ⟶ 6 element.

** Deformation of axially loaded member :-



~~from~~ hook's law :-

$\sigma \propto \epsilon$

$$\boxed{\sigma = E\epsilon}$$

$$\frac{\text{Load}}{\text{Initial Area}} = E \times \frac{\text{Change in Length}}{\text{Original Length}}$$

$$\frac{P}{A} = E\frac{\delta}{L}$$

$$\boxed{\delta = \frac{PL}{AE}}$$ **

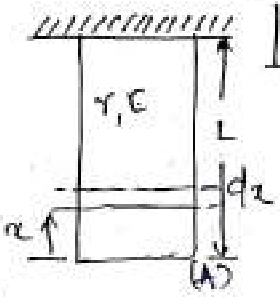
continue...

Over length "l"

P, A & E is constant.

*** Deformation of rectangular bar due to self weight:-

If Total weight of bar = W.



$\boxed{\gamma = \rho \cdot g}$ → weight density

At a distance x take a mass less streap of length dx

$$\boxed{d\delta_{strip} = \frac{P_x \cdot dx}{AE}}$$

$P_x = M_x \cdot g$
$= \rho \cdot V_x \cdot g$
$= \rho (A \cdot x)\, g$
$P_x = \gamma A x$

$$\Rightarrow d\delta_{strip} = \frac{\gamma \cdot A \cdot x \cdot dx}{\cancel{A} \cdot E}$$

$$\boxed{d\delta_{strip} = \frac{\gamma}{E} \cdot x\, dx} \quad \ldots\ldots (1)$$

deformation for strip.

For Entire length "we Integrate".

$$\int_0^{\delta} d\delta = \int_0^{l} \frac{\gamma}{E} \cdot x \cdot dx$$

$$\delta = \frac{\gamma}{E}\left[\frac{x^2}{2}\right]_0^{l}$$

$$\boxed{\delta = \frac{\gamma l^2}{2E}}$$ *** (formula-1)

$$\boxed{\delta = \frac{WL}{2AE}}$$ (formula-2)

$$\delta = \frac{\frac{W}{AL} \times l^2}{2E}$$

If weight of the bar is W.

$W = M \cdot g$
$= \rho \cdot V \cdot g = \gamma A L$

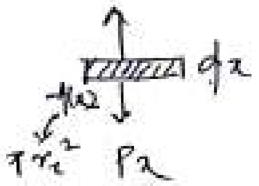
$$\boxed{\gamma = \frac{W}{A \cdot L}}$$

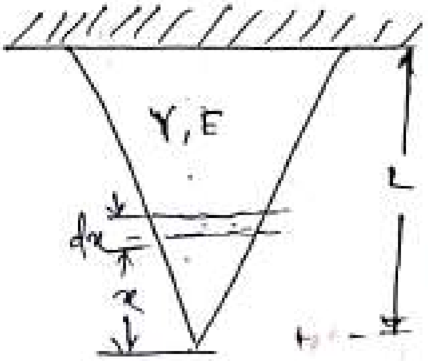
Note:-

*** Note:-

$\delta = \frac{Wl}{2AE}$ is not used for comparison, because "W" is not a independent quantity. and so $\delta = \frac{\gamma l^2}{2E}$ is used for comparison. for deformation.

Deformation of conical bar due to self weight :-





$$d\delta_{strip} = \frac{P_x \cdot dx}{A_x \cdot E}$$

$$P_x = m_x \cdot g$$

$$\Rightarrow P_x = \rho \cdot V_x \cdot g$$

$$\boxed{P_x = \gamma \frac{1}{3} A_x \cdot x}$$

$$d\delta_{strip} = \frac{\frac{1}{3}\gamma \cancel{A_x} \cdot x \cdot dx}{\cancel{A_x} \cdot E} \Rightarrow \frac{\gamma}{3E} \cdot x \cdot dx$$

for all over length.

$$\int_0^\delta d\delta = \frac{\gamma}{3E}\int_0^l x \cdot dx$$

$$\delta = \frac{\gamma}{3E}\left[\frac{x^2}{2}\right]_0^l = \frac{\gamma}{3E}\cdot\frac{l^2}{2}$$

$$\boxed{\delta = \frac{\gamma l^2}{6E}}$$ * ← conclusion

Total weight of the bar = W

$$W = M \cdot g$$

$$= \rho \cdot V \cdot g$$

$$= \gamma \cdot \frac{1}{3} A \cdot L$$

$$\boxed{\gamma = \frac{3W}{AL}}$$

$$\delta = \frac{\frac{3W}{AL} \times L^2}{6AE} = \boxed{\delta = \frac{WL}{2AE}}$$ **

Question :-

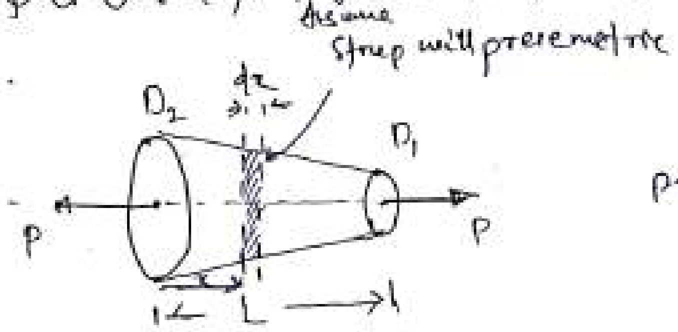
For the same material same length the deformation of rectangular bar is 'n' times than that of conical bar. Determine value of 'n'?

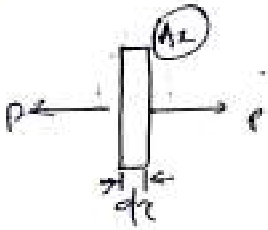
a) 3
b) 1/3
c) 1
d) none of these.

Soln $\frac{\delta_{rec}}{\delta_{con}} = \frac{\cancel{\gamma}\cancel{L^2}/2\cancel{E}}{\cancel{\gamma}\cancel{L^2}/6\cancel{E}3}$

= 3

* Deformation of tapered circular rod due to axial load:





$$d\delta_{strip} = \frac{P\cdot dx}{A_x\cdot E} = \frac{P\cdot dx}{\frac{\pi}{4}D_x^2\cdot E}$$

$$d\delta_{strip} = \frac{4P}{\pi E}\cdot\frac{dx}{(D_2-kx)^2}$$

for whole body.

$$\int_0^\delta d\delta = \frac{4P}{\pi E}\int_0^L \frac{dx}{(D_2-kx)^2}$$

$$\delta = \frac{4P}{\pi E}\left[\frac{(D_2-kx)^{-2+1}}{(-2+1)(-k)}\right]_0^L$$

$$\delta = \frac{4P}{\pi E k}\left[\frac{1}{D_x}\right]_0^L$$

$$\delta = \frac{4P}{\pi E k}\left[\frac{1}{D_1}-\frac{1}{D_2}\right]$$

$$\delta = \frac{4P}{\pi E\frac{(D_2-D_1)}{L}}\left(\frac{D_2-D_1}{D_1 D_2}\right)$$ $\because k = D_2 - D_1$

$$\delta = \frac{4PL}{\pi E D_1 D_2}$$

$$\boxed{\delta = \frac{4PL}{\left(\frac{\pi}{4}D_1 D_2\right)E}}\ **$$

a) $x = 0$, $D_x = D_2$

$x = L$, $D_x = D_1$

From 0–L, $\Delta D_x = D_2 - D_1$

for 1m length, $\Delta D_x = \frac{D_2-D_1}{L}$

for xm length, $\Delta D_x = \frac{(D_2-D_1)x}{L}$

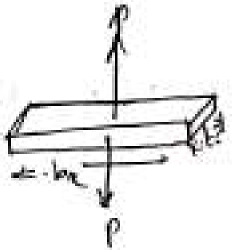
$$\boxed{\Delta D_x = k.x}$$

At $x = x_m$, $\boxed{D_x = D_2 - kx}$

Note

$$\int (a+bx)^n = \frac{(a+bx)^{n+1}}{(n+1)\,b}$$

Deformation of tapered rectangular bar due to axial loading:-



$$\boxed{b_x = b_1 + \left(\frac{b_2-b_1}{l}\right)\cdot x}$$

$$b_x = b_1 + Kx$$

Note:- $\int \frac{1}{x} = \ln x$

$$d\delta_x = \frac{P.dx}{A_x \cdot E}$$

$$d\delta_x = \frac{P.dx}{b_x \cdot t \cdot E}$$

$$d\delta_x = \frac{P}{tE}\cdot\frac{dx}{(b_1+Kx)}$$

for whole bar we Integrate:-

$$\int_0^{\delta} d\delta_x = \frac{P}{tE}\int_0^{L}\frac{dx}{(b_1+Kx)}$$

$$\Rightarrow \delta = \frac{P}{tE}\left[\frac{\ln(b+Kx)}{K}\right]_0^{l}$$

$$= \frac{P}{tE.K}\left[\ln b_x\right]_0^{l}$$

$$= \frac{P.L}{tE.(b_2-b_1)}\left[\ln b_2 - \ln b_1\right] \;**$$

$$\boxed{\delta = \frac{PL}{[t(b_2-b_1)]\cdot E}\times\ln\left(\frac{b_2}{b_1}\right)}$$

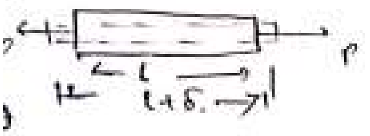
STRAIN * Topic-2

- It is a parameter which defines intensity of internal deformation.
- It is a tensor quantity. for its definition it required one magnatude two direction.

$$\boxed{\epsilon = \frac{\text{Change in length}}{\text{Original length}}}$$

Type of Strain

Normal strain



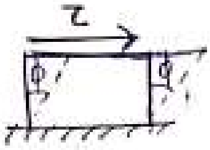
strain due to normal axial loading.

Size changes ~~Size~~ shape is ~~Shap~~ constant.

↓

This phenomina is refered as "**dialition**".

Shear Strain.



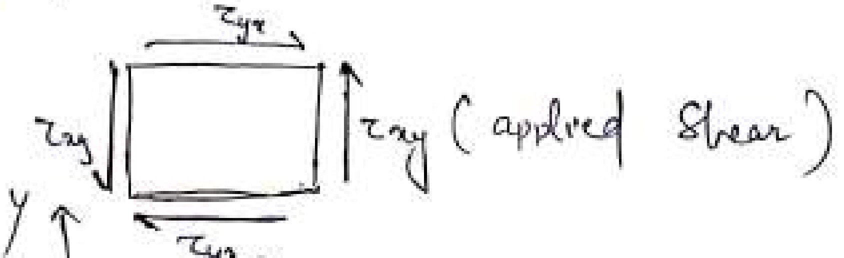
→ strain due to shear loading.

→ Due to shear shape change.

↓

This phenomena is refered as "**distortion**".

2. SHEAR STRAIN :-



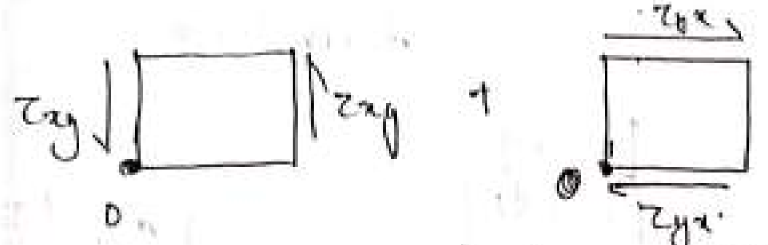
∵ From equality of shear stress

$$\boxed{\tau_{xy} = \tau_{yx}}$$

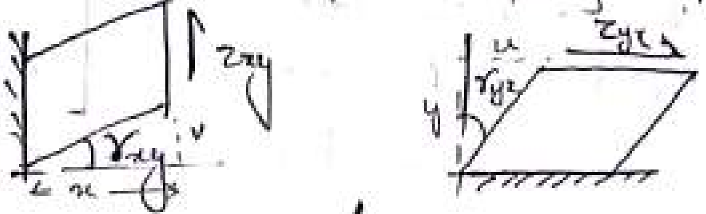
(~~Complementary shear~~)

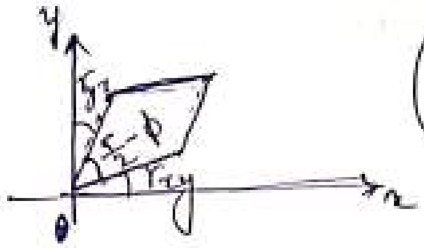
(Complimentery shear)

for analysis it divided into 2 parts, to analysed the distorted shape.



taking "O" point will fix





(distorted shape)

+ strain

$$\boxed{\phi_{xy} = \gamma_{xy} + \gamma_{yx}}$$ (Shear strain is ϕ_{xy})

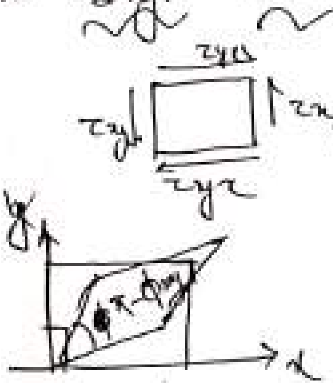
$\phi_{xy} = 2\gamma_{xy}$ or $2\gamma_{yx}$

Shear Modulus of Rigidity (G, C) :-

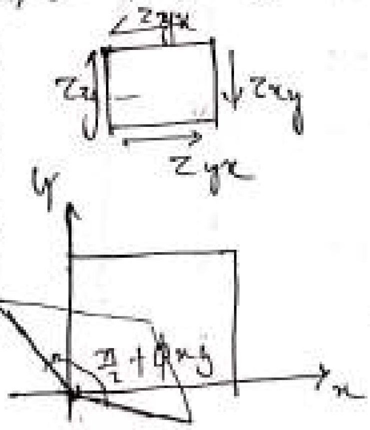
$$G / C = \frac{\text{Shear Stress}}{\text{Shear strain}}$$

$$\boxed{G = \frac{\tau_{xy} \text{ or } \tau_{yx}}{\phi_{xy}}}$$ **

** Sign Convention for Shear Strain :-



$$\boxed{\phi_{xy} = +\text{ve shear strain}}$$ **



$$\boxed{\phi_{xy} = -\text{ve shear strain}}$$ **

** STRAIN TENSOR:-

Given

Stress tensor.

$$\sigma_{ij} : \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \rightarrow$$

$\boxed{\sigma \rightarrow \epsilon}$

$\boxed{\tau \rightarrow \phi/_2}$

shear stress

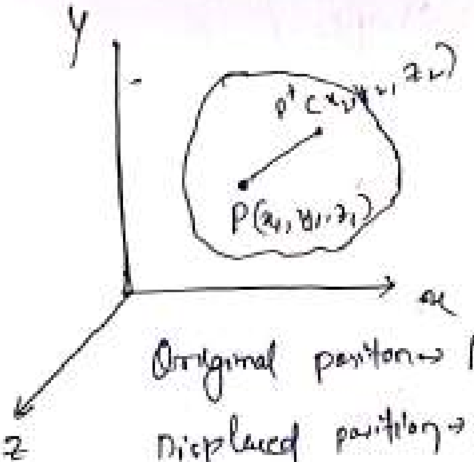
$\tau_{xy} \rightarrow \gamma_{xy}$

$\tau_{xy} \rightarrow \frac{\phi_{xy}}{2}$

Strain tensor

$$\epsilon_{ij} = \begin{bmatrix} \epsilon_{xx} & \frac{\phi_{xy}}{2} & \frac{\phi_{xz}}{2} \\ \frac{\phi_{yx}}{2} & \epsilon_{yy} & \frac{\phi_{yz}}{2} \\ \frac{\phi_{zx}}{2} & \frac{\phi_{zy}}{2} & \epsilon_{zz} \end{bmatrix}$$

imp ** DIFFERENTIAL FORM OF STRAIN:-



Original position → $P(x_1, y_1, z_1)$

Displaced position → $P'(x_2, y_2, z_2)$

Displacement → u → "x" dirⁿ → funⁿ(x, y, z)

v → "y" dirⁿ → funⁿ(x, y, z)

w → "z" dirⁿ → funⁿ(x, y, z)

Normal strain:-

$$\epsilon_{xx} = \frac{\text{Change in length}}{\text{length in } x} = \frac{u}{x} = \frac{\partial u}{\partial x}$$

$$\boxed{\begin{array}{l} \epsilon_{xx} = \frac{\partial u}{\partial x} \\ \text{Similarly} \\ \epsilon_{yy} = \frac{\partial v}{\partial y} \\ \epsilon_{zz} = \frac{\partial w}{\partial z} \end{array}}$$ **

Shear Strain

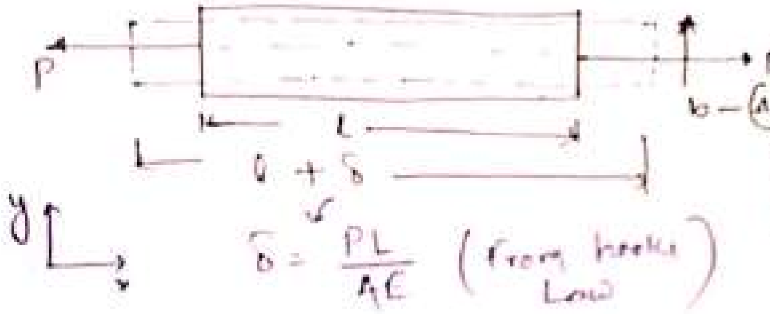
$$\phi_{xy} = \gamma_{xy} + \gamma_{yx}$$

$$\boxed{\phi_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}}$$

$$\boxed{\phi_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}}$$

$$\boxed{\phi_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}}$$

*** POISSON'S RATIO :-



$\delta = \frac{PL}{AE}$ (from hooke Law)

b – Δb → (this we found using poisson ratio)

↳ effect of longitudinal force in the lateral/transverse direction.

Note:-

→ Poisson's ratio is a factor which denots the effect of longitudinal force in lateral direction.

→ It is represented by μ, σ, $\frac{1}{m}$.

Mathematically

$$\mu = -\frac{\text{lateral strain}}{\text{longitudinal strain}} = -\frac{\epsilon_{yy}}{\epsilon_{xx}} = -\frac{\left[\frac{\Delta b}{b}\right]}{\left[\frac{\delta}{L}\right]}$$

* Range of "μ".

* for Isotropic material $0 < \mu < 0.5$ (Engineering material)

cork, glass

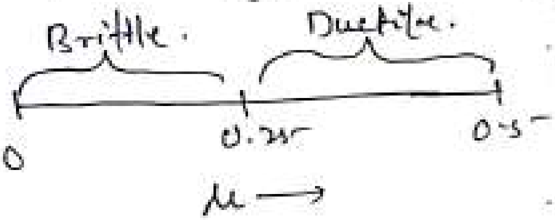
$\boxed{\mu = 0}$ → No change in lateral direction due to change long direction.

$\boxed{\mu = 0.5}$ → for incompressible fluids. Rubber.

for Polymer ($-1 \le \mu \le 0.5$)

-ve poisson's ratio ⇒ ~~Same~~ both the direction same nature of deformation.

for Metal ($1/4 < \mu < \frac{1}{3}$).



Note

μ (Increase) ~~then~~

⇒ ductility (Increase)

brittleness (decrease)

Material	μ
Gold	0.44
Copper	0.355
Bronze	0.35
Stainless Steel	0.35
aluminium	0.33
Steel	0.288
Wrought Iron	0.278
Cast Iron	0.270
Concrete	0.2

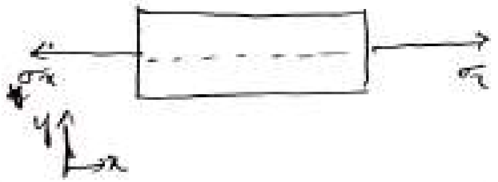
(Most ductile material) — Gold

Cast Iron → (exception) (Brittle material) (0–0.25 range)

Concrete (0.1 – 0.2)

ANALYSIS OF NORMAL STRAIN :-

1D analysis (loading):-



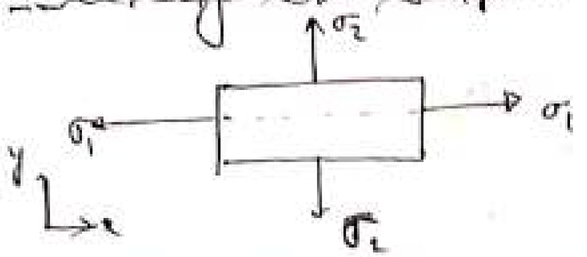
$$\boxed{\epsilon_x = \frac{\sigma_x}{E}}$$ (hook's law)

$$\epsilon_y : \mu = \frac{-\epsilon_y}{\epsilon_x} \Rightarrow \epsilon_y = -\mu \epsilon_x$$

$$\Rightarrow \boxed{\epsilon_y = \frac{-\mu \sigma_x}{E}}.$$ (from Poisson Ratio)

If σx applied in longitudinal direction then strain in longitudinal direction is (σ/E) and strain in transverse direction is (−μ.σ/E).

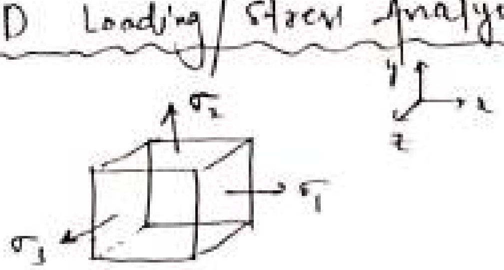
2D loading Stress Analysis ! -



2*

$$\boxed{\begin{aligned} \epsilon_x &= \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \\ \epsilon_y &= -\mu\frac{\sigma_1}{E} + \frac{\sigma_2}{E} = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} \end{aligned}}$$

3D Loading / Stress Analysis ! -



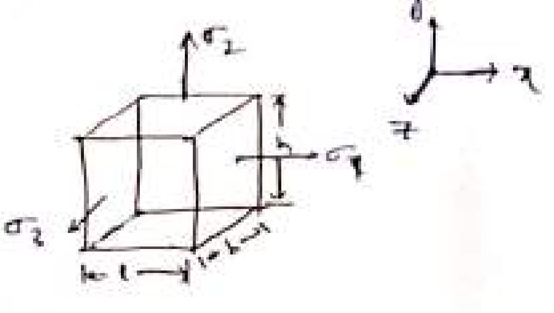
***

$$\boxed{\begin{aligned} \epsilon_x &= \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} \\ \epsilon_y &= \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_3}{E} \\ \epsilon_z &= \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \end{aligned}}$$

{ Tensile stress → +ve
Compressive stress → −ve. }

** VOLUMETRIC STRAIN (ϵ_v) : -

$$\epsilon_v = \frac{\text{Change in volume}}{\text{Original volume.}}$$



V = volume of cuboid = l.b.h

$$\Delta V = \Delta l.b.h + l\,\Delta b.h + l.b\,\Delta h$$

$$\epsilon_v = \frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}$$

$$\boxed{\epsilon_v = \epsilon_x + \epsilon_z + \epsilon_y}$$ **

$$\epsilon_x = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_y = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_z = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

Continue....

$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

$$\boxed{\epsilon_v = \frac{\sigma_1 + \sigma_2 + \sigma_3}{E}(1-2\mu)} \; **$$

e!-

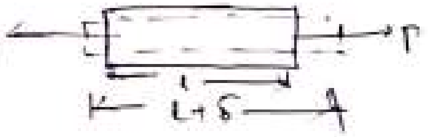
for incompressible fluids

$$\rightarrow \boxed{\Delta V = 0}$$
$$\downarrow$$
$$\boxed{\epsilon_v = 0} \longrightarrow \boxed{\frac{\sigma_1+\sigma_2+\sigma_3}{E}(1-2\mu) = 0}$$
$$\downarrow$$
$$\boxed{1-2\mu = 0} \Rightarrow \boxed{\mu = 1/2 = 0.5}$$

Relation between E, G, K and Poisson's Ratio(μ) :-

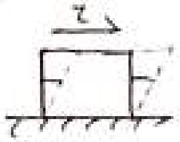
Young's modulus of elasticity (E) :-

$$E = \frac{\text{Linear stress}}{\text{Linear strain}} = \frac{(P/A)}{(\delta/l)} = \frac{\sigma}{\epsilon}$$



Shear Modulus of Rigidity (G, C) :-

$$G/C = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$$



Bulk modulus of Elasticity (k) :-

$$K = \frac{\text{Volumetric Stress}}{\text{Volumetric strain}}$$

$$K = \frac{P}{(\Delta V/V)}$$

Continued....

Relation's :-

$$\boxed{\begin{aligned} E &= 2G(1+\mu) \quad \cdots ① \\ E &= 3K(1-2\mu) \quad \cdots ② \\ E &= \frac{9GK}{G+3K} \quad \cdots ③ \end{aligned}}$$

from eqⁿ-1-2

$$2G(1+\mu) = 3K(1-2\mu)$$

$$2G + 2G\mu = 3K - 6K\mu$$

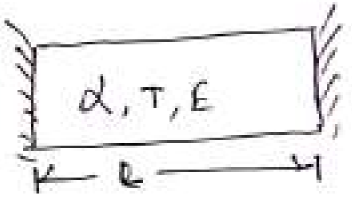
$$\mu(2G+6K) = 3K - 2G$$

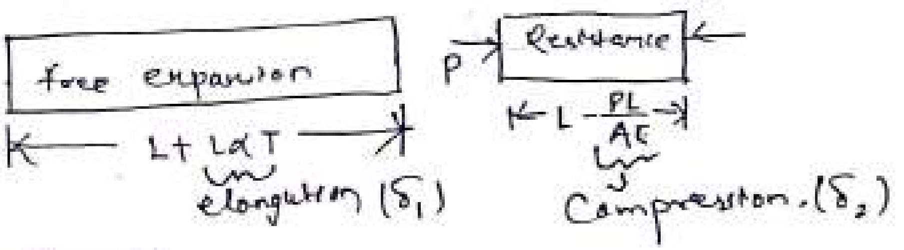
$$\boxed{\mu = \frac{3K-2G}{6K+2G}}$$

Thermal Stress (Topic-3) (Lecture-13)

~~Prism~~ Prismatic and non-prismatic Section :-

Case-1: Prismatic Section :-





- Due to fix end.

$$\delta_1 = \delta_2$$

$$L\alpha t = \frac{PL}{AE}$$

$$\boxed{\sigma_T = E\alpha T}$$

↳ It is a axial stress.

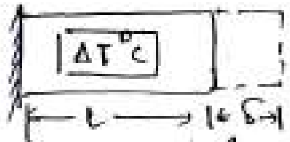
Topic-3! THERMAL STRESS !-

* Co-efficient of thermal expansion (α) :-

→ It is defined as thermal strain per unit change in temperature when subjected to free expansion.

$$\boxed{\alpha = \frac{\epsilon_T}{\Delta T^\circ C}} = \frac{\delta/L}{\Delta T}$$

$$\boxed{\delta = L\alpha . \Delta T}$$



free Expansion

*

$$\boxed{\begin{aligned} \alpha_{Al} &= 24 \times 10^{-6} /^\circ C \\ \downarrow & \quad \uparrow 5^\circ \\ \alpha_{Br} &= 19 \times 10^{-6} /^\circ C \\ \downarrow & \quad \uparrow 4 \\ \alpha_{Cu} &= 15 \times 10^{-6} /^\circ C \\ \downarrow & \quad \uparrow 3 \\ \alpha_{St} &= 12 \times 10^{-6} /^\circ C \end{aligned}}$$

Note :-

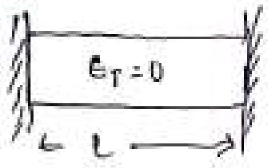
→ It is a Material property.

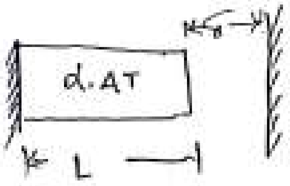
→

* THERMAL STRAIN! -

$$\epsilon_T \text{ (thermal strain)} = \frac{\text{Change in length}}{\text{Original length.}}$$

Fixed beam





No thermal strain!

so, $\boxed{\epsilon_T = 0}$

free expansion $= L\alpha . \Delta D$

If (i) $L\alpha . \Delta T > \delta$

$$\boxed{\epsilon_T = \delta/L}$$

(ii) $L\alpha . \Delta T < \delta$

$$\epsilon_T = \frac{L\alpha . \Delta T}{L}$$

$$\boxed{\epsilon_T = \alpha . \Delta T}$$

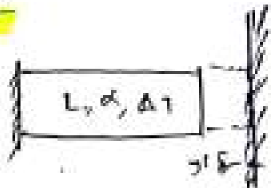
↳ Example of free expansion.

** THERMAL STRESS !-

Case-1

↳ $\delta = L\alpha \cdot \Delta t$
↓
Fee expansion → No thermal stress

Case-2



$$\boxed{L\alpha \cdot \Delta T > \delta}$$

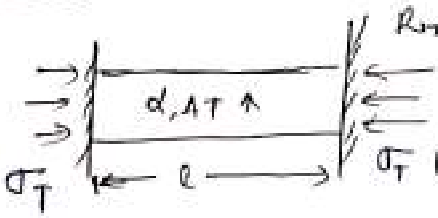
$$\sigma_T = E \times (\epsilon_T)_{prevented}$$

$$= E \times \frac{\text{Change in length prevented}}{\text{Initial length}}$$

$$\boxed{\sigma_T = E\,\frac{(L\alpha \cdot \Delta T - \delta)}{L}}$$
***

Case-3

Rise in temperature.



σ_T (Compressive in nature)

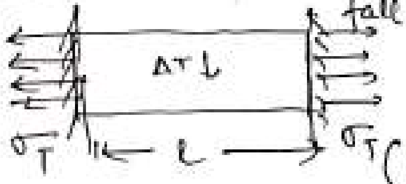
$$\sigma_T = E \times (\epsilon_T)_{prevented}$$

$$\sigma_T = E \times \frac{(\cancel{L}\alpha \cdot \Delta T - \delta^0)}{\cancel{L}}$$

$$\boxed{\sigma_T = E\alpha \cdot \Delta T}$$
***

Case-4

fall in temp.



σ_T (Tensile in nature)

*** Note.

Rise in temp. → compressive thermal stress

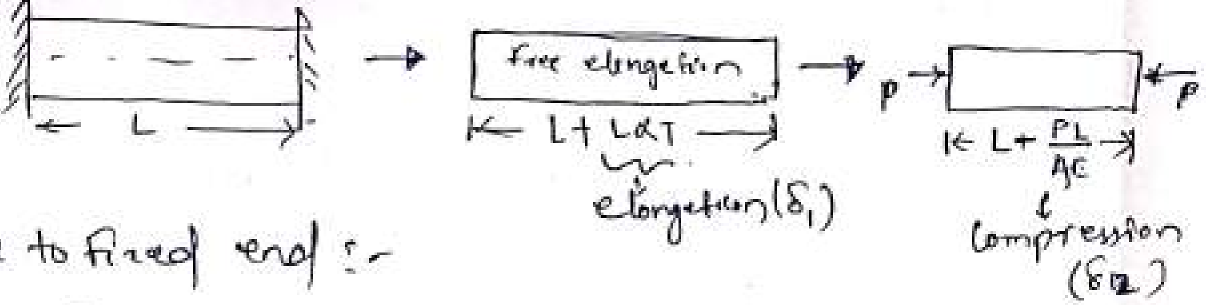
fall in temp → Tensile

te !- Stresses

- Mechanical / Primary Stress
  - → Due to application of External load.
- Thermal / Secondary Stress
  - → Develops due to change in temperature.

2 * Tharmal Stress on Presamatric and non Presmetric section :-

* Case-1 (Presmatric Section) :-



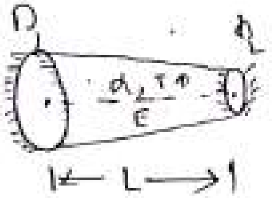
Due to fixed end :-

$$\delta_1 = \delta_2$$

$$L\alpha T = \frac{PL}{AE} \qquad \because \frac{P}{A} = \sigma_T$$

$$\boxed{\sigma_T = E\alpha T}$$

* Case-2 : (Non Presmatic Section) :-



for free expansion.

$\boxed{\delta_1 = L\cdot\alpha\cdot T}$ elongation.

for resistance :-

$$\boxed{\delta_2 = \frac{PL}{\frac{\pi}{4}(d_1 d_2)\cdot E}}$$

Compression.

for the fixed condition.

$$\delta_1 = \delta_2$$

$$\boxed{L\cdot\alpha\cdot T = \frac{P\cdot L}{\frac{\pi}{4}(D_1 D_2)\cdot E}}$$

~~Maximum stress → $\frac{P}{\frac{\pi}{4}D_2^2}$~~

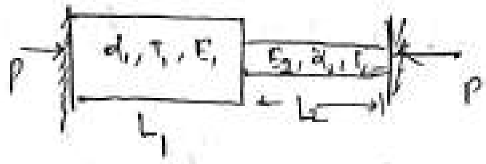
Minimum stress ~~$\frac{P}{\frac{\pi}{4}}$~~

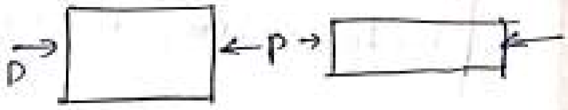
hen,

Maximum Stress → $\frac{P}{\frac{\pi}{4}D_2^2}$

Minimum stress → $\frac{P}{\frac{\pi}{4}\cdot D_1^2}$

Case-1 (~~composite section~~) :-





Free Expansion.

$$\boxed{\delta_1 = L_1\alpha_1 T_1 + L_2\alpha_1 T_2}$$ elongation.

Resistance

$$\boxed{\delta_2 = \frac{P.L_1}{A_1E_1} + \frac{PL_2}{A_2E_2}}$$ compression.

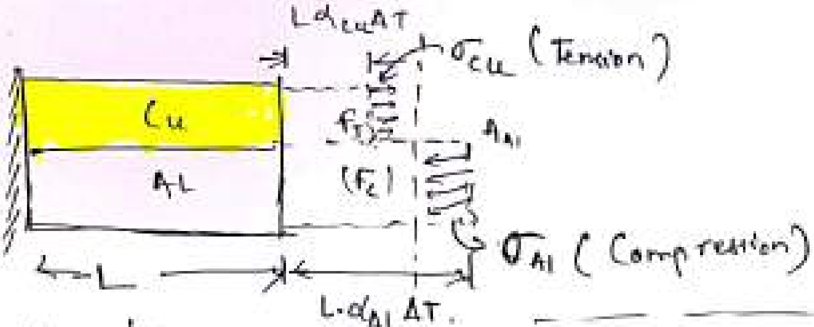
for the fixed condition :-

$$\boxed{\delta_1 = \delta_2}$$

$$L_1\alpha_1 T_1 + L_2\alpha_2 T_2 = \frac{P.L_1}{A_1E_1} + \frac{P.L_2}{A_2E_2}$$

$\Rightarrow P = ?$

Thermal Stress in Composite Section :-



e-1 Rise in temp.

$\alpha_{Al} > \alpha_{cu}$

Equilibrium condn:

$F_T = F_c$

$$\boxed{\sigma_{cu} A_{cu} = \sigma_{Al} A_{Al}} \quad \text{———} ①$$

Note:-
Due to rise in temperature material with higher α is sub subjected to compression and one with lower " α " is subjected to tension.

~~Among~~ 2 Unknown are given here and only one eqn is determine ~~eqn~~, so we need a compatibility eqn.

continue...

Compatibility eqⁿ

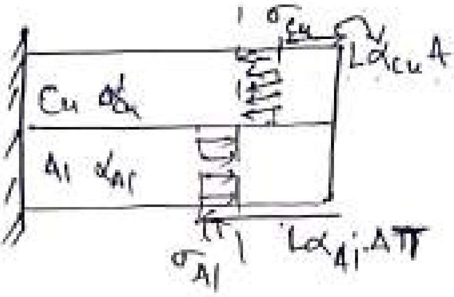
Deformation in Cu = Deformation in Al

$$l\,\alpha_{Cu}\,\Delta T + \left(\frac{\sigma_{Cu}}{E_{Cu}} \times l\right) = l\,\alpha_{Al}\,\Delta T - \left(\frac{\sigma_{Al}}{E_{Al}} \times l\right)$$

***

$$\boxed{(\alpha_{Al} - \alpha_{Cu})\,\Delta T = \frac{\sigma_{Cu}}{E_{Cu}} + \frac{\sigma_{Al}}{E_{Al}}} \quad \text{---} ②$$

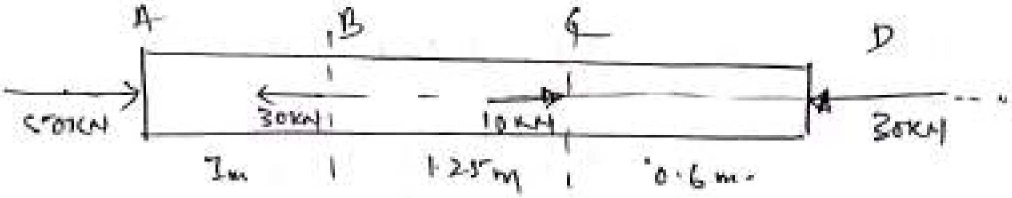
Case-2 . Fall in temp:-



Note- Material with greater α will be subjected to tension material with lower "α" subjected to compression due to fall in temperature.

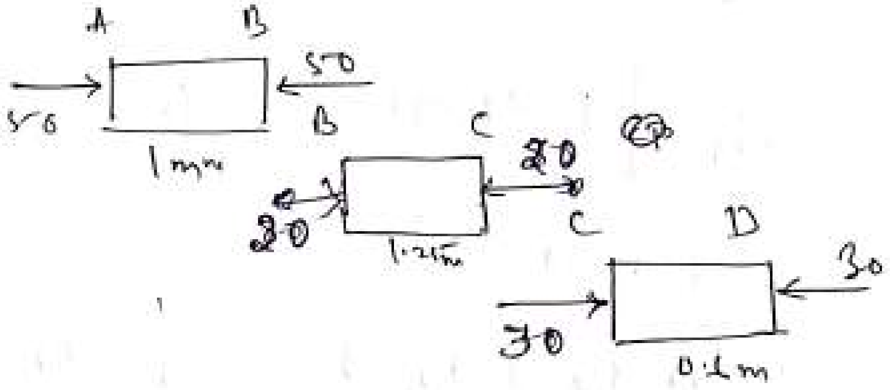
(Axial loading)

1.) ~~fosop~~ For the loading diagram in figure determine the deformation in the bar. ($G = 200$ Gpa , $A = 5 \text{ cm}^2$



$\Sigma f_x = 0$

$50 + 10 - 30 - 30 = 0$



$$\delta L = \delta L_1 + \delta L_2 + \delta L_3$$

$$= -\frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} - \frac{P_3 A_3}{AE}$$

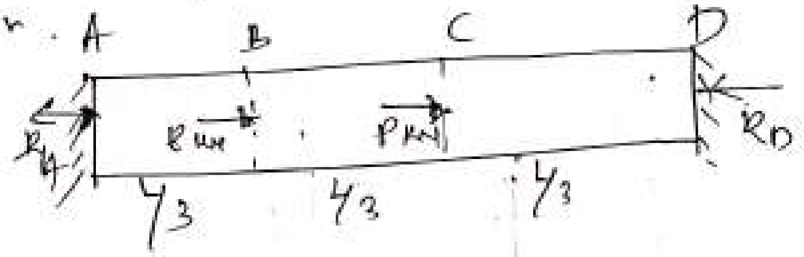
$$= \frac{P_2 L_2 - P_1 L_1 - P_3 L_3}{AE} = \frac{(20 \times 1.25) - (50 \times 1) - (30 \times 0.6)}{5 \times 200}$$

$$= \frac{(20 \times 1.25) - (50 \times 1) - (20 \times 0.2)}{5 \times 10^{-4} \times 200 \times 10^{9} \text{ N/m}^2}$$

$$= -0.93 \times 10^{-7} \text{ m}$$

$$= -0.93 \text{ mm} = 0.92 \text{ (compressive)}$$

2) For the fixed bar shown in figure determine the support reaction.



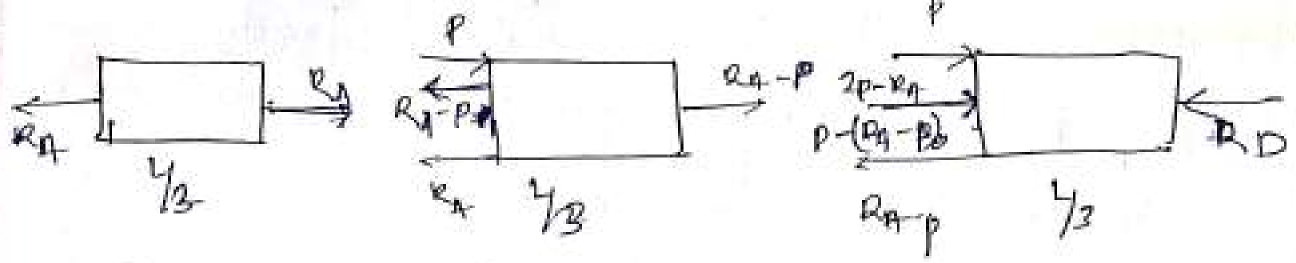
$\Sigma f_x = 0$

$R_A \pm P \pm P \pm R_D = 0$

$R_A \pm R_D = +2P$

~~$R_A = R_B + 2P - R_A$~~

$$\boxed{R_D = 2P - R_A}$$



$$\delta_{AD} = 0 \Rightarrow \delta_{AB} + \delta_{BC} + \delta_{CD} = 0$$

$$\Rightarrow \frac{P_1 l_1}{AE} + \frac{P_2 L_2}{AE} + \frac{P_3 L_3}{AE} = 0$$

$$\Rightarrow \frac{l}{3AE}\left[P_1 + P_2 + P_3\right] = 0$$

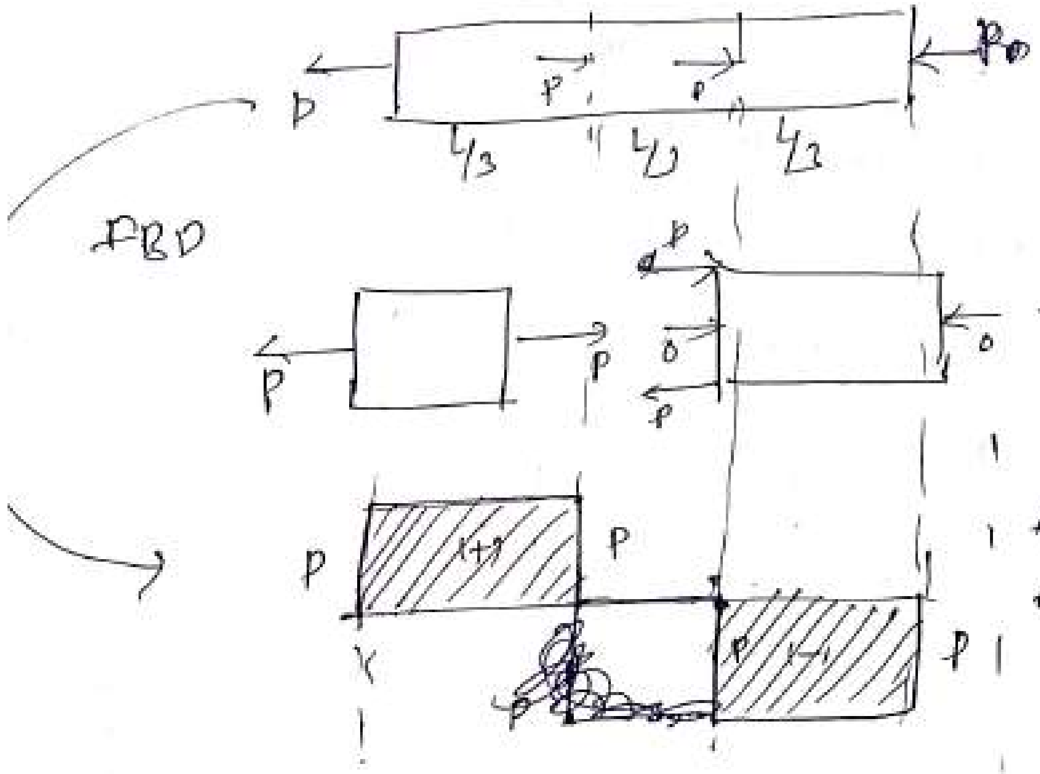
$$\Rightarrow P_1 + P_2 + P_3 = 0$$

$$\Rightarrow R_A + R_A - P - (2P - R_A) = 0$$

$3R_A = 3P$ | $R_D = 2P - R_A$

$\boxed{R_A = P}$ | $= 2P - P = P$

* Axial thrust diagram :-

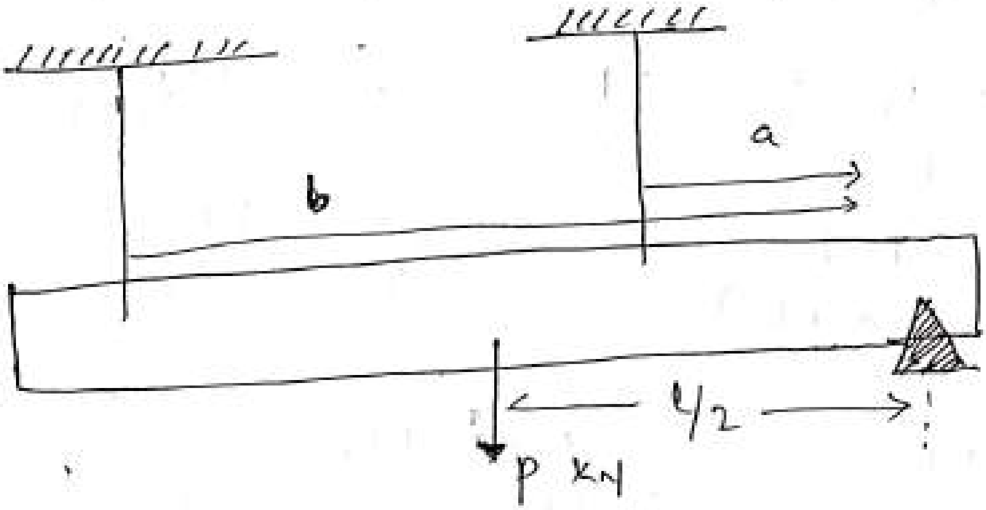


Axial thrust in AB = +P

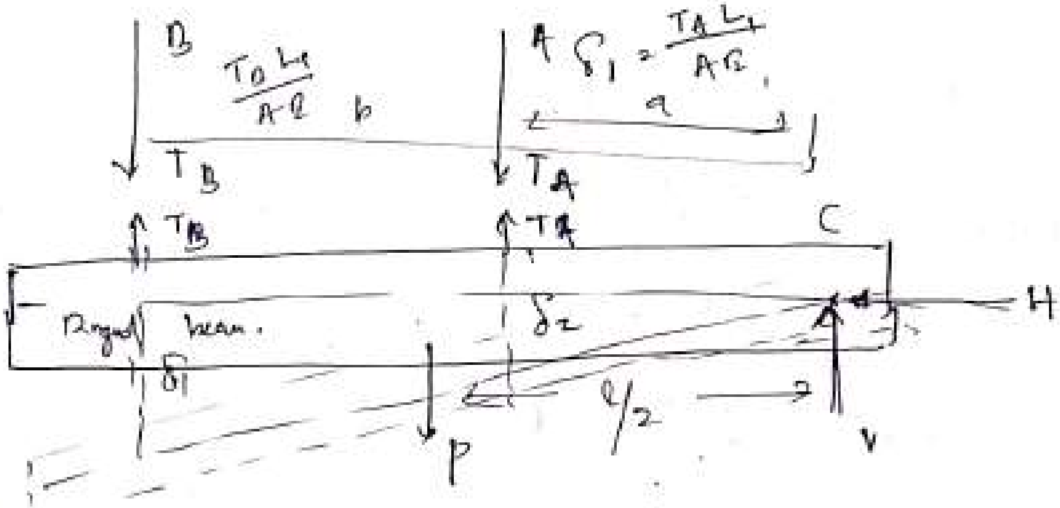
Axial " " BC = 0

" " " = −P

for the rigid beam show in figure determ tension in the cable A and B ?



FBD



$\Sigma M_C = 0$ ↻ +ve, ↺ -ve

$$\boxed{T_A \times a + T_B \times b - P \times \frac{L}{2} = 0} \text{ ——— } ①$$

compatibility eqn by using rigidity.

$$\tan\theta : \frac{\delta_1}{a} = \frac{\delta_2}{b}$$

$$\frac{T_A \cdot \cancel{L_1}}{\cancel{AE} \cdot a} = \frac{T_B \cdot \cancel{L_1}}{\cancel{AE} \cdot b}$$

$$T_A = \frac{T_B \times a}{b} \Rightarrow \boxed{T_A = T_B \left(\frac{a}{b}\right)} \text{ — } ②$$

Put ② in ①

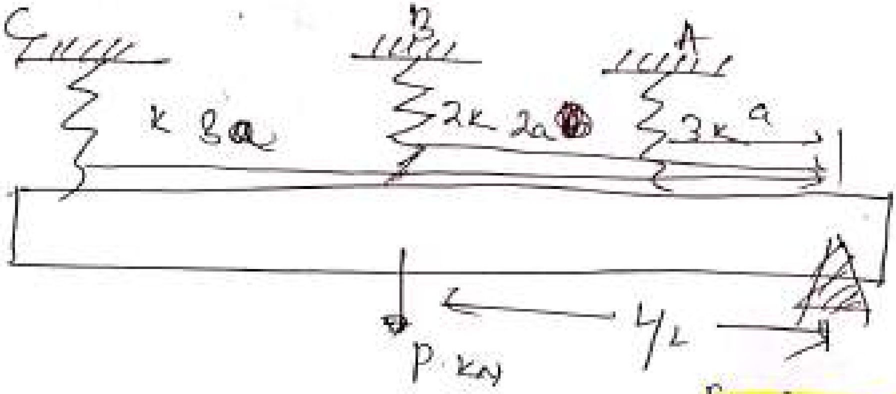
$$T_B \cdot \left(\frac{a^2}{b}\right) + T_B \cdot b = \frac{PL}{2} \quad ; \quad T_A = T_B\left(\frac{a}{b}\right)$$

$$T_B\left(\frac{a^2+b^2}{b}\right) = \frac{PL}{2} \quad ; \quad = \frac{PL \cdot \cancel{b}}{2(a^2+b^2)}\left(\frac{a}{\cancel{b}}\right)$$

$$T_B = \frac{PL \cdot b}{2(a^2+b^2)} \quad , \quad T_A = \frac{PL \cdot a}{2(a^2+b^2)}$$

$$\boxed{T_A = \frac{PL \cdot a}{2(a^2+b^2)} \quad , \quad T_B = \frac{PL \cdot b}{2(a^2+b^2)}} \text{ Ans}$$

Q.4 for the rigid beam shown in the figure. determine the ratios of forces in spring A, B & C?



for Spring

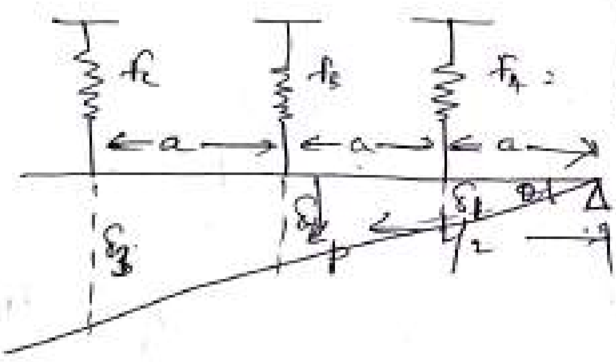
force = stiffness × deflection

$$\boxed{f = k\delta}$$

$$\boxed{\delta = \frac{f}{k}}$$

Solⁿ

FBD



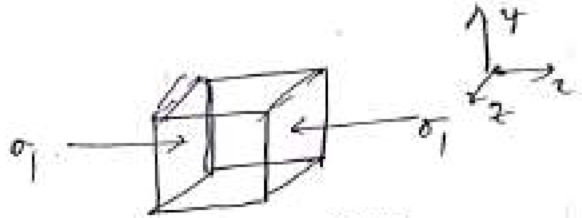
$$\tan\theta = \frac{\delta_1}{a} = \frac{\delta_2}{2a} = \frac{\delta_3}{3a}$$

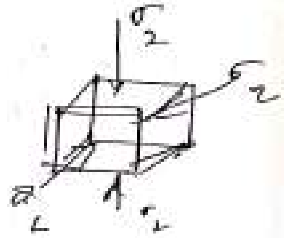
$$\tan\theta = \frac{f_A}{2K\cdot a} = \frac{f_B}{4Ka} = \frac{f_C}{k\cdot 3a}$$

Ans $\boxed{f_A : f_B : f_C = 3 : 4 : 3}$ Ans

:- Poisson Ratio :-

A cubical block is subjected to comp. load of sigma σ_1 in x ~~[illegible]~~ direction due to which strain is develop y and z direction to reduce the strain to half of magnitude comp. stress of sigma. σ_2 is applied in y and z dir. Determine the relation betn σ_1 and σ_2.





Strain that develop in y & z

$$\epsilon_{y_1} = \epsilon_{z_1} = -\mu\frac{(-\sigma_1)}{E} = \mu\frac{\sigma_1}{E}$$

$$\epsilon_y = \frac{-\sigma_2}{E} + \mu\frac{\sigma_1}{E} + \mu\frac{\sigma_2}{E}$$

$$\epsilon_{y_2} = -\frac{\sigma_2}{E} + \mu\frac{\sigma_1}{E} + \mu\frac{\sigma_2}{E}$$

$$\epsilon_{z_2} = -\frac{\sigma_2}{E} + \mu\frac{\sigma_1}{E} + \mu\frac{\sigma_2}{E}$$

$$\frac{\epsilon_{y_1}}{2} = \epsilon_{y_2} \quad \text{or} \quad \frac{\epsilon_{z_1}}{2} = \epsilon_{z_2}$$

$$\frac{\mu\sigma_1}{2\cdot E} = -\frac{\sigma_2}{E} + \mu\frac{\sigma_1}{16} + \mu\frac{\sigma_2}{E}$$

$$-\frac{\mu\sigma_1}{2} = -\sigma_2(1-\mu)$$

Ans

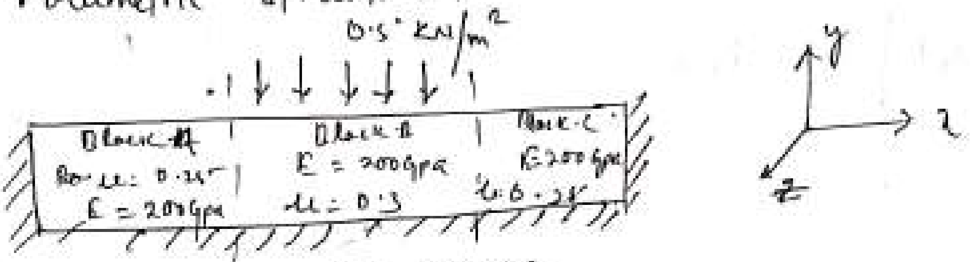
$$\boxed{\sigma_2 = \frac{\mu\sigma_1}{2(1-\mu)}}$$

Q.2 for the block shown in figure, determine.

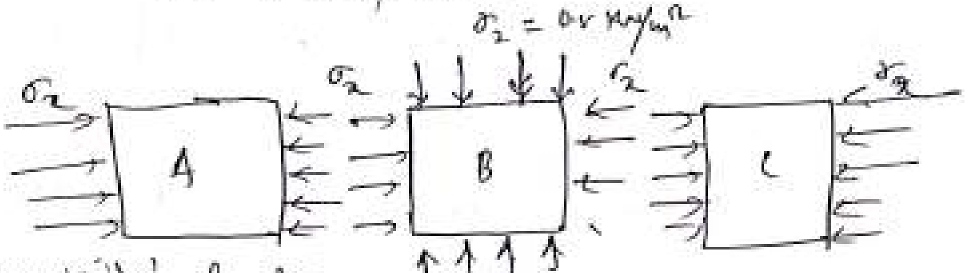
A) Stress in x direction

B) Strain in block B in x, y, & z direction

C) Volumetric strain "B".



Solⁿ



A) Stress in 'x' direction

$$\epsilon_x = 0$$

$$\Rightarrow \epsilon_{A_x} + \epsilon_{B_x} + \epsilon_{C_x} = 0$$

$$\Rightarrow -\frac{\sigma_x}{E_A} + \left[\frac{\sigma_x}{E_B} + \mu_B\left(\frac{\sigma_z}{E_B}\right)\right] + \mu_C\cdot\frac{\sigma_x}{E_C} = 0$$

$$\Rightarrow \frac{\sigma_x}{200}\left[-1 \;\phi\; \cancel{+1} + 0.3 + 0.25\right] = 0$$

$$\Rightarrow 3\sigma_x + \mu_B\sigma_z = 0$$

$$\Rightarrow 3\sigma_x = \mu_B\cdot\sigma_z.$$

$$= \frac{0.3\times0.5}{3} = 0.05 \text{ kN/m}^2$$

$$\boxed{\sigma_x = 0.5 \text{ kN/m}^2}$$

B. for Blo B

$$\sigma_x = \frac{\sigma_x}{E} - \mu\frac{\cancel{\sigma_y}^{0}}{E} - \mu\frac{\sigma_z}{E}$$

$$= -\frac{0.05\times10^3}{200\times10^9} + 0.3\left(\frac{0.5\times10^3}{200\times10^9}\right)$$

$$= 0.5\times10^9$$

$\epsilon_y = \frac{\cancel{\sigma_y}^{0}}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_z}{E}$

$= \frac{0.3 \times 10^3}{200 \times 10^9} \left(0.05 + 0.5 \right) =$ ~~0.0825 · 0.825 ×10^9~~ 0.825×10^{-9}

$\epsilon_z = \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \cancel{\mu \frac{\sigma_y}{E}}^{0}$

$= -\frac{0.5 \times 10^3}{200 \times 10^9} + 0.3 \left(\frac{0.05 \times 10^3}{200 \times 10^9} \right) = -2.425 \times 10$

. Volumetric strain

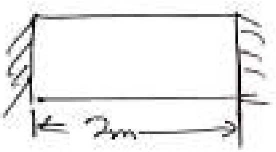
$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

$= (0.5 \cancel{\times 200} + 0.825 - 0.2425) \times 10^{-9}$

$= 1.08 \times 10^{-9}$ Ans

Thermal stress

Q.1



$\alpha = 10 \times 10^{-6}$ °/c

E = 200 gpa

$\Delta T = 15°C.$

Determine thermal stress.

Soln

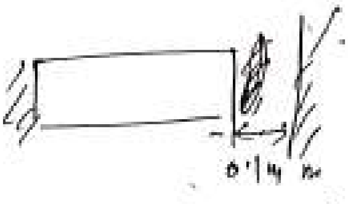
thermal stress $= E\alpha\Delta T$

$= 10 \times 10^{-6} \times 200 \times 10^9 \times 15$

$= 30 \times 10^6$ N/mm.

$=$ 30 Mpa.

Q.2)

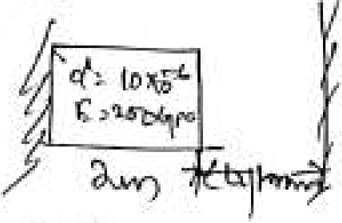


determine thermal stress.

Soln $L\alpha\Delta T = 2 \times 10^3 \times 200 \times 10^3 \times 10 \times \quad = 0.3$mm.

$\delta =$ 0.1mm.

2)



thermal stress ?

Sol^n.

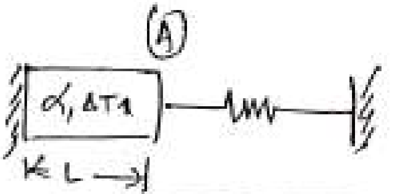
$$L\alpha\cdot\Delta T = 2\times10^{3}\times10\times10^{-6}\times15 = 0.3\,mm$$

$$\delta = 0.1\,mm$$

$$\sigma_T = E\times\epsilon_T \text{ prior}.$$

$$= 200\times10^{3}\left(\frac{0.3-0.1}{2\times10^{3}}\right) = 20\,mpa \text{ (Ans)}$$

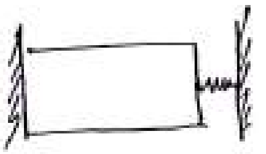
3)

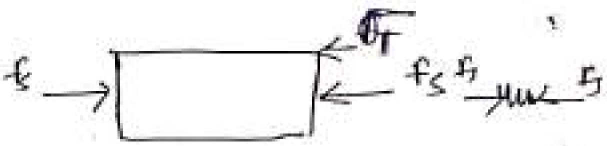


If $\boxed{L\alpha\Delta T > \delta_{spring}}$

then determine thermal stress & total stress.

Sol^n





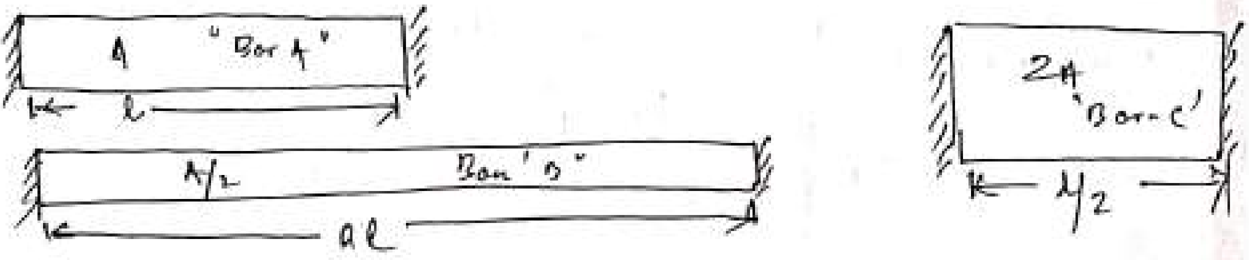
thermal stress $\sigma_T = E\times(\epsilon_T)_{\text{prevented}}$

$$\boxed{\sigma_T = E\times\left(\frac{L\alpha\Delta T - \delta_{spring}}{L}\right)}$$

$$\boxed{\sigma_{mech} = \frac{f_s\,\delta_{spring}}{A} = \frac{k\cdot\delta_{sp}}{A}}$$

$$\boxed{\sigma_{Total} = \sigma_{th} + \sigma_{mech}.}$$

Q.3 for the three fixed bar shown in the figure subjected to same temprature rise and made of same material deterr the ratio of stress develope in bar A. B and C.

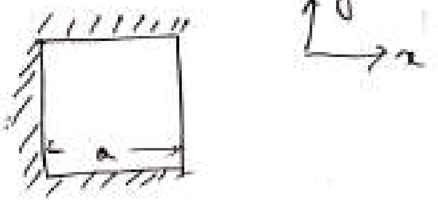


Soln.

Ratio of the thermal stress

$$\sigma_{T_A} : \sigma_{T_B} : \sigma_{T_C} = 1 : 1 : 1.$$

Q.4. A square plate of side a show in the figure is subjected to temprature rise of "T". Determine the final length of bar in the ~~⊕~~ x direction.



Soln

No stress in 'x' direction

Deformation in 'x' direction due to temp. chang in 'x' direction = $a \cdot \alpha \cdot T$ (elongation).

Thermal stress in "y" direction = $E\alpha \cdot \Delta T = \sigma_{Ty}$. (Comp.)

thermal strain in "y" direction = $\mu \frac{\sigma_y}{E}$

$$= \frac{\mu E \alpha T}{E} = \mu \alpha T$$

$$= a \cdot \mu \alpha T \text{ (elongation)}$$

Total deformation:-

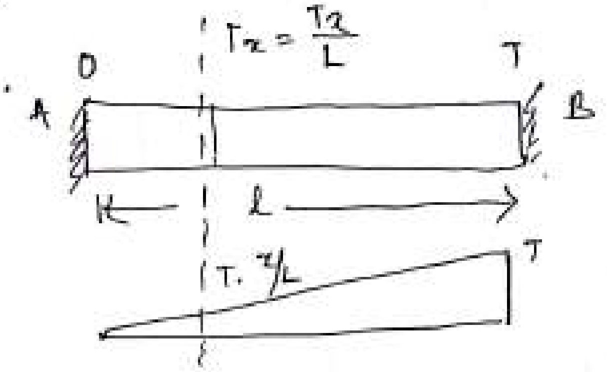
$$= a\alpha T + a \cdot \mu \alpha T$$

~~Final length~~ $= a\alpha T(1+\mu)$ (~~for~~

$$\boxed{\text{Final length} = a + a\alpha T(1+\mu)}$$ Ans

Q.5- A fixed bar of length 'L' is subjected to temperature rise such that it is 0 at A and T at B at a distance x from A is given be T·x/L. Determine the thermal stress develope in a bar.

Sol.



take a small strip from A ~~from A~~ of "x" over length dx
temp chang is ~~constt~~ constant.

$$d\delta_{strip} = \frac{dx \cdot \alpha \cdot Tx}{L}$$

$$\boxed{d\delta = \frac{\alpha T \cdot x \cdot dx}{L}}$$

δ for entire length.

$$\int_0^\delta d\delta = \int_0^L \frac{\alpha T \cdot x \cdot dx}{L}$$

$$\delta = \frac{\alpha T}{L} \int_0^L x\,dx$$

$$\delta = \frac{\alpha T}{L}\left[\frac{x^2}{2}\right]_0^L = \frac{\alpha T}{\cancel{L}}\left[\frac{L^{\cancel{2}}}{2}\right]$$

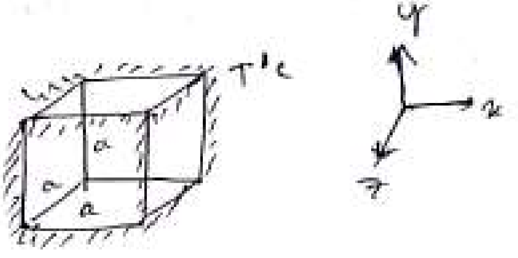
$$\boxed{\delta = \frac{\alpha T \cdot L}{2}}$$

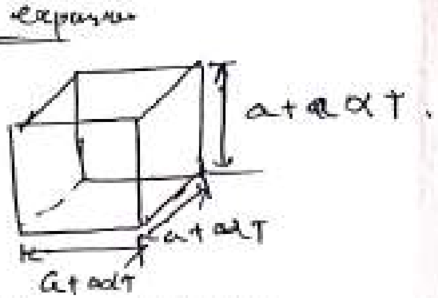
$$\sigma_T = E \times \frac{L\alpha \cancel{L}}{2 \cdot \cancel{L}}$$

$$\boxed{\sigma_T = \frac{E\alpha T}{2}}$$ Ans

Q. 1. cube of side 'a' fixed at the all the side subjected to temp. rise of T, Determine the thermal stress developed in the bar.

Sol-1





Expansion in x, y, z direction $= \delta_1 = a \cdot \alpha \cdot T$

Due to Restrain :-

Thermal strain in x, y, z $= \cancel{E\alpha T} - \frac{\sigma}{E} + \mu\frac{\sigma}{E} + \mu\frac{\sigma}{E}$

thermal deformation $= \left(-\frac{\sigma}{E} + \mu\frac{\sigma}{E} + \mu\frac{\sigma}{E}\right) \cdot a$

Compression $: \frac{\sigma}{E}(1-2\mu) \cdot a = \delta_2$

So $\delta_1 = \delta_2$

$\cancel{a} \cdot \alpha \cdot T = \frac{\sigma}{E}(1-2\mu) \cdot \cancel{a}$

$$\boxed{\sigma = \frac{E\alpha T}{\cancel{a}1-2\mu}}$$ Ans

Sol-2

Case-1 $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

$= \frac{\cancel{a}\alpha T}{\cancel{a}} \times 3$

$= 3\alpha T$ (exp.)

Case-2 $\epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$

$= \frac{(-\sigma) \times 3}{E}(1-2\mu)$

$= \frac{(-3\sigma)}{E}(1-2\mu)$

elongation = Compression.

$\cancel{3}\alpha T = \frac{\cancel{3}\sigma}{E}(1-2\mu$

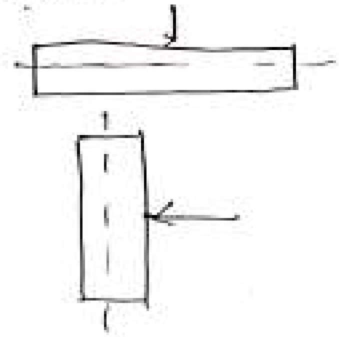
$$\boxed{\sigma_T = \frac{E\alpha T}{1-2\mu}}$$

Ans

Lect-14

CH-3 SFD and BMD

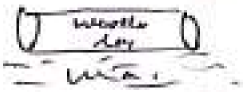
* BEAM! -



Example! -

→ Bat hitting the ball is a beam.

→ Wooden log flooting in water is also a beam



→ Vertical member subjected to a longitudinal load is also a beam.

→ ~~Vertical~~ Horizantal member which subjected to a transverse load is a beam.

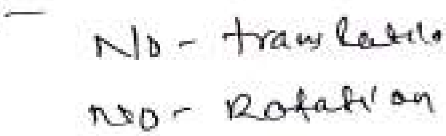
Definition! -

Any member subjected to transverse load is reffered as Beam.

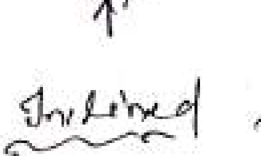
* Type of SUPPORT! -

1) Roller Support :- not allow the beam to move in axial direction of support.

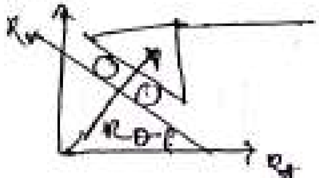
2. Hinged or Pinned! - No translation is allowed
free ~~static~~ rotation.



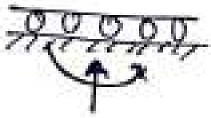
3. Fixed Support! - No-translation
No-Rotation.



4. Inclined Roller! - → looks like a roller but behave like a hinged.



5) Guided Roller :-



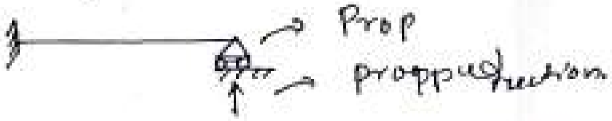
* Type Of Beam :-

1) Simply supported beam →

2) Cantilever Beam →

3) fixed Beam.

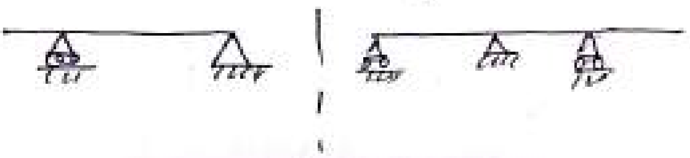
4) Propped cantilever Beam.



5) Contineous Beam

6) Over hanging Beam → Portion of the beam beyond the support.

Single Overhang | Double Overhang



* Beams with internal hinges

~~Non resistance~~
No resistance to moment

Additional equilibrium condition ← $\boxed{BM = 0}$ ← No resistance to moment

* DETERMINACY & INDETERMINACY OF BEAM :-

* Determinate Beam :- number equilibrium eqn are sufficient to determine the support reaction

$\boxed{D_s = 0}$

Indeterminate Beam!-

Number of equilibrium eqn are not suffic of determine the support ~~eqn~~ reaction.

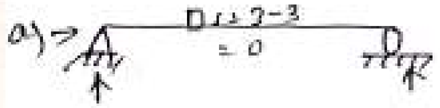
$$\boxed{D_s > 0}$$

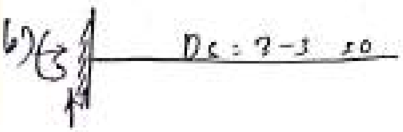
$$\boxed{D_s = r - s}$$

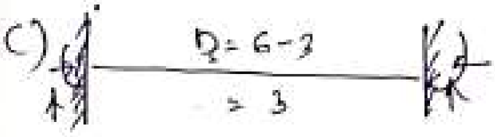
r = no of ~~equilibrium~~ unKnown eqn reaction eq

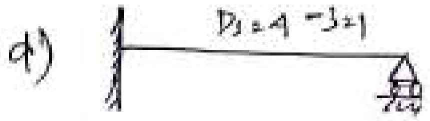
s = no of equilibrium eqn.

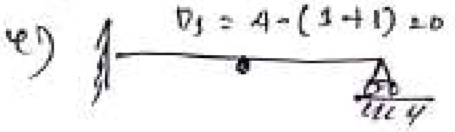
Example:-











Determinate → a, b, c.

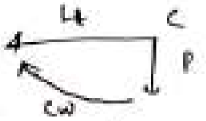
Interminate → c, d

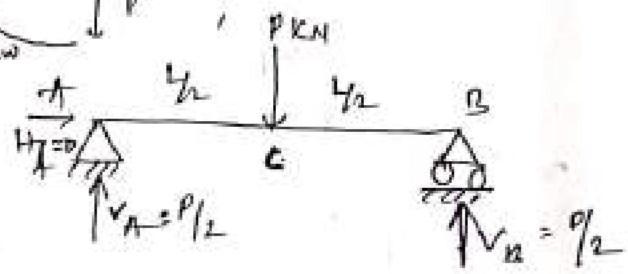
Note:-

In strength of material SFD and BMD is drawn only for determinate beam where as in structural Analysis, SFD and BMD is drawn for indeterminate Beam.

Lec-16

SUPPORT REACTION!-





- $\Sigma f_x = 0$

  (→ +ve, ← -ve)

  $$\boxed{H_A = 0}$$

- $\Sigma f_y = 0$

  (↑ +ve, ↓ -ve)

  $$\boxed{V_A + V_B = P} \quad — ①$$

- $\Sigma M_z = 0$

  $\Sigma M_{z_A} = 0$ (↻ +ve, ↺ -ve)

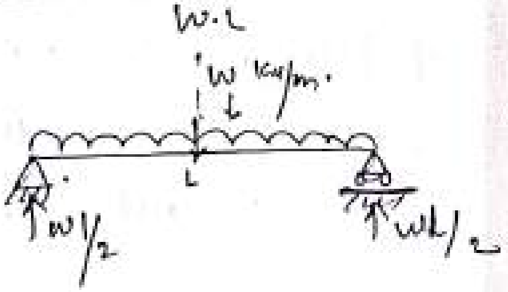
1) $P \cdot \frac{L}{2} - V_B \cdot L = 0$

2) $\boxed{V_B = \frac{P}{2}}$

$\boxed{V_A = \frac{P}{2}}$

②

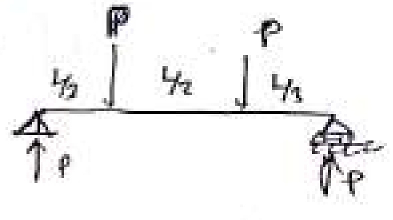
total load = Area under load * intensity diagram.



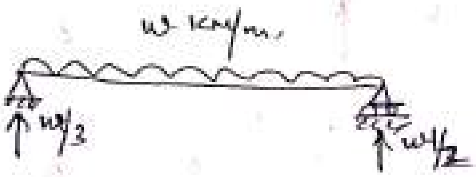
Shortcut - I : Symmetry :-

i)

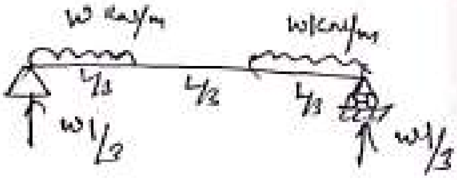
iii)



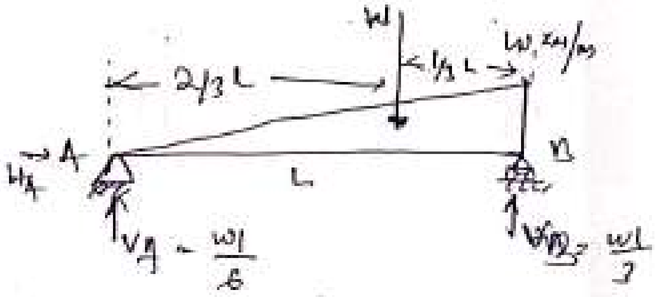
ii)



iv)



③



total load = $\frac{1}{2} \times w.L = W$

$\Sigma F_x = 0$

$\Rightarrow \boxed{H_A = 0}$

$\Sigma F_y = 0$ (↑+)(↓-)

$V_A - \frac{wl}{2} V_B = 0$

$\boxed{V_A + V_B = \frac{wl}{2}}$ → ①

$\Sigma M_{z_A} = 0$ (+↺, -↻)

$-L.V_B + \frac{2Wl}{3} = 0$

$V_B = \frac{2W.\cancel{l}}{3\cancel{l}}$

$= \cancel{2} \cdot \frac{wl}{3 \cdot \cancel{2}}$

$\boxed{V_B = \frac{wl}{3}}$, $V_A = \frac{wl}{2} - \frac{wl}{3} =$ [illegible]

$\boxed{V_A = \frac{wl}{6}}$

$\boxed{\frac{1}{3} + \frac{1}{6} = \frac{1}{2}}$

$$\sigma_{n\,max/min} = \frac{5+4}{2} \pm \sqrt{\left(\frac{5-4}{2}\right)^2 + (3)^2}$$

$$= 4.5 \pm \sqrt{\frac{1}{4} + 9}$$

$$= 4.5 \pm \sqrt{0.25 + 9}$$

$$= 4.5 \pm \sqrt{9.25}$$

$$= 4.5 \pm 3.02.$$

$$\boxed{\begin{aligned}\sigma_{p_1} &= 7.52 \text{ Mpa} \\ \sigma_{p_2} &= 1.48 \text{ Mpa}\end{aligned}}$$

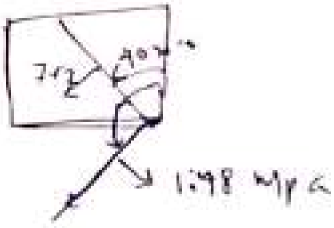
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 3}{5-(4)}$$

$$\theta_{p_1} = \frac{1}{2} \times \tan^{-1} 6$$

$$= 40.268^\circ$$

$$\theta_{p_2} = \theta_{p_1} + 90^\circ$$

$$= 130.268^\circ$$



Cross check

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cdot \cos\theta$$

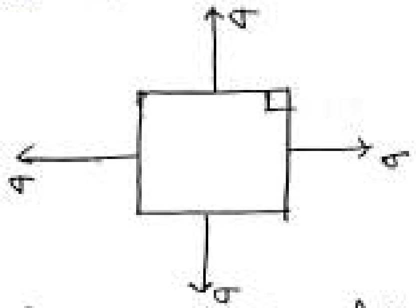
~~$= 5 \times \cos^2 40.268 + 3$~~

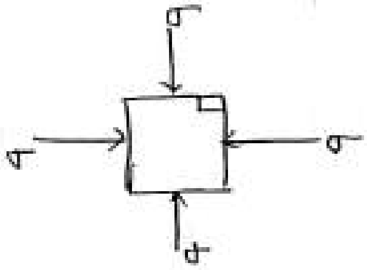
for θ_p ~~45~~ 40.268°

$$\boxed{\sigma_n = 7.52 \text{ mpa}}$$

× HYDROSTATIC LOADING :-

↳ Normal stresses on ⊥ʳ plane are equal and ~~same nature~~ of same ~~[illegible]~~ nature and Shear stress on these planes are equal to zero.





concept

Q.1 In Case of hydrostatic loading, Determine no. of principle plane? -

$$\tau_{x'y'} = (\overset{\sigma}{\cancel{\sigma_y}} - \overset{\sigma}{\cancel{\sigma_x}}) \frac{\sin 2\theta}{2} + \overset{0}{\tau_{xy}} \cos 2\theta$$

↓

for any value of 'θ',

↓

$\tau_{x'y'} = 0$

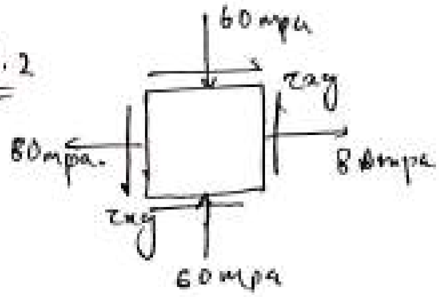
↓

Their exist infinite θ at which $\tau_{x'y'} = 0$

↓

There exist ~~infinite~~ Infinite ~~[illegible]~~ principle plane. (for static Loading)

Q.2



If ~~[illegible]~~ one of the principle stress 100 mpa. then determine.

i) τxy

(i) Value of other principle plane.

Solⁿ

$\sigma_x = +80$ mpa

$\sigma_y = -60$ mpa.

i) $\sigma_{P_1}/\sigma_{P_2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$100 = \frac{80-60}{2} + \sqrt{\left(\frac{80+60}{2}\right)^2 + \tau_{xy}^2}$$

$$90^2 = 70^2 + \tau_{xy}^2$$

$$\tau_{xy} = \sqrt{90^2 - 70^2} = \sqrt{(90+70)(90-70)}$$

$$\tau_{xy} = \sqrt{160 \times 20} = 40\sqrt{2} \text{ mpa.} \qquad \boxed{\sqrt{2} = 1.414}$$

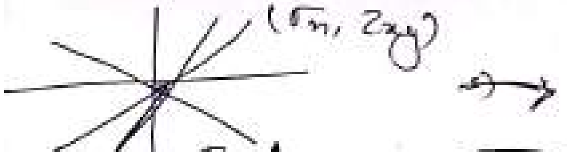
$$\tau_{xy} = 56.56 \text{ Mpa}$$

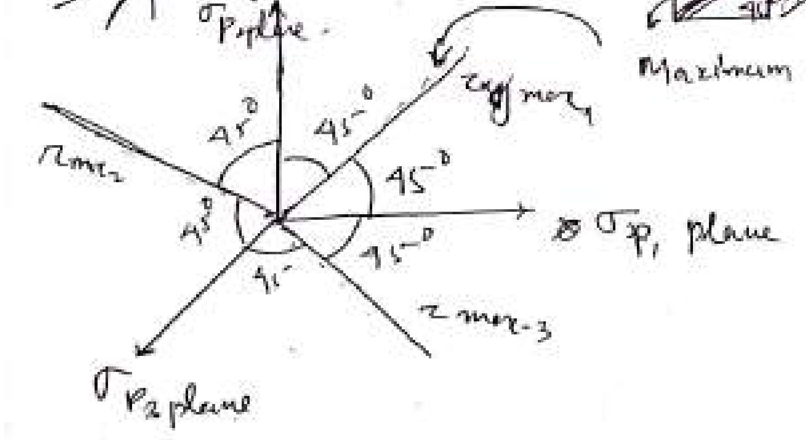
$$\sigma_{p_2} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

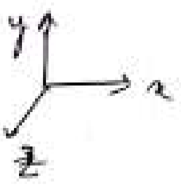
$$\sigma_{p_2} = \frac{80-60}{2} - 90$$

$$= 10 - 90 = -80 \text{ Mpa.}$$

MAXIMUM SHEAR PLANE :-

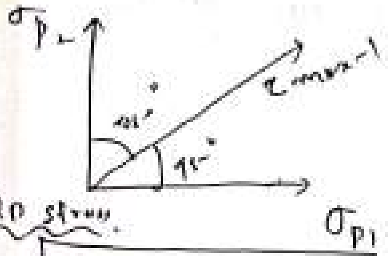






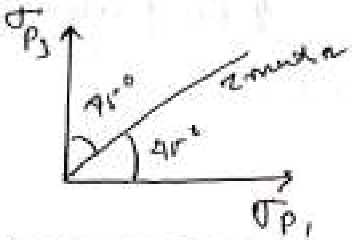
Definition:- From the infinite number of plane passing through a point, there exist 3 mutually perpendicular plane at which shear stress will be maximum. Such plane are referred as maximum / shear plane.

- It is equally inclined to the principle plane.

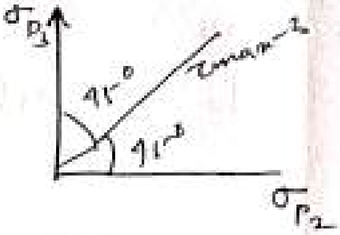


2D stress.

$$\boxed{\tau_{max_1} = \frac{\sigma_{p_1}-\sigma_{p_2}}{2}}$$



$$\boxed{\tau_{max_2} = \frac{\sigma_{p_2}-\sigma_{p}}{2}}$$



$$\boxed{\tau_{max_3} = \frac{\sigma_{p_2}-\sigma_{p_3}}{2}}$$

↳ In plane maximum shear stress.

** Maximum Shear stress @ point = Max^{m}(τ_{max_1}, τ_{max_2}, τ_{max_3})

→ Location of maximum shear plane:-

$$\tau_{x'y'} = (\sigma_y-\sigma_x)\frac{\sin 2\theta}{2} + \tau_{xy}\cos 2\theta$$

for $\tau_{x'y'}$ to be maximum.

$$\frac{d(\tau_{x'y'})}{d\theta} = 0$$

$$\Rightarrow \frac{d}{d\tau}\left[(\sigma_y-\sigma_x)\frac{\sin 2\theta}{2} + \tau_{xy}\cos 2\theta\right] = 0$$

$$\Rightarrow (\sigma_y-\sigma_x)\frac{\cancel{2}\cos 2\theta}{\cancel{2}} \cancel{+} - \tau_{xy}\,2\sin 2\theta = 0$$

$$\Rightarrow (\sigma_y-\sigma_x)\cos 2\theta = 2\tau_{xy}\sin 2\theta$$

$$\Rightarrow \boxed{\tan 2\theta = \frac{\sigma_y-\sigma_x}{2\tau_{xy}}}$$

$$\Rightarrow \boxed{\tan 2\theta_\tau = \frac{-1}{\left(\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right)}}$$

* Prove that maximum shear plane is equally inclined to the principle plane ! -

$$\Rightarrow \tan 2\theta_\tau = \frac{-1}{\left(\frac{2\tau xy}{\sigma_x - \sigma_y}\right)}$$

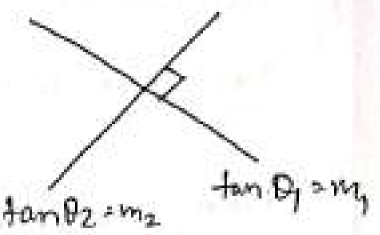
$$\Rightarrow \tan 2\theta_\tau = \frac{-1}{\tan 2\theta_p}$$

$$\Rightarrow \tan 2\theta_\tau \cdot \tan 2\theta_p = -1$$

$$|2\theta_p - 2\theta_\tau| = 90^\circ$$

$$\boxed{|\theta_p - \theta_\tau| = 45^\circ}$$ ... proved

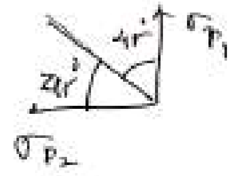
Basics Concept



$\tan\theta_1 \cdot \tan\theta_2 = -1$

$m_1 \cdot m_2 = -1$

$|\theta_2 - \theta_1| = 90^\circ$



-25

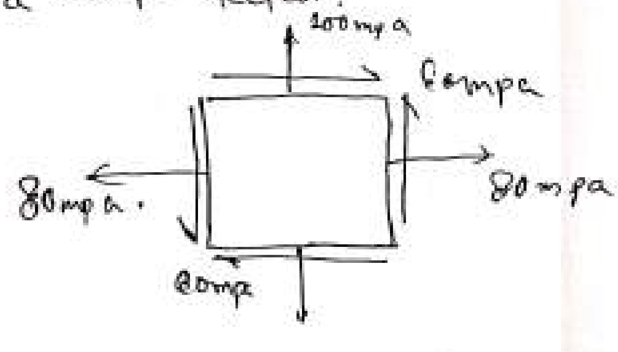
Numericals

Q.1 for the structural element shown in the figure, determine.

a) Principal stress and maximum shear stress.

b) Location of Principal plane and maximum shear plane

c) Show all the stresses with a neat sketch.



Solⁿ $\sigma_x = 80$ mpa

$\sigma_y = 200$ mpa.

$\tau_{xy} = 60$ mpa.

a) Principal strain

$$\sigma_{P_1}/\sigma_{P_2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 - \tau xy^2}$$

$$= \frac{80 + 200}{2} \pm \sqrt{\left(\frac{80 - 200}{2}\right)^2 - (60)^2}$$

$$= 140 \pm \sqrt{(-60)^2 - 60)^2}$$

$$= 140 \pm 60\sqrt{2}$$

$$\boxed{\begin{aligned}\sigma_{p_1} &= 140 + 60\sqrt{2} \\ &= 224.85 \text{ mpa}\end{aligned}}, \quad \sigma_{p_2} = 140 - 60\sqrt{2}$$

$$\boxed{\sigma_{p_2} = \cancel{80}\ \cancel{180}\ 55.15}$$

$$\tan \theta_{p_1} = \frac{2\tau_{xy}}{\sigma_x - \cancel{200}\, \sigma_y}$$

$$\theta_{p_1} = \frac{1}{2}\tan^{-1}\left(\frac{2 \times 60}{80 - 200}\right) = -22.5^\circ$$

$$\theta_{p_2} = -22.5^\circ + 90$$

$$\boxed{\theta_{p_2} = 67.5^\circ}, \quad \boxed{\theta_{p_1} = -22.5^\circ}$$

$$\tau_{max} = \frac{\sigma_{p_1} - \sigma_{p_2}}{2}$$

$$= \frac{224.85 - 55.15}{2}$$

$$\boxed{\tau_{max} = 84.85}$$

$$\tan 2\theta_\tau = \frac{-1}{\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)} = \frac{-1}{-1} = 1$$

$$\theta_\tau = \frac{1}{2}\tan^{-1}(1) = 22.5^\circ$$

ross check

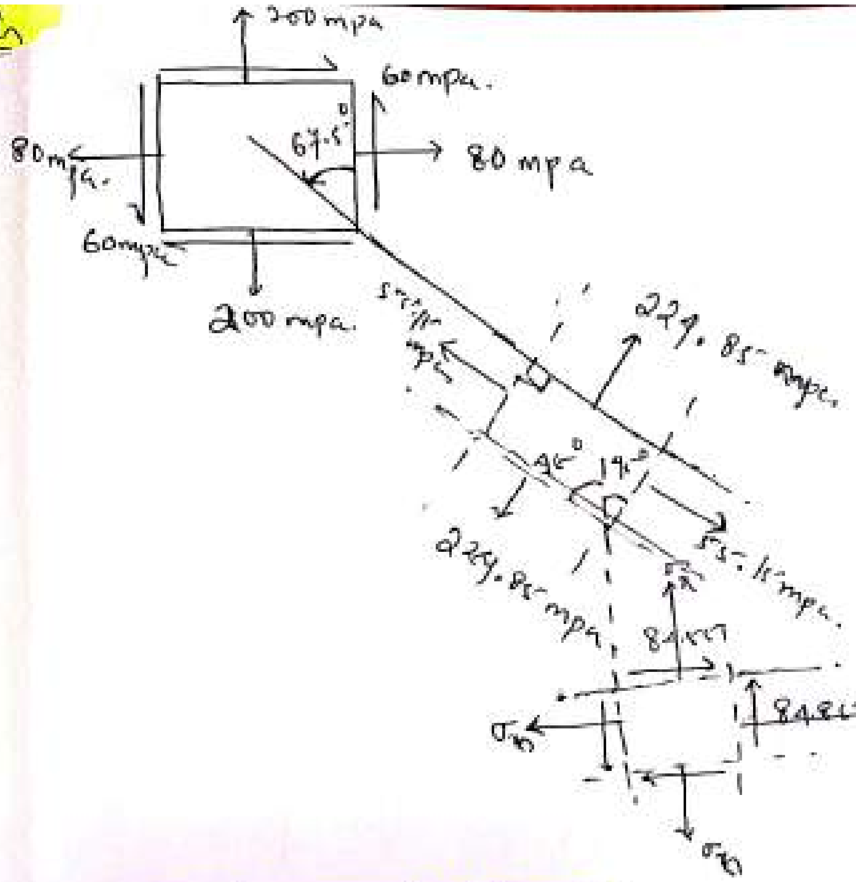
$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\cos\theta \cdot \sin\theta$$

$$= 80 \cos^2 67.5^\circ + 200\,(\sin 67.5^\circ)^2 + 2 \times 60 \times \cos 67.5 \cdot \sin 67.5$$

$$= 224.85 \text{ mpa} \cdot (\sigma_{p_1}, \text{ for } \theta = 67.5)$$

so, $(\sigma_{p_2}$, for $\theta = -22.5)$

Sketch



$\boxed{\sigma_n = 140 \text{ mpa}}$

Stress Invarient Concept:-

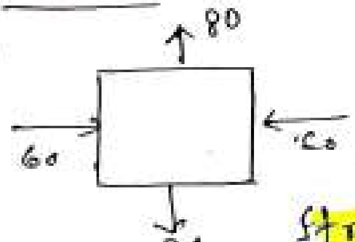
Summation of normal stress on a perpendicular plane at a point is constant.

$$\boxed{\sigma_x + \sigma_y = \sigma_{p_1} + \sigma_{p_2} = \sigma_n + \sigma_n}$$

$$\sigma_n = \frac{200+80}{2} = \frac{280}{2} = 140 \text{ mpa. (constant)}$$

Shortcuts:-

4)



$\sigma_{p_1} = 100$ Mpa.

$\sigma_{p_2} = ?$

Stress Invarient

$$\boxed{\sigma_{p_1} + \sigma_{p_2} = \sigma_x + \sigma_y}$$

$$100 + \sigma_{p_2} = 60 + 80$$

$$\sigma_{p_2} = 20 - 100$$

$$= -80 \text{ mpa}$$

B.

①
$$\tau_{max} = \frac{\sigma_{p_1} - \sigma_{p_2}}{2}$$

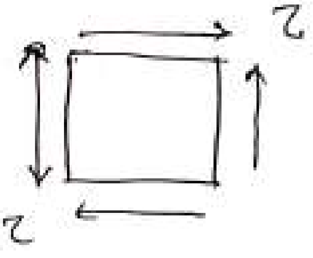
$$= \frac{\left(\cancel{\frac{\sigma_x + \sigma_y}{2}}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} - \left[\left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x \mp \sigma_y}{2}\right)^2 + \tau_{xy}^2}\right.}{2}$$

$$\frac{\cancel{2}\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 - \tau_{xy}^2}}{\cancel{2}}$$

$$\boxed{\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 - \tau_{xy}^2}}$$

PURE SHEAR CONDITION :-

* An ~~ae~~ eliment having only shear stress in its plane. is known ~~as~~ as pure stress condition.



Q.2 for the structural element, determine.

A) Principle stresses & maxm shear stresses

B) Location of principle plane & maximum shear plane.

C) Show all the stresses with net sketch.

solⁿ

$$\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = \tau$$

$$\sigma_{p_1}/\sigma_{p_2} = \cancel{\frac{\sigma_x + \sigma_y}{2}}^{\,0} \pm \sqrt{\cancel{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2} + \tau_{xy}^2}$$

$$\sigma_{pp_1}/\sigma_{p_2} \quad \pm \tau$$

$$\sigma_{p_1} = \tau, \quad \sigma_{p_2} = -\tau.$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$= \frac{2\tau}{0} = \infty$$

$$\theta_{p_1} = \frac{1}{2}\tan^{-1}(\infty) = 45^\circ$$

$$\theta_{p_2} = 45 + 90^\circ = 135^\circ$$

$$\tau_{max} = \frac{\sigma_{p_1} - \sigma_{p_2}}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

$$= \frac{\tau - (-\tau)}{2} = \frac{\cancel{2}\tau}{\cancel{2}}$$

$$\boxed{\tau_{max} = \tau}$$

$$\tan 2\theta_\tau = \frac{-1}{\left(\frac{-\tau_{xy}}{\sigma_x - \sigma_y}\right)} = \frac{-1}{\frac{-2\cdot}{0}} = \frac{0}{-2} = 0$$

$$\theta_\tau = \frac{1}{2}\tan^{-1}(0) =$$

$$\boxed{\theta_\tau = 0}$$

ran <u>Check</u>

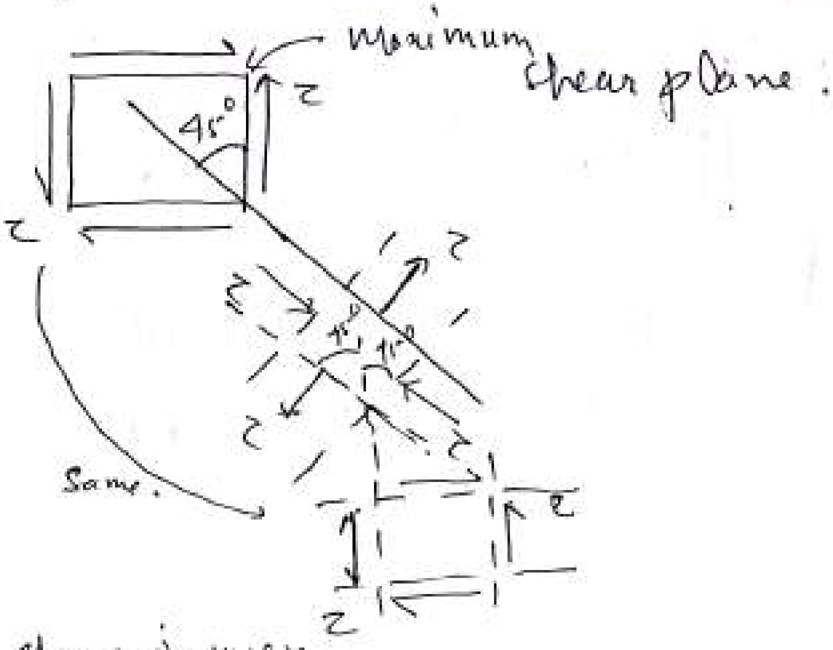
$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \cos^2\theta + 2\tau_{xy}\cos\theta\sin\theta$$

$$= 0 + 0 + \cancel{2}\tau \cdot \frac{1}{\cancel{2}}$$

$$\sigma_n = \tau, \boxed{\tau_{p_1}, \theta_{p_1} = 45}$$

$$\boxed{\tau_{p_2}, \theta_{p_2}\ 135^\circ}$$

Sketch of Stresses



from stress invarien:

$$\sigma_x + \sigma_y = \sigma_{p_1} + \sigma_{p_2} = \sigma_n + \sigma_n$$

$$0 + 0 = \tau - \tau = 2\sigma_n$$

$$\boxed{\sigma_n - 0}$$

V.imp for objective

NOTE (Conclusion For Pure Shear Condition) :-

* In pure shear Condition,

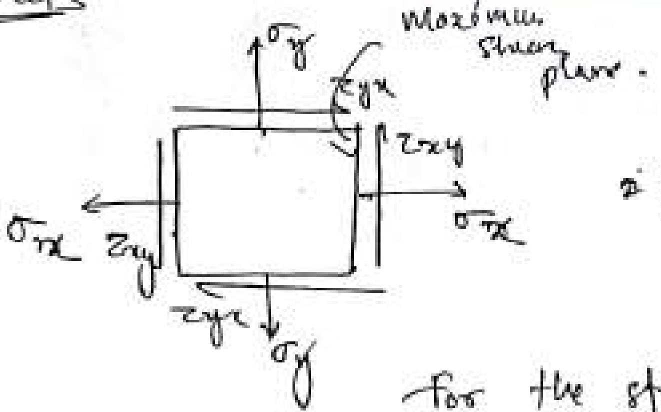
i) Major principle stress, Minor principal stress and maximum shear stress all has same value but their directio are different.

ii) Principal plane are inclined at an angle of 45° and 135° degree.

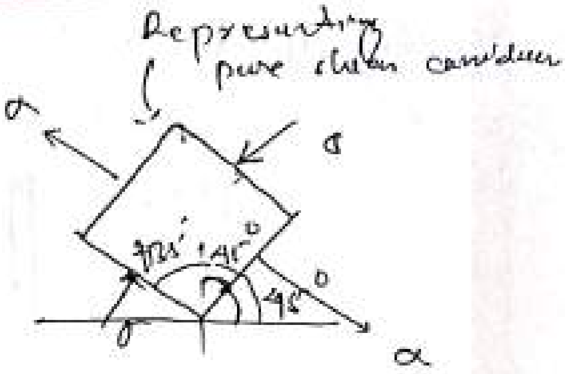
iii) Normal stress on the maximum shear plane is zero.

Numericals

Q-1



$\approx$



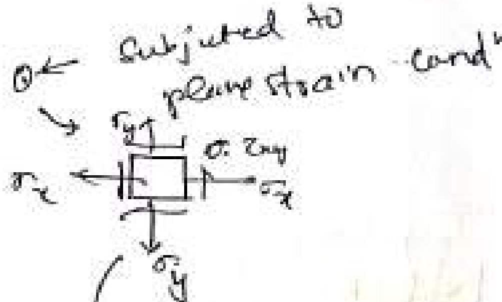
For the structural element @ a point, determine, τ_{xy}, σ_x & σ_y ?

Soln

$\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = \sigma$

* Plane Stress Condition :-

Principal stress one of the direction is equall to zero.



$\boxed{\sigma_{P_3} = 0}$

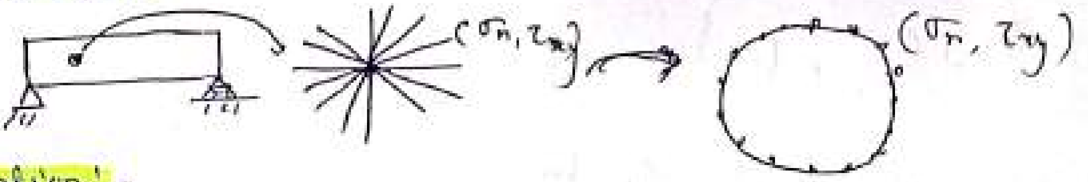
* Plane Strain Condition :-

Principle strain in one direction is equall to zero.

$\boxed{\epsilon_{P_3} = 0}$

GRAPHICAL METHOD (Mohr Circle Method) :-

* Mohr Circle !-



Defination!-

From a Point infinite plane can pass having a definate Value of normal and shear stress representation of these plane as a point on the circle is referred as Mohr' Circle.

* Its defined for a Point.
* Each point on a Mohr Circle is represents a defina value of normal and shear stress.

* Along the x - axis → Normal stress
  Along the y-axis → shear stress.

* Equation of Mohr' Circle :-

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \cos\theta \cdot \cos\theta \quad — ①$$

$$\tau_{x'y'} = (\sigma_y - \sigma_x)\frac{\sin 2\theta}{2} + \tau_{xy} \cos\theta \quad — ②$$

* from rearranging the eq-1 & 2

i) $$\boxed{\left[\sigma_n - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right]^2 + \left[(\tau_{x'y'} - 0)\right]^2 = \left[\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}\right]^2}$$

ii) $$\boxed{(x-a) + (y-b) = r^2} — eq^n \text{ of circle.}$$

$x = \sigma_n$, $y = \tau_{x'y'}$, $r = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$a = \frac{\sigma_x+\sigma_y}{2}$, $b = 0$

Continue.!-

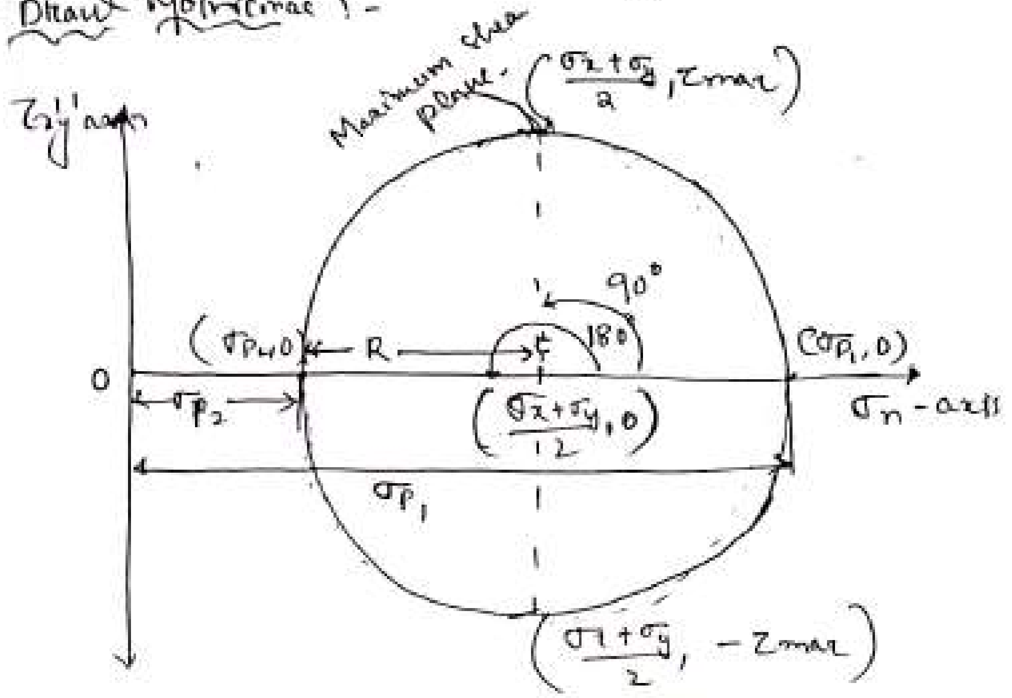
- **Centre of Mohr Circle**

$$\left(\frac{\sigma_x + \sigma_y}{2} , 0 \right)$$

- **Radius of Mohr Circle.**

$$\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

- **Draw Mohr circle :-**



- Diameter of Mohr circle. $= \sigma_{p_1} - \sigma_{p_2}$
- Radius of Mohr Circle

$$\boxed{\begin{aligned} &= \frac{\sigma_{p_1} - \sigma_{p_2}}{2} \\ &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \\ &= \tau_{max} \end{aligned}}$$ ***

- O – C distance.

$$C_x = \sigma_{p_2} + R$$
$$= \sigma_{p_2} + \frac{\sigma_{p_1} - \sigma_{p_2}}{2}$$
$$= \frac{2\sigma_{p_2} + \sigma_{p_1} - \sigma_{p_2}}{2}$$

$$\boxed{C_x = \frac{\sigma_{p_1} + \sigma_{p_2}}{2}}$$

Co-ordinate of centre.

$$C = \left[\frac{\sigma_x + \sigma_y}{2}, 0 \right] = \left[\frac{\sigma_{p_1} + \sigma_{p_2}}{2}, 0 \right]$$

Stress Invariant :-

$$\sigma_x + \sigma_y = \sigma_{p_1} + \sigma_{p_2} = \boxed{\sigma_n + \sigma_n}$$ Maximum shear plane.

σ_n → normal stress on maximum shear plane.

$$\boxed{\sigma_n = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_{p_1} + \sigma_{p_2}}{2}}$$

In structural Element :-

Angle between principle plane.

= 90°

Angle between Maximum shear plane and principle plane

= 90°

In a Mohr Circle :-

Angle between principle plane.

= 180°

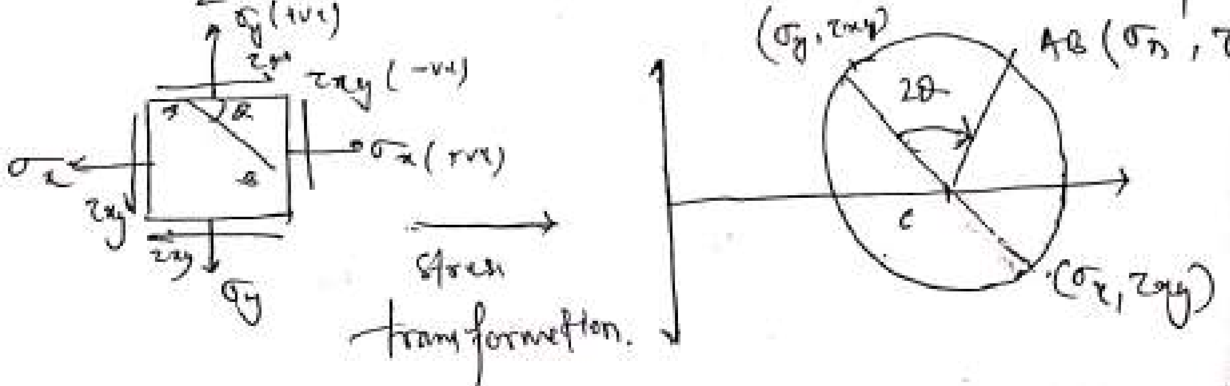
Angle between maximum shear plane and principle

= 90°

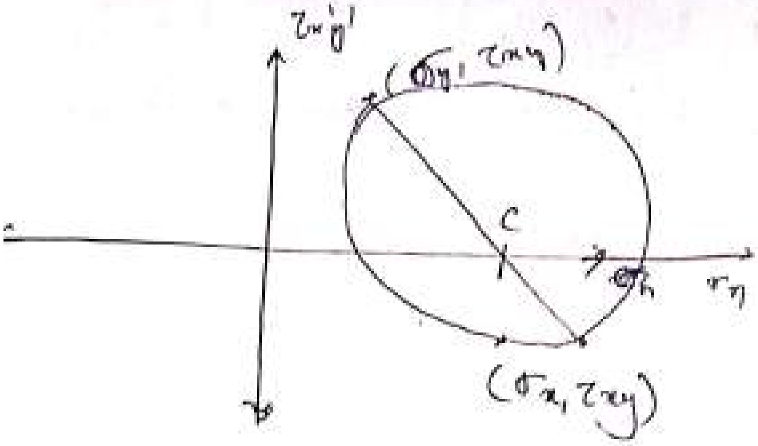
Q.:- If angle between 2 planes in structural element is θ, than angle between them in Mohr Circle will be —

a) $\frac{\theta}{2}$ [✓ b) 2θ] c) $\frac{\theta}{4}$ d) 4θ

Q.2



Sign Convension to draw Mohr Circle :-



σ (Normal)

tensile → +ve

Compressive → −ve

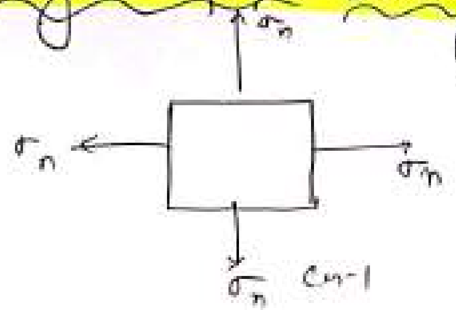
Shear (τ)

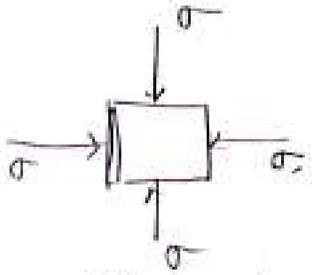
Clockwise → +ve

Anticlockwise → −ve

Mohr Circle for Standerd Case :-

Hydro Static loading





(or)

$\sigma_x = \sigma = \sigma_{p_1}$

$\sigma_y = \sigma = \sigma_{p_2}$

$\tau_{xy} = 0$

Case-2

σ

$C = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{\sigma_{p_1} + \sigma_{p_2}}{2}, 0\right)$

$C = (\sigma, 0)$

$R = \sigma_{p_1} - \sigma_{p_2} / 2 = 0$

↓

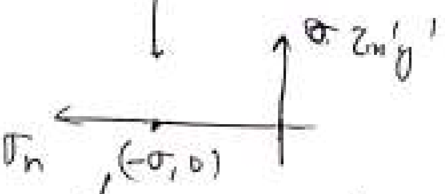
Point mohr Circle

$C = (-\sigma, 0)$

$R = 0$

↓

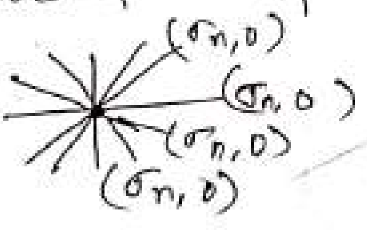
Point mohr Circle.



All the points of the circle get concentrated at the same point having same value of normal ~~value~~ stress.

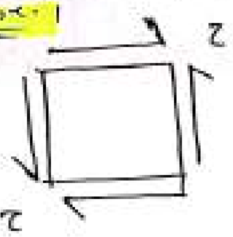
↓

Represent infinite principle plane.

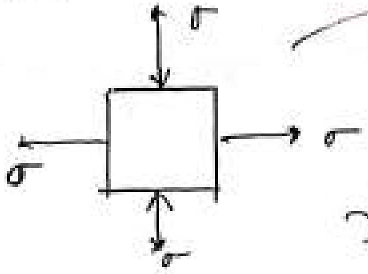


ii) Pure shear Condition :-

a) Case-1



:



Case-2

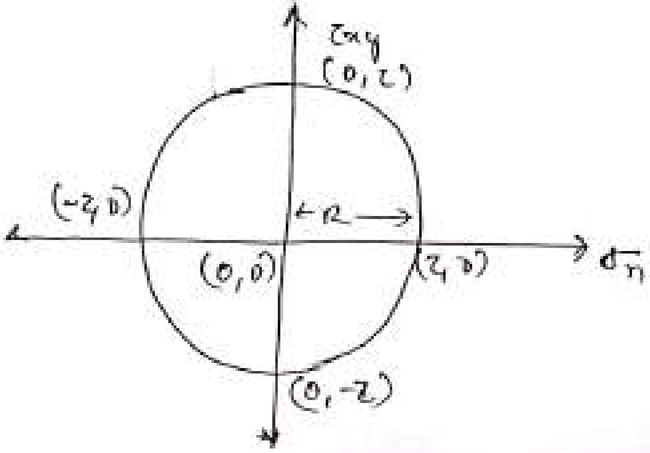
$\sigma_x = 0$

$\sigma_y = 0$

$\tau_{xy} = \tau = \tau_{max} = R$

$C = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = (0,0)$

a)



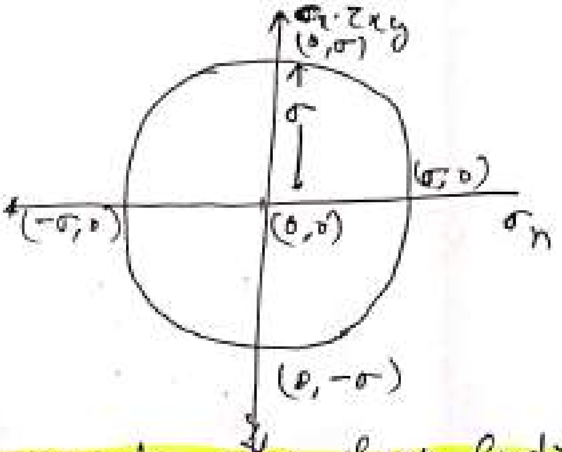
$\sigma_x = +\sigma = \sigma_{p_1}$

$\sigma_y = -\sigma = \sigma_{p_2}$

$\tau_{xy} = 0$

Radius $= \frac{\sigma_{p_1} - \sigma_{p_2}}{2} = \sigma$

$C : \left(\frac{\sigma_x - \sigma_y}{2}, 0\right) = (0,0)$

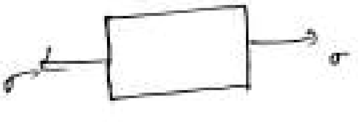


Note:- Both case-1 & case-2 are represents pure shear Conditio

Problem (Numerical) :-

Draw Mohr Circle)

i)



Sol:

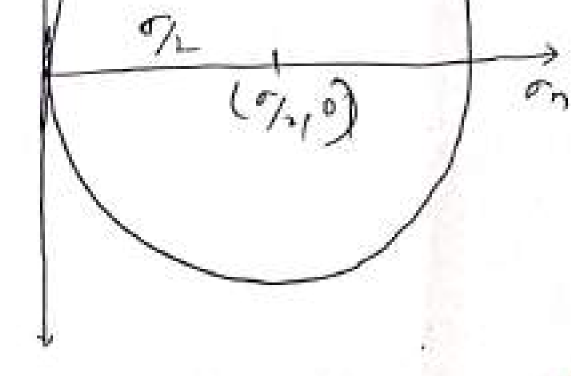
$\sigma_x = \sigma$

$\sigma_y = 0$

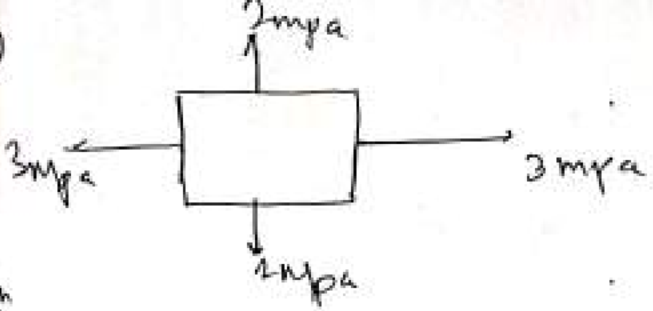
$\tau_{xy} = 0$

$C = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{\sigma}{2}, 0\right)$

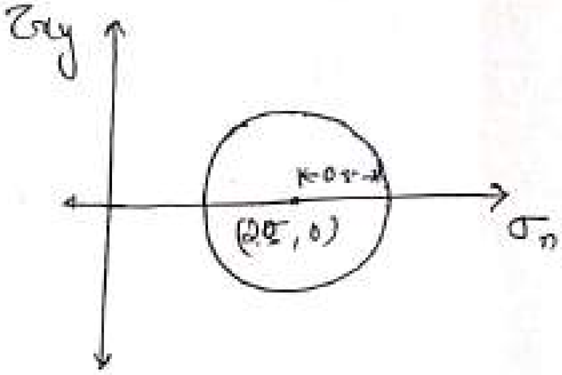
$R = \tau_{max} = \frac{\sigma_{p_1} - \sigma_{p_2}}{2} = \frac{\sigma}{2}$



(i)



Sol



$\sigma_x = 3mpa = \sigma_{p_1}$

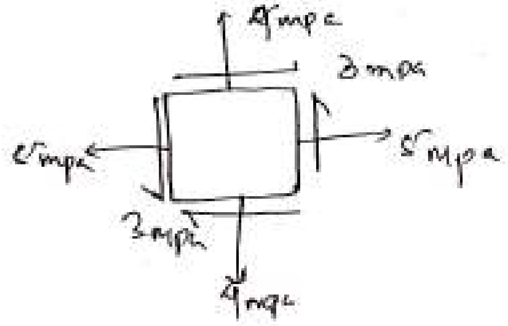
$\sigma_y = 2mpa = \sigma_{p_2}$

$\tau_{xy} = 0$

$C = \left(\frac{\sigma_x+\sigma_y}{2}, 0\right) = (2.5, 0)$

$R = \left(\frac{\sigma_{p_1}-\sigma_{p_2}}{2}\right) = 0.5$

(ii)



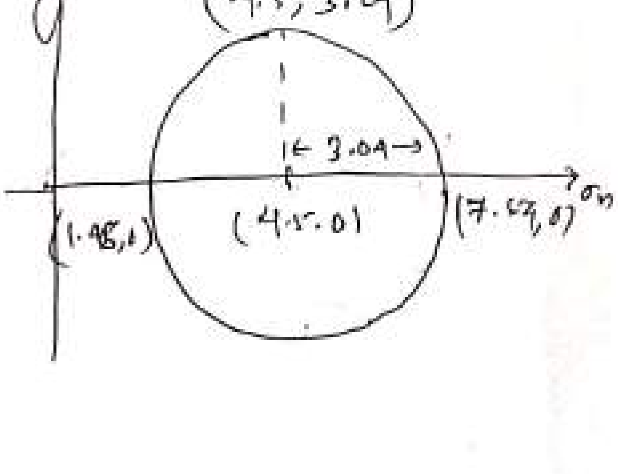
Sol

$\sigma_x = 5\,mpa$

$\sigma_y = 4\,mpa.$

$\tau_{xy} = 3mpa.$

$C = \left(\frac{\sigma_x+\sigma_y}{2}, 0\right) = \left(\frac{5+4}{2}, 0\right) = (4.5, 0)$



$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{0.5 + 3^2}$

$= \sqrt{9.25}$

$= 3.04.$

-4-4

Unit-2 STRAIN TRANSFORMATION AND PRINCIPAL STRAIN

Stress tensor Matrix

Strain tensor matrix

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \longrightarrow \epsilon_{ij} : \begin{bmatrix} \epsilon_{xx} & \frac{\phi_{xy}}{2} & \frac{\phi_{xz}}{2} \\ \frac{\phi_{yx}}{2} & \epsilon_{yy} & \frac{\phi_{yz}}{2} \\ \frac{\phi_{zx}}{2} & \frac{\phi_{xy}}{2} & \epsilon_{zz} \end{bmatrix}$$

**

$\sigma \rightarrow \epsilon$

$\tau \rightarrow \frac{\phi}{2}$

$\theta \rightarrow$ w.r.t horizontal

Anticlock wise +ve

* stress trasformation ⟶ Strain transformation *

1) $\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$

* $\epsilon_n = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \phi_{xy} \sin\theta . \cos\theta$

2) $\tau_{x'y'} = (\sigma_y - \sigma_x) \frac{\sin 2\theta}{2} + \tau_{xy} \cos 2\theta$

$\frac{\phi_{x'y'}}{2} = (\epsilon_y - \epsilon_x) \frac{\sin 2\theta}{2} + \frac{\phi_{xy}}{2} . \cos 2\theta$

* $\phi_{x'y'} = (\epsilon_y - \epsilon_x) \sin 2\theta + \phi_{xy} \cos 2\theta$

3. $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y .}$

* $\tan 2\phi_p = \frac{\phi_{xy}}{\sigma_x - \sigma_y}$

4. $\sigma_{p_1}/\sigma_{p_2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$

* $\epsilon_{p_1}/\epsilon_{p_2} = \frac{\epsilon_x + \epsilon_y}{} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{}\right)^2}$

5. $\tau_{max} = \frac{\sigma_{p_1} - \sigma_{p_2}}{2}$

$\frac{\phi_{max}}{2} = \frac{\epsilon_{p_1} - \epsilon_{p_2}}{2}$

* $\boxed{\phi_{max} = \epsilon_{p_1} - \epsilon_{p_2}}$

6. $\tan 2\theta_\tau = \frac{-1}{\frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$

* $\tan 2\theta_\tau = \frac{-1}{\left(\frac{\phi_{xy}}{\epsilon_x - \epsilon_y}\right)}$

7) Mohr Circle :-

Stress

$C = \left(\frac{\sigma_x - \sigma_y}{2}, 0\right)$

Radius = τ_{max}.

Strain:-

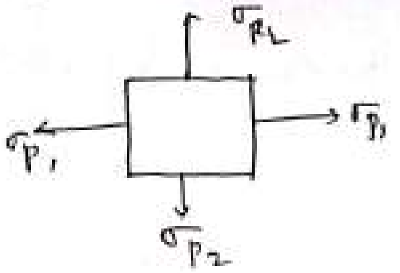
$OC = \left(\frac{\epsilon_x - \epsilon_y}{2}, 0\right)$

Radius = $\frac{\phi_{max}}{2}$ $\because$ $\boxed{\phi_{max} = \text{Diameter}}$

Note:-

Maximum shear stress is Radius of Mohr Circle where as Maximum shear stain is Diameter of mohr. circle.

Principal Stress in terms of principle strain !-



$$\boxed{\epsilon_{p_1} = \frac{\sigma_{p_1}}{E} - \mu \frac{\sigma_{p_2}}{E}} \text{ — ①}$$

$$\boxed{\epsilon_{p_2} = \frac{\sigma_{p_2}}{E} - \mu \frac{\sigma_{p_2}}{E}} \text{ — ②}$$

Multiply ② with μ and add to ①

$$\epsilon_{p_1} + \mu \epsilon_{p_2} = \frac{\sigma_{p_1}}{E} - \mu^2 \frac{\sigma_{p_1}}{E}$$

$$(\epsilon_{p_1} + \mu \epsilon_{p_2}) = \left(\frac{1-\mu^2}{E}\right) \sigma_{p_2}$$

$$\boxed{\sigma_{p_1} = \frac{E}{1-\mu^2} (\epsilon_{p_1} + \mu \epsilon_{p_2})}$$

$$\boxed{\sigma_{p_2} = \frac{E}{1-\mu^2} (\epsilon_{p_2} + \mu \epsilon_{p_1})}$$

- In plane stress Condition !- $(\sigma_{p_3} = 0)$

$$\epsilon_{p_3} = \overset{0}{\cancel{\frac{\sigma_{p_3}}{E}}} - \mu \frac{\sigma_{p_1}}{E} - \mu \frac{\sigma_{p_2}}{E}$$

$$\epsilon_{p_3} = \frac{-\mu}{E} (\sigma_{p_1} + \sigma_{p_2})$$

$$\epsilon_{p_3} = \frac{-\mu}{E} \left[\frac{E}{1-\mu^2} (\epsilon_{p_1} + \mu \epsilon_{p_2}) + \frac{E}{1-\mu^2} (\epsilon_{p_2} + \mu \epsilon_{p_1}) \right.$$

$$= \frac{-\mu}{E} \times \frac{E}{1-\mu^2} [\epsilon_{p_1} + \mu \epsilon_{p_2} + \mu \epsilon_{p_1} + \epsilon_{p_2})$$

$$= \frac{-\mu}{\cancel{E}} \times \frac{\cancel{E}}{1-\mu^2} (1+\mu) (\epsilon_{p_1} - \epsilon_{p_2})$$

$$\boxed{\epsilon_{p_3} = \frac{-\mu}{1-\mu} (\epsilon_{p_1} + \epsilon_{p_2})}$$

* STRAIN ROSETTE * (v imp for interview)

→ To determine Principal stress in ~~that~~ pratically Principle strain are required

→ To determine the principal strain, normal & shear should be known.

Determination of strain in field {
- Normal strain → Normal strain gauge → very sensative (Highly accurate)
- Shear strain → Can not measure by shear strain gauge → Not as sensative

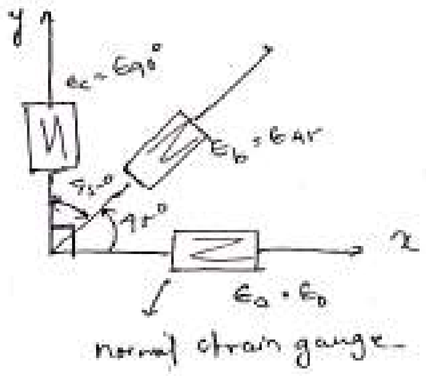
Definition :-

** To find the shear strain We use 3 normal strain gauges such a arrangement is ~~referred~~ referred as strain Rosette.

* strain Rosette ~~set~~

- 45° SR
- Delta SR
- star SR

1) 45° SR



Given :.

$\epsilon_0 = \epsilon_x$; ϕ_{xy} = Unknown

$\epsilon_{90} = \epsilon_y$

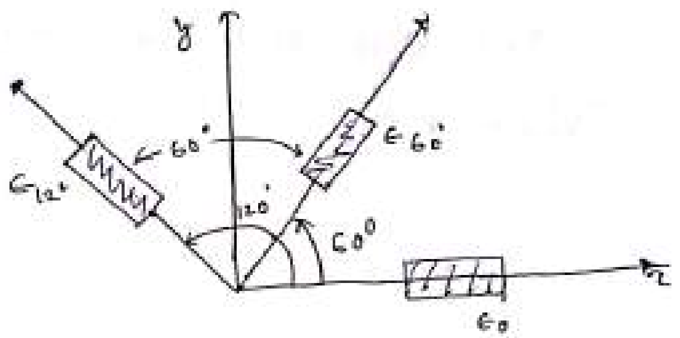
$$\epsilon_{P_1}/\epsilon_{P_2} = \frac{\epsilon_x+\epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2+\left(\frac{\phi_{xy}}{2}\right)^2}$$

Eqn from rant

$$\boxed{\epsilon_{45°} = \epsilon_0 \cos^2 45° + \epsilon_{90} \sin^2 45° + \phi_{xy} \cos 45° . \sin 45°}$$

find ϕ_{xy} from this eqn.

2. Delta Strain Rosette :-



$\epsilon_0 = \epsilon_x$, $\epsilon_y = \epsilon_{90} =$ Unknown.
ϵ_y & ϕ_{xy} Unknown.
2 Unknown 2 eq^n require.

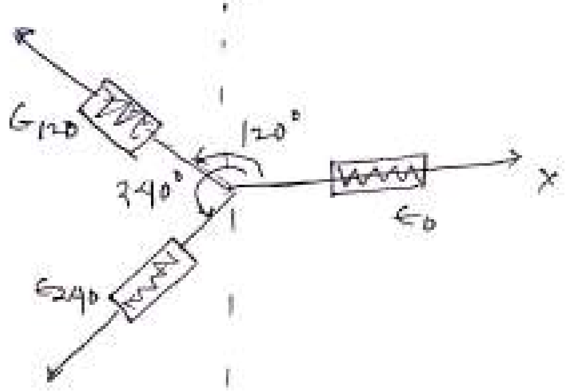
Eq^n are

Eq^n-1 → $\epsilon_{60} = \epsilon_0 \cos^2 60 + \epsilon_{90} \sin^2 60 + \phi_{xy} \cos 60 . \sin 60$

Eq^n-2 → $\epsilon_{120} = \epsilon_0 \cos^2 120 + \epsilon_{90} \sin^2 120 + \phi_{xy} \cos 120 . \sin 120 .$

from eq^n-1 & 2 $\epsilon_{90} = \epsilon_y$ & ϕ_{xy} to be determined.

3. Star Strain Rosette



$\epsilon_0 = \epsilon_x$, $\epsilon_y = \epsilon_{90} =$ Unknown
$\phi_{xy} =$ unknown
2 Unknown 2eq^n Required

Eq^n. Are

Eq^n-1 → $\epsilon_{120} = \epsilon_0 \cos^2 120 + \epsilon_{90} \sin^2 120 + \phi_{xy} \sin 120 . \cos 120$

Eq^n-2 → $\epsilon_{240} = \epsilon_0 \cos^2 240 + \epsilon_{90} \sin^2 240 + \phi_{xy} \sin 240 . \cos 240 .$

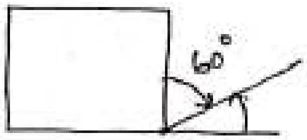
from eq^n-1 & 2 $\epsilon_{90} = \epsilon_y$ and ϕ_{xy} to be determined.

* Strain Invariant :-

$$\boxed{\epsilon_x + \epsilon_y = \epsilon_{p_1} + \epsilon_{p_2} \pm \epsilon_n + \epsilon}$$ [illegible]

Numericals ~~Method~~ !.-

Q.1



Determine strain on an Incline plane AB?

$\epsilon_x = 1000$

$\epsilon_y = -600$

$\phi_{xy} = 800$

Soln

$\theta_{AB} = 90 - 60 = 30°$

$$* \quad \epsilon_{AB} = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \phi_{xy} \cos\theta \cdot \sin\theta$$

$$= 1000(\cos 30)^2 - 600(\sin 30)^2 + 800 \cdot \cos 30 \sin 30.$$

$$= \overset{250}{\cancel{1000}} \times \frac{3}{\cancel{4}} - \overset{150}{\cancel{600}} \times \frac{1}{\cancel{4}} + \overset{200}{\cancel{800}} \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= 750 - 150 + 200\sqrt{3}$$

$$= 600 + 346$$

$$= 946.$$

$$\phi_{x'y'} = (\epsilon_y - \epsilon_x) \sin 2\theta + \phi_{xy} \cos 2\theta$$

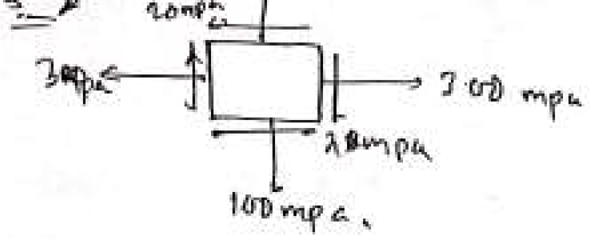
$$= (1000 - 600 - 1000) \cdot \sin 60 + 800 \times \cos 60°$$

$$= -1600 \times \frac{\sqrt{3}}{2} + 800 \times \frac{1}{\cancel{2}}$$

$$\Rightarrow -800\sqrt{3} + 400$$

$$\Rightarrow -\cancel{88} \cdot 1384 + 400$$

$$= -984.$$



Structural element is subjected to plane stress condition. Determine in plane max^m shear stress & maximum shear stress?

Sol^n

Given

$\sigma_x = 300$ Mpa.

$\sigma_y = 100$ mpa.

$\tau_{xy} = -20$ mpa

~~$\sigma_{p_1} = \frac{\sigma_x+\sigma_y}{2} \pm$~~ $\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+(\tau_{xy})^2}$

$$\tau_{max_1} = \frac{\sigma_{p_1}-\sigma_{p_2}}{2} = $$

$$= \sqrt{\left(\frac{300-100}{2}\right)^2+(20)^2}$$

$$\tau_{max_1} = 101.98 \text{ mpa.}$$

If the q^n is asking only for in plane maximum shear stress

$$\sigma_{p_1}/\sigma_{p_2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$= \frac{300+100}{2} \pm \sqrt{\left(\frac{300-100}{2}\right)^2+(20)^2}$$

$$= 200 \pm 101.98$$

$\sigma_{p_1} = 200+101.98 = 301.98$ Mpa

$\sigma_{p_2} = 200-101.98 = 98.02$ Mpa

$\sigma_{p_3} = 0$ ∵ from eq^n given subjected to plane strain condition.

Maximum shear at a point :-

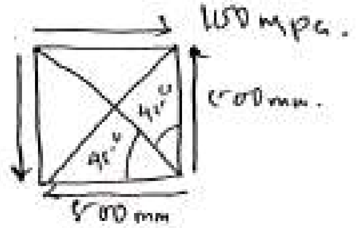
$$\tau_{max} = \frac{\sigma_{p_1}-\sigma_{p_2}}{2} = 101.98$$

$$\tau_{max_2} = \frac{\sigma_{p_1} - \sigma_{p_3}}{2} = \frac{301.98.0}{2} = 150.99 \text{ Mpa.}$$

$$\tau_{max_3} = \frac{\sigma_{p_2} - \sigma_{p_3}}{2} = \frac{98.67 - 0}{2} = 49.09 \text{ Mpa.}$$

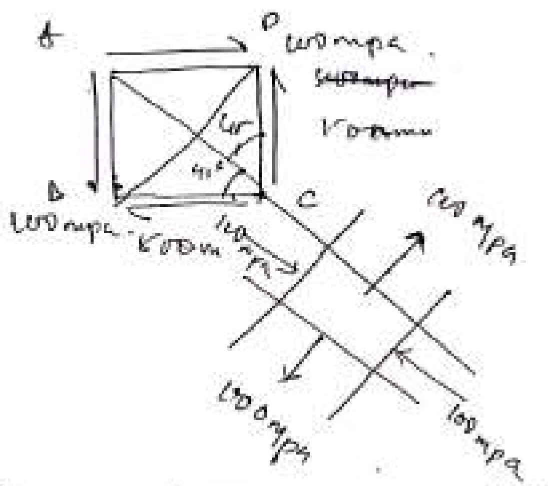
Maximum shear stress at point = τ_{max_2} = 150.99 Mpa.

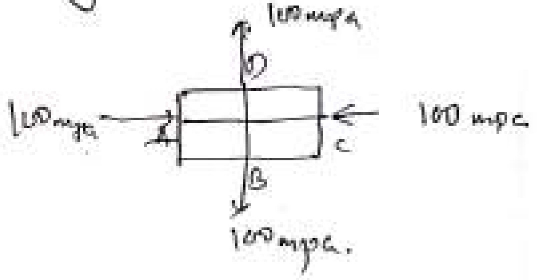
Q.3



If $\mu = 0.2$, E = 200 Gpa.
Determine deformed Length of the diagonals.

Sol^n





$$\epsilon_{ac} = 0 - \cancel{\sigma}\frac{100}{E} - \mu\left(\frac{100}{E}\right)$$

$$= -\frac{100}{E}(1+\mu)$$

$$\Delta l_{ac} = \frac{-100}{E}(1+\mu) \times L$$

$$= \frac{-100}{\cancel{2}00 \times 10^3}(1+0.2) \times \overset{250}{\cancel{500}}\sqrt{2}$$

$$= 1.2 \times 250 \times \sqrt{2}$$

$$= -0.429 \text{ mm.}$$

$$\boxed{\Delta l_{ac} = -0.429}$$

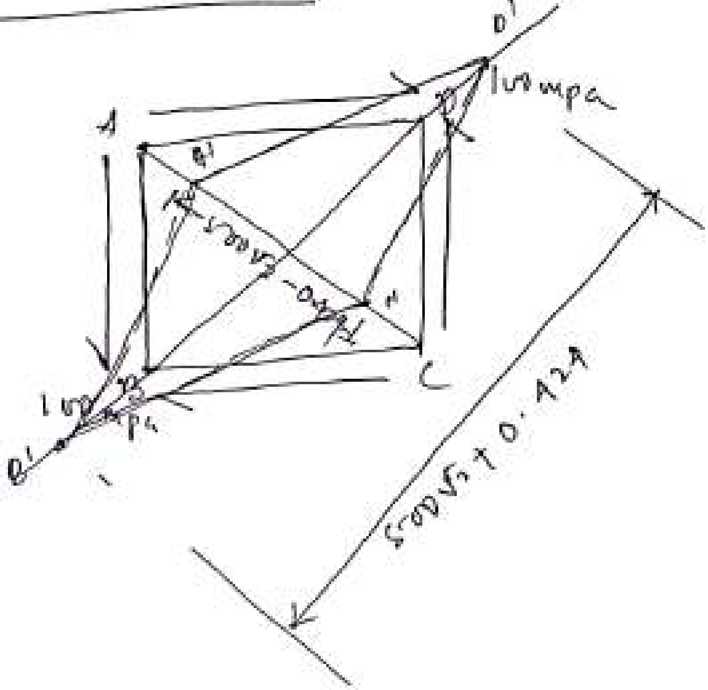
$$\epsilon_{BD} = \frac{100}{E} - \mu\left(\frac{-100}{E}\right)$$

$$= \frac{100}{E}(1+\mu)$$

$$\Delta L_{BD} = \frac{100}{E}(1+\mu) \times L_{BD}$$

$$\boxed{\Delta L_{BD} = 0.429\,mm}$$

<u>Deformated Diagram</u>



Strain Invariant

$$\boxed{\epsilon_x + \epsilon_y = \epsilon_{P_1} + \epsilon_{P_2} = \epsilon_n + \epsilon_n}$$

The simulation of normal strain on a perpendicular plane at a point is ~~same~~ a constant.

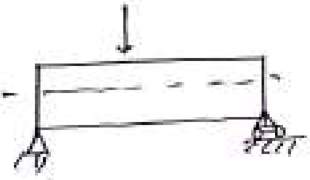
CH-1 BENDING & SHEAR

Lec-31

Topic-1 BENDING:-

- Basic of Bending:-

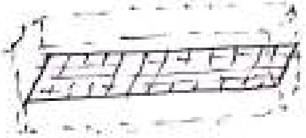
* When transverse load acts on the structure It will bend.



* NUTRAL AXIS/FIBER AND NUTRAL SURFACE/PLANE:-

A → before banding
A → After banding

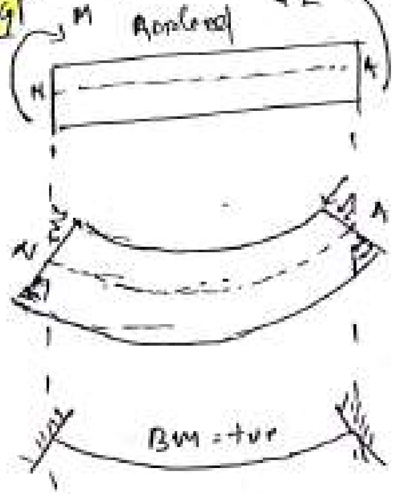
* **Neutral surface** :- A plane where dimention will not change upon application of banding.



* Nutral axis:- Itersection of neutral surface and transverse Cross section

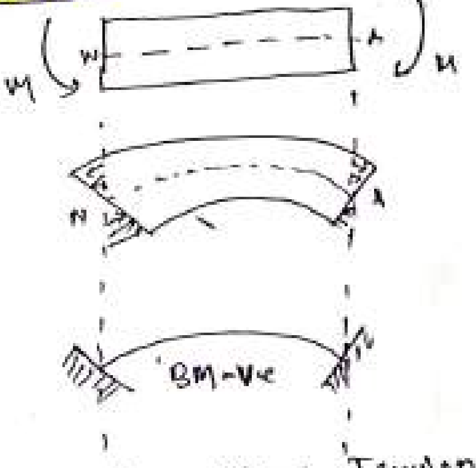
* SAGGING AND HOGGING BENDING MOMENT:-

SAGGING



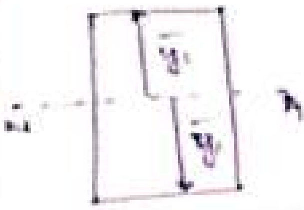
fibar above NA → Compression.
Fivre below NA → Tension

HOGGING



fiber above NA → Tension.
Fibre below NA → Compression.

SYMMETRICAL AND ASYMMETRICAL C/S :-

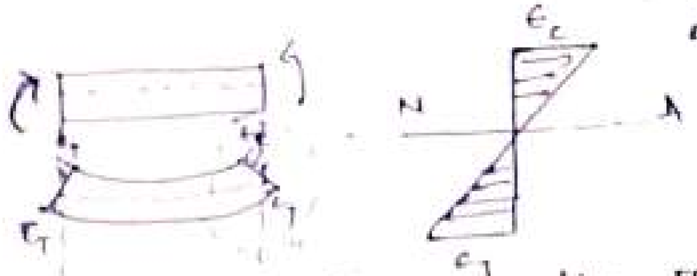


$\boxed{\bar{y}_1 = \bar{y}_2}$ → Symmetrical C/S

$\boxed{\bar{y}_1 \neq \bar{y}_2}$ → Asymmetrical C/S

* ASSUMPTION IN BENDING :-

1. Material is homogeneous, Isotropic and linearly elastic.
2. Before C/S is plane before and after bending.



Normal strain at the C/S varies linearly.

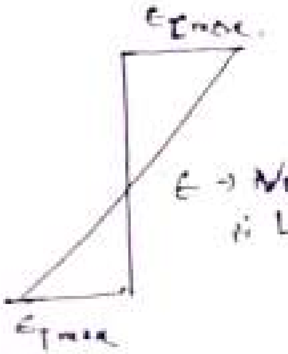
∵ Plane section before bending remains plain even after bending.

3. Young's Modulus of elasticity is same in tension and compression.

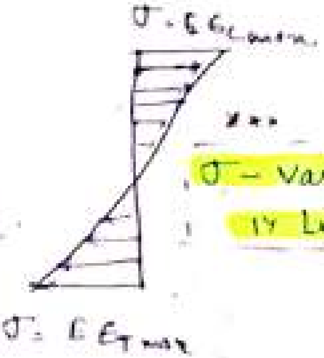
Hook's Law Valid

$\sigma \propto \epsilon$

$\boxed{\sigma = E\epsilon}$



ε → Variation is Linear.



σ – Variation is Linear.

Note - Q : MSQ

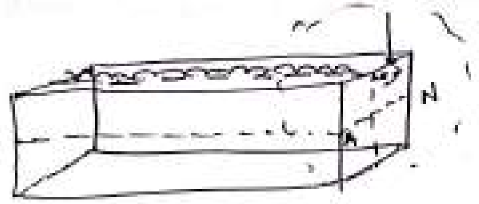
Plane section before bending remains plane even after bending this implies.

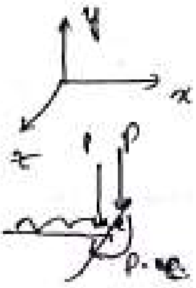
a) Strain variation is Linear
b) Strain variation is non-linear.
c) stress variation is non-linear
d) Stress variation is Linear

Ans :- a & d → both the Stress and Strain variation is Linear.

4. Material ~ Prismatic!

5. C/s & Loading must be Symmetrical.

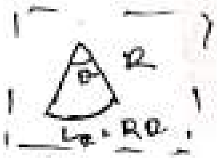




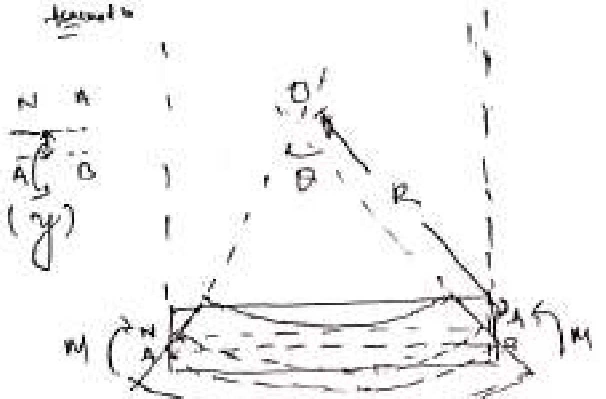
Note
** If ~~net~~ loading is not symmetric along with the bending moment twisting moment will also develop making the design uneconomical.

Note
** If cross section is not symmetrical, the load should pass through shear centre, so that no twisting develope.

** BENDING EQUATION :-



Length of arc



Bending in the form of arc of circle.

taking cross section

force on elementary Area dA.

$= \sigma_{AB} . dA.$

Moment of the elementary force about NA = $\boxed{\sigma_{AB} \cdot y \cdot dA = dM}$

Integration

$$\int dM = \frac{E}{R} \int y^2 . dA$$

$$M = \frac{E}{R} \times I_{NA}$$

$$\boxed{\frac{M}{I_{NA}} = \frac{E}{R}} \quad —②$$

** 2nd

Strain in AB

$$\epsilon_{AB} = \frac{L_{A'B'} - L_{AB}}{L_{AB}}$$

$$\boxed{\begin{aligned} L_{A'B'} &= (R+y)\theta \\ L_{AB} = L_{NA} &= R.\theta \end{aligned}}$$ **

so,

$$\epsilon_{AB} = \frac{(R+y)\theta - R\theta}{R\theta}$$

$$= \frac{\cancel{R\theta} + y\cancel{\theta} - \cancel{R\theta}}{R\cancel{\theta}}$$

$$\epsilon_{AB} = \frac{y}{R}$$

**

$$\boxed{\frac{\sigma_{AB}}{E} = \frac{y}{R}} \quad —①$$

from hooks law

$\sigma = E\epsilon$

$\epsilon = \frac{\sigma_{AB}}{E}$

(part-I)

$$\boxed{\sigma_{AB} = \frac{E}{R} . y}$$

{ $\int y^2 . dA$ = second Moment of area = Moment of inertia at N from Eqn - ① & ②

$$\boxed{\frac{M}{I_{NA}} = \frac{\sigma_{AB}}{y} = \frac{E}{R}}$$

simple bending Eqn

where :-

M → Bending moment @ section x-x

I_{NA} → Moment of Inertia of the cross section about NA.

σ_{AB} → Indirect Normal stress / flexural stress / Bending stress at a distance y from NA.

E → Young's modulus of elasticity

R → Radius of Curvature.

$\frac{1}{R}$ → Curvature.

For Straight Line.

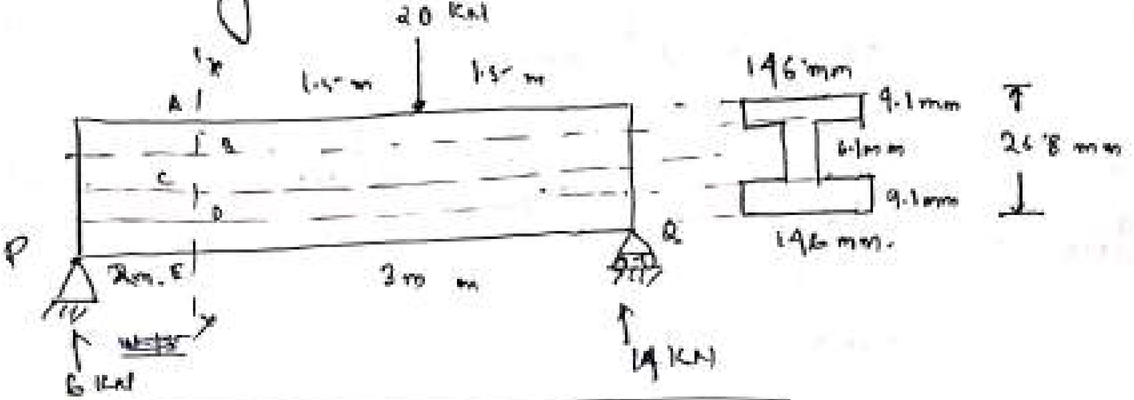
Radius of curvature → ∞

Curvature → 0

△
Im 0.2
0 - 0.1
0.12

→ Numericals on Bending

Q.1 for the loading diagram shown in figure, Determine. the bending stress at A, B, C, D, E at section x-x ?



Sol

$$\boxed{\text{Bending Stress } \sigma = \frac{M}{I} \times y}$$

M → BM at section x-x

I → Moment of inertia about NA

y → distance of the fiber at neutral stress.

$$V_{RP} = \frac{\overset{4}{\cancel{20}} \times 1.5}{\cancel{5}} = 6 \text{ KN}$$

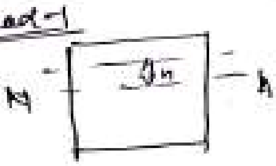
$$V_{RQ} = \frac{\overset{4}{\cancel{20}} \times 3.5}{\cancel{5}} = 14 \text{ KN}$$

Bending Moment at section x-x $M_{xx} = 6 \times 2 = 12$ KN-m.

$$\boxed{BM_{xx} = 12 \text{ KN-m}}$$

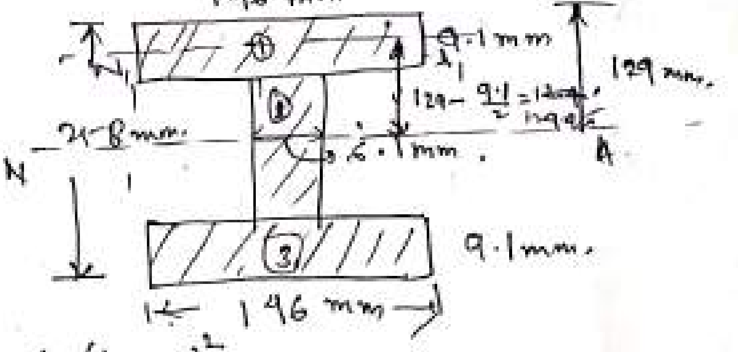
Object MOI cal^n

Method-1



$$I_{NA} = \frac{bd^3}{12}$$

$$\boxed{I_{xx} = I_{NA} + Ah^2}$$



for section ①

$$I_{NA} = \frac{146 \times (9.1)^2}{12} + (146 \times 9.1) \times (124.5)^2$$

for sec ③ = same as ① section. for sematry

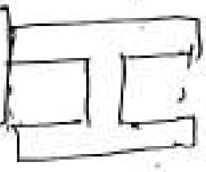
$$\text{for section ② } = \frac{[268 - (2 \times 9.1)]^3 \times 6.1}{12} = \frac{6.1 \times 239.8^3}{12}$$

$$I_{NA} = 2\left[\frac{146 \times 9.1^2}{12} + (146 \times 9.1) \times 129.45^2)\right] + \frac{6.1 \times 239.8^3}{12}$$

$$= 48.1 \times 10^6 \text{ mm}^4$$

(for objective) Method-2

$$I_{NA} = \frac{146 \times 358^3}{12} - \cancel{2}\left[\frac{\left(\frac{146-6.1}{\cancel{2}}\right) \times 239.8^3}{12}\right]$$



$$= \cancel{146\times}\ 48.1 \times 10^6 \text{ mm}^4.$$

× Bending stress calⁿ

ⓐ A: $\sigma_A = \frac{M}{I} \times y_A = \frac{12 \times 10^6}{48.1 \times 10^6} \times \left(\frac{258}{2}\right)$

$= 32.182$ Mpa. (C)

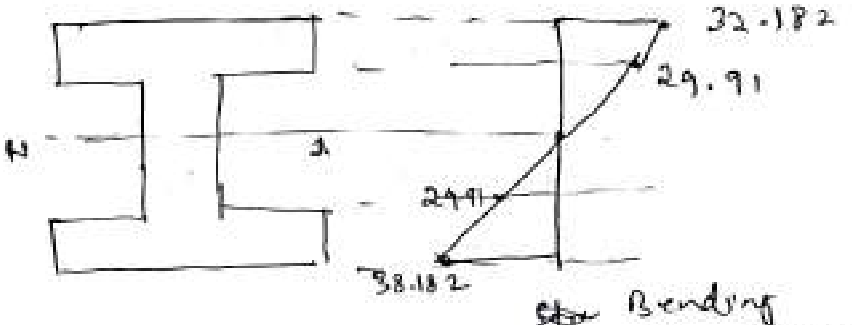
ⓐ B: $\sigma_B = \frac{M}{I} \times y_B = \frac{12 \times 10^6}{48.1 \times 10^6} \times \left(\frac{258 - (9.1 \times 2)}{2}\right)$

$= 29.91$ MPa. (C)

ⓐ C: $\sigma_C = \frac{M}{I} \times \overset{0}{y_C} = 0$

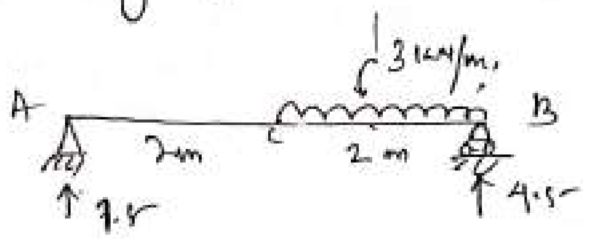
ⓐ D, $\sigma_D = \frac{M}{I} \times y_D = 29.91$ Mpa (T) $\boxed{y_B = y_D}$

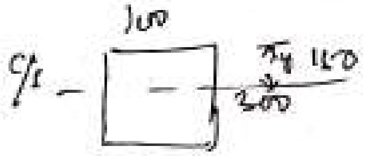
ⓐ E, $\sigma_E = \frac{M}{I} \times y_E = 32.182$ (T) $\boxed{y_E = y_A}$



~~str~~ Bending stress diagram.

Q.2:- for the beam shown in the figure determine the maximum bendig stress in the beam. C/s ~ 100 mm X 300 mm ?





Sol^n

$\sigma = \frac{M}{I} \times y$

σ = function of (M, y)

$$\boxed{\sigma_{max} = \frac{M_{max} \times y_{max}}{I_{NA}}}$$

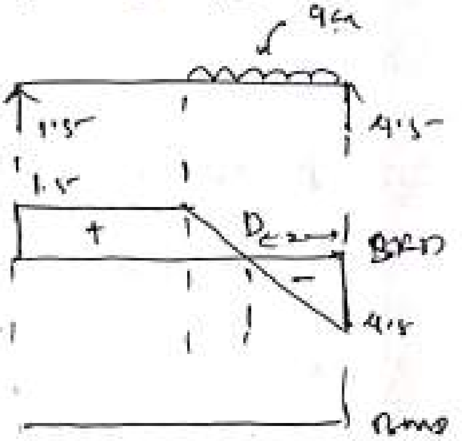
$\sigma_{max} = I_{NA} = \frac{100 \times 300^3}{12} = \cancel{7\cdots}\ mm^4$
225000000 mm^2

$\sigma_{max} = \frac{\overset{2.25}{\cancel{3.37}} \times 10^3}{2\cancel{2}25000000} \times 150$

$\boxed{\sigma_{max} = 1.15 \ mpa}$

$R_A = \frac{\cancel{6}^{3} \times 1}{\cancel{4}\ 2} = 1.5$ kN.

$R_B = \cancel{3}\ 4.5$



for D & ~~3~~

for SF = 0

$-4.5 + \cancel{3}x$

$x = \frac{4.5}{3} = \cancel{0.75}m.$ 1.5m

BM @ D

$= 4.5 \times \overset{1.15}{\cancel{0.75}} - \cancel{3}\ 0.75 \times \frac{\overset{1.5}{\cancel{0.75}}}{2}$
2.25

$M_D = \cancel{3.375}$ KN-m.

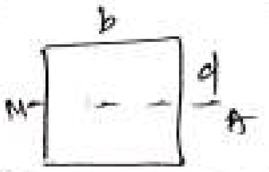
$\boxed{M_{max} = \cancel{3.375}\ \underset{2.25}{}}$ KN-m.

---

* SECTION MODULUS (Z):-

$\frac{M}{I} = \frac{\sigma}{y} = \frac{\sigma_{max}}{y_{max}}$

$M = \sigma_{max} \times \left(\frac{I}{y_{max}}\right)$ ← Section Modulus,



$$\boxed{Z = \frac{I}{y_{max}}}$$

$= \frac{bd^3}{12} / d/2$

$$\boxed{Z = \frac{bd^2}{6}}$$

Moment Carrying Capacity	=	Maximum bending stress	×	Section Modulus

If material same.

$\sigma_{max_1} = \sigma_{max_2}$

$\boxed{Z_1 > Z_2}$

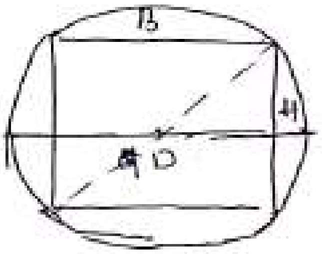
$\boxed{M_1 > M_2}$

Note! –

** In bending, greater the section modulus, greater will be strength of beam.

Numericals

Q.3 A ~~section~~ rectangular wooden block is to be cut for a circular c/s of diameter "D" such that the bending strength is maximum. Determine the dimension on the rectangular block.



Soln If bending strength maximum.

so, Z is maximum.

= $\frac{B.H^2}{6}$ is maximum.

⇒ $B^2+H^2=D^2$, $H^2=D^2-B^2$

⇒ $Z=\frac{B(D^2-B^2)}{6}$

For Z to be maximum.

$\frac{dZ}{dB}=0$, $\frac{d}{dB}\left(\frac{BD^2-B^3}{6}\right)=0$

⇒ $\frac{D^2}{6}-\frac{\cancel{3}B^2}{\cancel{6}\,2}=0$

⇒ $B^2=\frac{D^2}{3}$

$B=\sqrt{\frac{D^2}{3}}$ $\boxed{B=\frac{D}{\sqrt{3}}}$ *

$B^2+H^2=D^2$

$\frac{D^2}{3}+H^2=D^2$

$H^2=\frac{2}{3}D^2$

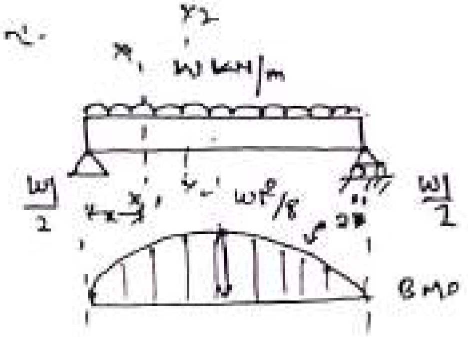
$\boxed{H=\sqrt{\frac{2}{3}}D}$ *

CL-3
Le-35

BEAM OF UNIFORM STRENGTH!-

* Maximum bending Stress at any section of the beam is a Constant.

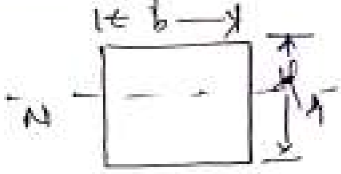
Explanation:-



$$\boxed{BM_{xx} = \frac{wl \cdot x}{2} - w.x\left(\frac{x}{2}\right)}$$

$$\boxed{M_{x_1-x_1} \neq M_{x_2-x_2}}$$

* If beam is prismatic



$$\sigma_{b\,max} = \frac{M}{I} \times y_{max}$$

Bending Moment along the beam is variable.

$$\sigma_{b\,max_{x_1x_1}} = \sigma_{b\,max_{x_2x_2}}$$

$$\frac{M_{x_1x_1}}{I_{NA}} \times y_{max_{x_1-x_1}} = \frac{M_{x_2-x_2}}{I_{NA}} \times y_{max_{x_2-x_2}}$$

$$\boxed{\frac{y_{max_{x_1-x_1}}}{I_{NA\,x_1-x_1}} \neq \frac{y_{max_{x_2-x_2}}}{I_{NA\,x_2-x_2}}}$$

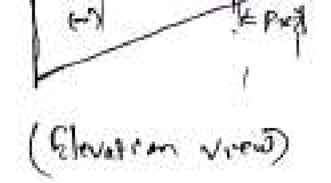
$$\boxed{Z_{x_1x_1} \neq Z_{x_2-x_2}}$$

* At all the section Z will be constant.

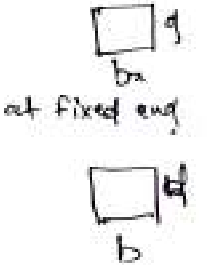
Note* For getting beam of uniform strength, if BM along the length of the beam is variable, then the beam should be non prismatic

** For Analysis:-

Rectangular Cantilever beam subjected to point Load at free end.



Any section



Case-I (width varies)

$$\sigma_{max} = \frac{M}{I} \times y_{max}$$

$$M = \frac{\sigma_{max}}{y_{max}} \times I$$

$$= \frac{\sigma_{max}}{d/2} \times \frac{bd^3}{12\,6}$$

$$\boxed{M = \sigma_{max} \times \frac{bd^2}{6}}$$

so, $\boxed{M \propto b}$

It for the statment.

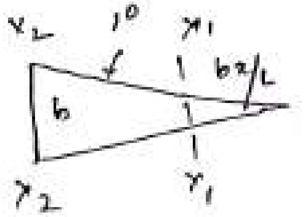
$$\sigma_{b\max_{x_1-x_1}} = \sigma_{b\max_{x_2-x_2}}$$

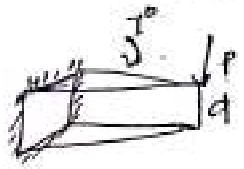
$$\frac{(-P.x)}{\frac{b_x.d^3}{12}} \times \frac{d}{2} = \frac{-P.L}{\frac{bd^3}{12}} \times \frac{d}{2}$$

$$\boxed{b_x = \frac{b}{L}\times x}$$

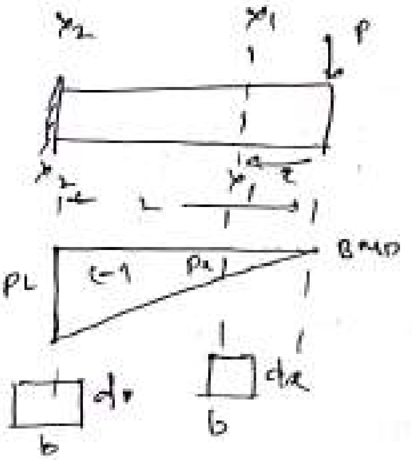
curve like ∵ $y = mx$

PLan





Case-2 (depth is variable):-



$$\sigma_{b\max_{x_1-x_1}} = \sigma_{b\max_{x_2-x_2}}$$

$$\frac{-P.x}{\frac{b.d_x^3}{12}}\times\frac{d_x}{2} = \frac{-P.L}{\frac{bd^3}{12}}\times\frac{d}{2}$$

$$d_x^2 = \frac{x}{L}\cdot d^2$$

$$\boxed{d_x = \frac{d}{\sqrt{L}}\times\sqrt{x}} \rightarrow \text{Parabolic variation}$$

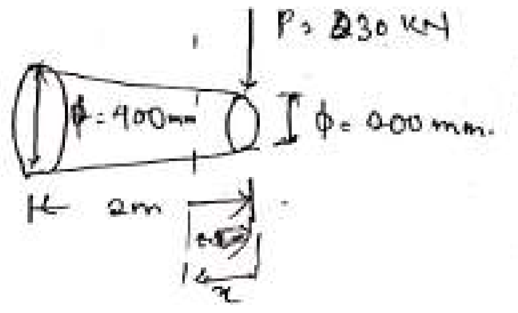
For beam of uniform strength:-

→ If width is constant, depth varies parabolically.

→ If depth is constant, width varies Linearly.

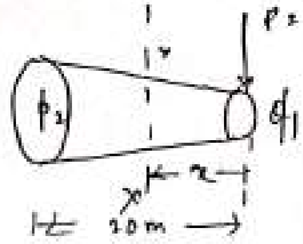
Numericals

Q1



Determine the maxima bending stress anyght at the beam.

$\underline{Sol^n}$



ⓐ $x-x$, $D_x = D_1 + \left(\frac{D_2 - D_1}{L}\right)x$

$$\boxed{D_x = D_1 + Kx}$$

σ_{max} @ $x-x$

$$\sigma_{max} = \frac{M_{x-x}}{I_{x-x}} \times y_{max\,x-x}$$

$$= \frac{-P.x}{\frac{\pi d_x^4}{64}} \times \frac{d_x}{2} = \frac{-\cancel{32x}}{}$$

$$\boxed{\sigma_{max} = \frac{-32Px}{\pi . d_x^3}}$$

for σ_{max} ~~f~~orto b. maximum for entair crase section.

$$\frac{d(\sigma_{max})}{dx} = 0$$

$$\Rightarrow \frac{d}{dx}\left[\frac{32P}{\pi}\left(\frac{x}{(D_1+Kx)^3}\right)\right] = 0$$

$$\Rightarrow \frac{32P}{\pi}\left[\frac{(D_1+Kx)^3.1 - x.3(D_1+Kx).K}{(D_1+Kx)^6}\right] = 0$$

$$\Rightarrow \frac{32P}{\pi}\left[\frac{(D_1+Kx)^2}{(D_1+Kx)^6} \times (D_1 + Kx - 3Kx)\right] = 0$$

$$\Rightarrow D_1 - 2Kx = 0$$

$$\boxed{x = \frac{D_1}{2K}} = \frac{200}{2\left(\frac{400-200}{2}\right)} = 1m$$

$$\boxed{ⓑ\ x = 1m\ ;\ d_x = 300\ mm}$$

$$\sigma_{max} = \frac{-32Px}{\pi d_x^3} = \frac{-32 \times 30 \times 10^3 \times 1 \times 10^3}{\pi (300)^3} = -11.32$$

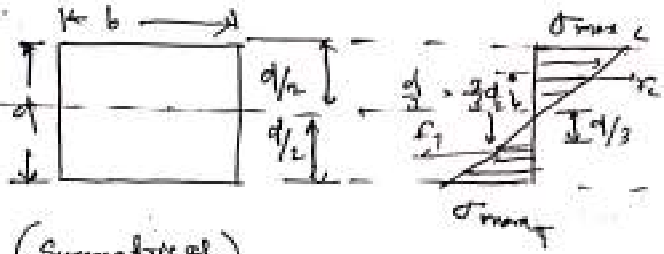
$$\boxed{\sigma_{max} = 11.32}$$ (compression)

* MOMENT CARRING CAPACITY OF A C/S) -

(Delivation)

* Ractangular C/S

Derivation.



(Symmatrical C/S)

$$\boxed{\sigma_{max_{C}} = \sigma_{max_{T}}}$$

$\boxed{f_C = f_T}$ (due to equlibrium body)

Moment Carring Capacity $= f_C \times LA = f_T \times LA.$

$\boxed{LA = \frac{2d}{3}}$, $f_C = f_T$ = Average stress × Area.

$= \left(\frac{Max^m + Min^m\ stress}{2}\right) \times \frac{bd}{2}$

$f_C = f_T = \left(\frac{\sigma_{max} + 0}{2}\right) \times \frac{bd}{2}$

* Moment carring capacity cal^n

$$M_{NA} = \left(\sigma_{max} \times \frac{bd}{4}\right) \times \frac{2d}{3}$$

$$= \boxed{M_{NA} = \sigma_{max} \times \frac{bd^2}{6}}$$ for Ractangular C/S.

Direct formula

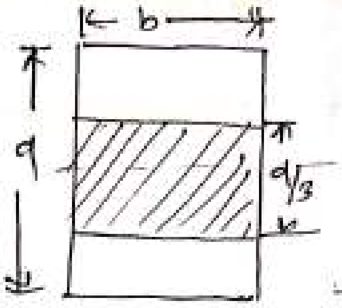
$$\boxed{M = \sigma_{max} \times Z}$$

$$= \sigma_{max} \times \frac{I}{y_{max}}$$

$$\boxed{M = \sigma_{max} \times \frac{bd^2}{6}}$$

Numericals:-

For a rectangular C/s width 'b' and depth 'b' moment capacity of is given M. Determine the moment carrying capacity of middle 1/3 portion:-

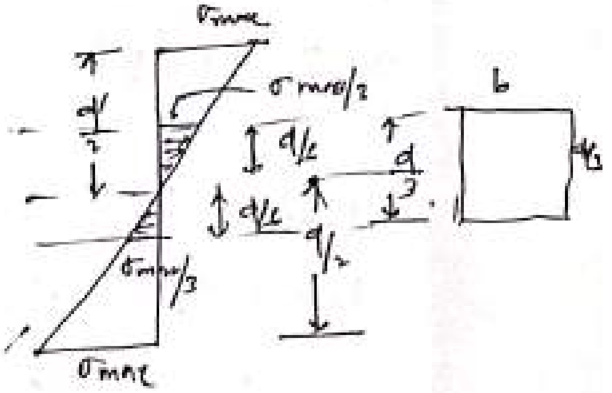


Sol^n

for Complete C/section

shortcut

$$\boxed{M = \sigma_{max} \times \frac{bd^2}{6}}$$



$$M_1 = \frac{\sigma_{max}}{3} \times \frac{b \times (d/3)^2}{6}$$

$$\boxed{M_1 = \frac{1}{27} \times \sigma_{max} \times \frac{bd^2}{6}}$$

$$\boxed{M_1 = \frac{M}{27}}$$

General method (for the d/3 section)

$$F_C = F_T = \left(\frac{M_{max} + M_{min}}{2}\right) \times Area$$

$$= \left[\frac{\frac{\sigma_{max}}{3} + 0}{2}\right] \times \frac{bd}{6}$$

$$= \frac{\sigma_{max}}{6} \times \frac{bd}{6}$$

$$M'_{NA} = F \times LA$$

$$= \frac{\sigma_{max}}{6} \times \frac{bd}{\cancel{6}3} \times \frac{\cancel{2}d}{9}.$$

$$M'_{NA.} = \left(\sigma_{max} \times \frac{bd}{6}\right) \times \frac{1}{27}$$

$$\boxed{M'_{NA} = \frac{M}{27}}$$

✻ FLINCHED BEAM!-

* Beam where c/s is made of different material.

$\boxed{\text{Moment carrying Capacity} = \sigma_{max} \times Z}$

Maximum BM: $\boxed{\frac{wl^2}{8} = \sigma_{max} \times \frac{bd^3}{16}}$

Case-1

* Material same : (σ is constant)

Length of beam is same.

$\boxed{w\uparrow} \rightarrow \boxed{M\uparrow} \rightarrow \boxed{Z}\uparrow \rightarrow \boxed{d\uparrow}$

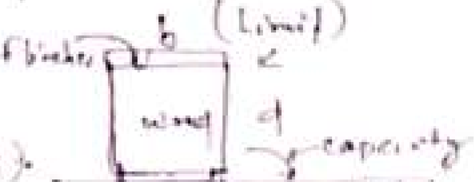
$\boxed{w\uparrow \cdot d\uparrow}$

* So, - Since "d" cannot be increased beyond a limit,

~~stronger~~ flinched beam is used.

Case-1

→ flinches are made up of stronger material. (for max moment).



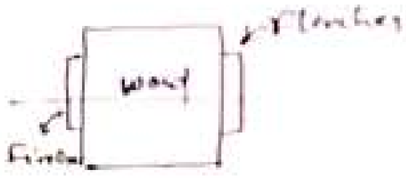
$\boxed{M = \sigma_{max} \times \frac{bd^2}{6}}$

$\boxed{M_1 - M} = \sigma_{max} \times \frac{bdt^2}{2}$

$\boxed{M_{original} > M}$

for design.

Case-2

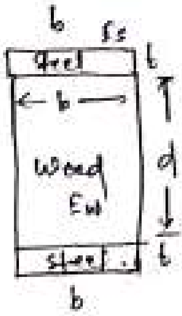


Note

To increase the moment carrying capacity of a structure and to work within limit of c/s dimension flinches of strength greater than the original material is applied at the cross-section either at the top and bottom or at the sides.

* Case-I | When flitches are attached to top and buttom.

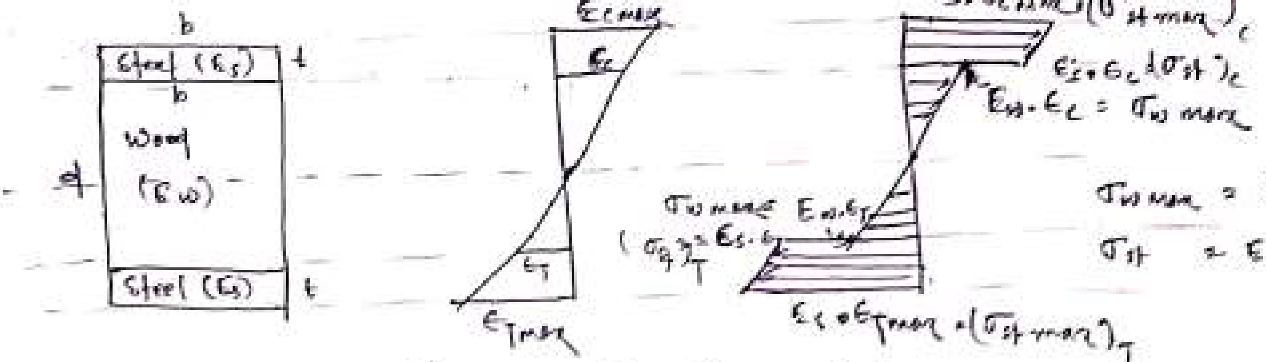
C/S



$$\boxed{E_s > E_w}$$

$$\boxed{\text{Modular ratio} = \frac{E_s}{E_w}} > 1$$

$$\boxed{\sigma = E \cdot \epsilon}$$



$\sigma_{w\,max} = E_w \cdot \epsilon_c$

$\sigma_{st} = E_s \cdot c$

(ε ϵ-distribution is linear)

(σ- distribution)

$\sigma_{w\,max} = E_w \cdot \epsilon_c$

$\sigma_{st} = E_s \cdot \epsilon_c$

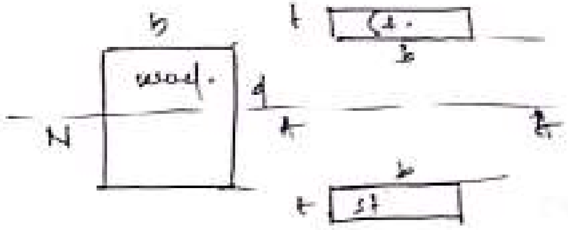
$$\text{Ratio of} = \frac{\sigma_{st}}{\sigma_{wood\,max}} = \frac{E_s}{E_w} = M$$

$$\boxed{\sigma_{st} = \sigma_{wood\,max} \cdot M} \quad **$$

$$\boxed{\sigma_{t\,max} = \frac{\sigma_{st}}{(d/2)}\left(\frac{d}{2} + t\right)} \quad **$$

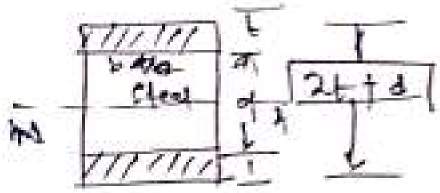
Method-1

* Moment carrying Capacity :-



$M = M_{wood} + M_{steel}$

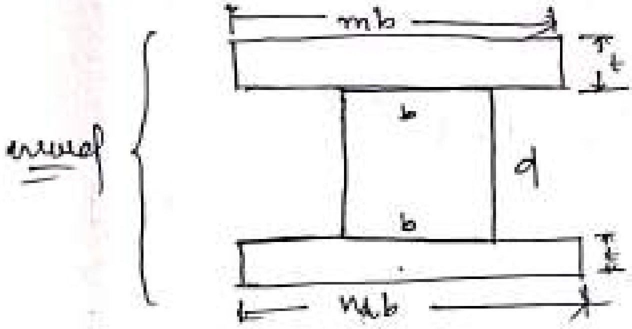
$$\boxed{M_{wood} = \sigma_{w\,max} \times \frac{bd^2}{6}}$$



$$\boxed{\sigma_{steel} = \sigma_{s\,max} \times \frac{b \times (d+2t)^2}{6} - \sigma_{st} \times \frac{bd^2}{6}}$$

Method-2 Equivalent area ~~[illegible]~~ Method !-

Equevelent wood



Steel → wood

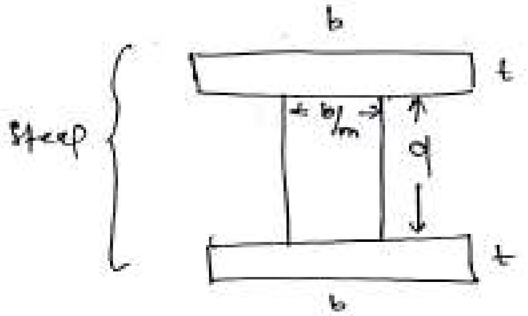
stronger (m times)

$I_s \to m I_s = W$

$\frac{bt^3}{12} \to \boxed{m}.\frac{bt^3}{12}$

$$\boxed{M = \sigma_{wood\,max} \times \frac{I_{NA}}{y_{max}}}$$

Equevent Steel :-



Wood → steel

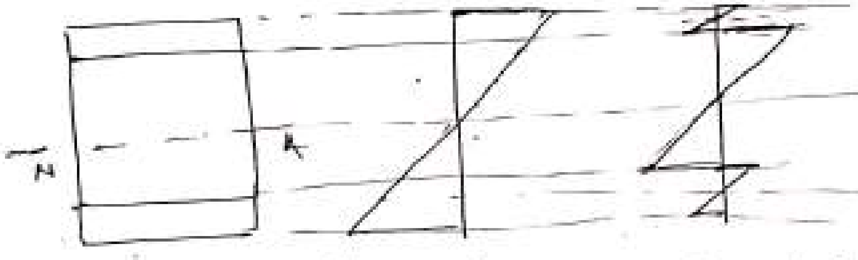
↓

weaker → stronger.

$I \to \frac{I}{m}$

$$\boxed{M = \sigma_{steel\,max} \times \frac{I_{NA}}{y_{max}}}$$

Note : Strain - variation !-

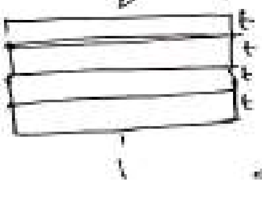


ε-Variation for bonded c/s

ε-variation for non bonded c/s

Question-1 'n' ~~[illegible]~~ non plates of thickness 't' and width 'b' are arranged in parallel at the c/s. determine the ratio of moment of their inertia when they bonded to that when they are unbonded.

Sol^n



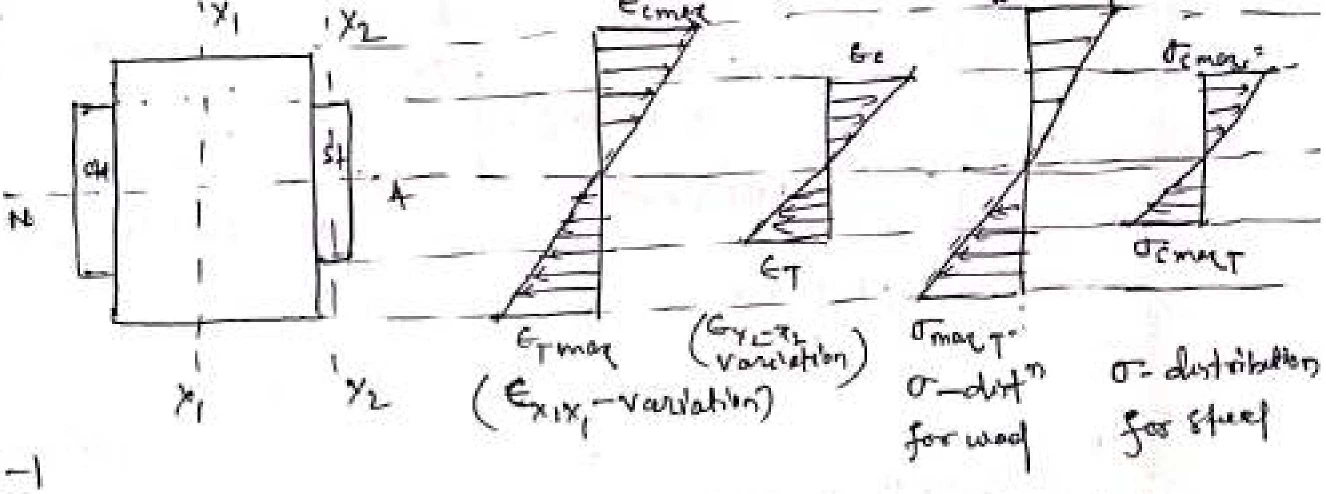
$I_{bonded} = \frac{b\times(nt)^3}{12} = \frac{n^3.bt^3}{12}$

$I_{unbonded} = \frac{n.(b.t^3)}{12} = \frac{n.b.t^3}{12}$

$\text{Ratio} = \frac{I_{bonded}}{I_{unbonded}} = \frac{\frac{n^3 bt^3}{12}}{\frac{nbt^3}{12}}$ $\boxed{n^2 : 1}$

* Case-2

When flinches are Attached at side!-



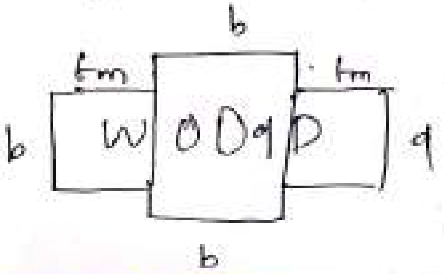
Method-1

$$M = M_{wood} + M_{steel}$$

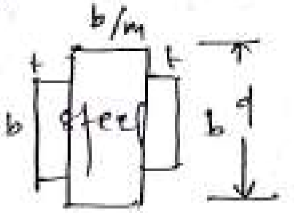
$$M = \left(\sigma_{wmax} \times \frac{bd^2}{6}\right) + \left[\left(\sigma_{stmax} \times \frac{tb^2}{6}\right) \times 2\right]$$

Method-2

Equivalent wood

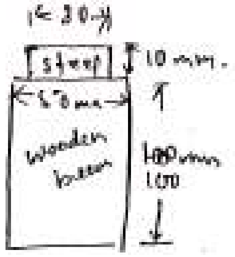


Equivalent Steel



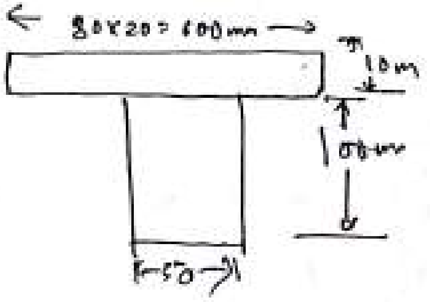
Numerical's ON Bending!-

Q.



If $\sigma_{wood\,max} = 6$ Mpa and $m = 20$ Determine Moment carrying capacity of the c/s using Equalent Area Method.

Solⁿ Convert interms of wood.



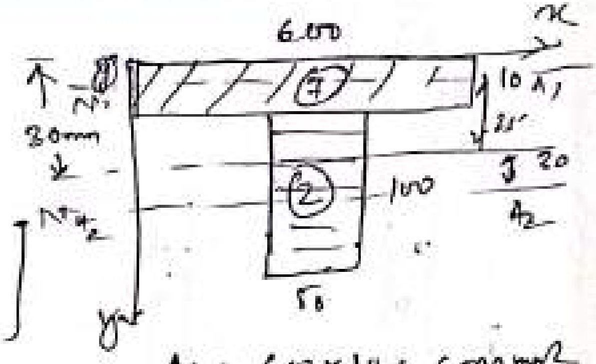
$$M = \sigma_{wood\,max} \times Z$$

$$\boxed{Z = \frac{I_{NA}}{y_{max}}}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \left[\frac{\cancel{(600\times10)}(6000\times5) + (5000\times60)}{6000+5000} \right]$$

$$= \frac{30+300}{11} = \frac{330}{11} = 30\text{mm.}$$



$A_1 = 600\times10 = 6000\text{mm}^2$

$A_2 = 50\times100 = 5000\text{mm}^2$

$y_1 = 5\text{mm.}$

$y_2 = 50+10 = 60\text{mm}$

$$\underline{\underline{I_{NA}}} \text{ for} = \left[\frac{600\times10^3}{12} + 6000\times25^2 \right] + \left[\frac{50\times100^3}{12} + (50\times100)\times30^2 \right]$$

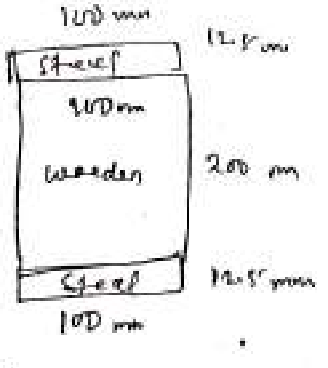
$$= 12.46\times10^6 \text{ mm}^4$$

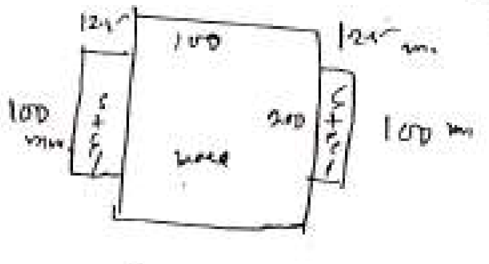
$$M = (\sigma_w)_{max} \times \frac{I_{NA}}{y_{max}}$$

$$= \left(6 \times \frac{12.46\times10^6}{80} \right) \left[\frac{N\text{-mm}}{10^6} \right]$$

$$= 0.93453 \text{ kNm} \quad \overset{\downarrow}{(kN-m)}$$

Q.2





Case-2

Case-1

$\sigma_{wood}\ max = 6\text{Mpa.}$

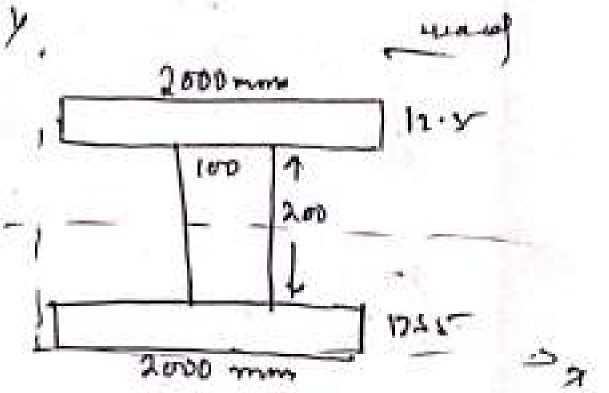
$\frac{E_s}{E_w} = 20$

Compare Moment carrying Capacity of Case-1 and 2.

Soln convert to wood.

Case-1

$$M = (\sigma_w)_{max} \times \frac{I_{NA}}{y_{max}}$$



$\bar{y} = 112.5$ mm.

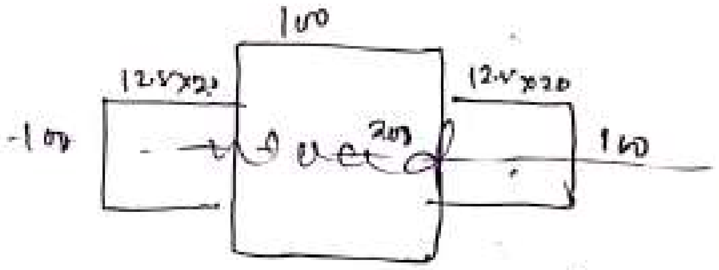
$$I_{NA} = \frac{2000 \times (225)^3}{12} - 2\left(\frac{\frac{(2000-100)}{2} \times (200)^3}{12}\right)$$

$I_{NA} = \overset{631.77}{\cancel{72.63}} \times 10^6 \; mm^4$

$M_1 = 6 \times \dfrac{\overset{631.77}{\cancel{72.63}} \times 10^6}{112.5 \times 10^6}$ (kN-m)

$= ~~3.8736~~ \; 33.6941$ kN-m.

Case-2



$$I_{NA} = \frac{12.5 \times 20 \times 100^3}{12} \times 2 + \frac{100 \times 200^3}{12}$$

$= 1088.33 \; mm^4$

$M_2 = 6 \times \dfrac{108.833 \times 10^6}{100 \times 10^6}$

$= 6.4998$ kN-m.

$$\left\{ \begin{array}{l} M_1 = 33.694 \text{ kN-m} \\ M_2 = 6.4998 \text{ kN-m} \end{array} \right\} \boxed{M_1 > M_2}$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Property of circle.

Products of [illegible] are same

[illegible]

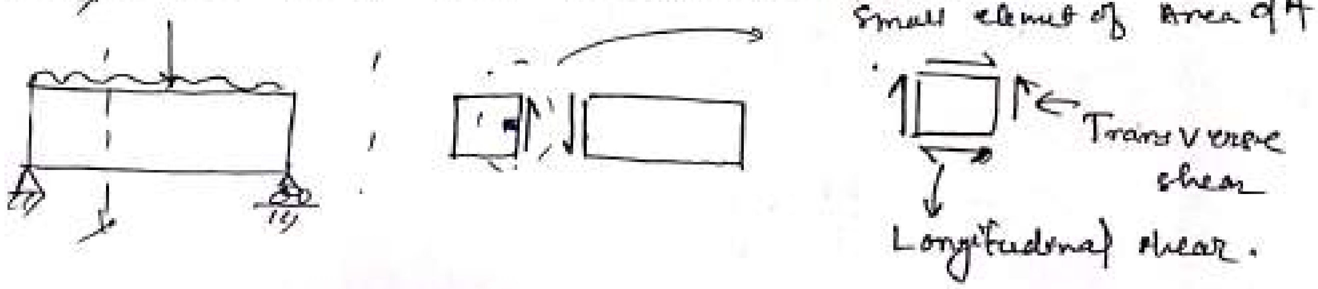
[illegible]

Topic-3 {SHEAR}-

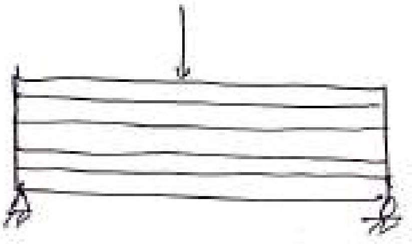
Lectur 36.

Shear Stress

Longitudinal Shear and transverse Shear !-



* Mechanism Of ~~loading~~ longitudinal shear due to Bending :-



No bond

Individual Bending

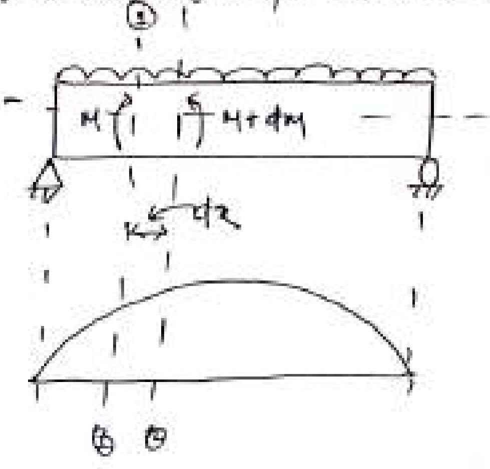
① → ↗ 4 → ⑤

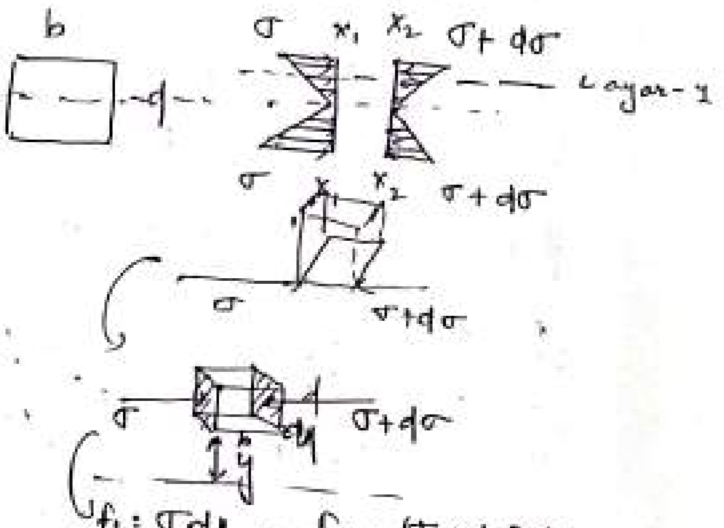
② → 1 5 → ↗

③ → 1 ~~&~~ 2, (up)
5, 4, (down)

Bonded Beam (shear develope due to bending)

* Derivation of shear Stress !-





$f_1 = \sigma dA$, $f_2 = (\sigma + dA) dA$

Net force along x axis

$$= (\sigma + d\sigma) dA - \sigma . dA$$

$$= d\sigma . dA$$

Contnue !-

Balanced shear force : $\tau . dx . b$

If Net force = 0

$$\boxed{d\sigma . dA = \tau . dx . B}$$

Now,

$$\sigma = \frac{M}{I} \times y$$

$$d\sigma = \frac{dM}{I} \times y$$

$$\frac{dM}{I_A} \times y . dA = \tau dx . B$$

$$\int_0 \tau = \frac{dM}{dx} \int_0 \frac{y . dA}{I . b}$$

$\because \int y . dA$ = first moment of Area

$= A\bar{y}$

$\because \frac{dM}{dx} = S.F = V$

$$\boxed{\tau = \frac{V}{Ib} \int y . dA}$$

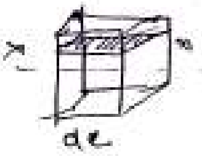
** $\boxed{\tau = \frac{V}{Ib} . A\bar{y}}$ (shear stress)

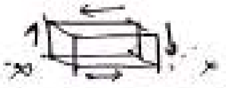
V → Shear force at a section x-x

I → Moment of Inertia about N.A.

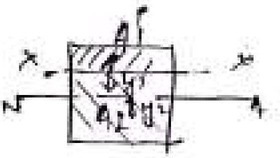
b = width of the section..

$A . \bar{y}$ = Moment of Area.





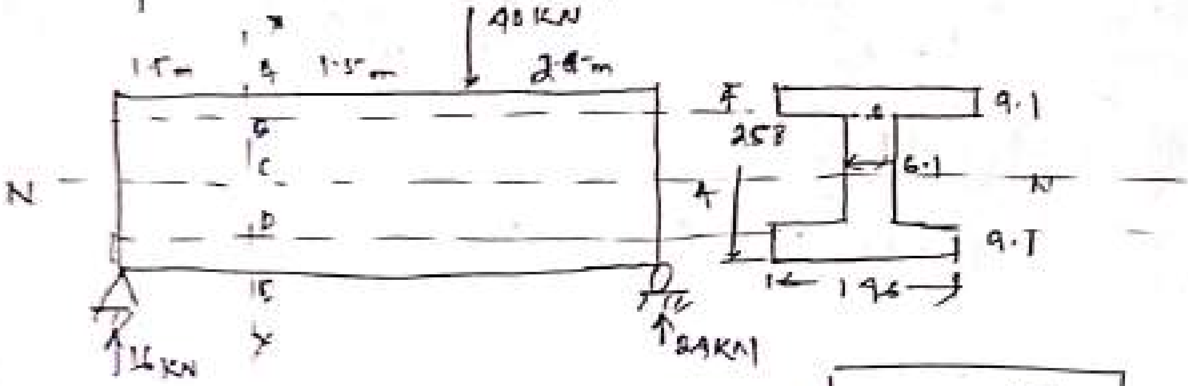
$$\boxed{\tau = \frac{VA\bar{y}}{Ib}}$$



$$\boxed{A_1\bar{y}_1 = A_2\bar{y}_2}$$

Always

Q.1 for the beam shown in the figure, determine shear stres at point A, B, C, D and E at a section x-x.



Sol^n

$I_{NA} = ~~89~~\ 48.1 \times 10^6\ mm^4$ $\qquad$ $\boxed{\tau = \frac{VA\bar{y}}{Ib}}$

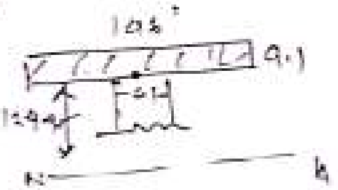
$V_{xx} = 16\ KN$

At point A $\quad \underline{A=0} \quad \tau = 0$

At point E $\quad \underline{\bar{y}=0} \quad \tau = 0$

$\boxed{\tau_A = \tau_E = 0}$

At point B :



4.55
+119.9
= 124.45

τ_B flange : $\frac{VA\bar{y}}{I.b}$

$$= \frac{16 \times 10^3 \times (146 \times 9.1) \times 124.45}{48.1 \times 10^6 \times 146} = 0.3767\ Mpa.$$

$$\tau_B\ web = \frac{V.A.\bar{y}}{I\ b} = \frac{16 \times 10^3 \times (146 \times 9.1) \times 124.45}{48.1 \times 10^6 \times 6.1}$$

$$= 9.016\ Mpa$$

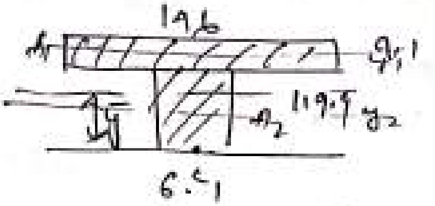
τ_D flange $= 0.3767$ Mpa

τ_D web $= 9.016$ Mpa.

$\tau_C =$

for C

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$



$$A\bar{y} = A_1 y_1 + A_2 y_2$$

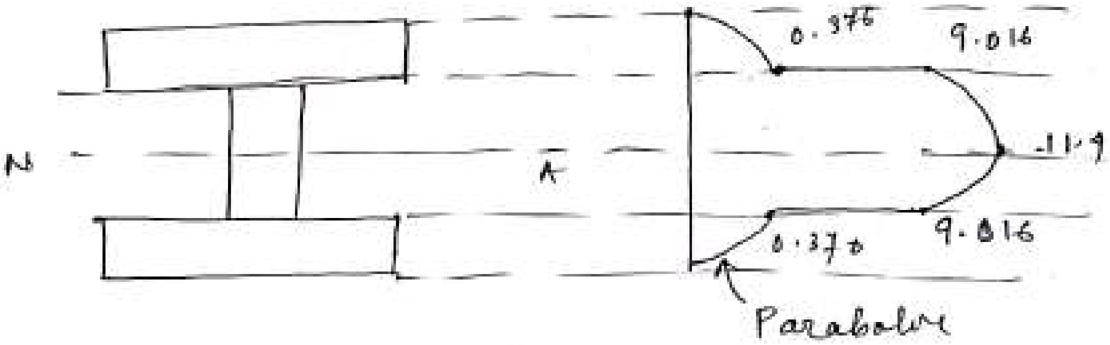
$$= \left[(146 \times 9.1)(129.45)\right] + \left[(119.9 \times 6.1) \times \frac{119.9}{2}\right]$$

$$= 209191.1 \text{ mm}^3$$

$$\tau_c = \frac{VA\bar{y}}{Ib}$$

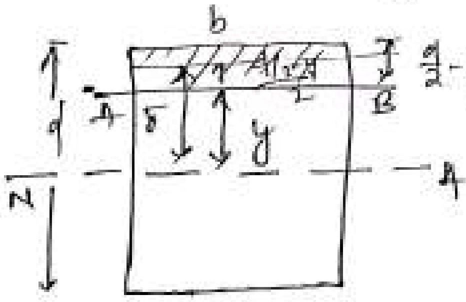
$$= \frac{16 \times 10^3 \times 209191.1}{48.1 \times 10^6 \times 6.1} = 11.4 \text{ mpa}$$

* shear stress distribution :-



* WHY THE Shear Stress Distribution is PARABOLIC :-

Considered of Rectangular C/s :-



$$\bar{y} = y + \frac{\frac{d}{2} - y}{2}$$

$$= \frac{2y + \frac{d}{2} - y}{2}$$

$$\bar{y} = \frac{y + \frac{d}{2}}{2}$$

$$\tau = \frac{V \cdot A\bar{y}}{Ib}$$

$$= \frac{V \times \left(\frac{d}{2} - y\right) \cdot \cancel{b}\left(\frac{y + \frac{d}{2}}{\cancel{2}}\right)}{\frac{bd^3}{\cancel{12}_{6}} \times \cancel{b}}$$

$$\boxed{\tau = \frac{6V}{bd^3}\left(\frac{d^2}{4} - \boxed{y^2}\right)}$$

y^2 → Indicates the Parabolic ~~[illegible]~~ variation

(I) Shear Stress distribution for Rectangular C/s :-

$$\boxed{\tau = \frac{6V}{bd^3}\left(\frac{d^2}{2} - y^2\right)}$$

Location of min^m shear stress

$\tau = 0$

$$\frac{6V}{bd^3}\left(\frac{d^2}{2} - y^2\right) = 0$$

$$\frac{d^2}{4} - y^2 = 0$$

$$\boxed{y = \pm\frac{d}{2}}$$

$$\boxed{\tau_{min} = 0}$$

Location of max^m shear stress

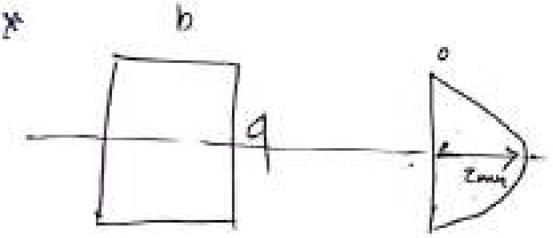
$$\frac{d\tau}{dy} = 0$$

$$\boxed{y = 0}$$

$$\tau_{max} = \frac{6V}{bd^3}\times\frac{d^2}{4}$$

$$\boxed{\tau_{max} = \frac{3V}{2bd}}$$

**

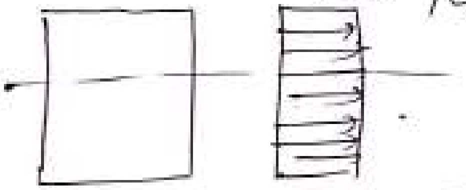


$$\boxed{\tau_{max} = \frac{3V}{2bd}}$$

Shear stress distribution ← due to bending

**

$\tau_{av} = V/c/s$ Area.



Due to ~~for~~ transverse shear stress distribution

$$\boxed{\tau_{av} = \frac{V}{bd}} \quad \frac{\text{Force}}{\text{Area}}$$

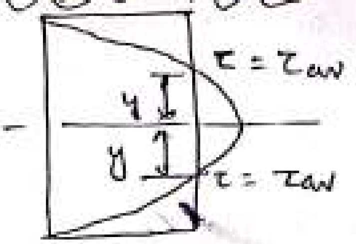
$$\frac{\tau_{max}}{\tau_{av}} = \frac{\frac{3}{2}\frac{V}{bd}}{V/bd}$$

$$\boxed{\tau_{max} = \frac{3}{2}\cdot\tau_{av}}$$

τmax → due to bending

τav → Nominal shear
or
due to transverse loading

* Distance when τ = τav



τ = τav

$$\frac{6V}{bd^3}\left(\frac{d^2}{4}-y^2\right) = \frac{V}{bd}$$

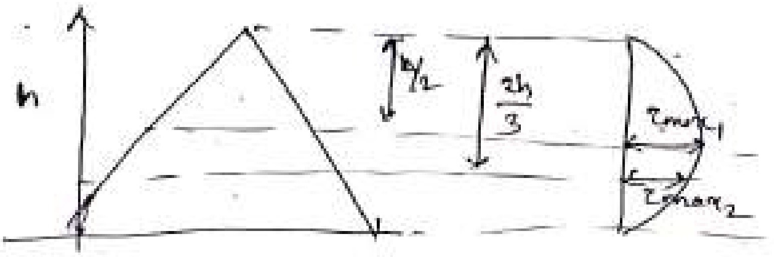
$$\frac{d^2}{4}-y^2 = \frac{d^2}{6}$$

$$y^2 = \frac{d^2}{4}-\frac{d^2}{6}$$

$$\boxed{y = \frac{d}{2\sqrt{3}}}$$

Distance of from N.A when shear stress distribution due to Bending and transverse loading will be same.

② Shear Stress Distribution for triangular C/s :-

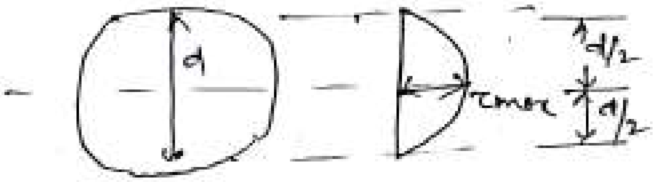


~~$\tau_{max_1} = \frac{3}{2}$~~

$$\boxed{\begin{aligned}\tau_{max_1} &= \frac{3}{2}\,\tau_{av}\\ \tau_{max_2} &= \frac{4}{3}\,\tau_{av}\end{aligned}}$$

$$\boxed{\tau_{av} = \frac{V}{\frac{1}{2}\times b\cdot h}}$$

③ Shear stress distribution for Circular C/s :-

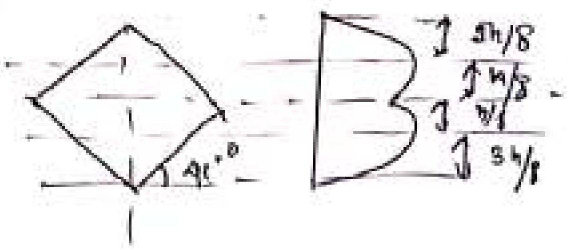


$$\boxed{\tau_{max} = \frac{4}{3}\,\tau_{av}}$$

~~$\tau_{av} = \frac{V}{\frac{1}{2}bh}$~~

$$\boxed{\tau_{av} = \frac{V}{\frac{\pi}{4}d^2}}$$

④ Shear Stress distribution for dimond C/s :-



$$\boxed{\tau_{max} = \frac{9}{8}\cdot\tau_{av}}$$

$$\tau_{av} = \frac{V}{C/s\ Area}.$$

$$\boxed{\tau_{av} = \frac{V}{a^2}}$$

Lec-37

POINTS TO DRAW SHEAR STREE DISTRIBUTION :-

1) $\boxed{\tau = \frac{VA\bar{y}}{Ib}}$ → As we move towards centre for a section, ($A\bar{y}$) will increase, τ will increase.

$$\boxed{\tau \propto A\bar{y}}$$

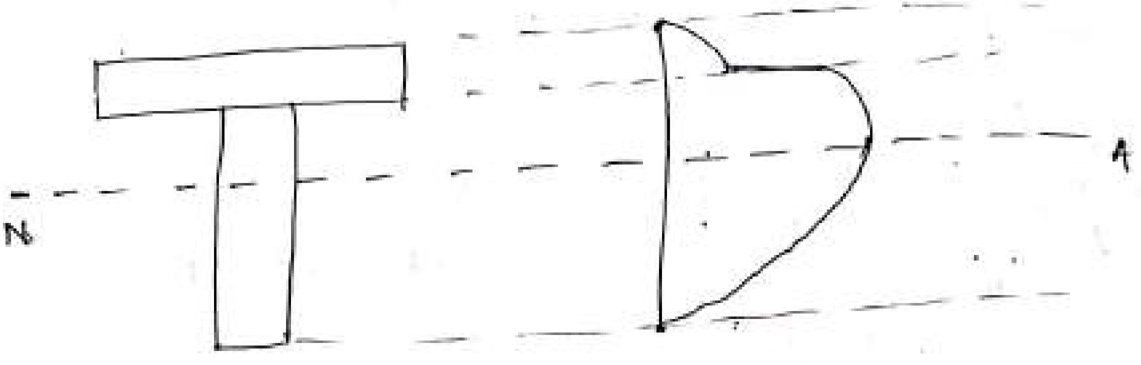
2) If there is sudden change (Increase or decrease) of width of C/s, then there is sudden change of shear stress.

$$\tau = \frac{VA\bar{y}}{I,b} \Rightarrow \boxed{\tau \propto \frac{1}{b}}$$

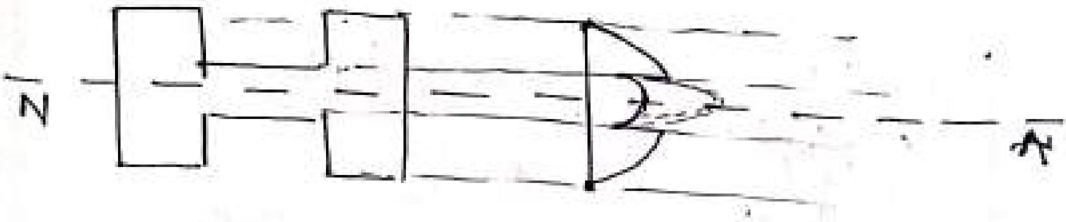
3) The variation is Parabolic.

** Shear Stress Distribution for Various Section :-

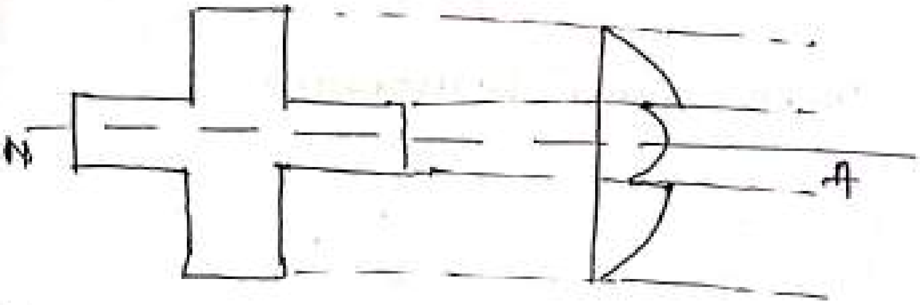
1) T-section



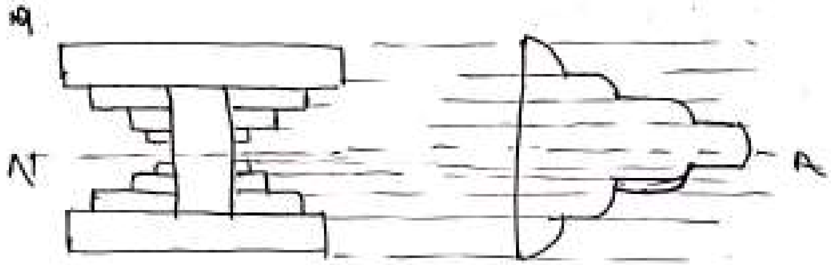
2) H-Section



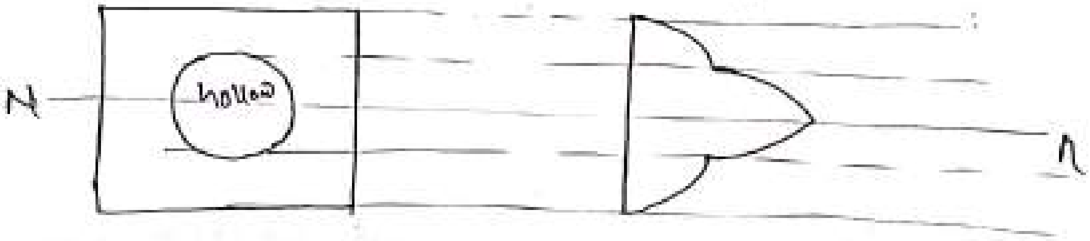
3) + - Section



4)

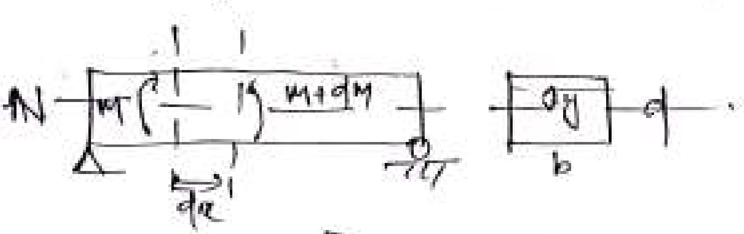


⑤



** SHEAR FLOW :-

** Shear flow Mean Shear force per Unit Length.



$$df = d\sigma . dA$$

$$df = \frac{dM}{I} . y . dA$$

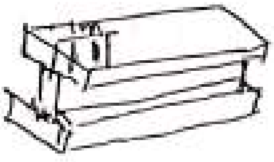
$$\frac{df}{dx} = \frac{dM}{dx} \cancel{\frac{Iy}{I} . dA} \times \frac{y}{I} . dA$$

$$\frac{\text{Shear force}}{\text{Length}} = \frac{dM}{dx}\cdot\frac{1}{\cancel{y}I}\int y\cdot dA$$

$$\boxed{\frac{SF}{\text{length}} = \frac{V\cdot A\bar{y}}{I}} = \cancel{Q}q^*$$

Significent of q^*

Rolled steel section.

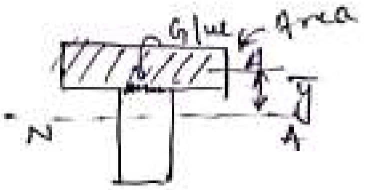


$\frac{SF}{1m} = 20kN$

1 bolt = 5kN (Capacity of bolt)

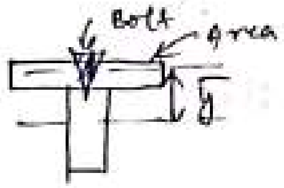
No of bolt = $\frac{20}{5}$ = 4 nos per metre.

Q.

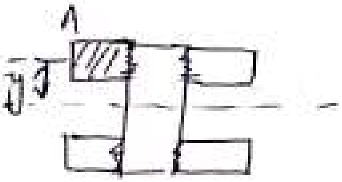


$q^* = \frac{VA\bar{y}}{I}$

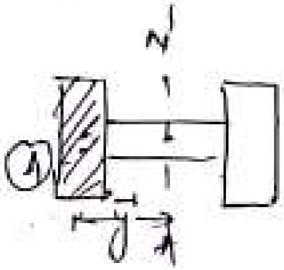
②



③



④

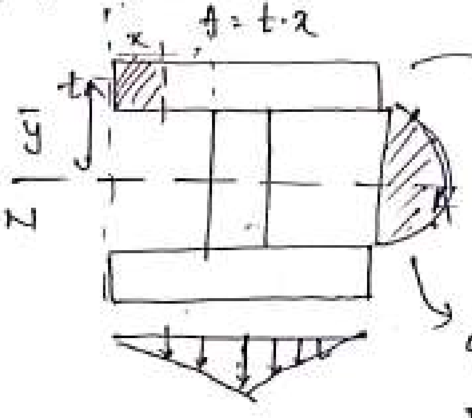


** Shear flow Diagram:-

Take I - Section

$$\boxed{q^* = \frac{VA\bar{y}}{I}}$$

$A = t\cdot x$



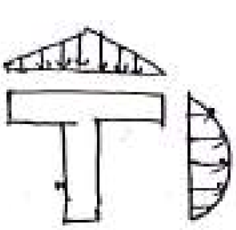
$* \; q^* = \frac{V}{I}\times t\cdot x\times\bar{y}$

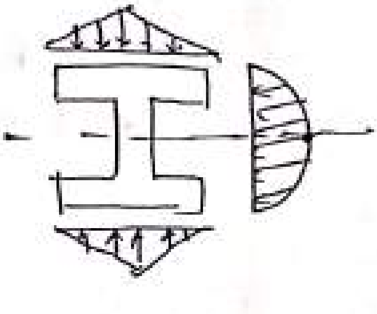
$$\boxed{q^* \propto x}$$

$q^* = \frac{V}{I}\cdot A\bar{y}$

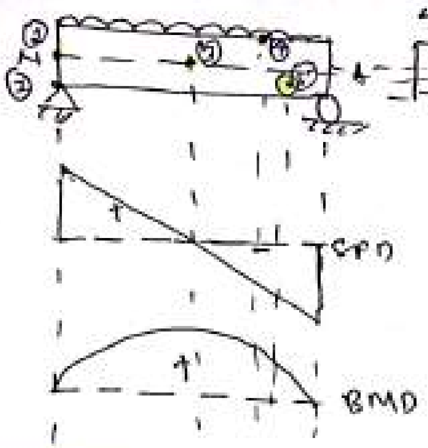
$$\boxed{q^* \propto A\bar{y}}$$

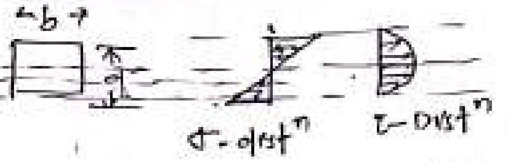
Tee Section





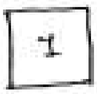
✡ *-* STRUCTURAL ELEMENT DUE TO BENDING & SHEAR:-





$= \frac{M}{I} \cdot y$ $\quad = \frac{VA\bar{y}}{Ib}$

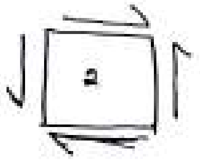
Take specific element-1



zero shear. due to no zero B.M

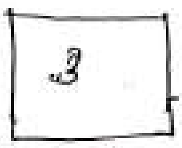
No-stress Element

Element-2



(pure shear element)

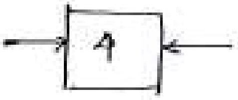
3rd Element



SF=0, shear stress=0
(No-bending stress)

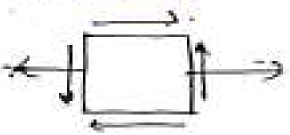
(No stress element)

Element-4



no shear stress.
Bending stress exist!

Element-5

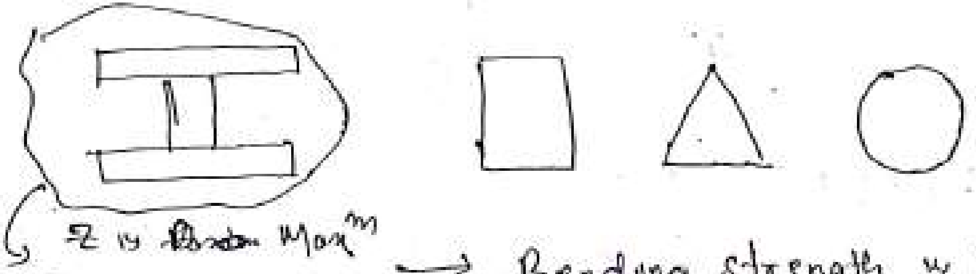


tensile bending stress
shear stress

Interview Q

** Why I-section is most efficient section!-

for the same area

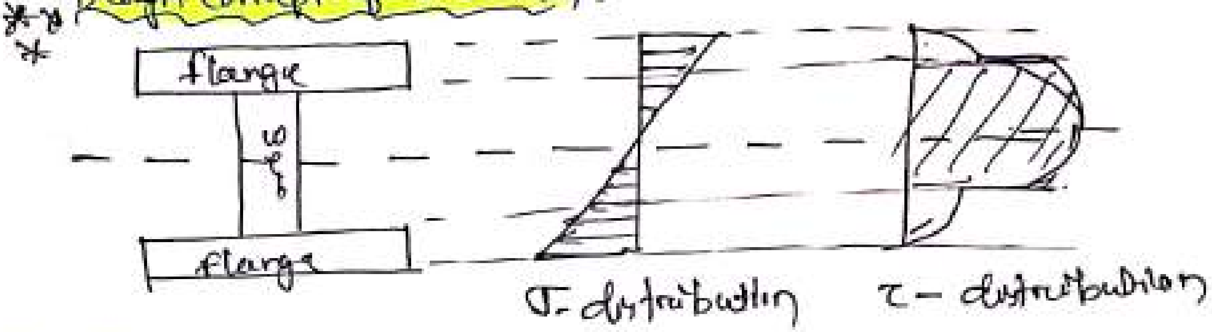


Z is maxᵐ for I section → Bending strength is high.
↓
Most efficient & Most economical section.

Note :-

for Design, Bending stress is considered whereas shear and axial stress are neglected.

** Design concept of I section :-**

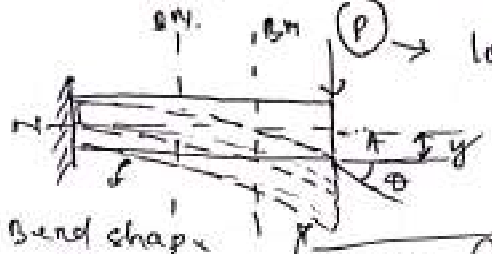


Note

- * 70 - 80 % of bending stress is resisted by flange, So flange is designed for bending stress.
- * 70-80% of shear stress is resisted by web, so, the web is designed for shear stress.

CH-6: SLOP & DEFLECTION OF BEAM:

6.1* 1) DEFLECTION & slope:-



→ load is within proportionality limit

* Bend shape is due to BM @ a section

Curve = Circular Arc

y = deflection
θ = Slope

** Explanation

BM @ BC = 0
BM @ AB = -Px

6.2* EQUATION OF DEFLECTED CURVE! - (Elastic Curve)

***

$$\boxed{\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}}$$

$L = \frac{I'm}{2y}$

y is in (mm)

y = very less

$\frac{dy}{dx}$ is also very less

$\left(\frac{dy}{dx}\right)^2$ is very very less

$\cancel{\left(\frac{dy}{dx}\right)^2} \approx 0$

$$\boxed{\frac{1}{R} = \frac{d^2y}{dx^2}}$$

from bending relation

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

*** $\boxed{\frac{M}{EI} = \frac{1}{R}}$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\boxed{M = EI \cdot \frac{d^2y}{dx^2}}$$

Very Important

y = deflection

$\frac{dy}{dx}$ = slope = θ

$\frac{d^2y}{dx^2} = \frac{M}{EI}$

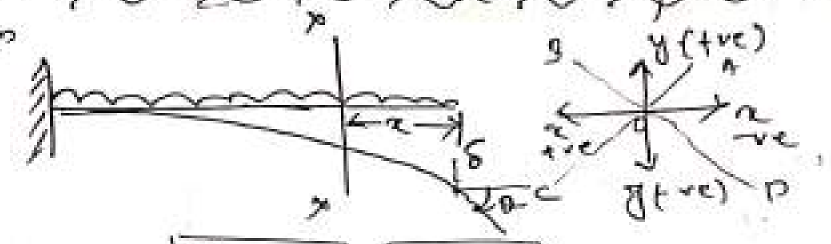
$\frac{d^3y}{dy^3} = \frac{d}{dx}\left(\frac{M}{EI}\right) = \frac{dM}{dx} \cdot \frac{1}{EI} = \frac{V}{EI}$

$\frac{d^4y}{dy^4} = \frac{d}{dx}\left(\frac{V}{EI}\right) = \frac{1}{EI}\left(\frac{dV}{dx}\right) = \frac{-w}{EI}$

* Sign Convention for Slope & Deflection ! -

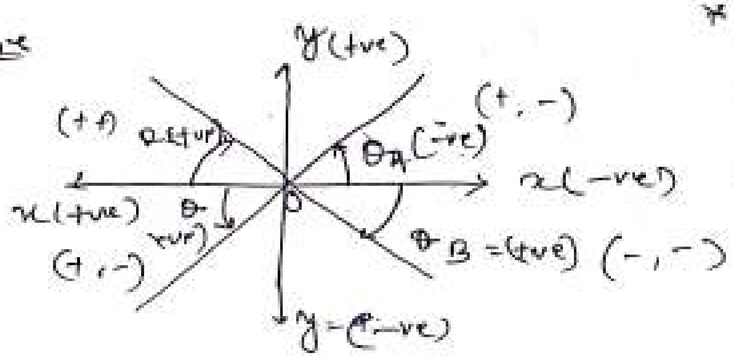
① Deflection

Right to left



$\boxed{\delta = -ve}$ $\boxed{\theta = +ve}$

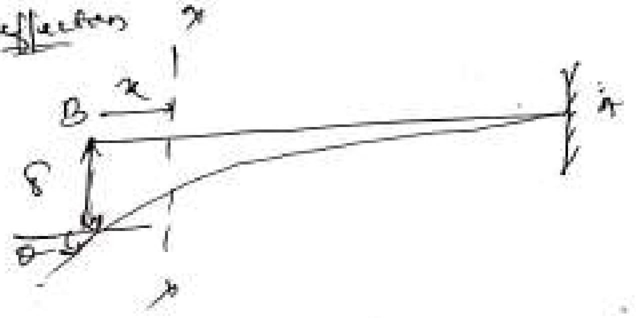
Slope



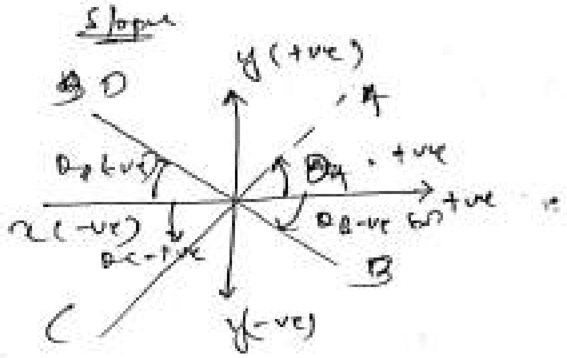
** Clockwise rotation = +ve
Anticlockwise = -ve

② Deflection

Left to right



Slope

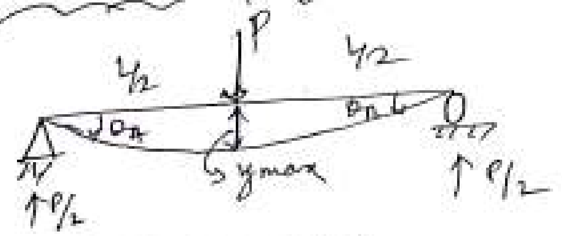


** Anticlock wise = +ve
Clock wise = -ve

δ = -ve
θ = +ve

6.2* STANDED FORMOLA FOR SLOPE & DEFLECTION ! -

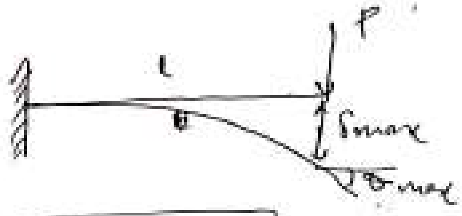
①



$$\theta_A = \theta_B = \frac{PL^2}{16EI}$$

$$\delta_{max} = \frac{PL^3}{48EI}$$

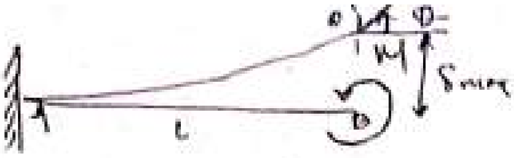
②



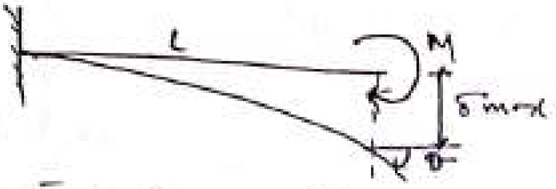
$$\theta_{max} = \frac{Pl^2}{2EI}$$

$$\delta_{max} = \frac{Pl^3}{3EI}$$

③



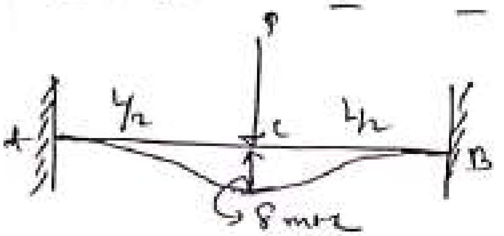
$$\boxed{\theta = \frac{Ml}{EI}}$$



$$\boxed{\delta_{max} = \frac{Ml^2}{2EI}}$$

(Lec-39)

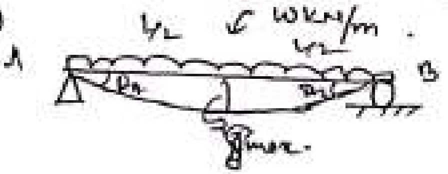
④



$$\boxed{\theta_A = \theta_B = \theta_C = 0}$$

$$\boxed{\delta_{max} = \frac{1}{4}\left(\frac{Pl^3}{48EI}\right)}$$

⑤

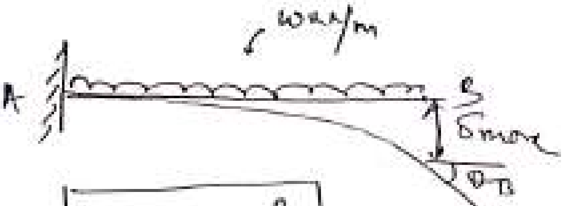


$\theta_C = 0$

$$\boxed{\theta_A = \theta_B = \frac{wl^3}{24EI}}$$

$$\boxed{\delta_{max} = \frac{5}{384}\cdot\frac{wl^4}{EI}}$$

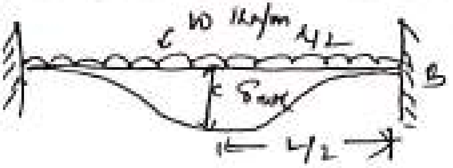
⑥



$$\boxed{\theta_B = \frac{wl^3}{6EI}}$$

$$\boxed{\delta_B = \frac{wl^4}{8EI}}$$

⑦

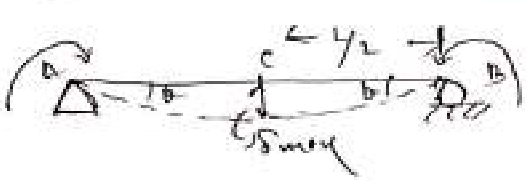


$$\boxed{\theta_A = \theta_B = \theta_C = 0}$$

$$\boxed{\delta_{max} = \frac{1}{5}\left(\frac{5}{384}\cdot\frac{wl^4}{EI}\right)}$$

$$\boxed{\delta_{max} = \frac{wl^4}{384EI}}$$

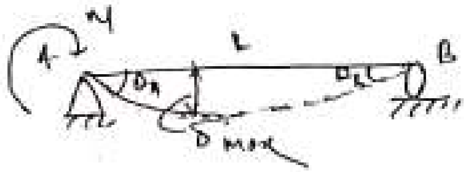
8)



$$\boxed{\theta_A = \theta_B = \frac{ML}{2EI}}$$

$$\boxed{\delta_{max} = \frac{Ml^2}{8EI}}$$

⑨



$$\boxed{\theta_A = \frac{Ml}{3EI}}$$

$$\boxed{\theta_B = \frac{Ml}{6EI}}$$

$$\boxed{\delta_{max} = \frac{Ml^2}{9\sqrt{3}EI}}$$

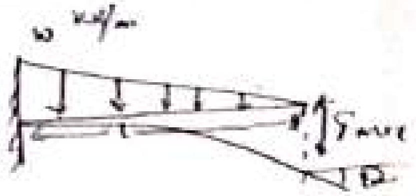
$$\boxed{\delta_{max} \text{ at } x < l/2}$$

10)

$$\boxed{\theta_A = \theta_B = \frac{Ml}{24EI}}$$

$$\boxed{\theta_C = \frac{Ml}{12EI}}$$

(11)



$$\boxed{\theta = \frac{wl^3}{24EI}}$$

$$\boxed{\delta_{max} = \frac{wl^4}{30EI}}$$

Remembering Short cut.

$\theta \rightarrow$ l, l^2, l^3

$$\xrightarrow{} \text{N/m}\quad \frac{\boxed{(w)\times l^3} \text{ or } \boxed{(P)\times l^2} \text{ or } \boxed{M\times l}}{\boxed{EI}} \text{ Radian}$$

$\hookrightarrow \frac{N}{m^2}\times m^4 = N\text{-}m^2$

$\delta \rightarrow$ $\frac{l^2}{l^3}$, l^4

$$\boxed{N/m}\frac{\boxed{w\cdot l^4} \text{ or } \boxed{P\cdot l^3} \text{ or } \boxed{M\cdot l^2}}{\boxed{EI}} = m.$$

$\hookrightarrow N\text{-}m^2$

* METHOD TO ~~DE~~ FIND SLOPE AND DEFLECTION

✓ 1) Double Integration Method
2) Maculay's Method
✓ 3) Strain Energy Method (Best method) (accurate Reasult)
✓ 4) Moment Area Method
✓ 5) Conjugate Beam Method.
6) Superposition Theorom
7) Maxwell Reciprocal theorom.
** 8) Betti's Theorom.

Ex 2) SUPERPOSITION THEOROM METHOD :-

Super position theorem! Condition! -

If ., { i) Loading is applied within proportionality limit.

ii) Load function (SF, BM, Axial stress, Slope, Deflection, Angle of twist) varies linearly with the load applied.

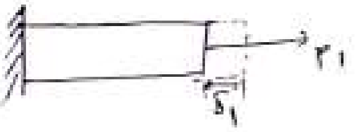
theorem

then,

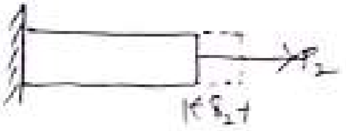
Summation of effect of individual loading at a point is equal to effect of multiple loading at the same point.

Example

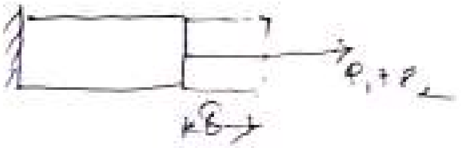
1)



2)



3)



$$\delta = \delta_1 + \delta_2$$

$$\frac{(P_1+P_2)L}{AE} = \frac{P_1 L}{AE} + \frac{P_2 L}{AE}$$

$$\boxed{(P_1+P_2) = P_1 + P_2}$$

$$\text{So, } \boxed{\delta \propto P}$$

Strain energy

for sec^n 1 → $U_1 = \frac{1}{2} \cdot P_1 \delta_1 = \frac{1}{2} P_1 \cdot \frac{P_1 L}{AE} = \frac{P^2 L}{2AE}$

2 → $U_2 = \frac{P_2^2 L}{2AE}$

3 → $U = \frac{(P_1-P_2)^2 L}{2AE}$

here $\boxed{U \propto P^2}$ → So, Superposition theorem can not be applied for the strain energy.

continue :-

So, $U_1+U_2 = \frac{P_1^2 l}{2AE} + \frac{P_2^2 L}{2AE} = \frac{(P_1^2+P_2^2).L}{2AE}$

$$U = \frac{(P_1+P_2)^2 L}{2AE} = \frac{(P_1^2+P_2^2+2.P_1P_2)l}{2AE}.$$

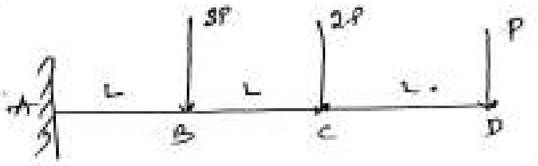
from eqn have been

$\boxed{U > U_1+U_2}$ & superposition principle is not valid.

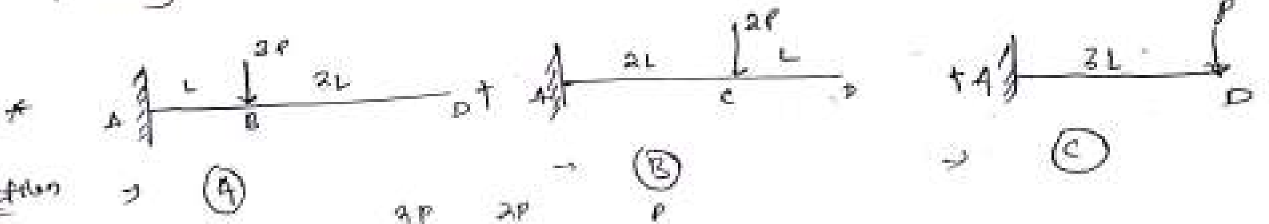
Lec-40

* NUMERICALS USING SUPERPOSITION METHOD:-

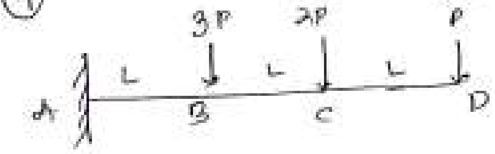
Q.1 Determine the slope and deflection at the free end.



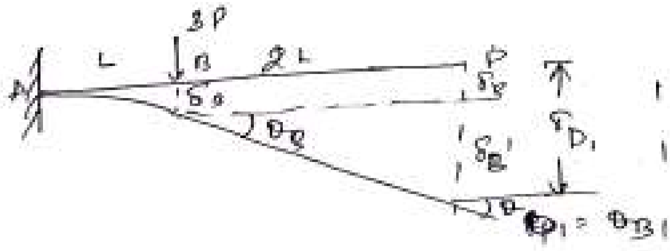
Sol^n According to super position theorem.



~~Sol~~ =



Section
Ⓐ



for small angle
$\tan\theta \approx \sin\theta \approx \theta$

$$\theta_B = \frac{\delta_B'}{2L}$$

$$\boxed{\theta_B = \frac{3P(L)^2}{2EI}} = \theta_{D_1}$$

$$\delta_B' = \theta_B . 2L$$

$$\delta_B' = \frac{3PL^2}{2EI} . 2L$$

$$\boxed{\delta_B = \frac{3P(L)^3}{3EI}}$$

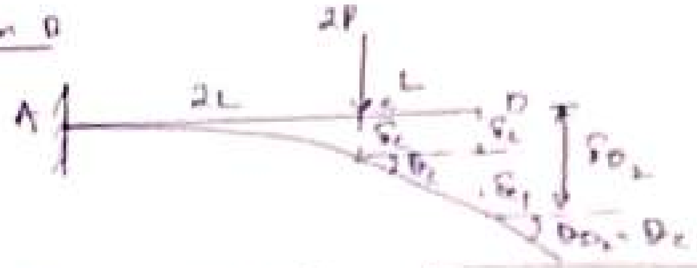
$$\boxed{\delta_B' = \frac{3PL^3}{EI}}$$

$$\theta_{D_1} = \theta_B + \theta_{B_1}$$

$$= \frac{PL^3}{EI} + \frac{3PL^3}{EI}$$

$$\boxed{\theta_{D_1} = \frac{4PL^3}{EI}}$$

Section B



$$\theta_c = \frac{2P(2L)^2}{2 \cdot EI} \Rightarrow \boxed{\theta_c = \frac{4PL^2}{EI} = \theta_{D_2}}$$

$$\delta_c = \frac{2P(2L)^3}{3 \cdot EI} = \frac{16PL^3}{3EI}$$

$$\delta_c' = \theta_c \times L = \frac{4PL^2}{EI} \times L = \frac{4PL^3}{EI}$$

$$\delta_{D_2} = \delta_c + \delta_c' = \frac{16PL^3}{3EI} + \frac{4PL^3}{EI}$$

$$\boxed{\delta_{D_2} = \frac{28}{3}\frac{PL^3}{EI}}$$

Section - C

$$\theta_{D_3} = \frac{P.(3L)^2}{2EI} \Rightarrow \boxed{\theta_{D_3} = \frac{9PL^2}{2EI}}$$

$$\delta_{D_3} = \frac{P.(3L)^3}{3EI} = \boxed{\frac{9PL^3}{3EI} = \delta_{D_3}}$$

$\theta_D = \theta_{D_1} + \theta_{D_2} + \theta_{D_3}$

$= \frac{3}{2}\frac{PL^2}{EI} + \frac{4PL^2}{EI} + \frac{9PL^2}{2EL}$

$$\boxed{\theta_D = \frac{10PL^2}{2EI}}$$ Ans

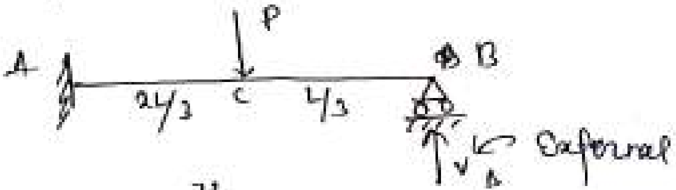
$\delta_D = \delta_{D_1} + \delta_{D_2} + \delta_{D_3}$

$= \frac{4PL^3}{EI} + \frac{28}{3}\frac{PL^3}{EI} + \frac{9PL^3}{EI}$

$= \frac{12PL^3 + 28PL^3 + 27PL^3}{3EI}$

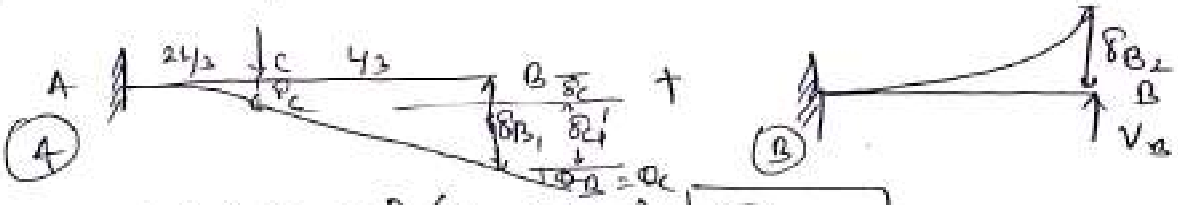
$$\boxed{\delta_D = \frac{67PL^3}{3EI}}$$ Ans

Q.2) Determine the prop reaction?



??

Sol^n



Compatability eq^n (condition) $\boxed{\delta_B = 0}$

See (A)

for δ_{B_1}

$\theta_c = \frac{P\left(\frac{2L}{3}\right)^2}{2EI} = \boxed{\frac{2PL^2}{9EI} = \theta_B}$

$\delta_c = \frac{P.\left(\frac{2L}{3}\right)^3}{3EI} = \frac{8PL^3}{81EI}$

$\delta_c' = \theta_c . \frac{L}{3} = \frac{2PL^2}{9EI} . \frac{L}{3} = \frac{2PL^3}{27EI}$

$\delta_{B_1} = \delta_c + \delta_c' = \frac{8PL^3}{81EI} + \frac{2PL^3}{27EI} = \boxed{\frac{14PL^3}{81EI} = \delta_{B_2}}$

Position (B)

$$\delta_{B_2} = \frac{VL^3}{3EI}$$

Now

$$\delta_{B_1} = \delta_{B_2}$$

$$\frac{14}{81\times 2}\cdot\frac{PL^3}{EI} = \frac{VL^3}{3EI}$$

$$\boxed{V = \frac{14P}{27}}$$ Ans

Note
Extra

*Check for Load transfer! -

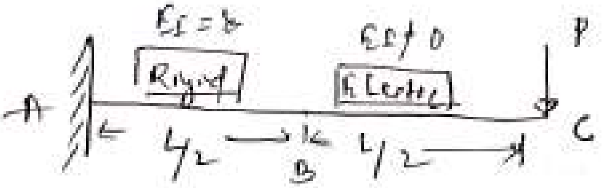
$$\delta_{B_1} = \frac{PL^3}{3EI} - \frac{\left(\frac{PL}{3}\right)\cdot L^2}{2EI}$$

$$= \frac{PL^3}{EI}\left(\frac{1}{3} - \frac{1}{6}\right)$$

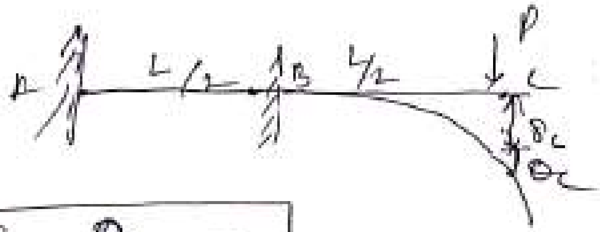
$$= \frac{PL^3}{6EI}$$ So here θ_B is [illegible]

This is Wrong (N)of Same Applicability

Q.3 Determine the slope & Deflection at B & C?



Sol^n

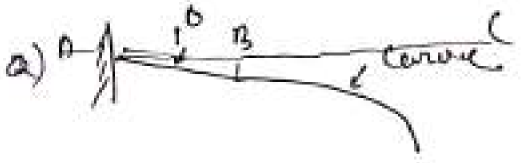


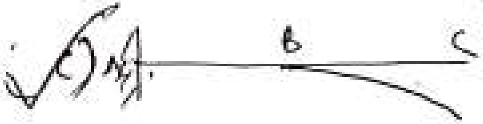
$$\boxed{\delta_B = \theta_C = 0}$$

$$\theta_C = \frac{P\left(\frac{L}{2}\right)^2}{2EI} \Rightarrow \boxed{\theta_C = \frac{PL^2}{8EI}}$$

$$\theta_C = PL^2$$

Option for deflected shape





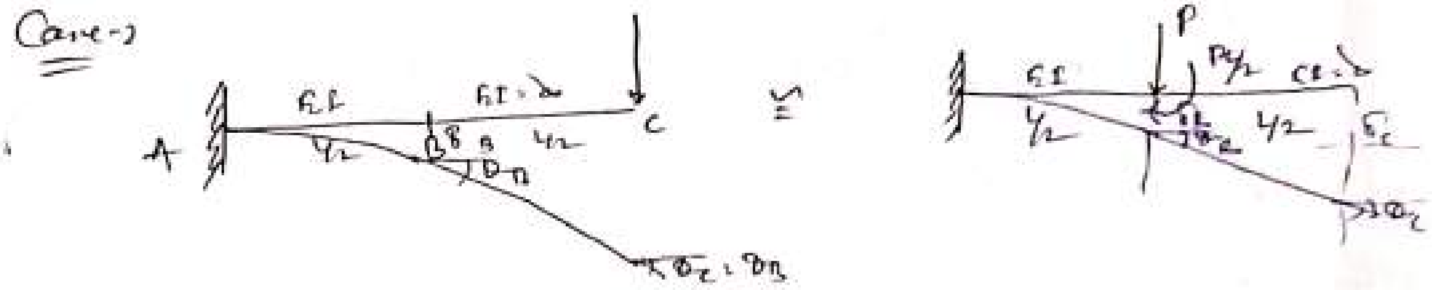
d) none of these

Ans (C)

$$\delta_C = \frac{P\left(\frac{L}{2}\right)^3}{3EI}$$

1) $$\boxed{\delta_C = \frac{PL^3}{24EI}}$$ (Ans)

Case-2



$$\theta_B = \frac{P\left(L/2\right)^2}{2EI} + \frac{\left(\frac{PL}{2}\right)\left(\frac{L}{2}\right)}{EI}$$

$$= \frac{PL^2}{8EI} + \frac{PL^2}{4EI} = \boxed{\frac{3PL^2}{8EI} = \theta_B}$$

$$\delta_B = \frac{P(L/2)^3}{3EI} + \frac{\left(\frac{PL}{2}\right)\left(\frac{L}{2}\right)^3}{2EI}$$

$$= \frac{PL^3}{24EI} + \frac{PL^3}{16EI} = \boxed{\frac{5PL^3}{48EI} = \delta_B}$$

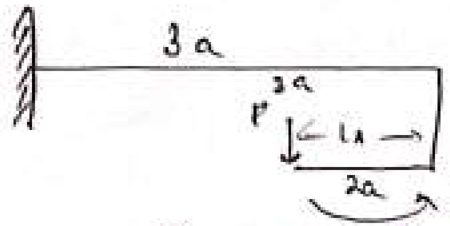
$$\delta_B' = \theta_B \times \frac{L}{2}$$

$$= \frac{3PL^2}{8EI} \times \frac{1}{2} = \frac{3PL^2}{16EI}$$

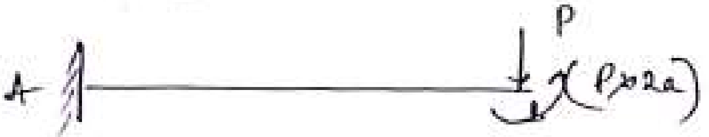
$$\delta_C = \delta_B + \delta_B' = \frac{5PL^3}{48EI} + \frac{3PL^3}{16EI}$$

$$\boxed{\delta_C = \frac{7}{24} \cdot \frac{PL^3}{EI}}$$ $$\boxed{\theta_B = \theta_C = \frac{3PL^2}{8EI}}$$

Q.5 Determine the slope and deflection at B?



Solⁿ Transfer the Load



$$\theta_B = \frac{P(3a)^2}{2EI} - \frac{(P\times 2a)\times(3a)}{EI}$$

$$= \frac{9}{2}\frac{Pa^2}{EI} - \frac{6Pa^2}{EI}$$

$$\boxed{\theta_B = -1.5\frac{Pa^2}{EI}} \text{ (Sag)} (\circlearrowleft)$$

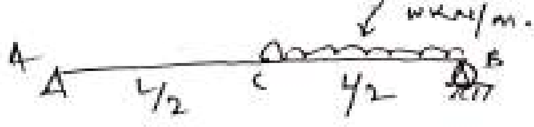
$$\delta_B = \frac{P(3a)^3}{3EI} - \frac{(P\times 2a)(3a)^2}{2EI}$$

$$= \frac{9Pa^3}{EI} - \frac{9Pa^3}{EI} = 0$$

$$\boxed{\delta_B = 0}$$

Lec-41

Q.6 Determine Deflection at C?

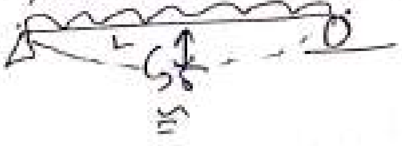


Solⁿ

$$\delta_c = \frac{1}{2}\left(\frac{5}{384}\cdot\frac{wl^4}{EI}\right)$$

$$\boxed{\delta_c = \frac{5}{768}\cdot\frac{wl^4}{EI}}$$ (Ans)

From superposition theorem



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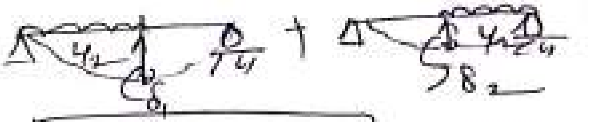
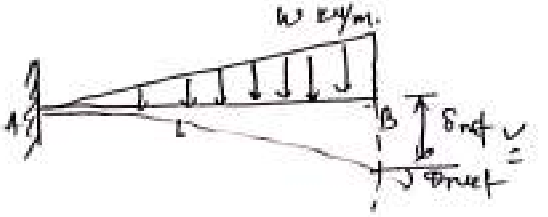


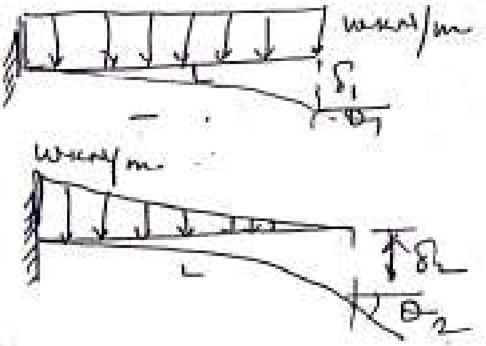
$$\boxed{\delta = \delta_1 + \delta_2}$$ ∵ $\boxed{\delta_1 = \delta_2}$

$\delta = 2\delta_1$

$$\boxed{\delta_1 = \delta/2}$$

7) determine the slope and deflection at the free end?





Sol

$$\delta_1 = \frac{wl^4}{8EI} \;;\; \delta_2 = \frac{wl^4}{30EI}$$

$$\theta_1 = \frac{wl^3}{6EI} \;;\; \theta_2 = \frac{wl^3}{24EI}$$

$$\theta_{net} = \frac{Wl^3}{6EI} - \frac{wl^3}{24EI} = \frac{3}{24} \cdot \frac{WL^3}{EI}$$

$$\boxed{\delta_{net} : \frac{WL^3}{8EI}}$$

(Ans)

$$\delta_{net} = \frac{Wl^4}{8EI} - \frac{WL^2}{30EI} \Rightarrow$$

$$\boxed{\delta_{net} = \frac{11}{120} \cdot \frac{WL^4}{EI}}$$

2. STRAIN ENERGY METHOD :-

* ASSUMPTION :-

i) No loss of energy

ii) Load is within the proportionality limit.

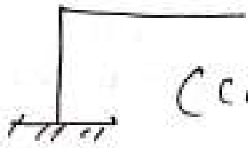
iii) Self weight of member is neglected.

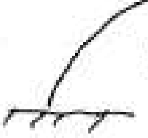
* ADVANTAGE :-

i) Most accurate method

ii) Can be used for both prismatic and nonprismatic beam.

3. Can be used for finding slope and deflection in cantilever arch and frames.


(Cantilever frame)


(Cantilever Arch)

* STRAIN ENERGY :-

* Strain energy due to axial loading

$$U = \frac{1}{2} \cdot P \cdot \delta$$

$$= \frac{1}{2} \times P \times \frac{Pl}{AE}$$

$$\boxed{U = \frac{P^2 L}{2AE}} \quad \text{or} \quad \boxed{U = \frac{\sigma^2}{2E} \cdot V}$$

* Strain Energy due to bending :-

Area dA

$$\boxed{\sigma = \frac{M}{I} \times y} \qquad \boxed{dv = dA \cdot dx}$$

Strain Energy for elementary volume $= \frac{\sigma^2}{2E} \cdot dv$

$$dU = \frac{M^2}{I^2 2E} \times y^2 \times dA \cdot dx$$

Integrate both side.

$$U = \iint \frac{M^2}{I^2 \cdot 2E} \times y^2 \times dA \cdot dx \qquad \boxed{U = \int_0^L \frac{M^2 dx}{2EI}}$$

$$= \int \frac{M^2 dx}{I^2 2E} \times I = \int_0^L \frac{M^2 \cdot dx}{2EI}$$

* REGIDITY, STIFFNESS AND FLEXIBLITY

Loading	Strain Energy	Regidity	Stiffness	flexiblity
Axial	$\frac{P^2L}{2AE}$	AE	$\frac{AE}{L}$	L/AE
flexural	$\int \frac{M^2dx}{2EI}$	EI	$\frac{EI}{L}$	L/EI
Shear	$\frac{V^2L}{2AG.}$	AG	$\frac{AG}{L}$	L/AG
Torsional	$\int \frac{T^2dx}{2GI_p}$	GI_p	$\frac{GI_p}{L}$	L/GI_p

We know for Axial

$\delta : \frac{PL}{AE}$

$AE \propto \frac{1}{\delta}$

$\frac{AE}{L} \propto \frac{1}{\delta}$

$\frac{L}{AE} \propto \delta$

$$\left\{ \begin{array}{l} AE \uparrow \Rightarrow \delta \downarrow \\ \frac{AE}{L} \uparrow \Rightarrow \delta \downarrow \\ \frac{L}{AE} \uparrow \Rightarrow \delta \uparrow \end{array} \right\}$$

$$\boxed{\delta : \frac{PL}{\text{Rigidity}}}$$

$$\boxed{\delta = \frac{P}{\text{Stiffness}}}$$

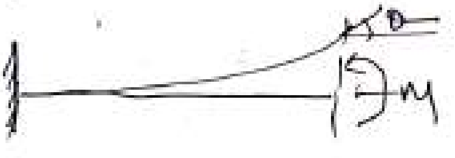
$$\boxed{\delta = P \times \text{flexibility}}$$

for spring

$$\boxed{\underset{\text{Load}}{F} = \underset{\text{Stiffness}}{K} \cdot \delta} \rightarrow \text{Deformation}$$

** CASTEGILANO'S THEOREM FOR FINDING SLOPE

for slop → "It state that ~~energy~~ slope at a point is equall to partial derivative of strain energy with respect to concentrated moment acting at that point."



$$\boxed{\theta : \frac{\partial U}{\partial M}}$$

or,

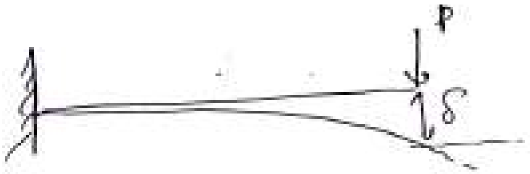
$$U = \int \frac{M^2 dx}{2EI}$$

$$\frac{\partial U}{\partial M} = \int \frac{(2M_x)\left(\frac{\partial M_x}{\partial M}\right)\cdot dx}{2EI}$$

$$\boxed{\theta = \frac{\int M_x \cdot \frac{\partial M_x}{\partial M} \cdot dx}{EI}}$$

for deflection :-

" It state that deflection at any point " equal to partial derivative of strain energy w.r.t " concentrated point load acting at that point."



$$\boxed{\delta = \frac{\partial U}{\partial P}}$$

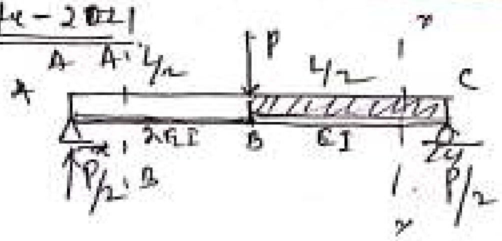
$$\delta = \frac{\partial U}{\partial P}$$

$$U = \int \frac{M_x^2 \cdot dx}{2EI}$$

$$\frac{\partial U}{\partial P} = \int \frac{\cancel{2}M_x \cdot \frac{\partial M_x}{\partial P} \cdot dx}{\cancel{2}EI}$$

$$\boxed{\delta = \frac{\int M_x \cdot \frac{\partial M_x}{\partial P} \cdot dx}{EI}}$$

Q. Gate-2021



$\boxed{L, E, I, P}$ Given

Sol^n

$$\boxed{\int \frac{M_x \cdot \frac{\partial M}{\partial y} \cdot dx}{EI}}$$

EI = Constant.

Mx eq^n is Constant.

M_x for AB $= \frac{Px}{2} \rightarrow \frac{\partial M_x}{\partial P} = \frac{x}{2}$

M_x for BC $= \frac{Px}{2} \rightarrow \frac{\partial M_x}{\partial P} = \frac{x}{2}$

$$\delta_B = \delta_{AB} + \delta_{BC}$$

For, $\delta_{AB} = \int_0^{L/2} \frac{M_x \cdot \frac{\partial M_x}{\partial P} \cdot dx}{2EI}$

$$= \int_0^{L/2} \frac{\frac{Px}{2} \cdot \frac{x}{2} \cdot dx}{2EI}$$

$$= \frac{P}{8EI} \int_0^{L/2} x^2 \cdot dx$$

$$= \frac{P}{8EI} \left[\frac{x^3}{3} \right]_0^{L/2}$$

$$= \frac{PL^3}{192EI}$$

For $\delta_{BC} = \int_0^{L/2} \frac{M_x \cdot \frac{\partial M_x}{\partial P} \cdot dx}{EI}$

$$= \int_0^{L/2} \frac{\frac{Px}{2} \cdot \frac{x}{2} \cdot dx}{EI}$$

$$= \frac{PL^3}{96EI}$$

$$\delta_B = \delta_{AB} + \delta_{BC}$$

$$= \frac{PL^3}{192EI} + \frac{PL^3}{96EI}$$

$$= \frac{3PL^3}{192EI}$$

$$\boxed{\delta_B = \frac{PL^3}{64EI}}$$

imp

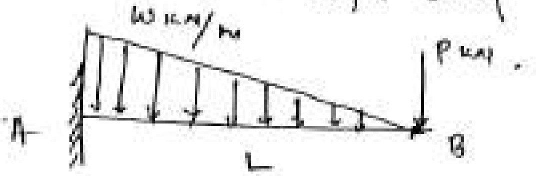
* PSEUDO LOAD *

Def

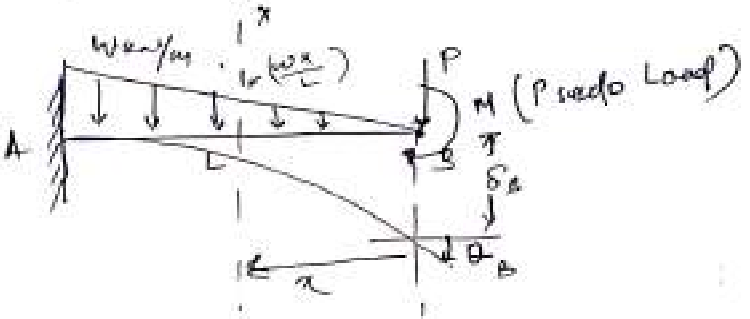
If at a point slope and ~~eff~~ deflection is to be found & concentrated moment & concentrated point Load are not acting at that point, then an imaginary load (Pseudo load) is applied at that point. before finding support reaction and bending eqn, after finding slope and deflection we ceit equals to zero.

NUMERICALS ON STRAIN ENERGY METHOD:-

Q.1) Determine the slope and deflection at the free end?



Sol^n



$$M_{xx} = -Px - M - \left[\left(\frac{1}{2}\times\frac{wx}{l}\cdot x\right)\cdot\frac{x}{3}\right]$$

$$= -\left[M + Px + \frac{wx^3}{6l}\right]$$

$$\boxed{\frac{\partial M_x}{\partial M} = -1}$$

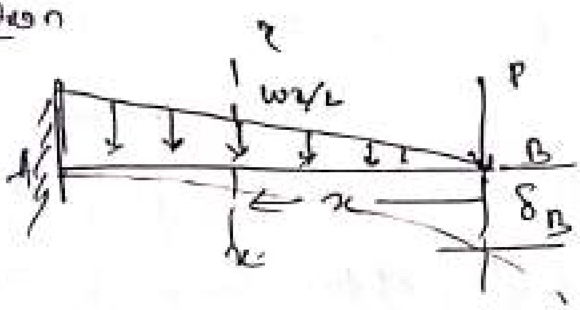
$$\theta_B = \int_0^l \frac{+\left(\overset{0}{M} + Px + \frac{wx^2}{6l}\right)\times(-1)\cdot dx}{EI}$$

$$= \frac{1}{EI}\int_0^L \left(Px + \frac{wx^3}{6l}\right)\cdot dx$$

$$= \frac{1}{EI}\left[P\left[\frac{x^2}{2}\right]_0^L + \frac{w}{6l}\left[\frac{x^4}{4}\right]\right]$$

$$\boxed{\theta_B = \frac{PL^2}{2EI} + \frac{wL^3}{24EI}}$$ Ans

<u>For Deflection</u>



$$M_{xx} = -P.x - \frac{1}{2}\left(\frac{wx}{L} \times x\right) \times \frac{x}{3}$$

$$M_{xx} = -\left[Px + \frac{wx^3}{6L}\right]$$

$$\boxed{\frac{\partial M_{xx}}{\partial P} = -x}$$

$$\delta_B = \int_0^L \frac{M_x . \frac{\partial M_x}{\partial P} . dx}{EI}$$

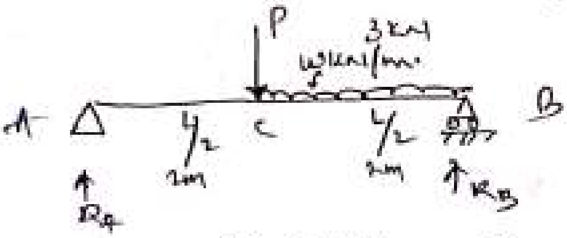
$$= \int_0^L \frac{-\left[Px + \frac{wx^3}{6L}\right] \times -x . . dx}{EI}$$

$$= \frac{1}{EI} \int_0^L \left(Px^2 + \frac{wx^4}{6L}\right) dx$$

$$= \frac{1}{EI} \left[P\frac{x^3}{3} + \frac{wx^5}{6L . 5}\right]_0^L$$

$$\boxed{\delta_B = \frac{PL^3}{3EI} + \frac{wl^4}{30EI}}$$ Ans

Q.2) Determine the deflection ~~of~~ at C ?



Sol^n

$$R_A = \frac{(3\times2)\times1}{\cancel{4}\,2} + \frac{P}{2} = 1.5 + \frac{P}{2}$$

$$R_B = \frac{3(2\times3)\times3}{\cancel{4}\,2} + \frac{P}{\cancel{4}\,2} = 4.5 + \frac{P}{2}$$

for AC

$$M_{x-x} = \left(1.5 + \frac{P}{2}\right)\cdot x$$

for BC

$$M_{x_2-x_2} = \left(\frac{P}{2} + 4.5\right)x - \frac{wx^2}{2}$$

$$\boxed{\delta_C = \delta_{C_{AB}} + \delta_{C_{BC}}}$$

for AC

$$\frac{\partial M_{x-x}}{\partial P} = \frac{\partial\left(1.5 + \frac{P}{2}\right)x}{dP} = \frac{x}{2}.$$

$$\delta_{C_{AB}} = \int_0^{\cancel{l}/2} \frac{\left(1.5 + \overset{0}{\cancel{\frac{P}{2}}}\right)x \cdot \frac{x}{2}\cdot dx}{EI}$$

$$= \frac{1.5}{2EI}\int_0^2 x^2\,dx$$

$$= \frac{1.5}{2EI}\left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{\cancel{1.5}}{\cancel{2}EI}\times\frac{\cancel{8}\,4\,2}{\cancel{3}\,\cancel{2}} \cdot \boxed{\frac{2}{EI} = \delta_{C_{AC}}}$$

For BC

$$\delta_{c_{BC}} = \frac{\partial M_x}{dp} = \frac{x}{2}$$

$$M_x = \left(\frac{P}{2} + 4.5\right)x - \frac{wx^2}{2}$$

$$\delta_{c_{BC}} = \int_0^2 \frac{M_x \cdot \frac{\partial M_x}{dP} \cdot dx}{EI}$$

$$= \int_0^2 \frac{\left[\left(\overset{0}{\cancel{\frac{P}{2}}} + 4.5\right)x - \frac{wx^2}{2}\right] \cdot \frac{x}{2} \cdot dx}{EI}$$

$$- \cancel{4.5\, \overset{\times 3}{w} x^3}$$

$$= \int_0^2 \frac{\left(\frac{4.5x^2}{2} - \frac{3}{4}x^3\right) \cdot dx}{EI}$$

$$= \frac{1}{EI}\left[\frac{4.5}{2} \times \frac{x^3}{3} - \frac{3}{4} \cdot \frac{x^4}{4}\right]_0^2$$

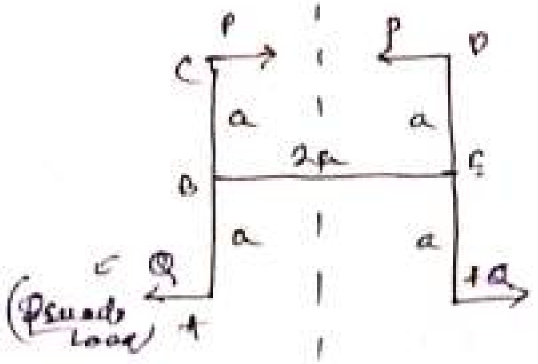
$$= \frac{1}{EI}\left[\frac{\overset{3}{\cancel{4.5}}}{\cancel{2}} \times \frac{\overset{2}{\cancel{8}}4}{\cancel{3}\cancel{2}} - \frac{3}{4} \times \frac{16^4}{4}\right]$$

$$\delta_{c_{BC}} = \frac{3}{EI}$$

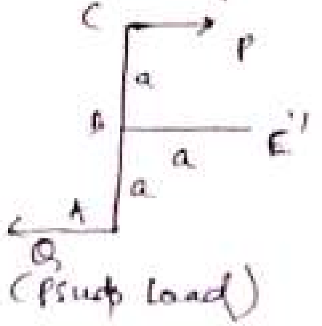
$$\delta_c = \left(\frac{2}{EI} + \frac{3}{EI}\right) = \frac{5}{EI}$$

$$\boxed{\delta_c = \frac{5}{EI}}$$

Q.3 Determine the relative displacement between A and F?

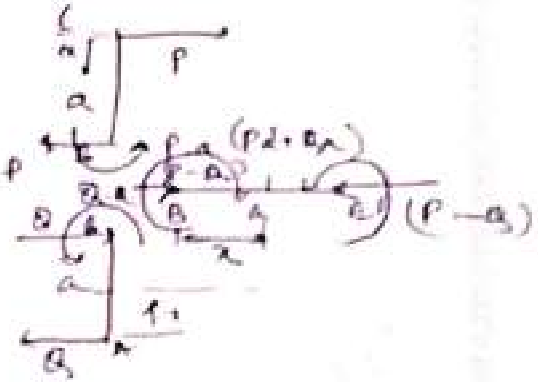


Sol^n



$\boxed{\delta_{HA}}$

FBD →



* Bending eq^n for

M_x for BC : $-Px$

M_x for AB $= +Qx$

M_x for BE $= (Pa + Qa)$

for BC

$M_x = -Px$

$\frac{\partial M_x}{\partial Q} = 0$

$\boxed{\delta_{AH_{BC}} = 0}$

for AB

$M_x = Qx$

$\frac{\partial M}{\partial Q} = x$

$$\delta_{AH_{AB}} = \int_0^a \frac{M_x \frac{\partial M_x}{\partial Q} \cdot dx}{EI}$$

$$= \int_0^a \frac{\overset{0}{Q}(x) \cdot x\, dx}{EI} = 0$$

$\boxed{\delta_{AH_{AB}} = 0}$

for BE'

$$M_x = (P+Q)a.$$

$$\frac{\partial M_x}{\partial Q} = a$$

$$\partial_{4H(BE')} = \int_0^a \frac{[(P+\overset{0}{\cancel{Q}})a] \cdot a \cdot dx}{EI}$$

$$= \frac{Pa^2}{EI} \times [x]_0^a$$

$$\boxed{\partial_{4H(BE')} = \frac{Pa^3}{EI}}$$

$$\boxed{\delta_{4H} = \frac{Pa^3}{EI}}$$

Total relative displacement bet^n 4 & r.

$$= 2 \times \frac{Pa^3}{EI}$$ Ans

L-43

3| MOMENT AREA METHOD :

* Advantage :-

Can be used for both prismatic and non prismatic beam.

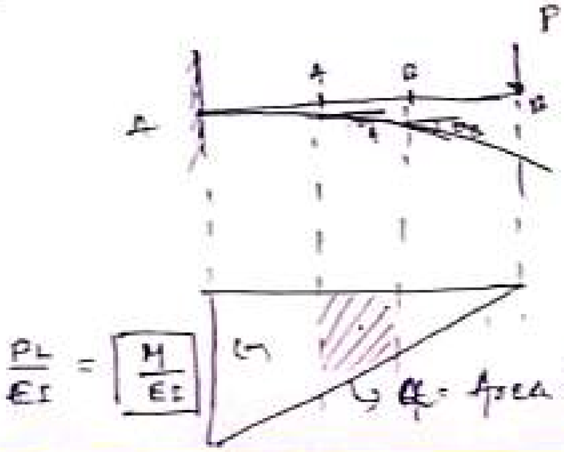
* Dis Advantage :-

It can be ~~o~~ used only when ~~only~~ area and centroid of M/EI diagram can be found.

** Moh'r theorem for Slop

It state that " different of slope between two points A & B is the area of M/EI diagram between A and B."

Explanation :-



then

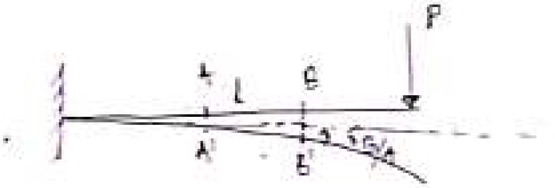
$$\boxed{\theta_B - \theta_A = a}$$

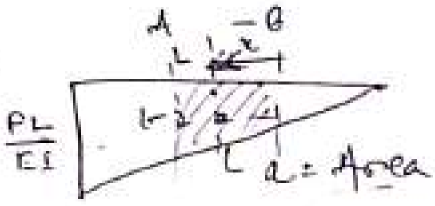
↳ a - Area between A&B of M/EI Diagram

* Mohr's theorem for Deflection :-

** It state that " deflection of B w.r.t tangent at A is given by moment of area of M/EI diagram between A and B taken about B".

Explanation





$\delta_{B/A}$ = Deflection of B w.r.t tangent @ A

$$\boxed{\delta_{B/A} = a \cdot \bar{x}}$$

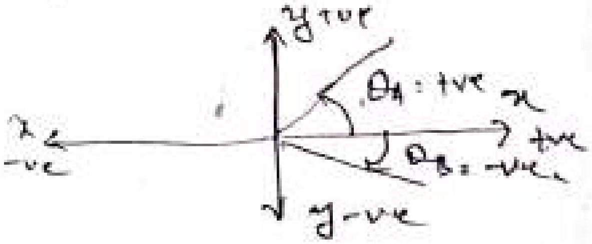
* Sign Convension

$\frac{M}{EI}$ diagram → +ve then, Area = +ve

$\frac{M}{EI}$ diagram → -ve then, Area = -ve

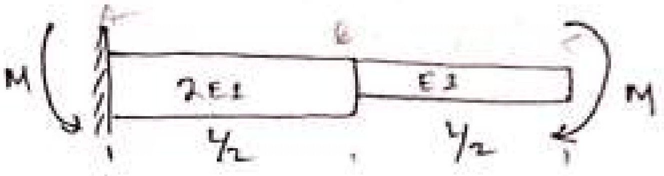
Clockwise = -ve

Anti Clockwise = +ve

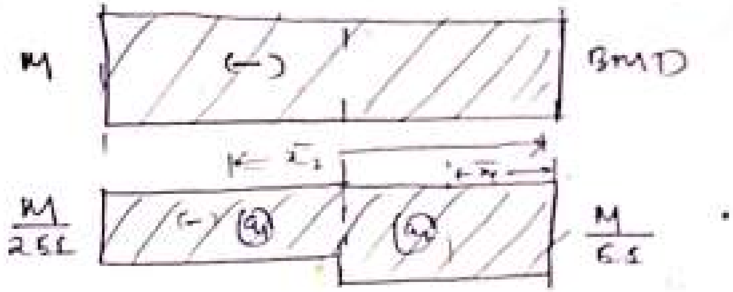


> NUMERICAL ON MOMENT DISTRIBUTION

Q.1 Determine the slope and deflection at the free end?



Sol^n



$\theta_C - \theta_A$ = Area of $\frac{M}{EI}$ diagram b/w A & C.

$$\theta_C - \theta_A = \left[-\left(\frac{M}{2EI} \times \frac{L}{2}\right) - \left(\frac{M}{EI} \times \frac{L}{2}\right)\right]$$

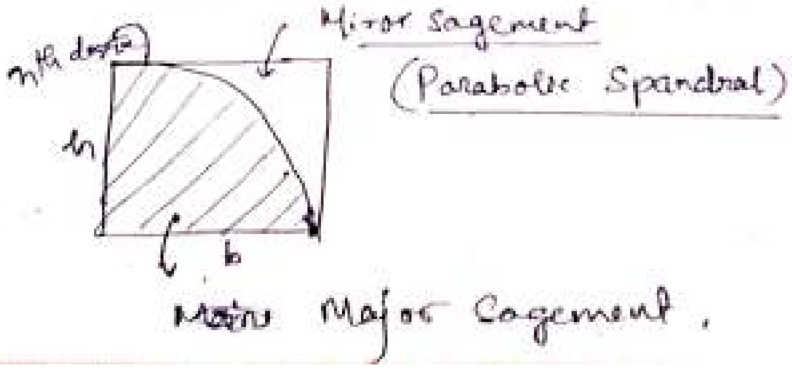
$$= -\frac{ML}{EI}\left(\frac{1}{4} + \frac{1}{2}\right)$$

$$\boxed{\theta_C = \frac{-3ML}{4EI}} \ (\circlearrowright)$$

$$\delta_{C/A} = \delta_C = \left[-\left\{\left(\frac{M}{2EI} \times \frac{L}{2}\right)\left(\frac{L}{4} + \frac{L}{2}\right)\right\} - \left\{\left(\frac{M}{EI} \times \frac{L}{2}\right) \times \frac{L}{4}\right\}\right]$$

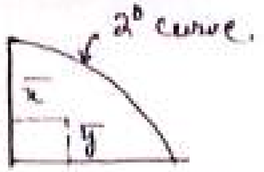
$$\boxed{\delta_C = \frac{-5ML^2}{16EI}}$$

* AREA AND CENTROID FOR nth DEGREE CURVE.



1) 2° Curve

a)

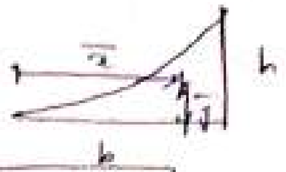


$\boxed{\text{Area} = \frac{2}{3} bh}$

$\boxed{\bar{x} = \frac{3}{8} b}$

$\boxed{\bar{y} = \frac{2}{5} \cdot h}$

b)



$\boxed{\text{Area} = \frac{1}{3} bh}$

$\boxed{\bar{x} = \frac{3}{4} b}$

$\boxed{\bar{y} = \frac{3}{10} h}$

2) n^{th} Degree

Major Segement

$\boxed{\text{Area} = \left(\frac{n}{n+1}\right) \cdot b \cdot h}$

$\boxed{\bar{x} = \frac{n+1}{2(n+2)} \cdot b}$

$\boxed{\bar{y} = \left(\frac{n}{2n+1}\right) \cdot h}$

Minor Segement

$\boxed{\text{Area} = \frac{1}{n+1} bh}$

$\boxed{\bar{x} = \left(\frac{n+1}{n+2}\right) b}$

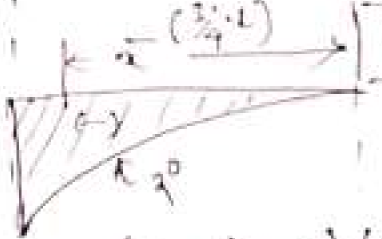
$\boxed{\bar{y} = \frac{n+1}{2(n+1)} \cdot h}$

NUMERICALS

Q.1 Determine prop reaction using Moment Area Method.

$\boxed{\delta_B = 0}$

Sol



$\boxed{\delta_{B_1} = \delta_{B_2}}$

$$\delta_{B_1} = \delta_{B/A} = \left(\frac{1}{3} \cdot \frac{wl^2}{2EI} \times l\right) \times \left(\frac{3}{4} \cdot l\right)$$

$$\boxed{\delta_{B_1} = -wl^4/8EI}$$

$$\delta_{B_2} = \delta_{B/A} = \left(\frac{1}{2} \times \frac{VL}{EI} \times L\right) \times \left(\frac{2}{3} \cdot L\right)$$

$$\boxed{\delta_{B_2} = \frac{VL^3}{3EI}}$$

$$\boxed{\delta_{B_1} = \delta_{B_2}} \quad \boxed{\delta_{B_1} + \delta_{B_2} = 0} \Rightarrow \frac{-Wl^4}{8EI} + \frac{V L^3}{3EI} =$$

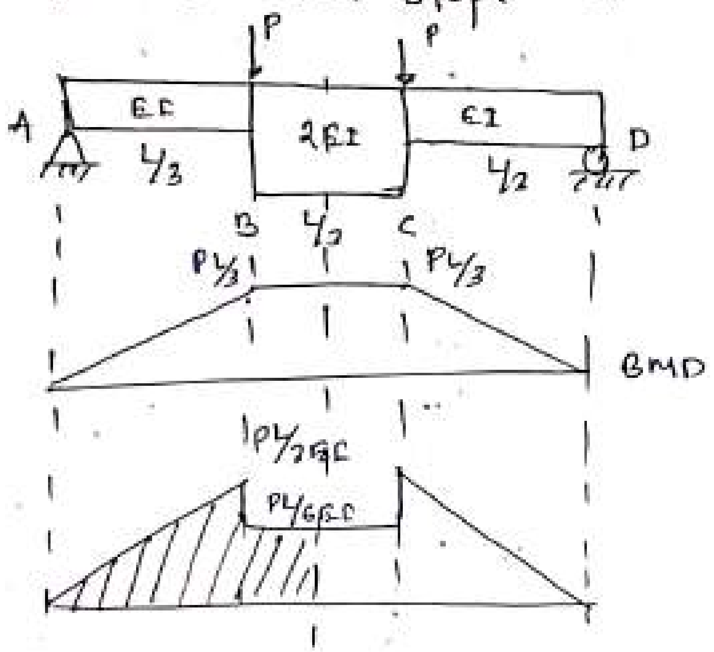
$$\frac{Wl^4}{8EI} = \frac{V l^3}{3EI}$$

$$\boxed{V = \frac{3W}{8}}$$

Ans

Q.3 Determine the slope at the ends ?

Soln



$$\theta_c - \theta_A = +\left(\frac{1}{3} \times \frac{Pl}{3EI} \times \frac{l}{3}\right) + \left(\frac{PL}{6EI} \times \frac{l}{6}\right)$$

$$-\theta_A = \frac{PL^2}{18\,EI} + \frac{PL^2}{36\,EI}$$

$$\boxed{\theta_A = -\frac{PL^2}{12\,EI}} \ (\curvearrowright)$$

$$\boxed{\theta_D = \theta_A = \frac{PL^2}{12\,EI}}$$

Lec-47

4: CONJUGATE BEAM METHOD :-

→ It can be applied both for prismatic & non-prismatic beam.

→ Can be used when area and centroid of $\frac{M}{EI}$ diagram can be found.

CONGOGATE BEAM :-

→ *** It is a imaginary beam whose loading diagram is $\frac{M}{AE}$ diagram of the ~~real~~ real beam.

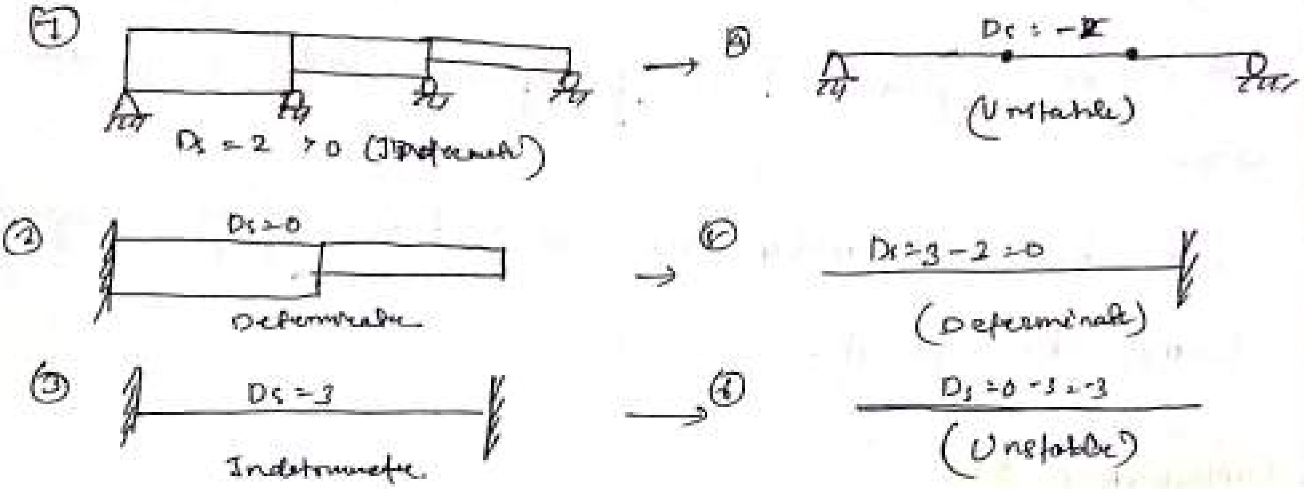
→ If $\frac{M}{EI}$ diagram (+ve) → Loading is Upward.

If $\frac{M}{EI}$ diagram (-ve) → Loading is downward.

***

Supports In Real beam	Corresponding Change in Immaginary Beam
fixed End	Free End
free End	fixed End
Simply supported End	Unchanged
Internal hinged	Roller Support
Intermediate Roller support	Internal hinged.

REAL BEAM ⟶ CONJUGATE BEAM



$$\boxed{D_s = r - s}$$

$D_s > 0$ (Stat Indeterminate)

$D_s = 0$ (Determinate)

$D_s < 0$ (Unstable)

Note

a) If Real beam is indeterminated; Cangugetae beam is ustable.

b) If Real beam is determinate; Canjugate beam det

* **Mohr's theorom for slope** :-

** It states that " Slope at any point in the real beam is equal to shear force at that point in the Cangugate Beam."

$$\boxed{\theta_{\text{Real beam}} = (S.F)_{\text{canjugate beam}}}$$

* **Mohr's theorom for Deflection** :-

** It state that, " Deflection at any point in the real beam is equal to Bending Momnt at that point in the cangugate Beam."

$$\boxed{(y)_{\text{Real beam}} = (B.M)_{\text{conjugate beam}}}$$

Trick to remember

$$\boxed{y_{R.B} = M_{C.B}}$$

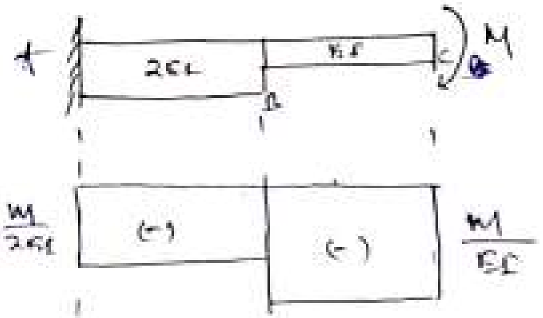
$$\boxed{\theta_{R.B} = \left(\frac{dy}{dx}\right)_{RB} = \left(\frac{dM}{dx}\right)_{C.B} = (V)_{C.B}}$$

$$\Rightarrow \boxed{\theta_{R.B} = V_{C.B}}$$

→ Numericals:

Q.1 Determine the slope and deflection at free end?

Sol^n



Conjugate beam

$$SF\ @\ C = -\frac{M}{2EI}\times\frac{L}{2} - \frac{M}{EI}\times\frac{L}{2}$$

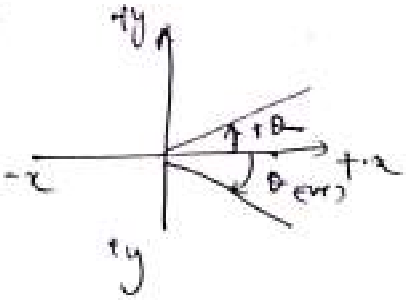
$$\boxed{SF\ @\ C = \frac{-3}{4}\frac{ML}{EI}} = \theta_{C_{RB}}$$

$$BM\ @\ C = -\left[\left(\frac{M}{2EI}\times\frac{L}{2}\right)\left(\frac{L}{2}+\frac{L}{4}\right)\right] - \left[\left(\frac{M}{EI}\times\frac{L}{2}\right)\times\frac{L}{4}\right]$$

$$\boxed{BM\ @\ C = \frac{-5}{16}\frac{ML^2}{EI} = \delta_{C_{R.B}}}$$

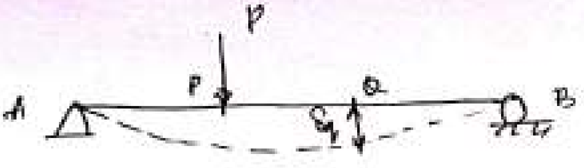
$$\boxed{\theta_C = \frac{-3}{4}\frac{ML}{EI}}\ (↻)$$

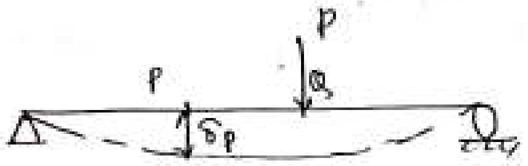
$$\boxed{\delta_C = \frac{-5}{16}\frac{ML^2}{EI}}\ (\downarrow)$$



Lec-45

5: MAXWELL RECIPROCAL THEOREM:-



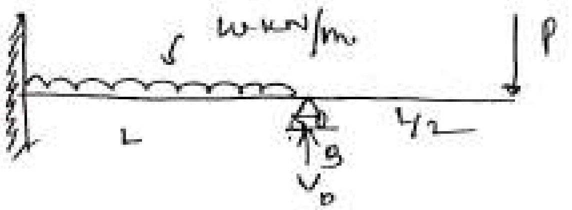


$$\boxed{P_P \times \delta_P = P_Q \times \delta_Q}$$

If $\boxed{P_P = P_Q}$

$\boxed{\delta_P = \delta_Q}$

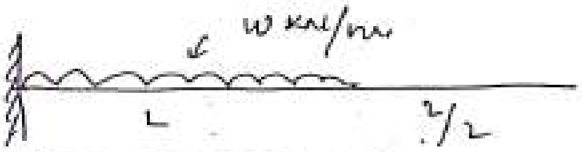
Numerical for Understanding:-



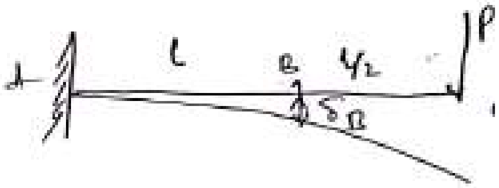
* Determine the reaction @ B?

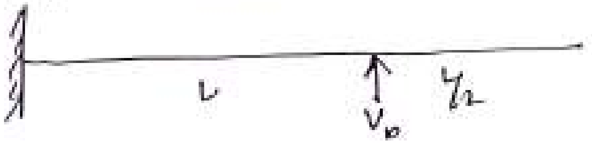
Sol^n

↶



$$\boxed{\delta_{B_1} = \frac{wL^4}{8EI}}\ (\downarrow)$$

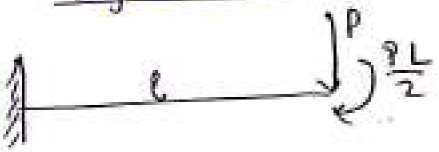




$$\boxed{\delta_{B_2} = \frac{V_b \cdot L^3}{3EI}}\ \uparrow$$

Method-1

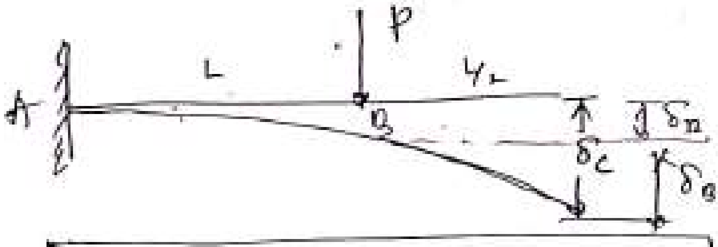
Shift Load



$$\delta_{B_3} = \frac{PL^3}{3EI} + \frac{\left(\frac{PL}{2}\right)\cdot}{2E}$$

$$\boxed{\delta_{B_3} = \frac{7}{12}\frac{PL^3}{EI}}$$

* Method-2 (Maxwell Reciprocal theorem)



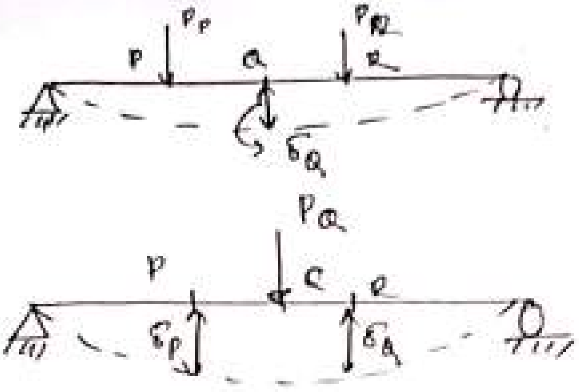
$$\boxed{\delta_{B_3} = \delta_C = \frac{7}{12} \cdot \frac{PL^3}{EI}}$$

$$\delta_{B_1} + \delta_{B_3} = \delta_{B_2}$$

$$\frac{WL^4}{8EI} + \frac{7}{12}\frac{PL^3}{12EI} = \frac{VL^3}{3EI}$$

$$\boxed{V = \frac{3}{8}WL + \frac{7}{4}\cdot P}$$ Ans

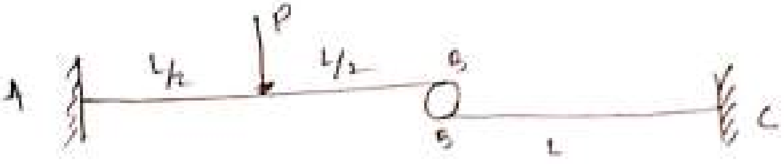
* BETTI'S THEOROM



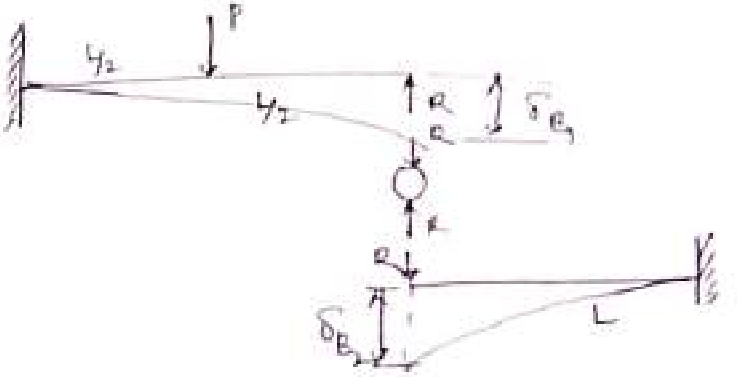
$$\boxed{P_P \times \delta_P + P_R \times \delta_R = P_Q \times \delta_Q}$$

All Over Topic Numerical

Q.1 Determine the deflection at point "B" ?



sol^n



$$\delta_{B_2} = \frac{RL^3}{3EI}$$

$$\boxed{\delta_{B_1} = \delta_B}$$

$$\frac{PL^3}{24EI} + \frac{PL^3}{16EI} - \frac{RL^3}{3EI} = \frac{RL^3}{3EI}$$

$$P\left(\frac{5}{48}\right) = \frac{2R}{3}$$

$$\boxed{R = \frac{5}{32}P}$$

$$\delta_{B_1} = \left[\frac{P \times \left(\frac{L}{2}\right)^3}{3EI} + \frac{P\left(L/2\right)^2}{2EI} \times \frac{L}{2}\right] - \frac{RL^3}{3EI}$$

$$\delta_{B_1} = \frac{PL^3}{24EI} + \frac{PL^3}{16EI} - \frac{R \cdot L^3}{3EI}$$

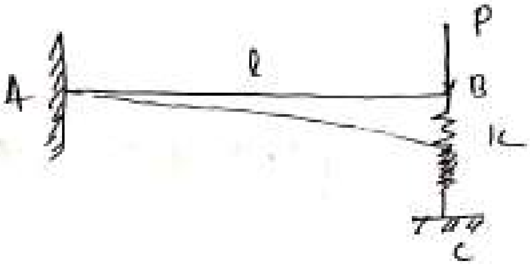
Deflection at B

$$\Rightarrow \delta_B = \frac{RL^3}{3EI}$$

$$= \frac{5}{32} P \times \frac{L^3}{3EI}$$

$$\boxed{\delta_B = \frac{5}{96} \cdot \frac{PL^3}{EI}}$$

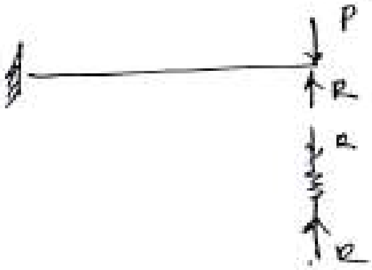
Note **Q. 2 Gate – 2015**



$\boxed{k = \frac{EI}{2l^3}}$

Determine the ~~de~~ compression in the spring.

soln



$$\delta_B = \delta_{spring}$$

$$\delta_B = \frac{(P-R)L^3}{3EI} = \frac{R}{k}$$

$$\Rightarrow \frac{(P-R)\cancel{L^3}}{3EI} = \frac{R\,2L^{\cancel{3}}}{\cancel{EI}}$$

$$\Rightarrow P - R = 6R$$

$$P = 7R$$

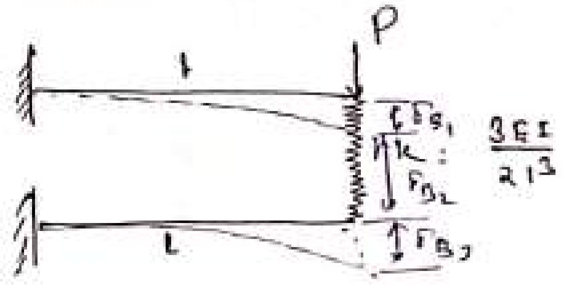
$$\boxed{R = P/7}$$

$$\delta_{spring} = \frac{R}{k} = \frac{P/7}{\frac{EI}{2l^3}} =$$

(2B)

$$\boxed{\delta_{spring} = \frac{2PL^3}{7EI}}$$ Ans

Q.3 Gate 2013

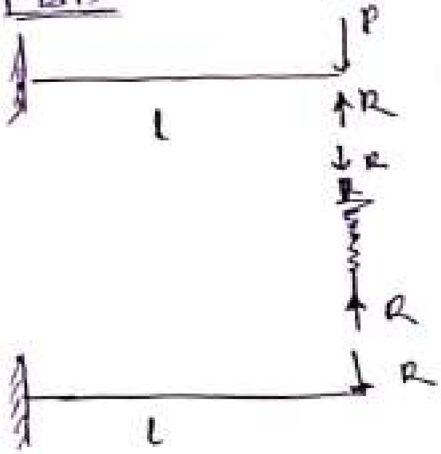


What fraction of load 'P' is transmitted to the spring?

Sol^n

$$\boxed{\delta_{B_1} = \delta_{B_2} + \delta_{B_3}}$$

FBD



$$\delta_{B_1} = \frac{(P-R)l^3}{3EI}$$

$$\delta_{B_2} = \frac{R}{k} = \cancel{3EI}\ \frac{2l^3 \cdot R}{3EI}$$

$$\delta_{B_3} = \frac{R \cdot l^3}{3EI}$$

$$\frac{(P-R)\cancel{l^3}}{\cancel{3EI}} = \frac{2R\cancel{l^3}}{\cancel{3EI}} + \frac{Rl^3}{3EI}$$

$$\boxed{R = \frac{P}{4}}$$

% trasforce = $\frac{R}{P} \times 100 = \frac{P/4}{P} \times 100^{25}$

$$\boxed{\%\ \text{transfer} = 25\%}$$

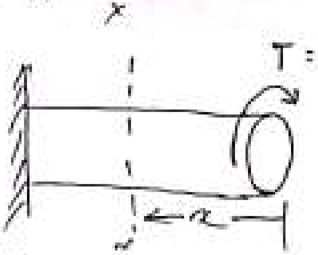
CH-7

Lecture-46

{TORSION}

Introduction

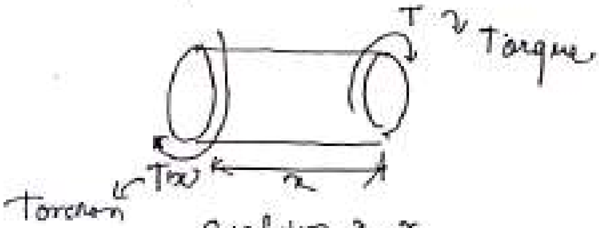
* Tourque & torsion



T : External twisting Moment

T = torque.

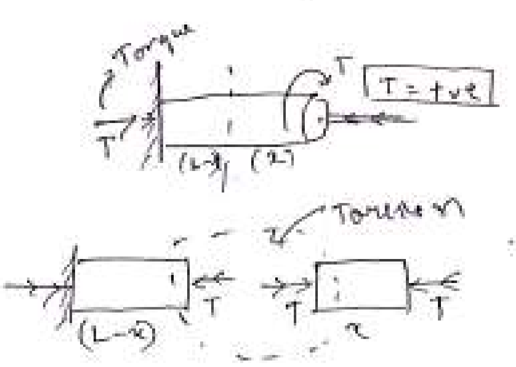
Torsion: It is the internal resistance in the form of twisting to resist the torque.

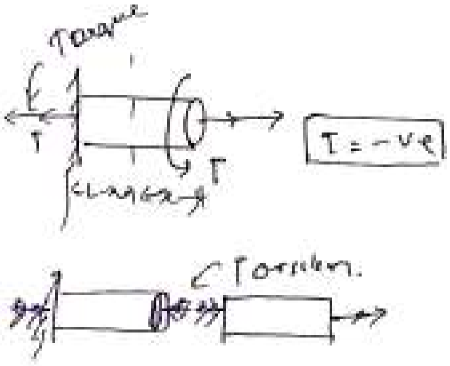


section x-x

* Sign Convention for Torsion

→ Using Right hand rule, curl the fingers in the direction of twist such that thum point ~~towards~~ in the direction of moment if it is towards the C/S then tourque is "positive" else "-ve".



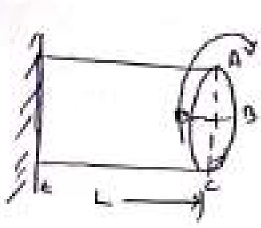


* ASSUMPTION IN TORSION :-

1) Material is homogeneous, Isotropic and linearly elastic.

2) Plane section before twisting remains plane even after twisting.

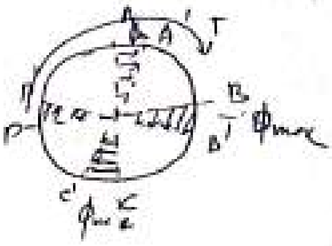
Explanation - P(2)

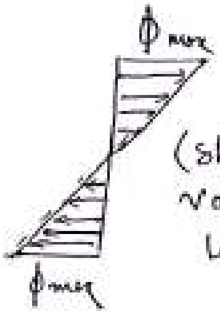


T: Shear strain induced at the c/s

↓

Variation linear





(Shear strain variation linear.) ⇒

(Shear stress distribution is also linear)

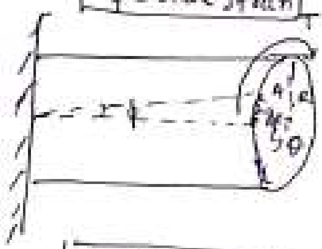
$$\boxed{\tau_{max} = G.\phi_{max}}$$

3: C/s is Circular; either hallow or solid circular.

V. imp

→ ** TORSIONAL EQUATION :-

$\boxed{\phi = \text{shear strain}}$



$$\boxed{AA' = L\phi}$$

$$\boxed{AA' = R\theta}$$

$$\boxed{L\phi = R\theta} \quad *** \quad —①$$

Now

$$\tau = G.\phi$$

$$\boxed{\phi = \frac{\tau_{max}}{G}} \quad —②$$

Put eqn ② in ①

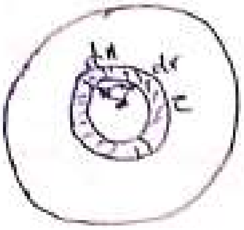
$$L \cdot \frac{\tau_{max}}{G} = R\theta$$

$$\boxed{\frac{\tau_{max}}{R} = \frac{G\theta}{L}} \quad ***$$

$$\boxed{\frac{\tau_{max}}{R} = \frac{\tau}{r}}$$ (due to linear variation)

$$\boxed{\frac{\tau}{R} = \frac{G\theta}{L}} \quad ***$$

$\boxed{\text{part - I}}$



Shear force acting on the strip

$$\boxed{dF = \tau \cdot dA}$$

Moment at the center due to dF.

$$\boxed{dT = (\tau \cdot dA) \cdot r}$$

for strip

$$dT = \frac{\tau_{max}}{R} \cdot r^2 \cdot dA \qquad \because \left\{ \tau = \frac{\tau_{max}}{R} \cdot r \right\}$$

for the total area C/s

$$\int dT = \frac{\tau_{max}}{R} \cdot \int_0^r r^2 \cdot dA \qquad \because \left\{ \int_0^r r^2 \cdot dA = I_p \right\}$$

$$T = \frac{\tau_{max}}{R} \times I_p$$

$$\Rightarrow \boxed{\frac{T}{I_p} = \frac{\tau_{max}}{R}}$$

[part - 2]

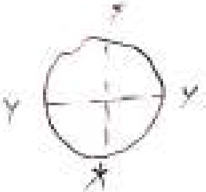
From Eq^n-1 & eq^n 2

**

$$\boxed{\frac{T}{I_p} = \frac{\tau}{r} = \frac{G\theta}{l}}$$ [Torsional eq^n]

T = twisting moment @ the section x-x.

I_p : Polar Moment of Inertia.



$$\boxed{I_p = I_{xx} + I_{yy}}$$

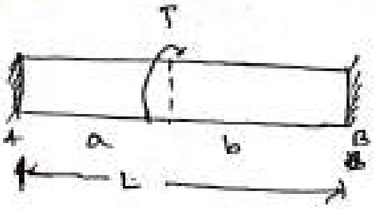
τ = Shear stress develops due to twisting at a radial distance "r".

G = Shear Modulus of Rigidity.

θ = Angle of twist, over the length 'l'.

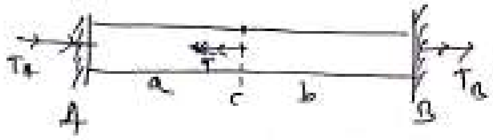
Numericals

Q.1



for the fixed shaft shown figure, determine the force reaction.

Soln



Equilibrium Eqn

$$\boxed{T_A + T_B = T}$$

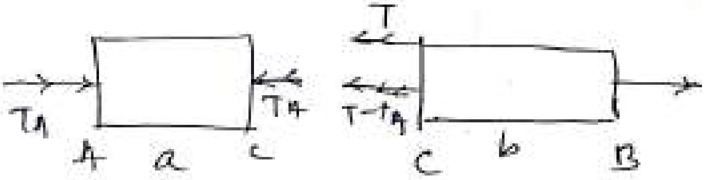
* Compatability condition required:-

$$\boxed{\theta_{AB} = 0}$$

$$\therefore \boxed{\theta = \frac{TL}{G I_p}}$$

$$\boxed{\theta_B - \theta_A = 0}$$

$$\boxed{\theta_{AC} + \theta_{CB} = 0}$$



$$\frac{T_A \cdot a}{G I_p} + \frac{[-(T - T_A) b]}{G I_p} = 0$$

$$T_A (a+b) - T \cdot b = 0$$

$$\boxed{T_A = \frac{T.b}{L}}$$

$$T_B = T - T_A = T - \frac{T.b}{l}$$

$$\boxed{T_B = \frac{T.a}{l}}$$

ce - 47
*
4/30

POLAR SECTION MODULUS !-

It gives the strength of the beam in twist

$$\frac{T}{I_p} = \frac{\tau}{r} = \frac{G\theta}{l} = \frac{\tau_{max}}{R}$$

$$\frac{T}{I_p} = \frac{\tau_{max}}{R}$$

$$T = \frac{\tau_{max} \times I_p}{R}$$

$$\boxed{T = \tau_{max} \cdot Z_p}$$

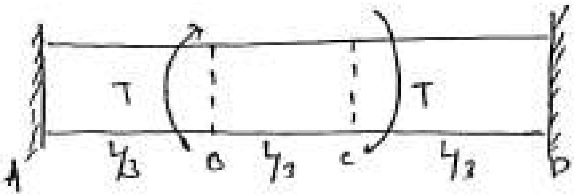
· Torsional Moment Carrying Capacity.

$$\boxed{\text{Torsional Moment Carrying Capacity} = \text{Max}^n \text{ shear stress} \times \text{Polar section Modulus}}$$

$$\boxed{Z_p = \frac{I_p}{R}}$$

Numericals:-

Q.7



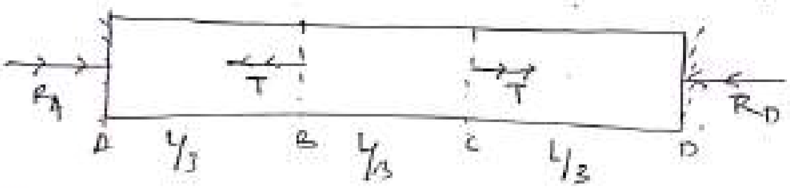
for the structural element shown in figure, Determine:-

a) Torsional reaction

b) Torsional Moment diagram.

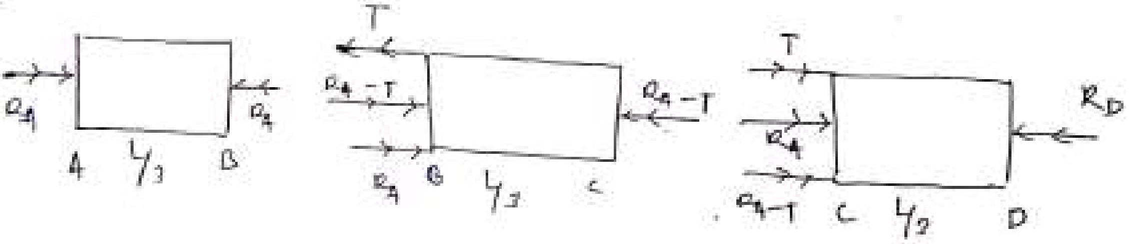
c) Angle of twist vs x diagram.

Sol^n



$$\boxed{R_A = R_D}$$

F.B.D



$\theta_{AD} = 0$

$$\boxed{G \cdot I_p = \text{Constant}}$$

$\theta_{AB} + \theta_{BC} + \theta_{CD} = 0$

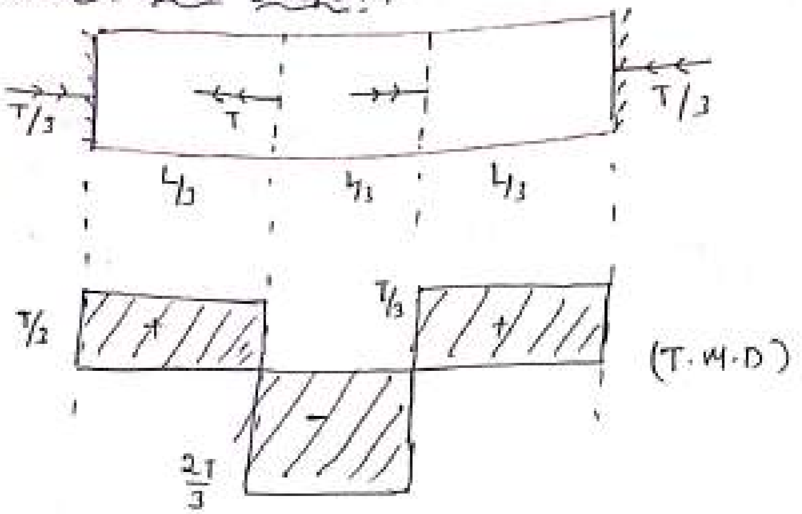
$$\frac{R_A \cdot L/3}{G \cdot I_p} + \frac{(R_A - T)\times \frac{L}{3}}{G I_p} + \frac{R_A \times L/3}{G \cdot I_p} = 0$$

$R_A + R_A - T + R_A = 0$

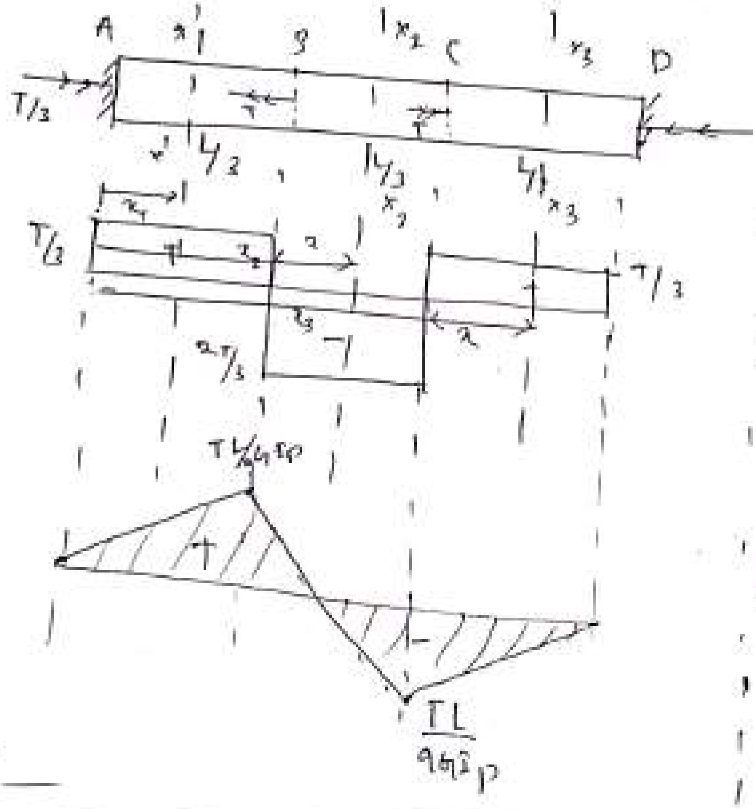
$3R_A = T$

$$\boxed{R_A = \frac{T}{3}} \qquad \boxed{R_D = \frac{T}{3}}$$

- Torsional Moment diagram :-



- θ vs x. diagram



$\theta = \dfrac{TL}{9GI_p}$

$\boxed{\dfrac{T}{9GI_p} = \text{Constant}}$

For portion AB

$\theta_{Ax_1} = \theta_{x_1} - \overset{0}{\cancel{\theta_A}} = \dfrac{\frac{T}{3} \times x}{GI_p}$

For . C

$\theta_{AB} = \theta_B = \dfrac{\frac{T}{3} \cdot \frac{L}{3}}{GI_p}$

$\boxed{\theta_B = \dfrac{TL}{9GI_p}}$

(Linear variation)

For BC Portion

$\theta_{Bx_2} = \dfrac{\left(\frac{-2T}{3}\right) \times x}{GI_p}$

$\theta_{x_2} - \theta_B = \dfrac{-2Tx}{3GI_p}$

$\theta x_2 = \theta_B - \dfrac{2Tx}{3GI_p}$

For C

$\theta_{x_2} = \dfrac{TL}{9GI_p} - \dfrac{2 \cdot T \cdot \frac{L}{3}}{3GI_p}$

$= \dfrac{TL}{9GI_p} - \dfrac{2TL}{9GI_p}$

$\boxed{\theta_C = \dfrac{-TL}{9GI_p}}$ (Linear variation)

For CD portion

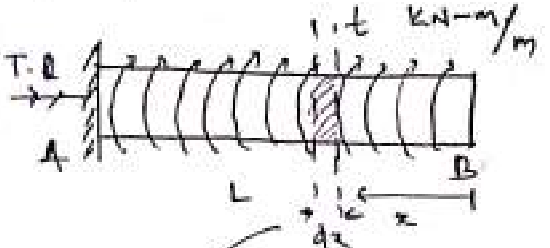
$\theta_{Cx_3} = \theta_{x_3} - \theta_C$

$= \dfrac{\frac{T}{3}x}{GI_p}$

$\theta_{x_3} = \dfrac{-TL}{9GI_p} + \dfrac{Tx}{3GI_p}$

$\theta_D = \dfrac{-TL}{9GI_p} + \dfrac{TL}{9GI_p}$

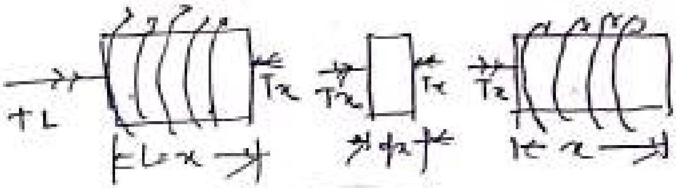
Q.2 A cantilever shaft is subjected to uniformly distributed torque throughout ~~the~~ its length. Determine the angle of twist at the free end.



Soln

Over this strap of length dx, torsion is assumed to be constant.

$$d\theta = \frac{T.dx}{G.I_p}$$



F.B.D

Torsion on the strip.

$$T_{strip} = t.x$$

For strip

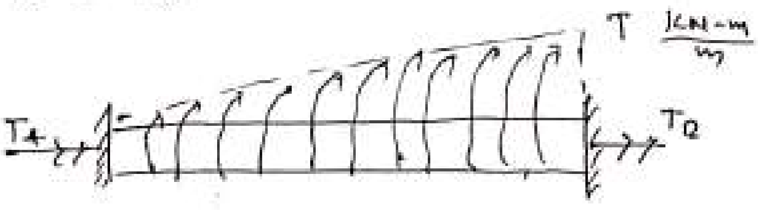
$$\boxed{d\theta = \frac{(t.x)(dx)}{G I_p}}$$

$$\int_{\theta_B}^{\theta_A} d\theta = \frac{t}{G I_p}\int_0^L x.dx$$

$$\cancel{\theta_A}^{0} - \theta_B = \frac{tL^2}{2GI_p}$$

$$\boxed{\theta_D = \frac{-TL^2}{2GI_p}}$$

Q.3 A fixed shaft is subjected to uvl torque. Determine the torsional reaction at the support.



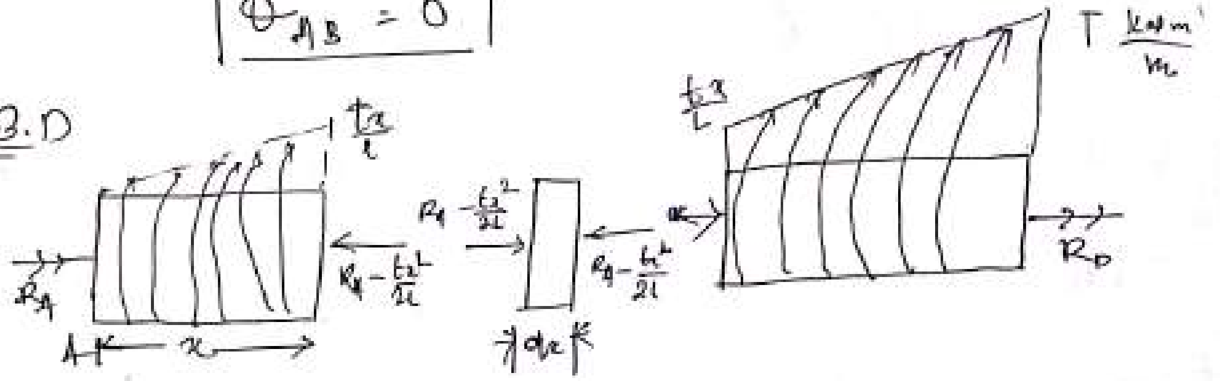
Solⁿ Equilibrium Eqⁿ

$$\boxed{T_A + T_B = \frac{tL}{2}}$$

Compatability Eqⁿ:

$$\boxed{\theta_{AB} = 0}$$

F.B.D



$\longrightarrow R_A$

$\longleftarrow \frac{1}{2} \times \frac{tx}{l} \times x = \frac{tx^2}{2L}$ $\quad \longrightarrow R_A - \frac{tx^2}{2L}$

for strip 'dx'

$$d\theta = \frac{\left(R_A - \frac{tx^2}{2L}\right) \cdot dx}{G I_p}$$

$$\int_{\theta_A}^{\theta_B} d\theta = \frac{\left(R_A - \frac{tx^2}{2l}\right) \cdot dx}{G I_p}$$

$$\overset{0}{\theta_B} - \overset{0}{\theta_A} = \frac{1}{G \cdot I_p}\left(R_A \cdot l - \frac{t}{2l} \times \frac{l^3}{3}\right)$$

$$R_A \cdot l - \frac{tl^2}{6} = 0 \quad ; \quad R_B = \frac{tl}{2} - \frac{tL}{6}$$

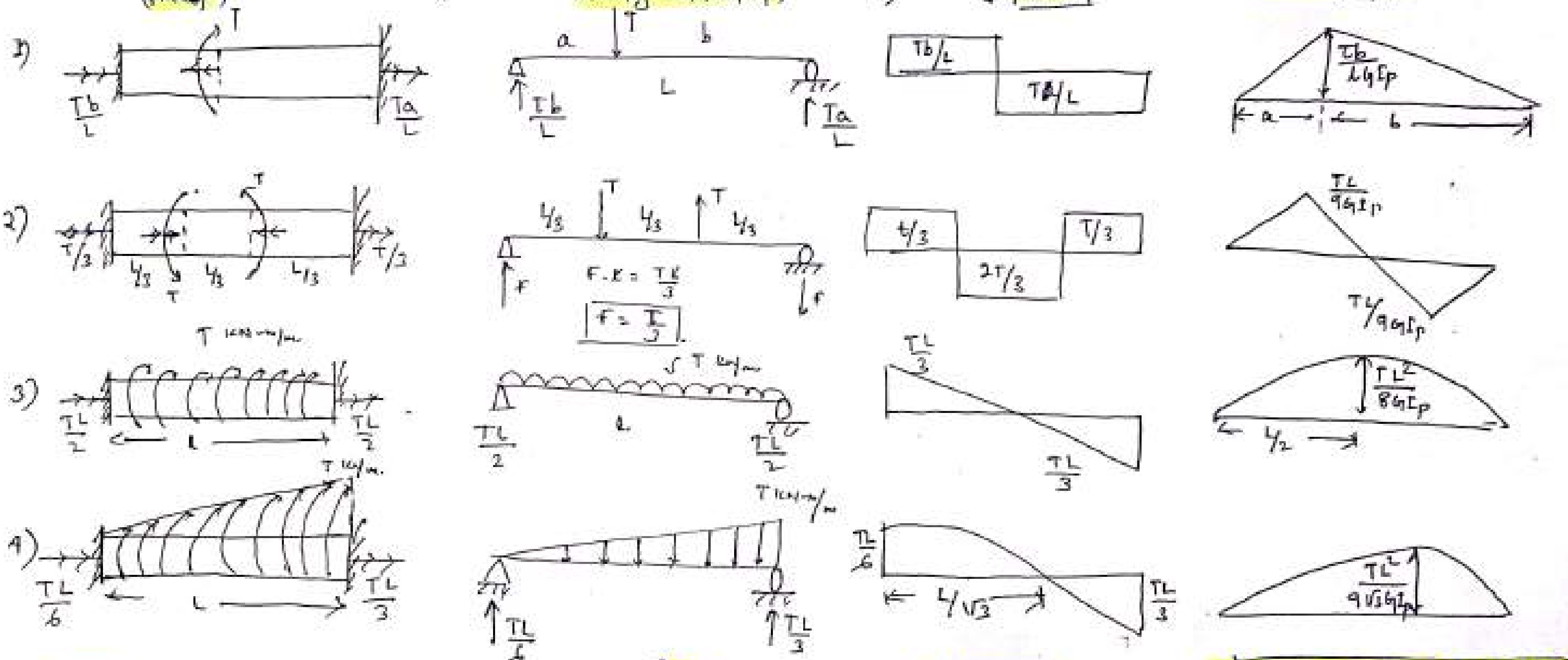
$$L R_A = \frac{tl^2}{6}$$

$$\boxed{R_A = \frac{tL}{6}} \qquad \boxed{R_B = \frac{tL}{3}}$$

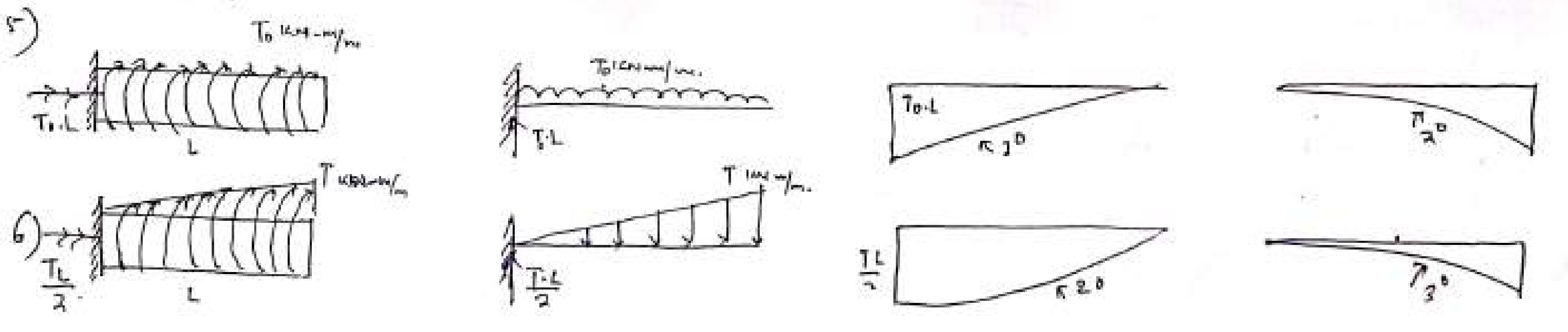
Ans

Shortcut for finding Support reaction, TMD and twisting vs x-dig.

Loading Condition (fixed) → Support reactions (Simply Supported) → T.M.D ≈ SFD → Angle of twist vs x-diagram. ≈ BMD/GIp



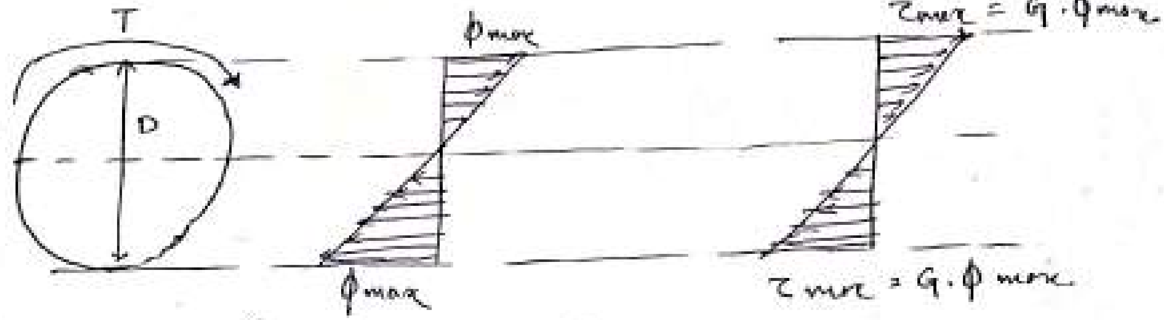
(Cantilever shaft) → (Cantilever Beam shaft) → ≈ SFD → ≈ Mirror image of BMD/GIp



✱ SHEAR STRESS DISTRIBUTION FOR VARIOUS C/S DUE TO TOR

1) SOLID CIRCULAR C/S

$\boxed{\tau = G\phi}$

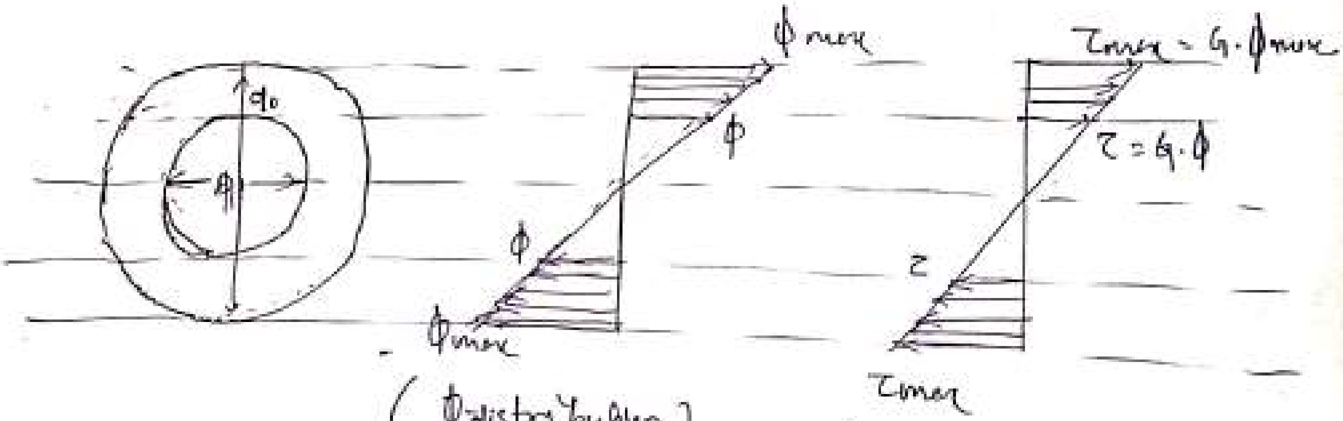


(φ - distribution)

$$I_p = I_{xx} + I_{yy}$$
$$= 2 \times \frac{\pi d^4}{64}$$
$$\boxed{I_p = \frac{\pi d^4}{32}}$$

$$Z_p = \frac{I_p}{R_3}$$
$$= \frac{\pi d^4/32}{d/2}$$
$$\boxed{Z_p = \frac{\pi d^3}{16}}$$

2) HOLLOW CIRCULAR C/S



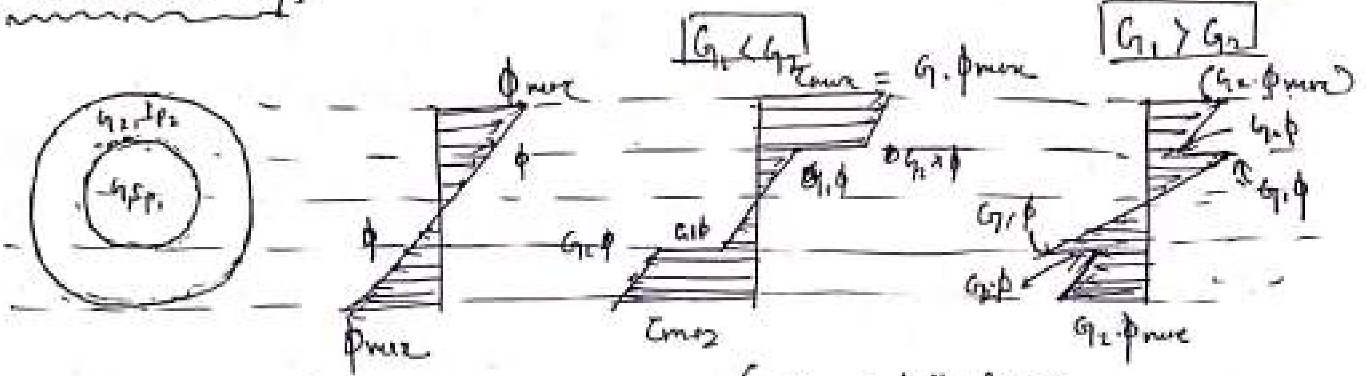
(φ distribution)

(τ-distribution)

$$I_p = I_{xx} + I_{yy}$$
$$= 2\,\frac{\pi}{64}\left(d_0^4 - d_i^4\right)$$
$$\boxed{I_p = \frac{\pi}{32}\left(d_0^4 - d_i^4\right)}$$

$$Z_p = \frac{I_p}{R_1} = \frac{I_p}{d_0/2}$$
$$\boxed{Z_p = \frac{\pi}{16 d_0}\left(d_0^4 - d_i^4\right)}$$

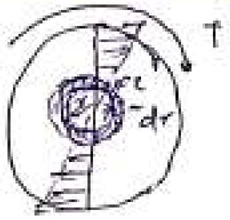
3. COMPOSITE C/S



(ϕ - distribution) (τ - distribution)

Lec-49

** STRAIN ENERGY STORED IN CIRCULAR SHAFT

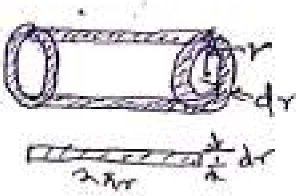


$$\boxed{\text{Strain Energy Due to shear} = \frac{\tau^2}{2G} \times \text{volume.}}$$

for, τ = Constant, G = Constant.

length of shaft is 'L'

Strain Energy stored in the strip $= \frac{\tau^2}{2G} \times dv.$



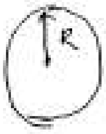
$$du = \frac{\tau^2}{2G} (2\pi r \cdot dr) \times L$$

$$\boxed{dA = 2\pi r \cdot dr}$$

$$dv = (2\pi r \cdot dr) \times L$$

$$\boxed{dU = \frac{\tau^2}{G} \times \pi . L \times r . dr}$$

(A) Strain Energy Stored in Solid Circular Shaft



for small strip $\boxed{du = \frac{\tau^2}{G} \times \pi L \times r . dr}$ — ①

from torsional eqn

$$\frac{T}{I_p} = \frac{\tau}{r} \Rightarrow \boxed{\tau = \frac{T}{I_p} \cdot r} \quad \text{— ②}$$

* Put ② in ①

$$\boxed{du = \frac{T^2}{I_p^2 G} \times \pi L \times r^3 dr}$$ for strip.

for Entire ~~Volume~~ C/s we Integrate.

$$\Rightarrow \int_0^U du = \frac{T^2}{I_p^2 G} \times \pi L \times \int_0^R r^3 . dr.$$

$$\Rightarrow U = \frac{T^2}{I_p^2 G} \times \pi L \times \frac{R^4}{4}$$

$$\because \left\{ I_p = \frac{\pi d^4}{32} = \frac{\pi \times}{2} \quad \boxed{I_p = \frac{\pi R^4}{2}} \right.$$

$$\therefore \frac{\pi R^4}{4} = \frac{I_p}{2}$$

$$U = \frac{T^2}{I_p^2 . G} . L \times \frac{I_p}{2}$$

*** Imp

$$\boxed{U = \frac{T^2 . L}{2 G I_p}}$$

(Strain Energy)

* Strain Energy In terms of maximum shear (τ_{max}).

$$\boxed{du = \frac{\tau^2}{G} \times \pi L \times r . dr} \text{ — ①}$$

~~Now~~ We Know

$$\frac{\tau}{r} = \frac{\tau_{max}}{R}$$

$$\boxed{\tau = \frac{\tau_{max}}{R} . r.} \text{ — ②}$$

Put ② in ①

$$dU = \frac{\tau_{max}^2}{G . R^2} . \pi L . r^3 . dr$$

for Entire Volume

$$\int_0^U dU = \frac{\tau_{max}^2}{G R^2} . \pi L . \int_0^R r^3 . dr$$

$$U = \frac{\tau_{max}^2}{G . R^2} \times \pi L . \frac{R^4}{4}$$

$$\left\{ \quad \because \boxed{\pi R^2 L = V} \right.$$

$$U = \frac{\tau_{max}^2}{4 G} . V$$

***** Imp

$$\boxed{\frac{U}{V} = \frac{\tau_{max}^2}{4 G}}$$

***

B. Strain Energy for Hollow Circle C/s

$$\boxed{dU = \frac{\tau^2}{G} \times \pi . L \times r . dr}$$ ...(1)

$$\frac{\tau}{r} = \frac{\tau_{max}}{R_0} \Rightarrow \boxed{\tau = \frac{\tau_{max}}{R_0} . \cancel{R}\, r}$$ ...(2)

Substitut ① - ②.

$$dU = \frac{\tau_{max}^2}{G . R_0^2} \times \pi . L \times r^3 . dr$$

for entire C/s we itigrate.

$$\int_0^U dU = \frac{\tau_{max}^2}{G R_0^2} \times \pi . L \times \int_{R_i}^{R_0} r^3 . dr$$

$$U = \frac{\tau_{max}^2}{G R_0^2} . \pi . L \times \left(\frac{R_0^4 - R_i^4}{4} \right)$$ (Volume of hollow circular)

$$U = \frac{\tau_{max}^2}{G R_0^2} \times [\pi . L (R_0^2 - R_i^2)] . \left[\frac{R_0^2 + R_i^2}{4} \right] \left\{ \because V = \pi L (R_0^2 - R_i^2 \right.$$

** *

Imp

$$\boxed{U = \frac{\tau_{max}^2}{4 G} . \left(1 + \frac{R_i^2}{R_0^2} \right) . V}$$

Note:-

for the Same material used, the strain Energy due to torsion well be. more in the hollow circular C/s as compared to Solid circular C/s.

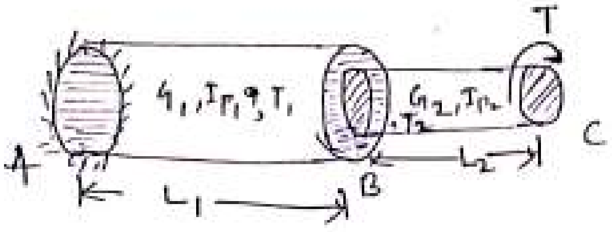
3.7 CONNECTIONS IN SHAFTS

Connection In shaft

(a) Shaft in Series

(b) Shaft in parallel

A) SHAFTS IN SERIES:-



From equilibrium.

$$\boxed{T_1 = T_2 = T}$$

For twist

$$\boxed{\theta_{AC} = \theta_{AB} + \theta_{BC}}$$

$$\boxed{\theta_{AC} = \frac{T.L_1}{G_1 I_{p_1}} + \frac{T L_2}{G_2 I_{p_2}}}$$

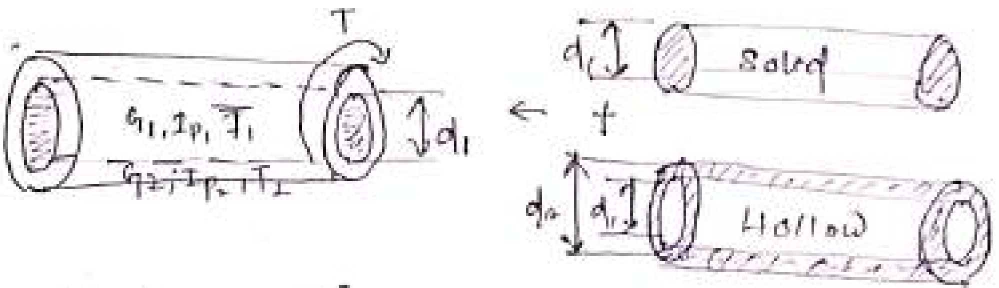
In AB

$$\boxed{\tau_{max} = \frac{T_1}{Z_{p_1}}}$$

In BC

$$\boxed{\tau_{max} = \frac{T_2}{Z_{p_2}}}$$

(B) SHAFTS IN PARALLEL:-



- From Equilibrium eqn

$$\boxed{T_1 + T_2 = T}$$

Maximum shear

$$\boxed{\tau_{max_1} = \frac{T_1}{Z_{p_1}}}$$

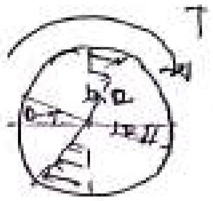
$$\boxed{\tau_{max_2} = \frac{T_2}{Z_{p_2}}}$$

- Compatibility Condition

$$\boxed{\theta_1 = \theta_2} \Rightarrow \boxed{\frac{T_1 . \cancel{L}}{G_1 I_{p_1}} = \frac{T_2 . \cancel{L}}{G_1 I_{p_1}}} \Rightarrow \boxed{\frac{T_1}{G_1 I_{p_1}} = \frac{T_2}{G_2 I_{p_2}}}$$

** POWER TRANSMITTED IN A SHAFT

* Power = Rate of workdone.



$$\boxed{W = T.\theta}$$

$$= \frac{d}{dt}(T.\theta)$$

$$= T.\left(\frac{d\theta}{dt}\right)$$

** $\boxed{P = T.\omega}$ $\boxed{\omega = \text{Angular Velocity}}$

* If N = revolution per second

$$\boxed{\omega = 2\pi N}$$

N = revolution per minute.

$$\boxed{\omega = \frac{2\pi N}{60}}$$

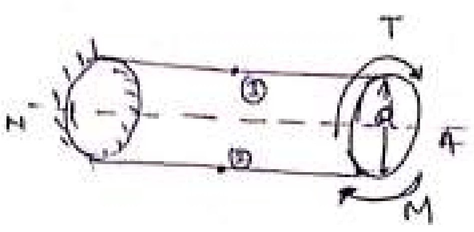
* UNITS (Important)

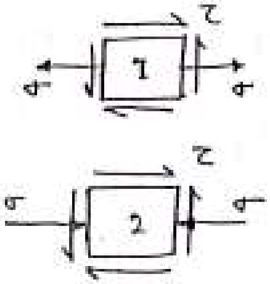
T	Work done	Power.
N-m	$\boxed{N-m}$ or Joule (J)	$\left(\frac{N-m}{s}\right)$ or Watt (W)
KN-m	$\boxed{KN-m}$ or (KJ)	$\left(\frac{KN-m}{s}\right)$ or KW

~~1-hor~~ One hortz Power (1Hp)

$$\boxed{\begin{aligned} 1hp &= 746 \text{ Watt} \\ &= 0.746 \text{ KW.} \end{aligned}}$$

EQUIVELENT BENDING AND TWISTING MOMENT





Due to bending $\sigma = \frac{M}{Z} = \frac{M}{\frac{\pi d^3}{32}} = \boxed{\frac{32M}{\pi d^3} = \sigma}$

Due to torsion $\tau = \frac{T}{Z_p} = \frac{T}{\frac{\pi d^3}{16}} = \boxed{\frac{16T}{\pi d^3} = \tau}$

* For Case-I (Element I)

$$\sigma_x = \sigma, \quad \sigma_y = 0, \quad \tau_{xy} = \tau$$

$$\sigma_{P_1}/\sigma_{P_2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+(\tau_{xy})^2}$$

$$= \frac{\sigma+0}{2} \pm \sqrt{\left(\frac{\sigma-0}{2}\right)^2+\tau^2}$$

$$= \frac{16M}{\pi d^3} \pm \sqrt{\left(\frac{16M}{\pi d^3}\right)^2+\left(\frac{16T}{\pi d^3}\right)^2}$$

$$\boxed{\sigma_{P_1}/\sigma_{P_2} = \frac{16}{\pi d^3}\left(M \pm \sqrt{M^2+T^2}\right)}$$

* Maximum normal stress :-

$$\boxed{\sigma_{max} = \frac{16}{\pi d^3}\left(M+\sqrt{M^2+T^2}\right)}$$

* Maximum Shear Stress :-

$$\tau_{max} = \frac{\sigma_{P_1}+\sigma_{P_2}}{2} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+(\tau_{xy})^2}$$

$$\boxed{\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2+T^2}}$$

Similarly

* for Case-2 (Element-2)

$\sigma_x = -\sigma$, $\sigma_y = 0$, τ_{xy}, τ

$$\boxed{\sigma_{P_1} / \sigma_{P_2} = \frac{16}{\pi d^3}\left(-M \pm \sqrt{M^2 + T^2}\right)}$$

* Maximum normal stress:-

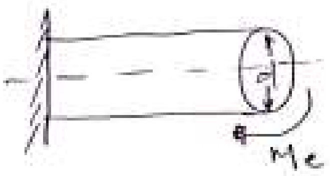
$$\boxed{\sigma_{max} = \frac{16}{\pi \cdot d^3}\left(M + \sqrt{M^2 + T^2}\right)}$$

* Maximum shear stress:-

$$\boxed{\tau_{max} = \frac{16}{\pi d^3}\left(\sqrt{M^2 + T^2}\right)}$$

* EQUIVALENT BENDING MOMENT :-

⇒ Maximum normal stress due to combine bending and twisting will be equal to Maximum normal stress due to equivalent bending.



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Max^m normal stress due to bending & Twisting	=	Max^m normal stress due to equivalent bending

Mathematically

$$\frac{16}{\pi d^3}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\overset{2}{32} Me}{\pi d^3}$$

***

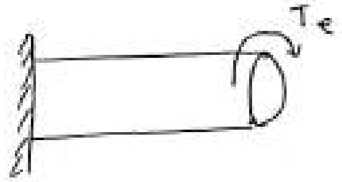
$$\boxed{Me = \frac{M + \sqrt{M^2 + T^2}}{2}}$$

(V. imp formula)

$$\sigma_{max_{Me}} = \frac{Me}{Z} = \frac{Me}{\frac{\pi d^3}{32}}$$

$$\boxed{\sigma_{max_{Me}} = \frac{32 Me}{\pi d^3}}$$

* EQUIVALENT TWISTING MOMENT :-



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Maximum shear stress due to combined bending & twisting	=	Max^m shear stress due to equivalent twisting

$$\tau_{max_{Te}} = \frac{T_e}{Z_p}$$

$$= \frac{T_e}{\frac{\pi d^3}{16}}$$

$$\boxed{\tau_{max_{Te}} = \frac{16 T_e}{\pi d^3}}$$

* Mathematically

$$\frac{16}{\pi d^3}\left(\sqrt{M^2+T^2}\right) = \frac{16}{\pi d^3} \cdot T_e$$

***

$$\boxed{T_e = \sqrt{M^2+T^2}}$$

V. imp. formula

* NUMERICALS :-

Q.1 A solid shaft is rotated at 800 Rpm and power transmitted to the shaft is 800 kW. If the maximum shear stress will be ~~attoed~~ allowed is 75 MPa. then.

a) Determine the diameter of shaft

B) If the solid shaft is to be replaced by hollow circular shaft such as diameter = 0.6 d_0 then determine the dia of shaft.

c) Also determine the %age saving the weight of the shaft.

Sol^n

a) $P = T \cdot \omega$

$$\overset{2}{\cancel{8}}00 \times 10^3 = \frac{T \cdot 2\pi \times 800}{60}$$

$$T = \frac{\overset{7.5}{\cancel{100}}}{\pi} \times 10^3 \cdot N\text{-}m$$

$$T = \tau_{max} \cdot Z_p$$

$$\frac{\overset{7.5}{\cancel{100}}}{\pi} \times 10^3 \times 10^3 = 75 \times \frac{\pi d^3}{16}$$

$$d = \sqrt[3]{\frac{16 \times \cancel{8}}{10 \times \pi^2} \times 10^6}$$

$= \cancel{137.39}\ 54.52\,mm$

~~$T = \tau_{max} \times Z_p$~~

b) $T = \tau_{max} \times Z_p$

$$\Rightarrow \frac{7.5}{\pi} \times 10^3 \times 10^3 = 75 \times \frac{\pi}{16 \times d_0}\left(d_0^4 - d_i^4\right)$$

$$\Rightarrow \frac{7.5}{\pi} \times 10^6 \times \frac{16}{75 \times \pi} = \frac{d_0^{\overset{3}{\cancel{4}}}}{\cancel{d_0}}\left(1 - 0.6^4\right)$$

$$\Rightarrow \quad d_0 = \sqrt[3]{\frac{7.5 \times 10^6 \times 16}{\pi^2 \times 75 \times (1 - 0.6^4)}}$$

$$\boxed{d_0 = 57.1 \text{ mm}}$$

$d_i = 0.6 \times d_0 \Rightarrow 57.1 \times 0.6$

$$\boxed{d_i = 34.26 \text{mm}}$$

c) $\%$age Saving $= \frac{W_1 - W_2}{W_1} \times 100\%$

$W = m.g$

$= \rho.V.g.$

$= \rho.A.L.g$

$$\boxed{W = \gamma.A.L}$$

$\%$age Saving $= \frac{\cancel{\gamma}.A_1.\cancel{L} - \cancel{\gamma}.A_2.\cancel{L}}{\cancel{\gamma}.A_1.\cancel{L}} \times 100\%$

$= \frac{A_1 - A_2}{A_1} \times 100\%$

$= \frac{\cancel{\frac{\pi}{4}} \cdot d^2 - \cancel{\frac{\pi}{4}}(d_0^2 - d_i^2)}{\cancel{\frac{\pi}{4}} \cdot d^2} \times 100\%$

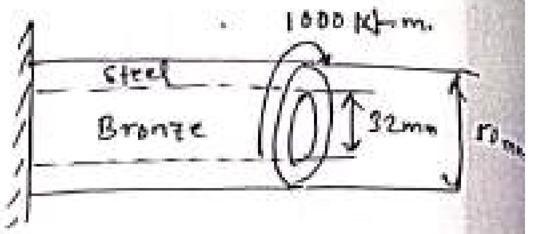
$\%$age Saving $= \frac{d^2 - (d_0^2 - d_i^2)}{d^2} \times 100$

$= \frac{(54.52)^2 - [(57.1)^2 - (34.26)^2]}{(54.52)^2} \times 100$

$= 29.79 \simeq 30\%$

Q.2) A solid bronze shaft of 32mm dia is embedded inside a hollow steel shaft of inner dia 32mm and external dia of 50 mm. The assembly is subjected to combined torque of 1000 N-m. If $G_s = 2G_b$ then determine the maximum stress developed in steel and bras

Sol^n from Equilibrium Eq^n



$$\boxed{T_s + T_B = 1000}$$

from Compatibility Condition

$$\boxed{\theta_s = \theta_B}$$

$$\frac{T_s \times L}{G_s \times I_{P_s}} = \frac{T_B \cdot L}{G_B \cdot I_{P_B}}$$

$$\boxed{\frac{T_s}{T_B} = \frac{G_s}{G_B} \times \frac{I_{P_s}}{I_{P_B}}}$$

$$\frac{T_s}{T_B} = \frac{2 \cdot \cancel{G_B}}{\cancel{G_B}} \times \frac{\cancel{\frac{\pi}{32}} \times (50^4 - 32^4)}{\cancel{\frac{\pi}{32}} \times 32^4}$$

$$\boxed{\frac{T_s}{T_B} = 9.92} \Rightarrow \boxed{T_s = 9.92\, T_B}$$

Now,

$$T_s + T_B = 1000$$

$$9.92 T_B + T_B = 1000$$

$$10.92 T_B = 1000$$

$$T_B = \frac{1000}{10.92} = 91.57 \text{ N-m.}$$

$$T_s = 1000 - 91.57 = 908.43 \text{ N-m.}$$

$$\tau_{max_B} = \frac{T_B}{Z_{p_B}} = \frac{91.57 \times 10^3}{\frac{\pi \times 32^3}{16}} =$$

$$\boxed{\tau_{max_B} = 14.23 \text{ Mpa}}$$ Ans

$$\tau_{max_S} = \frac{T_S}{Z_{p_S}} = \frac{908.43 \times 10^3}{\frac{\pi}{16}\left(\frac{50^4 - 32^4}{50}\right)}$$

$$\boxed{\tau_{max_S} = 44.47 \simeq 44.5 \text{ Mpa}}$$ Ans

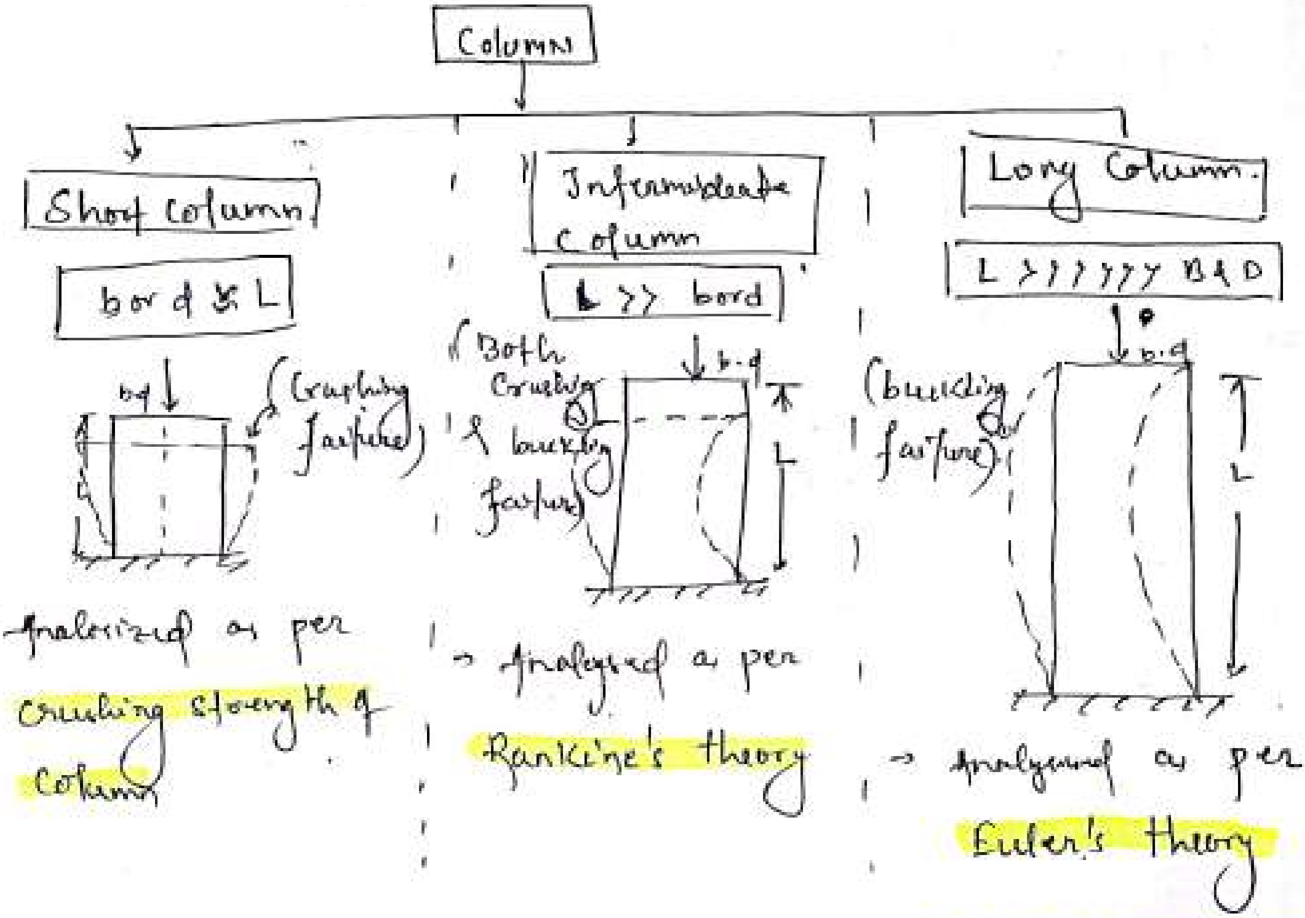
CH8 : COLUMN

Intro

* **Column :-**

It is a vertical member subjected to compressive loading which can be concentric or eccentric such that compressive or tensile stress developed at the cross section.

* **Types of Column :-**



* Analysed as per crushing strength of column

→ Analysed as per Rankine's theory

→ Analysed as per Euler's theory

Note :- For the design of column ~~the~~ is used Modified Rankine's theory as [illegible] ~~&~~ Indian standard.

[TH-1]

** EULER'S THEORY :-

* Assumptions :-

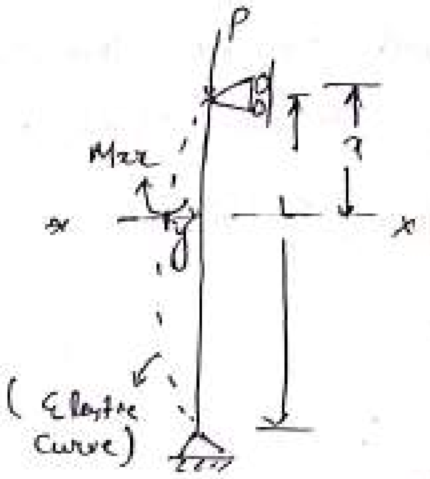
1) The material is homogeneous, isotropic and linearly elastic.
2) Column is long (slender).
3) The loading is perfectly axial.

Cont...

9) Before the loading, Column is perfectly straight.

10) Column fails due to buckling.

*, *, * Analysis of Euler's Theory :-



$$\Sigma M_{xx} = 0 \quad (+2, B)$$

$$P.y + M_{xx} = 0$$

$$P.y + EI . \frac{d^2y}{dx^2} = 0$$

$$\boxed{\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0} \quad — ①$$

* General solⁿ of the eqⁿ ① .

$$\boxed{y = C_1 \cos\sqrt{\frac{P}{EI}} . x + C_2 . \sin\sqrt{\frac{P}{EI}} . x}$$

* Boundary Condition :-

1) $\boxed{x = 0 , y = 0}$

2) $\boxed{C_1 = 0}$

3) $(x = L, y = 0)$

∴) $0 = C_2 . \sin\sqrt{\frac{P}{EI}} \times L$

∴) $\sin\sqrt{\frac{P}{EI}} \times L = 0$

∴) $\sqrt{\frac{P}{EI}} \times L = \sin^{-1}(0)$

∴) $\sqrt{\frac{P}{EI}} \times L = n\pi$

***

$$\boxed{P_b = \frac{n^2\pi^2 EI}{L^2}}$$

↳ (buckling Load)

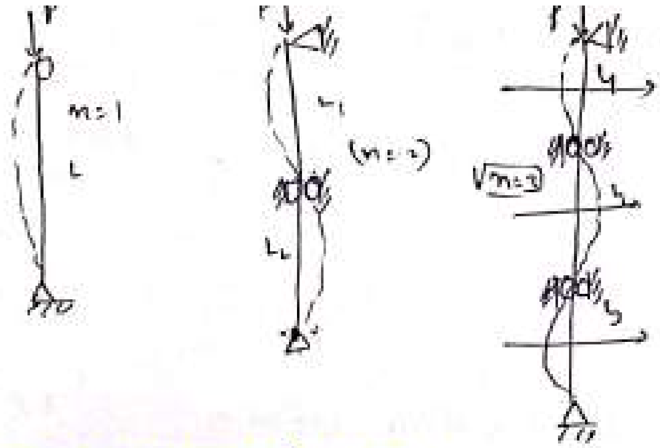
* P_b → buckling load of column

L → Length of column.

E → Youngs Modulus of elasticity

I → Minimum Moment of Inertia.

N → Number of loops



$P_1 = \frac{\pi^2 EI}{L_1^2}$

$P_2 = \frac{\pi^2 EI}{L_2^2}$

$P_3 = \frac{\pi^2 EI}{L_3^2}$

If $l_1 > l_2 > l_3$

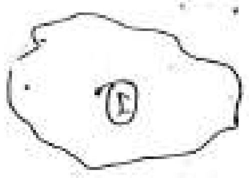
then $P_1 < P_2 < P_3$

$\boxed{P.\text{design} = \text{min}^m \text{ of } (P_1, P_2, P_3)}$

* Modified Eulers theory

$$\boxed{P_b = \frac{\pi^2 E I_{min}}{L_{eff}^2}}$$

** RADIUS OF GYRATION :-



$I = \int r^2 \cdot dA$

$I = r^2 \int dA$

$I = r^2 \cdot A$

$$\boxed{r = \sqrt{\frac{I}{A}}}$$

$$\boxed{I_{min} = r_{min}^2 \cdot A}$$

Definition :-

If a body has a moment of inertia 'I' about a point and entair mass is distributed at at the circumference of the ring such a way that inertia at the centre of ring is same moment of inertia of "I" then the radius of the ring is therefore referred as radius of gyration.

$I_{min} = r_{min}^2 \cdot A$.

$P_b = \frac{\pi^2 E A \cdot r_{min}^2}{L_{eff}^2}$

$\sigma_b = \frac{P_b}{A} = \frac{\pi^2 E}{(L_{eff}/r_{min})^2}$

** $\boxed{\sigma_b = \frac{\pi^2 E}{\lambda^2}}$

Buckling Stress.

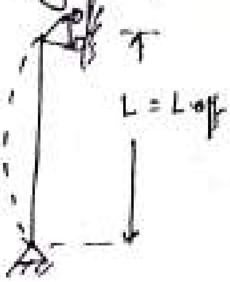
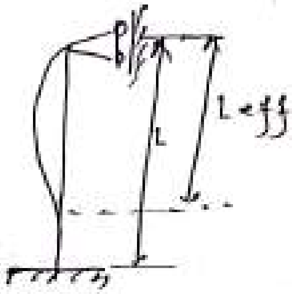
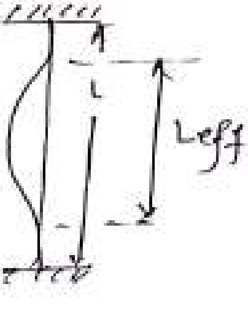
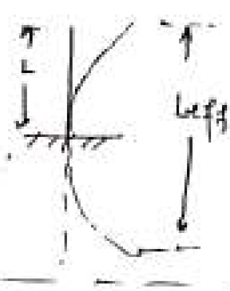
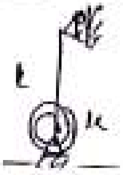
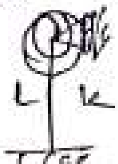
** λ = Slenderness Ratio.

$$\boxed{\lambda = \frac{L_{eff}}{r_{min}}}$$

Note :- The slenderness Ratio is independent of the material used and depending only on the dimentions of column.

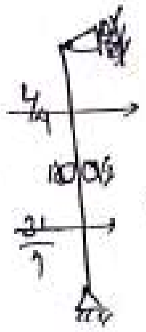
** EFFECTIVE LENGTH OF COLUMN

→ It is the distance between points of zero bending Moment.

End conditions	Loading diagram	Effective L
1) Both end are simply Suported		$\boxed{L_{eff} = L}$
2) When One end fixed and another end simply suported!		$\boxed{L_{eff} = \frac{L}{\sqrt{2}}}$
3) When Both the end are fixed.		$\boxed{L_{eff} = \frac{L}{2}}$
4.) One end is fixed and another end free		$\boxed{L_{eff} = 2L}$
5) Rotational Stiffner (k) $(0 < k < \infty)$ →0 → free rotation ∞ → Rigid	 $(0 < K < \infty)$	$K \to \infty \to L_{eff} = L/\sqrt{2}$ $K \to 0 \to L_{eff} = L$ $\boxed{\frac{L}{\sqrt{2}} < L_{eff} < L}$
		$K \to \infty \to L_{eff} = L/2$ $K \to 0 \to L_{eff} = L/\sqrt{2}$ $\boxed{\frac{L}{2} < L_{eff} < \frac{L}{\sqrt{2}}}$

Find effective length

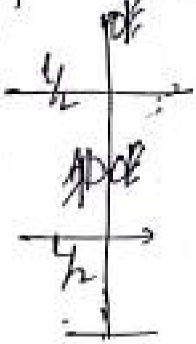
$\boxed{P \propto \frac{1}{L^2}} \rightarrow \boxed{L\uparrow \rightarrow P\downarrow}$



$\frac{L}{4} = L_e$

$\frac{3L}{4} = L_{eff}$

$\boxed{L_{eff} = \frac{3L}{4}}$



$L_e = \frac{L}{2}$

$L_e = \frac{L}{2\sqrt{2}}$

$\boxed{L_{eff} = \frac{L}{2}}$

* DEMERIT'S OF EULER'S THEORY :-

1) Euler assumed the column to be long, but column can be short & intermediate.

2) Euler assumed the loading to be axial, but load can also act at ~~some~~ an eccentricity.

3) Euler assumed the failure to be buckling but failure can be due to crushing also.

V. Imp. **For Validation of Euler's Theory :-**

* $\boxed{\text{Crushing Load carrying Capacity} > \text{Buckling load carrying Capacity}}$

$\Rightarrow \sigma_c \cdot A > \frac{\pi^2 E}{\lambda^2} \times A$

$\Rightarrow \sigma_c > \frac{\pi^2 E}{\lambda^2}$

$\boxed{\lambda^2 > \frac{\pi^2 E}{\sigma_c}}$

$\lambda^2 > \lambda_c^2$

$\boxed{\lambda > \lambda_c}$

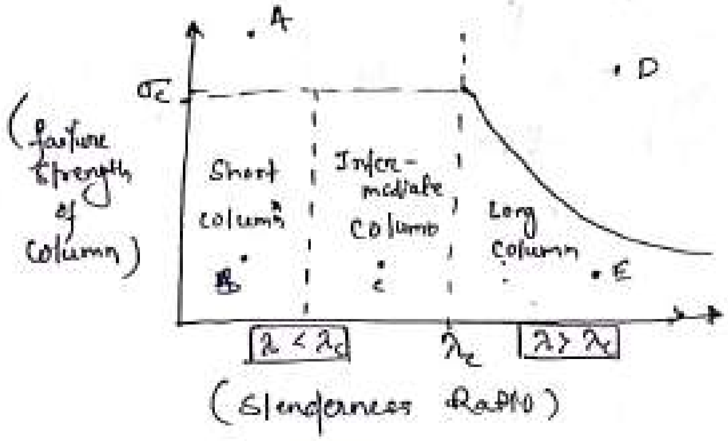
λ_c = Critical slenderness ratio

$\boxed{\lambda_c = \sqrt{\frac{\pi^2 E}{\sigma_c}}}$

* λ_c is a Material properties

* It is independent of column dimension.

** EULER'S FAILURE CURVE



$\sigma_b = \frac{\pi^2 E}{\lambda^2}$

$\to \boxed{\sigma_b \propto \frac{1}{\lambda^2}}$

↓

hyperbolic variation.

A → Short & Unsafe

B → Short & Safe

C → Intermediate & Safe

D → Long & Unsafe

E → Long & safe.

TH- 2. RANKINE'S THEORY :-

* It is the theory used for the Intermediate column which fails in combined buckling and crushing.

* Rankine's theory in the modified form is used by Indian Standard for the column design.

Analysis

* Theory Explanation!-

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_b}$$

$$P_e = \frac{P_c \cdot P_b}{P_c + P_b}$$

$$\boxed{P_e = \frac{P_c}{1 + \frac{P_E}{P_b}}}$$

$$P = \frac{\sigma_c A}{1 + \frac{\sigma_c \cdot A}{\frac{\pi^2 E}{\lambda^2} \cdot A}}$$

$$P = \frac{\sigma_c \cdot A}{1 + \left(\frac{\sigma_c}{\pi^2 E}\right)\lambda^2}$$

**

$$\boxed{P = \frac{\sigma_c \cdot A}{1 + \alpha \cdot \lambda^2}}$$

when,

α = Rankine's Constant.

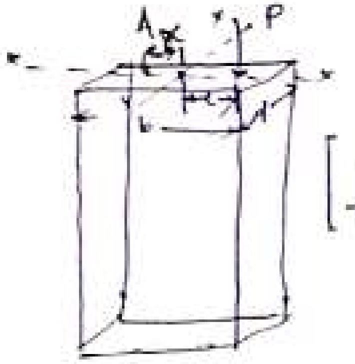
$$\boxed{\alpha = \frac{\sigma_c}{\pi^2 E}} \Rightarrow \alpha = \frac{1}{\frac{\pi^2 E}{\sigma_c}}$$

$$\boxed{\alpha = \frac{1}{\lambda_c^2}} \; **$$

V important

Lec-13

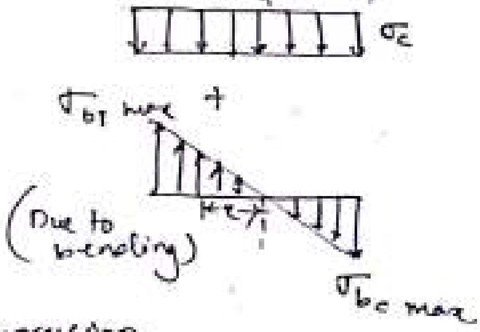
** AXIAL LOADING AND UNIAXIAL LOADING :-



[Comp.(+ve)
Tension (-ve)]

(Due to axial load)

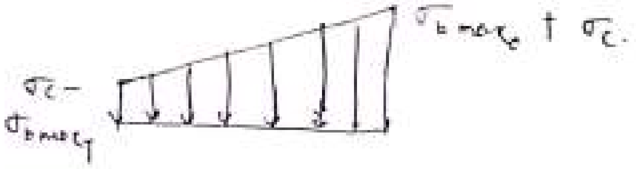
$\sigma_c = \frac{P}{A} = \frac{P}{bd}$



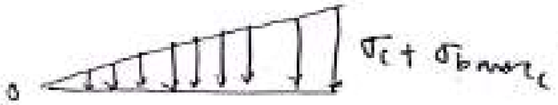
* Due to bending → on the right of yy → compression.

* Resultant diagram :-

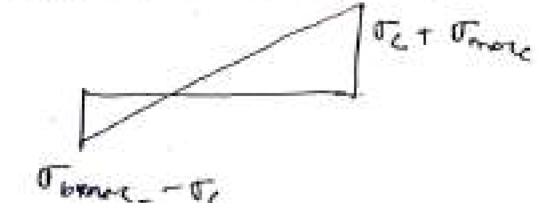
i) $\boxed{\sigma_c > \sigma_{b\,max}}$



(ii) $\boxed{\sigma_c = \sigma_{b\,max}}$



(iii) $\boxed{\sigma_c < \sigma_{max}}$



$$\boxed{\sigma_b = \frac{\sigma_{b\,max}}{b/2} \times x}$$

$$= \frac{M_{yy}}{Z \times \frac{b}{2}} \times x$$

Col^n Resultant of Stress!-

$$\sigma_R \cdot \cancel{A} = \sigma_c \cdot \cancel{A} \pm \sigma_b \cdot A$$

$$= \frac{P}{A} \pm \frac{M_{yy}}{I} \cdot \cancel{y}\, x$$

$$= \frac{P}{b.d} \pm \frac{P.e}{\frac{db^3}{12}} \times x$$

** *
$$\boxed{\sigma_R = \frac{P}{bd} \pm \frac{12Pe}{db^3} \cdot x}$$

$$\sigma_{R\max} = \frac{P}{bd} \pm \frac{12Pe}{db^3} \times \frac{b}{2}$$

$$\sigma_{R\max} = \frac{P}{b.d} + \frac{6Pe}{bd^2}$$

**
$$\boxed{\sigma_{R\max} = \frac{P}{b.d}\left(1 + \frac{6e}{b}\right)}$$

**
$$\boxed{\sigma_{R\min} = \frac{P}{b.d}\left(1 - \frac{6e}{b}\right)}$$

*** Column are generaly made up of brittle material an sience brittle materials are week in tension, thairfor it is design such that tens maximum tensile stress in column is zero.

(A)
(Rectangular C/s)

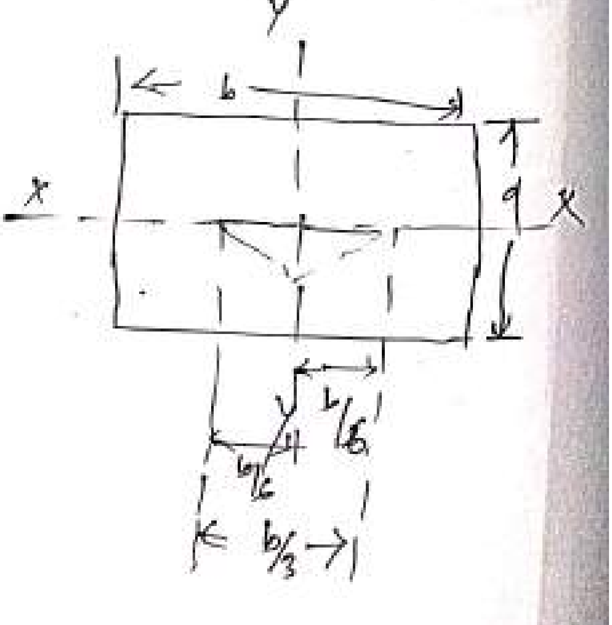
**
$$\boxed{\sigma_{R\min} \geq 0}$$

$$\frac{P}{bd}\left(1 - \frac{6e}{b}\right) \geq 0$$

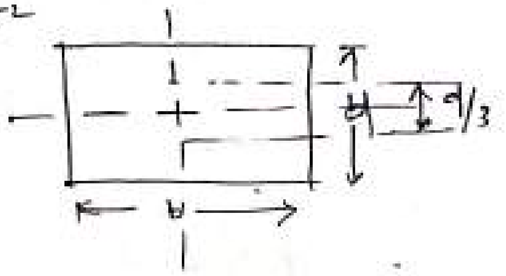
$$1 - \frac{6e}{b} \geq 0$$

***
$$\boxed{e \leq \frac{b}{6}}$$

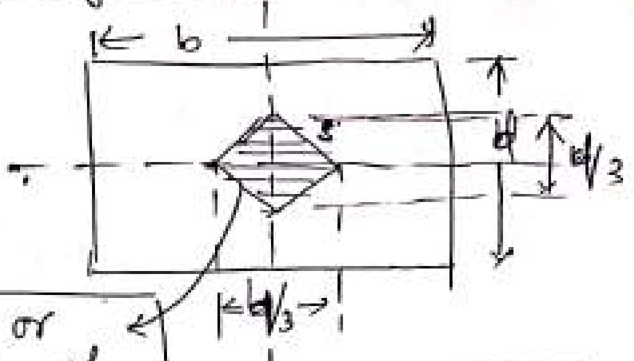
Case-1



Case 2



I) Rectangular C/s



Shape
↓
Rombus

← Core or Kern of Column

$$\boxed{S = \frac{\sqrt{b^2 + d^2}}{6}}$$

Moore One third Rule! -

For a rectangular Cross section if the load is applied along middle one third length along the axis then resultant tensile stress is equals to zero.

Core or Kern of a Column! -

It is the region over which if load is applied resultant tensile stress in column is zero.

(2) Solid Circular C/s

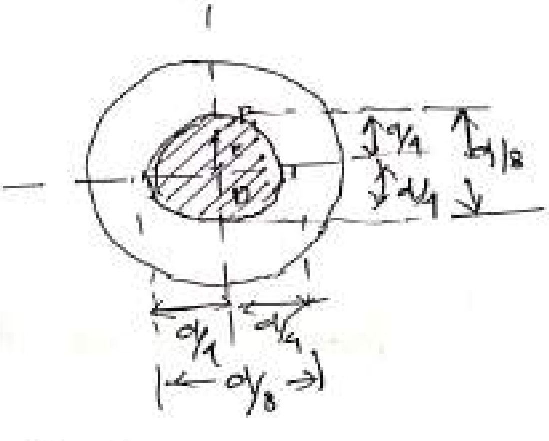
$\Rightarrow \quad \sigma_{Rmin} \geq 0$

$\Rightarrow \quad \frac{P}{A} - \frac{Pe}{Z} \geq 0$

$\Rightarrow \quad \frac{P}{\frac{\pi}{4} \cdot D^2} - \frac{Pe}{\frac{\pi D^3}{32}} \geq 0$

$\Rightarrow \quad \frac{4P}{\pi D^2}\left(1 - \frac{8e}{D}\right) \geq 0$

$$\boxed{e \leq \frac{D}{8}}$$

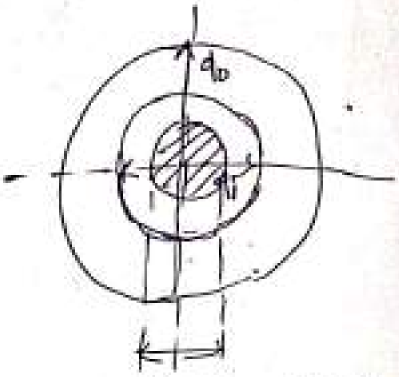


Circular

$$\boxed{Dia = \frac{D}{4}}$$

3. Hollow Circular C/s

$$\frac{P}{A} - \frac{Pe}{Z} \geq 0$$



$$\frac{P}{\frac{\pi}{4}(D_0^2 - D_i^2)} - \frac{P.e}{\frac{\pi}{32 D_0}(D_0^4 - D_i^4)} \geq 0$$

$$\frac{4P}{\pi(D_0^2 - D_i^2)}\left[1 - \frac{8 D_0 . e}{(D_0^2 + D_i^2)}\right] \geq 0 \qquad \frac{D_0}{4}\left[1+\left(\frac{D_i}{D_0}\right)^2\right]$$

$$e \leq \frac{D_0^2 + D_i^2}{8 D_0}$$

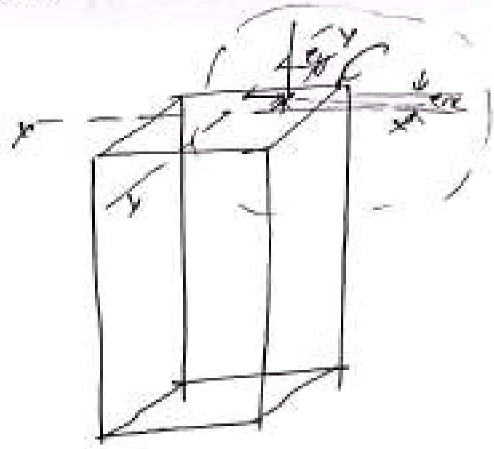
$$\boxed{e \leq \frac{D_0}{8}\left(1+\left(\frac{D_i}{D_0}\right)^2\right)}$$

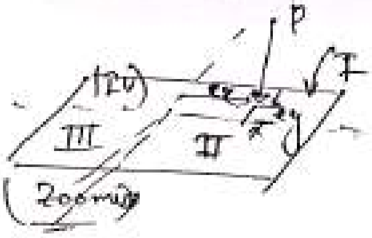
* Shape of Core or Kern

Shape of C/s	Shape of Cor/Kern
Rectangular C/s or I-section	Rhombus
Square C/s	Square
Solid circular	Circular
Hollow circular	Circular

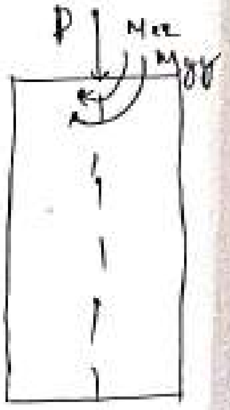
5.4

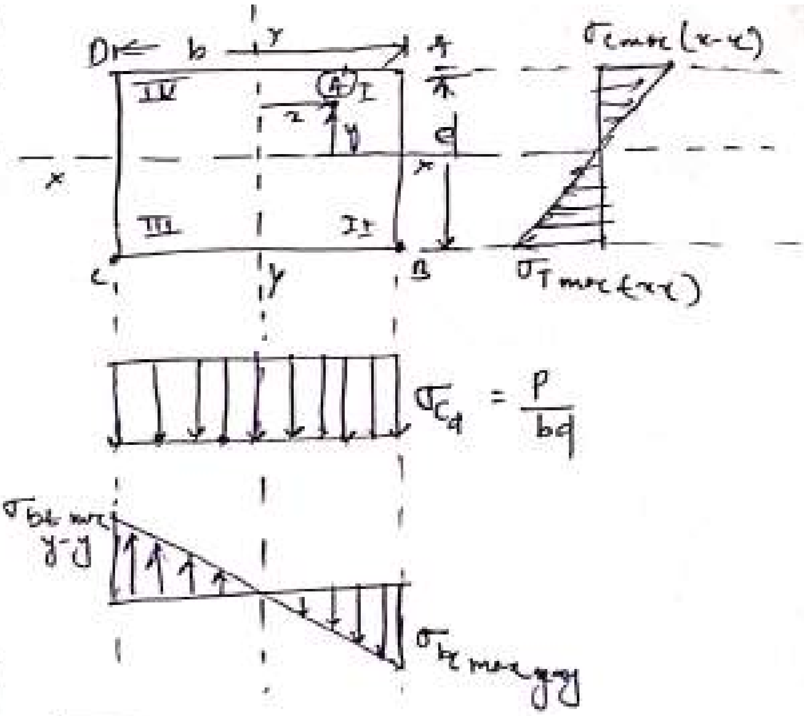
* AXIAL LOADING AND BIAXIAL BENDING :-





$$M_{yy} = P.e_x$$
$$M_{xx} = P.e_y$$





* $\sigma_{R_A}' = \frac{P}{b \cdot d} + \frac{M_{yy}}{I_{yy}} \cdot x + \frac{M_{xx}}{I_{xx}} \cdot y$

$$= \frac{P}{b \cdot d} + \frac{P \cdot e_x}{\frac{db^3}{12}} \times x + \frac{P \cdot e_y}{\frac{bd^3}{12}} \times y$$

xx

$$\sigma_{R_A} = \frac{P}{b \cdot d} + \frac{M_{yy}}{Z_{yy}} + \frac{M_{xx}}{Z_{xx}}$$

$$= \frac{P}{bd} + \frac{P \cdot e_x}{\frac{db^2}{6}} + \frac{P \cdot e_y}{\frac{bd^2}{6}}$$

xxx xx

$$\boxed{\sigma_{R_A} = \frac{P}{b \cdot d} + \frac{6 P e_x}{db^2} + \frac{6 P e_y}{bd^2}} \rightarrow \boxed{\sigma_{R_{Max}}}$$

Point

$$\boxed{\sigma_{R_B} = \frac{P}{bd} + \frac{6 P e_x}{db^2} - \frac{6 P \cdot e_y}{bd^2}}$$

$$\boxed{\sigma_{R_C} = \frac{P}{bd} - \frac{6 P \cdot e_x}{db^2} - \frac{6 P \cdot e_y}{bd^2}} \rightarrow \boxed{\sigma_{R_{min}}}$$

$$\boxed{\sigma_{R_D} = \frac{P}{b \cdot d} - \frac{6 P \cdot e_x}{db^2} + \frac{6 P \cdot e_y}{bd^2}}$$

CH-8: PRESSURE VESSEL

* DEFINATIONS:-

→ It is a vessel or container used to store the fluids at a pressure either more then or less then atmospheric pressure.

Note:-

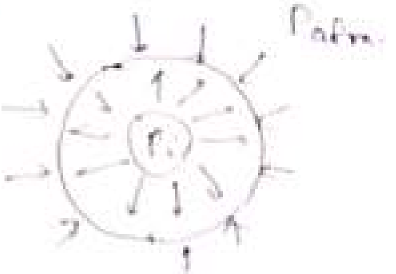
More then atmospheric pressure → Gauge pressure

Less then atmospheric pressure → Vaccume pressure

* Type of Pressure Vessel

- THIN P.V
  - thickness / internal dia : $\frac{t}{d} < \frac{1}{10}$ to $\frac{1}{15}$
- THICK P.V.
  - $\frac{t}{d} > \frac{1}{10}$ to $\frac{1}{15}$

** INTERNAL PRESSURE AND EXTERNAL PRESSURE



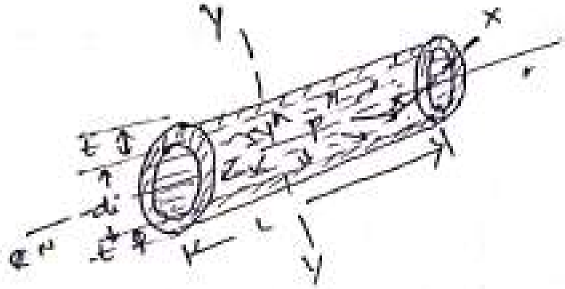
P > Patm → Internal pressure (Pi - Patm)

P < Patm → External pressure (Patm - Pi)

** STRESSES IN PRESSURE VESSEL :-

1) Hoop Stress / Circumferential stress / Meridional Stress
2) Longitudinal stress
3) Radial stress.

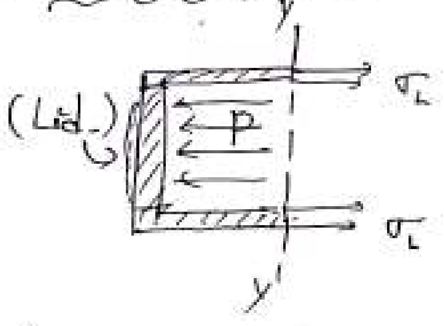
1) HOOP STRESS:-



(C/s X->)

* Hoop stress → It is a circumferential stress
  - → Due to internal pressure it is tensile in nature
  - → Due to external pressure it is Compressi in nature.
  - → ~~Due~~ In thin P.V hoop stress is Constan throughout the thickness. but in thick pressure vessel it varies hiperbolically
  - → Due to hoop stress Diameter will chang

2.) LONGITUDINAL DIRECTION :-



→ for developing of Longitudinal stre Lid should be closed.

(C/s y-y)

→ Due to longitudinal stress the Length of P.V will change.

→ ~~It is~~ a stress acts in longitudinal direction.

→ Due to internal pressure, longitudinal stress is tensile in nature

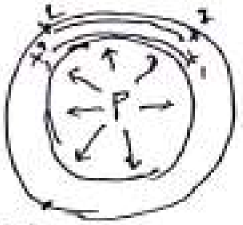
→ Due to external pressure, Longitudinal stress is Compressive in nature.

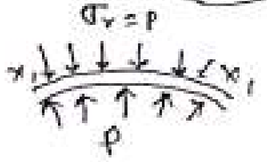
→ In thin and thick pressure vessel it is Constant throughout thickness.

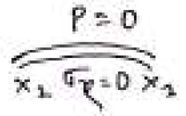
3. RADIAL STRESS

→ Stress acts in radial direction.

* for Internal pressure :-

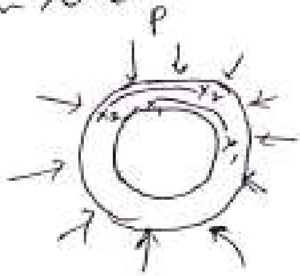


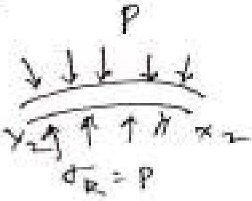


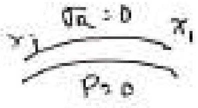


→ Due to internal pressure, radial stress is compressive in nature with maximum (P) at inner surface. ~~or~~ and minimum at outer surface.

* for External Pressure :-





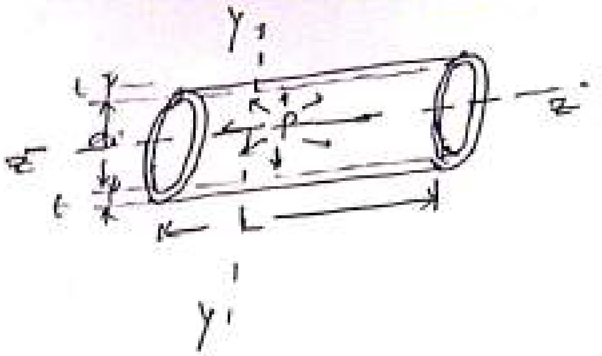


→ Due to external pressure, radial stress is compressive in nature with maximum (P) at outer surface and minimum(0) at inner surface.

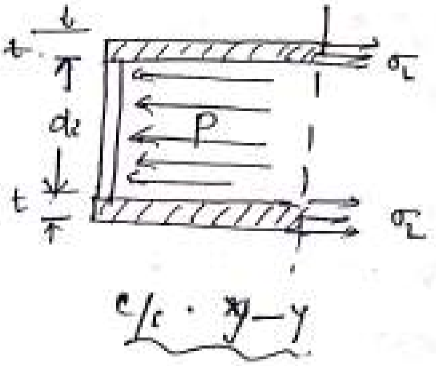
** → Due to Radial stress thickness of Pressure vessel changes.

**

THIN CYLINDER :-



1) LONGITUDENAL STRESS :-



Bursting force (B.F) $= P \times \left(\frac{\pi}{4} \cdot d^2\right)$

Resisting force (R.F) $= \sigma_L \cdot (\pi d \cdot t)$

- If body need to be in equilibrium

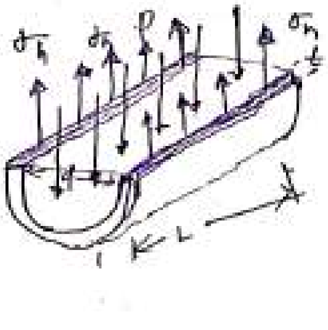
R.F. = B.F.

$\sigma_L \times (\pi \cdot dt) = p\left(\frac{\pi}{4} \cdot d^2\right)$

**

$$\boxed{\sigma_L = \frac{Pd}{4t}}$$

B) HOOP STRESS



Bursting force (B.F) $= P \cdot (d \cdot L)$

Resisting force (R.F) $= \sigma_h \, (2Lt)$

- For equilibrium.

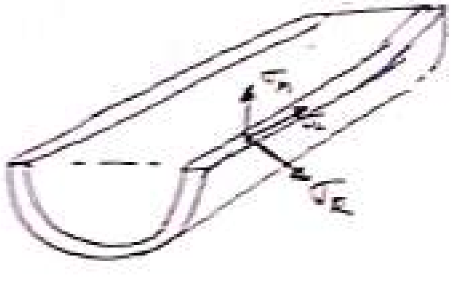
BF = RF

$P \cdot d \cdot L = \sigma_h \cdot 2 \cdot Lt$

**

$$\boxed{\sigma_h = \frac{Pd}{2t}}$$

Note (v imp)



- $\sigma_h, \sigma_L, \sigma_R$ – are mutually perpendicular to each other and no shear stress is acting, Hence these are principle stresses.

$\boxed{\sigma_h = \frac{Pd}{2t}}$; $\boxed{\sigma_L = \frac{Pd}{4t}}$; (hyperbolic) $\boxed{\sigma_R = 0 \text{ to } 0}$

$\downarrow$ $= \sigma_{P_1}$; $\downarrow$ $= \sigma_{P_2}$

Assume ($\sigma_R = 0$)

$\boxed{\sigma_h, \sigma_L >> \sigma_R}$ $\rightarrow$ $\boxed{\sigma_R = 0}$ $\left\{ \begin{array}{l} \frac{t}{d} < \frac{1}{10} \text{ to } \frac{1}{15} \\ \frac{d}{t} \text{ or } > 10 \text{ to } 15 \end{array} \right.$

$\boxed{\sigma_{P_3} = 0}$

= Maximum Shear Stress

$$\tau_{max_1} = \frac{\sigma_{P_1} - \sigma_{P_2}}{2} = \frac{\frac{Pd}{2t} - \frac{Pd}{4t}}{2} = \frac{Pd}{8t}$$

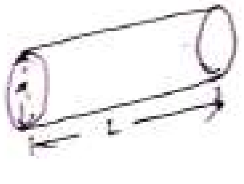
$$\tau_{max_2} = \frac{\sigma_{P_1} - \sigma_{P_3}}{2} = \frac{\frac{Pd}{2t} - 0}{2} = \frac{Pd}{4t}$$

$$\tau_{max_3} = \frac{\sigma_{P_2} - \sigma_{P_3}}{2} = \frac{\frac{Pd}{4t} - 0}{2} = \frac{Pd}{8t}$$

$\tau_{max} = \text{max}^m$ of $(\tau_{max_1}, \tau_{max_2}, \tau_{max_3})$

$$\boxed{\tau_{max} = \frac{Pd}{4t}}$$

= Volumetric Strain :-



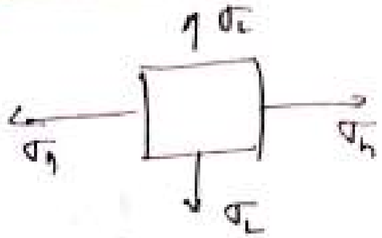
Volume $V = \frac{\pi d^2 L}{4}$

$$\Delta V = \frac{\pi}{4} \times 2d\,\Delta d \times L + \frac{\pi}{4} \cdot d^2 \cdot \Delta L$$

$$\epsilon_V = \frac{\Delta V}{V} = 2\frac{\Delta d}{d} + \frac{\Delta L}{L}$$

$$\boxed{\epsilon_V = 2\epsilon_d + \epsilon_L}$$

- Strain in diameter



$$\epsilon_d = \frac{\sigma_h}{E} - \mu\frac{\sigma_L}{E}$$

$$= \frac{Pd}{2tE} - \mu\frac{Pd}{4tE}$$

** $\boxed{\epsilon_d = \frac{Pd}{4tE}(2-\mu)}$

- Strain in length

$$\epsilon_L = \frac{\sigma_L}{E} - \mu\frac{\sigma_h}{E}$$

$$= \frac{Pd}{4tE} - \mu\frac{Pd}{2tE}$$

** $\boxed{\epsilon_L = \frac{Pd}{4tE}(1-2\mu)}$

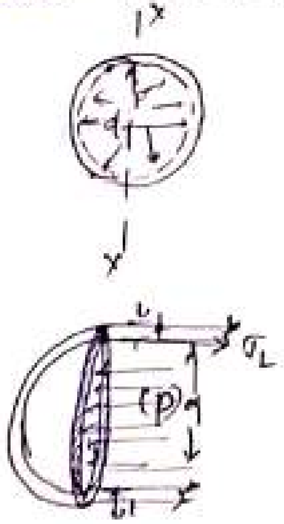
- Volumetric strain

$$\boxed{\epsilon_v = 2\epsilon_d + \epsilon_L}$$

$$\epsilon_v = 2\times\frac{Pd}{4tE}(2-\mu) + \frac{Pd}{4tE}(1-2\mu)$$

** $\boxed{\epsilon_v = \frac{Pd}{4tE}(5-4\mu)}$

** THIN SPHERE:



Here Hoop & Longitudinal stress

$$\boxed{\sigma_h = \sigma_L}$$

$$B.F = P\cdot\frac{\pi}{4}\cdot d^2$$

$$R.F = \sigma_L\cdot(\pi\cdot dt)$$

~~B.F~~ = $\boxed{BF = RF}$

$$P\cdot\frac{\pi}{4}d^2 = \sigma_L\times\pi dt$$

$$\boxed{\sigma_L = \frac{Pd}{4t}}$$

and also

$$\boxed{\sigma_h = \frac{Pd}{4t}}$$

[illegible] Maximum Shear stress

$$\tau_{max_1} = \frac{\sigma_{P_1} + \sigma_{P_2}}{2} = \frac{\frac{Pd}{4t} - \frac{Pd}{4t}}{2} = 0$$

$$\tau_{max_2} = \frac{\sigma_{P_2} - \sigma_{P_3}}{2} = \frac{\frac{Pd}{4t} - 0}{2} = \frac{Pd}{8t}$$

$$\tau_{max_3} = \frac{\sigma_{P_1} - \sigma_{P_1}}{2} = \frac{Pd}{8t}$$

***

$$\boxed{\tau_{max} = \frac{Pd}{8t}}$$

2 Volumetric Strain

$$V = \frac{\pi d^3}{6}$$

$$\Delta V = \frac{\pi}{6} \cdot 3d^2 \cdot \Delta d$$

$$\epsilon_v = \frac{\Delta V}{V} = \frac{3\Delta d}{d}$$

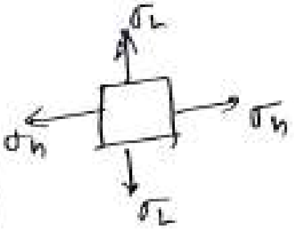
$$\boxed{\epsilon_v = 3\epsilon_d}$$

***

$$\boxed{\epsilon_v = \frac{3Pd}{4tE}(1-\mu)}$$

$$\epsilon_d = \frac{\sigma_h}{E} - \mu\frac{\sigma_L}{E}$$

$$\boxed{\epsilon_d = \frac{Pd}{4tE}(1-\mu)}$$



Unit
Topic/CH-89

: SPRING :

The main function of spring is to deform under the load and to absorb energy and once the load is removed it regains to its original position.

- Practical application:- When energy is to absorve and release during shock or vibrational loading.. Eg- automobiles, railway, lift. etc.

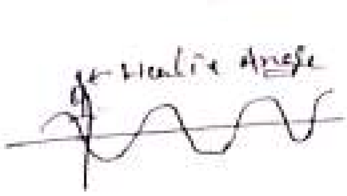
* Type Of Spring

1. Leaf spring

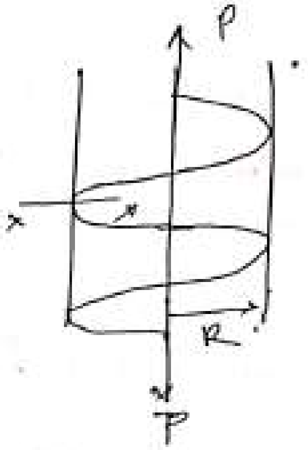
2. helical spring

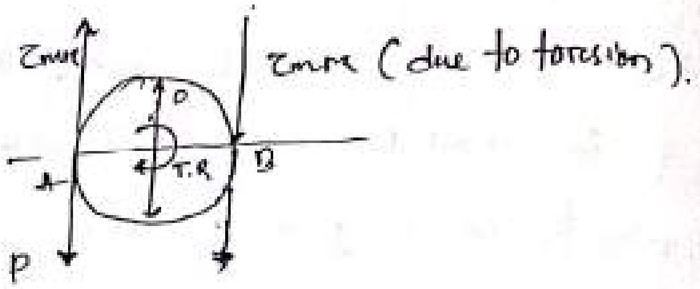
   a) Close coil H. spring or Torsional spring
   - → Twisting moment is pre-domenent
   - → Helix angel < 10°

   b) Open coil H. spring or Bending spring
   - → Bending is predominent.
   - → Helix angle > 10°



2.8 : CLOUSED COIL HELICAL SPRING





P → act's tangential to C/S

> Shear stress due to 'P' load = $\frac{P}{\left(\frac{\pi}{4}\times d^2\right)}$

> shear stress due to Torsion :-

$$\tau_{max} = \frac{T}{Z_P} = \frac{T\cdot R}{\frac{\pi d^3}{16}} = \frac{16TR}{\pi d^3}$$

** Resultant shear stress

$$\tau_{R_{outer}} = -\frac{P}{\frac{\pi}{4}d^2} + \frac{16TR}{\pi d^3}$$

$$\boxed{\tau_{R_{outer}} = \frac{16PR}{\pi d^3}\left(1-\frac{4d}{R}\right)}$$ & . $\left\{\frac{4d}{R} \rightarrow \text{direct shear}\right\}$

$$\tau_{R_{inner}} = \frac{P}{\frac{\pi}{4}d^2} + \frac{16PR}{\pi d^3}$$

$$\boxed{\tau_{R_{inner}} = \frac{16PR}{\pi d^3}\left(1+\frac{4d}{R}\right)}$$

$R >>>> d$

$\frac{4d}{R} \rightarrow 0$

↳ is neglected

$$\boxed{\tau_R = \frac{16PR}{\pi d^3}}$$

***

Note:-

Direct shear is neglected and only Indirect shear is considerd.

* Strain Energy Stored In Spring :-

String energy stored due to torsion = $\frac{T^2 L}{2 G I_p}$

$$U = \frac{(P\cdot R)^2 \times (2\pi R n)}{2\cdot G \times \frac{\pi d^4}{32}}$$

$$\boxed{U_{spring} = \frac{32 P^2 R^3 n}{G d^4}}$$

***

* Axial deformation in Spring :-

According to Castiglano's theorem.

$$\delta = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P}\left(\frac{32 P^2 R^2 \cdot n}{G d^4}\right)$$

***

$$\boxed{\delta_{spring} = \frac{64 P R^3 n}{G d^4}}$$

* Angle of twist :-

-> From torsional eq^n

$$\theta = \frac{T\cdot L}{G I_p} = \frac{P\cdot R\,(2\pi R n)}{G \times \frac{\pi d^4}{32}}$$

***

$$\boxed{\theta_{spring} = \frac{64 P R^2 n}{G\cdot d^4}}$$

* Stiffness of spring

$$P = K\delta$$

$$K = \text{Stiffness} = \frac{P}{\delta} = \frac{P}{\frac{64 P R^3 n}{G d^4}} \Rightarrow$$

***

$$\boxed{K = \frac{G d^4}{64 R^3 n}}$$

** PROOF LOAD :-

→ Maximum load spring can resist without plastic deformation.

We Know

$$\tau_{max} = \frac{16PR}{\pi d^3}$$

***
$$\boxed{P = \frac{\pi d^3}{16 \cdot R} \cdot \tau_{max}}$$

where

τ_{max} → Material properties.

Note :-

1) If spring is cut in 2 equal part, then stiffne
of the individual spring will be. ?

a.) half
[✓ b)] twice
c) one-fourth
d) four forth

Ans :

$$K = \frac{Gd^3}{64R^3 n}$$

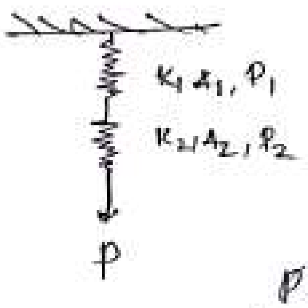
$$\boxed{K \propto \frac{1}{n}}$$

when spring is cu
in two parts no.
loops will be half

Hence, stiffness will becomes twice.

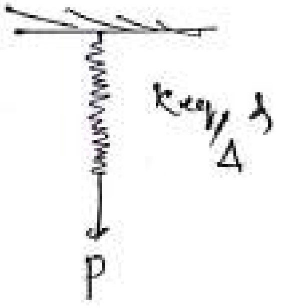
** CONNECTIONS IN SPRINGS

A) Series Connection of spring :-



$$\boxed{P_1 = P_2 = P}$$

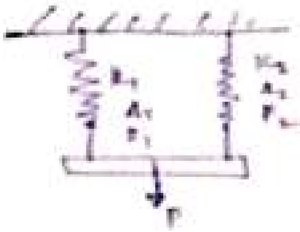
$$\boxed{\Delta = \Delta_1 + \Delta_2}$$

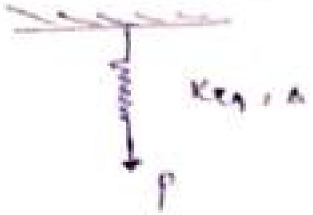


$$\frac{P}{K_{eq}} = \frac{P_1}{K_1} + \frac{P_2}{K_2}$$

***
$$\boxed{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}}$$

B) Springs in Parallel:-





$$\boxed{P = P_1 + P_2}$$

$$\boxed{\Delta = \Delta_1 = \Delta_2}$$

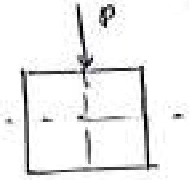
$$K_{eq} \times \Delta = K_1 \times \Delta_1 + K_2 \times \Delta_2$$

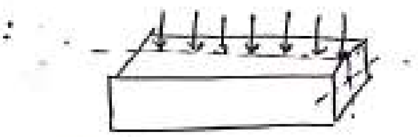
$$\Rightarrow \boxed{K_{eq} = K_1 + K_2}$$

Topic-10 :
Learning

* SHEAR CENTRE *

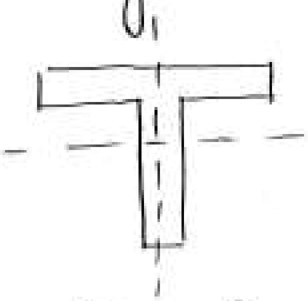
* It is the point through which if the resultant loading is applied, then resultant twisting moment is equall to zero.

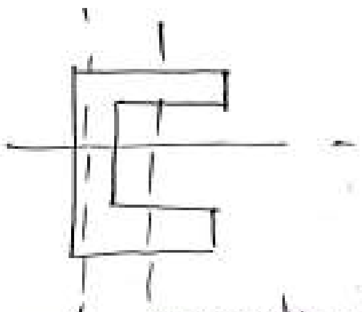




⇒ ~~If~~ (C/s is symmetric)

⇒ If C/s is symmetric, apply the load at the axis of symmetry.



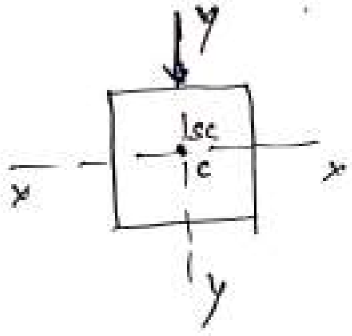


→ If the C/s is not symmetric, then the load should be pass through shear centre so, that no twisting moment will developed.

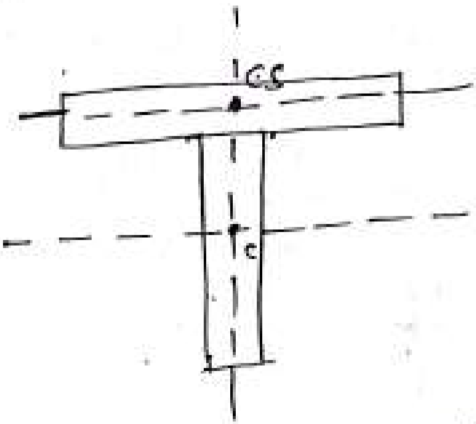
** POINTS to DETERMINE SHEAR CENTRES :-

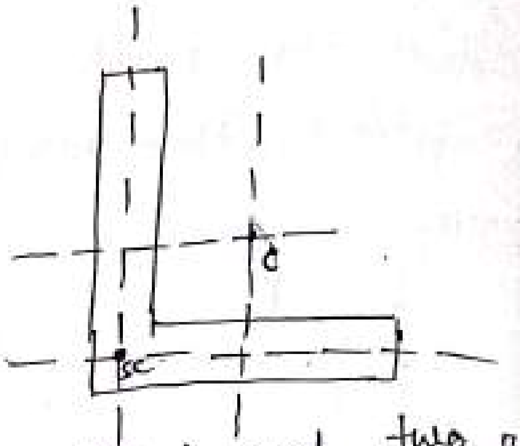
1) When C/s is symmetrical about both the axis :-

→ When C/s is symmetrical to both the axis the shear centre Co-incides with the centroid



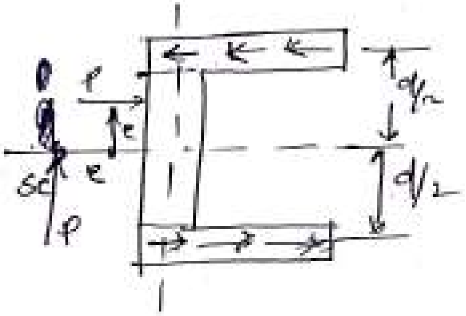
B) When c/s is Symmetrical about one axis :-





* When Cross Section is not symmetrical and two recta are meeting then the meeting points of the centre of two rectangles is reffered ~~as~~ as shear Centre.

C) CHANNAL C/s.

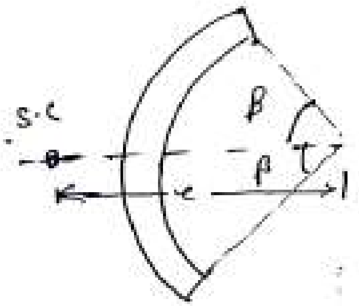


$$Pe = F \cdot \frac{d}{2}$$

$$e = \frac{F \times \frac{d}{2}}{P}$$

$$\boxed{e = \frac{tb^2 d^2}{4I}}$$

D) ARC OF ½ CIRCLE :-



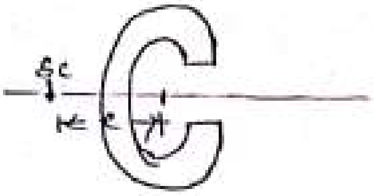
$$\boxed{e = \frac{2r(\sin\beta - \beta\cos\beta)}{\beta - \sin\beta \cdot \cos\beta}}$$

for Semicircular Arc

$$e = \frac{2r(1-0)}{\frac{\pi}{2} - 0}$$

$$\boxed{e = \frac{4r}{\pi}}$$

6. Slit Section:-



$\boxed{e = 2R}$

Less

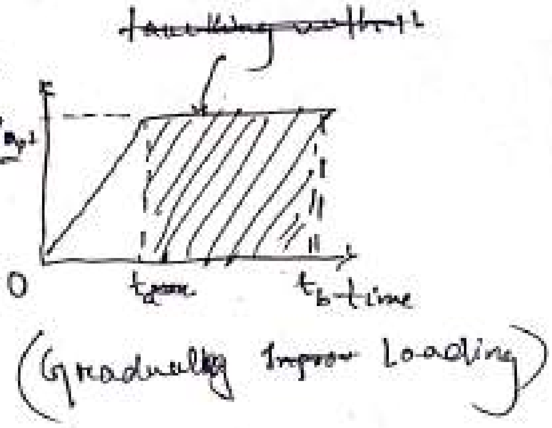
CH-11 : THEORY OF FAILURE :-

* Design under static Load :-

σ_{ut} = tensile strength

σ_{uc} = Compressive strength.

FOS = factor of safety.



(Gradually Improv Loading)

* The theory of failure is can study the zone of $t_a - t_b$.

[illegible]

$$\boxed{\frac{\sigma_{up}}{FOS} = \sigma_{premsible}}$$

types * 5- types theory of failure :

1) Rankine's theory (Maximum principal stress theory)
2) ST. venand's theory (Maximum principal strain theory)
3) Guest & Tresca's Theory (Maximum shear stress theory)
4) Haigh's Theory (Maximum strain Energy theory)
5) Von-Mises -Henky theory (Maximum shear strain energy Distrortion energy theory)

I. RANKENE'S OR MAXIMUM PRINCIPAL STRESS THEORY :-

(Useful for · Brittle material)

~~for failure~~

σ_1 , σ_2 , σ_3

σ_1 (+) → Max Tensile

σ_3 → Max - Compressive.

* ~~for failure~~ for tension

$$\boxed{\sigma_1 > \sigma_{ut}}$$

Compression

$$\boxed{\sigma_3 > \sigma_{uc}}$$

for sho' safty

$$\boxed{\sigma_1 \le \sigma_{ut}}$$

&

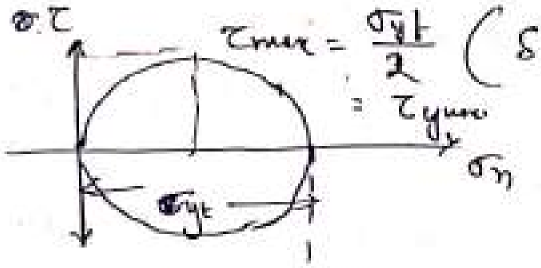
$$\boxed{\sigma_3 \le \sigma_{uc}}$$

} for brittle material

$$\boxed{\sigma_3 \le \sigma_{yt}}$$

$$\boxed{\sigma_3 \le \sigma_{yc}}$$

} for ductile material



$\tau_{max} = \frac{\sigma_{yt}}{2}$ (Shear strength of material from single tension test)
$= \tau_{yield}$

(τ_{yield} yield shear strength)

* For Safty

$$\boxed{(\tau_{max})_{abs} \leq (\tau_{max})_{yield}}$$

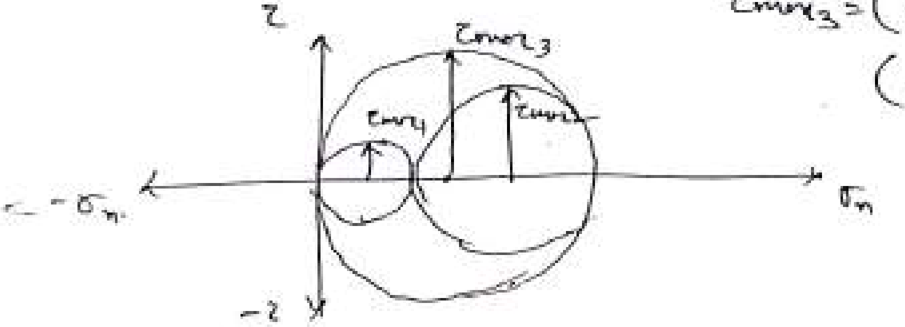
or (Acount to fos)

$$\boxed{(\tau_{max})_{abs} \leq (\tau_{permissible})}$$

Assumption:-

Yield shear strength is half of the yield strength in tension. $\boxed{(\tau_{max})_{abs} \leq \tau_y = \frac{\sigma_{yt}}{2}}$

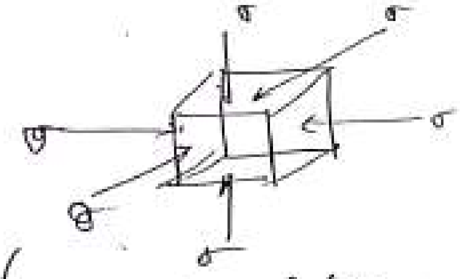
Mohr's Circle



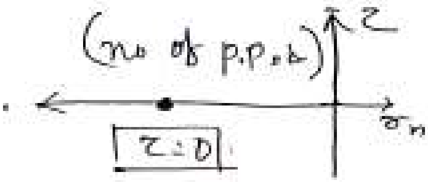
$\tau_{max_3} = (\tau_{max})_{abs} \leq (\tau_{max})_{yield}$
$(\tau_{max})_{abs} \leq \tau_{permissible}$

* Limitation *

For hydrostatic loading there no shear stress is developed so the theory should not be applicable.



(equal and like stress in 3D)



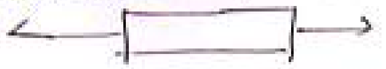
(Under hydrostatic loading condⁿ)
(Mohr's Circle)
(MSST) (will not valid)

3. ~~Max~~ Von-Mises-Hencky Theory (Maximum Strain Energy / Distortion Energy theory) (Hencky theory)

(Distortion Energy/unit volume for tri axial situation)

$$\boxed{U = \frac{1}{12G}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]}$$

→ Simple tension test (strain Energy) /
(for ductile material)



$\sigma = \sigma_{yt}$

$\sigma_1 = \sigma = \sigma_{yt}$

$\sigma_2 = \sigma_3 = 0$

→ Maximum Distortion Energy / simple unit volume for simple tension test.

$$= \frac{1}{12G}\, 2 .\sigma_1^2 = \frac{\sigma_1^2}{6G} = \frac{\sigma_{yt}^2}{6G}$$

So, for safety Condition

$$\boxed{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 \leq 2\sigma_{yt}^2}$$

with fos

$$\frac{\sigma_{ut}}{fos} = \sigma_{per}$$

$$\boxed{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 \leq 2\sigma_{per}^2}$$

* for Pure Shear :-

(Plane stress Condition)

Mohr's Circle

$$\boxed{\tau^2 + \tau^2 + 4\tau^2 \le 2\sigma_{yt}^2}$$

for $\boxed{fos = 1}$

$$\boxed{6\tau^2 \le 2\sigma_{yt}^2}$$

for critical case.

$$\tau = \sqrt{\frac{\sigma_{yt}^2}{3}}$$

$$\tau = \frac{\sigma_{yt}}{\sqrt{3}}$$

** * $\boxed{\tau = 0.577\,\sigma_{yt}}$

↳ Maximum shear stress allowed As per (MDET ~~[illegible]~~)

$$\boxed{(\tau_{max})_{ultimate} = 0.577 \cdot \sigma_{yt}}$$

↳ strength of the material in shear As per (MDET)

** **Note**:-

- * Rankine's theory → Brittle Material.
- * Guest & Tresca's → MSST → Ductile ~~[illegible]~~ Material
  ↳ Easy to use
- * Von - mises & Hencky → MDET → Ductile material
  ↳ ~~More Accurate~~
  More accurate

4. ST. VENENT'S THEORY (Maximum Principle Strain theory):

$\boxed{\sigma_1, \sigma_2, \sigma_3}$

$\boxed{\epsilon_1, \epsilon}$

Maximum principle strain!

$$\boxed{\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}}$$

* Principle strain in simple tension test.

$$\boxed{\epsilon_1 = \frac{\sigma}{E} = \frac{\sigma_{yt}}{E}}$$

* For failure.

$$\boxed{\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} \leq \frac{\sigma_{yt}}{E}}$$

with FOS

$$\sigma_{per} = \frac{\sigma_{yt}}{FOS}$$

$$\boxed{\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} \leq \frac{\sigma_{per}}{E}}$$

** ~~Low~~ Accuracy is very low, this method is over estimate the value of stress.

5: HEGH's THEORY (Maximum Total Strain Energy theory):

Given → $\sigma_1, \sigma_2, \sigma_3, E, \mu$.

* Maximum total strain ~~engy~~ energy per unit volume:-

$$\boxed{U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\cdot\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]}$$

From simple tension test:-

$$\sigma_{tot} = \sigma_1 = \sigma_{yt}$$

~~Max^m Strain Energy~~ :

Max^m total strain Energy in simple tension test

$$\boxed{U_T = \frac{\sigma_{ut}^2}{2E}}$$

For Safty

$$\boxed{u = \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\cdot\sigma_2+\sigma_2\cdot\sigma_3+\sigma_3\cdot\sigma_1)\right] \leq \frac{\sigma_{yt}^2}{2E}}$$

With FOS

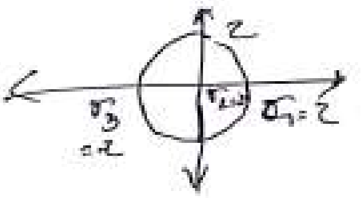
$$\frac{1}{\cancel{2E}}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\cdot\sigma_2+\sigma_2\cdot\sigma_3+\sigma_3\cdot\sigma_1)\right] \leq \frac{\sigma_{per}^2}{\cancel{2E}}$$

$$\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\cdot\sigma_3)+\sigma_3\cdot\sigma_1)\right] \leq \sigma_{per}^2$$

* For plan Stain Condition! -

* PURE SHEAR

$\sigma_1 = \tau,\ \sigma_2 = \tau,\ \sigma_3 = 0$



$$\tau^2+\tau^2+2\mu\tau^2 \leq \sigma_{yt}^2$$

$$\boxed{2\tau^2(1+\mu) \leq \sigma_{yt}^2}$$

For steel

CH-6 : SLOPE AND DEFLECTION

* METHODS OF DETERMINING SLOPE AND DEFLECTION

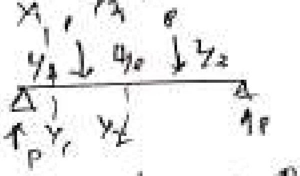
1) Double Integration Method.
2) Macaulay's Method.

1) DOUBLE INTEGRATION METHOD :-

i) It can be applied only for prismatic beam.

ii) It can not be used when bending eqn varies along the length of the beam. when load and or Concentrated moment acts on the span.

* Example

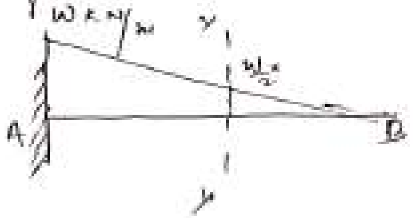


$M_{xx} = Px$

$M_{x_2x_2} = Px - P(x - l/2)$.

Numericals

Q.1 For a cantilever beam shown in figure, determine slope and deflection at free end.



l — w

$x \to ?$ $\frac{w}{L}$

$? = \frac{wx}{L}$

$\theta = \frac{dy}{dx}$

$\to y$

Soln

$$M_{xx} = -\left(\frac{1}{2} \times \frac{wx}{L} \times x\right) \times \frac{x}{3} = -\frac{wx^3}{6L}$$

$$EI \frac{d^2y}{dx^2} = \frac{-wx^2}{6L} \quad \text{——} ①$$

Integrate eqn 1 from both ends.

$$EI \frac{dy}{dx} = -\frac{w}{6L} \times \frac{x^4}{4} + C_1 \quad \text{——} ②$$

Integrating eqⁿ ② on both sides

$$EI\,y = \frac{-w}{24l} \cdot \frac{x^5}{5} + C_1 x + C_2 \quad \text{——} \quad ③$$

- We know

Boundaries Conditions are

at $x = L,\ y = 0$,

@ $x = L,\ \frac{dy}{dx} = 0$

Using boundries condition.

× ⓐ $x = L, \quad \frac{dy}{dx} = 0$

$$EI \cdot 0 = \frac{-wx^4}{24l} \times \frac{l^4}{4} + C_1$$

$$\boxed{C_1 = \frac{wl^3}{24}}$$

× ⓑ $x = l, \quad y = 0$

$$EI \cdot 0 = -\frac{w}{24l} \cdot \frac{l^5}{5} + \frac{wl^5}{24} \times L + C_2$$

$$C_2 = -\frac{wl^4}{24} + \frac{wl^4}{120}$$

$$= \frac{(-5+1)\,wl^4}{120}$$

$$\boxed{C_2 = -\frac{wl^4}{30}}$$

** eqⁿ-1 (Put eq Constant on eqⁿ)

eqⁿ-1 $EI \cdot \frac{dy}{dx} = \frac{-wx^3}{24l} + \frac{wl^3}{24}$

eqⁿ-2 $EI\,y = \frac{-wx^5}{120l} + \frac{wl^3}{24} \cdot x - \frac{wl^4}{30}$

$x=0$
Slope and deflection at 'B' (free end)

(i) $x=0$ Slope

$$EI \cdot \frac{dy}{dx} = \frac{wl^3}{24}$$

$$\frac{dy}{dx} = \boxed{\frac{wl^3}{24EI} = \theta_B}$$ (clock wise)

(ii) @ $x=0$, Deflection.

$$EI \cdot y = -\frac{wl^4}{30}$$

$$\boxed{y_B = \frac{-wl^4}{30EI}}$$ (downward)

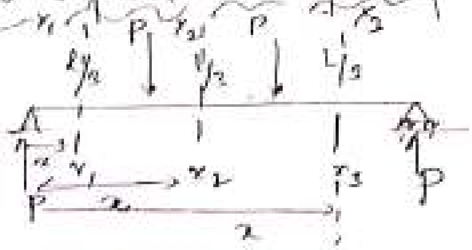
Method

2: MACAULAY's METHOD

i) The method can be used only for prismatic beam.

ii) The method is an improvement over double integration method as it can be used when bending eq^n along the length of the beam changes.

* How improve the method. Explanation with Example.



BM $M_{x_1 x_1} = Px$

$$M_{x_2 x_2} = P.x - P.(x - L/3)$$

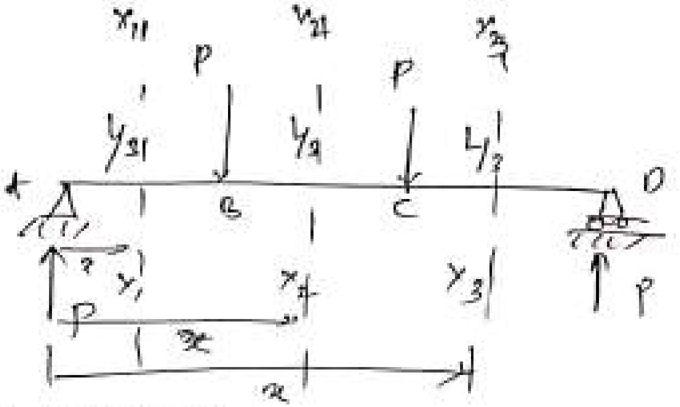
$$M_{x_3 x_3} = P.x - P(x - L/3) - P\left(x - \frac{2L}{3}\right)$$

POINTS TO USE MACAULAY'S METHOD

① Take the section just before the end of the beam and write bending eqn., Put the brackets of the each term to indicate whether the next term is valid or not. The term is invalid the ~~bracket value~~ bracket value is negative or zero.

Illustrations

①



By ① section x_1x_1 -

$M_{x_1x_1} = Px$

$M_{x_2x_2} = Px - P(x - L/3)$

$M_{x_3x_3} = Px - P(x - L/3) - P\left(x - \frac{2L}{3}\right)$

(point-1)

Step-1 (Section selection)

According to Macaulay method. (from ← Left - Right)

①st , ②nd , ③rd

$M_{x_3x_3} : Px \,/ - P(x - L/3) - / P\left(x - \frac{2L}{3}\right)$

- In AB region $(0 \le x \le L/3) \Rightarrow$ 1st is valid.
- BC region $\left(L/3 \le x \le \frac{2L}{3}\right) \rightarrow$ 1st and 2nd is valid.
- CD region $(2L/3 \le x \le L) \rightarrow$ all the terms are valid.

point-2

~~Step~~-2 : (Integration)

$M_{xx} = Px \,/ - P(x - L/3) \,/ - P(x - 2L/3)$

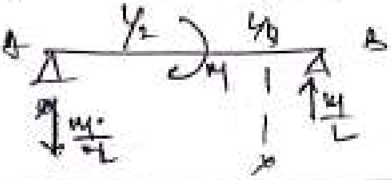
$EI . \frac{d^2y}{dx^2} = Px \,/ - P(x - L/3) \,/ - P(x - 2L/3)$

Integrate both side

$$\boxed{EI\cdot\frac{dy}{dx} = \frac{Px^2}{2} + C_1 \;/\; - P.\frac{(x-L/3)^2}{2} \;/\; - \frac{P\left(x-2L/3\right)^2}{2}}$$

point-3 ~~Step-2~~ : If concentrated Moment is present.

(Integration)



② Ac

$M_{x_1x_1} = \frac{-M}{L}\cdot x$

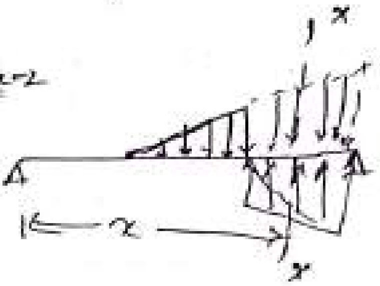
$M_{x_2x_2} = \frac{-M}{L}\cdot x + M$

$$\boxed{M_{xx} = \frac{-M}{L}\cdot x \;/\; + M\left(x-\frac{L}{2}\right)^0}$$

Point-4 : If UDL & UVL start and End within the span.

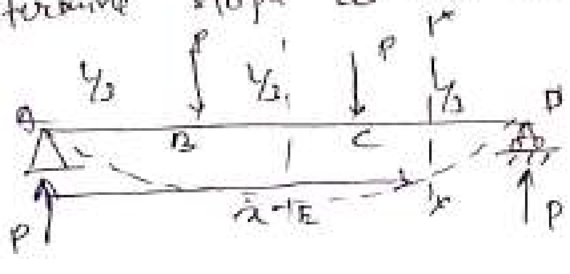
Case-1

Case-2



* Numericals !—

① for the simply supported beam shown in the figure, determine slope at the end and deflection at the centre?



Sol^n

$$M_{xx} = P.x \;/\; - P\left(x-\frac{L}{3}\right) \;/\; - P\left(x-\frac{2L}{3}\right) \quad — ①$$

$$\boxed{EI\cdot\frac{d^2y}{dx^2} = Px \;/\; - P\left(x-\frac{L}{3}\right) \;/\; - P\left(x-\frac{2L}{3}\right)} \quad — ①$$

* Intigrate One bathe side of eqⁿ ① for slope

$$EI \cdot \frac{dy}{dx} = \frac{Px^2}{2} + C_1 \;/\; - P \frac{(x-\frac{L}{3})^2}{2} \;/\; - P \frac{(x-\frac{2L}{3})^2}{2}$$

— ②

* Integrate eqⁿ ② on both side to get deflection

$$EI \cdot y = \frac{Px^3}{6} + C_1x + C_2 \;/\; - P.\frac{(x-\frac{L}{3})^3}{6} \;/\; - P.\frac{(x-\frac{2L}{3})^3}{6}$$

--- ③

* Boundrie Condition

i) $x=0,\ y=0$

ii) $x = L/2,\ \frac{dy}{dx} = 0$

iii) $x = L,\ y = 0$

* Using boundrie Condition

i) $x=0,\ y=0$

$$0 = 0 + 0 + C_2$$

$$\boxed{C_2 = 0}$$

ii) $x = L/2,\ \frac{dy}{dx} = 0$

$$0 = \frac{P(L/2)^2}{2} + C_1 - \frac{P.(\frac{L}{2}-\frac{L}{3})^2}{2}$$

$$C_1 = \frac{-P.L^2}{8} + \frac{PL^2}{72}$$

$$C_1 = \frac{(-9+1)\cdot PL^2}{72}$$

$$\boxed{C_1 = \frac{-PL^2}{9}}$$

Sa. eq^n of slope

$$EI \cdot \frac{d^2y}{dx^2} = \frac{Px^2}{2} - \frac{PL^2}{9} \Big/ - \frac{P(x - L/3)^2}{2} \Big/ - \frac{P(x - \frac{2L}{3})^2}{2}$$

Deflection

$$EI \cdot y = \frac{Px^3}{6} - \frac{PL^2}{9} \cdot x \Big/ - \frac{P(x - L/3)^3}{6} \Big/ - \frac{(P - \frac{2L}{3})^3}{6}$$

Slope @ 'A' (x = 0)

$$EI \cdot \frac{dy}{dx} = -\frac{PL^2}{9}$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{-PL^2}{9EI} = \theta_A} \left(\text{Anti Clockwise } \circlearrowleft\right)$$

Slope @ 'D' (x = L)

$$EI \cdot \frac{dy}{dx} = \frac{PL^2}{2} - \frac{PL^2}{9} - \frac{4PL^2}{18} - \frac{PL^2}{18}$$

$$EI \cdot \frac{dy}{dx} = \frac{(9 - 2 - 4 - 1)PL^2}{18}$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{PL^2}{9EI} = \theta_D} \left(\text{Clockwise } \circlearrowright\right)$$

Deflection @ "centre" (C) (x = L/2)

$$EI \cdot y = \frac{P}{6}\left(\frac{L}{2}\right)^3 - \frac{PL^2}{9}\left(\frac{L}{2}\right) - \frac{P\left(\frac{L}{2} - \frac{L}{3}\right)^3}{6}$$

$$EI \cdot y = \frac{PL^3}{48} - \frac{PL^3}{18} - \frac{PL^3}{6 \times 6}$$

$$EI \cdot y = \frac{(27 - 72 - 1)PL^3}{6 \times 8 \times 3 \times 9}$$

$$EI \cdot y = \frac{-\cancel{46}^{23}}{\underset{24}{\cancel{48}} \times 27} \cdot PL^3$$

$$\therefore \boxed{y = \frac{-23}{648} \cdot \frac{PL^3}{EI}} \left(\downarrow \text{downword}\right)$$

CH-12

Centre of Gravity & Moment of Inertia.

C.G & MI

CENTRE OF GRAVITY :—

→ The position through which the whole weight of the body will act, irrespective of its position, is known as Centre of Gravity or C.G.

Centroid :—

→ The plain figure have only area, but no mass. The Centre of area of such 2D figure is called as Centroid.

→ the method of finding the C.G. and Centroid will same.

Method to determine C.G :—

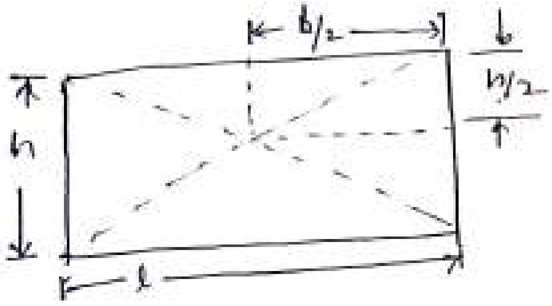
C.G or Centroid can be determined by :

1. Geometrical consideration.
2. Moment Method.
3. Graphical Method.

1. C.G. By Geometrical consideration :—

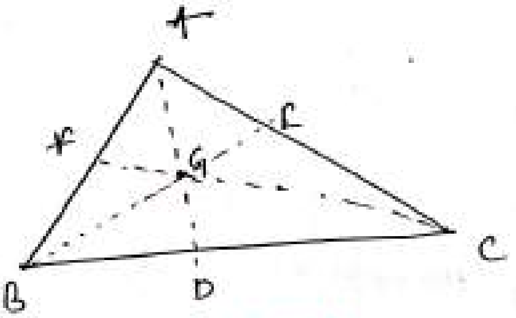
A. For 'Rectangle' :—

Point where diagonal meet each other.

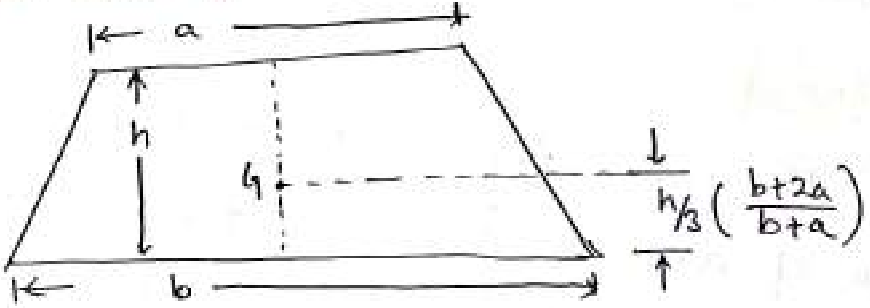


B. for triangular section:—

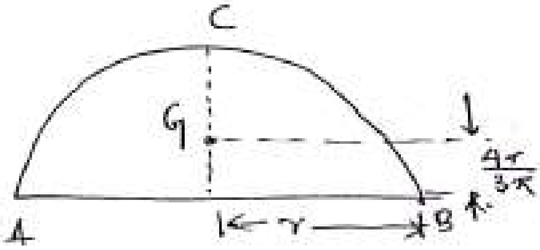
Point where 3 medians are Co-incide.



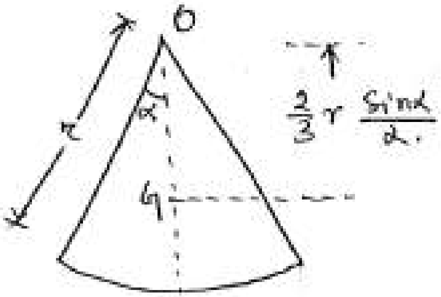
C. for trapizium with parallal side.



D. for semi-circle:—



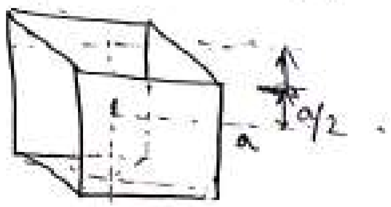
E. for Sector of circle:—



3D

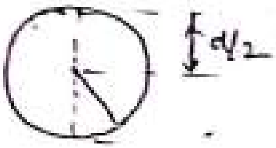
F. for Cube:—

1/2 from every face.



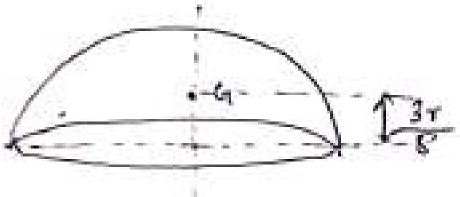
5) For Sphere

Distance $\frac{d}{2}$ from every point.



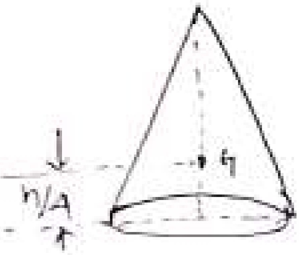
6. Hemisphere.

$\frac{3r}{8}$ from its Base.



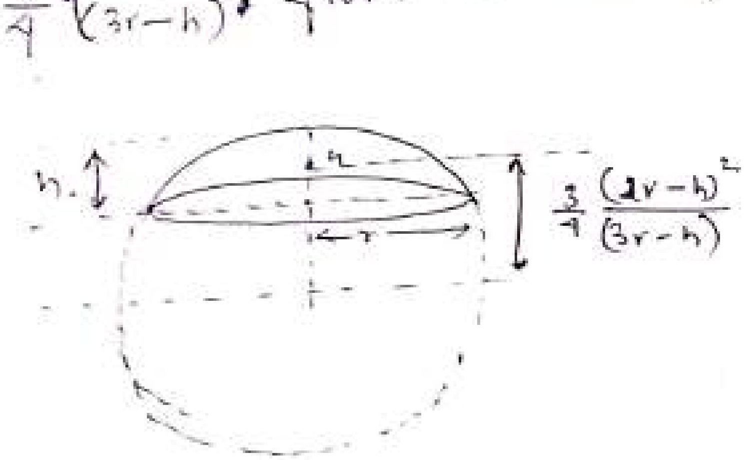
(7) For Right Circular Cone:—

$\frac{h}{4}$ from Base.

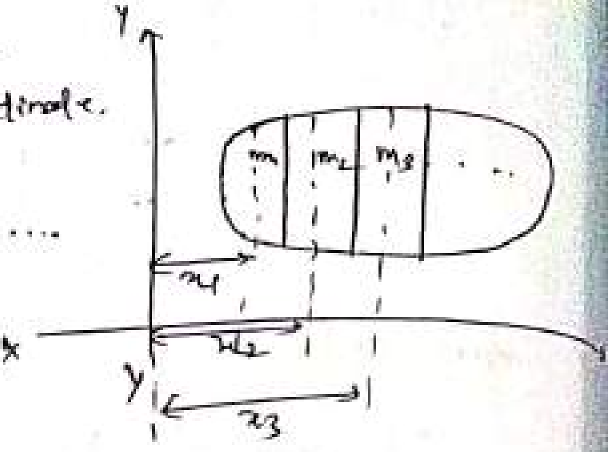


7. Segment of sphere of height (h).

$\frac{3}{4}\left\{\frac{(2r-h)^2}{(3r-h)}\right\}$ from the Centre of sphere.



2. C.G. By Moment :—



$m_1, m_2, m_3 ..$ → Mass of the system.
$(x_1, y_1), (x_2, y_2), (x_3, y_3)$ .... → co-ordinate of C.G.

~~[illegible]~~ .....

$M\bar{x} = m_1x_1 + m_2x_2 + m_3x_3$ .....

$$\boxed{\bar{x} = \frac{\Sigma m \cdot x}{M}}$$

$$\therefore \boxed{M = m_1 + m_2 + m_3 \; ....}$$

Similarly

$$\boxed{\bar{y} = \frac{\Sigma my}{M}}$$

Centre of Gravity of Plain figure :— (2D)

taking refference x-axis.

$$\boxed{\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 \cdots}{a_1 + a_2 + a_3 ..}}$$

taking refference Y-axis.

$$\boxed{\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 \cdots}{a_1 + a_2 + a_3 ..}}$$

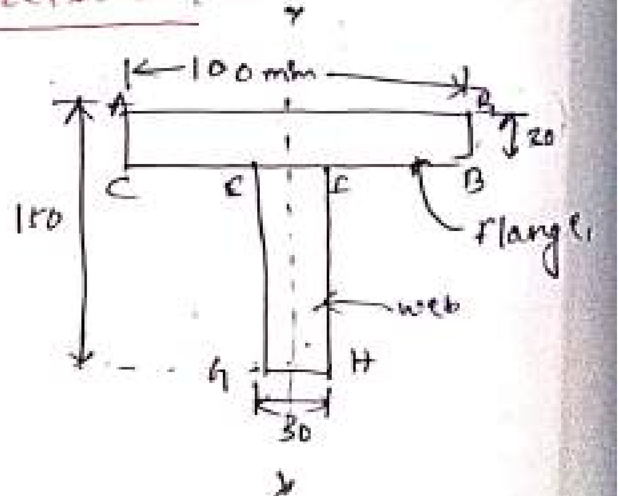
C.G. OF A UNSYMMETRICAL SECTIONS :—

①
Sol^n For Rec. ABCD

$a_1 = 100 \times 30 = 3000 \text{ mm}^2$

$\bar{y}_1 = \left(150 - \frac{30}{2}\right) = 150 - 15$
$= 135$ mm.

~~For R~~ $x_1 = \frac{100}{2} = 50$ mm.



For Rec APGH

$a_2 = (150-30) \times 30$

$= 120 \times 30$

$= 3600 \text{ mm}^2$

$x_2 = \frac{100}{2} = 50 \text{mm}$

$y_2 = \frac{120}{2} = 60 \text{mm.}$

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1+a_2} = \frac{3000 \times 50 + 50 \times 3600}{3000 + 3600}$$

$$= 50 \text{ mm.}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2} = \frac{3000 \times 135 + 60 \times 3600}{3000 + 3600}.$$

$$= 94.09 \simeq 94.1 \text{ mm..}$$

$$\boxed{\bar{x} = 50 \text{mm}, \ \bar{y} = 94.1 \text{ mm.}}$$ Ans

Centre of Gravity of Solid fig!— (3D)

$$\boxed{\bar{x} = \frac{v_1x_1 + v_2x_2 + v_3x_3 \cdots}{v_1+v_2+v_3 \cdots}}$$

$$\boxed{\bar{y} = \frac{v_1y_1 + v_2y_2 + v_3y_3 \cdots}{v_1+v_2+v_3 \cdots}}$$

$$\boxed{\bar{z} = \frac{v_1z_1 + v_2z_2 + v_3z_3 \cdots}{v_1+v_2+v_3 \cdots}}$$

C.G. of section with cut holes!—

$$\boxed{\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2}}$$

$$\boxed{\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}}$$

Assumption :-

* The ultimate strength in compression & tension are same for Brittle material and yield strength in compression and tension are same in Ductile material.

$\boxed{\sigma_{ut} = \sigma_{uc}}$ (Brittle material)

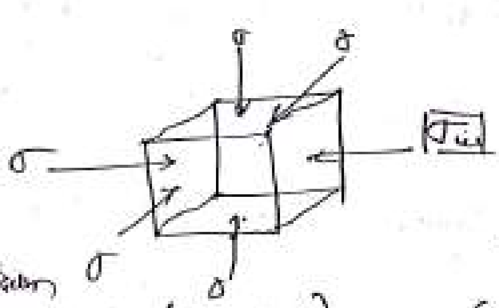
$\boxed{\sigma_{yt} = \sigma_{yc}}$ (Ductile material)

* Limitation / Demerit of Rankine's theory :-

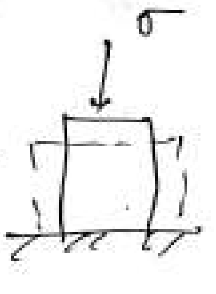
→ for hydrostatic loading condition the Rankine's theory will not valid.

because. $\boxed{\sigma_{uc} \neq \sigma_{ut}}$

(any ~~effor~~ of this will be maximum at the hydrostatic [illegible] loading)



(case-1)



(Cas-2)

Note:-

* Estimated ultimate compressive through simple compression test greater in (Case-1) as compare (Case-2)

2: Guest-Columb-THEORY (MAXIMUM SHEAR STRESS THEORY)

TRASCA'S THEORY

* Condition of failure

$\boxed{(\tau_{max})_{absolute} > \tau_{ys}}$ (Shear Strength) failure

↳ (Simple tension test)

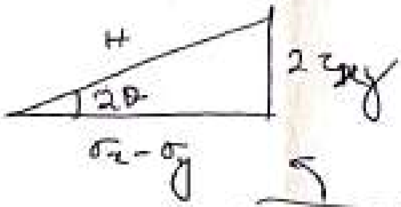
(Ductile ~~body~~ Material)

Prove that shear stress is zero on the principal plane !-

Shear at any plan at angle -θ.

$$\tau_{x'y'} = (\sigma_y - \sigma_x)\frac{\sin 2\theta}{2} + \tau_{xy}\cos 2\theta$$

$$\tau_{x'y'} = \left(\frac{\sigma_y - \sigma_x}{\cancel{2}}\right)\cdot\frac{\cancel{2}\tau_{xy}}{H} + \tau_{xy}\cdot\left(\frac{\sigma_x-\sigma_y}{H}\right)$$



from $\boxed{\tan 2\theta}$

$$\tau_{x'y'} = \frac{\tau_{xy}}{H}\left(\cancel{\sigma_y} - \cancel{\sigma_x} + \cancel{\sigma_x} + \cancel{\sigma_y}\right)$$

$$\boxed{\tau_{x'y'} = 0}$$ ... proved

formule for principal strese !-

$$\boxed{\sigma_{p_1}/\sigma_{p_2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

σ_{p_1} → + sign → Major principle stress / Maximum tensile stress / minimum compressive stress

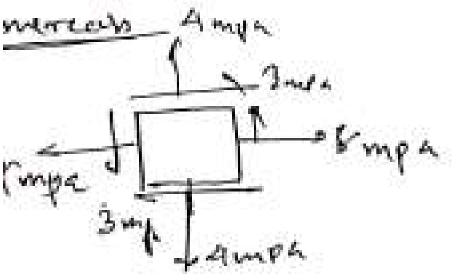
σ_{p_2} → - sign → Minor principal stress / Maximum compressive stress / min^m tensile stress.

(4, 8) → $\sigma_{p_1} = 8$, $\sigma_{p_2} = 4$

$\boxed{-4, 6}$ → $\sigma_{p_1} = 6$, $\sigma_{p_2} = -4$

$\boxed{-2, -4}$ → $\sigma_{p_1} = -2$, $\sigma_{p_2} = -4$

merecals



In this structural element in the fig det determine. i) principle stress
ii) principlane.

$$\sigma_{p_1}/\sigma_{p_2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

σ_x = 8mpa, σ_y = 4mpa, τ_{xy} = 3mpa.

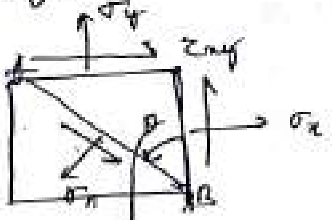
!-

- Principal plane are perpendicular to each other

Principal stresses are the maximum or minimum. value of normal stress that can act at a plane.

Location of principal plane !-

$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\,\tau_{xy} \sin\theta \cdot \cos\theta$



Location of plane AB

For σ_n to be maximum and minimum.

$$\boxed{\frac{d(\sigma_n)}{d\theta} = 0}$$

$$\because \boxed{2\sin\theta \cdot \cos\theta = \sin 2\theta}$$

$\sigma_n \Rightarrow \sigma_x [2 . \cos\theta (-\sin\theta)] + \sigma_y (2\sin\theta \cdot \cos\theta) + \tau_{xy} (2\cos 2\theta) = 0$

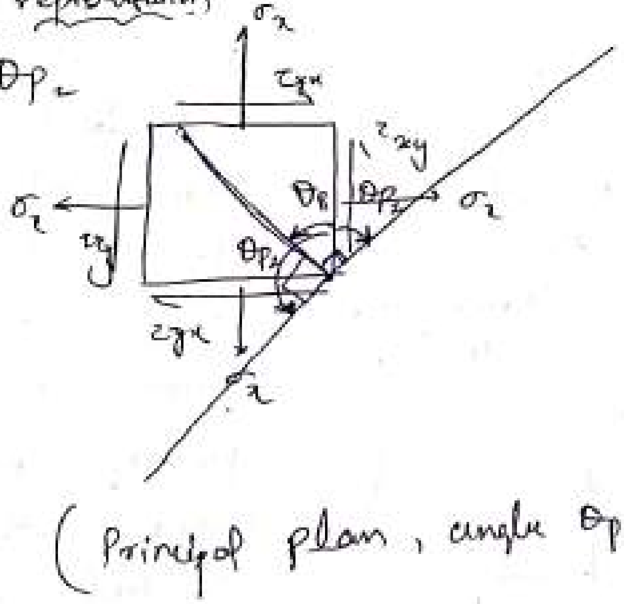
$\Rightarrow \quad 2\sin\theta \cdot \cos\theta (\sigma_y - \sigma_x) + \tau_{xy}\, 2 . \cos 2\theta = 0$

$\tau_{xy} \cdot 2 . \cos 2\theta = \sin 2\theta (\sigma_x - \sigma_y)$

$$*** \quad \boxed{\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}} \text{ — } \boxed{2\theta} \rightarrow \theta_{p_1}$$

$$** \quad \boxed{\theta_{p_2} = \theta_{p_1} \pm 90}$$

graphical representation θ_{p_1}, θ_{p_2}



(Principal plan , angle $\theta_{p_1}, \theta_{p_2}$)

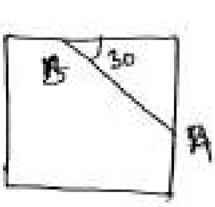
$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \cos\theta \cdot \sin\theta$

$120 = 50(\cos 60^\circ)^2 + 35.6(\cos 60)^2 + 2 . \cos 30 . \sin 30\ \tau_{xy}$

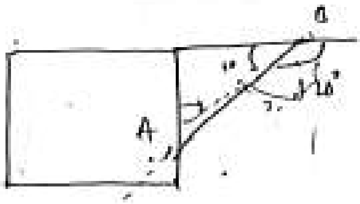
$120 = \overset{17.5}{\cancel{50}} \times \frac{1}{\cancel{4}} + \overset{8.9}{\cancel{35.6}} \times \frac{3}{\cancel{4}} + \cancel{2} . \frac{1}{\cancel{2}} . \frac{\sqrt{3}}{2} \times \tau_{xy}$

$\tau_{xy} = \frac{120 - 12.5 - 26.7}{\sqrt{3}} \times 2 = 93.29$ mpa.

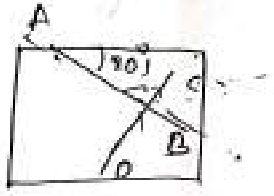
Determine θ θ = 0.18



$\theta_{AB} = 60^\circ$

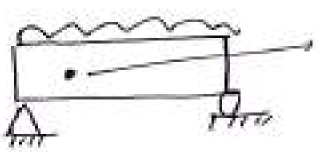


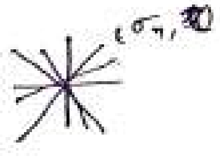
$\theta_{AB} = -30^\circ$
$= +150^\circ$



$\theta_{AB} = 70^\circ$
$\theta_{CD} = -20^\circ$

Principal Stress :-





3 plane : → 3D
2 Planes → 2D

$\tau_{xy}' = 0$

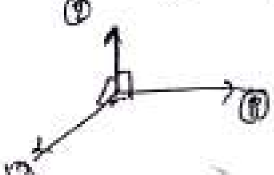
σ_n

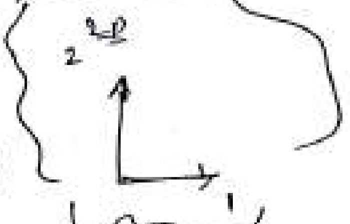
this plane are principle planes.

principle stress

Max^m or min^m

Mutually ⊥^r to each other



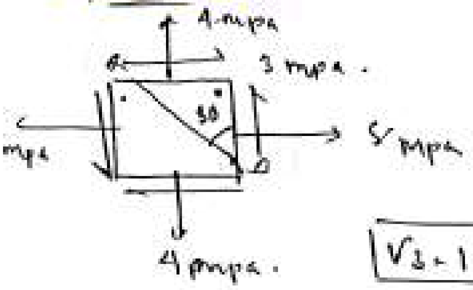


all analysis in 2D.

Defination :-

from A point infinite number plane can pass, Each plane has definate value of normal and shear stress. from the infinite number of plane their exist 3 plane in 3D and 2 plane in 2D at which shear stress is equall to zero. Such plane are referred as principal plane and the normal stress on this plane are reffered as principal stress.

Numerical



$\boxed{\sqrt{3} = 1.73}$

Determine normal & shear stress on inclined plan AB?

$\sigma_x = 5$ (mpa) $\sigma_y = 4$, $\tau_{xy} = \cancel{3}$, $\theta = 30°$

$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\cos\theta\cdot\sin\theta$

$= 5\times(\cos 30)^2 + 4\cdot(\sin 30°)^2 + 2\cdot 3\cdot\cos 30\cdot\sin\theta$

$= 5\left(\frac{\sqrt{3}}{2}\right)^2 + 4\left(\frac{1}{\cancel{2}}\right)^2 + \cancel{2}\cdot \cancel{3}\cdot \frac{\sqrt{3}}{\cancel{2}}\times\frac{1}{2}$

$\Rightarrow \overset{2.5}{\cancel{5}}\times\frac{3}{\cancel{2}} + \cancel{4}\frac{1}{\cancel{4}} +$

$\Rightarrow 7.5 + 1 + 1.5 \times 1.73$

$\Rightarrow 7$ 2.59 Mpa

$\tau_{x'y'} = (\sigma_y - \sigma_x)\frac{\sin 2\theta}{2} + \tau_{xy}\cos 2\theta$

$= (4-5)\frac{\sin 60}{2} + 3\cdot\cos 60$

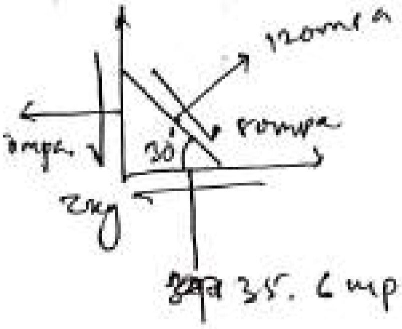
$\Rightarrow \overset{0.25}{-\cancel{1}}\times\frac{\sqrt{3}}{\cancel{4}} + \cancel{3}\times\frac{1}{\cancel{2}}$

$\Rightarrow -0.43 + 1.5$

$\Rightarrow 1.07$ mpa

$\boxed{\sigma_n = 1.75\text{mpa}\ \tau_{x'y'} = 1.07\text{ mpa.}}$

Gate-2021



Determine τ_{xy}?

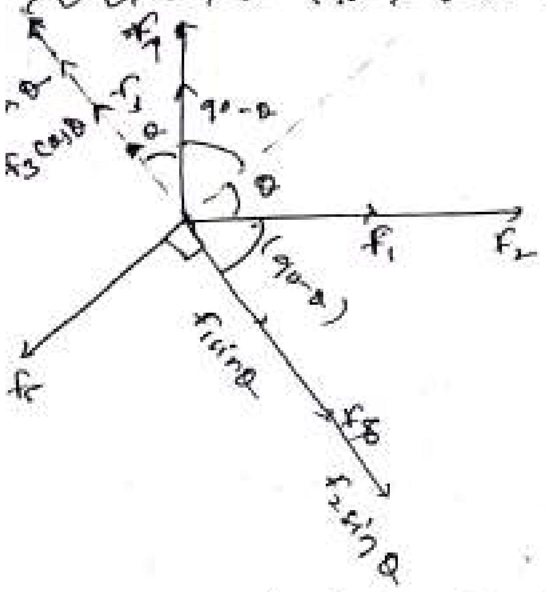
$\sigma_x = 50$ mpa

$\sigma_y = 35.6$ mpa.

$\sigma_n = 120$ mpa,

$\tau_{xy} = ?$

Determination of Shear Stress



Equilibrium :- taken in the direction of shear.

$$f_6 + f_1\sin\theta + f_2\sin\theta = f_3\cos\theta + f_4\cos\theta$$

$$f_6 = f_3\cos\theta + f_4\cos\theta - f_1\sin\theta - f_2\sin\theta$$

$$\tau_{x'y'}\cdot OB = \sigma_y\cdot AB\cos\theta + \tau_{xy}\cdot OA\cdot\cos\theta - \sigma_x\cdot OA\sin\theta - \tau_{yx}\cdot AB\cdot\sin\theta$$

$$\tau_{x'y'} = \sigma_y\sin\theta\cos\theta + \tau_{xy}\cos^2\theta - \sigma_x\cdot\cos\theta\cdot\sin\theta - \tau_{xy}\sin^2\theta$$

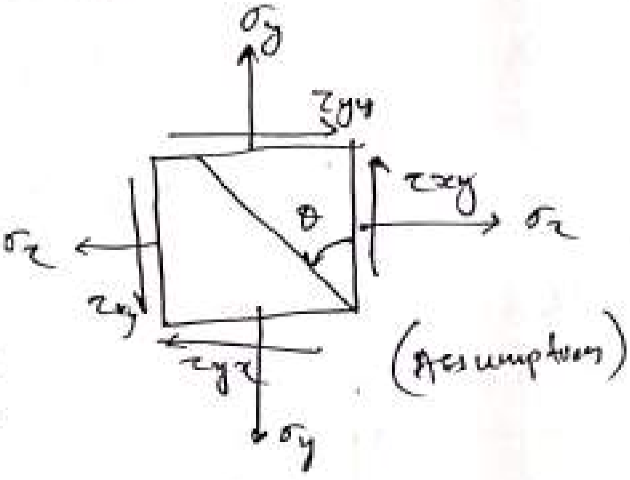
$$\tau_{x'y'} = (\sigma_y-\sigma_x)\sin\theta\cdot\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\boxed{\tau_{x'y'} = (\sigma_y-\sigma_x)\frac{\sin 2\theta}{2} + \tau_{xy}\cos 2\theta}$$

Shear stress

Sign Convention :-

Normal stress	Shear stress
	↑ ▣ ↓
Tensile → +ve	Anticlockwise → +ve
Compressive → -ve	Clockwise → -ve



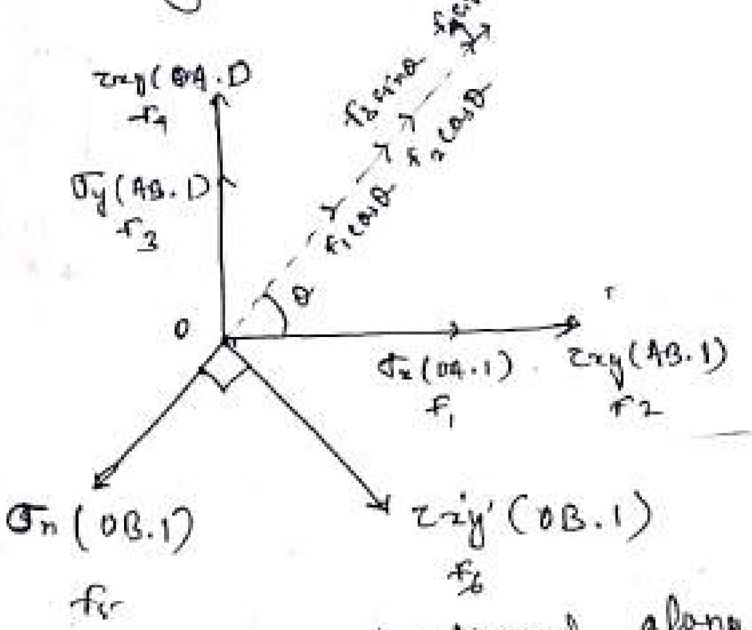
(Assumption)

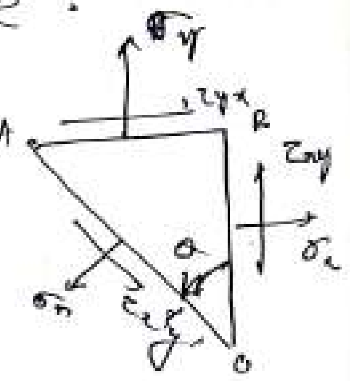
θ
w.r.t vertical

Anticlockwise → +ve

Clockwise → -ve

- Determining Normal Stress at plane "OB".





- Equilibrium Analysed along "σn".

$$F_5 = F_1\cos\theta + F_2\cos\theta + F_3\sin\theta + F_4\sin\theta$$

$$\Rightarrow \sigma_n(OB) = \sigma_x\, OA\cos\theta + \tau_{yx}\cdot AB\cdot\cos\theta + \sigma_y AB\cdot\sin\theta + \tau_{xy}\cdot OA\cdot\sin\theta$$

$$\Rightarrow \sigma_n = \sigma_x \frac{OA}{OB}\cdot\cos\theta + \tau_{yx}\frac{AB}{OB}\cdot\cos\theta + \sigma_y\cdot\frac{AB}{OB}\cdot\sin\theta + \tau_{xy}\cdot\frac{OA}{OB}\cdot\sin\theta$$

$$\Rightarrow \sigma_n = \sigma_x\cos^2\theta + \tau_{yx}\sin\theta\cos\theta + \sigma_y\sin^2\theta + \tau_{xy}\cos\theta\cdot\sin\theta$$

- We know from equality of shear stress.

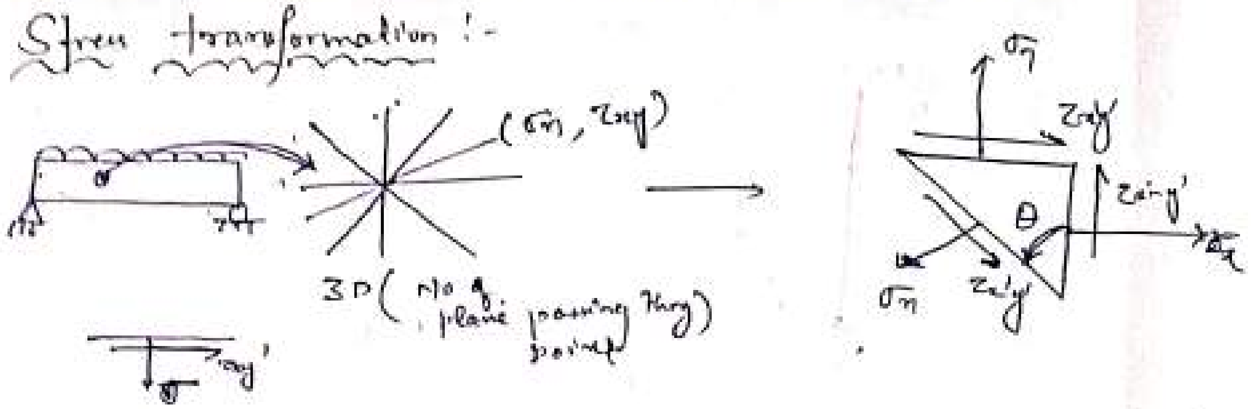
$$\boxed{\tau_{xy} = \tau_{yx}}$$

$$\boxed{\sigma_n = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\cos\theta\cdot\sin\theta}$$

normal stress.

(U-1 Unit-I) **Stress Transformation & Principal Stress/strain** (Lec-28)

Stress transformation :-



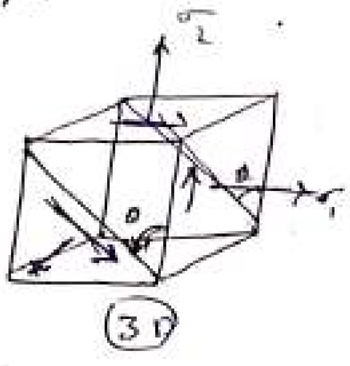
→ If normal and shear stress on mutually perpendicular planes are known, than normal and shear stress on any ~~point~~ plane inclined at a angle can be determined. This process is referred as stress transformation.

- It can be done in two Method.

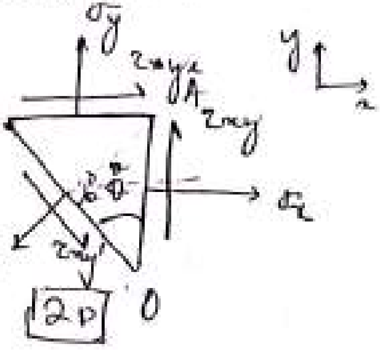
(i) Analytical Method

(ii) Graphical Method. (Mohr's Circle Method)

I) Analytical Method (2D analysis)



(3D)



(2D)

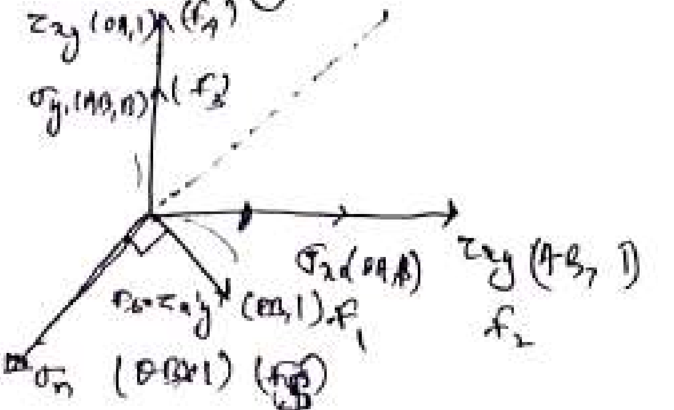
Assume unit thickness

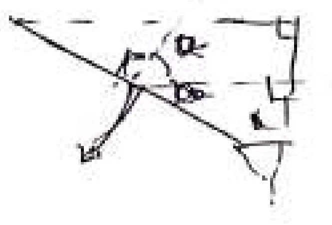
$\boxed{\sin\theta = \frac{AB}{OB}}$

$\boxed{\cos\theta = \frac{OA}{OB}}$

forces in x direction plane OA & OB ⇒ $\overrightarrow{\sigma_x (OA \times 1)}$, $\overrightarrow{\tau_{yx}(AB)}$

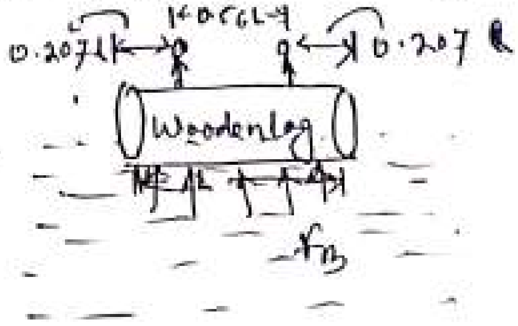
forces in y direction plane OA & OB → $\sigma_y (AB \times 1)\uparrow$, $\tau_{xy}(OA \times 1)\uparrow$



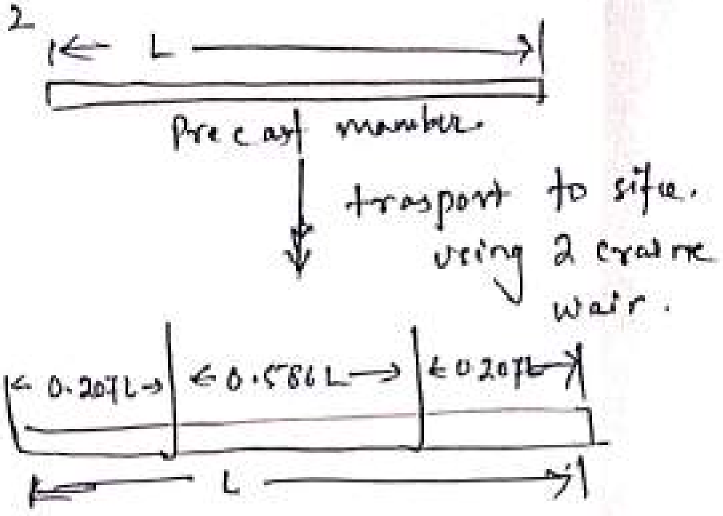


Continue →

Few Interview Questions



Q-2



—x—

Percent Overhang

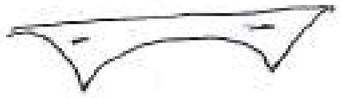
****

$$\boxed{\%\ \text{Overhang} = \frac{a}{L+2a} \times 100\%}$$

when $L = 2a$, $\boxed{\%\ \text{overhang} = 25\%}$ →

$L > 2a$, $\boxed{\%\ \text{Overhang} < 25\%}$ →

L ~~θ~~ $< 2a$, $\boxed{\%\ \text{Overhang} > x\%}$ →



Most Economical Beam

↓

for a given load ~~stress~~ stress is minimum.

↓

Maximum +ve ~~BM~~ BM = Maximum -ve Bending moment

$$\boxed{\text{Max +ve BM} = \text{Max -ve BM}}$$

in previous Case

$$\frac{\cancel{W}}{\cancel{8}_{4}}(L^2 - 4a^2) = \frac{\cancel{W}a^2}{\cancel{2}}$$

$$L^2 - 4a^2 = 4a^2$$

$$L^2 = 8a^2$$

$$\boxed{L = 2\sqrt{2}a}$$

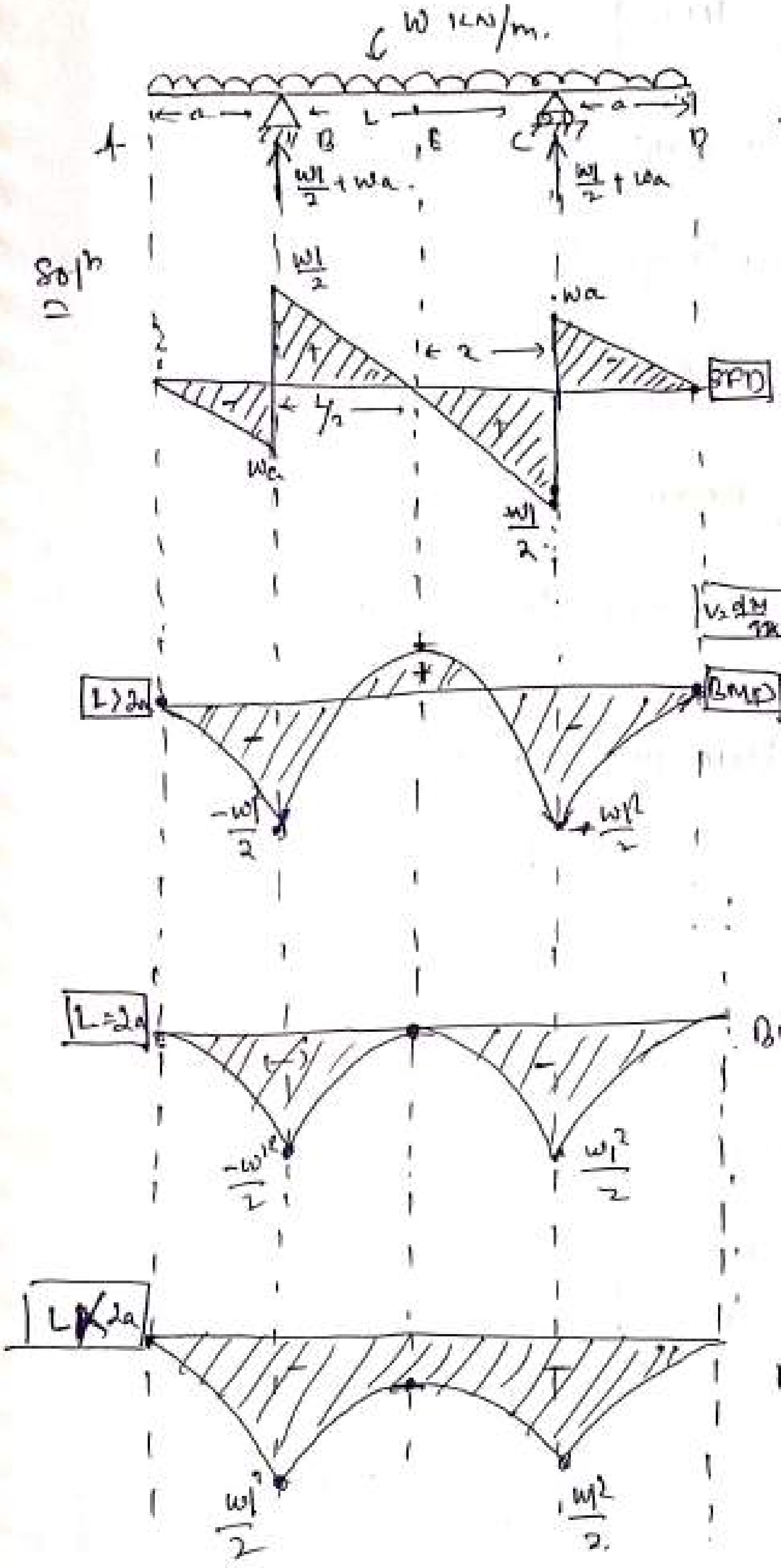
% Overhang :-

$$\frac{a}{L+2a} \times 100 = \frac{a}{(2\sqrt{2}a + 2a)} \times 100$$

$$= 20.71\%$$

R4 ≈ 20 percent

Q.2 For the double overhanging beam shown in the figure draw SFD & BMD, when (i) L > 2a
(ii) L = 2a
(iii) L < 2a



$R_A = R_D = \frac{w(L+2a)}{2}$

$= \frac{wl}{2} + wa$

$\boxed{L > 2a,\ L = 2a,\ L < 2a}$

For all the condition the SFD ~~dia~~ will be same.

Critical points.

A, B, C, D, E

$\boxed{M_A = M_D = 0}$

$BM_B = -w \times a \times \frac{a}{2}$

$\boxed{M_B = \frac{-wa^2}{2} = M_c}$

BM @ E = M_E

$M_E = \left(\frac{wl}{2} + wa\right) \times \frac{L}{2} - \left[w\left(a + \frac{L}{2}\right) \times \left(\frac{a + \frac{L}{2}}{2}\right)\right]$

$M_E = \frac{wl}{4}\left[(L+2a) - \frac{w}{2 \times 4}(L+2a)^2\right.$

$M_E = \left[\frac{w}{8}(L+2a)\right]\left[2L - (L+2a)\right]$

$M_E = \frac{w}{8}[L+2a][L-2a]$

$\boxed{M_E = \frac{w}{8}[L^2 - 4a^2]}$

× × × $M_2 - M_1$ = Area of shear force diagram. between ① & 2.

↳ L to R → SFD (+ve) → Area (+ve)

SFD (−ve) → Area (−ve)

from "B to A".

$M_B - \cancel{M_A}^{0} = \frac{1}{2}(10+15.5) \times 6^3$

$M_B = 15.5 \times 3 = 46.5$ KN-m.

$\boxed{M_B = 46.5 \text{ KN-m}}$

from "B to C".

$M_C - M_B = -\frac{1}{2}(1.5+9) \times 10^5$

$M_C - 46.5 = -52.5$

$M_C = -52.5 + 46.5$

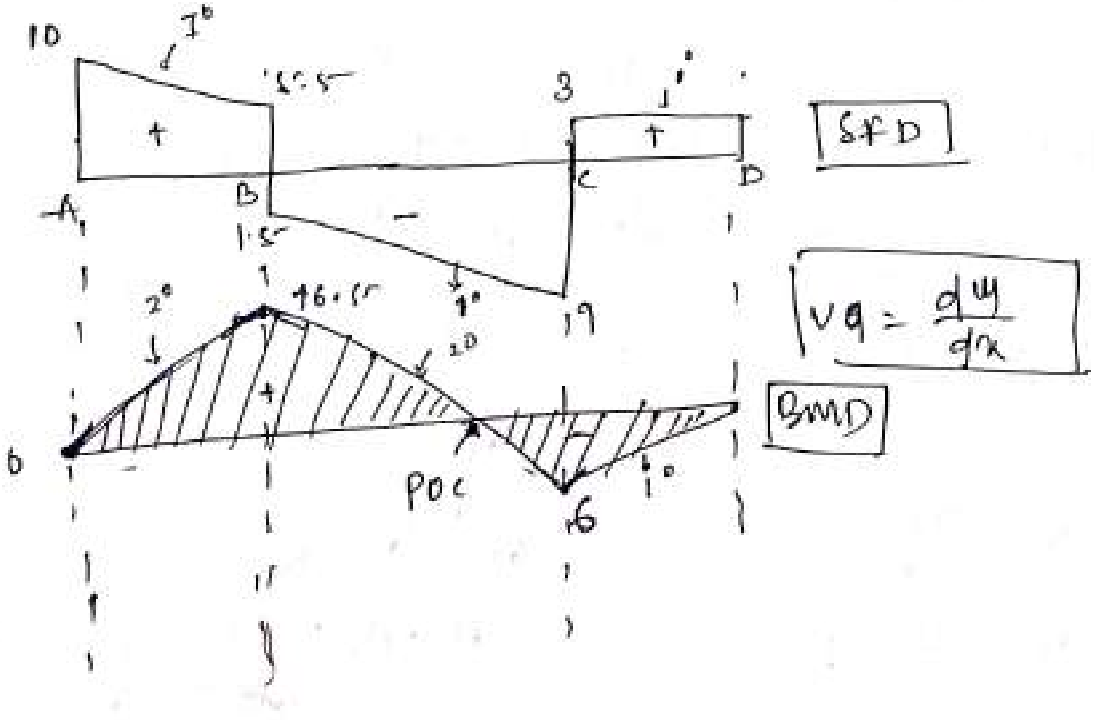
$\boxed{M_C = -6 \text{ KN-m.}}$

from "C to D"

$M_D - M_C = 3 \times 2.$

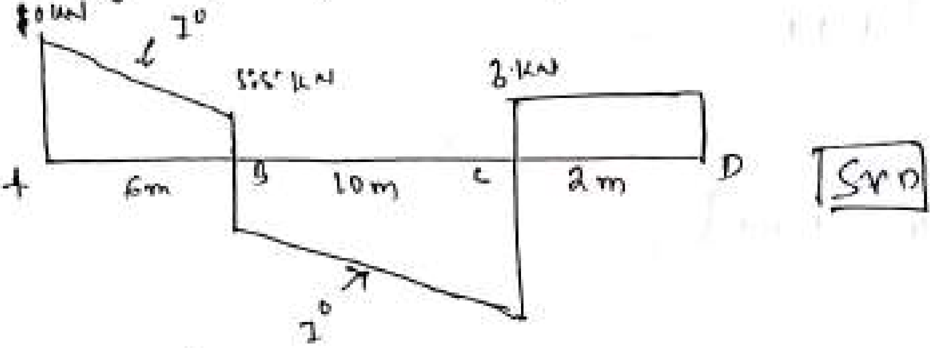
$M_D = 6 - M_C = 6 - 6 = 0$

$\boxed{M_D = 0}$

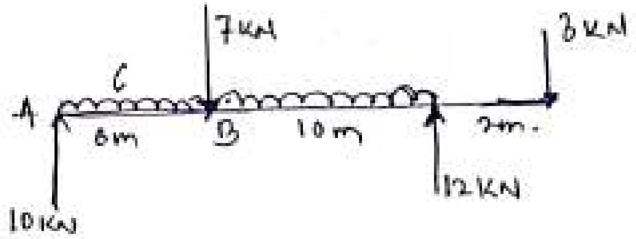


Lec-22 Application of SFD & BMD. (Conceptual & Important)

Q.1 for the SFD shown in the figure, draw the load intensity diagram and BMD?



Soln



$$w_1 = \frac{\text{Change in Load}}{\text{Length}} = \frac{10 - 5.5}{6} = 0.75 \text{ kN/m.}$$

$$w_2 = \frac{\text{Change in load}}{\text{Length}} = \frac{9 - 1.5}{10} = 0.75 \text{ kN/m.}$$

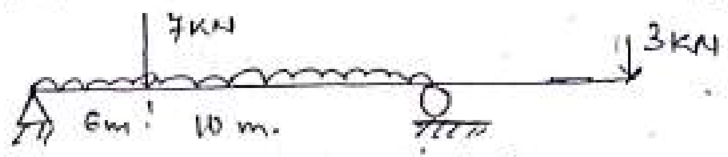
* Check for the End condition:-

BM ⓐ A Rnd to Left.

$$= 12\times16 - 3\times18 - (0.75\times16)\times\frac{16}{2} - 7\times6$$

$$= 192 - 54 - 96 - 42 = 0$$

* BM ⓐ A = 0 so the end is ~~Simply~~ (hinged or Roller) so the beam is simply supported.



[Final Load extent]

Note:-

$$V = \frac{dM}{dx}$$

$$\boxed{dM = V \cdot dx}$$

$$\int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} V \cdot dx$$

$M_2 - M_1$ = Area of shear force diagram.

Continue

BM cal^n

Critical Points :-

A, B, C, D, E, F

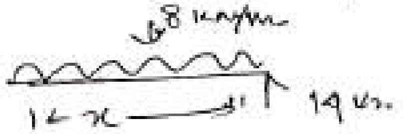
$\boxed{M_A = M_C = M_D = 0}$

BM @ B (Right - left)

$M_B = 14 \times 4 - 32 \times 2 = 56 - 64$

~~BM~~ $\boxed{M_B = -8 \text{ kN-m}}$

for F (Right - left.)



SF @ F = 0

$14 - 8x = 0$

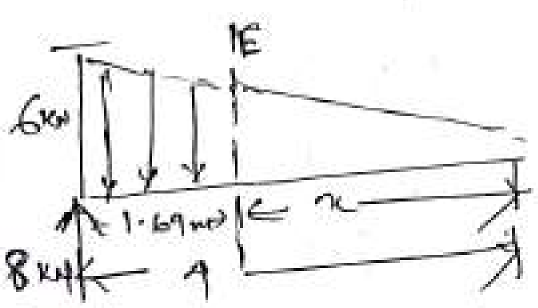
$x = \frac{14}{8} = \frac{7}{4} = 1.75 \text{ m.}$

BM @ F $= M_F = 14x - (8x)\frac{x}{2}$

$= 14(1.75) - \frac{\overset{4}{\cancel{8}}(1.75)^2}{\cancel{2}}$

$\boxed{M_F = 12.25 \text{ kN-m.}}$

For Point E ~~(~~(Left - Right)



$\frac{6}{4} = \frac{w}{x}$

$\boxed{w = 1.5x}$

SF @ F = 0.

$8 - \frac{1}{2}(6 + 1.5x)(4 - x) = 0$

$(6 + 1.5x)(4 - x) = 16$

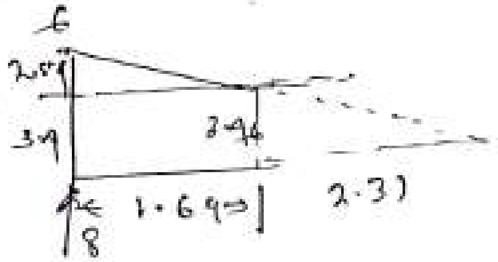
$24 - \cancel{6x} + \cancel{6x} - 1.5x^2 = 16$

$1.5x^2 = 24 - 16 = 8$

$x = \frac{8}{1.5} = \frac{4}{\sqrt{3}} = 2.31 \text{ m}$

AE = 4 - 2.31 = 1.69

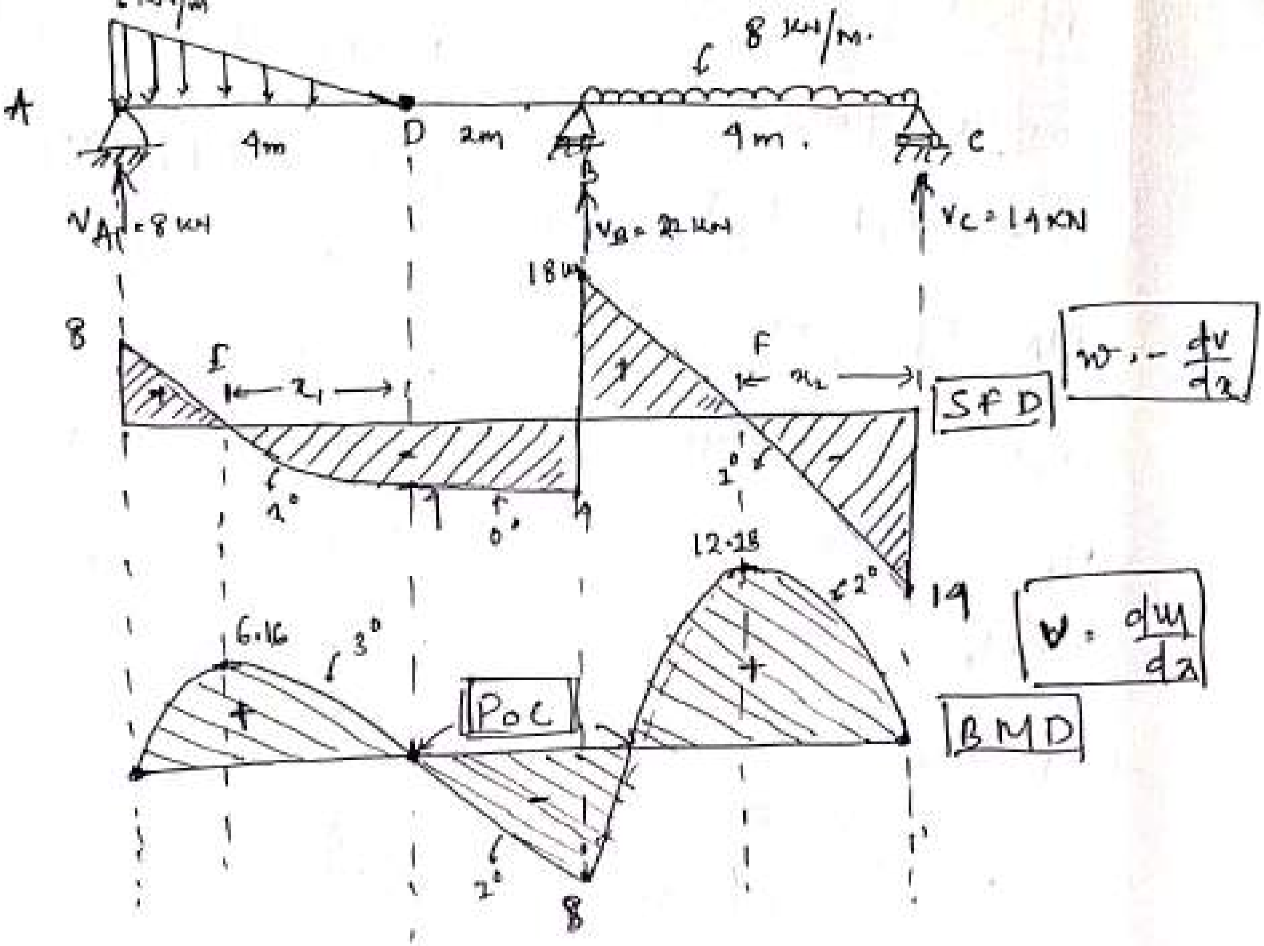
w = 1.69 × 1.5 = 3.46



$M_F = 8 \times 1.69 - (1.69 \times 3.4)\frac{1.69}{2} - \left(\frac{1}{2} \times 2.54 \times 1.69\right)\left(\frac{2 \times 1.69}{3}\right)$

$\boxed{M_F = 6.\cancel{8}16 \text{ kNm}}$

$\boxed{M_F = 6.16 \text{ kN-m}}$



$DS = r - s$

$= 4 - (3+1) = 0$ [determinate beam]

$F_x = 0$

$\boxed{H_A = 0}$

$\Sigma F_y = 0$ (↑+, ↓-)

$V_A - \frac{1}{2}(6 \times 4) + V_B - (8 \times 4) + V_C = 0$

$\boxed{V_A + V_B + V_C = 44 \text{ kN}}$ —— ①

BM @ D = 0 (Left - Right)

$V_A \times 4 - \cancel{12}^{4}\left(\frac{2}{3} \times 4\right) = 0$

$\boxed{V_A = 8}$

BM @ D = 0 (Right - Left)

$2V_B - 8 \times 4 \times \left(\frac{4}{2} + 2\right) - 6V_C = 0$

$2\ 2V_B - 64 \times 2 + 6V_C = 0$

$\boxed{V_B + 3V_C = 64}$ —— ②

$V_B + V_C = 44 - 8$

$\boxed{V_B + V_C = 32}$ —— ④

eqⁿ ② - eqⁿ ④

$2V_C = 64 - 36 = 28$

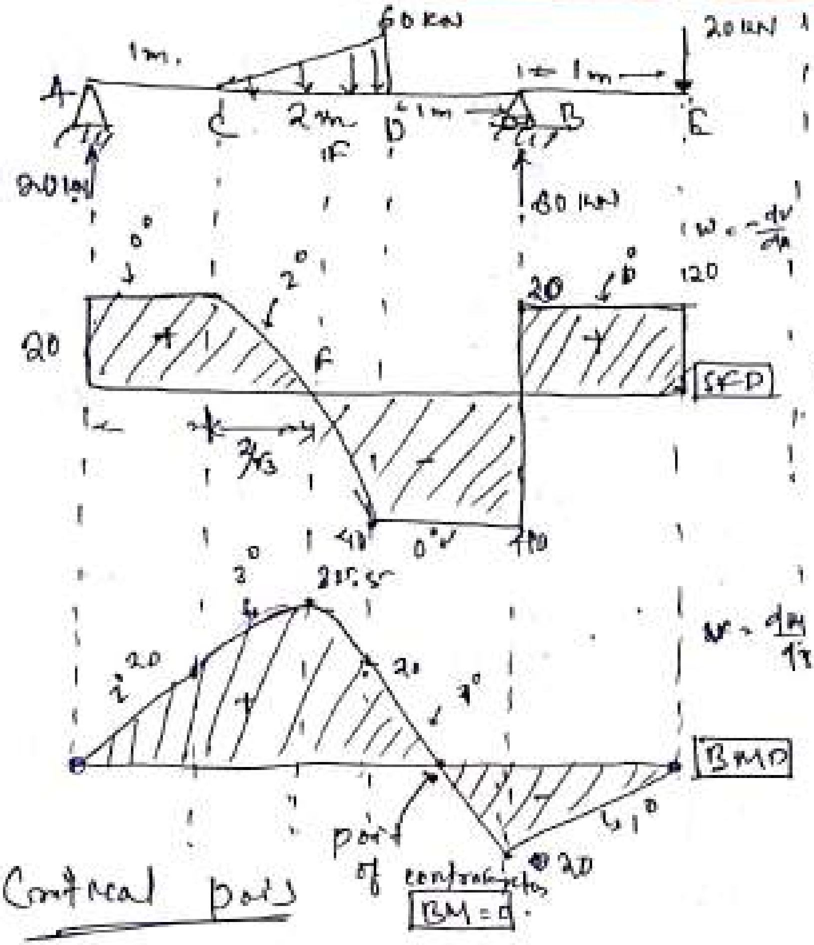
$\boxed{V_C = 14 \text{ KM}}$

$V_B = 32 - 14 \Rightarrow \boxed{V_B = 22}$

Continue

⑤

Q.



Bmc^n

Support reaction

$\Sigma F_y = 0$

$V_A + V_B = 60 + 20$

$\boxed{V_A + V_B = 80 \text{ kN}}$

$\Sigma M_B = 0$ (↻ + ↺ −)

$V_A \times 4 - 60 \times \left(\frac{2}{3} + 1\right) + 20 \times 1 = 0$

$4V_A - \cancel{60}^{20} \times \frac{5}{\cancel{3}} + 20 = 0$

$4V_A - 80 = 0$

$V_A = \frac{80}{4} = 20 \text{ kN}$

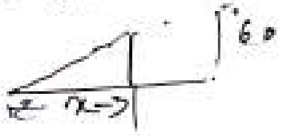
$V_B = 80 - V_A$

$= 80 - 20 = 60 \text{ kN}$

Critical points

A, B, C, D, E, F

Cal^n distance from C to F



$\frac{60}{2} \cancel{x} = \frac{6\cancel{3}w}{x}$

$\boxed{w = 30x}$

SF @ F = 0

$20 - \frac{1}{\cancel{2}} \times \cancel{30}^{15} x \times x = 0$

$15x^2 = 20$

$x^2 = \frac{20}{15} = \frac{4}{3}$

$\boxed{x = \frac{2}{\sqrt{3}}}$

BM cal^n

$M_A = M_E = 0$

Right - left

$M_B = -20 \times 1 = -20 \text{ kN m.}$

$M_D = -20 \times 2 + 60 \times 1$

$= -40 + 60 = 20 \text{ kN m.}$

left to right

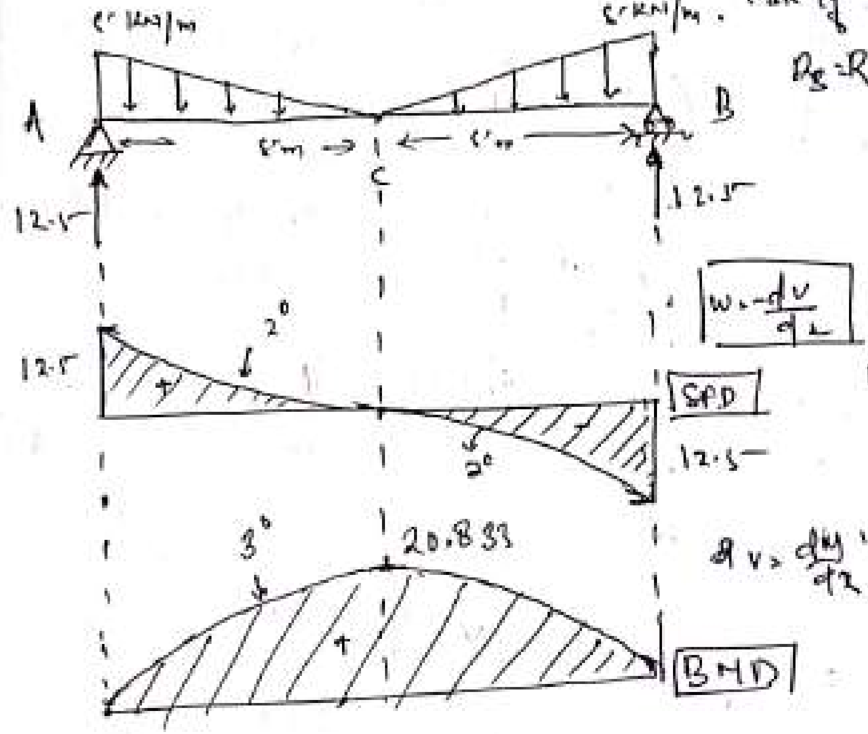
$M_C = 20 \times 1 = 20 \text{ kN m.}$

$M_f = 20(1+x) - \frac{1}{\cancel{2}} \times (x \cdot \cancel{30}^{5} x) \times \frac{x}{\cancel{3}}$

$= 20\left(1 + \frac{2}{\sqrt{3}}\right) - 5\left(\frac{2}{\sqrt{3}}\right)^3$

$= 20 \times 2.15 - 5 \times 1.5$

$= 35.5 \text{ kN m.}$



Case of symmetry

$R_B = R_A = \frac{1}{2} \times 5 \times 5 \times \cancel{\frac{3}{3}}$

$= \frac{25}{2} = 12.5$

<u>Critical points</u>

A, B, C.

$M_A = M_B = 0$

BM @ C $= (12.5 \times 5) - \left(12.5 \times (5) \times \frac{2}{3}\right)$

$= 12.5 \left(\frac{1}{3}\right) = 20.833$ kNm

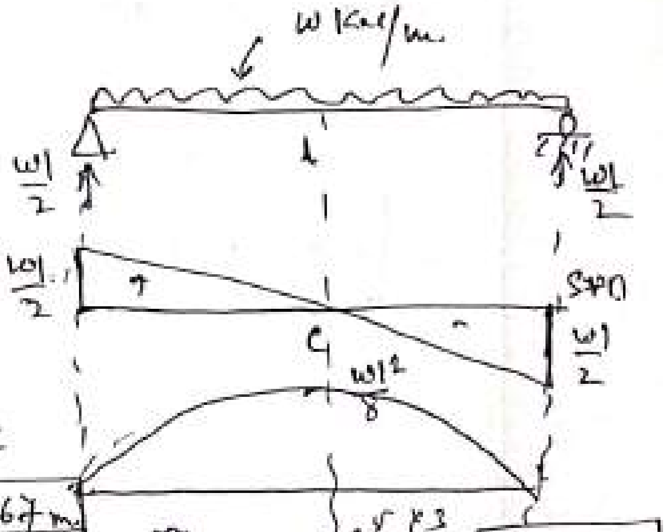
In the above problem, it proposed to replace the beam with another simply supported beam subjected to a UDL through out the span. such that design bending moment and designed shear force for both the beam are same. Determine the length and Load intensity of newly proposed beam..

Design shear force :- Maximum Magnitude of shear force in SF Diagram.

Design ~~of~~ Bending moment :- Maxᵐ Magnitude of bending moment in BM Diagram.

Design SF = 12.5 ~~kN/m~~ kN.

Design BM = $\frac{125}{6}$ kN~~/m~~-m



① $\frac{wl}{2} = 12.5$

$\boxed{wl = 25}$

$\boxed{w = \frac{25}{L}}$ —①

② $\frac{wl^2}{8} ~~x~~ = \frac{125}{6}$

$\frac{\frac{25}{\cancel{L}} \times l^2}{8} ~~x~~ = \frac{125}{6}$

$L = \frac{\overset{5}{\cancel{125}} \times 8}{\cancel{6} \times 25}$

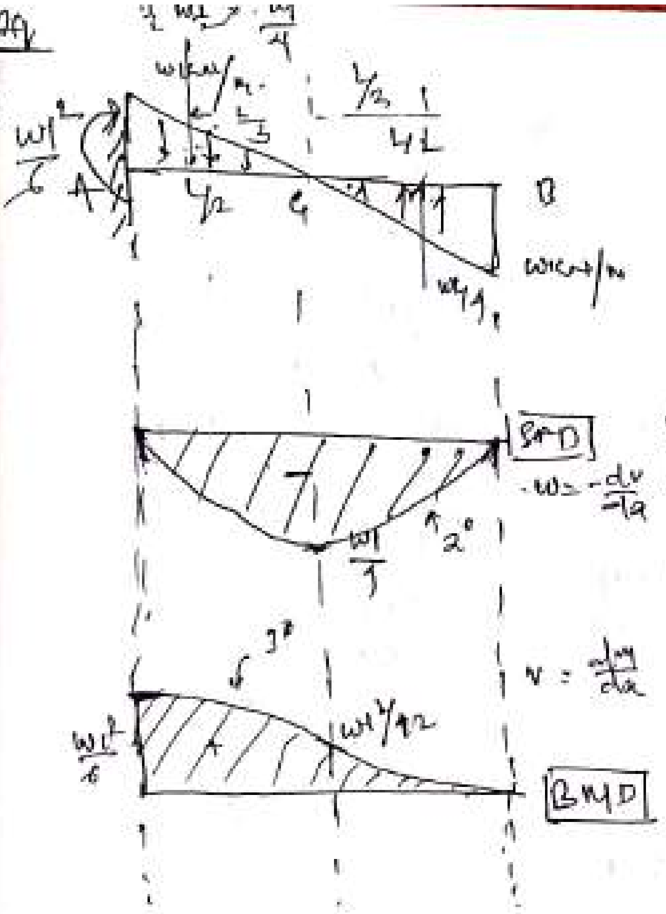
$= \frac{20}{3} = \boxed{6.67\,m}$

Ans

(ii) $w = \frac{\overset{5}{\cancel{25}} \times 3}{\cancel{40}\, 8} \times 8 = \boxed{3.75\text{ KN}}$ — (4w)

Level-29

①



Reaction

$M_A = \frac{wL}{\cancel{4}} \times \frac{\cancel{2}L}{3} = \frac{wl^2}{6}$

$R_A = +\frac{wl}{4} - \frac{wl}{4} = 0$

SF cal^n

Critical points.

A, B, C.

SF @ A = 0

SF @ B = 0

SF @ C = $\frac{wl}{4}$

BM cal^n

Critical points

A, B, C.

$M_A = \frac{wl^2}{6}$ | $M_C = \frac{wl^2}{6} - \frac{wl}{4} \times \frac{l}{3}$

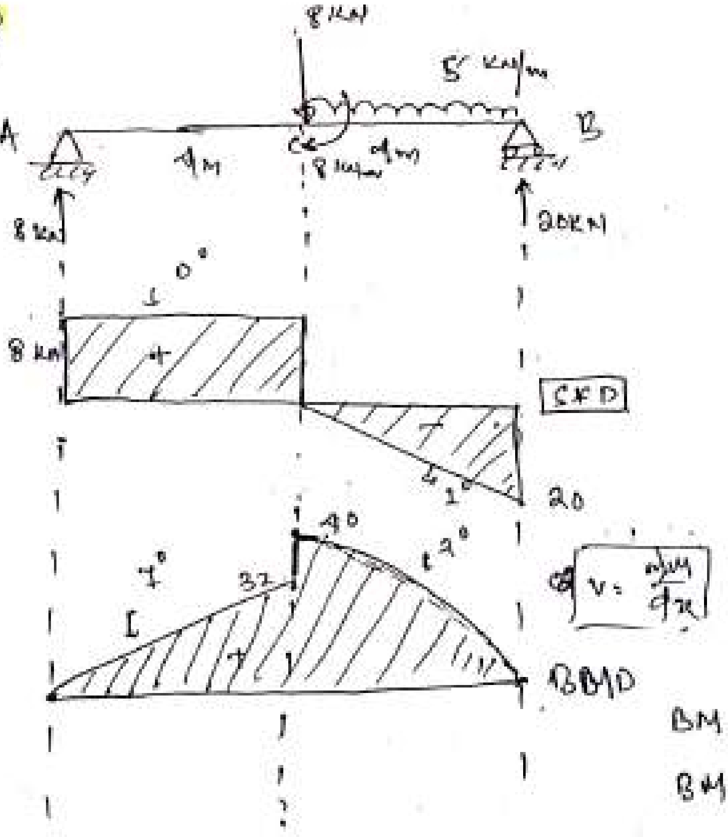
$= \frac{wl^2}{6} - \frac{wl^2}{12}$

$M_B = 0$

$= \frac{wl^2}{12}$

Lecture-20

q②



$R_A = \frac{8}{2} - \frac{8}{8} + \frac{20 \times 2}{8}$

$= 4 - 1 + 5 = 8$ kN

$R_B = \frac{8}{2} + \frac{8}{8} + \frac{20 \times 6}{8}$

$= 4 + 1 + 15 = 20$ kN

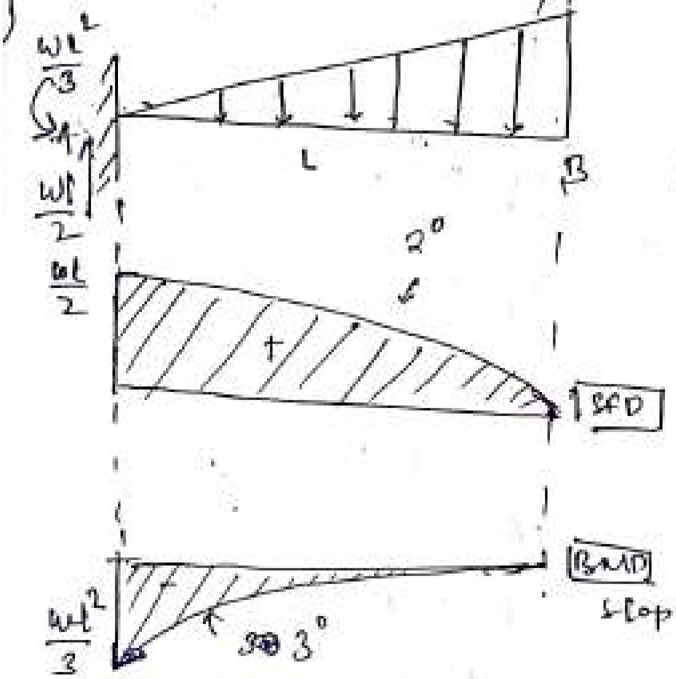
critical points

A, B, C.

$M_A = M_B = 0$

BM @ C^- = 8×4 = 32 kN-m

BM @ C^+ = 32+8 = 40 kN



$w = -\frac{dv}{dx}$

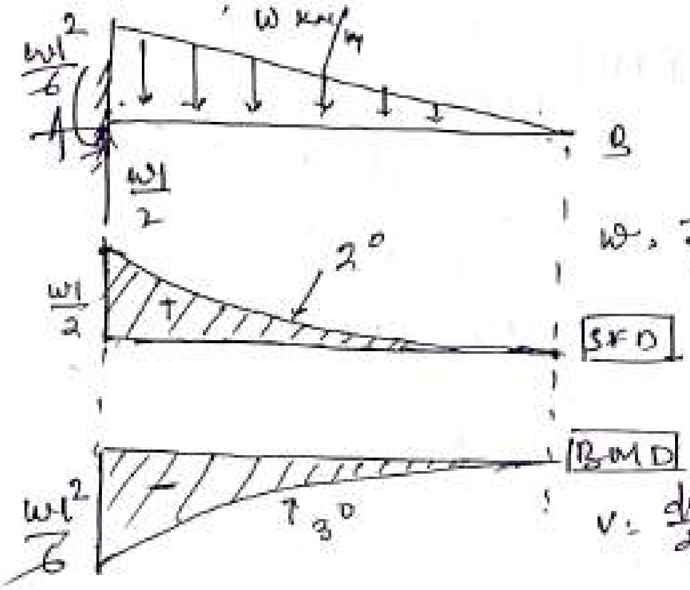
Critical parts

A, B

$M_A = -\frac{wl^2}{3}$

$M_B = 0$

slope = $V = \frac{dM}{dx}$



$w = \frac{-dv}{dx}$

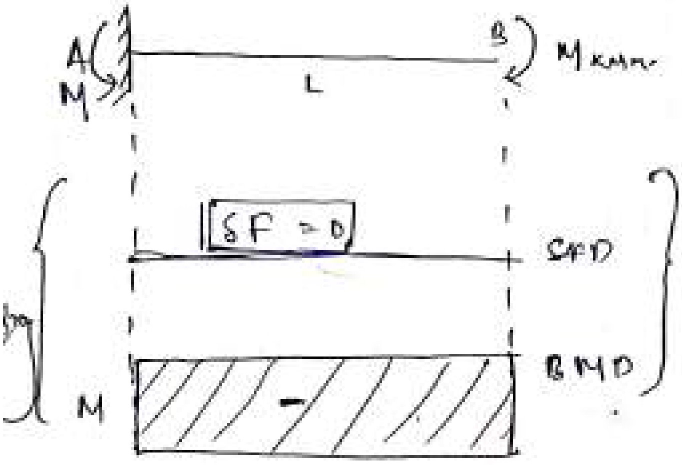
$V = \frac{dM}{dx}$

Critical parts

A, B

$M_A = -\frac{wl^2}{6}$

$M_B = 0$



Pure Bending

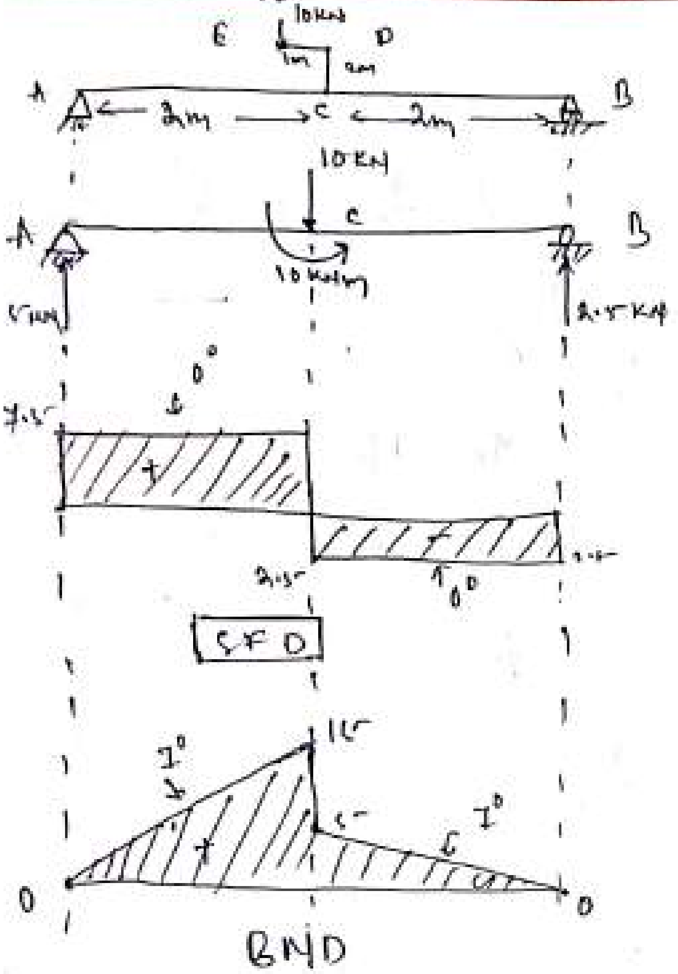
Critical part

A, B

$M_A = -M$

$M_B = -M$

Note:-
The case of SFD & BMD where the shear force will be zero and Bending moment will be constant is referred as Pure Bending



$H_A = 0$,

$R_A = \frac{10}{2} + \frac{10}{4} = 5 + 2.5$
$= 7.5$ kN

$R_B = \frac{10}{2} - \frac{10}{4} = 5 - 2.5$
$= 2.5$ kN

BM calⁿ

Critical points

A, B & C.

$M_A = 0$

$M_B = 0$

• BM @ C (not define)

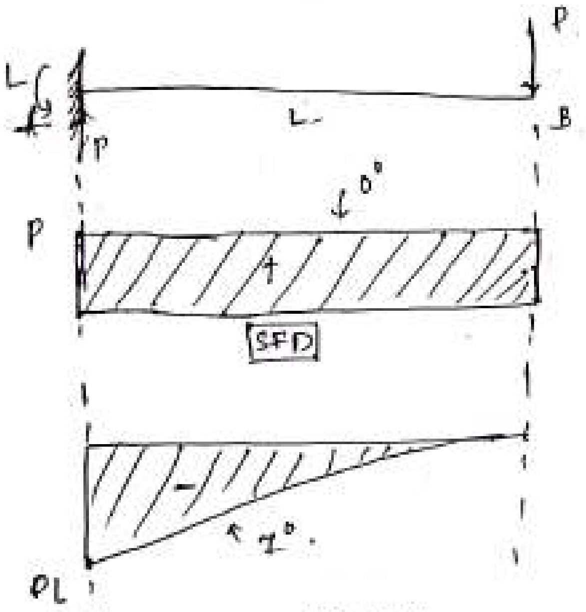
~~M_c~~

BM @ just left of C

$M_{c^-} = 7.5 \times 2 = 15$ kN-m

BM @ just Right of C.

$M_{c^+} = 7.5 \times 2 - 10 = 15 - 10$
$= 5$ kN-m

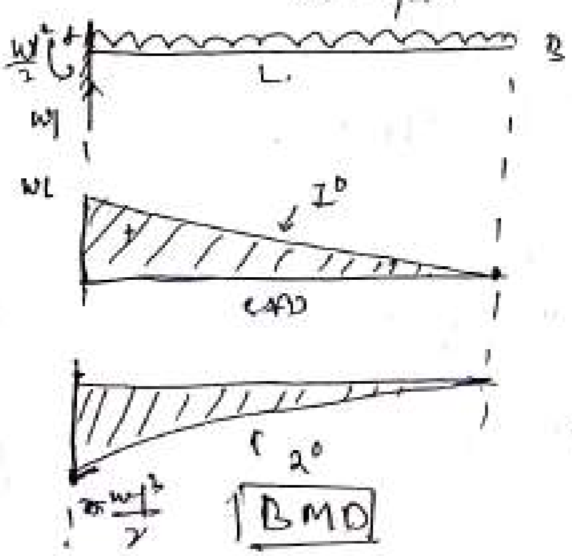


Critical points

A, B

$BM_A = -PL$

$BM_B = -PL + PL = 0$



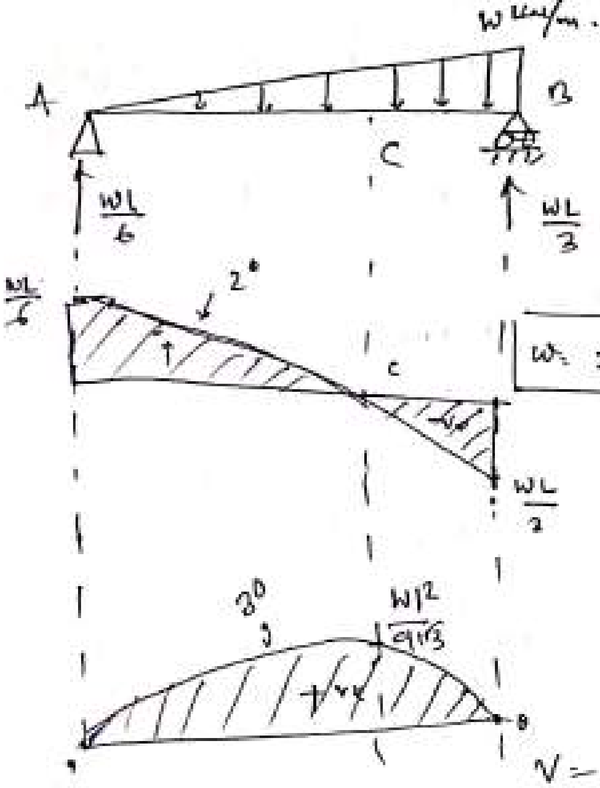
Critical point

A, B

$BM_A = -\frac{wl^2}{2}$.

$BM_B = 0$

$N = \frac{dy}{dx}$



$\boxed{w = \frac{-dv}{dx}}$

$V = \frac{dM}{dx}$

$\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

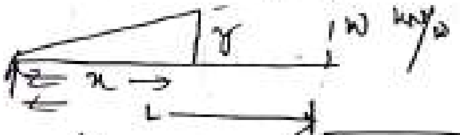
$+\frac{wl}{6} - \frac{wl}{2} = -\frac{wl}{3}$

BM

A, B, C

$M_A = M_B = 0$

from x distance from 'C'



$\frac{w}{L} = \frac{y}{x} \Rightarrow \boxed{y = \frac{wx}{L}}$

Shear force at C : 0

$\frac{wl}{6} - \left(\frac{1}{2} \times \frac{wx}{L} \times x\right) = 0$

$\frac{wl}{\cancel{6}\,3} = \frac{wx^2}{\cancel{2}l}$ $\boxed{x = \frac{L}{\sqrt{3}}}$

BM @ C

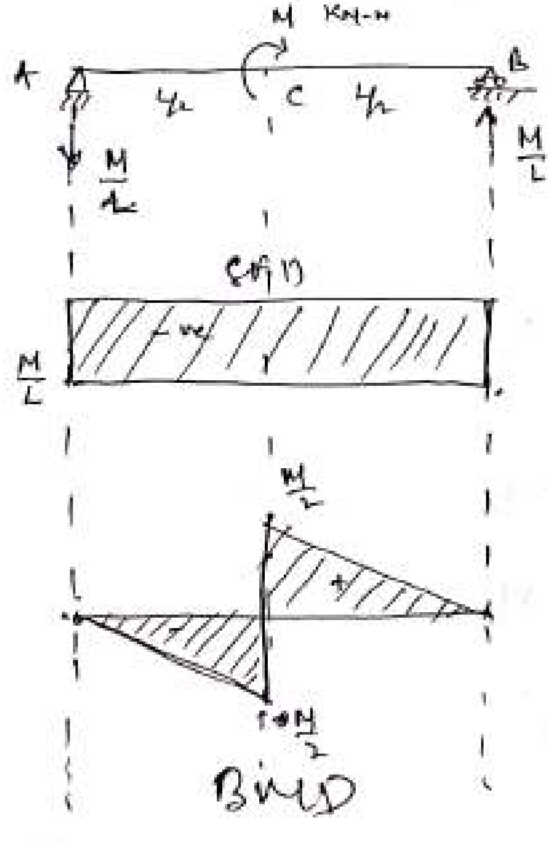
$\Rightarrow \frac{wL}{6} \times x - \left(\frac{1}{2} \times \frac{wx}{l} \cdot x\right) \times \frac{x}{3}$

$\Rightarrow \frac{w \cdot L}{6} \times \frac{L}{\sqrt{3}} - \frac{w}{6l} \times \left(\frac{l}{\sqrt{3}}\right)^3$

$= \frac{wl^2}{6\sqrt{3}} - \frac{wl^2}{18\sqrt{3}}$

$= \frac{wl^2(3-1)^{\cancel{2}}}{\cancel{18}_{9}\sqrt{3}} = \frac{wl^2}{9\sqrt{3}}$

$\boxed{M_C = \frac{wl^2}{9\sqrt{3}}}$



BM cal^n

Critical points

A, B, C

$\boxed{M_A = M_B = 0}$

for 'C'

ME.

Just left @ C^- = $M_{C^-} = -\frac{M}{L} \times \frac{L}{2}$

$= -\frac{M}{2}$

Just right of 'C' = $M_{C^+} = -\frac{M}{L} \times \frac{L}{2} + M$

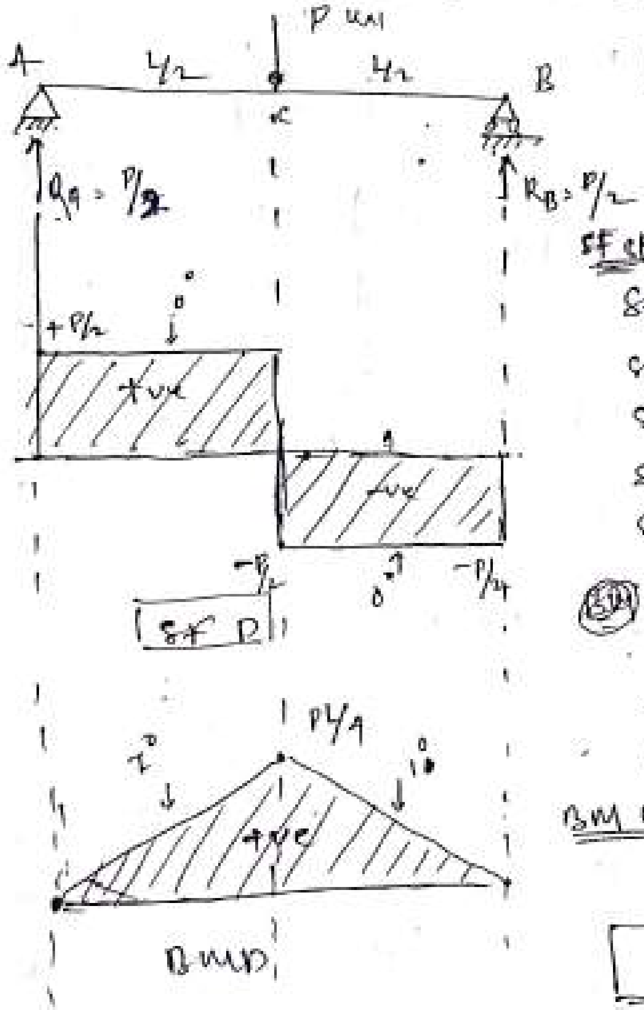
$= \frac{M}{2}$

Lect-18

Calculation of SFD & BMD $\boxed{\text{Q Level-1}}$ Q

①

Simply supported



(In SFD, critical point are checked only when load intensity changes it sign.)

SF eqn

SFD. A $= +\frac{P}{2}$

SF @ C = not defined

SF @ $C^- = +P/2$

SF @ $C^+ = -P/2$

SF @ B $= -P/2$

(SM) SFD → of any part M abscissa (y-axis) is indicates magnitude of SF.

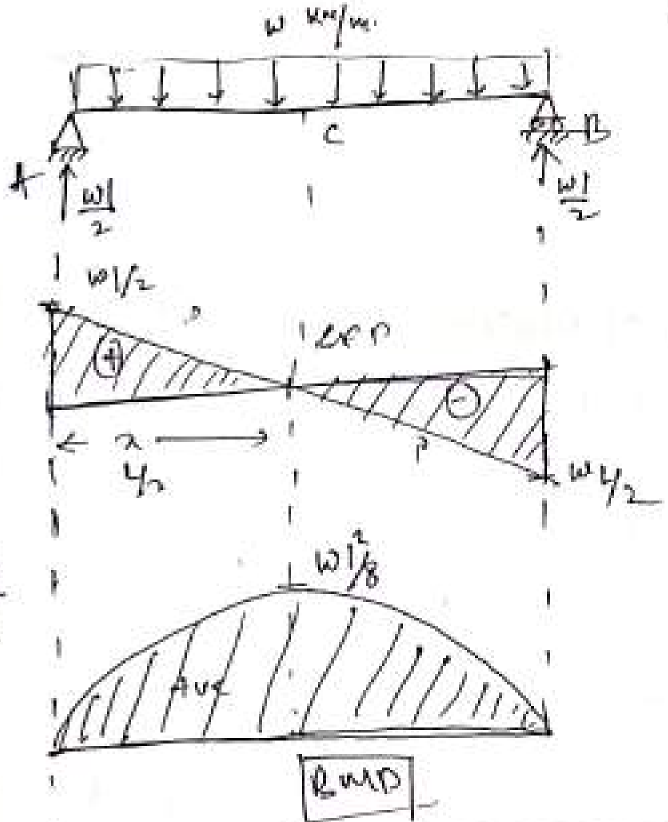
BM eqn critical part! -

A, B, C.

$$\boxed{M_A = 0 = M_B}$$

$$\boxed{M_C = +\frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}}$$

②



$V = \frac{dM}{dx}$

SF eqn

SF @ a $= +\frac{wl}{2}$

SF @ B $= -\frac{wl}{2}$

BM eqn

Critical path! -

A, B, C.

$$\boxed{M_A = M_B = 0}$$

Calcn of crit point c

At 'c' SF = 0

$\frac{wl}{2} - w.x = 0 \Rightarrow \boxed{x = \frac{l}{2}}$

$$\boxed{BM @ C = M_C = \frac{wl^2}{8}}$$

BM @ C $= +\frac{wl}{2} \times x - \frac{w.x^2}{2}$

$= \frac{w.l^2}{4} - \frac{w}{2} \cdot \frac{l^2}{4}$

$= \frac{wl^2}{4} - \frac{wl^2}{8} = \boxed{\frac{wl^2}{8}}$

4. Sign Convention :-

Left to Right		Right to Left	
SFD	↑ (upward) " +ve " ↓ (downward) " -ve "	SFD	↑ (-ve) ↓ (+ve)
BMD	↻ C.W (+ve) ↺ A.W (-ve)	BMD	↻ (-ve) ↺ (+ve)

5. SHAPE OF SFD And BMD :-

Type of Loading		Shape of SFD $\left(\frac{dv}{dx}\right)$			Shape of BMD $\left(\frac{dM}{dx}\right)$	
No Load (W=0)		$\frac{dv}{dx} = 0 \Rightarrow$	V = Constant	0° straight line.	$\frac{dM}{dx} = V = C$ $\boxed{M = Cx + C_1}$	1° straight line.
UDL	Load intensity (W) = Constant	$\frac{dv}{dx} = C$ $\boxed{V = Cx + C_1}$	1° straight line.		$\frac{dM}{dx} = Cx + C_1$ $M = Cx^2/_2 + C_1x + C_2$	2° square parabola
UVL	1°	2° square parabola.			3° cubic parabola	

6. CRITICAL POINTS :-

These are the point of which value of SFD and BMD should be known prior to the making of SFD & BMD.

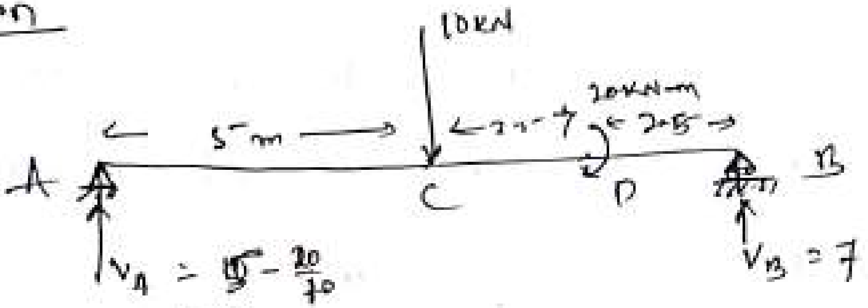
a) Start and end of the beam.

b) Start and end of the UDL & UVL.

c) Point where concentrated Point Load ~~and~~ or moment is acting.

d) Point where Load intensity diagram and SFD changes its sign.

Justification



$V_A = 5 - \frac{20}{10}$

$= 3$

$V_B = 7$

Point (a)

- At point "C" (just left)

$\boxed{SF @ C^-} = \boxed{+3 KN}$

$BM @ C^- = 3 \times 5 = 15 KNm$

- At point "C" (just Right)

$\boxed{SF @ C^+} = +3 - 10 = \boxed{-7 KN}$

$BM @ C^+ = 3 \times 5 = 15 KNm$

→ at point 'c' sudden change in SF value due to point Loading

Point (b)

- At point "D" (just left)

$SF @ D^- = -7 KN$

$\boxed{BM @ D^-} = 3 \times 7.5 - 10 \times 2.5$

$= \boxed{-2.5} KN-m.$

- At point "D" (just Right)

$SF @ D^+ = -7 KN$

$\boxed{BM @ D^-} = -2.5 + 20$

$= \boxed{17.5} KNm.$

→ At point D sudden change in BM value due to concentred Moment.

Note :-

** Point where concentrated point Load is acting, "SHEAR FORCE" is not defined.

** Point where Concentrated moment acting, "BENDING MOMENT" is not defined.

2. a) Point where load intensity diagram is changing its sign, "Shear force" is either maximum or minimum.

where, $w = 0$ & $v = f(x)$

$\boxed{w = -\frac{dv}{dx} = 0}$. $\boxed{\frac{dv}{dx} = 0}$ (for v to be maximum & minimum.)

b) Point where shear force diagram is changing its ~~s~~ sign, "Bending moment" is either maximum or minimum.

where, if $V = 0$, M = maximum or minimum.

$\boxed{\frac{dM}{dx} = 0}$

c) The point where "Bending Moment diagram" changing its sign, is referred as point of contra flexure.

$\boxed{BM = 0}$

Point of Inflection :-

↳ Point at which deflected curve changes its sign.

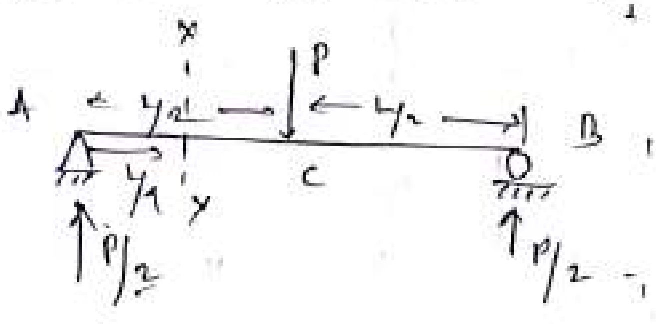
it mean
concave → convex
convex → concave.

3. a) Point where concentrated point Load is acting, there is sudden in SFD but no such change in BMD.

b) Point where concentrated moment, there is sudden change in BMD but no change in SFD.

Continue —

- Difference between Summetion Moment and Bending Moment :-



↻ +ve

$\boxed{\Sigma M_{xx} = 0}$ always for a equilibrium body.

$$\frac{P}{2}\times\frac{L}{4} + P\times\frac{L}{4} - \frac{P\times 32}{4} = 0$$

$$BM_{xx} = \frac{P}{2}\times\frac{L}{4} = \frac{PL}{8}$$

$$\boxed{BM_{xx} = \frac{PL}{8}}$$

L-17

V.imp

Points to draw SFD and BMD :-

I. Empirical relation between Load intensity (w), Shear force (V) and bending moment (M).

(i) $\boxed{w = \frac{-dv}{dx}}$ ** $v = f(x)$

$\frac{dv}{dx}$ = Slope of SFD = (Load intensity)

** slope of shear force diagram of a point indicates Load intensity at that point.

(i) $\boxed{V = \frac{dM}{dx}}$ SF

** Slope of the bending moment digram at a point indicates shear force at that point.

SHEAR FORCE & BENDING MOMENT

~~Straight lattle~~ →

Both are Internal forces
↓
forces In the form of Resistance.
↓
Resistance to applied Load

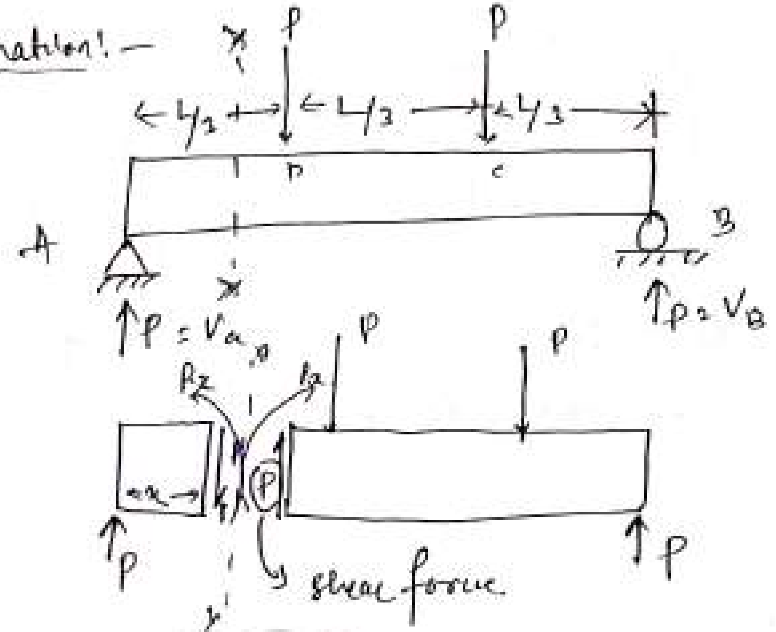
Shear force (V):-

Summation of all the transverse forces while moving left-Right or Right-left of that section.

Bending Moment (M):-

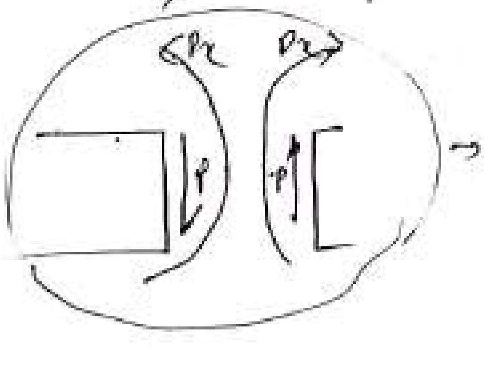
Summation of all the moment caused due to transverse force or concentrated moment while moving Left-Right or Right-left at that section.

Explanation:-



SF @ x-x = P

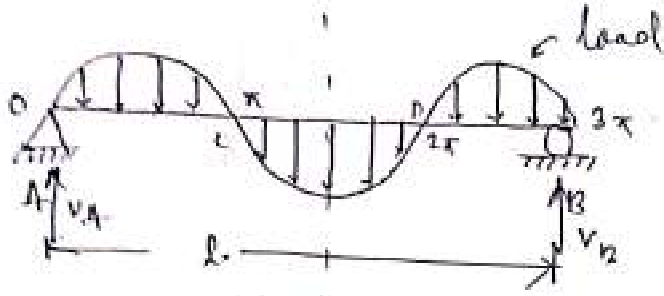
BM @ x-x = Px



→ Inter resistance forces.

1) A simply supported beam has a subjected to load intensity given $\sin(3\pi x/L)$. Determine the support reactions.

Sol^n Load $\boxed{w = \sin\left(\frac{3\pi x}{L}\right)}$



load intensity diagram.

$$V_A = V_B = \frac{\text{total load}}{2}$$

total load = Area under load intensity diagram

$$= \int_0^L \sin\left(\frac{3\pi . x}{L}\right) dx$$

$$V_A = V_B = \int_0^{\frac{L}{2}} \sin\left(\frac{3\pi x}{L}\right) . dx$$

$$= \frac{L}{3\pi}\left[-\cos\left(\frac{3\pi x}{L}\right)\right]_0^{L/2}$$

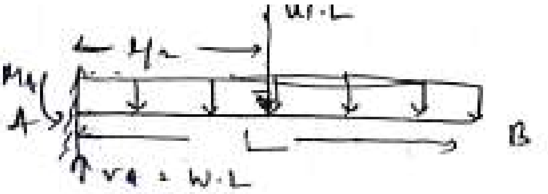
$$= \frac{L}{3\pi}\left[\cos\left(\frac{3\pi x}{L}\right)\right]_{L/2}^{0}$$

$$= \frac{L}{3\pi}\left[\cos 0^\circ - \cos\frac{3\pi . \cancel{L}}{\cancel{L} . 2}\right]$$

$$= \frac{L}{3\pi}\left[1 - \left(\cos\frac{3\pi}{2}\right)\right] \left\{\because \cos\frac{\pi}{2} = 0 \quad \cos 0^\circ = 1\right.$$

$$= \frac{L}{3\pi}(1 - 0) = \boxed{\frac{L}{3\pi} = V_A = V_B}$$

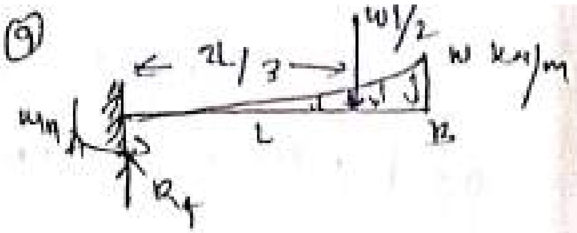
8.



$V_A = W.L$

$M_A = \frac{w.l^2}{2}$

⑨



$R_A = \frac{wl}{2}$

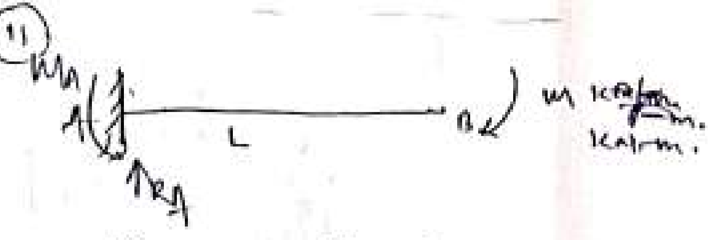
$M_A = \frac{wl}{\cancel{2}} \cdot \frac{\cancel{2}L}{3} = \frac{wl^2}{3}$

~~Q~~ ⑩

$R_A = \frac{wl}{2}$, ~~~~

$M_A = \frac{w.l}{2} \cdot \frac{l}{3} = \frac{wl^2}{6}$

⑪

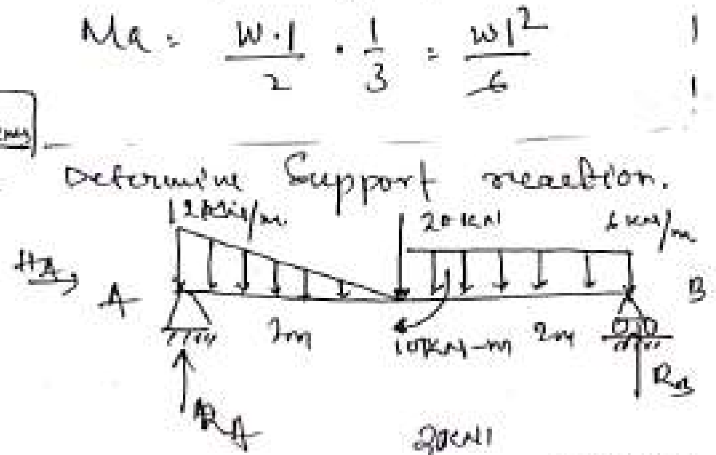


$R_A = 0$, $H_A = 0$

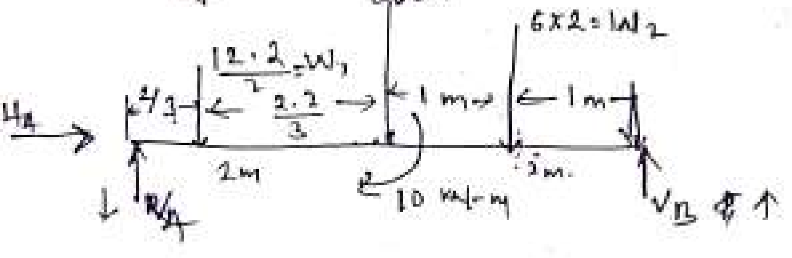
$M_A = M$

Problem

Q.1 Determine Support reaction.



sol^n

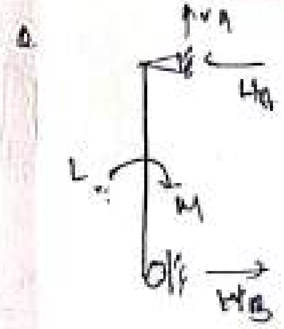


$$V_A = \frac{\overset{3}{\cancel{12}}\times 1}{\cancel{4}} + \frac{20}{\cancel{4}\,2} - \frac{10}{4}\times\cancel{2} + \frac{12\left(2+\frac{1}{3}\right)}{4} =$$

$$= 3 + \frac{\cancel{25}}{10} - \frac{\cancel{20}}{2.5} + \frac{\cancel{40}}{\cancel{4}}^{10} = 20.5 \text{ KN}$$

$$V_B = \frac{\cancel{12}\times\frac{2}{\cancel{3}}}{\cancel{4}} + 10 + 2.5 + \frac{3\times\overset{3}{\cancel{12}}}{\cancel{4}}$$

$$= 2 + 10 + 2.5 + 9 = 23.5 \text{ KN}$$

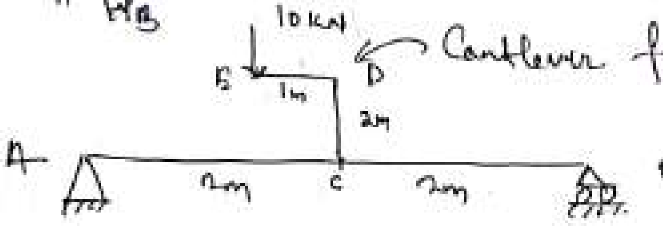


$V_A = 0$

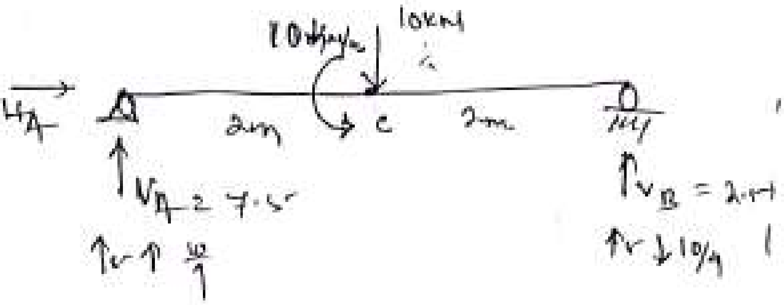
$H_A = \frac{M}{L}$

$H_B = -\frac{M}{L}$

⑤



Cantilever frame.



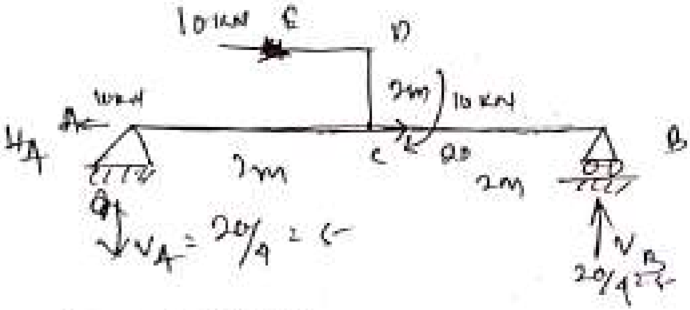
$V_A = \frac{10}{2} + \frac{10}{4}$

$= 5 + 2.5 = 7.5$

$V_B = \frac{10}{2} - \frac{10}{4}$

$= 5 - 2.5 = 2.5$

6)



$H_A = -10\text{kN}$

$V_A = -5\text{ KN}$

$V_B = 5\text{ KN}$

L16! Support reaction!.

⑦

$\Sigma F_x = 0$

$\boxed{H_A = 0}$

$\boxed{\Sigma F_y = 0}$ ↑+ve true

$V_A - P = 0$

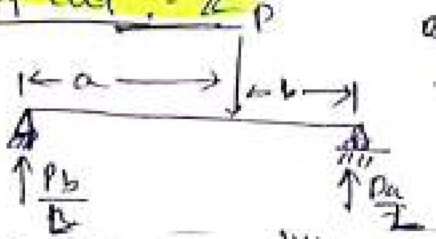
$\boxed{V_A = P}$

$\Sigma M_{B_A} = 0$ (+↻, ↺−ve)

$-M_{R_A} + P.L = 0$

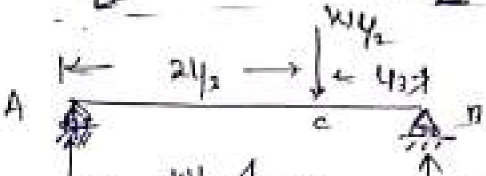
$\boxed{M_A = P.L}$

Short cut :- II



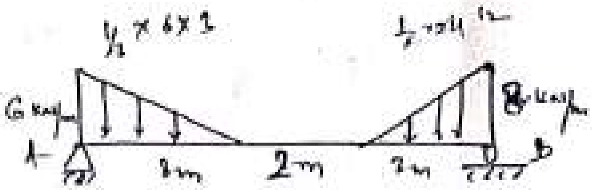
$R_A = \frac{Pb}{L}$

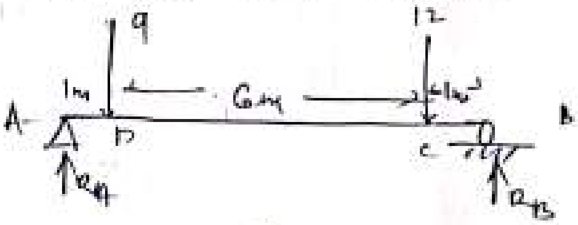
$R_B = \frac{Pa}{L}$



$V_A = \frac{\frac{wl}{2}\cdot\frac{l}{3}}{l} = \frac{wl}{6}$

$V_B = \frac{\frac{wl}{2}\cdot\frac{2l}{3}}{l} = \frac{wl}{3}$





$R_A = \frac{12 \times 1}{8}$ ~~$= \frac{9}{2} = 4.5$ kN~~

$+ \frac{9 \times 7}{8} = \frac{12 + 63}{8} = \frac{75}{8}$

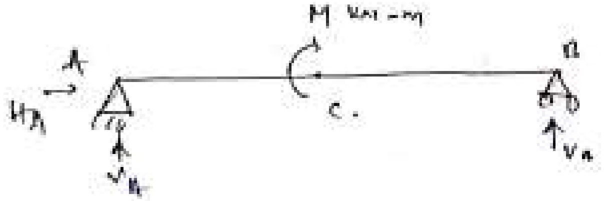
$= 9.375$ kN

$R_B = \frac{12 \times 7}{8} + \frac{9 \times 1}{8}$

$= \frac{84 + 9}{8} = \frac{93}{8}$

$= 11.625$

1)



$\Sigma F_x = 0$

$\boxed{H_A = 0}$

$\Sigma F_y = 0$ (↑+ve, ↓-ve)

$\boxed{V_A + V_B = 0}$

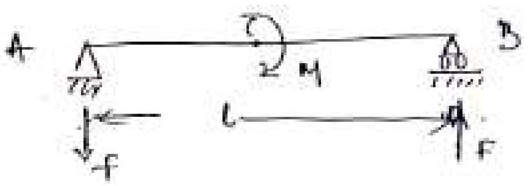
$\Sigma M_B = 0$ (↻+ve, ↺-ve)

$L V_A + M = 0$

$\boxed{V_A = \frac{-M}{L}}$

$\boxed{V_B = \frac{M}{L}}$

Short Cut - 3



$F \times L = M$

$\boxed{F = \frac{M}{L}}$

Moment

Balance Moment

~~Couple~~

→ Opposite moment — fixed end or fixed support

→ opposite couple. → two supports which can give resistance in same direction.

$M = f \times d$