

LECTURE NOTE
ON
CONSTRUCTION MANAGEMENT
FOR
DIPLOMA IN CIVIL ENGINEERING
(6TH SEMESTER STUDENTS)
AS PER SCTE&VT SYLLABUS



PREPARED BY:

Miss Sagarika Patel

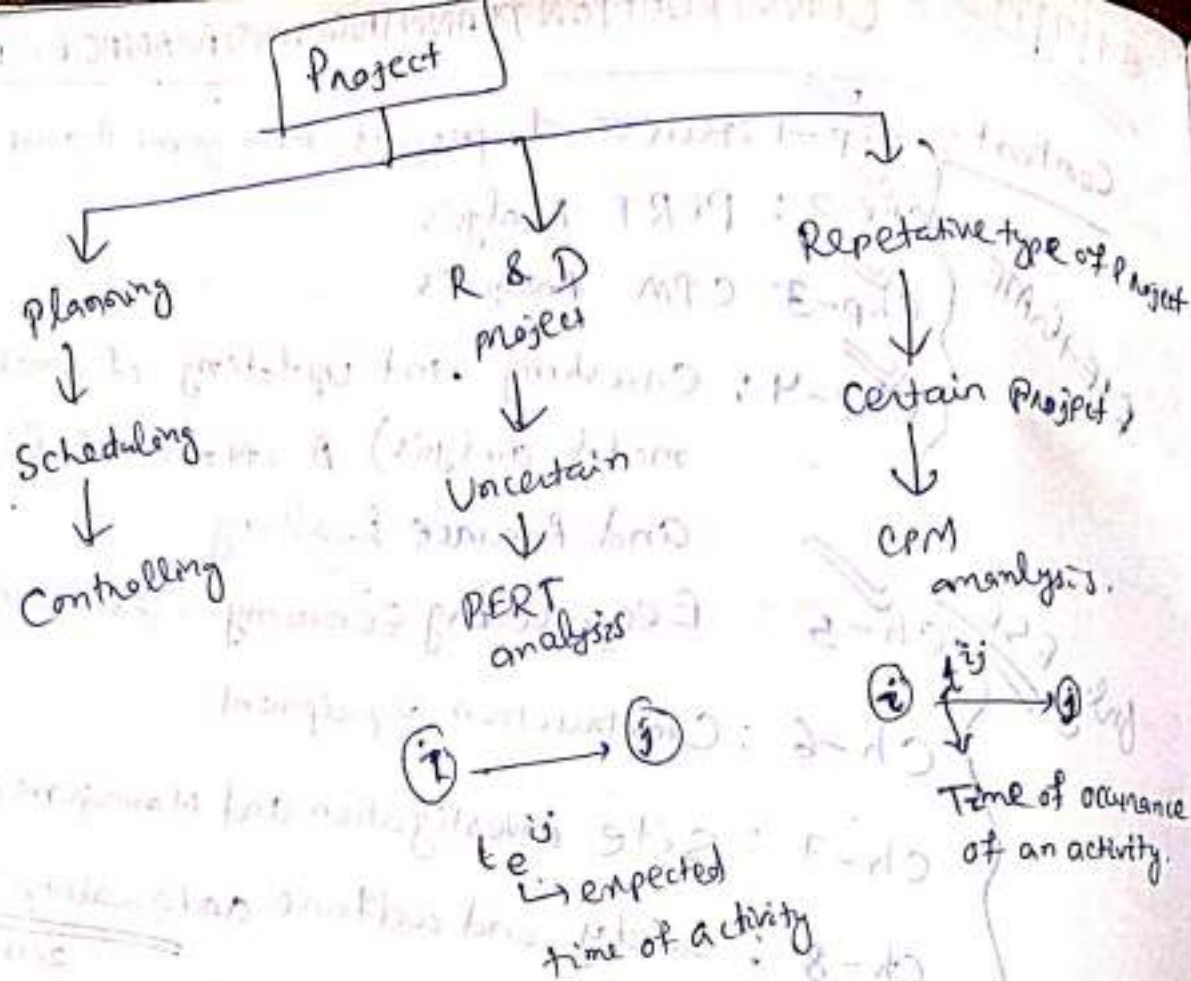
PTGF in Civil Engineering

Department of Civil Engineering

Government Polytechnic, Sambalpur (Rengali)

www.gpsambalpur.com

Intro:-



Project management:

It deals with material and human resources both to increase productivity of efficiency.

For completion of a project two basic things are required.

- ① Material resources
- ② Man power

Each project either it is big or small has 3 objectives:-

- (i) project should be completed in min time.
- (ii) It should use available man power & material resources without delay in project.
- (iii) Project should be completed in min of capital investment.

There 3 phases of project management.

① Project planning or planning. → This is the most important phase of project management. Planning involves defining the objectives of project, arrangement of job that must be performed, determining gross requirements for material, equipment, man power & preparing estimate of cost & durations for various jobs for satisfactory completion of the project.

Importance

→ It provides dirn.

→ It " " performance std.

② Scheduling or project scheduling

Scheduling is the allocation of resources such as time, material, equipment, man power resources in appropriate manner such that it results in efficient working.

Scheduling also involves sequencing of activities.

Note

In traditional ~~techniques~~ techniques the term planning & scheduling are performed as one step.

③ Project Controlling :- It is the process to identify critical (running exactly on schedule but not complete), Subcritical (before schedule) or non critical) or super critical (delayed) (late).
Critical → on time.

Activities

The critical & super critical activities are given extra attention so that network is updated.

Note → Planning & scheduling are performed before execution of the project whereas controlling is carried out during execution of the project.

Work break down structure

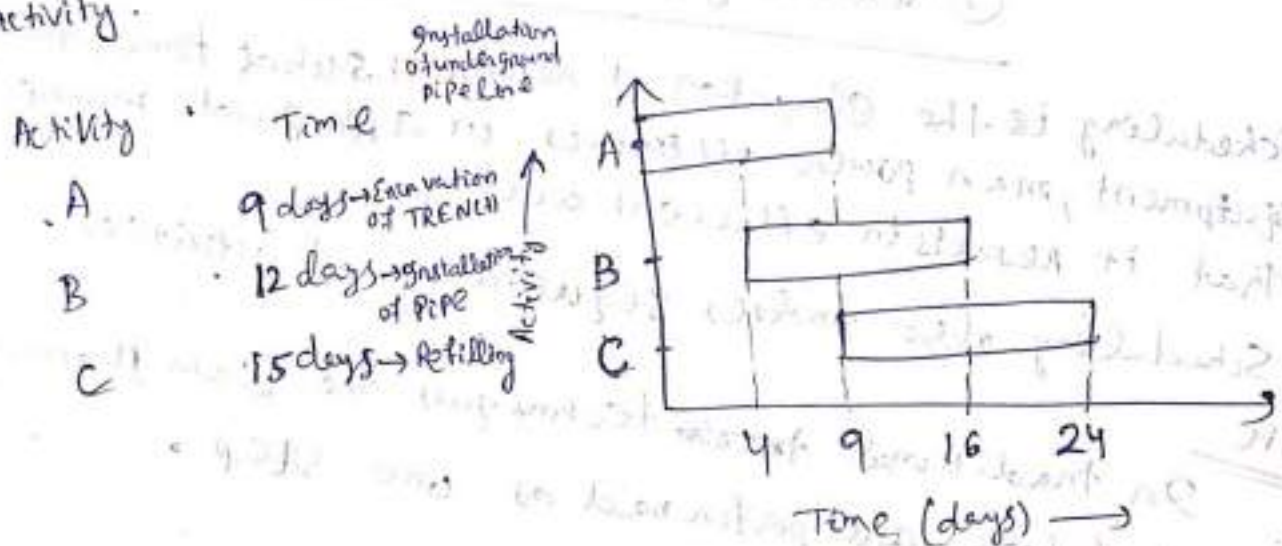
This is incremental & hierarchical decomposition of project phases. It is tree like str. It is also called top-down approach & is suitable for big & complex project.

Technique used in Project management:

① Bar chart:-

This is the oldest technique of project management and it was developed by Henry Gantt around 1900.

Hence it is also called Gantt chart. There are two co-ordinates on which horizontal axis represent time and vertical axis represent activity.



From the above bar chart, it is clear that activity C will start after completion of Activity A. But it is not clear that how activity A, B, C are interconnected to each other.

Limitation of bar chart

① Lack of degree of detail

In a bar chart only major activities are shown. Subactivities cannot be represent hence it is difficult to carry out the control. So it is not used for controlling purpose.

WBS for construction of 5 km. Village Road (flexible pavement)

Earthwork

- * Test of compaction by cone cutter.
- * Watering & Compaction
- * Soil Reclaiming
- * Soil cutting and filling
- * Field survey

Soil subgrade

- * Test of compaction by sand replacement method
- * Watering & Rolling
- * Installation of coarse sand on embankment

Sub-base course

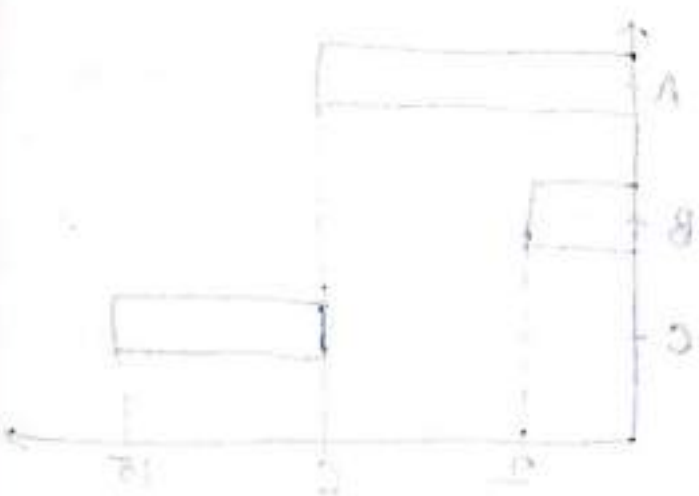
- * Test of compaction and test of grading by sieve analysis
- * Watering & Rolling
- * Install mix on surface
- * Prepare a mix of coarse sand and stone metal of flaky size.

Base course

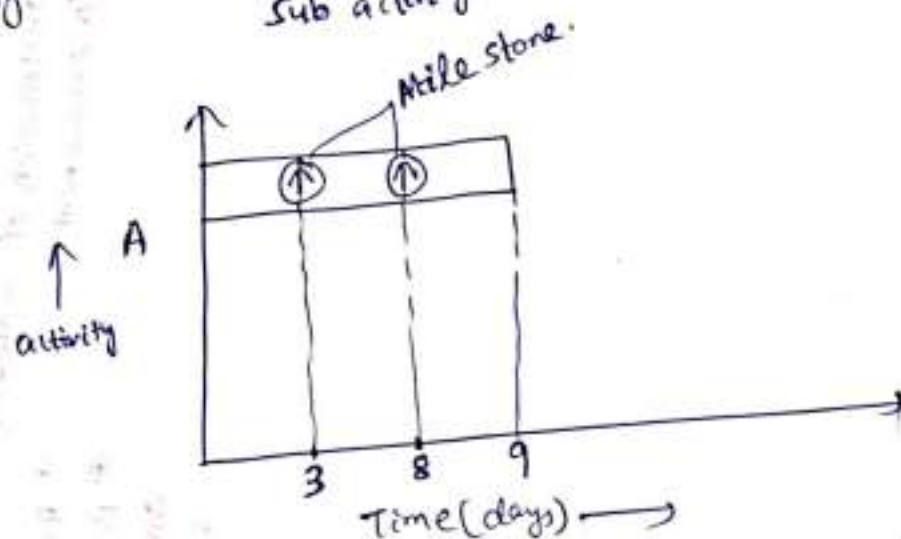
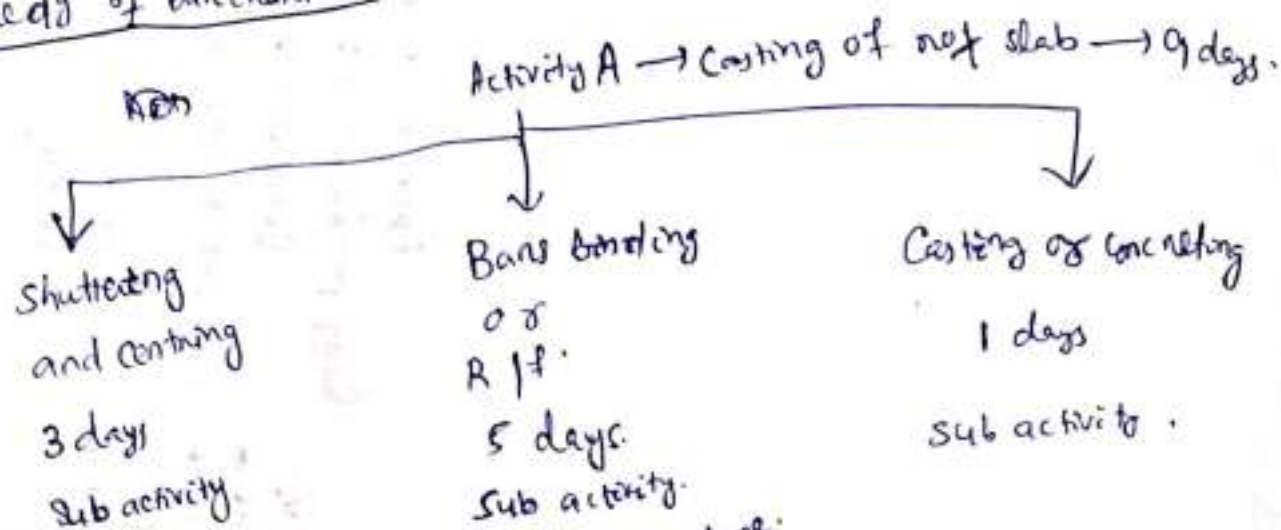
- * Test of compaction & gradation
- * Watering & compaction
- * Install morum on surface
- * Install stone metal on surface

Surface course

- * Test of thickness of Bitumen layer.
- * Seal coat & compaction by static roller
- * Tack coat & prime coat (PMC)
- * Prime Coat (SS)
- * Cleaning of surface by air compression



Remedy of bar chart :-



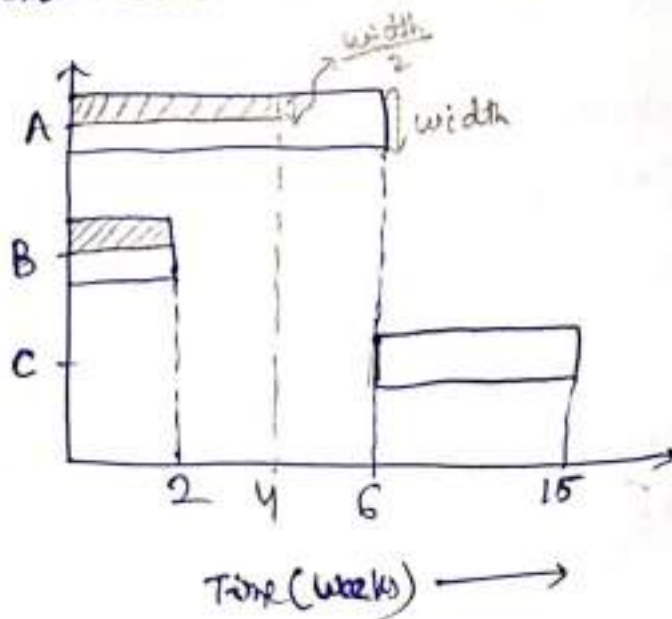
Milestone represent an event of occurrence of a sub activity at an intermediate stage of a activity.

② No provision to show project progress

Since project progress can't be shown ~~there~~ hence bar chart can't be used as controlling mechanism.

Review after 8 weeks

Activity A is incomplete and it has only 4 week work which has done. So it requires 2 more weeks. So, Activity A will complete at the end of 10 weeks. Activity C is completely dependent on A. So C will start after 10 weeks. So 'C' will finish in 19 weeks.



Activity B is complete.

$\therefore$ Project is delayed by 4 weeks, Hence rescheduling of activity 'C' is required.

③ Lack of activity interrelationship.

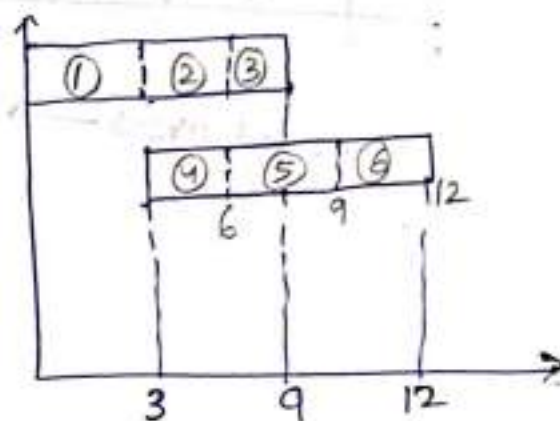
In bar chart, clear picture b/w activity relationship is not available hence it is difficult to identify their interdependency.

Remedy:-

This difficulty can be partially removed by breaking each activity into a no. of sub secⁿs. So that succeeding sub secⁿ will depend on completion of preceding sub secⁿ.

Breaking each activity into a number of equal sub activity.

Sub-secⁿ 4 will start after completion of sub secⁿ ①



④ No provision to account for time uncertainty
hence it is not suitable for new or Research and development types of project.

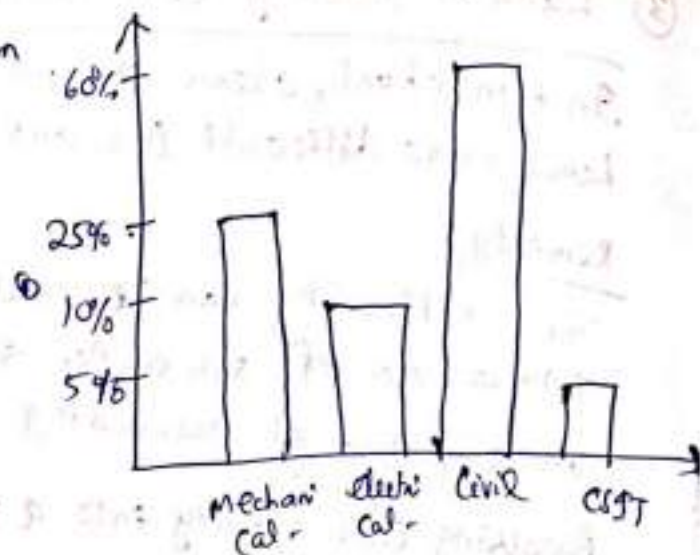
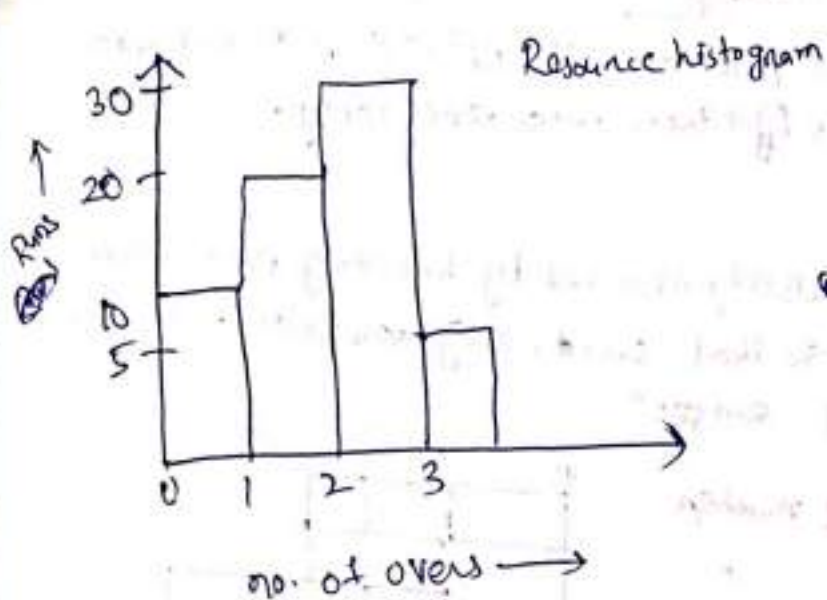
⑤ It is difficult to identify critical activity & difficult to find critical path. So that bar chart can't be used as controlling mechanism.

Milestone chart:

This is imp. to over gantt chart controlling can be better achieved with the help of milestone chart.

Resource histogram

Bar chart and resource histogram both are similar. The only diff. is that the continuous data and each column represent quantitative variable of particular group.



Bar chart

Bar chart represent data of a diff. grp.

④ Linked Bar chart

→ The end of preceeding activity is connected to the start of succeeding activity.

* It provides clear picture of activity relⁿship.

Dotted arrow (Dummy) are used to established their inter relⁿship.

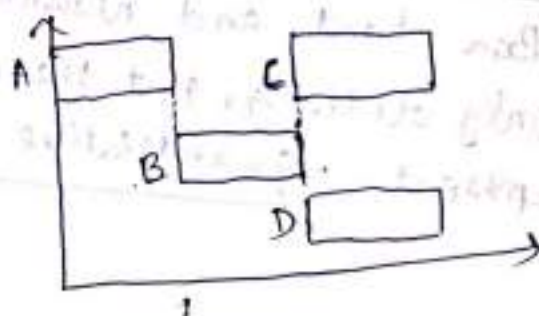
(Freedom) Floats → Can be easily calculated.

There is no need to monitor separate milestone.

It is easy to understand.

It can be used for controlling purpose.

Activity B will start after completion of activity A.



* Resources for individual Activities can be easily planned.

Drawback

- Uncertainty can't be accounted hence not suitable for new or research & development type of project.
- Critical path ~~can't~~ can't be calculated.

Network diagram

This is improvement over milestone chart & it is to be used for big project under complex cond^{ns}.

All limitations of bar chart are eliminated here. Network is a pictorial representation of all activities in which activities and their interrelationship are shown & dummies may be used to show their interrelationship if required.

Types of Network:

A - O - A

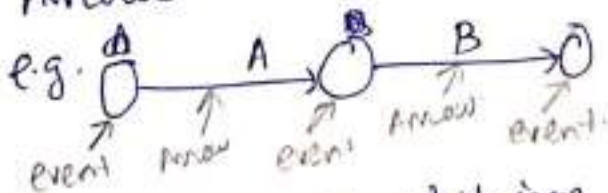
A - O - N

Diff. b/w A - O - A & A - O - N Network.

A - O - A

A - O - N

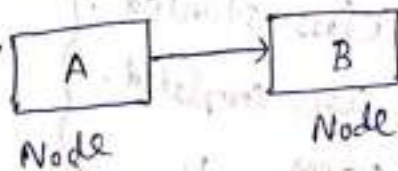
1. Activities are represented by Arrows.



2. The no. of arrow is exactly equal to no. of activity in a network.

1. Activities are represented by Nodes.

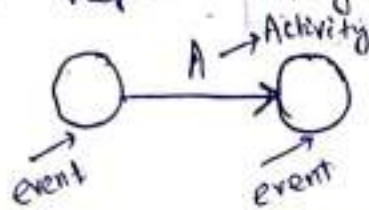
e.g.



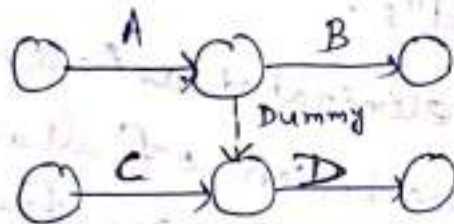
2. The no. of arrow will not be equal to no. of activity.

e.g. No. of activity = no. of arrow = 2
in above network.

3. Events are represented by Nodes.



4. Dummy are used to establish logical relationship if required.



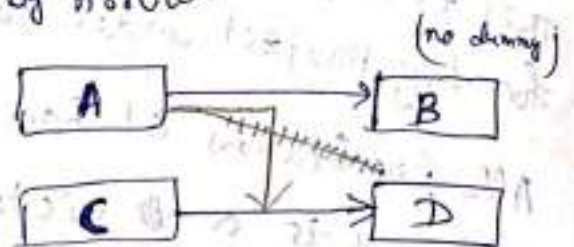
5. It is used for both PERT as well as CPM based project.
(Continued project)

6. Suitable for both R & D and repetitive type of project.

e.g. no. of activity $\neq$ no. of arrow
no. of arrow = 1
no. of activity = 2 in above network

3. Events have no place.

4. There is no role of dummy however there relationship is represented by Arrow.



5. It is used only for CPM based project.

6. Suitable for only repetitive type of project.

ELEMENTS OF A-O-A Network

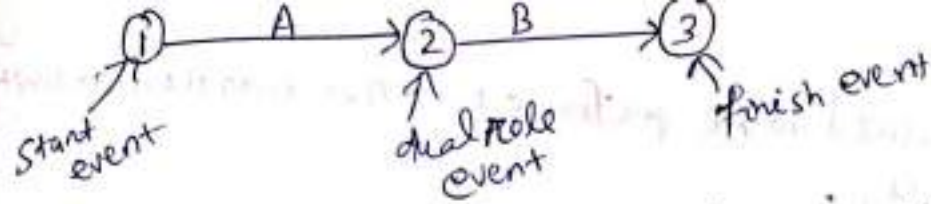
① Event : $\rightarrow$ An event is either start or completion stage of an activity.
It doesn't need any resources and any time for its occurrence.

Ex Class started } event.
Class completed }
CTPM " }
Slot started }

Types of event

- ① Start event
- ② finish "
- ③ intermediate event or dual node event.





event ② is finish event for activity A.

event ② is start event for activity B.

∴ ② is dual role event.

An event is represented by  Circle or  oval shape.

* In a network diagram there is only one start event & only one finish event & remaining all are dual role event.

② Activity →

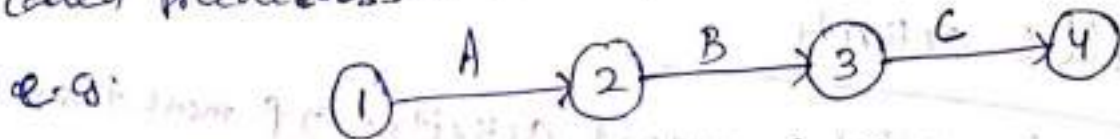
It is the actual performance of work which requires time & resources both for its completion. e.g. class is going on, slab is casting.

Types of activities

① Predecessor Activity.

Activities that are ~~needed~~ ^{to be} performed before another activities.

is called predecessor activity.



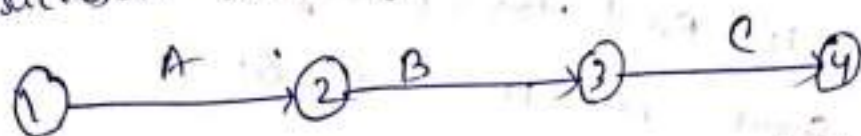
Activity A is predecessor of activity B.

" " " " " "

Note: Never say: activity A & activity B are predecessor of activity C for the above network.

② Successor activity

Activity that are required to be performed after another activity is called successor activity.

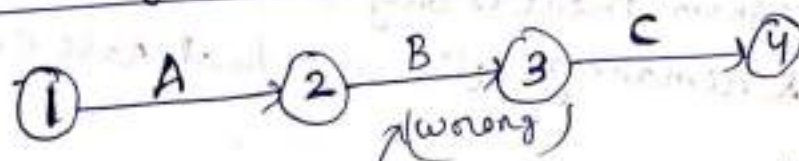


Activity B is successor of Activity A.

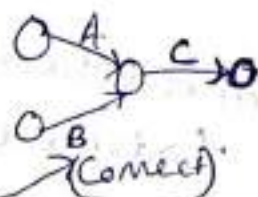
|| C || is
(unwanted elements)

" " B.

* Redundancy in a Network



Activity A & B are predecessor of activity C.
(Just immediate)



Successor
(Just front)

If more than one network is possible for a given statement then redundancy will exist in a network.

predecessor means immediate predecessor

& successor

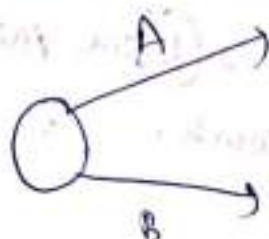
"

"

Successor

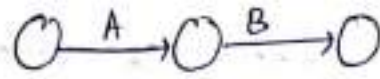
③ Parallel activity

This is also called Concurrent activity. If more than one activity will start at the same time then it is called concurrent activity or parallel activity.



Activity A & activity B are Concurrent

④ Serial Activity.



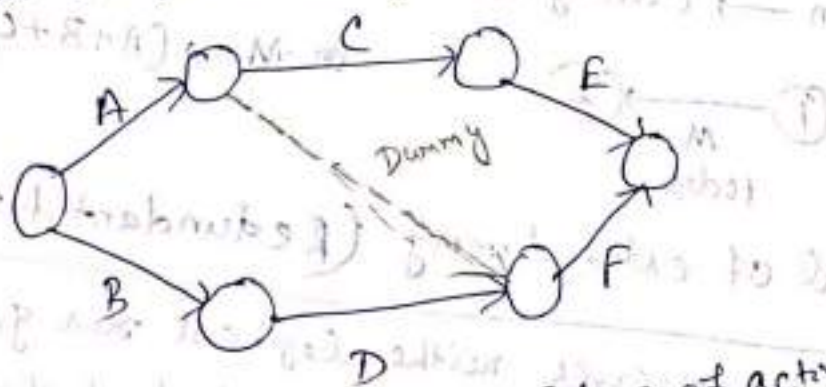
If one activity will start after completion of another activity then it is called serial activity.

⑤ Dummy activity

It is an activity which requires neither resources nor time for its completion. It is used to establish a logical relationship b/w two activities. Also ~~dummy~~ ^{are} ~~used~~ ^{used} for grammatical purpose.

⑥ Logical Purpose Activity Purpose.

Dummies are used to establish interrelationship b/w two activities in a logical way.



* Activity F will depend on completion of activity A & D.

* A precedes 'C' but restrains the occurrence of F.

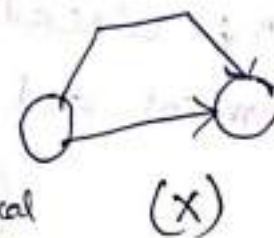
* F succeeds D but restrained by occurrence of A.

Grammatical Purpose:

Accⁿ to Network Rule more than one activity can't have common starting and finishing loads.

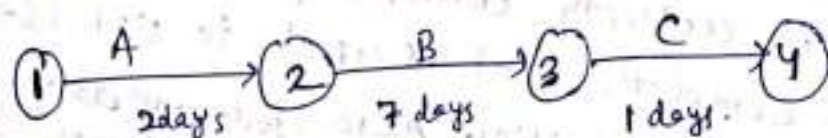


for grammatical purpose.



(X)

Note If events are added or removed then there will be no change in resources and time for completion of a project. But if activities are removed then time & resources both will change, hence activities can neither be added nor be removed from a network.



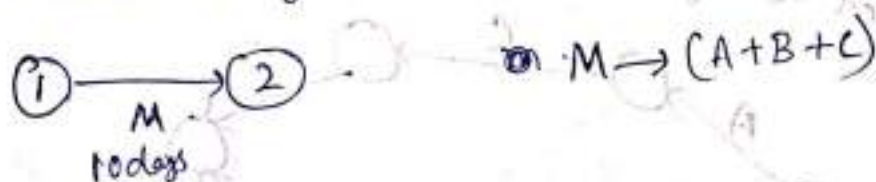
Casting of roof slab

A → shuttering & centering

B → Bar activity

C → concreting

m → casting of roof slab

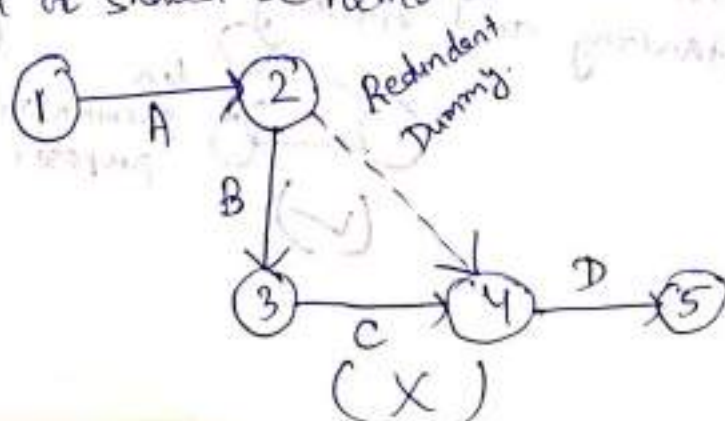


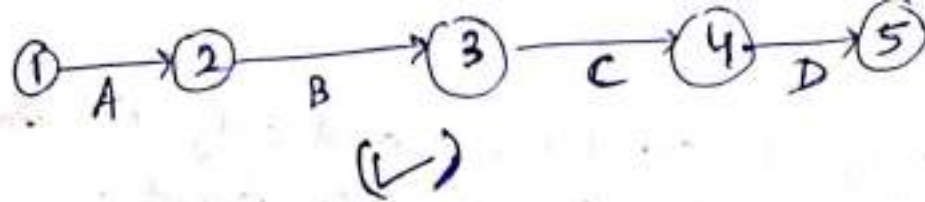
Removal of extra dummy (Redundant dummy)

If dummy serves neither logical nor grammatical purposes hence such a dummy is called redundant dummy & it should be removed from a network.

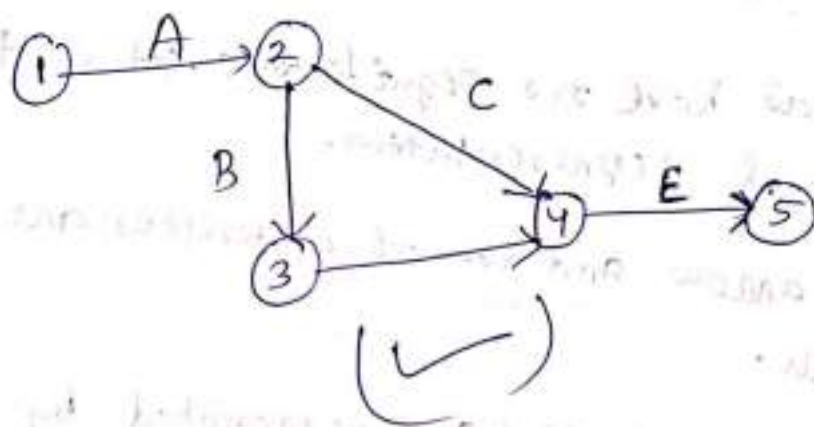
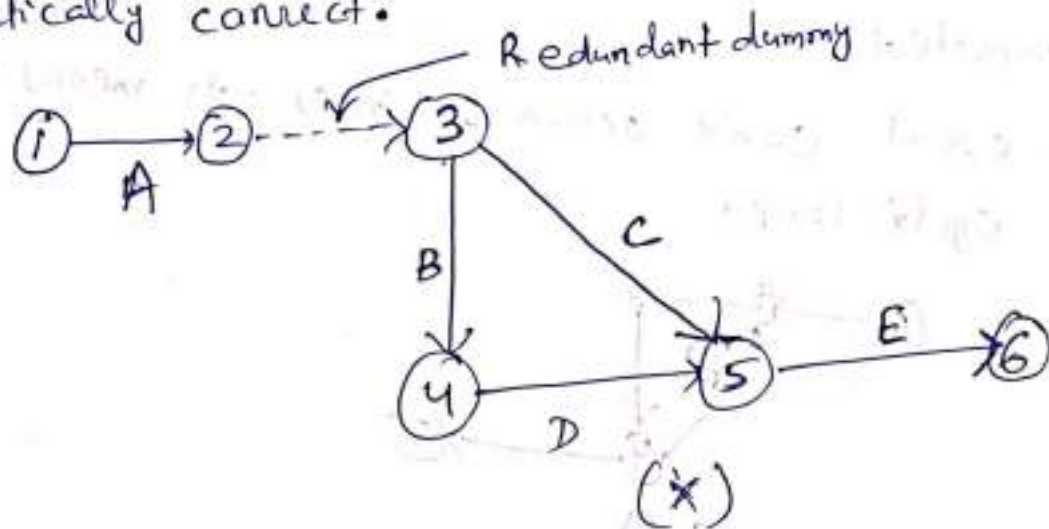
Guidelines for removal of **Redundant Dummy**:

- If dummy show the predecessor reln in a network but that reln is already established in a network then that dummy is redundant and it should be removed.

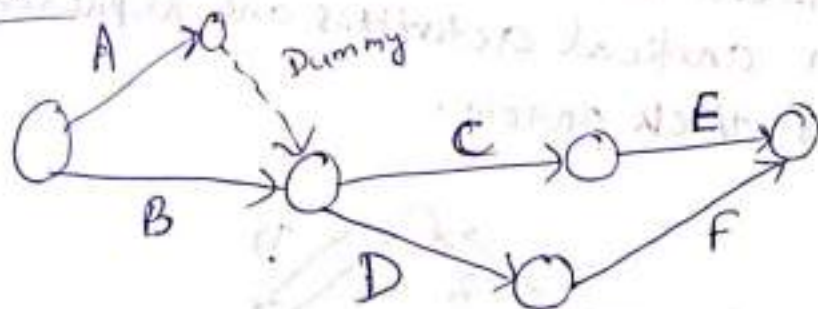




2. If dummy is the only incoming arrow to an event then it should be removed provided diagrams are logically and grammatically correct.

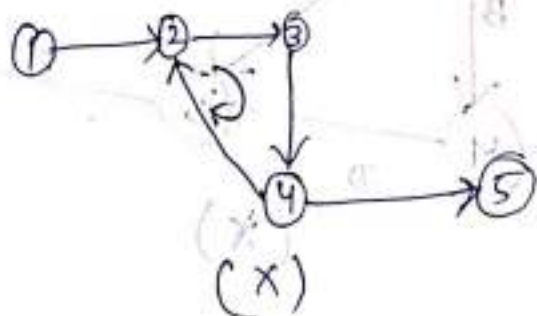


Logical Purpose

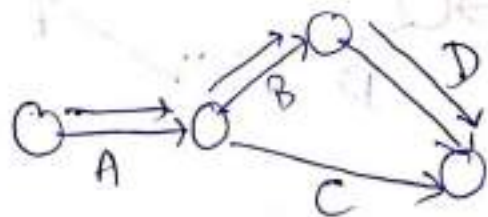


Network rule

1. There can be only one start event and only one finish event.
* remaining all are dual or node event.
2. An event can't occur till all preceding activities are completed.
3. An event can't occur 2 times, it means that there can't be cyclic loop.

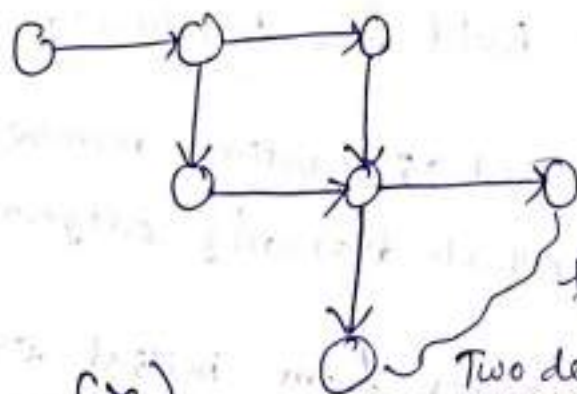


4. Length of arrow have no significance, it is taken only accⁿ to pictorial representation.
5. The no. of arrow and no. of activities are equal in A-O-A network.
6. The non-critical activities are represented by single arrow whereas critical activities are represented by double arrow or thick arrow.



Activity A, B & D are critical activity.

7. In a network there can be only one dead end or final event.



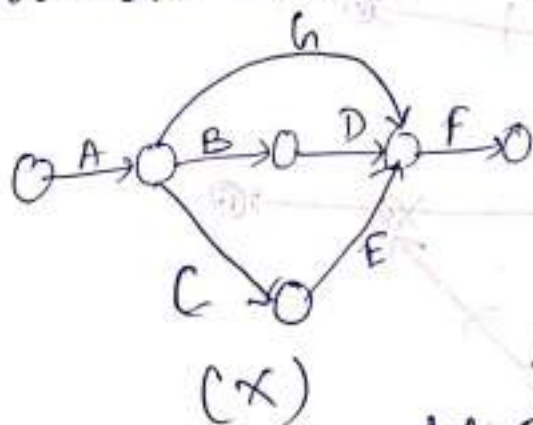
(X)

final event.

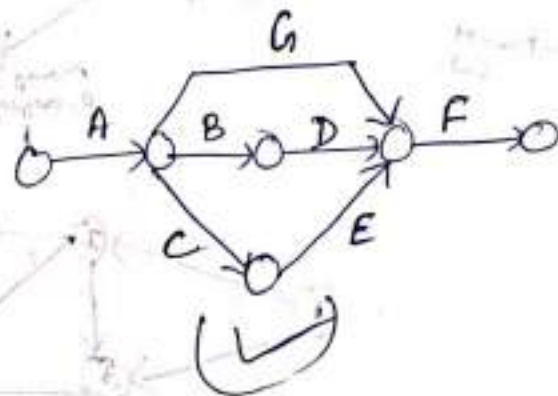
final ~~event~~ event
Two dead end (Dangling Error)

Conventions of network

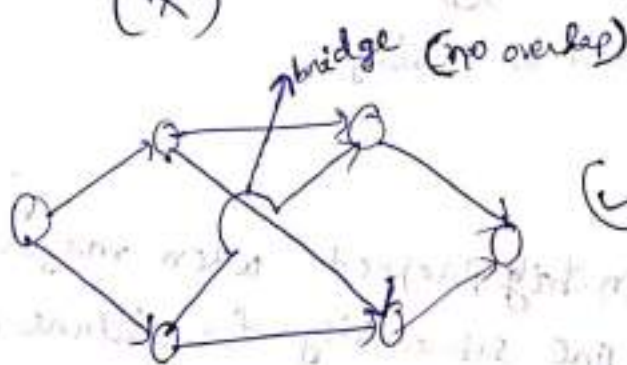
1. As far as possible an arrow should be a straight and ^{their} crossing should be avoided, but if not possible then an arrow should be bent and their crossing should be bridged.



(X)

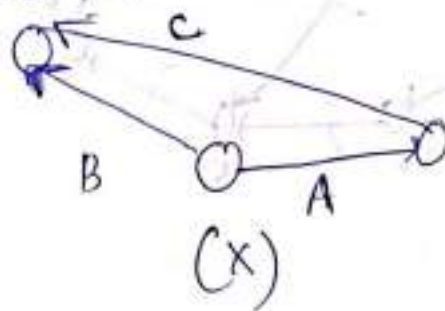


(✓)

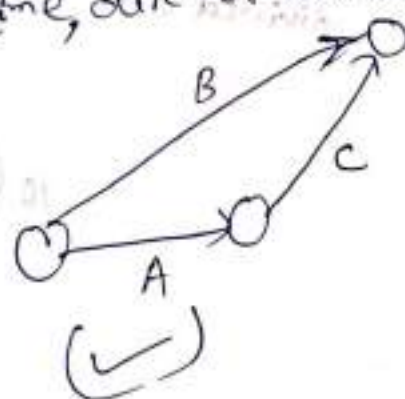


(✓)

2. As far as possible, with the flow of time, dirⁿ of arrow should be from left to right.



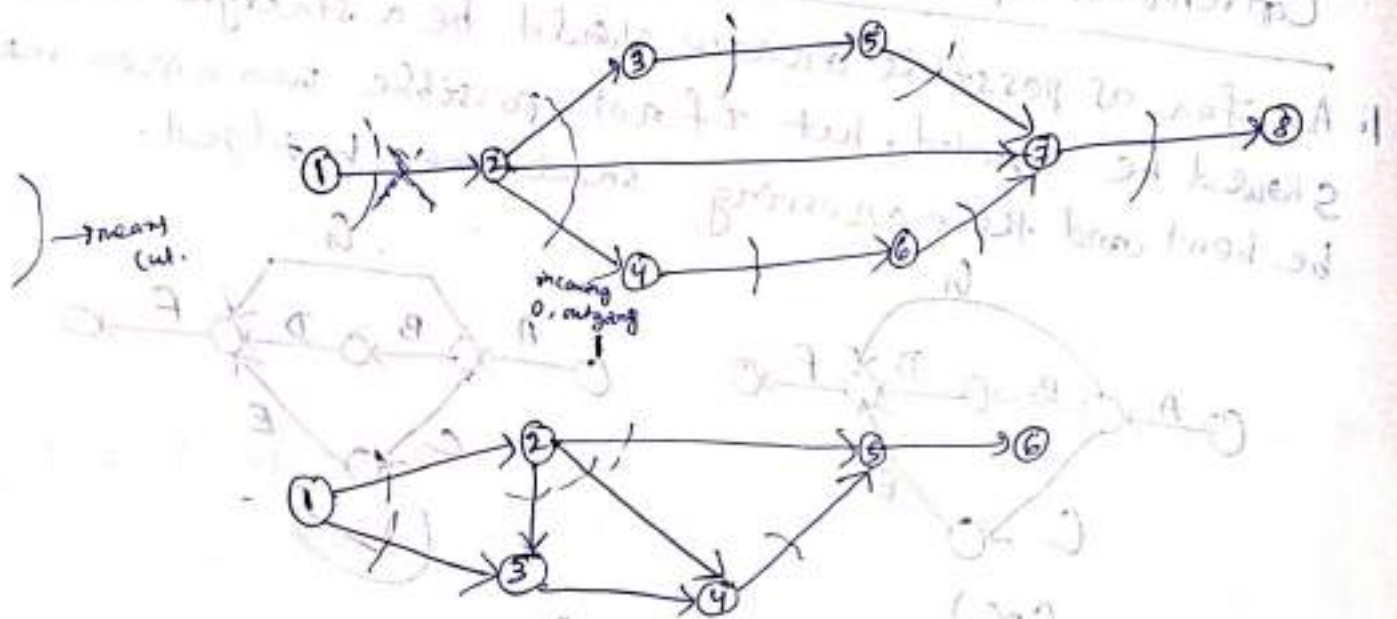
(X)



(✓)

D.R. FULKERSON'S RULE for Numbering the event.

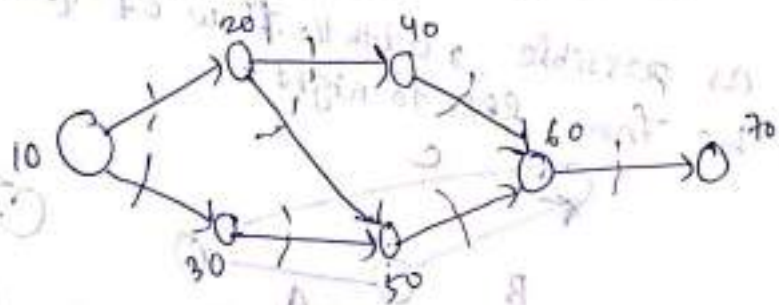
1. Initial event is numbered as starting number.
Initial event is that which has only outgoing arrows.
2. Ignore all outgoing arrows from initial event which has already numbered & create new initial event which will assign next no.
3. Repeat Step ② till all events are numbered.



If there is an incoming arrow then no numbering.

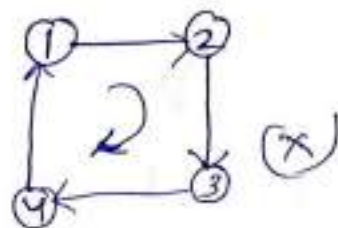
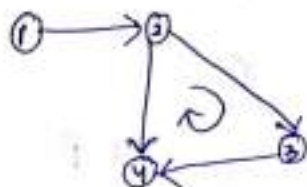
Skip Numbering

It is generally used in big project when major activities are divided into more than one sub activity for future opportunities.



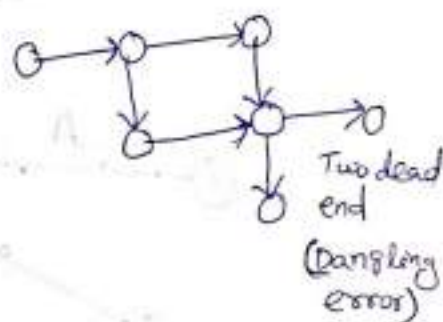
→ 1. Looping error or cyclic error

An event can't occur two times in a network so that there can't be a ^{cyclic} ~~typical~~ loop in a network.



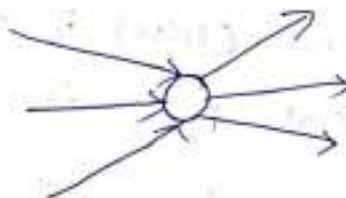
2. Dynamic error

In a network there can't be more than one dead end or can't be more than one final event.



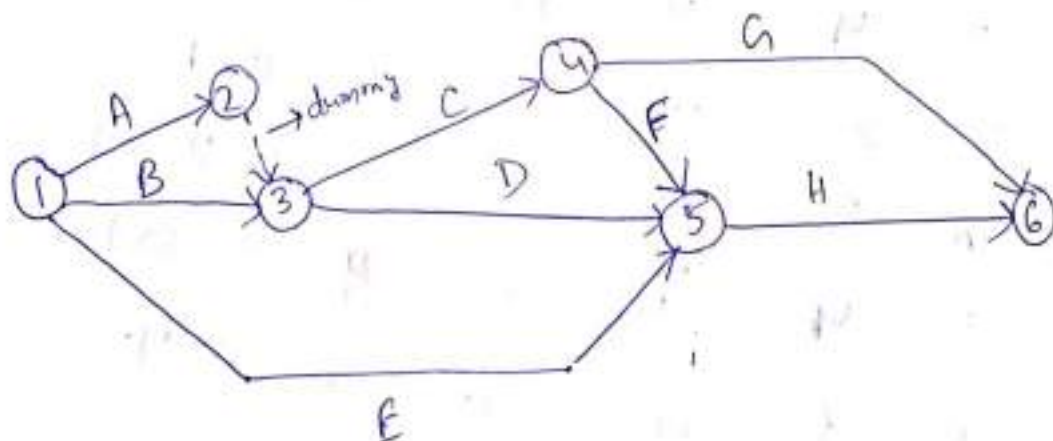
3. Wagon wheel error

In a given network more than ³ ~~one~~ activity completed at the same time and more than 3 activity ~~are~~ started at the same time. It is practically not possible, hence such type of error is called wagon wheel error.

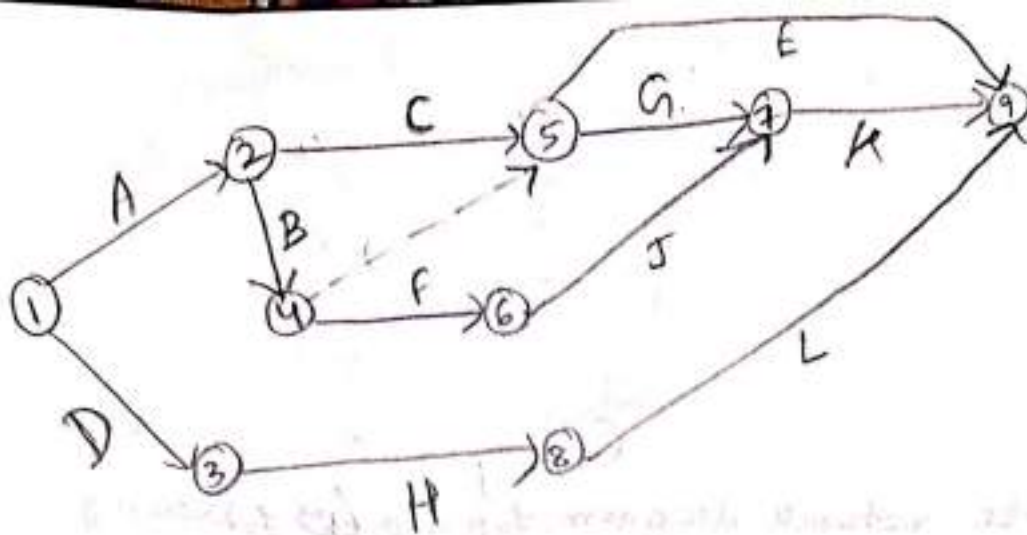


WB

cr2

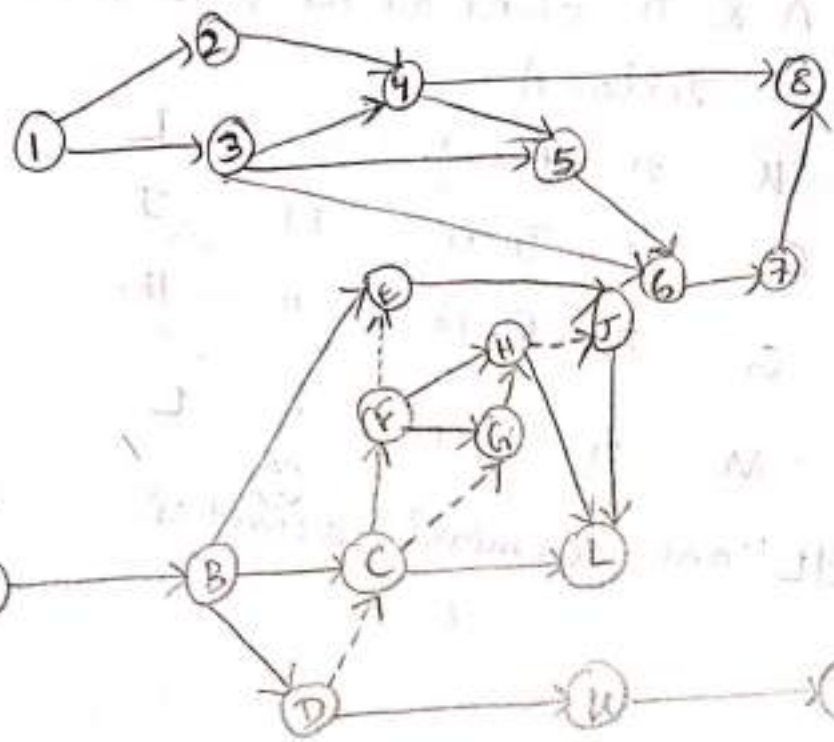


Q. In a particular project the following relationship occur
between the events.

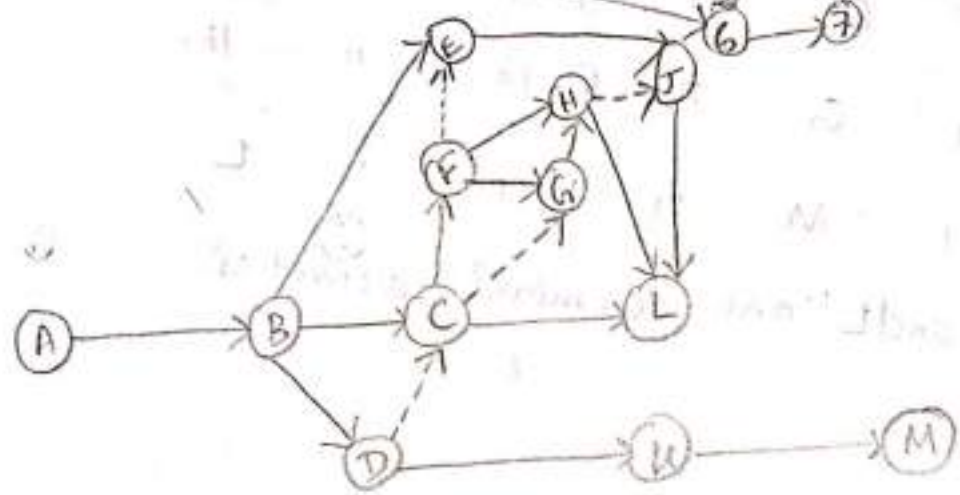


WB ch-2

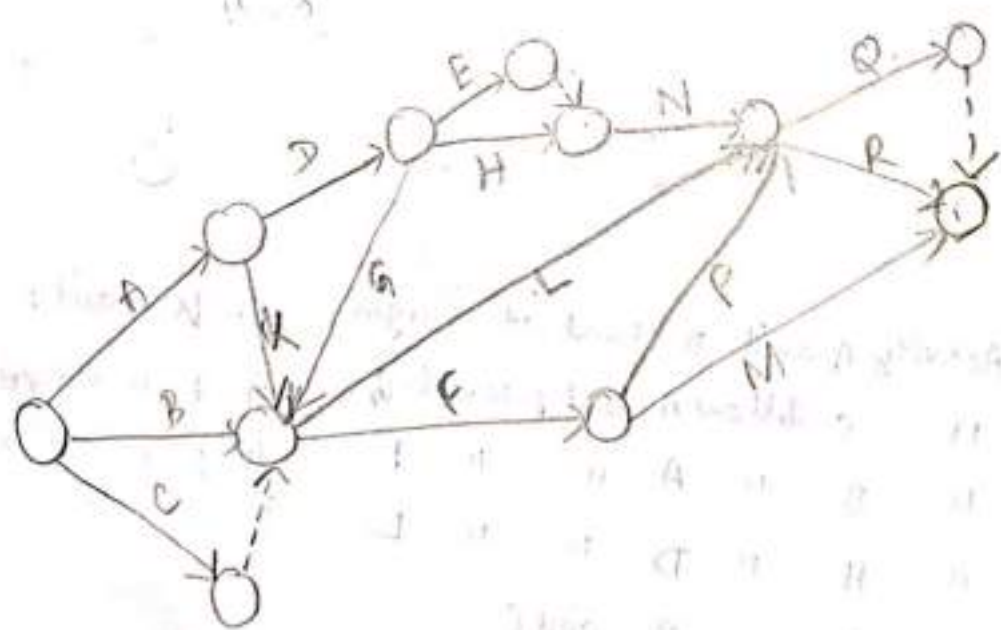
11.



T2.



12



PERT

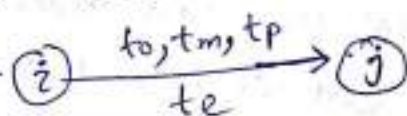
1. It stands for programme evaluation and review technique. It was first developed by U.S Navy department in 1958 for development of missile.

2. It is based on three time estimates for completion of an activity.

(1) Optimistic Time (t_o)

(2) Pessimistic time (t_p)

(3) (t_e or t_m) Most likely time



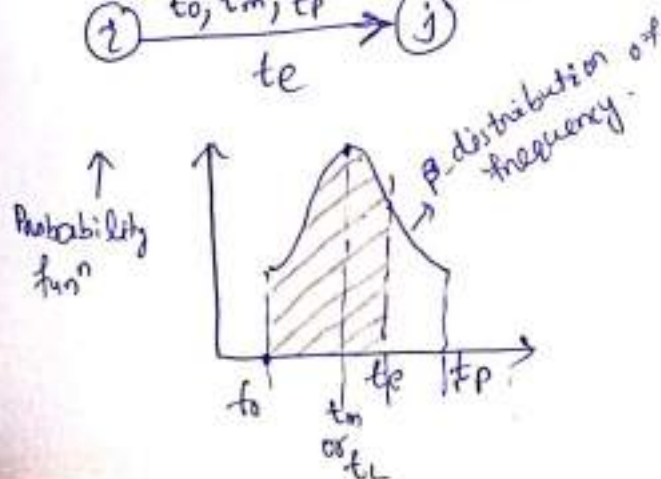
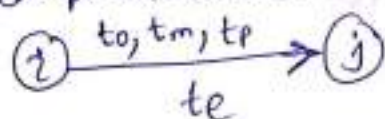
Based on the above three time estimates, mean expected completion time of an Activity is given by

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

t_e → expected mean time for completion of an activity.

Based on the mean expected time (t_e) probability of completion of an activity is only 50%.

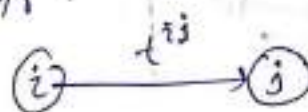
3. In PERT each activity is assumed to follow β -distribution of frequency



CPM

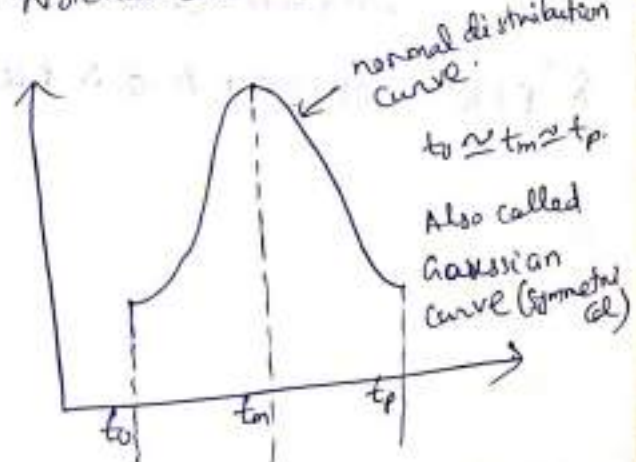
1. It stands for critical path method. It was first developed in 1956 by Kelly and Walker when a Herman company wanted to construct a chemical factory.

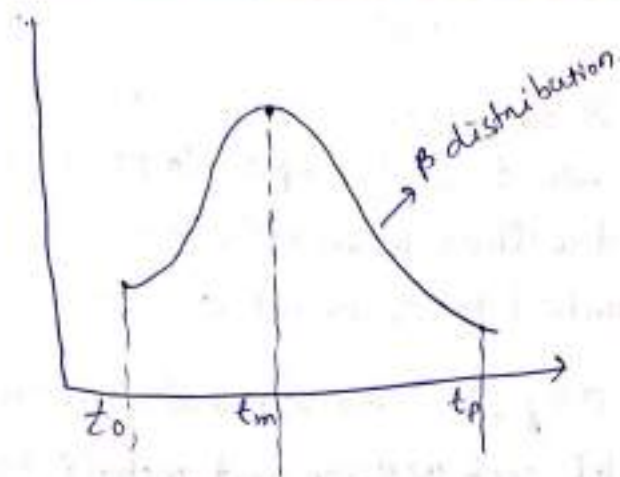
2. Only one time estimates for completion of each activity and probability of completion of an activity in this time is 100%.



t_{ij} → Time of completion of an activity

(3) In CPM, each activity is assumed to follow Normal distribution of frequency





std. deviation,

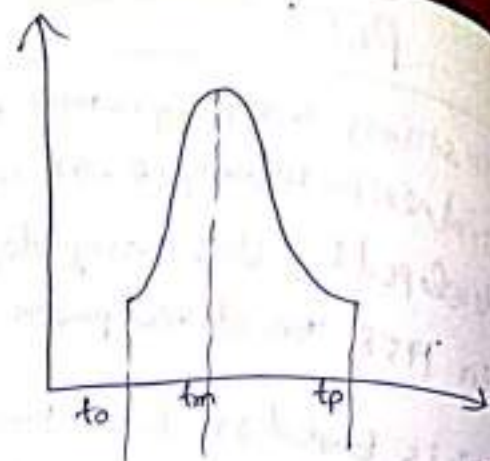
$$\sigma = \frac{t_p - t_o}{6}$$

5. In PERT, time is directly proportional to cost therefore to minimise the cost, time of completion should be minimum

6. PERT is generally used for New or Research and development type of project

7. PERT was event oriented Network.

8. PERT uses only A-o-A Network

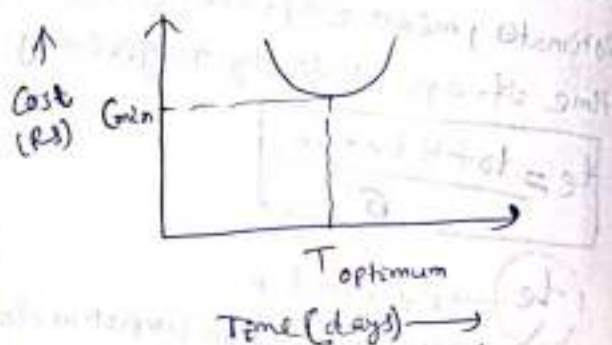


$$(t_m - t_o) \approx (t_p - t_m)$$

$$\sigma \approx 0$$

i.e. certain project.

5. In CPM, time cost model is analysed and min^m cost of project is obtained corresponding to optimum time.

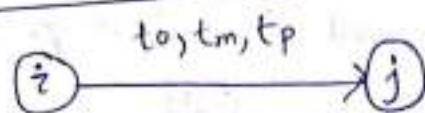


6. CPM is generally used for repetitive type of projects. Such as civil engineering construction project or Mechanical engineering production project.

7. CPM uses activity oriented Network.

8. CPM uses both A-o-A and A-o-N Network

PERT ANALYSIS



* It is based on three time estimates for each activity.

(i) Optimistic time (t_o): It is the min. time in which an activity is expected to complete when condⁿ are most favourable.

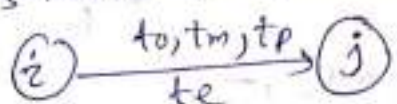
(ii) Pessimistic Time (t_p): It is the max^m time in which an activity is expected to complete under adverse condⁿs or when condⁿs are most unfavourable.

Note while estimating pessimistic time natural climatic anomaly, earthquake, floods, draughts, fire etc. are not considered.

(iii) Most likely time (t_m or t_c)

It is the normal time in which an activity is expected to complete under normal condⁿs.

It reflects mode on β -distribution of frequency curve.

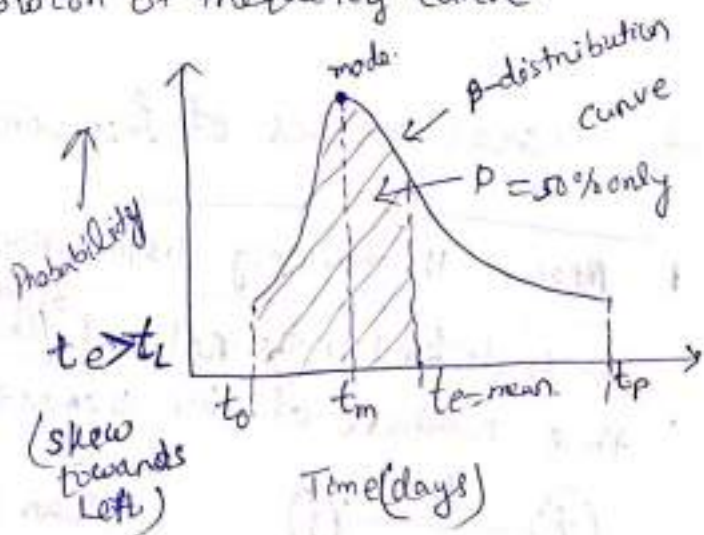


Based on above three time estimates mean expected completion time is taken is weighted avg of all three times.

$$t_e = \frac{1t_o + 4t_m + 1t_p}{1+4+1}$$

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

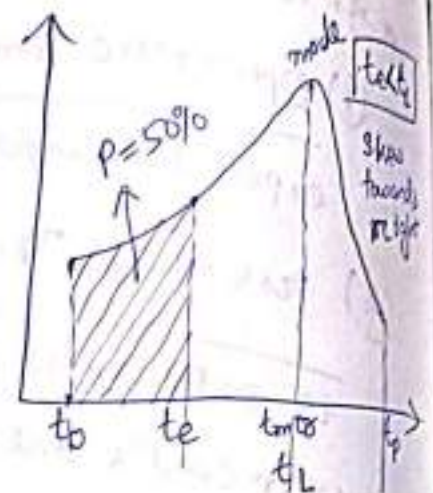
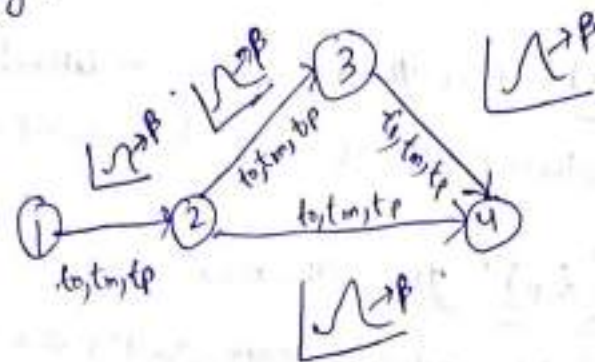
→ expected
→ mean time



The probability of completion of an activity in time t_e is only 50%.

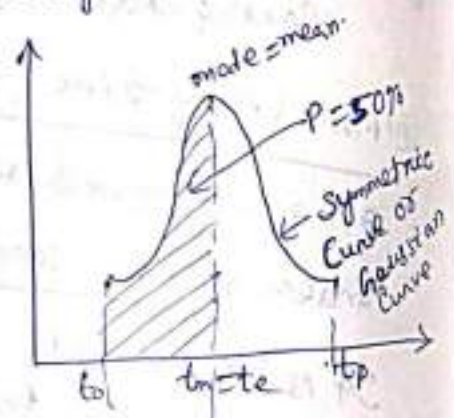
- * In β -distribution of frequency mean and mode are different.
- * In PERT analysis each activity is assumed to follow β -distribution of frequency.

Note



Note

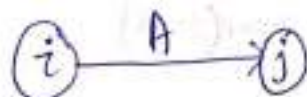
If mean and mode of a frequency curve coincide then such a frequency curve is called Normal frequency distribution curve.



* Characteristics of frequency curve :-

1. Mean of the frequency distribution:

It is defined as the ratio of the ^{algebraic sum of} time reqd. for completion of an activity to the number of time the activity is analysed.



t_1 = 1st time

t_2 = 2nd time

t_3 = 3rd time

t_n = n time.

$$\text{Mean time, } t_{\text{mean}} = \frac{t_1 + t_2 + \dots + t_n}{n}$$

$$\therefore t_{\text{mean}} = \frac{\sum_{i=1}^n t_i}{n}$$

Basic formula.

t_{mean} = expected completion time of an activity

$$t_{\text{mean}} \approx t_e \approx \frac{t_o + 4t_e + t_p}{6}$$

2. Deviation (δ) (Not in x m)

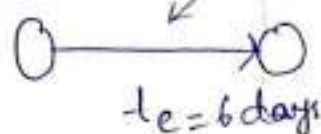
It is defined as the diff. of time need. for completion of an Activity with the mean of that activity.

once came in x m

$$\delta = (t_i - t_{\text{mean}}) \quad * * * *$$

$t_i = 10 \text{ days} = \text{Actual time.}$

e.g.



$$\therefore \delta = (10 - 6) = 4 \text{ days.}$$

3. Variance (σ^2)

* It is defined as mean of square of deviation.

* It is used to represent uncertainty or variability in completion of an Activity.

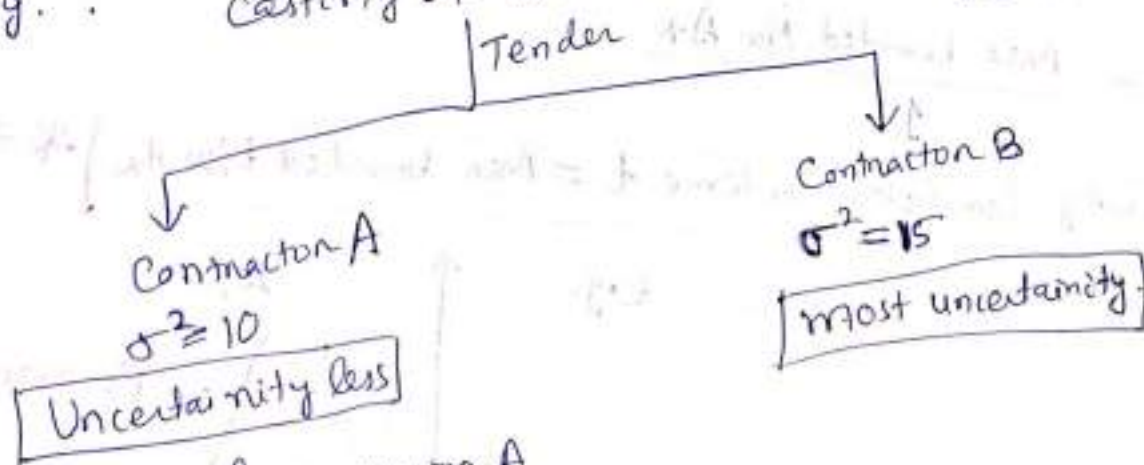
* It is square of standard deviation.

* Greater Variance means greater uncertainty in completion of an activity.

Variance, $\sigma^2 = \frac{\sum (t_i - t_{\text{mean}})^2}{n} \quad * * * * \text{IES 95}$

$\sigma^2 = \left(\frac{t_p - t_o}{6} \right)^2 \quad * * * * \text{for numerical.}$

e.g. . Casting of roof slab ESE-97



Prefer Contractor A
over Contractor B

Standard deviation (σ)

- * It is square root of Variance and its significance is similar to that of Variance.
- * It also represents uncertainty or variability in completion of an activity.

$$\sigma = \sqrt{\text{Variance}}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (t_i - t_{\text{mean}})^2}{n}}$$

* Probability Distribution Curve

Probability distribution curve is similar to frequency curve which is standardised in such a way that Area bounded by the curve is 1 unit.

Area bounded b/w ABC = 1 unit

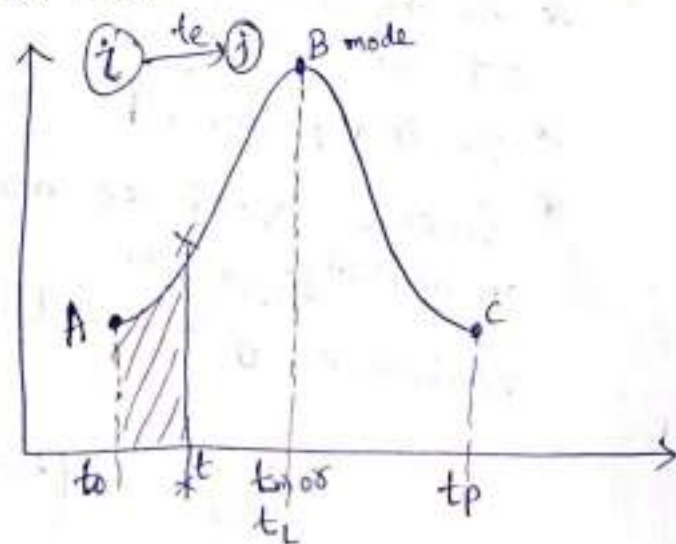
Probability of completion of an activity in time 't' is given by,

$$P = \frac{n(E)}{n(S)} \quad \text{Area}$$

$$= \frac{\text{Area bounded b/w Ax}}{\text{Area bounded b/w ABC}}$$

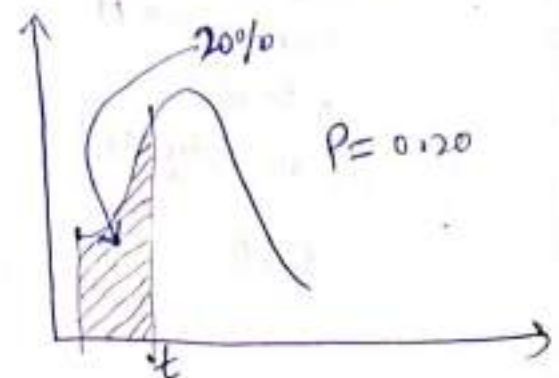
$$= \frac{\text{Area bounded b/w Ax}}{1}$$

Probability
funⁿ



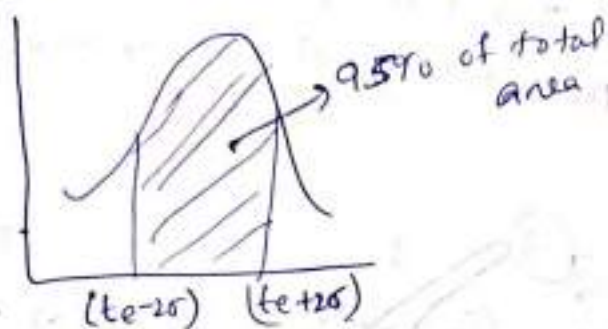
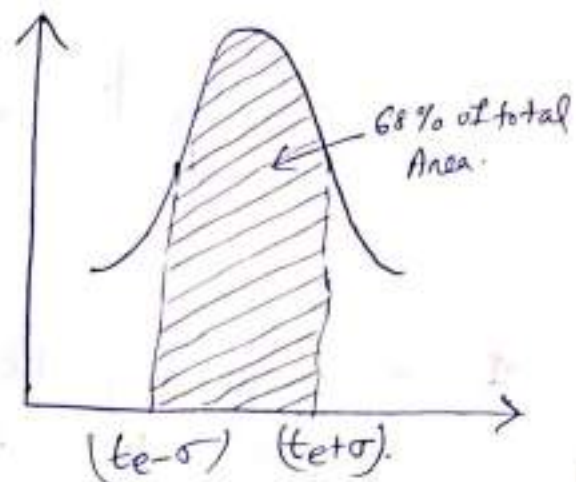
Probability Completion in time $t = \text{Area bounded b/w Ax}$ ****

e.g.



Properties of probability Distribution Curve:-

- * Area bounded b/w $(t_e - \sigma)$ to $(t_e + \sigma)$ is 68% of total area.
- * Area bounded b/w $(t_e - 2\sigma)$ to $(t_e + 2\sigma)$ is 95% of total area.

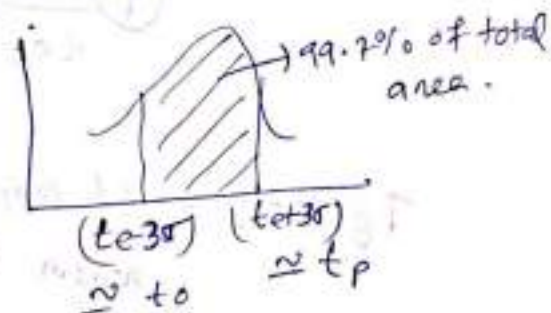


- * Area bounded b/w $(t_e - 3\sigma)$ to $(t_e + 3\sigma)$ is 99.7% of total area.

$$t_o = (t_e - 3\sigma)$$

$$t_p = (t_e + 3\sigma)$$

$$\therefore t_p - t_o = (t_e + 3\sigma) - (t_e - 3\sigma) = 6\sigma$$



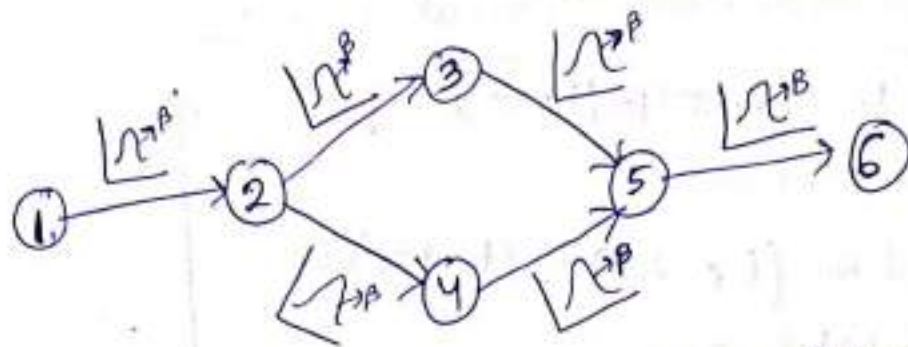
$$\therefore \sigma = \left(\frac{t_p - t_o}{6} \right) \quad \text{*****}$$

- * Normal range of time for completion of an activity is $(t_e - \sigma)$ to $(t_e + \sigma)$

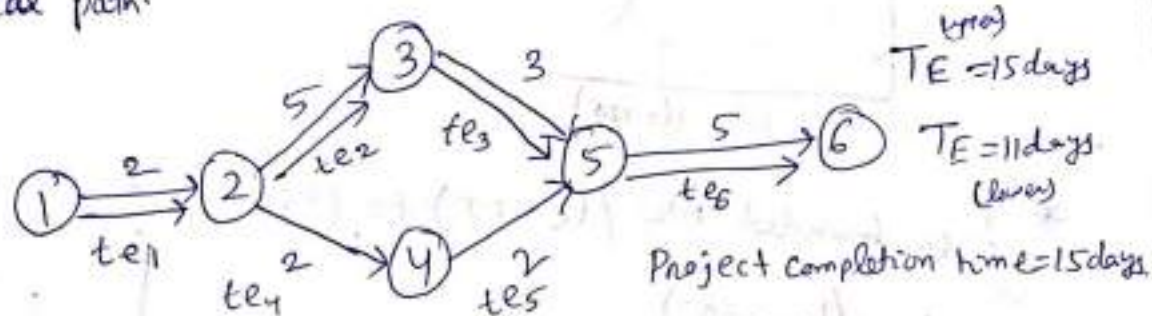
- * Max. Range of time for completion of an activity is $(t - 3\sigma)$ to $(t + 3\sigma)$

Central Limit Theorem:-

- ① If there are various activities on a project network which having β -distribution of frequency then entire project as whole will have normal distribution of frequency.



- ② The total expected project duration is equal to sum of expected completion time of those activities which are critical and located along critical path.



$T_E = 11 \text{ days} \rightarrow$ if project completion time is 11 days then some activity will remain incomplete.

$\therefore$ Activity ①-②, ②-③, ③-⑤ & ⑤-⑥ are critical activity

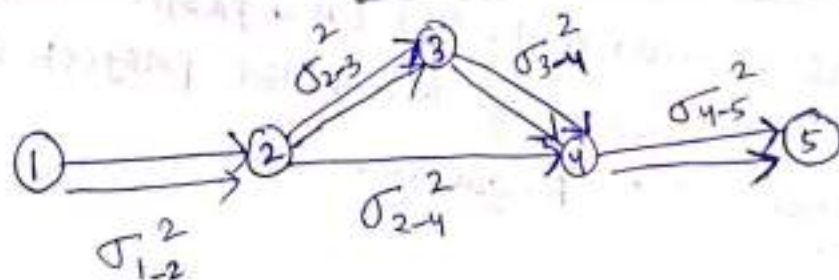
and 1-2-3-5-6 is critical path.

$$\therefore T_E = t_{e1} + t_{e2} + t_{e3} + t_{e6} = 2 + 5 + 3 + 5 = 15 \text{ days}$$

$T_E =$ expected completion duration of project

Probability of completion of a project in time T_E is only 50%.

- ③ The variance of the project is equal to sum of variances of all critical activities which are located along critical path.



Let's consider path, 1-2-3-4-5 is critical.

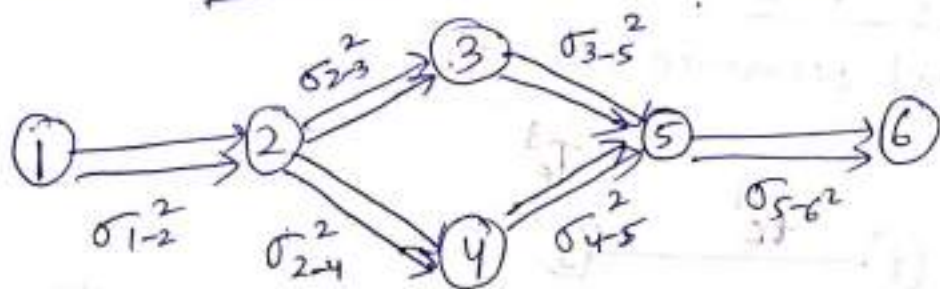
Variance of the project, $\sigma^2 = \sigma_{1-2}^2 + \sigma_{2-3}^2 + \sigma_{3-4}^2 + \sigma_{4-5}^2$ Along critical path.

standard deviation of project, $\sigma = \sqrt{\sigma^2}$

$$\therefore \sigma = \sqrt{\sigma_{1-2}^2 + \sigma_{2-3}^2 + \sigma_{3-4}^2 + \sigma_{4-5}^2} \text{ along critical path}$$

Imp

$$\sigma \neq \sigma_{1-2} + \sigma_{2-3} + \sigma_{3-4} + \sigma_{4-5}$$



Variance along critical path ① [1-2-3-5-6]

$$\sigma_1^2 = \sigma_{1-2}^2 + \sigma_{2-3}^2 + \sigma_{3-5}^2 + \sigma_{5-6}^2$$

Variance along critical path ② [1-2-4-5-6]

$$\sigma_2^2 = \sigma_{1-2}^2 + \sigma_{2-4}^2 + \sigma_{4-5}^2 + \sigma_{5-6}^2$$

$$\therefore \sigma_1^2 < \sigma_2^2$$

path ① is less uncertain.

path ② is most uncertain. select

Take most certain path as critical path.

If there are more than one critical path then priority is given to that path which have greater value of variances. but this path is most uncertain, hence extra attention are required further path.

All critical path having same project duration.

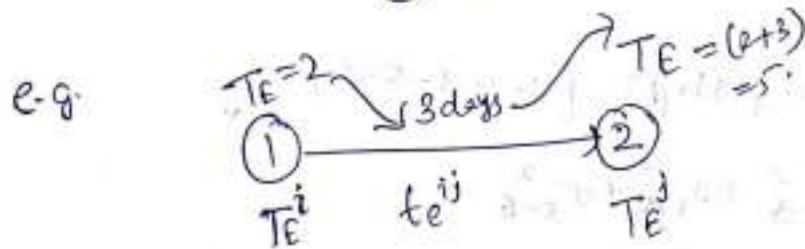
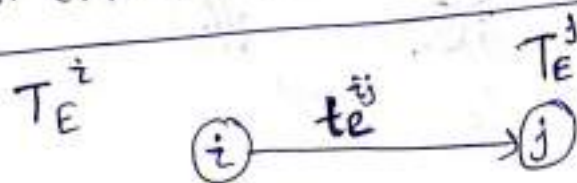
Critical path :- Critical path is time wise longest path connecting start event to finish event.

In a network there may be one critical path but all critical paths having same project duration.

Along the critical path each activity has just proper resources hence any delay in any activity will result in delays of the project. Hence along the critical path each activity and each event are critical. Hence total float for each activity is zero and slack for each event is zero.

Time calculation for event :-

① Earliest expected occurrence time (T_E)

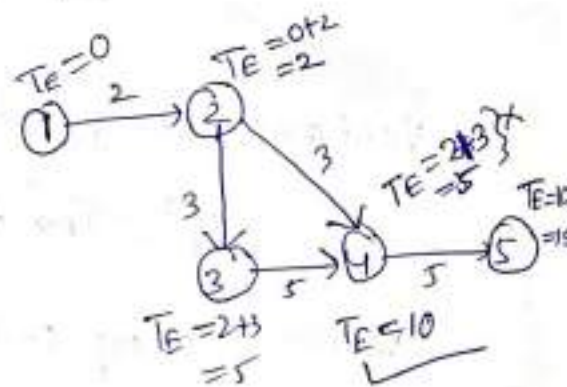


$$T_E^j = T_E^i + t_e^{ij}$$

$$T_E^j = (T_E^i + t_e^{ij})$$

$$T_E^j = (T_E^i + t_e^{ij})_{\max}$$

if more than one path
it is called by forward path.

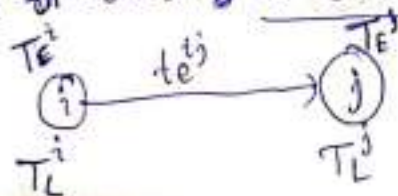


If T_E for the start event is not given then it is taken as zero.

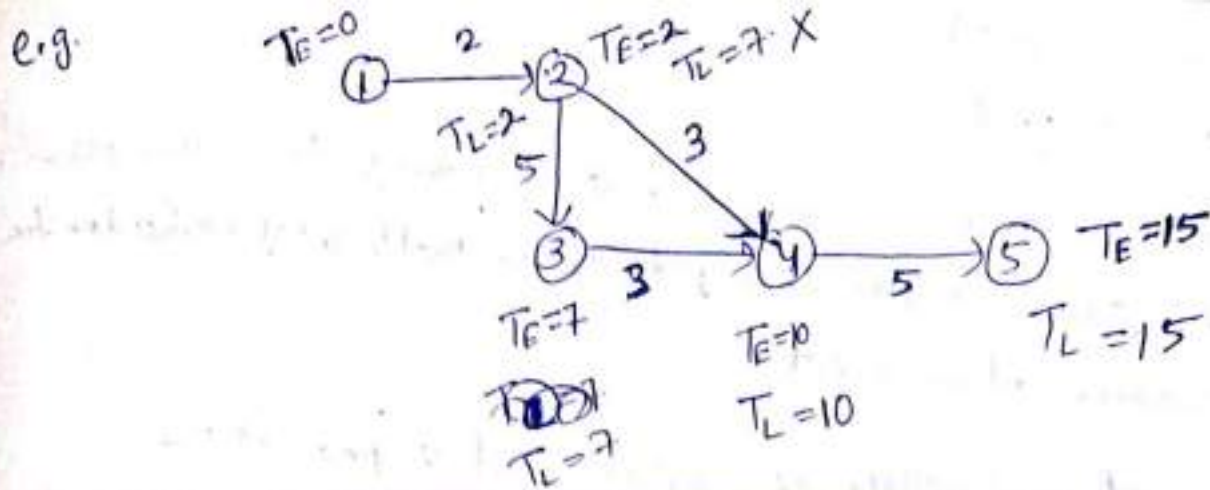
It is the time in which an event is expected to occur as early as possible.

② Latest allowable occurrence time

It is the time in which an event is expected to occur as late as possible.



$$T_L^i = T_L^j - t_e^{ij}$$



$$T_L^i = T_L^j - t_{ij}^{\text{min}}$$

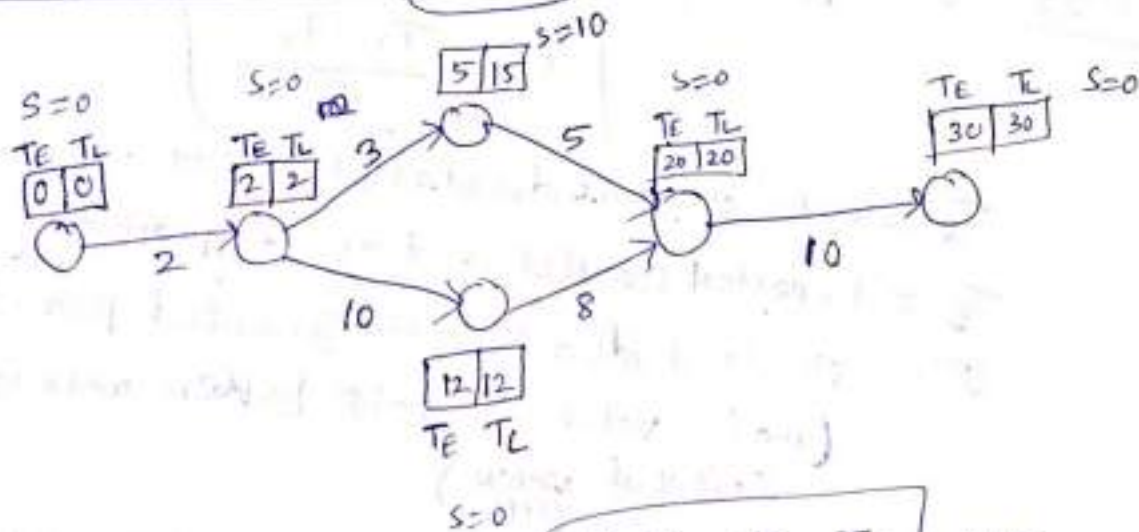
if more than one path.

$$T_L^i = T_L^j - t_{ij}$$

← Calculated by backward path

(random) SLACK(S) :→

Slack of an event represent the excess time which may be available to delay the occurrence of an event without affecting total project duration.



Slack = freedom of event. $\boxed{\text{Slack} = T_L - T_E}$ **

Case ① - If $\boxed{\text{Slack} = 0}$

then event is critical. Activity for occurrence of such event have just adequate resources. So any delay will result in delay of the project.

Case ② - If $\boxed{\text{Slack} > 0}$

then event is subcritical, than activity having enough resources for occurrence of such an event. (degree of freedom > 0 so that delay of this event

will not delay the project).

Case-③ $9 \neq \boxed{\text{slack} < 0}$

than event is supercritical, than activity having improper resources
So occurrence of event has already delayed. that's why no freedom for
late occurrence of an event.

* Calculation of probability of completion of a project: $\rightarrow$

Step ① Draw PERT Network and find T_E and T_C for each event.

Step ② Find slack for each event and then identify critical path. Paths connecting critical event (event of zero slack) is called critical path.

Step ③ Cal. Variance / S.D of project along critical path.

Step ④ Cal. ^{Probability} ~~Factor~~ Factor Z .

$$Z = \frac{T_s - T_E}{\sigma}$$

T_s = given schedule duration of project.

T_E = Expected completion time of project.

σ = standard deviation along critical path. (max^m)

(max^m value of σ is taken when more than one critical ~~value~~ path)

Step ⑤: Cal. probability of completion of project from value of probability factor (given)

Z	P
0	50%
1	84.13%
2	97.72%
3	99.87%

Z	P
0	50%
-1	15.87%
-2	2.28%
-3	0.13%

(130)

Intermediate value can be calculated by linear interpolation.

$$Q. 25. t_o = 25 \text{ min}$$

$$t_p = 48 \text{ min}$$

$$t_m \text{ or } t_L = 35 \text{ min.}$$

expected completion time,

$$t_e = \frac{t_o + 4t_m + t_p}{6} = 35.5 \text{ min. Ans.}$$

$$\text{Variance, } \sigma^2 = \left(\frac{t_p - t_o}{6} \right)^2 = \left(\frac{48 - 25}{6} \right)^2 = 14.69 \text{ min.}$$

$$\therefore \text{Standard deviation, } \sigma = \sqrt{14.69} = 3.83 \text{ min.}$$

$$\text{Time allowed, } = (t_e - 3\sigma) \text{ to } (t_e + 3\sigma)$$

$$= (35.5 - 3 \times 3.83) \text{ to}$$

$$(35.5 + 3 \times 3.83)$$

$$= 24.01 \text{ min to } 46.94 \text{ min}$$

Ans

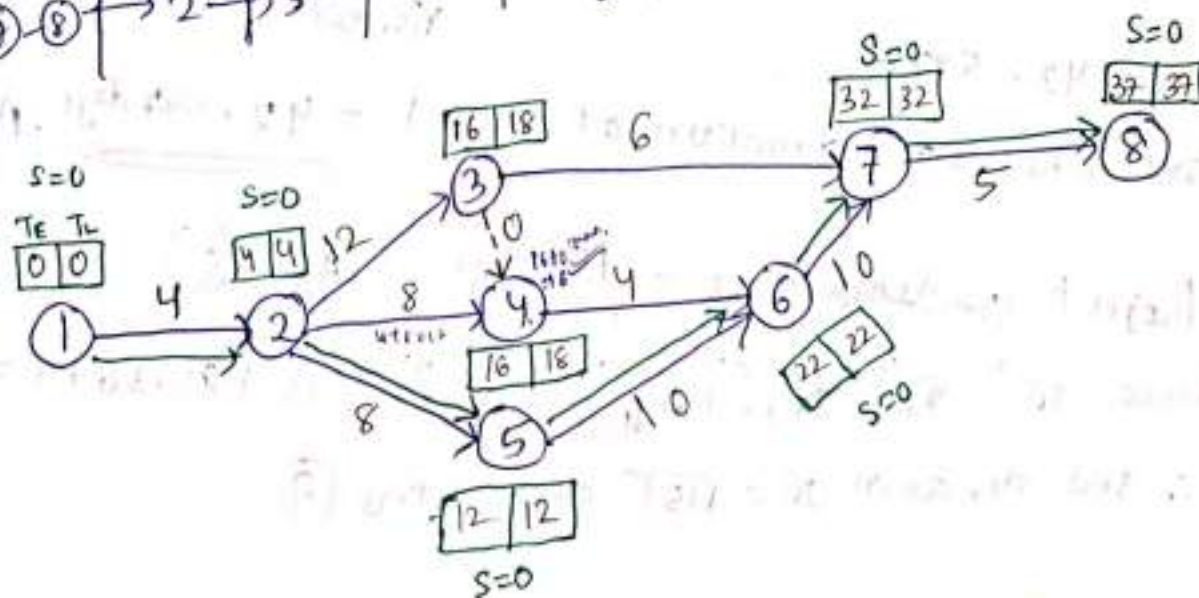
Q. Following is the activity relationship with their 3 time estimates. Draw a PERT network dig. & find

① critical path

② variance

③ what time did you allow. so that prob. of completion is 90%.

Activity	t_o	t_i	t_p	$t_e = \frac{t_o + 4t_m + t_p}{6}$	Std. deviation (σ)
①-②	2	4	6	4	0.67
②-③	8	11	20	12	2
②-④	4	7	16	8	2
②-⑤	3	6	17	8	2.33
③-⑥	3	5	13	6	1.67
③-⑦	0	0	0	0	0
④-⑥	2	3	10	4	1.33
⑤-⑥	4	9	20	10	2.67
⑥-⑦	7	10	13	10	1
⑦-⑧	2	5	8	5	1



(18) Ans

(i) Critical path is 1-2-5-6-7-8 Ans

(ii) Variance for the project = Sum of all variances along critical path

$$\Rightarrow \sigma^2 = \sigma_{1-2}^2 + \sigma_{2-5}^2 + \sigma_{5-6}^2 + \sigma_{6-7}^2 + \sigma_{7-8}^2$$

$$= (0.67)^2 + (2.33)^2 + (2.67)^2 + 1^2 + 1^2 = 15.0067$$

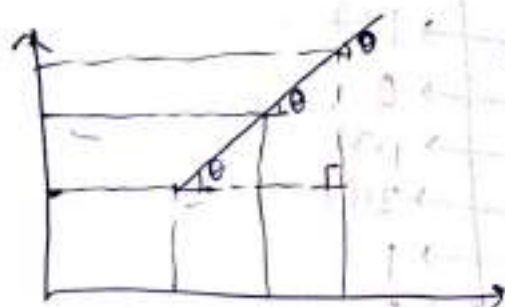
$$\therefore \text{Variance} = 15.0067 \quad \underline{\text{Ans}}$$

(iii) Probability of completion = 90% probability, $T_s = ?$

Project duration, $T_E = 37$ days.

Project factor Z for 90% probability is calculated by linear interpolation

Z	P
0	50%
1	84.13
?	90%
2	97.72



$$\tan \theta = \frac{97.72 - 84.13}{2 - 1} = \frac{90 - 84.13}{2 - 1}$$

$$\therefore Z = 1.432$$

$$\therefore Z = \frac{T_s - T_E}{\sigma}$$

$$\Rightarrow 1.432 = \frac{T_s - 37}{\sqrt{15.0067}}$$

$$\Rightarrow T_s = 42.547 \text{ days.}$$

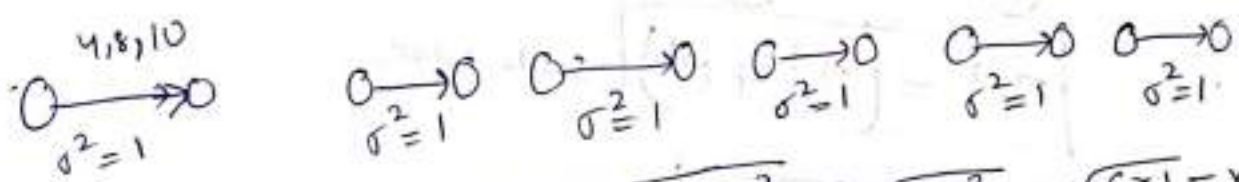
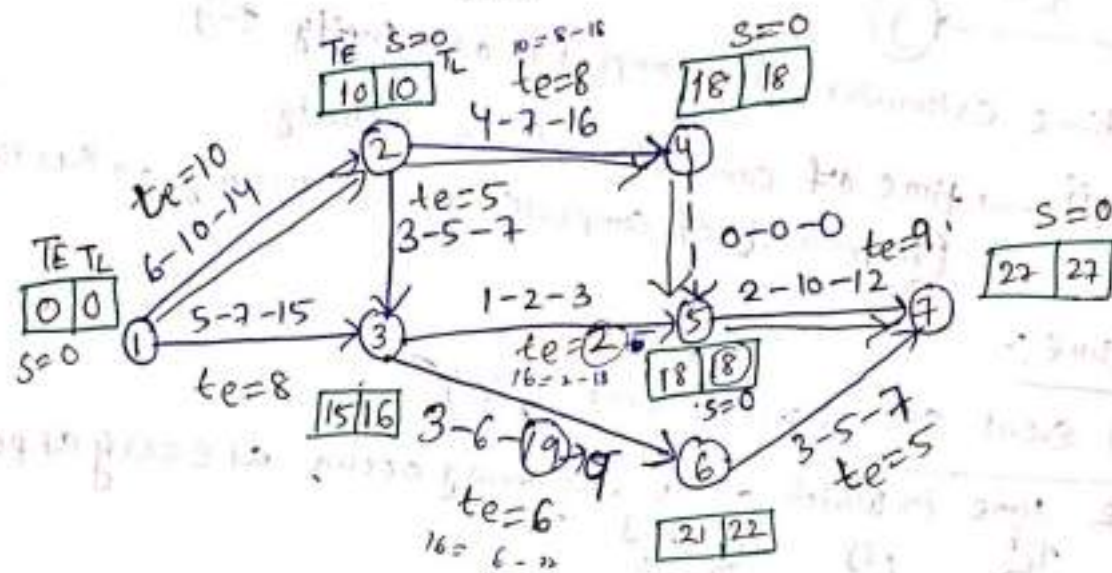
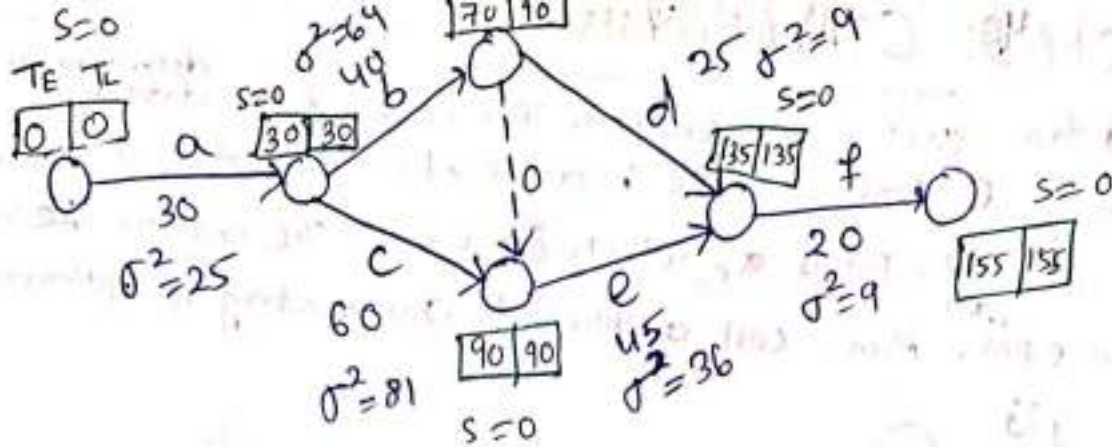
~~and~~ scheduled duration of project = 42.547 days. Ans.

Project completion time = 155 days Ans (8)

$$\text{Variance } \sigma^2 = \sigma_{1-2}^2 + \sigma_{2-1}^2 + \sigma_{4-5}^2 + \sigma_{5-6}^2 = 25 + 81 + 36 + 9 = 151$$

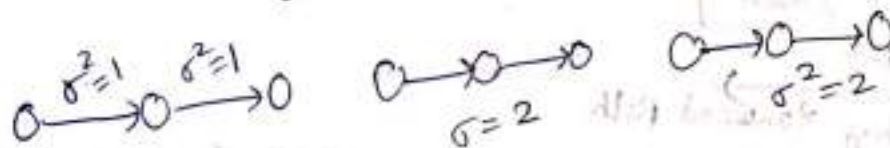
$$\therefore \text{std. deviation } \sigma = \sqrt{151} \text{ days.} \quad \underline{\text{Ans}} \quad (9)$$

20/10/22
Gate 2020
Q.8



Std. deviation, $\sigma = \sqrt{\sigma^2 + \sigma^2 + \dots + \sigma^2} = \sqrt{6\sigma^2} = \sqrt{6 \times 1} = \sqrt{6}$

$\therefore X_1 = \sqrt{6}$



Variance = $\sigma^2 + \sigma^2 + \sigma^2 = 2 + 2 + 2 = 6$

$\therefore$ Std. deviation = $\sqrt{\text{variance}} = \sqrt{6} = \sqrt{6} = X_2$

Ratio = $\frac{X_1}{X_2} = \frac{\sqrt{6}}{\sqrt{6}} = 1$

Q.27

Activity 1: TE=0, TL=4, TF=4, $\sigma^2=1$

Activity 2: TE=4, TL=4, TF=8, $\sigma^2=1$

Activity 3: TE=8, TL=4, TF=12, $\sigma^2=1$

Activity 4: TE=12, TL=4, TF=16, $\sigma^2=1$

Activity 5: TE=16, TL=4, TF=20, $\sigma^2=1$

Activity 6: TE=20, TL=4, TF=24, $\sigma^2=1$

Activity 7: TE=24, TL=4, TF=28, $\sigma^2=1$

Activity 8: TE=28, TL=4, TF=32, $\sigma^2=1$

Activity 9: TE=32, TL=4, TF=36, $\sigma^2=1$

Activity 10: TE=36, TL=4, TF=40, $\sigma^2=1$

Activity 11: TE=40, TL=4, TF=44, $\sigma^2=1$

Activity 12: TE=44, TL=4, TF=48, $\sigma^2=1$

Activity 13: TE=48, TL=4, TF=52, $\sigma^2=1$

Activity 14: TE=52, TL=4, TF=56, $\sigma^2=1$

Activity 15: TE=56, TL=4, TF=60, $\sigma^2=1$

Activity 16: TE=60, TL=4, TF=64, $\sigma^2=1$

Activity 17: TE=64, TL=4, TF=68, $\sigma^2=1$

Activity 18: TE=68, TL=4, TF=72, $\sigma^2=1$

Activity 19: TE=72, TL=4, TF=76, $\sigma^2=1$

Activity 20: TE=76, TL=4, TF=80, $\sigma^2=1$

Activity 21: TE=80, TL=4, TF=84, $\sigma^2=1$

Activity 22: TE=84, TL=4, TF=88, $\sigma^2=1$

Activity 23: TE=88, TL=4, TF=92, $\sigma^2=1$

Activity 24: TE=92, TL=4, TF=96, $\sigma^2=1$

Activity 25: TE=96, TL=4, TF=100, $\sigma^2=1$

Activity 26: TE=100, TL=4, TF=104, $\sigma^2=1$

Activity 27: TE=104, TL=4, TF=108, $\sigma^2=1$

Activity 28: TE=108, TL=4, TF=112, $\sigma^2=1$

Activity 29: TE=112, TL=4, TF=116, $\sigma^2=1$

Activity 30: TE=116, TL=4, TF=120, $\sigma^2=1$

Activity 31: TE=120, TL=4, TF=124, $\sigma^2=1$

Activity 32: TE=124, TL=4, TF=128, $\sigma^2=1$

Activity 33: TE=128, TL=4, TF=132, $\sigma^2=1$

Activity 34: TE=132, TL=4, TF=136, $\sigma^2=1$

Activity 35: TE=136, TL=4, TF=140, $\sigma^2=1$

Activity 36: TE=140, TL=4, TF=144, $\sigma^2=1$

Activity 37: TE=144, TL=4, TF=148, $\sigma^2=1$

Activity 38: TE=148, TL=4, TF=152, $\sigma^2=1$

Activity 39: TE=152, TL=4, TF=156, $\sigma^2=1$

Activity 40: TE=156, TL=4, TF=160, $\sigma^2=1$

Activity 41: TE=160, TL=4, TF=164, $\sigma^2=1$

Activity 42: TE=164, TL=4, TF=168, $\sigma^2=1$

Activity 43: TE=168, TL=4, TF=172, $\sigma^2=1$

Activity 44: TE=172, TL=4, TF=176, $\sigma^2=1$

Activity 45: TE=176, TL=4, TF=180, $\sigma^2=1$

Activity 46: TE=180, TL=4, TF=184, $\sigma^2=1$

Activity 47: TE=184, TL=4, TF=188, $\sigma^2=1$

Activity 48: TE=188, TL=4, TF=192, $\sigma^2=1$

Activity 49: TE=192, TL=4, TF=196, $\sigma^2=1$

Activity 50: TE=196, TL=4, TF=200, $\sigma^2=1$

Activity 51: TE=200, TL=4, TF=204, $\sigma^2=1$

Activity 52: TE=204, TL=4, TF=208, $\sigma^2=1$

Activity 53: TE=208, TL=4, TF=212, $\sigma^2=1$

Activity 54: TE=212, TL=4, TF=216, $\sigma^2=1$

Activity 55: TE=216, TL=4, TF=220, $\sigma^2=1$

Activity 56: TE=220, TL=4, TF=224, $\sigma^2=1$

Activity 57: TE=224, TL=4, TF=228, $\sigma^2=1$

Activity 58: TE=228, TL=4, TF=232, $\sigma^2=1$

Activity 59: TE=232, TL=4, TF=236, $\sigma^2=1$

Activity 60: TE=236, TL=4, TF=240, $\sigma^2=1$

Activity 61: TE=240, TL=4, TF=244, $\sigma^2=1$

Activity 62: TE=244, TL=4, TF=248, $\sigma^2=1$

Activity 63: TE=248, TL=4, TF=252, $\sigma^2=1$

Activity 64: TE=252, TL=4, TF=256, $\sigma^2=1$

Activity 65: TE=256, TL=4, TF=260, $\sigma^2=1$

Activity 66: TE=260, TL=4, TF=264, $\sigma^2=1$

Activity 67: TE=264, TL=4, TF=268, $\sigma^2=1$

Activity 68: TE=268, TL=4, TF=272, $\sigma^2=1$

Activity 69: TE=272, TL=4, TF=276, $\sigma^2=1$

Activity 70: TE=276, TL=4, TF=280, $\sigma^2=1$

Activity 71: TE=280, TL=4, TF=284, $\sigma^2=1$

Activity 72: TE=284, TL=4, TF=288, $\sigma^2=1$

Activity 73: TE=288, TL=4, TF=292, $\sigma^2=1$

Activity 74: TE=292, TL=4, TF=296, $\sigma^2=1$

Activity 75: TE=296, TL=4, TF=300, $\sigma^2=1$

Activity 76: TE=300, TL=4, TF=304, $\sigma^2=1$

Activity 77: TE=304, TL=4, TF=308, $\sigma^2=1$

Activity 78: TE=308, TL=4, TF=312, $\sigma^2=1$

Activity 79: TE=312, TL=4, TF=316, $\sigma^2=1$

Activity 80: TE=316, TL=4, TF=320, $\sigma^2=1$

Activity 81: TE=320, TL=4, TF=324, $\sigma^2=1$

Activity 82: TE=324, TL=4, TF=328, $\sigma^2=1$

Activity 83: TE=328, TL=4, TF=332, $\sigma^2=1$

Activity 84: TE=332, TL=4, TF=336, $\sigma^2=1$

Activity 85: TE=336, TL=4, TF=340, $\sigma^2=1$

Activity 86: TE=340, TL=4, TF=344, $\sigma^2=1$

Activity 87: TE=344, TL=4, TF=348, $\sigma^2=1$

Activity 88: TE=348, TL=4, TF=352, $\sigma^2=1$

Activity 89: TE=352, TL=4, TF=356, $\sigma^2=1$

Activity 90: TE=356, TL=4, TF=360, $\sigma^2=1$

Activity 91: TE=360, TL=4, TF=364, $\sigma^2=1$

Activity 92: TE=364, TL=4, TF=368, $\sigma^2=1$

Activity 93: TE=368, TL=4, TF=372, $\sigma^2=1$

Activity 94: TE=372, TL=4, TF=376, $\sigma^2=1$

Activity 95: TE=376, TL=4, TF=380, $\sigma^2=1$

Activity 96: TE=380, TL=4, TF=384, $\sigma^2=1$

Activity 97: TE=384, TL=4, TF=388, $\sigma^2=1$

Activity 98: TE=388, TL=4, TF=392, $\sigma^2=1$

Activity 99: TE=392, TL=4, TF=396, $\sigma^2=1$

Activity 100: TE=396, TL=4, TF=400, $\sigma^2=1$

Activity 101: TE=400, TL=4, TF=404, $\sigma^2=1$

Activity 102: TE=404, TL=4, TF=408, $\sigma^2=1$

Activity 103: TE=408, TL=4, TF=412, $\sigma^2=1$

Activity 104: TE=412, TL=4, TF=416, $\sigma^2=1$

Activity 105: TE=416, TL=4, TF=420, $\sigma^2=1$

Activity 106: TE=420, TL=4, TF=424, $\sigma^2=1$

Activity 107: TE=424, TL=4, TF=428, $\sigma^2=1$

Activity 108: TE=428, TL=4, TF=432, $\sigma^2=1$

Activity 109: TE=432, TL=4, TF=436, $\sigma^2=1$

Activity 110: TE=436, TL=4, TF=440, $\sigma^2=1$

Activity 111: TE=440, TL=4, TF=444, $\sigma^2=1$

Activity 112: TE=444, TL=4, TF=448, $\sigma^2=1$

Activity 113: TE=448, TL=4, TF=452, $\sigma^2=1$

Activity 114: TE=452, TL=4, TF=456, $\sigma^2=1$

Activity 115: TE=456, TL=4, TF=460, $\sigma^2=1$

Activity 116: TE=460, TL=4, TF=464, $\sigma^2=1$

Activity 117: TE=464, TL=4, TF=468, $\sigma^2=1$

Activity 118: TE=468, TL=4, TF=472, $\sigma^2=1$

Activity 119: TE=472, TL=4, TF=476, $\sigma^2=1$

Activity 120: TE=476, TL=4, TF=480, $\sigma^2=1$

Activity 121: TE=480, TL=4, TF=484, $\sigma^2=1$

Activity 122: TE=484, TL=4, TF=488, $\sigma^2=1$

Activity 123: TE=488, TL=4, TF=492, $\sigma^2=1$

Activity 124: TE=492, TL=4, TF=496, $\sigma^2=1$

Activity 125: TE=496, TL=4, TF=500, $\sigma^2=1$

Activity 126: TE=500, TL=4, TF=504, $\sigma^2=1$

Activity 127: TE=504, TL=4, TF=508, $\sigma^2=1$

Activity 128: TE=508, TL=4, TF=512, $\sigma^2=1$

Activity 129: TE=512, TL=4, TF=516, $\sigma^2=1$

Activity 130: TE=516, TL=4, TF=520, $\sigma^2=1$

Activity 131: TE=520, TL=4, TF=524, $\sigma^2=1$

Activity 132: TE=524, TL=4, TF=528, $\sigma^2=1$

Activity 133: TE=528, TL=4, TF=532, $\sigma^2=1$

Activity 134: TE=532, TL=4, TF=536, $\sigma^2=1$

Activity 135: TE=536, TL=4, TF=540, $\sigma^2=1$

Activity 136: TE=540, TL=4, TF=544, $\sigma^2=1$

Activity 137: TE=544, TL=4, TF=548, $\sigma^2=1$

Activity 138: TE=548, TL=4, TF=552, $\sigma^2=1$

Activity 139: TE=552, TL=4, TF=556, $\sigma^2=1$

Activity 140: TE=556, TL=4, TF=560, $\sigma^2=1$

Activity 141: TE=560, TL=4, TF=564, $\sigma^2=1$

Activity 142: TE=564, TL=4, TF=568, $\sigma^2=1$

Activity 143: TE=568, TL=4, TF=572, $\sigma^2=1$

Activity 144: TE=572, TL=4, TF=576, $\sigma^2=1$

Activity 145: TE=576, TL=4, TF=580, $\sigma^2=1$

Activity 146: TE=580, TL=4, TF=584, $\sigma^2=1$

Activity 147: TE=584, TL=4, TF=588, $\sigma^2=1$

Activity 148: TE=588, TL=4, TF=592, $\sigma^2=1$

Activity 149: TE=592, TL=4, TF=596, $\sigma^2=1$

Activity 150: TE=596, TL=4, TF=600, $\sigma^2=1$

Activity 151: TE=600, TL=4, TF=604, $\sigma^2=1$

Activity 152: TE=604, TL=4, TF=608, $\sigma^2=1$

Activity 153: TE=608, TL=4, TF=612, $\sigma^2=1$

Activity 154: TE=612, TL=4, TF=616, $\sigma^2=1$

Activity 155: TE=616, TL=4, TF=620, $\sigma^2=1$

Activity 156: TE=620, TL=4, TF=624, $\sigma^2=1$

Activity 157: TE=624, TL=4, TF=628, $\sigma^2=1$

Activity 158: TE=628, TL=4, TF=632, $\sigma^2=1$

Activity 159: TE=632, TL=4, TF=636, $\sigma^2=1$

Activity 160: TE=636, TL=4, TF=640, $\sigma^2=1$

Activity 161: TE=640, TL=4, TF=644, $\sigma^2=1$

Activity 162: TE=644, TL=4, TF=648, $\sigma^2=1$

Activity 163: TE=648, TL=4, TF=652, $\sigma^2=1$

Activity 164: TE=652, TL=4, TF=656, $\sigma^2=1$

Activity 165: TE=656, TL=4, TF=660, $\sigma^2=1$

Activity 166: TE=660, TL=4, TF=664, $\sigma^2=1$

Activity 167: TE=664, TL=4, TF=668, $\sigma^2=1$

Activity 168: TE=668, TL=4, TF=672, $\sigma^2=1$

Activity 169: TE=672, TL=4, TF=676, $\sigma^2=1$

Activity 170: TE=676, TL=4, TF=680, $\sigma^2=1$

Activity 171: TE=680, TL=4, TF=684, $\sigma^2=1$

Activity 172: TE=684, TL=4, TF=688, $\sigma^2=1$

Activity 173: TE=688, TL=4, TF=692, $\sigma^2=1$

Activity 174: TE=692, TL=4, TF=696, $\sigma^2=1$

Activity 175: TE=696, TL=4, TF=700, $\sigma^2=1$

Activity 176: TE=700, TL=4, TF=704, $\sigma^2=1$

Activity 177: TE=704, TL=4, TF=708, $\sigma^2=1$

Activity 178: TE=708, TL=4, TF=712, $\sigma^2=1$

Activity 179: TE=712, TL=4, TF=716, $\sigma^2=1$

Activity 180: TE=716, TL=4, TF=720, $\sigma^2=1$

Activity 181: TE=720, TL=4, TF=724, $\sigma^2=1$

Activity 182: TE=724, TL=4, TF=728, $\sigma^2=1$

Activity 183: TE=728, TL=4, TF=732, $\sigma^2=1$

Activity 184: TE=732, TL=4, TF=736, $\sigma^2=1$

Activity 185: TE=736, TL=4, TF=740, $\sigma^2=1$

Activity 186: TE=740, TL=4, TF=744, $\sigma^2=1$

Activity 187: TE=744, TL=4, TF=748, $\sigma^2=1$

Activity 188: TE=748, TL=4, TF=752, $\sigma^2=1$

Activity 189: TE=752, TL=4, TF=756, $\sigma^2=1$

Activity 190: TE=756, TL=4, TF=760, $\sigma^2=1$

Activity 191: TE=760, TL=4, TF=764, $\sigma^2=1$

Activity 192: TE=764, TL=4, TF=768, $\sigma^2=1$

Activity 193: TE=768, TL=4, TF=772, $\sigma^2=1$

Activity 194: TE=772, TL=4, TF=776, $\sigma^2=1$

Activity 195: TE=776, TL=4, TF=780, $\sigma^2=1$

Activity 196: TE=780, TL=4, TF=784, $\sigma^2=1$

Activity 197: TE=784, TL=4, TF=788, $\sigma^2=1$

Activity 198: TE=788, TL=4, TF=792, $\sigma^2=1$

Activity 199: TE=792, TL=4, TF=796, $\sigma^2=1$

Activity 200: TE=796, TL=4, TF=800, $\sigma^2=1$

Activity 201: TE=800, TL=4, TF=804, $\sigma^2=1$

Activity 202: TE=804, TL=4, TF=808, $\sigma^2=1$

Activity 203: TE=808, TL=4, TF=812, $\sigma^2=1$

Activity 204: TE=812, TL=4, TF=816, $\sigma^2=1$

Activity 205: TE=816, TL=4, TF=820, $\sigma^2=1$

Activity 206: TE=820, TL=4, TF=824, $\sigma^2=1$

Activity 207: TE=824, TL=4, TF=828, $\sigma^2=1$

Activity 208: TE=828, TL=4, TF=832, $\sigma^2=1$

Activity 209: TE=832, TL=4, TF=836, $\sigma^2=1$

Activity 210: TE=836, TL=4, TF=840, $\sigma^2=1$

Activity 211: TE=840, TL=4, TF=844, $\sigma^2=1$

Activity 212: TE=844, TL=4, TF=848, $\sigma^2=1$

Activity 213: TE=848, TL=4, TF=852, $\sigma^2=1$

Activity 214: TE=852, TL=4, TF=856, $\sigma^2=1$

Activity 215: TE=856, TL=4, TF=860, $\sigma^2=1$

Activity 216: TE=860, TL=4, TF=864, $\sigma^2=1$

Activity 217: TE=864, TL=4, TF=868, $\sigma^2=1$

Activity 218: TE=868, TL=4, TF=872, $\sigma^2=1$

Activity 219: TE=872, TL=4, TF=876, $\sigma^2=1$

Activity 220: TE=876, TL=4, TF=880, $\sigma^2=1$

Activity 221: TE=880, TL=4, TF=884, $\sigma^2=1$

Activity 222: TE=884, TL=4, TF=888, $\sigma^2=1$

Activity 223: TE=888, TL=4, TF=892, $\sigma^2=1$

Activity 224: TE=892, TL=4, TF=896, $\sigma^2=1$

Activity 225: TE=896, TL=4, TF=900, $\sigma^2=1$

Activity 226: TE=900, TL=4, TF=904, $\sigma^2=1$

Activity 227: TE=904, TL=4, TF=908, $\sigma^2=1$

Activity 228: TE=908, TL=4, TF=912, $\sigma^2=1$

Activity 229: TE=912, TL=4, TF=916, $\sigma^2=1$

Activity 230: TE=916, TL=4, TF=920, $\sigma^2=1$

Activity 231: TE=920, TL=4, TF=924, $\sigma^2=1$

Activity 232: TE=924, TL=4, TF=928, $\sigma^2=1$

Activity 233: TE=928, TL=4, TF=932, $\sigma^2=1$

Activity 234: TE=932, TL=4, TF=936, $\sigma^2=1$

Activity 235: TE=936, TL=4, TF=940, $\sigma^2=1$

Activity 236: TE=940, TL=4, TF=944, $\sigma^2=1$

Activity 237: TE=944, TL=4, TF=948, $\sigma^2=1$

Activity 238: TE=948, TL=4, TF=952, $\sigma^2=1$

Activity 239: TE=952, TL=4, TF=956, $\sigma^2=1$

Activity 240: TE=956, TL=4, TF=960, $\sigma^2=1$

Activity 241: TE=960, TL=4, TF=964, $\sigma^2=1$

Activity 242: TE=964, TL=4, TF=968, $\sigma^2=1$

Activity 243: TE=968, TL=4, TF=972, $\sigma^2=1$

Activity 244: TE=972, TL=4, TF=976, $\sigma^2=1$

Activity 245: TE=976, TL=4, TF=980, $\sigma^2=1$

Activity 246: TE=980, TL=4, TF=984, $\sigma^2=1$

Activity 247: TE=984, TL=4, TF=988, $\sigma^2=1$

Activity 248: TE=988, TL=4, TF=992, $\sigma^2=1$

Activity 249: TE=992, TL=4, TF=996, $\sigma^2=1$

Activity 250: TE=996, TL=4, TF=1000, $\sigma^2=1$

Activity 251: TE=1000, TL=4, TF=1004, $\sigma^2=1$

Activity 252: TE=1004, TL=4, TF=1008, $\sigma^2=1$

Activity 253: TE=1008, TL=4, TF=1012, $\sigma^2=1$

Activity 254: TE=1012, TL=4, TF=1016, $\sigma^2=1$

Activity 255: TE=1016, TL=4, TF=1020, $\sigma^2=1$

Activity 256: TE=1020, TL=4, TF=1024, $\sigma^2=1$

Activity 257: TE=1024, TL=4, TF=1028, $\sigma^2=1$

Activity 258: TE=1028, TL=4, TF=1032, $\sigma^2=1$

Activity 259: TE=1032, TL=4, TF=1036, $\sigma^2=1$

Activity 260: TE=1036, TL=4, TF=1040, $\sigma^2=1$

Activity 261: TE=1040, TL=4, TF=1044, $\sigma^2=1$

Activity 262: TE=1044, TL=4, TF=1048, $\sigma^2=1$

Activity 263: TE=1048, TL=4, TF=1052, $\sigma^2=1$

Activity 264: TE=1052, TL=4, TF=1056, $\sigma^2=1$

Activity 265: TE=1056, TL=4, TF=1060, $\sigma^2=1$

Activity 266: TE=1060, TL=4, TF=1064, $\sigma^2=1$

Activity 267: TE=1064, TL=4, TF=1068, $\sigma^2=1$

Activity 268: TE=1068, TL=4, TF=1072, $\sigma^2=1$

Activity 269: TE=1072, TL=4, TF=1076, $\sigma^2=1$

Activity 270: TE=1076, TL=4, TF=1080, $\sigma^2=1$

Activity 271: TE=1080, TL=4, TF=1084, $\sigma^2=1$

Activity 272: TE=1084, TL=4, TF=1088, $\sigma^2=1$

Activity 273: TE=1088, TL=4, TF=1092, $\sigma^2=1$

Activity 274: TE=1092, TL=4, TF=1096, $\sigma^2=1$

Activity 275: TE=1096, TL=4, TF=1100, $\sigma^2=1$

Activity 276: TE=1100, TL=4, TF=1104, $\sigma^2=1$

Activity 277: TE=1104, TL=4, TF=1108, $\sigma^2=1$

Activity 278: TE=1108, TL=4, TF=1112, $\sigma^2=1$

Activity 279: TE=1112, TL=4, TF=1116, $\sigma^2=1$

Activity 280: TE=1116, TL=4, TF=1120, $\sigma^2=1$

Activity 281: TE=1120, TL=4, TF=1124, $\sigma^2=1$

Activity 282: TE=1124, TL=4, TF=1128, $\sigma^2=1$

Activity 283: TE=1128, TL=4, TF=1132, $\sigma^2=1$

Activity 284: TE=1132, TL=4, TF=1136, $\sigma^2=1$

Activity 285: TE=1136, TL=4, TF=1140, $\sigma^2=1$

Activity 286: TE=1140, TL=4, TF=1144, $\sigma^2=1$

Activity 287: TE=1144, TL=4, TF=1148, $\sigma^2=1$

Activity 288: TE=1148, TL=4, TF=1152, $\sigma^2=1</$

Chp-4: CPM ANALYSIS

CPM stands for critical path method. This is based on ~~data~~ deterministic approach in which only one time estimate is made after completion of each activity. The network diagram in CPM is activity on event. The cost is the most imp. criteria in CPM. Min. cost is obtained corresponding to optimum time.



one time estimates for completion of activity i-j

t_{ij} → time of completion of an activity.
[probability of completion of an activity in this time is 100%]

Event time:-

1. Earliest event occurrence Time (T_E)

It is the time in which an event may occur as early as possible.



$$T_E^j = (T_E^i + t_{ij})$$

single activity

forward path.

$$T_E^j = (T_E^i + t_{ij})_{\max}$$

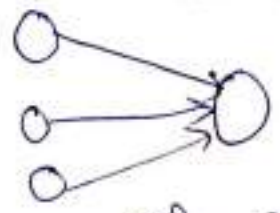
if more than one path

many activity

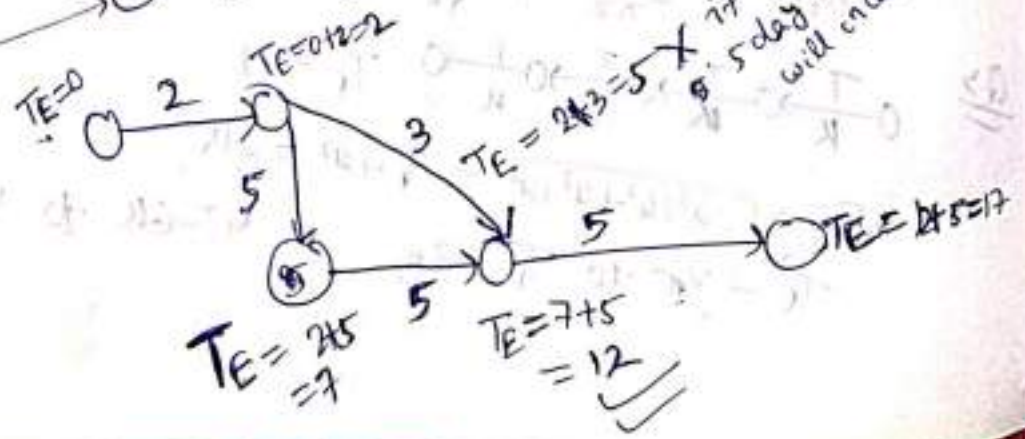
Calculated by forward path.

Imp. if T_E for initial event is not given then it is taken as zero.

e.g.

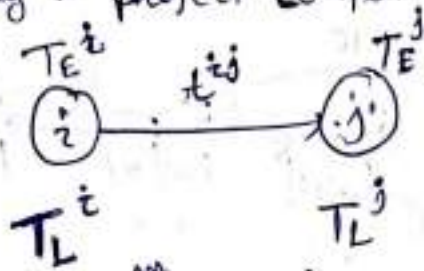


$$T_E^j = (T_E^i + t_{ij})_{\max}$$

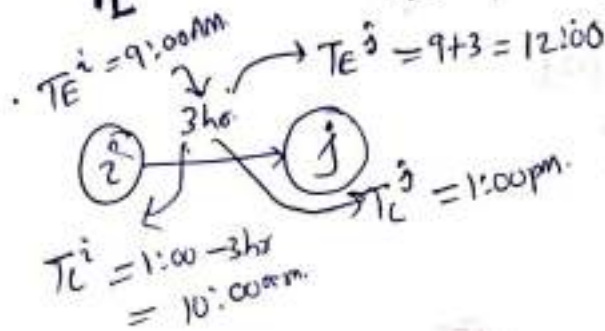


② Latest allowable Occurrence Time (T_L)

It is the latest ^(max) time by which an event may occur as late as possible without delay in project completion time.



e.g.



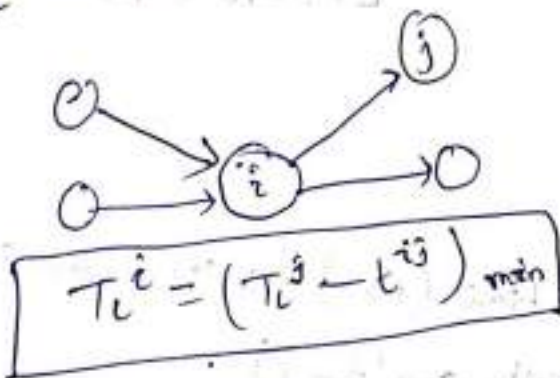
We have 4 hr. from 9:00 AM to 1:00 PM, we have to take 3 hr. at least.

T_E^i = initial event = 9:00

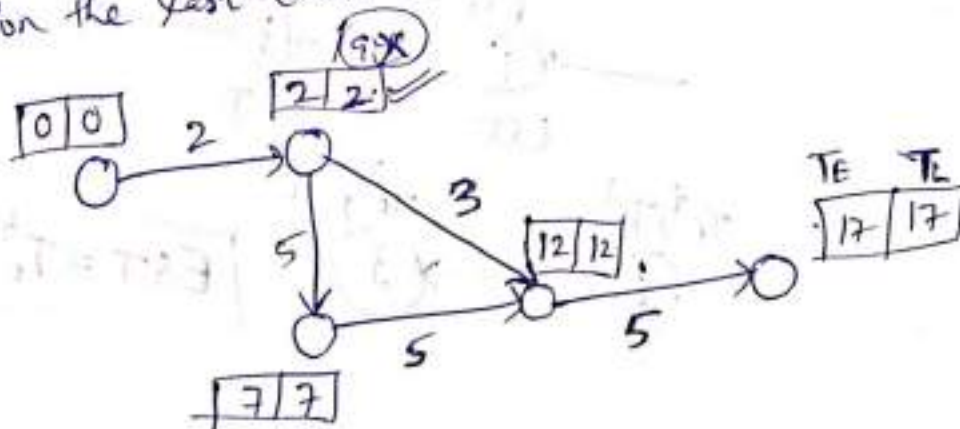
$T_E^j = 9 + 3 = 12:00$

$T_L^j = 1:00 \text{ PM}$

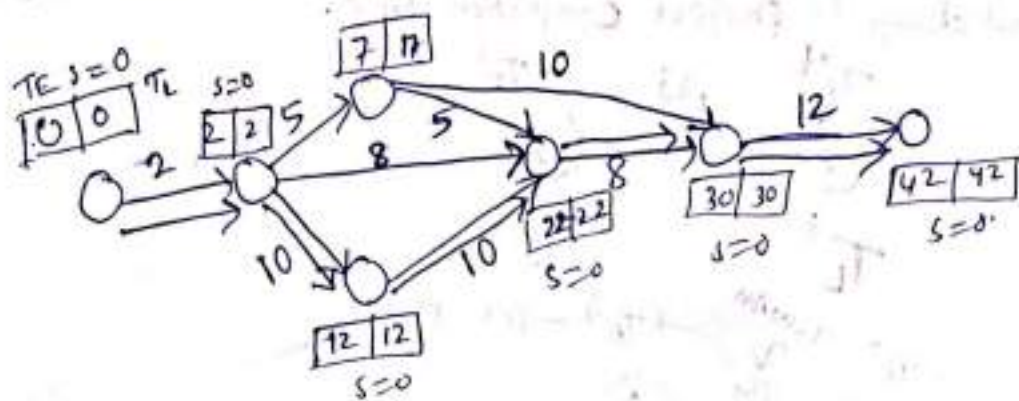
$T_L^i = 1:00 \text{ PM} - 3 \text{ hr} = 10:00 \text{ AM}$



T_L for the last event is taken as T_E for the last event.

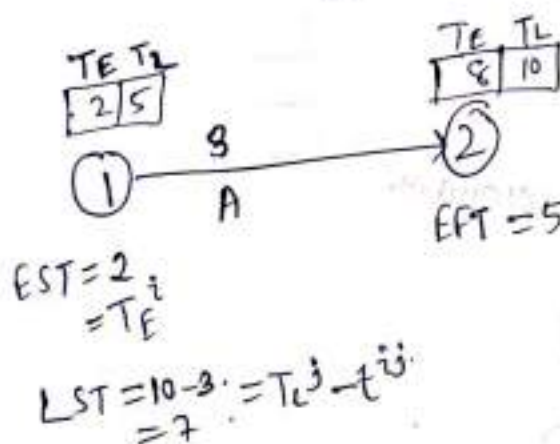
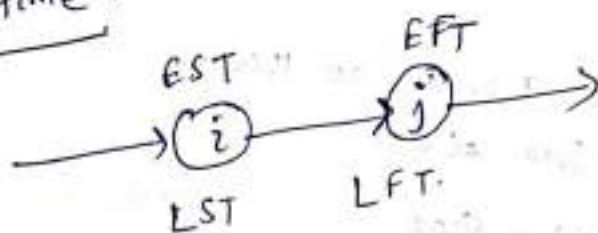


9x) If we will take a than project will complete in 24 days, 30 min time
2 days is considered for no delay in project.



⇒ is critical path.

Activity time
✓ es ①

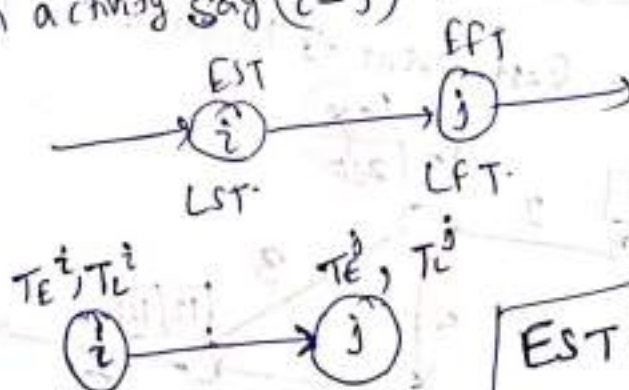


$$\begin{aligned} EST &= T_E^i \\ EFT &= T_E^i + t^i \\ LFT &= T_L^j \\ LST &= T_L^j - t^i \end{aligned}$$

$$\begin{aligned} EFT &= 5 = 2 + 3 = T_E^i + t^i \\ LFT &= 10 = T_L^j \end{aligned}$$

There are 4 activity time

① Earliest start time (EST) → It is the earliest possible start time for an activity say $(i-j)$.



$$EST = T_E^i$$

② Earliest finish time (EFT) $\rightarrow$ It is the earliest possible finish time for an activity. For EFT an activity should start at EST.

$$EFT = TE^i + t^{ij}$$

③ Latest finish time (LFT) $\rightarrow$ It is the time in which an activity may allow to finish as late as possible without delay in project duration time.



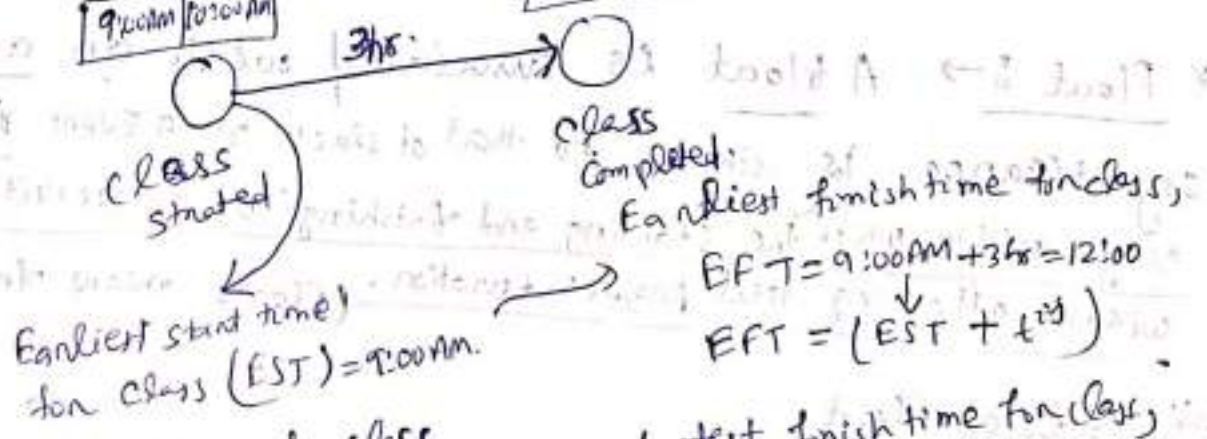
④ Latest start time (LST) $\rightarrow$ It is the latest start time for an activity, so that an activity should be completed as late as possible without delay in project completion time.

$$LST = TL^j - t^{ij}$$

✓ e.g. Refer e.g. ① prev. page.

TE	TL
9:00 AM	10:00 AM

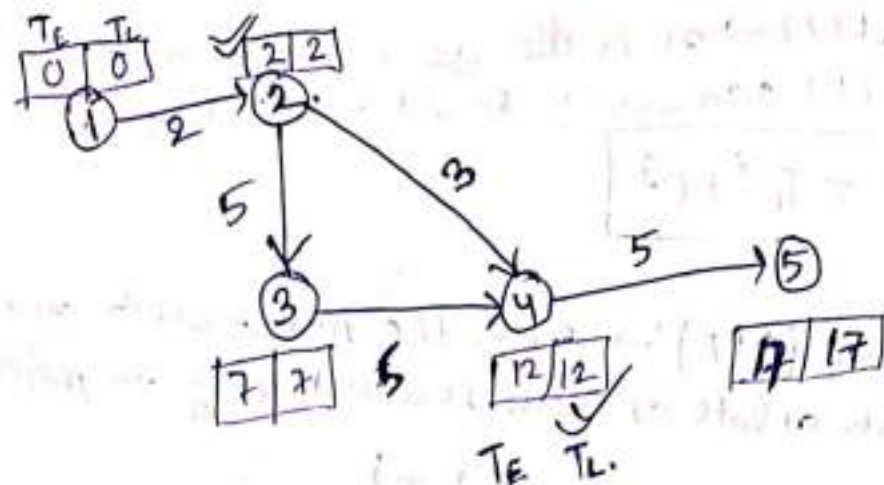
TE	TL
12:00	1:00 PM



Lat est start time for class
 $LST = 1:00 - 3hrs = 10:00 AM$

$$LST = LFT - t^{ij}$$

Q:



Find EST, EFT, LFT and
LST for activity
②-④

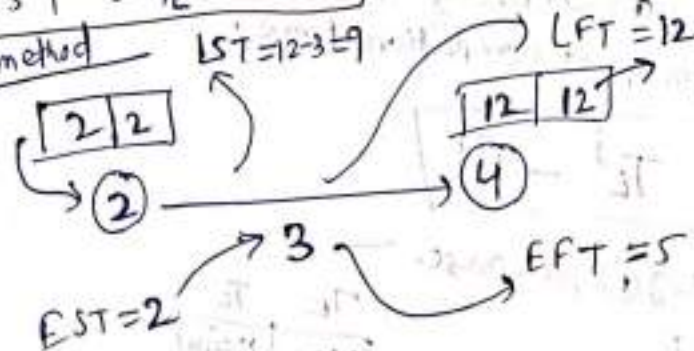
$$EST = T_E' = 2$$

$$EFT = T_E' + t^j = 2 + 3 = 5$$

$$LFT = T_L^j = 12$$

$$LST = T_L^j - t^j = 12 - 3 = 9$$

Alternate method

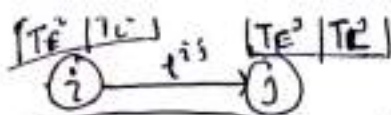


* Float $\rightarrow$ A float is associated with an activity and its significance is similar to that of slack in a event. Float denotes the range within which the starting and finishing of an activity may vary without affecting total project duration. (floats means freedom of activity)

* Types of Float

- ① Total float (FT)
- ② Free float (FF)
- ③ Independent float (FID)
- ④ Interfering " (FIN)

① Total float (FT) $\rightarrow$ It is the max^m time by which either starting or completion of an activity may delay without affecting schedule duration of project.



$$EST = TE^i$$

$$LST = (TL^i - t^{ij})$$

$$EFT = TE^i + t^{ij}$$

$$LFT = TL^j$$

Freedom = LST - EST
(starting delay) = starting delay.

Completion delay
= LFT - EFT
= $TL^j - TE^i - t^{ij}$

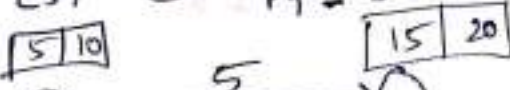
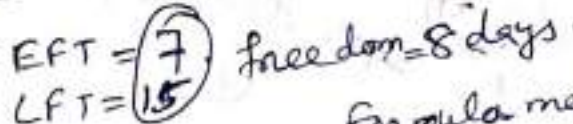
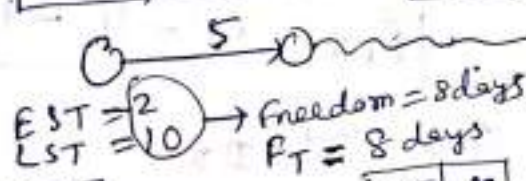
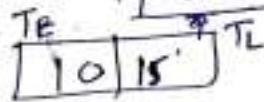
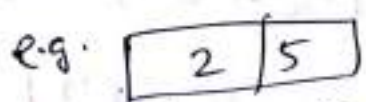
$$FT = TL^j - TE^i - t^{ij}$$

$\therefore$ Time & float = starting delay or freedom

$$= LST - EST$$

$$= TL^j - t^{ij} - TE^i =$$

$$TL^j - TE^i - t^{ij}$$

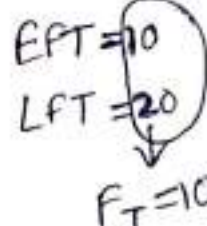
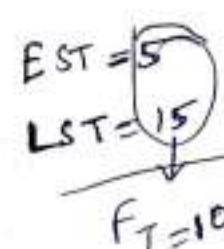


$FT = 10$

Formula method

$$FT = TL^j - TE^i - t^{ij}$$

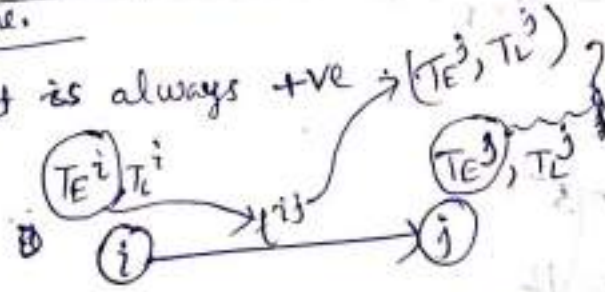
$$= 15 - 2 - 5 = 8 \text{ days}$$



② Free float (FF):

It is that portion of the total float which can be used by an activity for completion delay without affecting total project duration (TPD).
For calc of free float all activities must be started at their earliest start time.

Free float is always +ve $\rightarrow (TE^j, TL^j)$



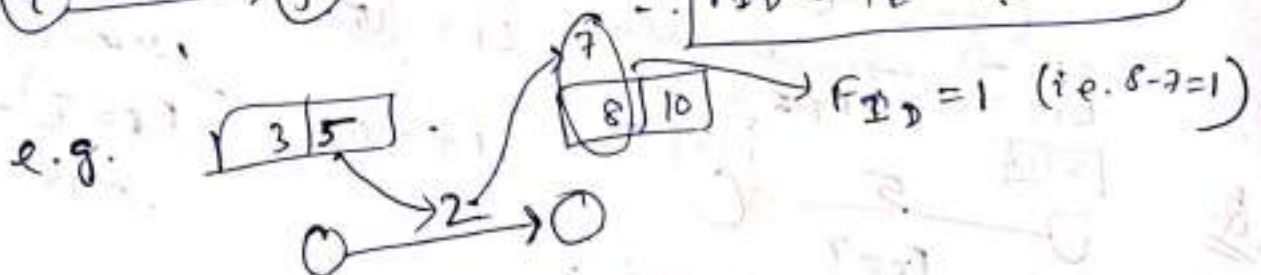
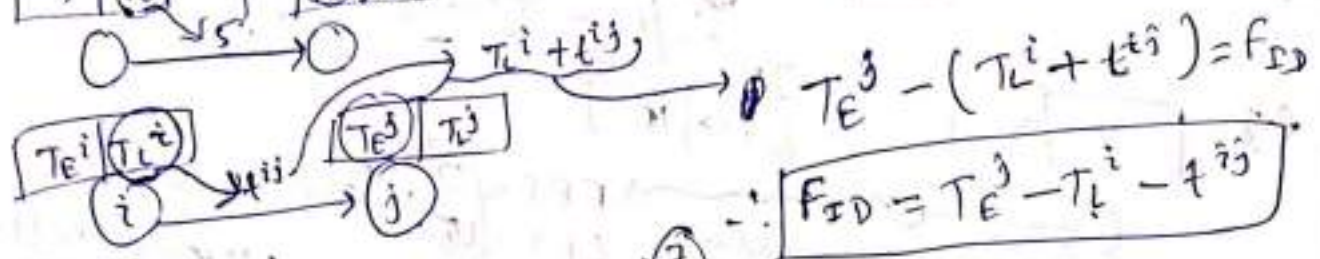
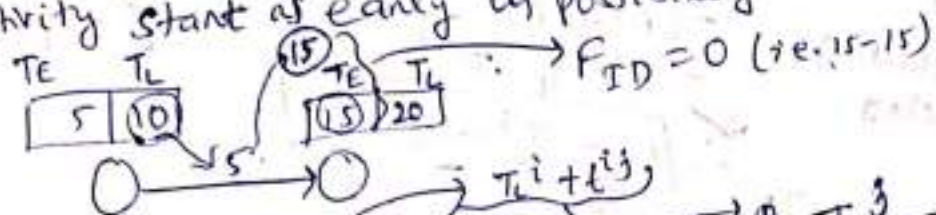
$$\text{Freedom} = FF = TE^j - (TE^i + t^{ij})$$

$$FF = TE^j - TE^i - t^{ij}$$

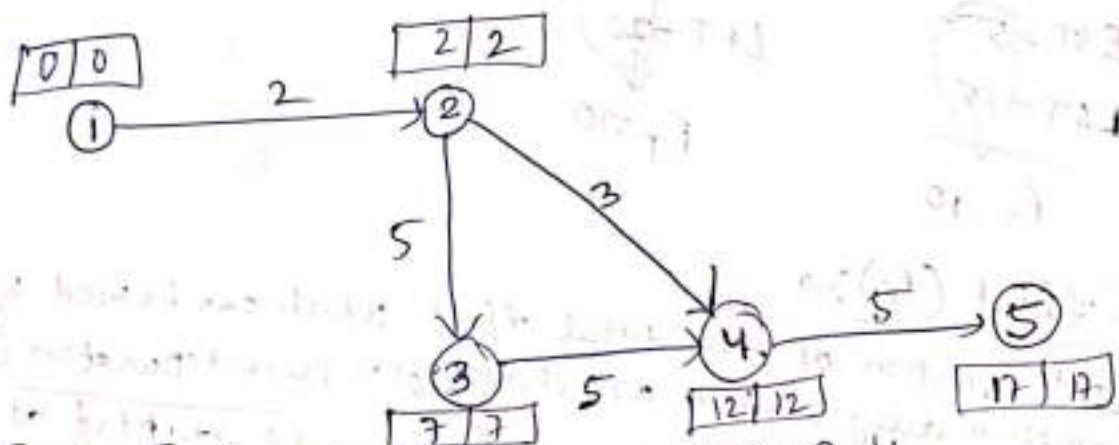
Free float affects preceding activity only, whereas total float affects succeeding as well as preceding activity both.

③ Independent float (F_{ID}):

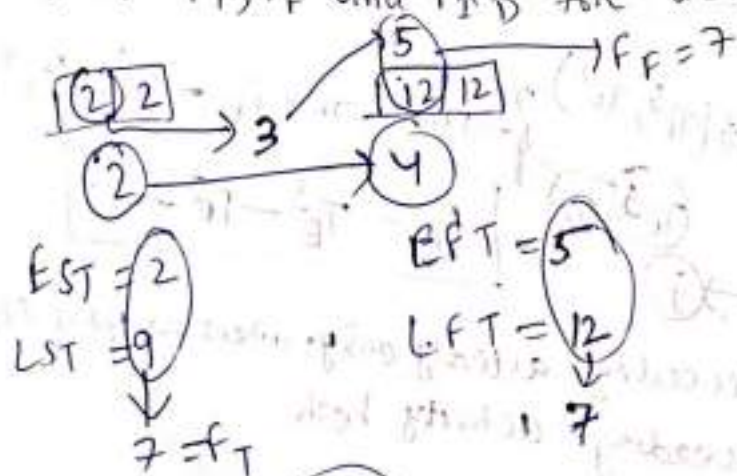
It gives the idea about ~~excess~~ ^{over} time ~~over~~ activity duration when preceding activity completing as late as possible & succeeding activity start as early as possible.



Q.



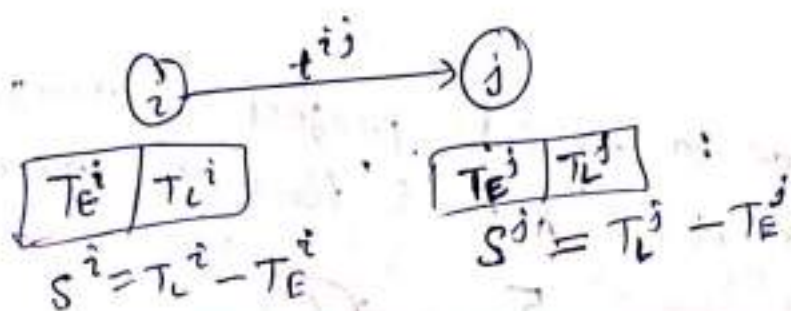
Find F_T, F_F and F_{ID} for activity 2-4



$F_T = 7$
 $F_F = 7$
 $F_{ID} = 7$

$$F_{FN} = F_T - F_F = T_L^j - T_E^i - t^{ij} - (T_E^j - T_E^i + t^{ij}) = T_L^j - T_E^j = S^j$$

$$F_{FN} = \text{Head event slack}$$



Formula trick

$$F_T = T_L^j - T_E^i - t^{ij}$$

$$F_F = T_E^j - T_E^i - t^{ij}$$

$$F_{FN} = T_E^j - T_L^i - t^{ij}$$



just remember.

Note
Independent float neither affect preceding activity nor affect succeeding activity. It only affect concern activity.

Critical path

The path which ~~also~~ connects activity means zero total float is called critical path. ($F_T = 0$)

Resources for such activity are just proper, hence any delay in any activity will result in delay of the project. It is time wise longest path.

Super critical path

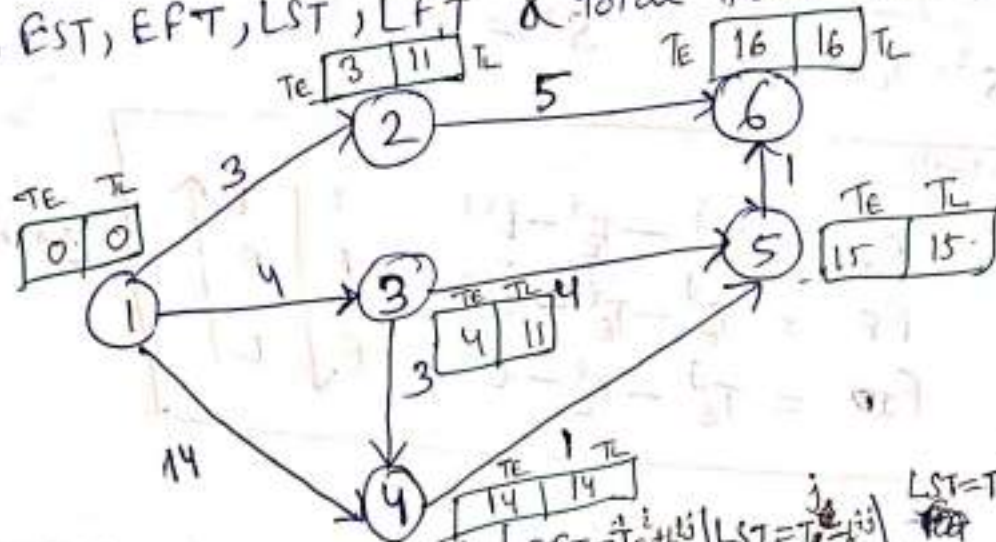
Path connecting activities which $T.F.(F_T) < 0$ is called supercritical path.

Resources in these path are insufficient.

Subcritical path

Path connecting activities with $FT > 0$ is called subcritical path. Resources in these path are sufficient & enough.

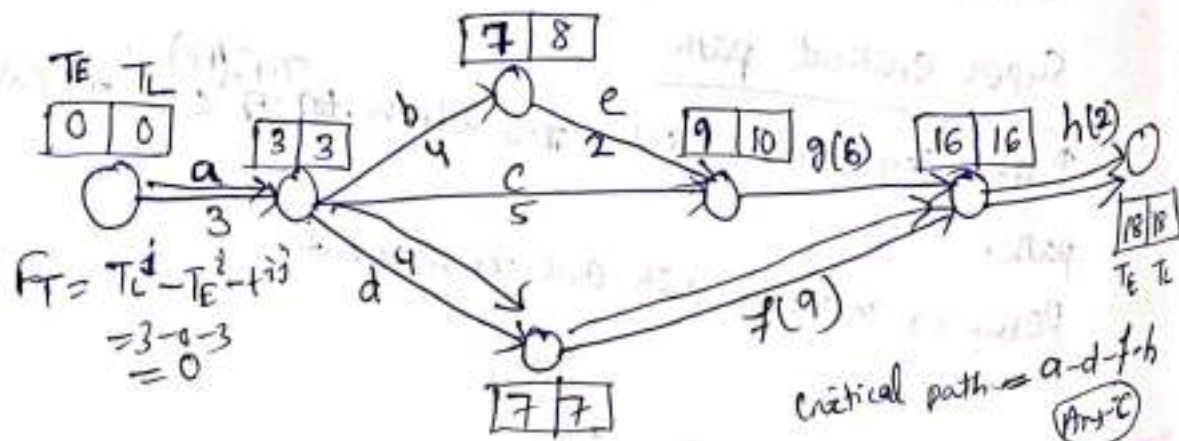
For the network show in fig. Cal. project duration, critical path, EST, EFT, LST, LFT & total float for each activity.



Activity	t_{ij}	EST = TE^i	EFT = $TE^i + t_{ij}$	LST = $TL^j - t_{ij}$	LFT = TL^j	FT = LFT - EFT	Remarks
①-②	3	0	3	8	11	8	
①-③	4	0	4	7	11	7	
①-④	14	0	14	0	14	0	critical
②-⑥	5	3	8	11	16	8	
③-④	3	4	7	11	14	7	
③-⑤	4	4	8	11	15	7	critical
④-⑤	1	14	15	14	15	0	critical
⑤-⑥	1	15	16	15	16	0	critical

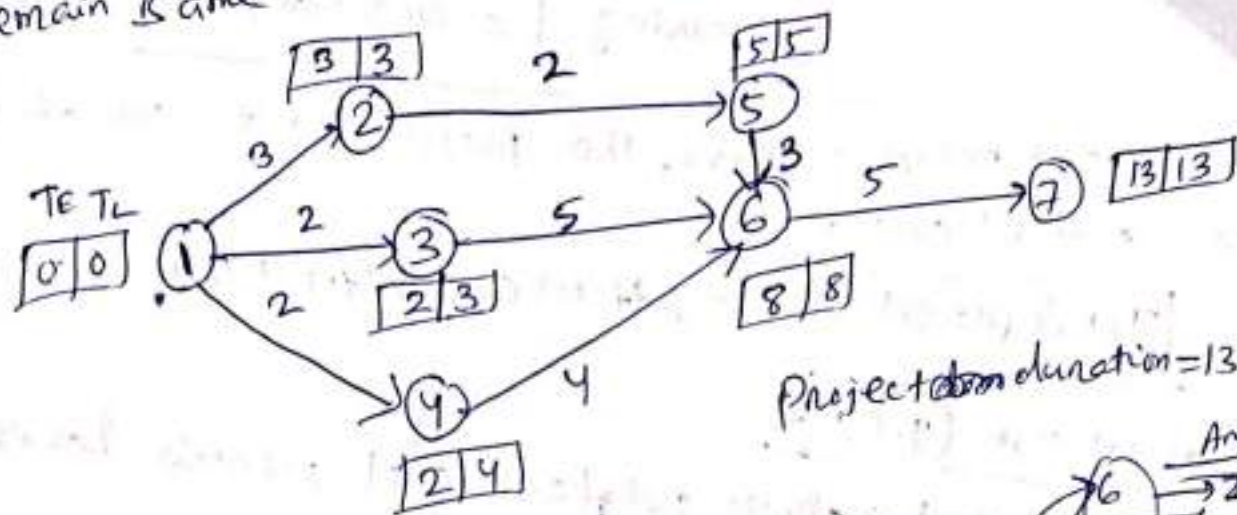
WB Ch-4

12-13

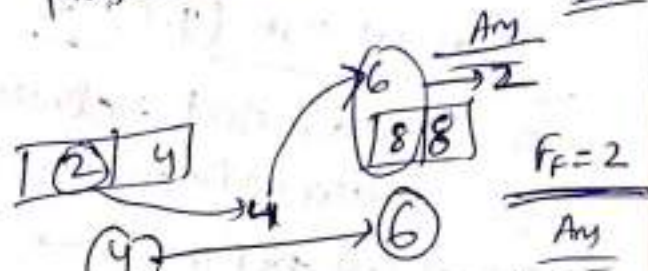


Q.13 Project duration will change to 19 days but critical path remain same.

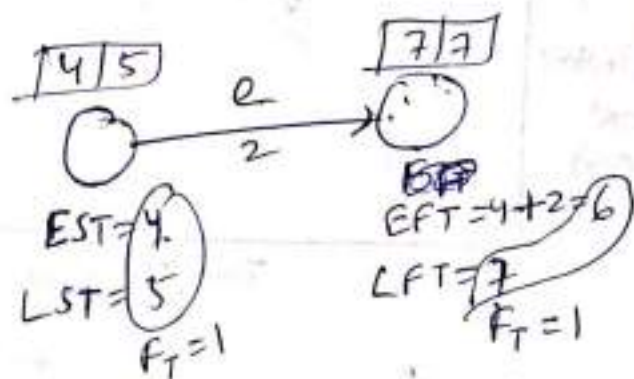
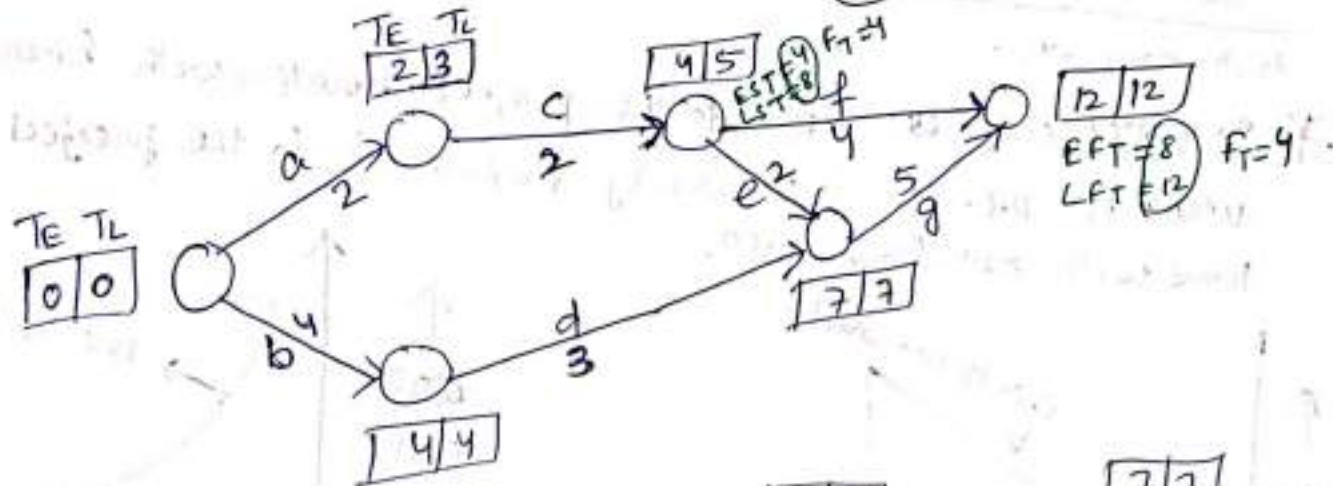
Q.17



Project duration = 13 days



Q.18



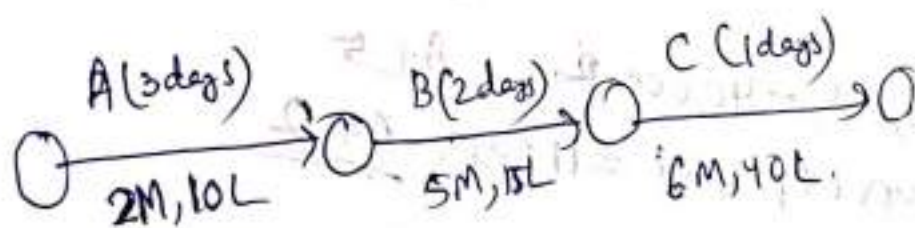
1 and 4

Resource allocation and Resource Levelling

Resource is a critical ^{variable} ~~value~~ which is required for completing the diff. activities of a project. This may be material resources, Man power resources, Money, time, etc.

Resource allocation means finding out which resources are to be allotted to which activity & in what quantity.

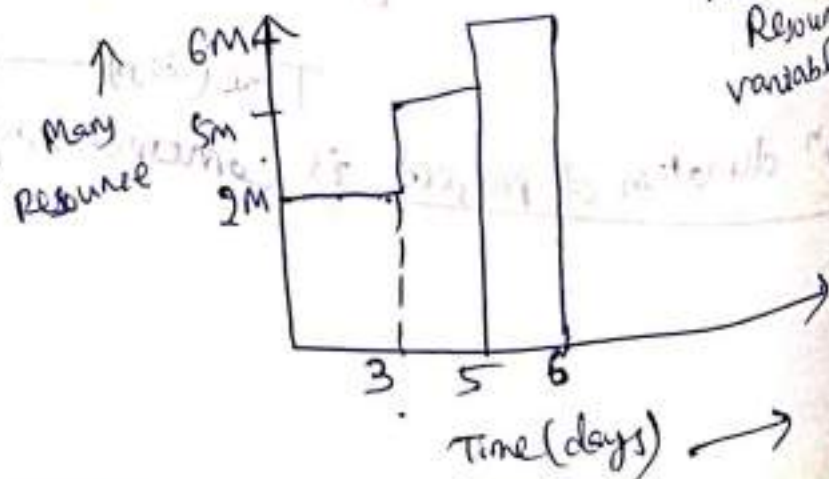
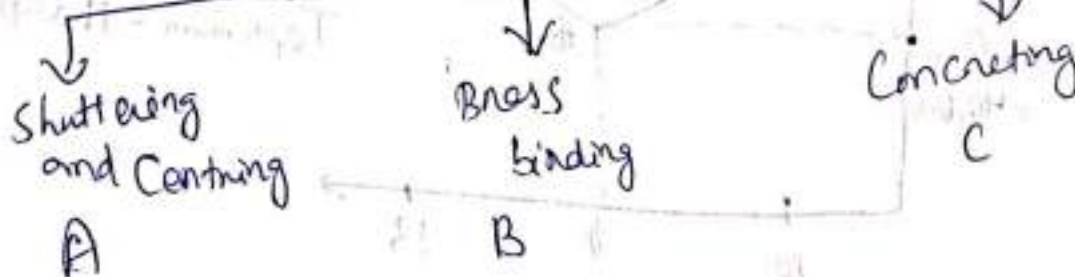
The dig. which show the ~~variation~~ variation in the requirement of resources ~~which~~ with time is called resource histogram.



M → Masons.

L → Labour.

Easting of Roof slab



Resources Histogram
Resources are variable here

Resource allocation can be done by 2 techniques

① Resources Smoothing

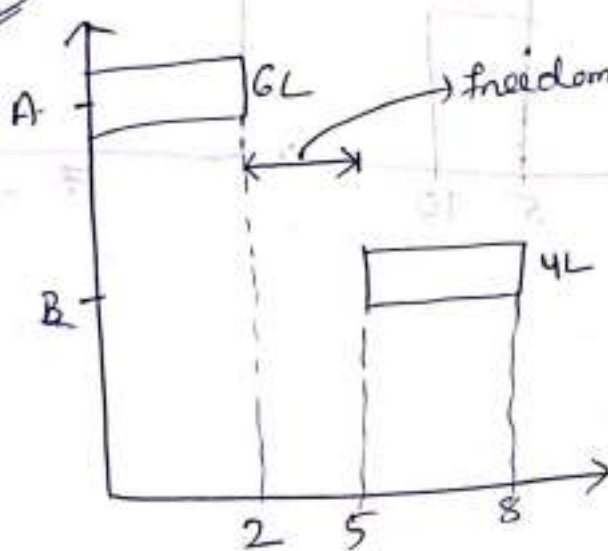
② Resource Levelling

① Resources Smoothing

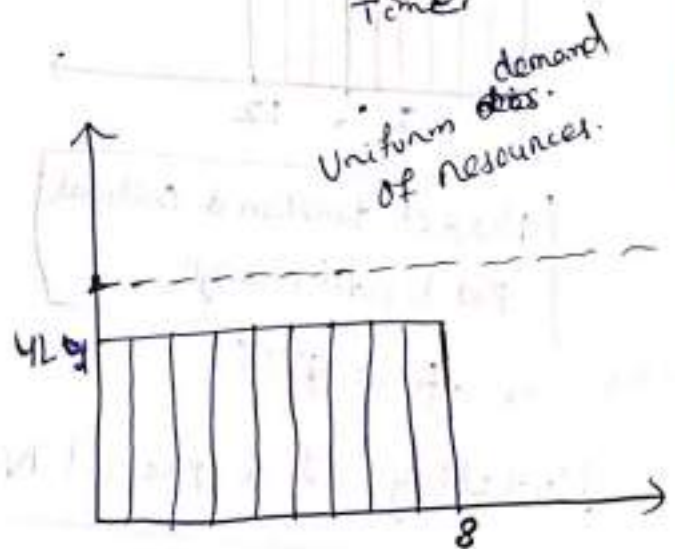
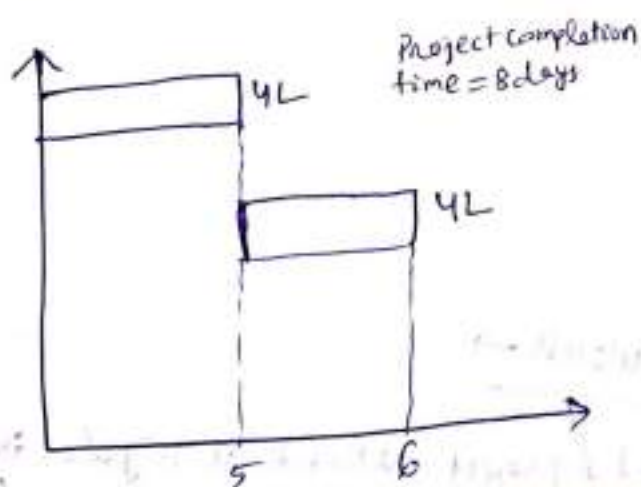
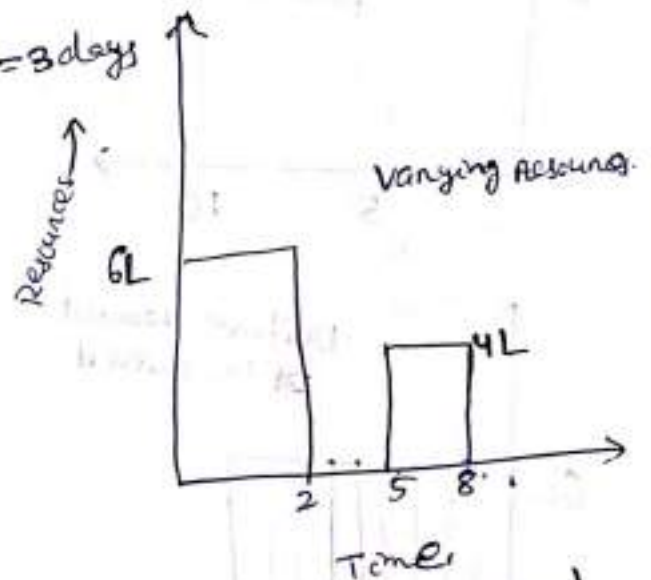
In this process of resource allocation the activities are delayed by their available total float in such a way that uniform demand of resources is achieved.

It is to be noted that resources in this case are considered ~~that~~ as unlimited & critical activities remain unchanged.

The project completion time & critical path also remain unchanged.



Project completion time = 8 days.

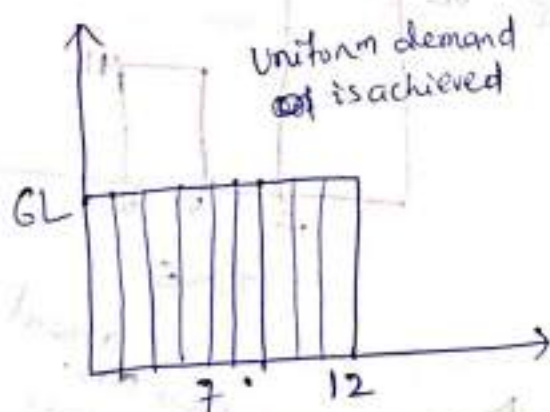
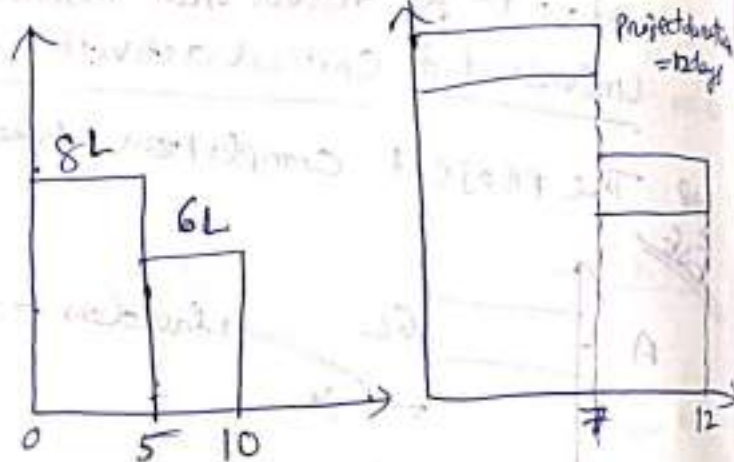
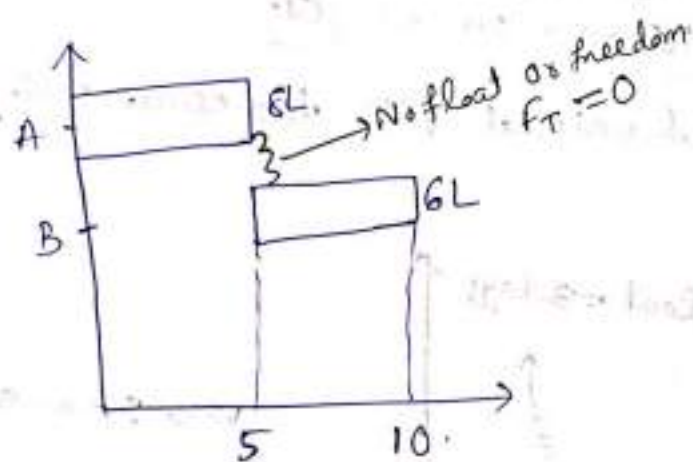


Project duration and critical path remain unchanged.

② Resource Levelling :-

In this process of resource ~~allocation~~ ^{elongation} the activities are rescheduled on such a way that the peak requirement at any point of time is less than the max. resources available.

In this process the resources are considered ~~to be~~ to be limited & Critical path as well as project ~~completion~~ ^{completion} time remain changed.



Project duration & critical path will change

Updating of a project Network →

Updating is the process of renew of a project network at regular interval in order to account for last moment change in program or any deviations in project.

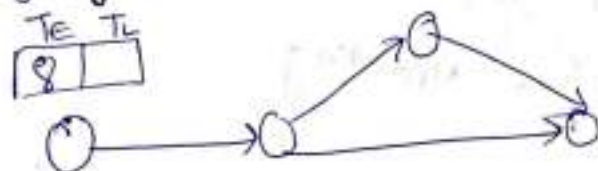
During updating network dig. is redrawn, dates and times are revised hence total project duration as well as ~~project~~ critical path will change.

Procedure of updating

Step

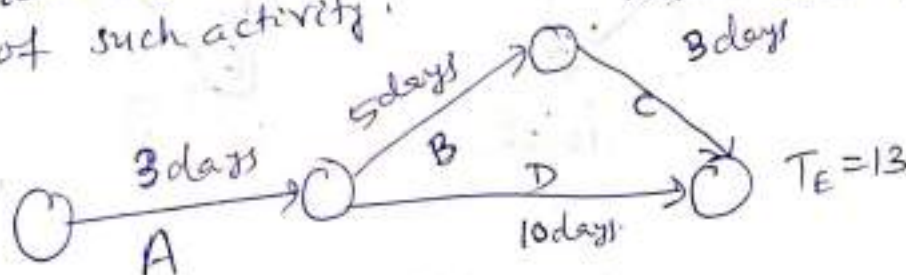
1. At the time of updating redraw the network dig. The start time of initial event = no. of days after which updating is done.

Review after 8 days.



2. On the date of updating determine how much time will further require for completion of each activity including ~~for~~ activities in progress or activities to a start and enter time required above the required activity.

3. If some activities are completed at the time of updating then further time required for that activity is 0 & enter time in front of such activity.
 project duration = 13 days

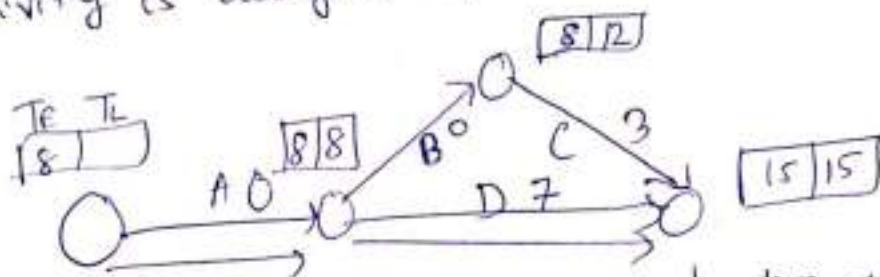


Review after 8 days.

- ① Activity A and activity B is completed.

- ② Activity C is on time.

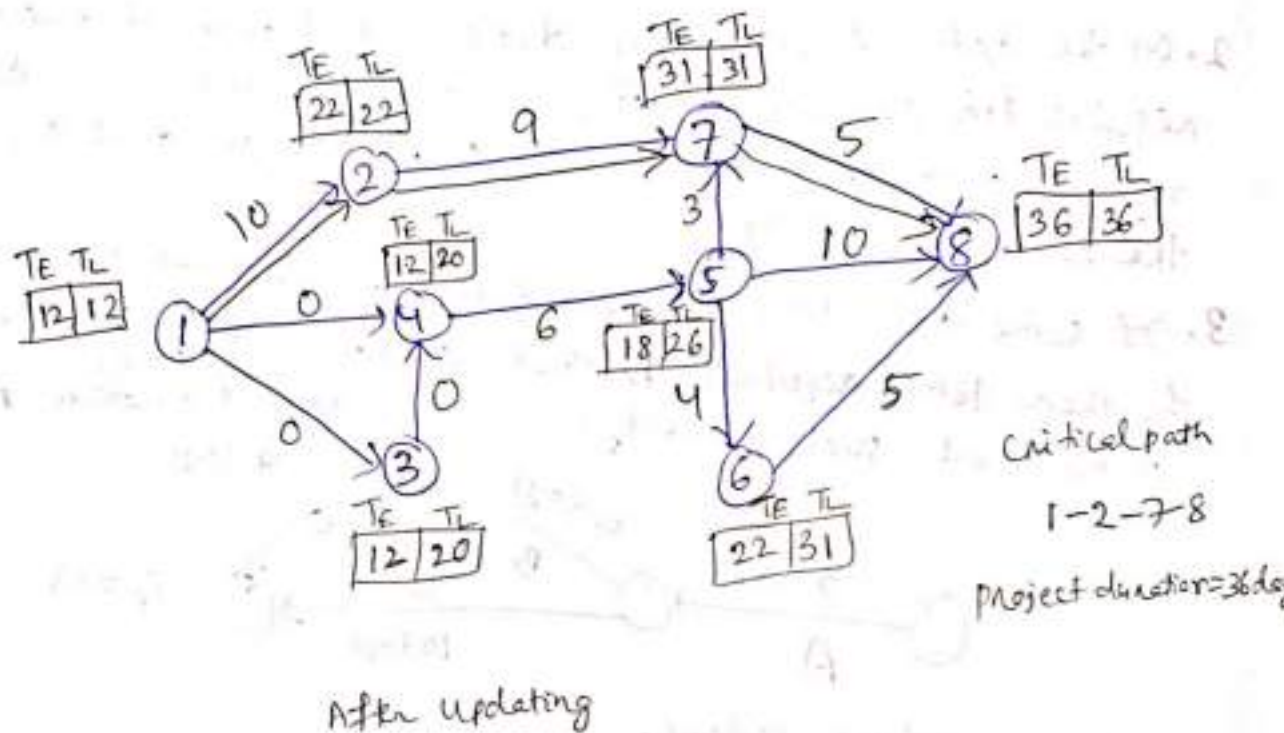
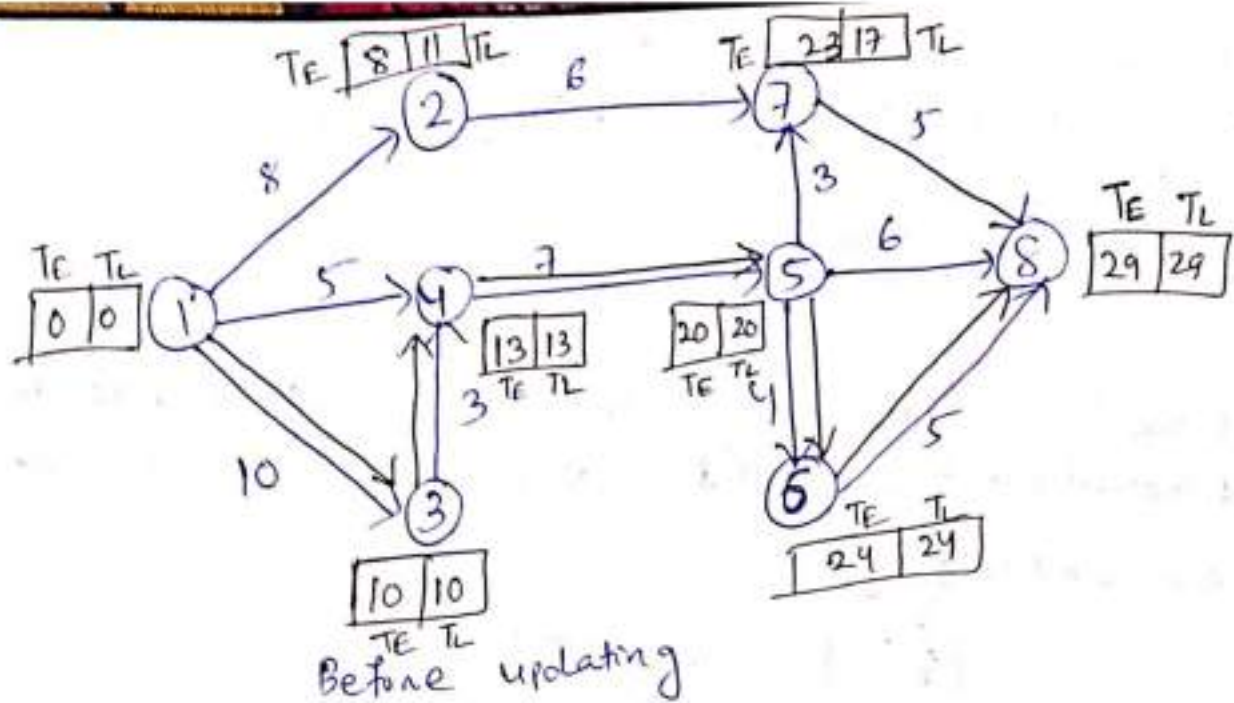
- ③ Activity is delayed drastically & it requires 7 more days



Now project duration = 15 days

- ④ Find TE and TL for each event and then find critical path and project duration.

WB
Q.14

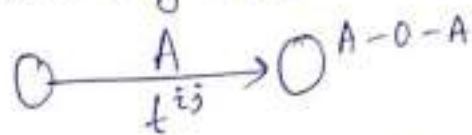


A-O-N Network

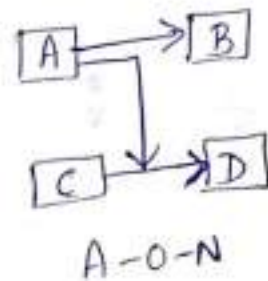
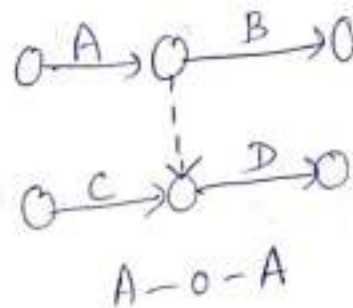
A-O-N stands for Activity Of Node Network. The network diagram is ~~not~~ self explanatory and it is activity oriented. It is suitable only for CPM based project.

Features of A-O-N Network

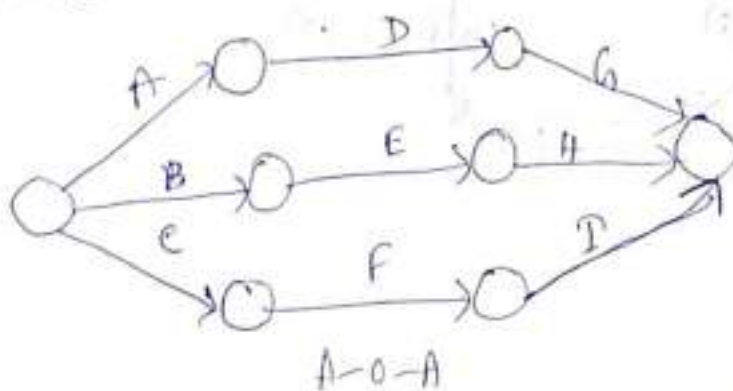
- In this network only activities are shown and events are no place.
- The network dig. is self explanatory and all activities time are shown in the network. (EST, EFT, LST, LFT)
- There is no role of dummy however their inter relationship is maintained by arrow.

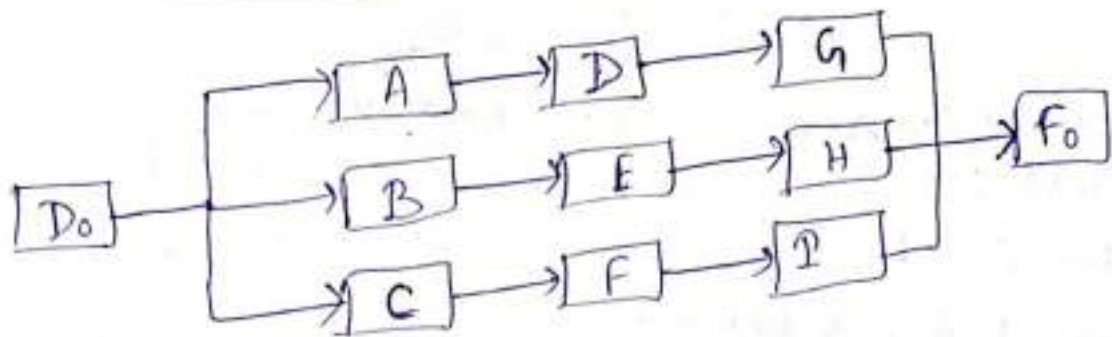


A-O-N



- If there are more than one activity which will start simultaneously then a new activity DEBUT (Do) has introduced. Similarly if more than one activity will finish ^{at the same time} then a new activity finish (Fo) has been introduced. Such activity doesn't require any resources and any time for its occurrence.

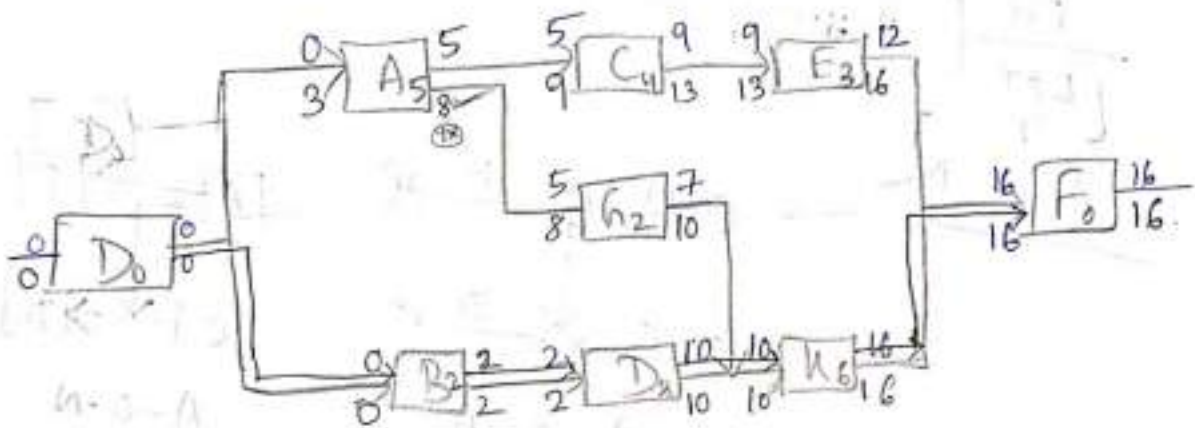
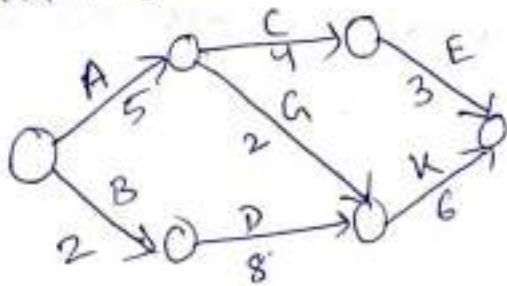




A-O-N

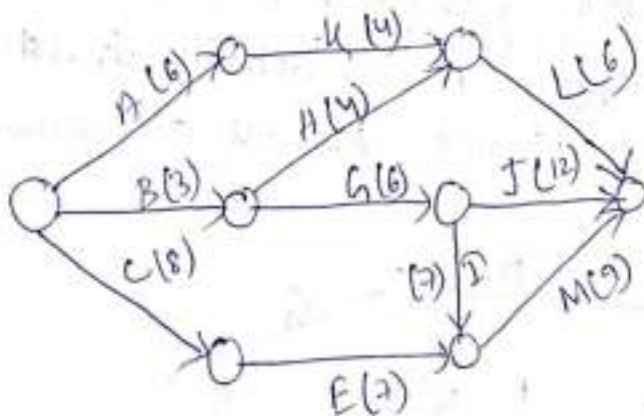
Q.

A-O-A network is shown in fig. Convert it into A-O-N network & find project duration and critical path.

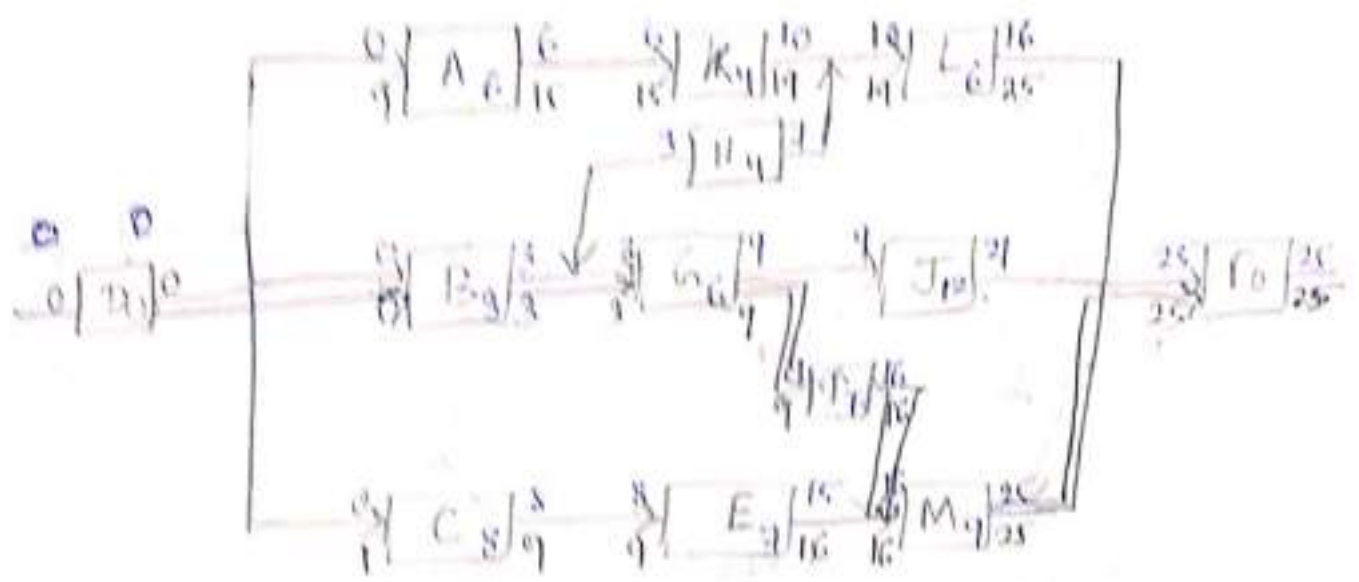


Critical path → B-D-K

Project duration = 16 days.



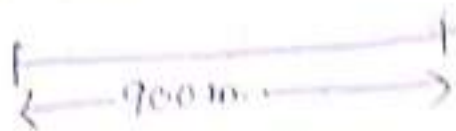
Q.



LADDER NETWORK

Ladder network is generally used when one activity is delayed due to another activity taking long duration. It is generally used to minimise time of completion of a project. In this method all activities are divided into equal sub activities.

Eg. Installation of underground pipeline in 900 m. length.



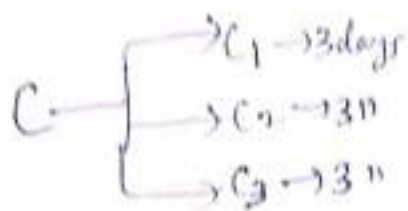
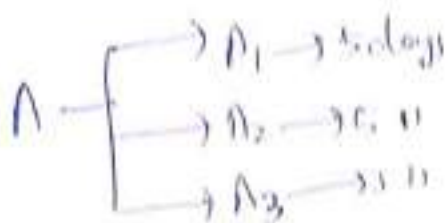
Activity

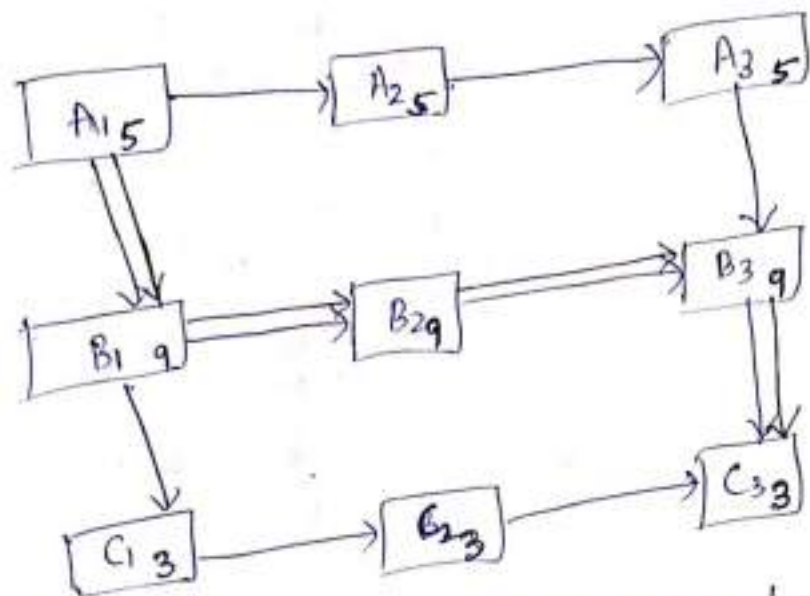
Excavation of trench $\rightarrow A \rightarrow 15 \text{ days}$

Installation of pipe line $\rightarrow B \rightarrow 27 \text{ days}$

Backfilling $\rightarrow C \rightarrow 9 \text{ days}$

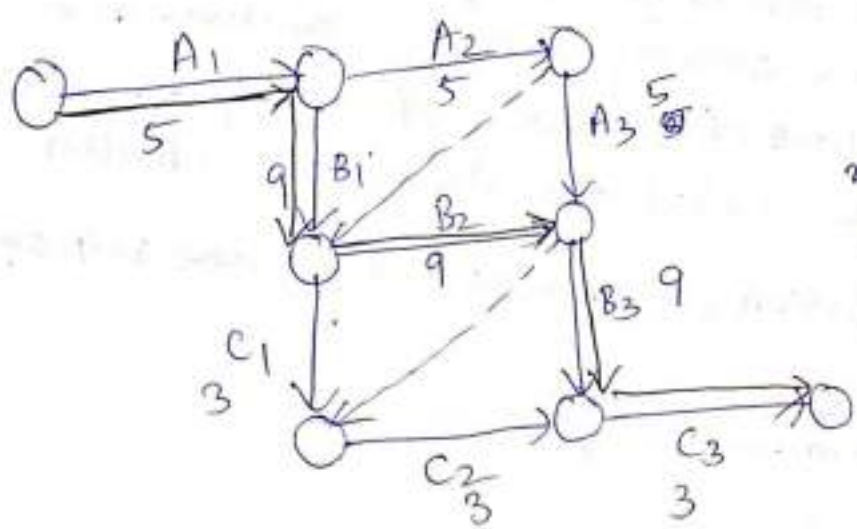
5 days = project duration





Project duration = 35 days
 saving = 51 - 35
 = 16 days.

Ladder network can be solved either by A-O-N network or by A-O-N network but preference is given to A-O-N analyser.



objectives.
~~objectives~~
 no. of dummy = 2
 project duration = 35 days.