LECTURE NOTE

ON

HYDRAULICS & IRRIGATION ENGG

FOR

DIPLOMA IN CIVIL ENGINEERING

(4TH SEMESTER STUDENTS)

AS PER SCTE&VT SYLLABUS

PREPARED BY:

Shri Pratyush Panda

PTGF in Civil Engineering

Department of Civil Engineering

Government Polytechnic, Sambalpur (Rengali)

www.gpsambalpur.com

Lec-1 : Dt- 13/03/2023

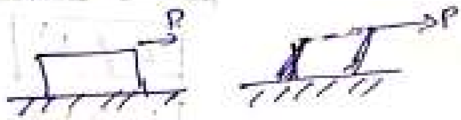
CH-1 : FLUID MECHANICS
FLUID PROPPERTIES PART-1

INTRO :

Matter :- Anything that has mass & occupy ~~space~~ space. are known as matters.

- Matter
  - Solid
    - Rigid Solid (Engineering Mechanics)
    - Deformable solid (Strength of Material)
  - Fluid (Fluid Mechanics)
    - Liquids
    - Gasses.

1. SOLID :- Solid are substance which have finite deformation under action of shear force and will try regain its its original position after removal of load.



2. FLUID :- fluids are the substance deformation continously under action of shear force.

Eg :- liquid or gas.

- Matter
  - Solid
  - Fluid

→ Solid show finite deformation

→ solid try to regain its original position after removal of load within elastic limit.

→ Fluid deform continously

→ Fluid doesnot even try to regain its original position.

Fluid

Liquids

→ Have free surface


$V_L = \frac{\pi}{4} D^2 H$

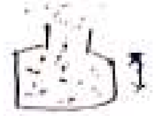
$V_L = \frac{\pi}{4} D^2 h$

→ No shape

→ Has own size (volume)

Gasses

→ Don't have free surface

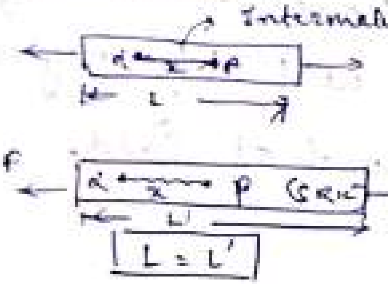

$V_L = V_g = \frac{\pi}{4} D^2 H$

→ No shape

→ No own size (volume)

SOLID

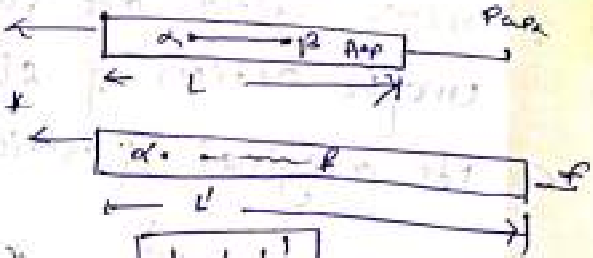
Rigid → (~~Ideal concept~~ Ideal concept)


Intermolecular force

(Any load/force)

$\boxed{L = L'}$

Deformable

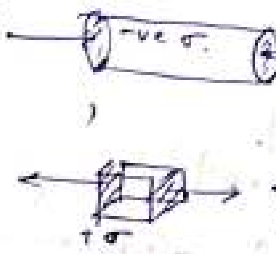

$\boxed{L \neq L'}$

$\boxed{L' > L}$

Force

Normal force

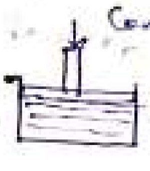
Solid {


$-ve\ \sigma$ Compressive force.

tensile force

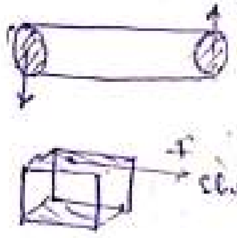
}

Liquid {

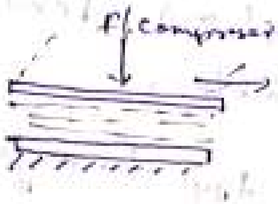

Compressive

}

Shear force


Shear force

Shear force.

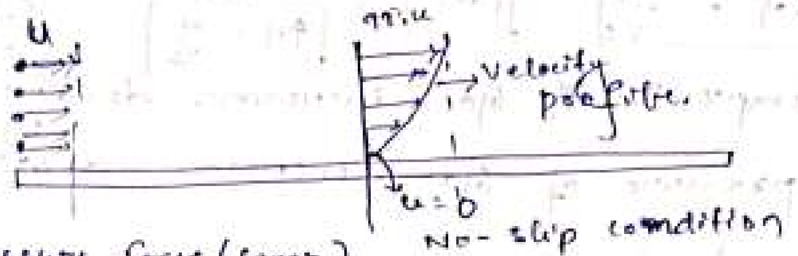

F compressive

F shear force

FLUID:- fluid are the substance which deform continuously under the action of shear force (No matter how small it is)

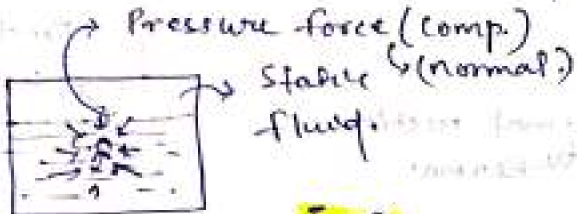
Eg:- ① Liquid & Gases are fluids.

→ fluids are zero memory substance.
↳ (not remember its starting shapes)

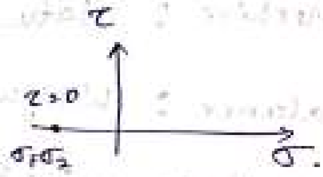


} velocity profile.

Note



$\tau = 0$
$\sigma_1 = \sigma_2$



Note:-
Conversely if there is no shear force implies no relative motion and the fluid is said to be at rest then is called a hydrostatic condition (A state of zero shear & and equal and alike normal stress).

Dt-14/03/2023
Lec-2

FLUID PROPERTIES:-

I. DENSITY / MASS DENSITY:-
→ It is a ratio of mass to volume.

$$\boxed{\rho = \frac{Mass}{Volume}}$$

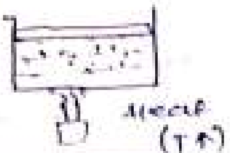
→ unit : Kg/m^3
→ Dimentional formula : $[ML^{-3}]$

→ for liquids : $\rho \approx$ Constant
→ for Gases : $\rho = f(P, T)$

{ P = pressure
T = temperature

For Gas case-1 | case-2

eg



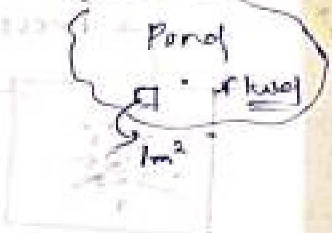
$\boxed{\downarrow\rho = \frac{M}{V\uparrow}}$ $\boxed{\rho \propto \frac{1}{T}}$

$PV = mRT$
$P \propto \frac{1}{V}$

$\boxed{\uparrow\rho = \frac{M}{V\downarrow}}$ $\boxed{\rho \propto P}$

→ As temperature of gas increases, density decreases.
→ As pressure of gas increases, density also increases.

• $\boxed{\rho = \frac{M}{V}}$



Working principles {
* Weighing Machine : Normal rection. ~~State Reason~~
* Spring Balance : Weight.
* ~~Sp~~ Physical Balance : Weight
}

Short Q. {
$\rho_{Hg} = 13600\ kg/m^3$
$\rho_{H_2O} = 1000\ kg/m^3$
$\rho_{oil} = 800\ kg/m^3$ / 860 kg/m^3
$\rho_{gly} = 1260\ kg/m^3$
ρ_{air} @ STP = $1.22\ Kg/m^3$
↳ standard temperature & pressure
(1atm, 273 K)
0°c
}

~~Normal atmosphere~~ ~~1 atm =~~

NTP (Normal temp. & Pressure) = (1atm, 293 K)
20°C

* at 4°C the density of water is maximum (1000 kg/m³)

Pressure

$\boxed{1 atm = 101325\ Pa}$ ∵ $\boxed{1 Pa = 10^{-3}\ kPa}$

$\boxed{1\ atm = 101.325 kPa}$

1 ...

Pressure in terms of head

$$\boxed{1\,atm \begin{cases} 10.3\,m \text{ of } H_2O \\ 760\,mm \text{ of } Hg \end{cases}}$$

$$\boxed{1\,pa = N/m^2 = kg/m\text{-}s^2}$$

$$\boxed{1\,Torr = 760\,mm \text{ of } Hg}$$

$$\boxed{1\,Kpa = 10^{-2}\,Bar}$$

$$\boxed{1\,pa = 10^{-5}\,bar}$$

$$\boxed{1\,atm = 1.0132\,bar}$$

$$\boxed{1\,bar = 10^5\,Pa}$$

2. SPECIFIC WEIGHT OR WEIGHT DENSITY (γ, w) :-

$$\boxed{\gamma = \frac{\text{Weight}}{\text{Volume}}} = \frac{M.g}{V} = \rho.g = N/m^3 ;\ \frac{kg-m}{s^2 \times m^2}$$

unit

S m/sec²

$$\boxed{\gamma = \rho.g}$$

→ Unit : $\boxed{N/m^3}$ (SI unit), $\boxed{kg/m^2\text{-}s^2}$ → MKS unit , $\boxed{gm/cm^2\text{-}s^2}$ → (C.G.S. system)

→ Dimentional formula = $[ML^{-2}T^{-2}]$

$\gamma_{H_2O} = 1000 \times 9.81$

$\boxed{\gamma_{H_2O} = 9810\ N/m^3}$ or $\boxed{\gamma_{H_2O} = 9.81\ kN/m^3}$ $\boxed{g_{moon} = \frac{1}{6} g_{earth}}$

* $\boxed{\gamma = f(P, T, g)}$

$$\boxed{\begin{matrix} 1\,kN = 1000\,N \\ 1\,N = 10^{-3}\,kN \end{matrix}}$$

$$\boxed{1\,N = 10^5\,Dyne}$$

3. SPECIFIC VOLUME (v) :-

$$\boxed{v = \frac{\text{Volume}}{\text{Mass}} = \frac{1}{\rho}}$$

~~unit: m^3/kg~~ Unit : m^3/kg

~~Dim L^3M^{-1}~~ Dimension = $[M^{-1}L^3]$

4. SPECIFIC GRAVITY OR RELATIVE DENSITY :-

$$\boxed{S = \frac{\rho_{fluid}}{\rho_{standard\ fluid}}}$$

→ It is a ratio of density of fluid to density of standard fluid.

→ Specific gravity is unit-less fluids property.

* For Liquids : $\rho_{st.\ fluid} = 1000\ kg/m^3$ (water)
* For gases : $\rho_{st\ fluid} = 1.22\ kg/m^3$ (air)

$$\boxed{\begin{aligned} S_{Hg} &= \frac{13600}{1000} = 13.6 \\ S_{H_2O} &= \frac{1000}{1000} = 1 \\ S_{oil} &= \frac{860}{1000} = 0.86 \end{aligned}}$$

* $S_f < 1$ → Lighter then water (oil)
* $S_f > 1$ → heavier then water (Hg, Glycerine)

* $\rho_{Hg} = 13600\ kg/m^3$

$$\rho_{Hg} = \underbrace{\boxed{13.6}}_{\text{specific Gravity}}\ gm/cm^3$$

gm/cc or kg/Lit or gm/m.Lit

↳ the constant value are the specific gravity

Calculation

$$= \frac{13.6 \times 10^{-3}}{(10^{-2})^3} = 13.6 \times 1000 = 13600\ kg/m^3$$

$$\rho_{H_2O} = \underbrace{\boxed{1}}_{\text{specific Gravity}}\ kg/Lit$$

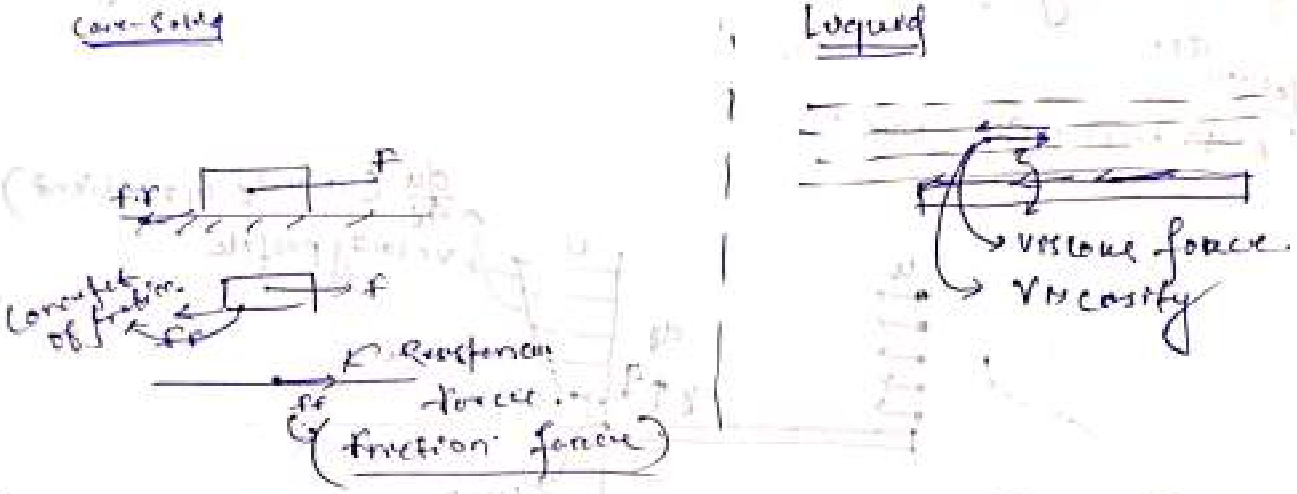
$$S = \frac{\rho_f}{\rho_{st}} = \frac{1\ kg}{10^{-3}\ m^3} = 1000\ kg/m^3$$

$$\boxed{\rho_{Hg} = \boxed{13.6}\ gm/m.lit}$$
↳ specific gravity

$$\boxed{\rho_{oil} = \boxed{0.86}\ kg/lit}$$
↳ specific gravity

→ Hydrometer is used to measure specific Gravity / ~~sed~~ Relative density. It is based on ~~Boyancy~~ Buoyancy concept.

VISCOSITY :-



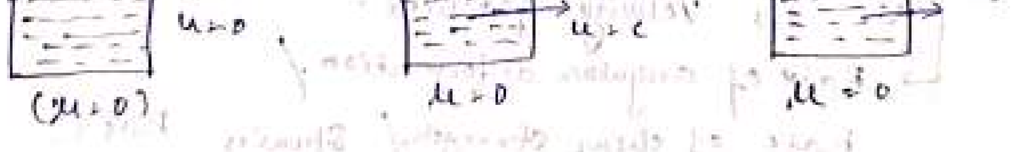
→ ~~P~~ Viscosity is nothing but Resistance to flow or Deformation.

Mathematically :-

→ It is a Internal resisting force ~~actio~~ acting between ~~to~~ two adjacent fluid particals when it is in motion.

→ If the fluids at rest, then viscosity will not come into the picture.

→ Viscosity is represents as 'μ'.



Note:-

$\boxed{\text{Maximum viscosity : Tar} \\ \text{Minimum Zero viscosity : Helium.}}$ → eg. (highest viscous fluids) → (zero viscous fluid)

$$\boxed{\mu_{H_2O} = 0.001} \rightarrow Pa\text{-}s$$

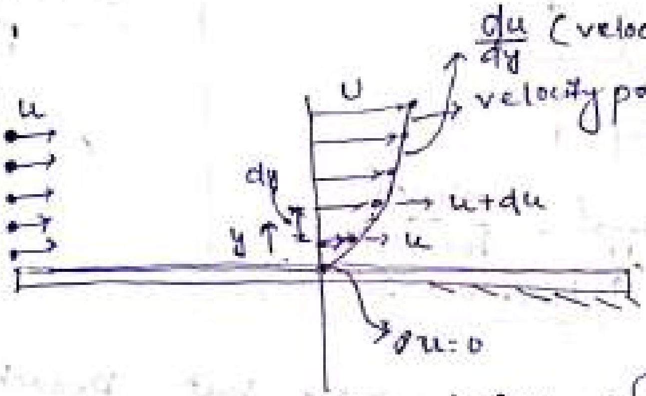
$$\boxed{\mu_{air} = 18 \times 10^{-6}} \rightarrow Pa\text{-}s$$

$$\frac{\mu_{H_2O}}{\mu_{air}} = 55.55$$

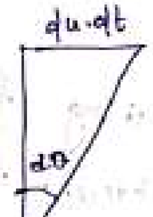
$$\boxed{\mu_{H_2O} = 55.55\, \mu_{air}}$$

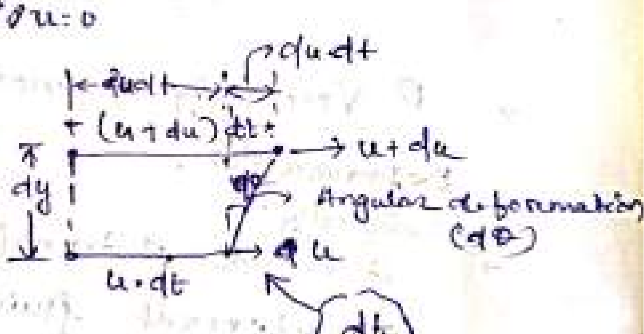
** viscosity of water is almost 55 times of viscosity of air

Dt-15/03/2021
Lec-3 # Viscosity Continue :-



$$\left\{ v = \frac{d}{t} \qquad d = v \cdot t \right\}$$





$$\tan d\theta = \frac{du \cdot dt}{dy}$$

∵ $\tan d\theta \approx d\theta$ (due to $d\theta$ is very small)

$$d\theta = \frac{du \cdot dt}{dy}$$

$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}} \rightarrow \frac{m/s}{m} \rightarrow \boxed{1/s = \text{unit} = s^{-1}}$$

↳ Velocity Gradient

↳ Rate of angular deformation / Rate of shear ~~strength~~ strain / Strain Rate.

NEWTON'S LAW OF VISCOSITY :-

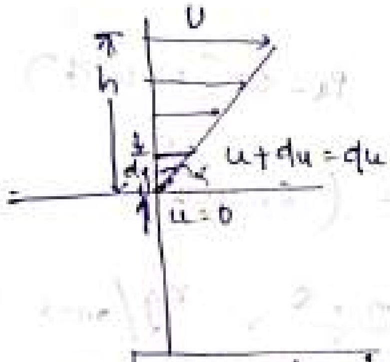
Shear stress ∝ Rate of shear strain

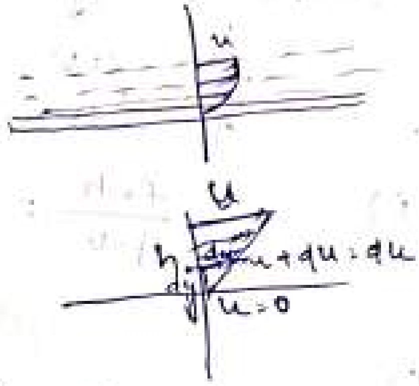
$$\tau \propto \frac{d\theta}{dt} \text{ or } \frac{du}{dy}$$

$$\boxed{\tau = \mu \frac{du}{dy} = \mu \frac{d\theta}{dt}}$$ Newton's Law of viscosity

↳ Proportionality constant / Dynamic Constant.
Constant of viscosity / Absolute viscosity.

* for a small fluid thickness we can considered linear velocity profile.





$$\tan\alpha = \boxed{\frac{U}{h} = \frac{du}{dy}}$$

from similar tringle

$$\boxed{\frac{du}{dy} = \frac{U}{h}}$$

$$\tau = \mu \cdot \frac{U}{h}$$

$$\boxed{\tau = \frac{F}{A} = \mu \frac{U}{h}}$$ (Linear profile)

where, (significance of parameter)

h = The pendicular distance betn fixed and moving plate.

V = Velocity ; F = Tangential shear force. (N)

μ = fluid property

A = Contacting area of fluid (A)

Numericals

Eg:-

$f = ?$

$$\boxed{\frac{F}{A} = \frac{\mu U}{h}}$$

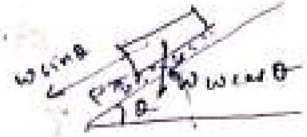
Sol^n

$A = \pi D L$

$v = r\omega$

$h = \frac{D' - D}{2}$

Eg



$A = xy$

$h = p$

$f = W\sin\theta$

Unit of viscosity :-

$$\frac{F}{A} = \frac{\mu V}{h}$$

i) $\mu = \frac{F \cdot h}{A \cdot V} = \frac{N \cdot m}{m^2 \cdot m} \times s = Pa\text{-}s$ (SI unit)

↳ $N\text{-}s/m^2$ (SI unit)

ii) $\mu = \frac{N\text{-}s}{m^2} = \frac{kg\text{-}m \cdot s}{s^2\, m^2} = kg/m\text{-}s$ (MKS unit)

$$\boxed{1\,N = 1\,kg \cdot \frac{m}{s^2}}$$

iii) $\mu = \frac{kg}{m\text{-}s} = \frac{1000\,gm}{100\,cm\text{-}s} = \frac{10\,gm}{cm\text{-}s}$

↓

(C.G.S unit)

$\frac{gm}{cm\text{-}s}$ → C.G.S unit

↳ Poise

Dimensional formula

: $[M L^{-1} T^{-1}]$

$\boxed{1\ kg/m\text{-}s = 10 \text{ poise}}$

$\boxed{1 \text{ poise} = \frac{1}{10}\ kg/m\text{-}s}$

$\boxed{1 \text{ poise} / \frac{1}{10} \text{ Pascal-s}}$

$\boxed{1 \text{ poise} = \frac{1}{10} \text{ Pascal-sec } Pa\text{-}s}$

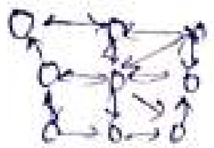
Note! There is no relation between density & Dynamic viscosity

Relation density → $\boxed{\rho_{Hg} > \rho_{gly} > \rho_{H_2O} > \rho_{oil} > \rho_{air}}$

Relation viscosity → $\boxed{\mu_{gly} > \mu_{oil} > \mu_{Hg} > \mu_{H_2O} > \mu_{air}.}$

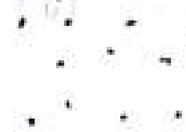
Note

Liquid



Inter molecular Bond (Cohesive force)

Gases



(Molecular Momentum) ($\because$ $M = m \cdot v$)

Kinamatic Viscosity (ν) :-

$$\boxed{\nu = \frac{\mu}{\rho}}$$

→ Momumentum diffusability

Unit

i) $\frac{kg/m\text{-}s}{kg/m^3} = m^2/s$ = stokes

Dimensional formula $[M^0 L^2 T^{-1}]$

Relation betn ν & ρ

$$\boxed{\nu \propto \frac{1}{\rho}}$$

*

$\boxed{1 \text{ stokes} = 10^{-4}\ m^2/s}$

$10^{-4}\ m^2/s = 1\ cm^2/s$

$\nu_{H_2O} = 10^{-6}\ m^2/s$

$\nu_{air} = 15 \times 10^{-6}\ m^2/s$

$\frac{\nu_{air}}{\nu_{H_2O}} = \frac{15 \times 10^{-6}}{10^{-6}}$

**

$$\boxed{\nu_{air} = 15 \cdot \nu_{H_2O}}$$

Definition → The ability of fluid particle to transfer momentum.

Unit

$\nu = \frac{m^2}{s} \rightarrow$ (MKS unit)

$\nu \rightarrow cm^2/s$ (C-G-S unit)

$1 \frac{m^2}{s} = 10^4 \; cm^2/s$

↳ C.G.S

↳ stoke

$$\boxed{1 \, m^2/s = 10^4 \text{ stoke}}$$

$\frac{1}{10^4} m^2/s = 1$ stoke.

$$\boxed{1 \text{ stoke} = 10^{-4} \, m^2/s}$$

VARIATION OF VISCOSITY WITH Respect to PRESSURE & TEMPERATURE :-

1. VARIATION OF VISCOSITY WITH PRESSURE :-

$$\boxed{\mu_{1000atm} = 2 \, \mu_{1atm}}$$

→ The variation of viscosity with respect to pressure is negligible.

2. VARIATION OF VISCOSITY WITH TEMPERATURE :-

Fluid

↓ Liquid (Cohesive force) (C.F)

↓ Gases (Molecular Momentum)

(1) Liquids :-

$T(\uparrow) \Rightarrow$ C.F $\downarrow$ $\Rightarrow$ flow become easy $\Rightarrow \mu(\downarrow)$

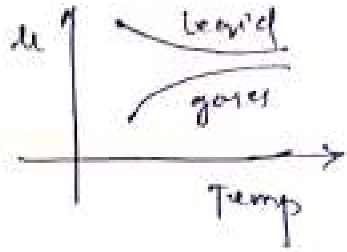
(2) Gases :-

$T(\uparrow) \Rightarrow$ Molecular momentum $(\uparrow) \Rightarrow$ flow becomes difficult $\Rightarrow \mu\uparrow$

* Kinetic Theory of gases :-

$$\boxed{\mu \propto \sqrt{T}}$$

$$\boxed{\nu \propto T\sqrt{T}}$$



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L - 4

~~VARATION~~ VARIATION of KINEMATIC VISCOSITY WITH TEMP :-

$$\boxed{\nu = \frac{\mu}{\rho}}$$

* Liquid

i) T - Increase :-

$$\downarrow\nu = \left(\frac{\mu\downarrow\downarrow}{\rho\downarrow\downarrow}\right)\downarrow$$

$$\boxed{T\uparrow = \nu\downarrow}$$

* Gases :-

$$(T\uparrow) \quad \nu = \left(\frac{\mu\uparrow\uparrow}{\rho\downarrow\downarrow}\right)\uparrow$$

$$\boxed{T\uparrow = \nu\uparrow}$$

Problem

(1) $\nu = 7.4 \times 10^{-7}\ m^2/s$

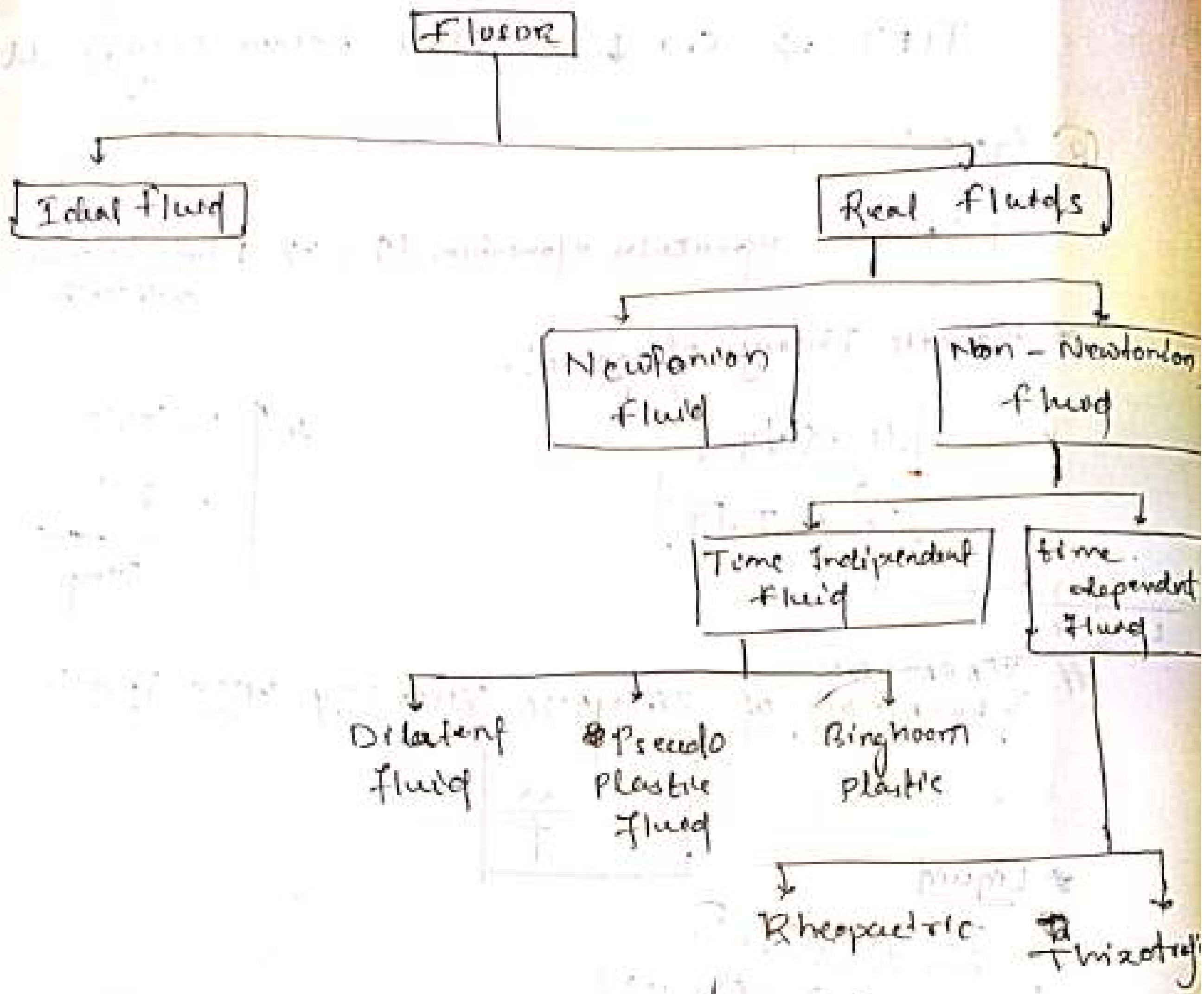
$S = 0.88$; $h = 0.5$ mm

$\rho = 880 kg/m^3$; $V = 0.5$ m/s

$\mu = \nu \cdot \rho$.

$$\frac{F}{A} = \mu \frac{U}{H} = \frac{\rho \cdot \nu \cdot U}{H} = \frac{880 \times 7.4 \times 10^{-7} \times 0.5}{0.5 \times 10^{-3}} = 0.651\ N/m^2$$

Types Of Fluid :-



① IDEAL FLUID :-

Ideal fluid is incompressible fluid and having zero viscosity. It is an imaginary fluid.

$\boxed{\mu = 0}$

$\boxed{\tau = 0}$

$\boxed{\text{Surface tension } (\sigma) = 0}$

$\boxed{\text{Compressibility } (\beta) = 0}$

$\boxed{\text{Bulk modulus of elasticity } (K) = \infty}$

$\boxed{\text{INCOMPRESSIBLE}}$

2. REAL FLUID:-

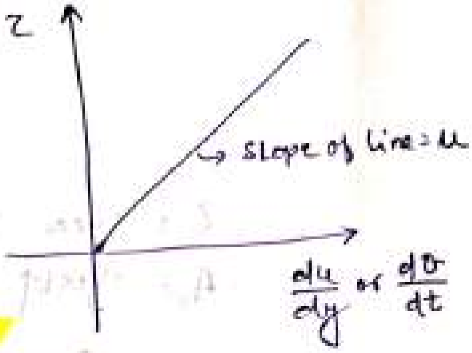
A: NEWTONIAN FLUIDS:-

The fluids which follow Newton's Law of viscosity are known as Newtonian fluids.

$$\boxed{\tau = \mu \frac{du}{dy}}$$

$y = m x$

y axis, slope, x axis

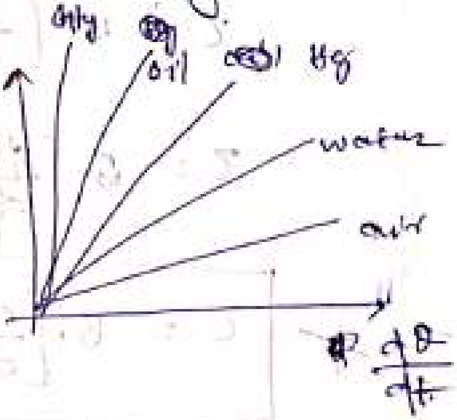


* All fluids which we are using in fluid mechanics are newtonian fluid.
* Eg:- Hg, oil, glycerine, H_2O, air are newtonian fluid.
* The study of non-newtonian fluid is called ~~Rheology~~ Rheology.
* In Newtonian fluid the dynamic viscosity constant (except temperature change).

case-1

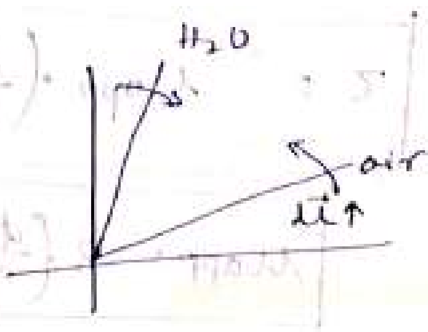
~~$\mu_{gly} > \mu_{oil} > \mu_{Hg} > \mu_{water} > \mu_{air}$~~

$\mu_{gly} > \mu_{oil} > \mu_{Hg} > \mu_{H_2O} > \mu_{air}$.



case-2

Temperature → Increase



B. NON - NEWTONIAN FLUID :-

B.(I) TIME INDEPENDENT FLUID :-

B.(I).1 DILATANT FLUID :- Non-Newtonian fluid follows

$$\boxed{\tau = A + B\left(\frac{du}{dy}\right)^n}$$

↳ OSTWALD DEWAELEY POWER LAW

τ : shear stress
A : yield stress / Threshold stress
B = Consistency Index
n : flow Index
$\frac{du}{dy}$: velocity gradient.

(a) DILATENT FLUID :-
[$n > 1$, $A = 0$]

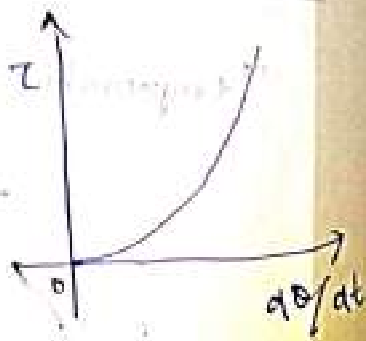
$$\tau = B\left(\frac{du}{dy}\right)^n$$

$$\tau = B\left(\frac{du}{dy}\right)\left(\frac{du}{dy}\right)^{n-1}$$

$$\boxed{\tau = B\cdot\left[\frac{du}{dy}\right]^{n-1}\cdot\left(\frac{du}{dy}\right)}$$

$$\boxed{\tau = \mu_{app}\cdot\left(\frac{du}{dy}\right)}$$

$$\boxed{\mu_{app} = B\left(\frac{du}{dy}\right)^{n-1}}$$



Case-1

- $n \uparrow \Rightarrow \left(\frac{du}{dy}\right)^{n-1} \uparrow \Rightarrow$ ~~μ_{app}~~ $\mu_{app} \uparrow \Rightarrow \tau \uparrow$

Example: Butter, Sugar & salt sol^n, ~~Ob~~ Oablek, quick sand
Suspension of rice starch, wet sand.

→ It is also known as shear thickening fluid.

(b) PSUDO PLASTIC FLUID :-

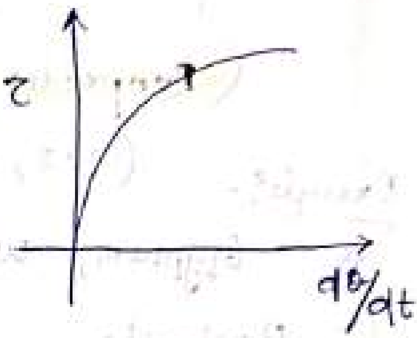
$$[\, n < 1 \,,\ A = 0 \,]$$

$$\tau = B \cdot \left(\frac{du}{dy}\right)^{n}$$

$$\tau = B \left(\frac{du}{dy}\right)^{n-1} \left(\frac{du}{dy}\right).$$

$$\boxed{\tau = \mu_{app} \left(\frac{du}{dy}\right)}$$

$$\boxed{\mu_{app} = B \cdot \left(\frac{du}{dy}\right)^{n-1}}$$



Condition

$$n < 1 \Rightarrow \left(\frac{du}{dy}\right)^{n-1} \downarrow \Rightarrow \mu_{app}(\downarrow) \Rightarrow \tau(\downarrow)$$

Example :- Paint, syrap, blood, Milk, etc.

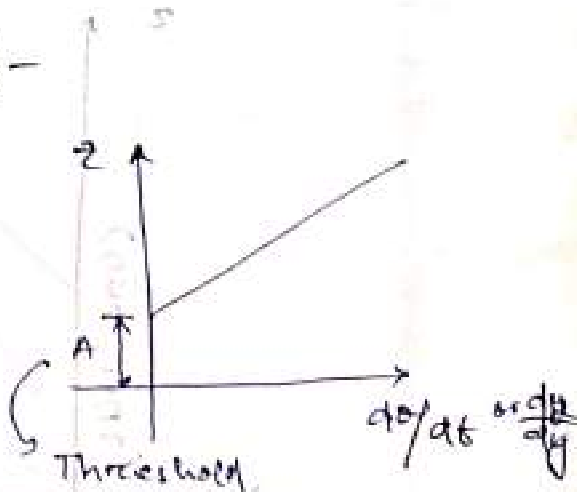
→ It is a shear thining fluid.

(C) ~~Bingh~~ BINGHAM PLASTIC FLUID :-

$$[\, n = 1,\ A \neq 0 \,]$$

$$\boxed{\tau = A + B \cdot \frac{du}{dy}}$$

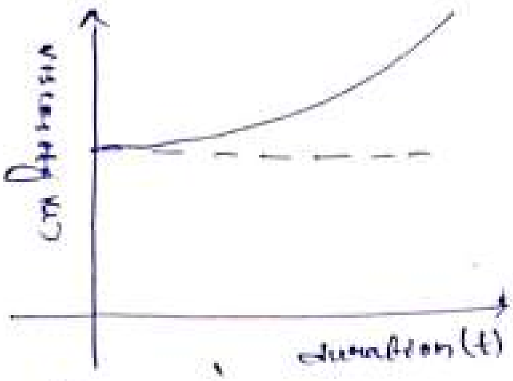
$$\boxed{y = c + mx}$$



→ Minimum amount of shear stress initially required for a fluide flow Threeshold shear stress. Clled stress.

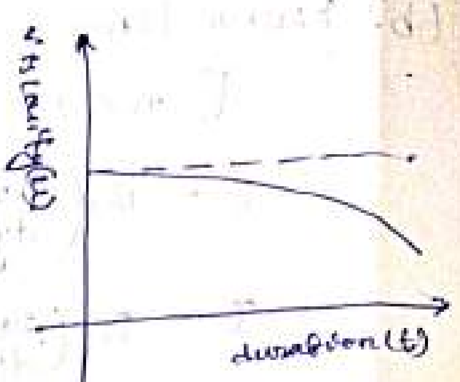
Example :- Toothpaste, cratchap, choacklate, ~~etc~~ etc. mustard, Clay, Jam,

B.2: TIME DEPENDENT FLUID :-



(Rheopectic fluid)
$(n > 1,\ A \neq 0)$

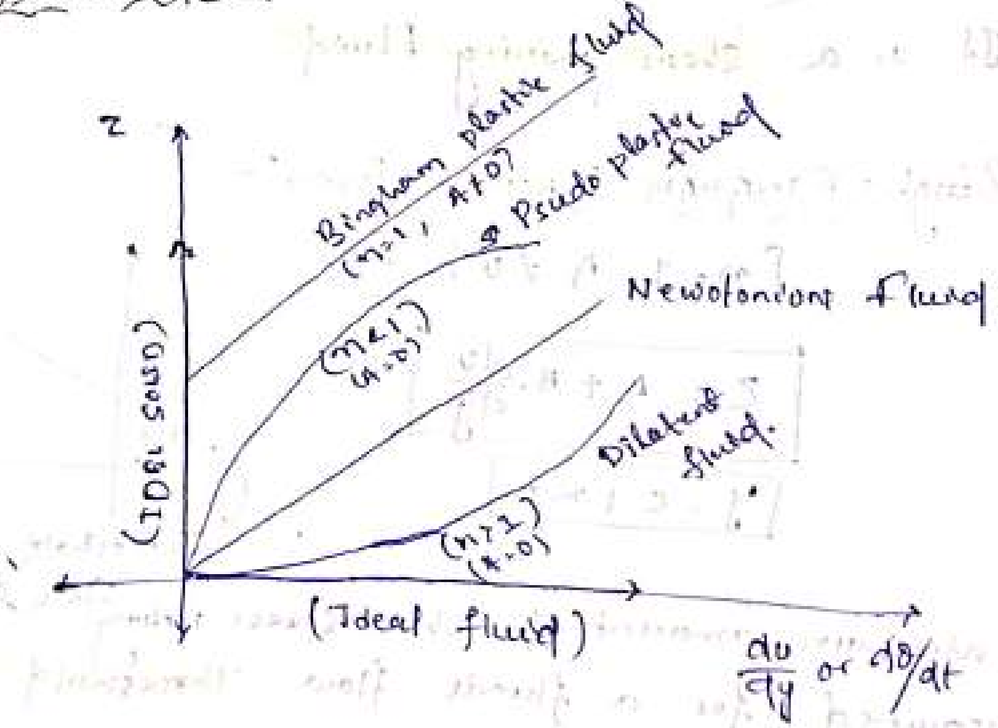
Example:-
Gypsum, whipping cream,
Bentonite



(Thixotropic fluid)
$(n < 1,\ A \neq 0)$

Example:-
Lipstick, printer Ink,
Hand Lotion

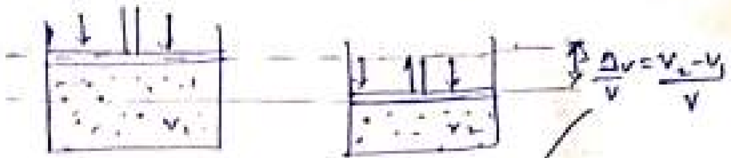
Note:-
COMBINED GRAPH:-



Dt- 17/03/2023
Class- 5th

COMPRESSIBILITY :-

→ Compressibility of fluid represents fractional change in volume per unit pressure.



Fractional change in volume

$$\frac{-\Delta v}{v} = \frac{v_2 - v_1}{v}$$

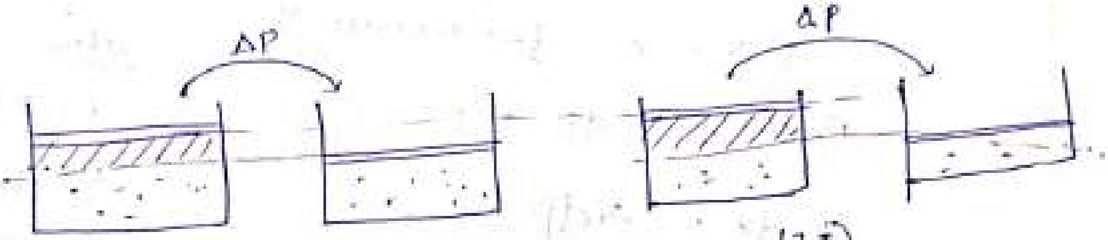
UNIT → rcp. m^2/N

$$\boxed{\beta = \frac{-\frac{\Delta v}{v}}{\Delta P}}$$

or

$$\boxed{\beta = \frac{-1}{v} \cdot \frac{dv}{dp}}$$

Example:-



$$\boxed{\beta_{II} > \beta_I}$$

Mass (m) = Constant

$$\rho \cdot v = C$$

$$\rho \cdot dv + v \cdot d\rho = 0$$

$$\rho \cdot dv = -v \cdot d\rho$$

$$\boxed{-\frac{dv}{v} = \frac{d\rho}{\rho}}$$

∵ $\frac{d}{dx} u \cdot v = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

or $\boxed{d(u \cdot v) = u \cdot dv + v \cdot du}$

$$\boxed{\beta = \frac{-1}{v} \cdot \frac{dv}{dp} = \frac{1}{\rho} \cdot \frac{d\rho}{dp}}$$

→ compressibility

$$\boxed{\text{Bulk modulus of elasticity } (K) = \frac{\Delta p}{\frac{-\Delta v}{v}}}$$

→ Pressure required for change per unit volume.

$$\boxed{\beta = -\frac{1}{v}\cdot\frac{dv}{dp} = \frac{1}{\rho}\cdot\frac{d\rho}{dp} = \frac{1}{K}}$$

* Liquids → Incompressible → $\boxed{dv = 0}$ $\boxed{d\rho = 0}$ → $\boxed{\beta = 0}$ $\boxed{K = \infty}$

* Gases → Compressible → $dv \neq 0$, $d\rho \neq 0$ → $\beta \neq 0$, $K \neq 0$

** Isothermal Compressibility :-

$\boxed{T = \text{Constant}}$,

$\boxed{PV = mRT}$ → Ideal Gas eqn

$PV = C$ } → one mole of gas.

where

R → Ideal Gas Constant

T = temp constant for

m = mass

$$P\cdot dv + v\cdot dP = 0$$

$$P\cdot dv = -v\,dP$$

$$\frac{-dv}{v\cdot dP} = \frac{1}{P}$$

$$\boxed{\beta_T = \frac{1}{P} = \frac{1}{K}}$$ $\boxed{P \propto K}$

Note:- As pressure increases bulk modulus Increases and compressibility decreases.

* Isentropic Compressivity :-

$$\boxed{PV^{\gamma} = C}$$

$$P\gamma\cdot V^{\gamma-1}\,dV + V^{\gamma}\cdot dP = 0 \quad \because \frac{d}{dx}x^{n} = nx^{n-1}$$

$$P\gamma\cdot\frac{V^{\gamma}}{V}\cdot dV = -V^{\gamma}\cdot dP$$

$$-\frac{1}{V}\cdot\frac{dV}{dP} = \frac{1}{\gamma\cdot P}$$ γ = Adiabatic Index.

$$\boxed{\beta_{Isen} = \frac{1}{\gamma\cdot P} = \frac{1}{K}}$$

Numerical

(i) Given $\beta = 5 \times 10^{-11}\ Pa$

$k = ?$

$k = \frac{1}{\beta} = \frac{1}{5 \times 10^{-11}} = \frac{10 \times 10^{10}}{5} = 2 \times 10^{10}\ N/m^2\ (N/m^2 (Pa))$

$= 20\ Gpa$

$\boxed{1\ Pa = 10^{-9}\ Gpa}$

(ii) ans → $k = p$ ⇒ $\beta_{H_2O} = \frac{1}{P} = \frac{1}{K}$

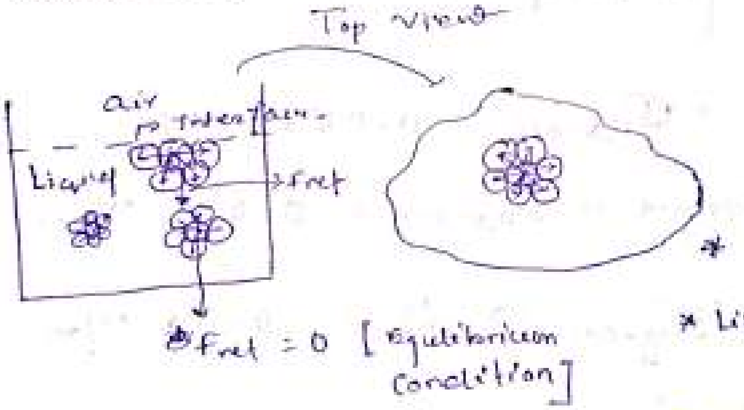
Note:-

$\beta_{air} = 10^{-5}\ Pa^{-1}$

$\beta_{H_2O} = 0.5 \times 10^{-9}\ Pa^{-1}$

$\boxed{\beta_{air} = 20,000\ \beta_{H_2O}}$

SURFACE TENSION:-



molecule

* Like - Like → Cohesive forces.
* Like - unlike → Adhesive force.

→ It is a property of surface and liquid interface.

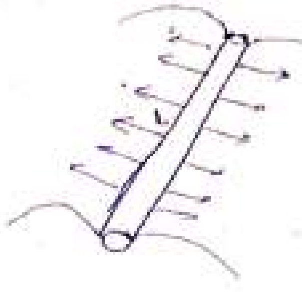
→ All the surface molecule are under the influence of net down word force (cohesive force) to overcome this downward force the surface molecules develope strong cohesive force with each other, So the whole liquid surface appear like a thin film which is in tension. and this tension is known as surface ~~tens~~ tension.

Example:

Steel needle can float, insects can walk.

* Mathematically :-

→ surface tension acts normal to the line of interface (length) and in the plane of liquid surface.



$F_s \propto$ length of interface.

$F_s \propto l$

$\boxed{F_s = \sigma \cdot l}$ → surface tension (fluid property)

$\boxed{\sigma = \frac{F_s}{L}}$ → [Binary Property]

Unit: $\boxed{N/m}$

Dimensional formula: $\frac{kg-m}{s^2-m} \to \frac{kg}{s^2} \to [MT^{-2}]$

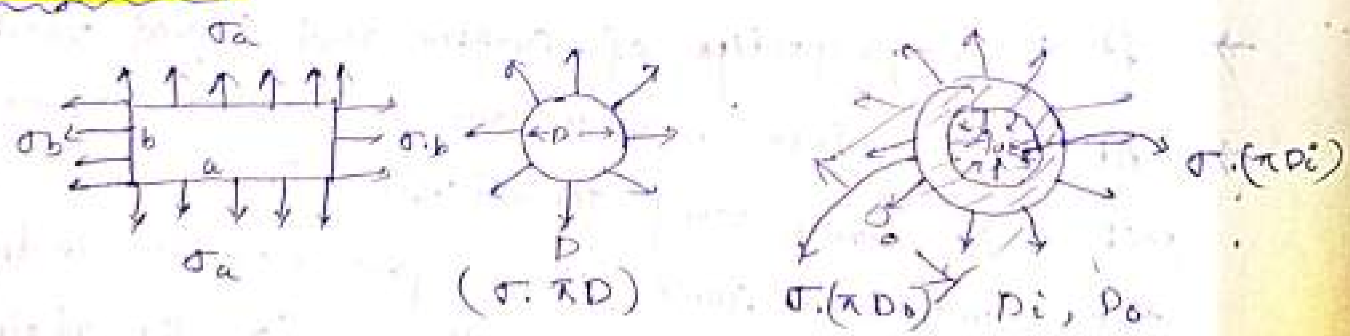
$\boxed{\text{As temp. } T(\uparrow) \Rightarrow CF\downarrow \Rightarrow \sigma\downarrow}$

* At $20^\circ C$ → $\sigma_{H_2O-air} = 0.0732$ N/m

* Soap Detergent → $\sigma_{H_2O-air} = 0.039$ N/m.

* σ_{Hg-air} @ $20^\circ C$ = 0.48 N/m.

→ For Different shapes :-



UNIT

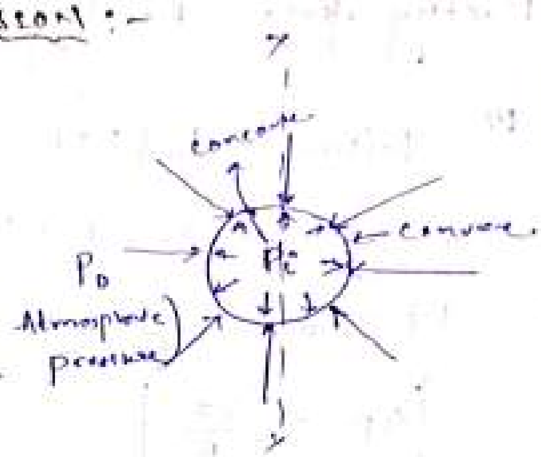
$$\sigma = \frac{N}{m} \times \frac{m}{m} = \frac{N\text{-}m}{m^2} = \frac{Joule(J)}{m^2} \Rightarrow J/m^2$$

$$\boxed{\sigma = \frac{N}{m} \text{ or } J/m^2}$$

Application of Surface Tension :-

① Liquid Droplet :-

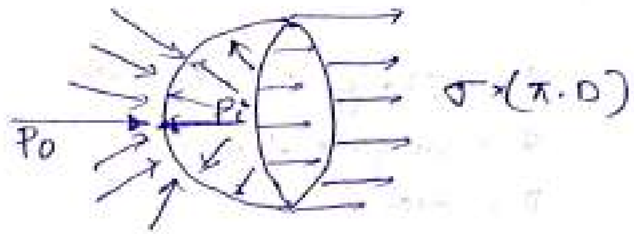
- Equilibrium Condition :-



Bursting force = Resisting force.

$$P_i \times \frac{\pi}{4} D^2 = \sigma . \pi D + P_o \times \frac{\pi}{4} . D^2$$

$$P_i . \frac{D}{4} = \sigma . + P_o . \frac{D}{4}$$

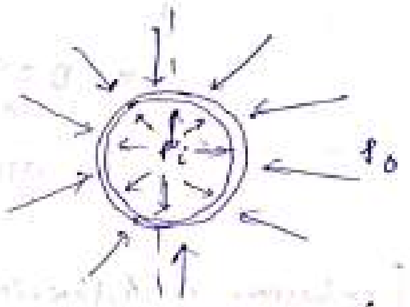


$$\boxed{P_i = \frac{4\sigma}{D} + P_o} \quad (P_i > P_o)$$

$$P_i - P_o = \frac{4\sigma}{D}$$

$$\boxed{\Delta P = \frac{4\sigma}{D}}$$ ← Liquid droplet. (σ = surface tension)

2. Soap Bubble :-

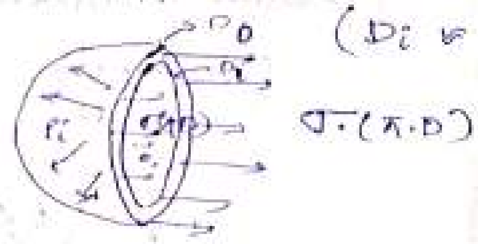


Equilibrium Condition

Bursting force = Resisting force.

$$P_i \times \frac{\pi}{4} D^2 = (2\sigma \pi D) + P_o \frac{\pi}{4} D^2$$

(D_i & $D_o = D$)



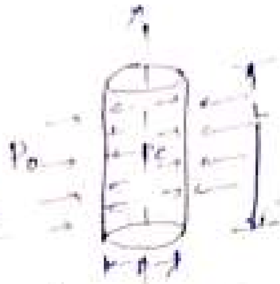
$$P_i = \frac{8\sigma}{D} + P_o$$

$$P_i - P_o = \frac{8\sigma}{D}$$

$$\boxed{\Delta P = \frac{8\sigma}{D}}$$

3. Liquid Jet :-

Equilibrium condition



→ Bursting force = Resisting force.

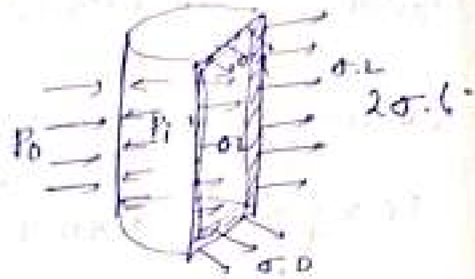
$L >>> D$

$$P_i(D \cdot L) = P_0(D \times L) + 2(\sigma D + \sigma L)$$

$$P_i DL = P_0 DL + 2\sigma L$$

$$P_i - P_0 = \frac{2\sigma}{D}$$

$$\boxed{\Delta p = \frac{2\sigma}{D}}$$

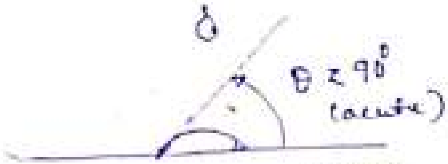


Δp = Change in pressure
σ = surface tension
D = dia.

D - 18/03/2023

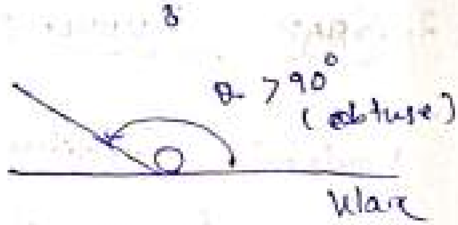
L-20 :-

+ WETTING & NON-WETTING SURFACE :-



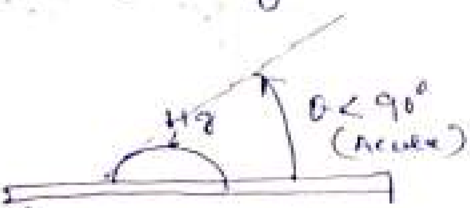
$\boxed{\text{cohesion} < \text{Adhesion}}$

(wetting)

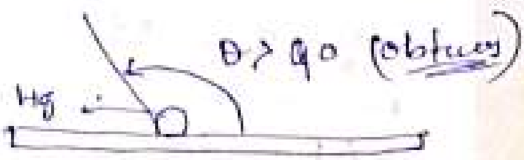


$\boxed{\text{Cohesion} > \text{Adhesion}}$

(Non-wetting)



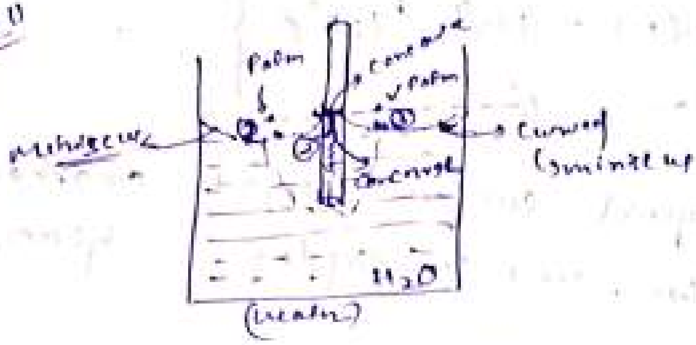
cohesive < adhesive
copper / silver / gold

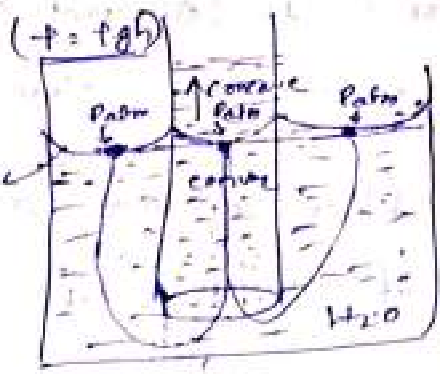


Eg glass

CAPILLARITY :-

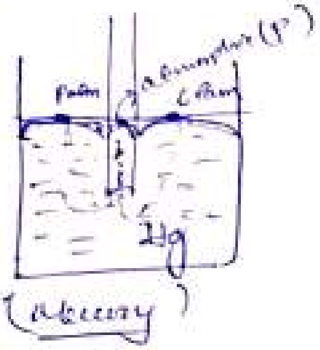
Case-1

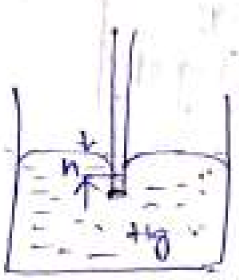




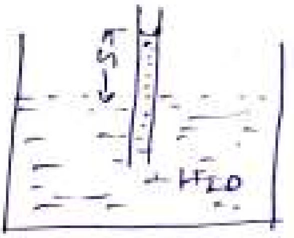
[concave P > convex P]

Case-2





(FALL)



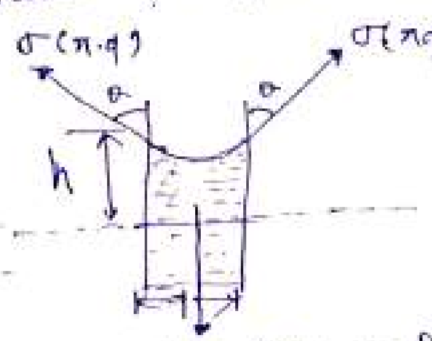
(RISE)

convex concave

→ The rise or fall of liquid in a small dia. tube (< 5 mm) due to surface tension is called capillarity.

Examples :-

Kerosene oil in stove, Liquid rises in a tree, etc.



Equilibrium condition :-

upward force = downward force

$$\rho \cdot \frac{\pi}{4} \cdot d^2 h \cdot g = \sigma \cdot (\pi d) \cos\theta$$

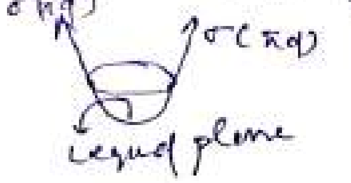
$W = mg = \rho \cdot v \cdot g$

$= \rho \frac{\pi}{4} \cdot d^2 \cdot hg$

Rise / fall
$$\boxed{h = \frac{4\sigma \cdot \cos\theta}{\rho \cdot g \cdot d}}$$

↳ +ve (Rise)

↳ −ve (fall)



θ values:-

$$\begin{cases} \text{for normal water}(\theta) = 8^\circ \\ \text{for Hg and glass}(\theta) = 132^\circ / 128^\circ \\ \text{pure water, clean glass} = 0^\circ \end{cases}$$

* When capillary a liquid surface supports another liquid of density (ρ_L), then rise in capillary is given by:-

$$\boxed{h = \frac{2\sigma\cos\theta}{(\rho-\rho_L)\cdot g\cdot r}}$$

Ch-2: FLUID STATIC:-

There are 3 conditions of fluid statics.

1. fluid is at rest
2. Ideal fluid condition
3. Rigid body ~~Moth~~ Motion → Translation, Rotation

* The fluids statics deals with gravity and pressure force.

TYPES OF PRESSURE:-

1) ATMOSPHERIC PRESSURE:-

The pressure exerted by environmental mass of atmosphere is called atmospheric pressure

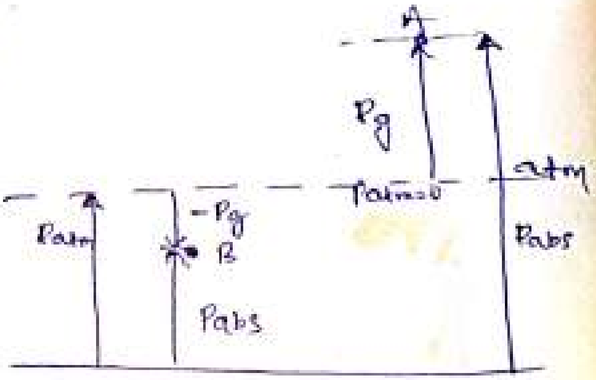
$1\,atm = 101.325\,kPa$

$1\,atm = 1.01\,bar$

$1\,atm = 760\,mm$ of Hg or $10.3\,m$ of H_2O

2. GAUGE PRESSURE (P_g):-

pressure which is above the atmospheric pressure is called Gauge pressure.



3. ABSOLUTE PRESSURE (P_{abs}):-

$$\boxed{P_{abs_A} = P_{atm} + P_g}$$

* Negative Gauge pressure ($-P_g$) is known as vaccume pressure

$$\boxed{P_{abs_B} = P_{atm} - P_g}$$

Note:-

- Pressure measuring device :-

Gauge Pressure → Piezometer.

Vacuum Pressure → Pirani Gauge, Mcleod Gauge.

Absolute Pressure → Manometers

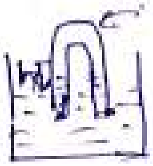
Atmospheric Pressure → Barometer.

BAROMETER:-

$$\boxed{P_{atm} = \rho \cdot g \cdot h}$$

P_{atm} → Barometric pressure.

ρ → Low vapour pressure, High density



- barometric liquid → Hg (taken)

PRESSURE HEAD:-

$$\boxed{P = \rho \cdot g \cdot h}$$

P → pressure.

$$\boxed{h = \frac{P}{\rho \cdot g}}$$

h → pressure head

Eg:- (i) $P = 1 atm$

$h = ?$ ____ mm of hg

(ii) $h =$ 10.3 m of water.

Sol^n (i) $h = \frac{P}{\rho \cdot g} = \frac{101325}{13600 \times 9.81} = 0.760$ m of hg

$$\boxed{h = 760 \text{ mm of hg}}$$

$h = \frac{P}{\rho \cdot g} = \frac{101325}{1000 \times 9.81} = \boxed{10.3 \text{ m of water} = h}$

Eg. 2

$P_B = 30$ cm of hg

find pressure in kpa = ?

Soln

$$h = \frac{P}{\rho \cdot g}$$

$$P = \rho \cdot g \cdot h = 13600 \times 9.81 \times 0.30 = 40025 \text{ Pa}$$

PASCAL'S LAW :-

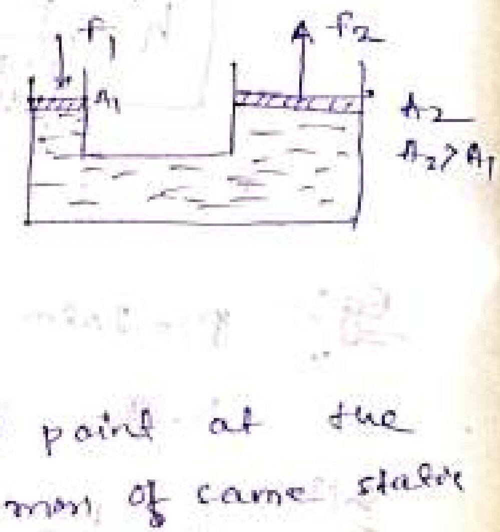
Ist law :

The pressure at a point in a static fluid is equall in all direction.

$$\boxed{P_x = P_y = P_z}$$

2nd Law :

External pressure exerted on fluid is distributed evenly throughout the fluid.



$$\boxed{P = \frac{F_1}{A_1} = \frac{F_2}{A_2}}$$

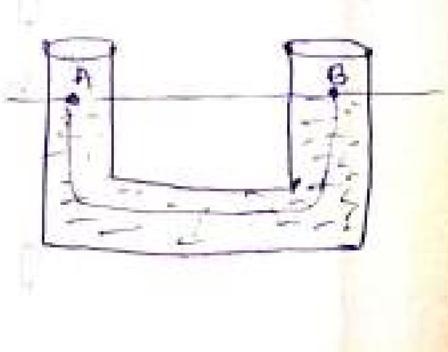
Pressure loss or head loss zero

Example :- Hydraulic lift.

3rd Law :-

Pressure at any ~~same~~ ~~elevation~~ point at the same elevation in a contineous mass of same static fluid is equals.

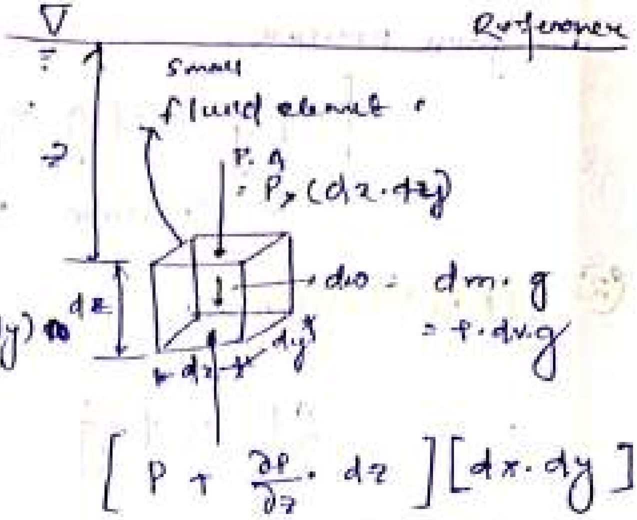
$$\boxed{P_A = P_B}$$



Dt - 20/03/2023

UNIT:- HYDROSTATIC LAW :-

$$dV = dx \cdot dy \cdot dz$$



* Equilibrium Condition :-

$\boxed{\text{Upward force} = \text{Downward force}}$

$$P(dx \cdot dy) + \rho(dx \cdot dy \cdot dz)g = \left(P + \frac{\partial p}{\partial z} \cdot dz\right)(dx \cdot dy)$$

$$\Rightarrow \cancel{P(dx \cdot dy)} + \rho \cdot (dx \cdot dy \cdot dz) \cdot g = \cancel{P(dx \cdot dy)} + \frac{\partial p}{\partial z} \cdot (dx \cdot dy \cdot dz)$$

$$\Rightarrow \rho \cdot (\cancel{dx \cdot dy \cdot dz}) \cdot g = \frac{\partial p}{\partial z} \cdot (\cancel{dx \cdot dy \cdot dz})$$

$$\Rightarrow \boxed{\frac{\partial p}{\partial z} = \rho \cdot g} \quad \text{(Depth)}$$

→ Hydrostatic Law.

$$\boxed{\frac{\partial p}{\partial z} = -\rho \cdot g} \quad \text{upward height}$$

Defination

the rate of change of increment in pressure is equall to the specific weight of the fluid at any point in a Static fluid system

① Liquid :- (ρ = Constant) :-

$$\frac{\partial p}{\partial z} = \rho \cdot g$$

$$\frac{\partial p}{\partial z} = \rho g$$

$$\partial p = \rho \cdot g \cdot \partial z$$

Integrating both side

$$\int_{P_{atm}}^{P_{abs}} dp = \int_0^z \rho \cdot g \cdot dz$$

$$\therefore [P]_{atm}^{abs} = \rho \cdot g \, [z]_0^z$$

$P_{abs} - P_{atm} = \rho g$

$$\boxed{P_{abs} - P_{atm} = \rho \cdot g \cdot z}$$

for gauge pressure. $\because P_{atm} = 0$

$$\boxed{P_g = \rho \cdot g \cdot z}$$

↳ pressure variation " ~~linear~~ linear w.r.t 'z'.

(2) Gases: - $(\rho \neq c)$

$\because$ Ideal gas eqⁿ

$P \cdot V = m \cdot RT \Rightarrow P = \frac{m}{V} \cdot RT$

$P = \rho RT$

$\rho = \frac{P}{R \cdot T}$

$$\frac{\partial P}{\partial z} = \rho \cdot g$$

$$\frac{\partial P}{\partial z} = \frac{P}{RT} g$$

$$\frac{\partial P}{P} = \frac{g}{RT} \partial z$$

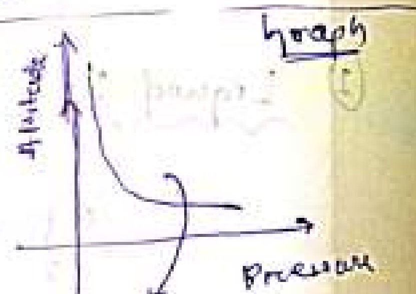
$$\int_{P_{atm}}^{P_{abs}} \frac{\partial P}{P} = \frac{g}{RT} \int_0^z \partial z$$

$$[\ln P]_{P_{atm}}^{P_{abs}} = \frac{g}{RT} [z]_0^z$$

$$\ln P_{abs} - \ln P_{atm} = \frac{g}{RT} [z - 0]$$

$$\ln \frac{P_{abs}}{P_{atm}} = \frac{gz}{RT}$$

Graph



First slowly, then steeply

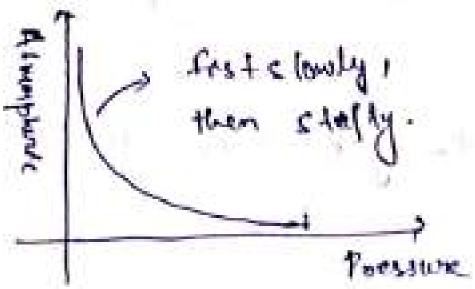
Antilogging both sides.

$$\frac{P_{abs}}{P_{atm}} = e^{\frac{gz}{RT}}$$

$$\boxed{P_{abs} = P_{atm} \cdot e^{\frac{gz}{RT}}}$$ (depth.)

$$\boxed{P_{abs} = -P_{atm} \cdot e^{\frac{gz}{RT}}}$$ (height)

↳ exponential variation.



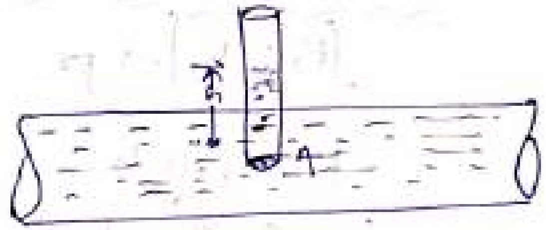
- In a static incompressible fluid pressure varies linearly with depth. in the case of compressible fluid, pressure varies exponential.

PRESSURE MEASURING DEVICES :-

1. Pizometer :-

It is a open ended tube, one end is placed where the pressure is to be measure & other end is open to atmosphere.

$$\boxed{P_g = \rho \cdot g \cdot h}$$

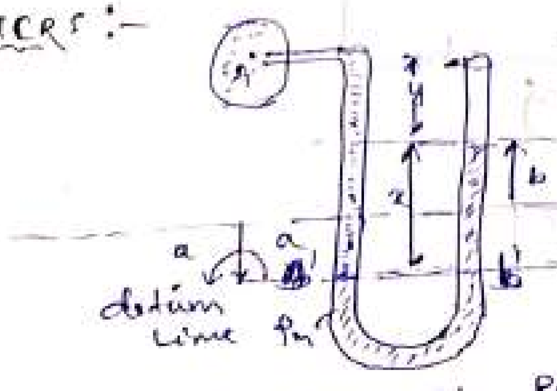


→ Not suitable for gas pressure measurement.

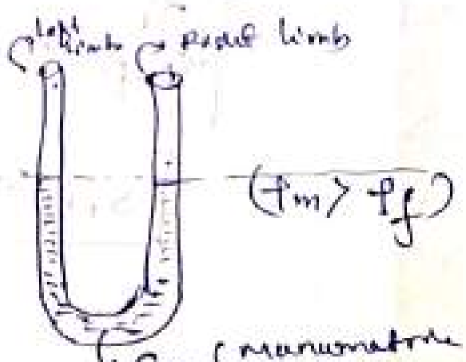
→ Not suitable for extreamly high pressure measurement.

→ Suitable for Low or Moderate pressure measurement.

2. MANOMETERS :-



x : Manomatric fluid deflection.



(Determination of pressure eq^n)

Pressure sign

$\boxed{\downarrow +ve, \uparrow -ve}$

Method-1

1.) Jumping Method :-

$$\boxed{P_A + \rho_f \cdot g\,(y+x) - \rho_m \cdot g \cdot x - P_{atm} = 0}$$

Method-2

Datum Line Method :-

a-b - datum line.

$$P_a = P_b$$

$$P_A + \rho_f \cdot g(x+y) = \rho_m \cdot g \cdot x + P_{atm}.$$

$$\boxed{P_A + \rho_f \cdot g(x+y) - \rho_m \cdot g \cdot x - P_{atm} = 0}$$

$\boxed{a+b = x}$ $\quad \because A_1 \neq A_2$

$\boxed{2a = x}$ $\quad \because A_1 = A_2$, $a = b$

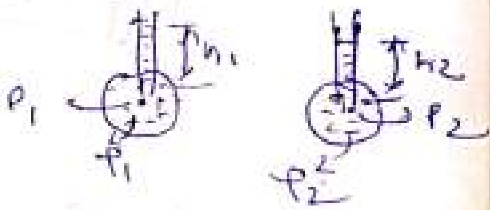
$\boxed{a = \frac{x}{2}}$

* Conversion of one fluid column into other :-

$\boxed{P_1 = P_2} \rightarrow$ pr

$$P = \rho \cdot g\, h$$

pressure head change $\leftarrow \boxed{h = \frac{P}{\rho \cdot g}}$



$$P_1 = P_2$$

$$\rho_1 \cancel{g} \cdot h_1 = \rho_2 \cancel{g} \cdot h_2$$

$$\boxed{\rho_1 h_1 = \rho_2 h_2}$$

$$\boxed{\frac{\rho_1}{\rho_{H_2O}} h_1 = \frac{\rho_2}{\rho_{H_2O}} \cdot h_2}$$

$$\boxed{S_1 h_1 = S_2 h_2}$$

Example!

Q. $P_A = 760$ mm of Hg ①

↳ ___ m. of H_2O ②

Ans — $\rho_1 h_1 = \rho_2 \cdot h_2$

$$13600 \times 760 = 1000 \times h_2$$

$$h_2 = \frac{13600 \times 760}{1000} = 10.33 \text{ m of } H_2O$$

Numericals :-

Example

P_A = 100 m of water (H_2O

___ m of Hg.

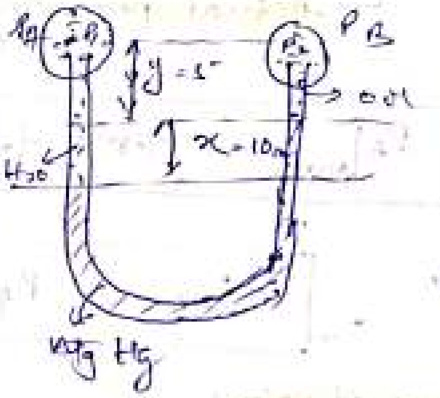
$$\rho_1 h_1 = \rho_2 h_2$$

$$\Rightarrow 1000 \times 100 = 13600 \times h_2$$

$$\Rightarrow h_2 = \frac{1000 \times 100}{13600} = 7.3 \text{ m of Hg}$$

Example-3

Q find $P_A - P_B$



Sol^n

$$P_A + \rho_{H_2O} g \cdot (0.15) - \rho_{Hg} \cdot g (0.1) - \rho_{oil} \cdot g\,(0.05) - P_B = 0$$

$$\Rightarrow P_A - P_B = \rho_{Hg} \cdot g (0.1) + \rho_{oil} \cdot g \cdot (0.05) - \rho_{H_2O}\, g (0.15)$$

$$= (13600 \times 9.81 \times 0.1) + (800 \times 9.81 \times 0.05) - (1000 \times 9.81 \times 0.15)$$

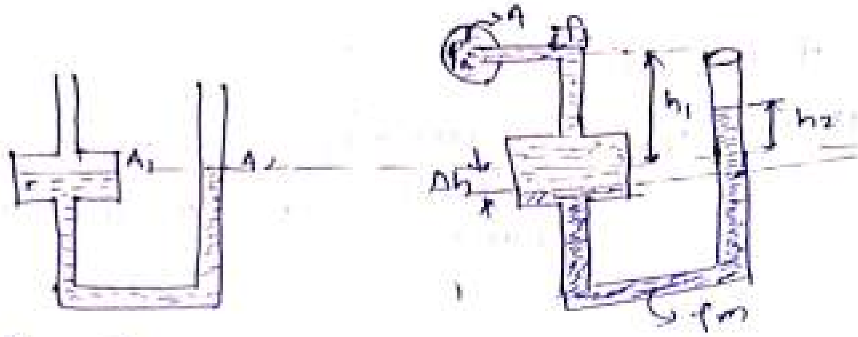
$$P_A - P_B = 13341.6 + 421.83 - 1471.5$$

$$= 12291.93 \text{ Pa}$$

$$P_A - P_B = 12.3 \text{ kPa}$$

3. Single Column Manometer :-

($P \neq P_{atm}$)



$$P = \frac{F}{A}$$

$$P \times (A) = F$$

Pressure eqn

$$P_A + \rho_f g(h_1 + \Delta h) - \rho_m g(\Delta h + h_2) - P_{atm} = 0$$

$$\boxed{P_A = \rho_m g(\Delta h + h_2) - \rho_f g(h_1 + \Delta h) + P_{atm}}$$

→ Actual pressure / Exact Pressure.

$$\boxed{P_A\big|_{approx} = \rho_m \cdot g\, h_2 - \rho_f \cdot g\, h_1 + P_{atm}}$$

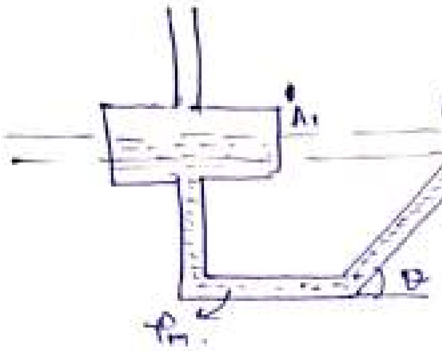
$$\boxed{\text{Error} = P_A - P_A\big|_{approx}}$$

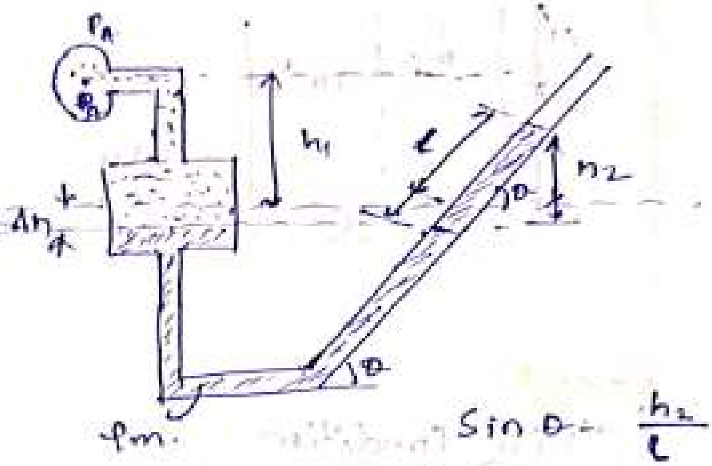
In terms of volume

$$\Delta h \times A_1 = h_2 \times A_2$$

$$\boxed{\Delta h = \frac{A_2}{A_1} \cdot h_2}$$

4. INCLINE COLUMN MANOMETER :-





$$\sin\theta = \frac{h_2}{l}$$

$$\boxed{h = l \cdot \sin\theta}$$

Pressure eqn

$$\boxed{P_A + \rho_f \cdot g\,(h_1 + \Delta h) - \rho_m \cdot g\,(\Delta h + h_2) - P_{atm} = 0}$$

$$h_2 = l\sin\theta$$

** $$\boxed{\text{Sensitivity} = \frac{1}{\sin\theta} = \frac{l}{h_2}}$$

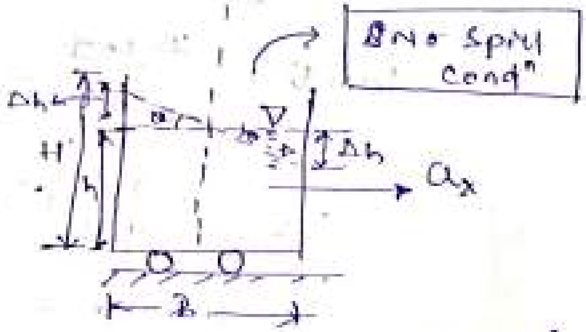
Dt - 21/03/2023
L-3 start

RIGID BODY MOTION :-

1. TRANSLATION
2. ROTATION. (Vortex Motion)

(1) TRANSLATION :-

case-1 : No-spill condition :-



$$\tan\theta = \frac{\Delta h}{B/2} = -\frac{a_x}{g}$$

$$\boxed{\tan\theta = \frac{-a_x}{g} = \frac{2\Delta h}{B}}$$

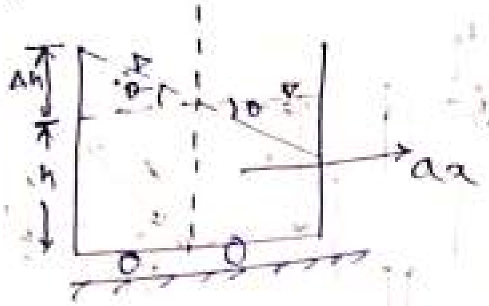
* fluid is not come out of container.

$a_x \rightarrow$ acceleration

$g \rightarrow$ gravitational const = 9.81 m/sec²

Case-2 :

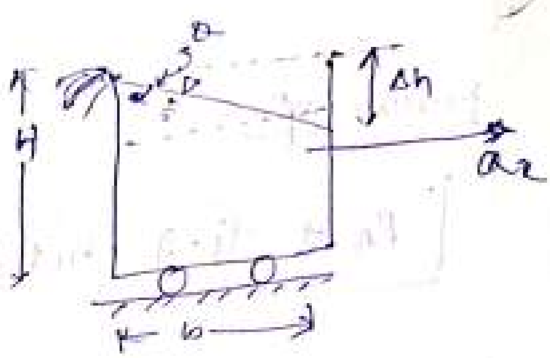
About to spill condition.



Case-3

Spill condition

$$\boxed{\tan\theta = \frac{-a_x}{g} = \frac{\Delta h}{B}}$$

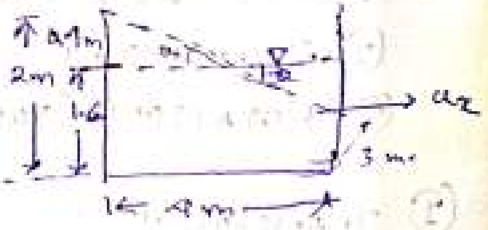


(4 × 3 × 2 m)

Numericals

Q. An open rectangular tank of dimension 4 m It is accelerated contain water to a height of 1.6 m. What is the possible maximum along its larger side without spill of water. acceleration

Sol^n about to spill



$\tan\theta = -\frac{a_x}{g} = \frac{2\Delta h}{B}$

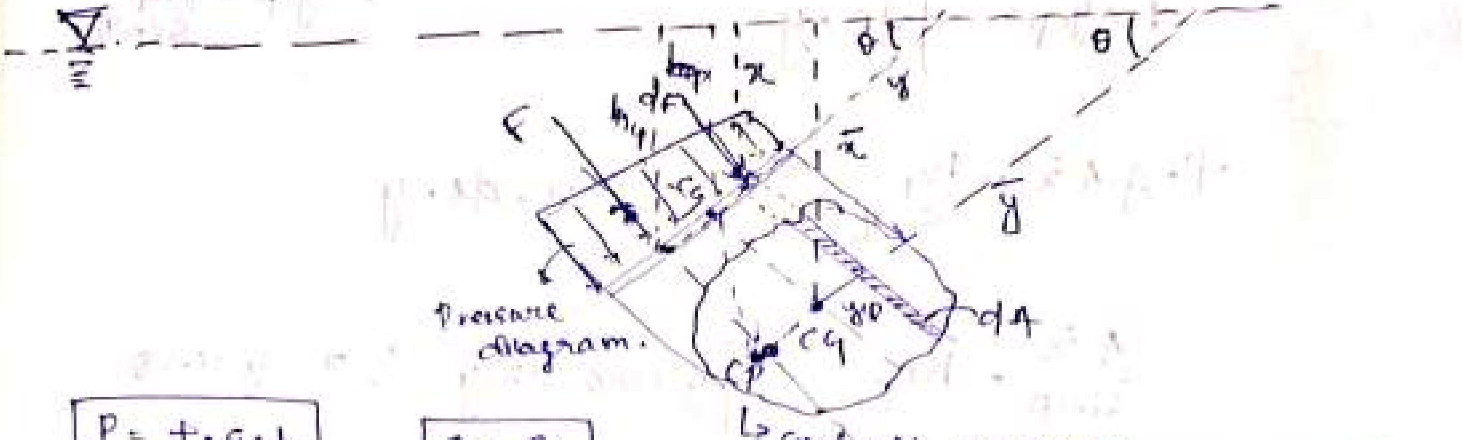
$\tan\theta = -\frac{a_x}{g} = \frac{2\Delta h}{B}$

$= \frac{a_x}{9.81} = \frac{2 \times 0.4}{4}$

$= a_x = \frac{0.4}{2} \times 9.81$

$$\boxed{a_x = 1.962 \ m/sec^2}$$

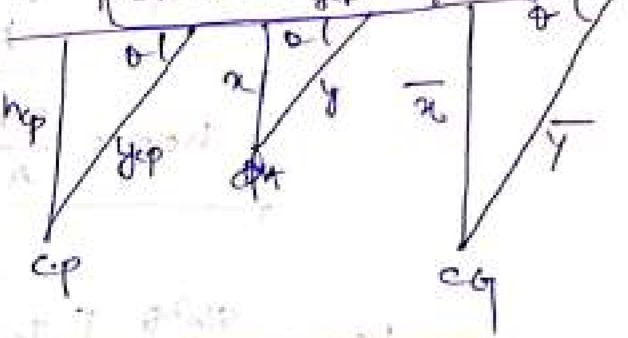
HYDRO-STATIC FORCE ON A PLANE SURFACE :-



↳ centre of pressure

$\boxed{P = \rho \cdot g \cdot h}$ $\boxed{P = F/A}$ $\boxed{F = P \cdot A}$

$\boxed{\sin\theta = \frac{h_{cp}}{y_{cp}} = \frac{x}{y} = \frac{\bar{x}}{\bar{y}}}$



Pressure for small area

$P = \rho g x$

for small area (force)

$dF = P \cdot dA$

$dF = \rho \cdot g\, x\, dA$

Integrating both side.

$F = \rho \cdot g \int x \cdot dA$

↳ first moment of area.

$$\boxed{\int x \cdot dA = \bar{x} A}$$

$$\boxed{F = \rho \cdot g \cdot A \bar{x}}$$

↳ Hydrostatic force.

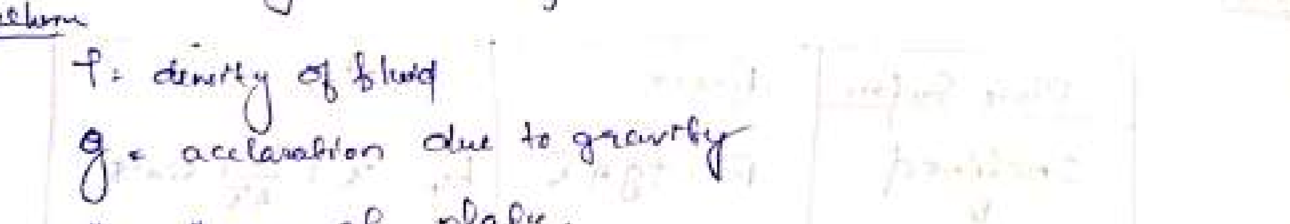
where

ρ = density of fluid

g = accelaration due to gravity

A = Area of plate.

$\bar{x}$ = ⊥ distance from CG to free surface.



$A = \frac{\pi}{4} D^2$ $\quad A = H^2$ $\quad A = H \cdot B$ $\quad A = \frac{1}{2} B \cdot H$

$\bar{x} = \frac{D}{2}$ $\quad \bar{x} = \frac{H}{2}$ $\quad \bar{x} = H/2$ $\quad \bar{h} = 2H/3$

$\boxed{I_{cg} = \frac{\pi}{64} D^4}$ $\quad \boxed{I_{cg} = \frac{H^4}{12}}$ $\quad \boxed{I_{cg} = \frac{BH^3}{12}}$ $\quad \boxed{I_{cg} = \frac{BH^3}{36}}$

Moment on the C.P

$$F \times y_{cp} = dF \times y$$

$$\because y_{cp} = \frac{h_{cp}}{\sin\theta}$$

$$F \times y_{cp} = \int dF \times y$$

$$\rho \cdot g \cdot A\bar{x} \times \frac{h_{cp}}{\sin\theta} = \int \rho \cdot g\, x \cdot dA \cdot y$$

$$\frac{A\bar{x}}{\sin\theta} \cdot h_{cp} = \int y \sin\theta \cdot dA \cdot y \quad \because x = y\sin\theta$$

$$h_{cp} = \frac{\sin^2\theta}{A\bar{x}} \int y^2 dA$$

↳ second moment of area I_0

$$\boxed{h_{cp} = \frac{\sin^2\theta}{A\bar{x}} [I_0]}$$

* Parallel axis theorem.

$$I_0 = I_{cg} + A\bar{y}^2$$

$$I_0 = I_{cg} + A\frac{\bar{x}^2}{\sin^2\theta}$$

$$h_{cp} = \frac{\sin^2\theta}{A\bar{x}} \left[I_{cg} + A\frac{\bar{x}^2}{\sin^2\theta} \right]$$

$$\boxed{h_{cp} = \bar{x} + \frac{I_{cg}}{A\bar{x}} \cdot \sin^2\theta}$$

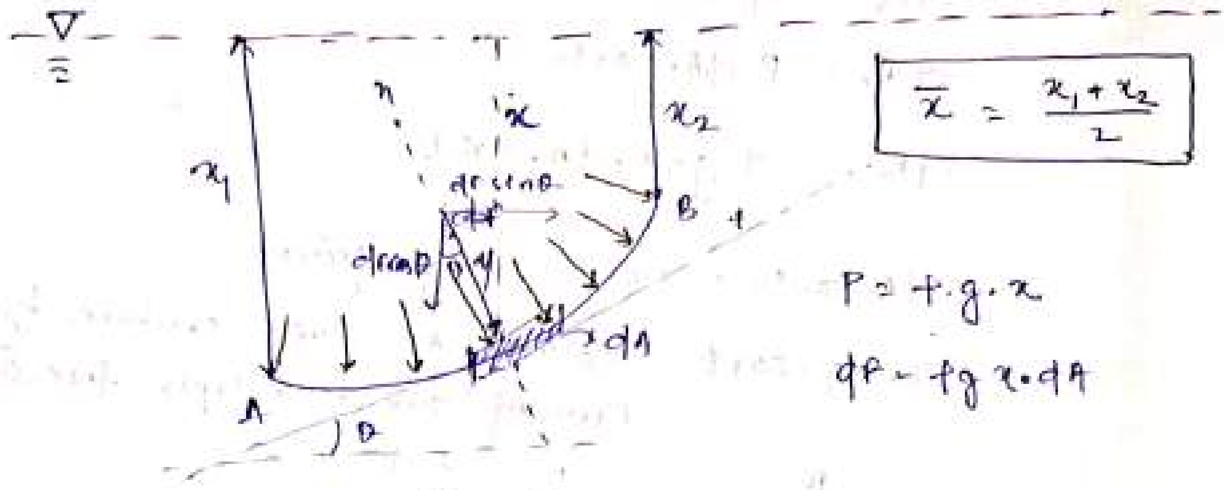
Note! The point where total hydrostatic force is called center of pressure C.P.

Short ~~...~~ Note:-

Imp

Plane Surface	force	h_{cp}
Inclined θ	$F = \rho g A\bar{x}$	$h_{cp} = \bar{x} + \frac{I_{cg}}{A\bar{x}} \sin^2\theta$
Vertical $\theta = 90°$	$F = \rho g A\bar{x}$	$h_{cp} = \bar{x} + \frac{I_{cg}}{A\bar{x}}$
Horizontal $\theta = 0°$	$F = \rho g A\bar{x}$	$h_{cp} = \bar{x}$ ↳ Center of pressure & C.G. coinside.

HYDRO-STATIC FORCE ON CURVED SURFACE :-

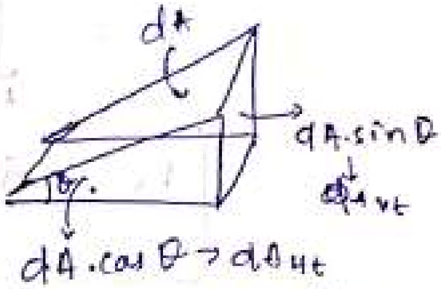


$$\bar{x} = \frac{x_1 + x_2}{2}$$

$$P = \rho \cdot g \cdot x$$

$$dF = \rho g\, x \cdot dA$$

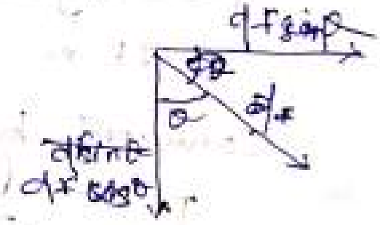
(I) Horizontal hydrostatic force :-



$$dF_H = dF \cdot \sin\theta$$

$$dF_H = \rho \cdot g\, x\, (dA \sin\theta)$$

↳ vertical projection of curved surface.



$$dF_H = \rho \cdot g \cdot dA_{vt}$$

$$F_H = \int \rho \cdot g \cdot x \cdot dA_{vt}$$

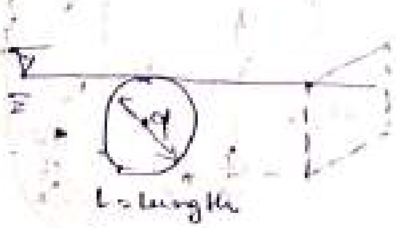
$$\boxed{F_H = \rho \cdot g\, A_{vt} \cdot \bar{x}_{vt}}$$

$A_{vt} \to$ vertical projection of curve surface Area.

$\bar{x}_{vt} \to$ Distance from CG to free surface of projected area.

Example :-

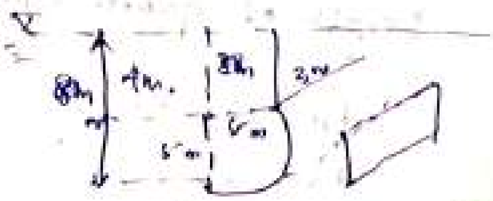
(1)



$$A_{vt} = L \times d$$

$$\bar{x}_{vt} = \frac{d}{2}$$

(2)



$$A_{vt} = 6 \times 3 = 18\ m^2$$

$$\bar{x}_{vt} = 6.5\ m$$

(2) Vertical Hydrostatic force:-

$$dF_v = dF \cdot \cos\theta$$

$$dF_v = P \cdot dA \cdot \cos\theta$$

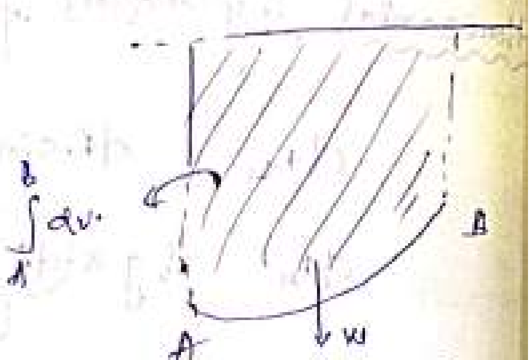
$$dF_v = \rho \cdot g \cdot x \cdot dA \cdot \cos\theta$$

$dA \cos\theta$ = Horizontal projection.

$x \cdot dA \cdot \cos\theta$ = Volume of fluid contain by curved surface upto free surface $= dV$

$$F_v = \int_A^B \rho g \cdot dV$$

$$\boxed{F_v = \rho \cdot g \int_A^B dV}$$



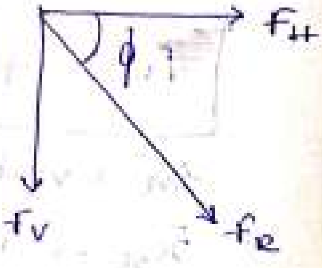
$W = mg$

Vertical hydrostatic force:-

It is a weight of the fluid contained by curved surface upto free surface. $W = \rho \cdot g \cdot V$

$$\boxed{F_R = \sqrt{F_H^2 + F_v^2}}$$

$$\boxed{\tan\phi = \frac{F_v}{F_H}}$$

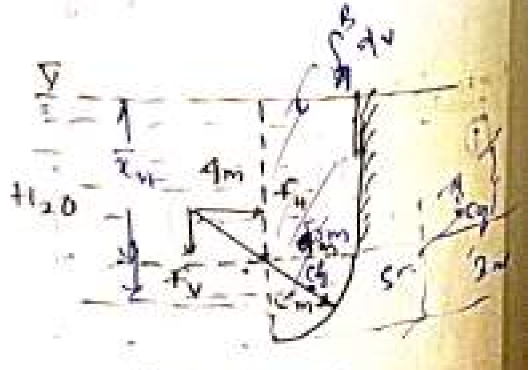


Example:-

Length of quarter Cylinder = 3m.

F_H = ___ kn

F_v = ___ ?



Soln

$$F_H = \rho \cdot g \cdot A_{vt} \, \bar{x}_{vt}$$

$A_{vt} = 3 \times 5$

$$= 1000 \times 9.81 \times (3 \times 5) \times \left(\frac{5}{2} + 4\right)$$

$\bar{x}_{vt} = \left(\frac{5}{2} + 4\right) = 6.5$

Vertical H-force

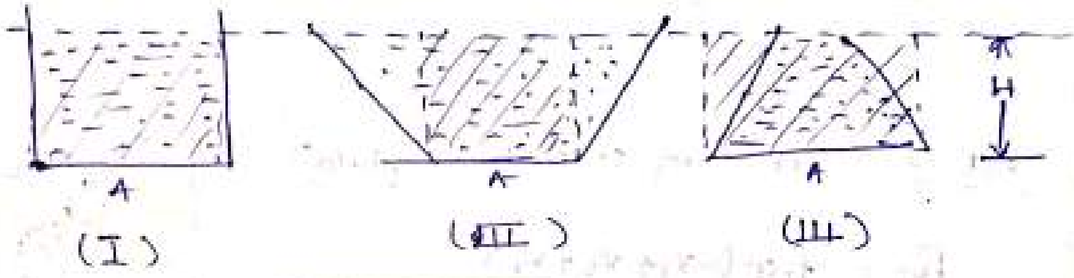
$$f_v = \rho \cdot g \int_A^B dv$$

$$f_v = 1000 \times 9.81 \times \left\{ \frac{\frac{\pi}{4} D^2 L}{4} + (5 \times 4 \times 3) \right\}$$

$$f_v = \frac{1000 \times 9.81}{1000} \left[\left(\frac{1}{4} \frac{\pi}{4} (10)^2 \times 3 \right) + (4 \times 5 \times 3) \right]$$

$$f_v = 1166.45 \text{ KN}$$

HYDROSTATIC PARADOX :-

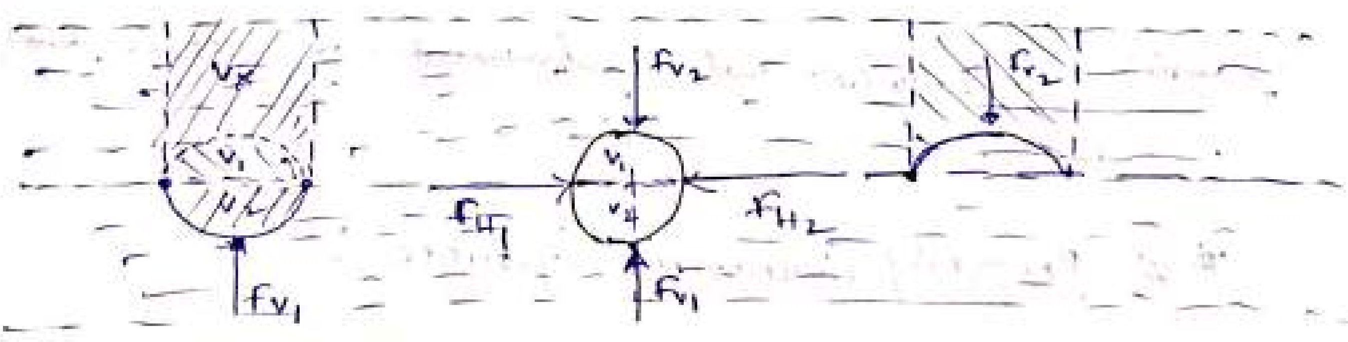


$$\boxed{W_{II} > W_I > W_{III}}$$
$$\boxed{f_{V_I} = f_{V_{II}} = f_{V_{III}}}$$
} Hydrostatic Paradox.

* Hydrostatic force may not be same as actual weight of the fluid.

COMPLETELY Submerged Cylinder:-

Case-1: submerged in same fluid



$$f_H = \rho \cdot g \cdot A_{vt} \cdot \bar{x}_{vt}$$

$$\boxed{f_{H_1} = f_{H_2}}$$

$$\boxed{f_{net/H} = 0}$$

Total Cylender Volume = V

$V = V_1 + V_2$

$F_{V_1} = \rho \cdot g (V_1 + V_2 + V_x)$

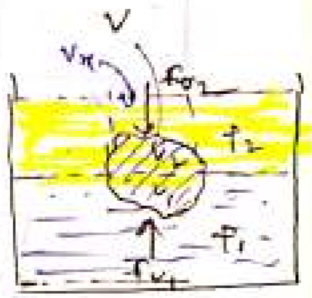
$F_{V_2} = \rho \cdot g (V_x)$

$F_{net}/_V = F_{V_1} - F_{V_2}$

$F_{net}/_V = \rho \cdot g (V_1 + V_2)$

$\boxed{F_{net}/_V = \rho \cdot g \,[\text{Volume of cylinder } (V)]}$

CASE-2 (Submerged in different fluid)



$\cancel{F_{V_1} = \rho \cdot g (V_1 + V_2 + V_x)}$

$F_{V_1} = \rho_1 g V_1 + \rho_2 g V_2 + \rho_2 g V_x$

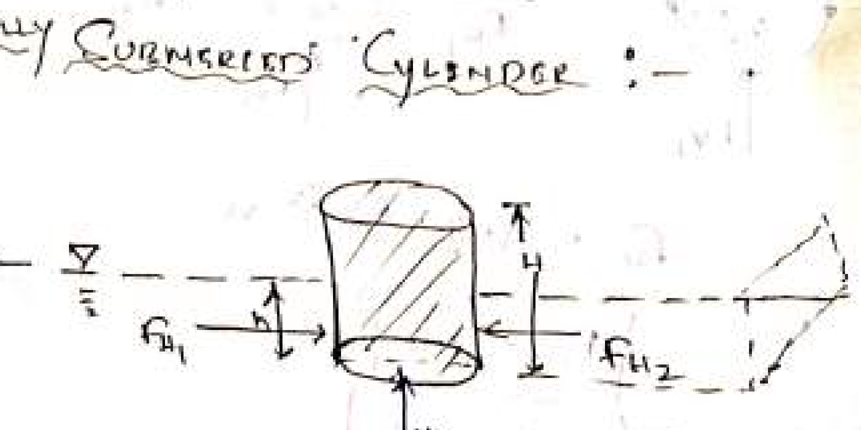
$F_{V_2} = \rho_2 g \cdot V_x$

$F_{net}/_V = F_{V_1} - F_{V_2}$

$\boxed{F_{net}/_V = \rho_1 g V_1 + \rho_2 g V_2}$

Example:- ~~A body 60's volume immersed in water and~~.
Define

PARTIALLY SUBMERGED CYLINDER :-



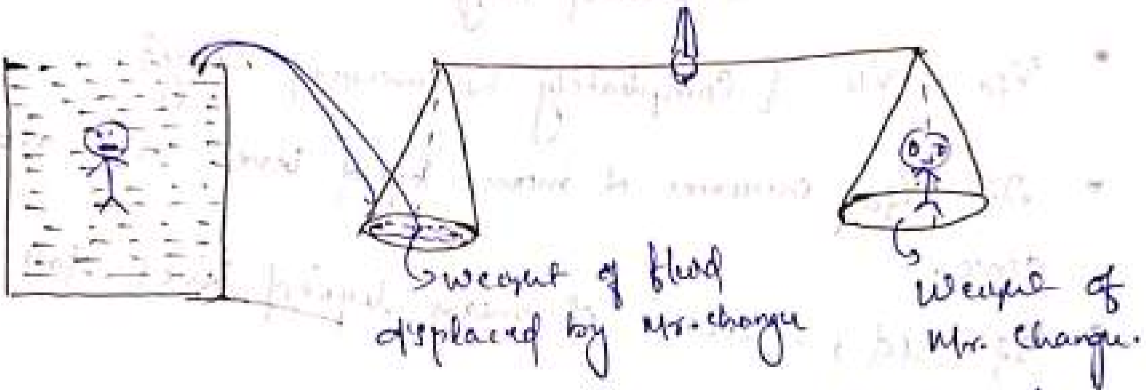
$$\boxed{f_{H_1} = f_{H_2}}$$

$$\boxed{f_{net}|_{H} = 0}$$

$$f_v = \rho \cdot g \int dV$$

$$\boxed{f_v = \rho \cdot g \, \frac{\pi}{4} D^2 h}$$

BUOYANCY & FLOATATION :-

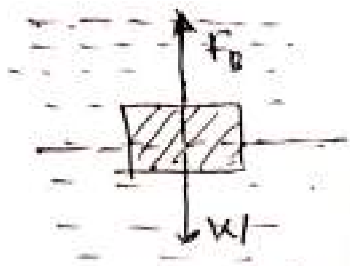


* When the body is submerged in a fluid either partially or completely the net vertical hydrostatic force is called Buoyancy force.

$$\boxed{f_B = \text{wt of fluid displaced by the body}}$$

According to Archimedes principle:-

$$\boxed{f_B = W_{body}}$$



Condition of equilibrium:-

(1) Vertical Equilibrium:-

$$f_B = W \quad \Rightarrow \quad \boxed{\rho_f \cdot g \cdot V_{fd} = \rho_b \cdot g \cdot V_b}$$

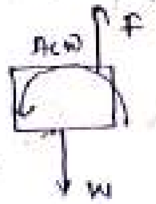
$$\rho_f \cdot g V_{fd} = M \cdot g$$

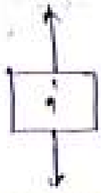
ρ_f = Density of fluid

V_{fd} = volume of fluid displ by body

2. Rotational Equilibrium:-

→ The line of action of buoyancy force & weight of the body must be same for rotational equilibrium.



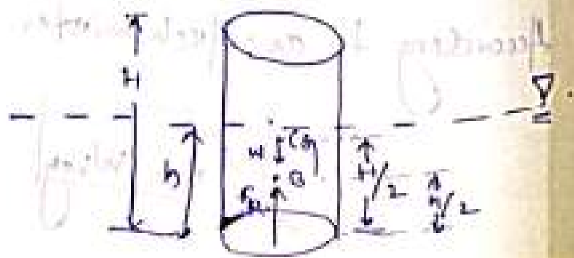


for Rotational equilibrium. Condition.

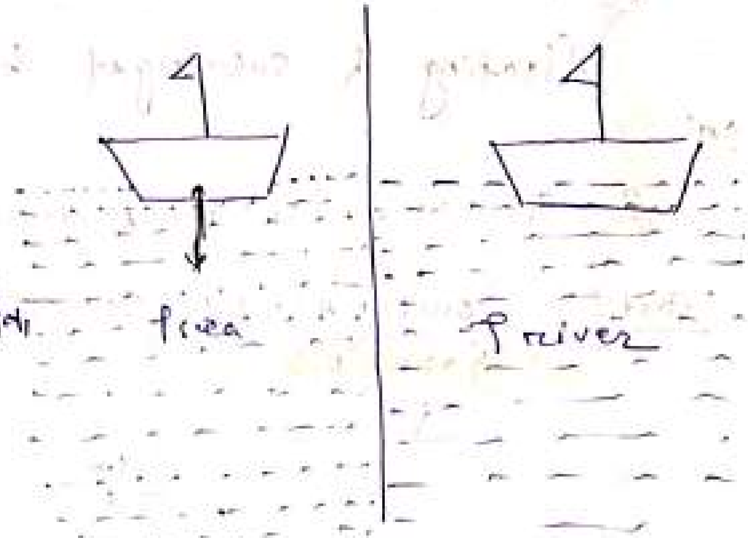
- $V_{fd} < V_b$ {Partially Submerged body}
- $V_{fd} = V_b$ {Completely Submerged body}
- If V_{fd}, increases it means liquid level rise and body sink.
- If V_{fd}, decrease it means liquid level fall and body will rise.

Date- 23/02/2023

* Centre of ~~Bo~~ Buoyancy:- Point of application of buoyancy is called centre of buoyancy. It is a centroid of displaced fluid.



Ex: Ship enters from sea to reaver, will it move up or down.



Case-1: Sea

$F_b = W$

$\rho_{sea} \cdot g \cdot V_{fd_s} = \rho_b g \cdot V_b$

$\Rightarrow V_{fd_s} = \frac{\rho_b \cdot V_b}{\rho_{sea}}$ —(1)

Case-2: Reaver

$F_b = W.$

$\rho_{river} \times g \times V_{fd_r} = \rho_b \cdot g \cdot V_b$

$V_{fd_r} = \frac{\rho_b \cdot V_b}{\rho_{river}}$ —(2).

($\rho_{sea} > \rho_{river}$)

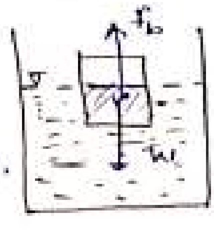
from eqⁿ (1) & (2)

$\Rightarrow \boxed{V_{fd_r} > V_{fd_s}}$

∴ Ship will move down.

Ex-2: An Ice block is kept in water, when it melt water level will rise or fall or will remain same?

Condition-1
Case-1: when Ice is Solid:—



weight of ice = weight of melted ice.

$\rho_{ice} g V_{ice} = \rho_w \cdot g \; V_w$ (added)

$\boxed{V_w(\text{added by ice}) = \frac{\rho_{ice} \cdot V_{ice}}{\rho_w}}$

Condition:—2

$F_b = W$

$\rho_w \cdot g \cdot V_{fdi} = \rho_{ice} \cdot g \cdot V_{ice}$

$\boxed{V_{fdi} = \frac{\rho_{ice} \cdot V_{ice}}{\rho_{water}}}$

$\boxed{V_{fice} = V_w \text{ (add)}}$

∴ Hence no change in water

Numericals

** SSC 2021 (1st Mark)

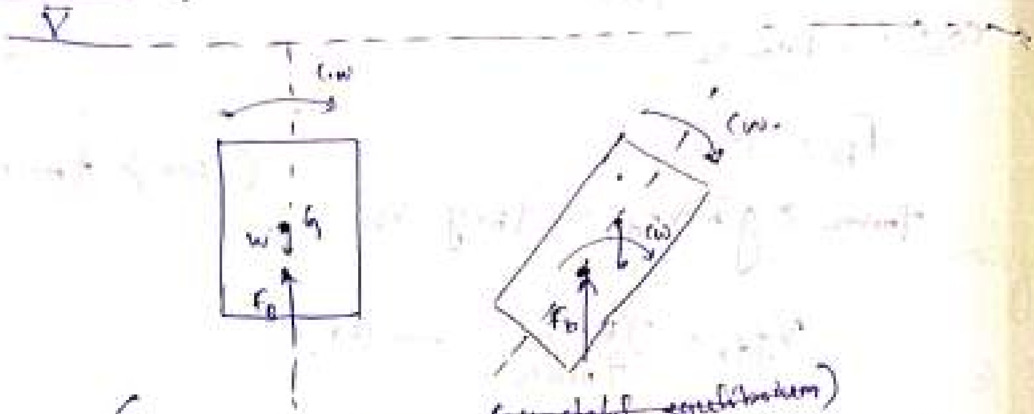
7. Describe the the condition of equilibrium of a floating & submerged body.

Sol:

CONDITION OF ROTATIONAL EQUILIBRIUM :-

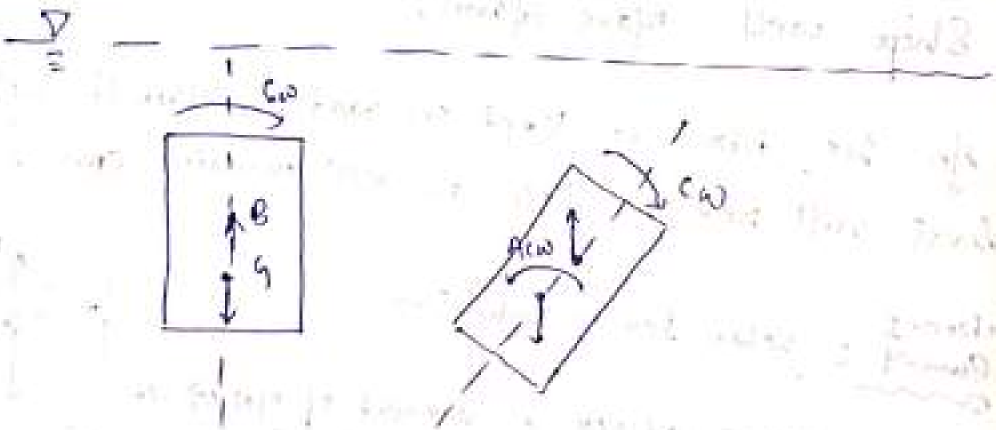
Case 1: COMPLETELY SUBMERGED BODY :-

a) "G above B" :-



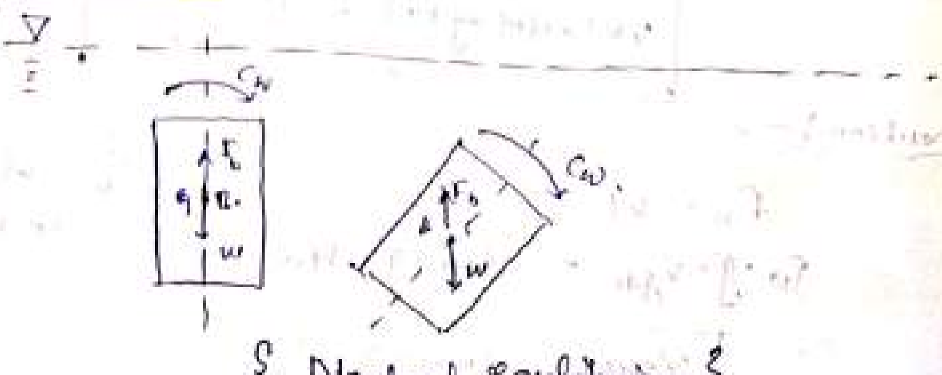
{Unstable Equilibrium} ~~(unstable equilibrium)~~

b) "G below B" :-



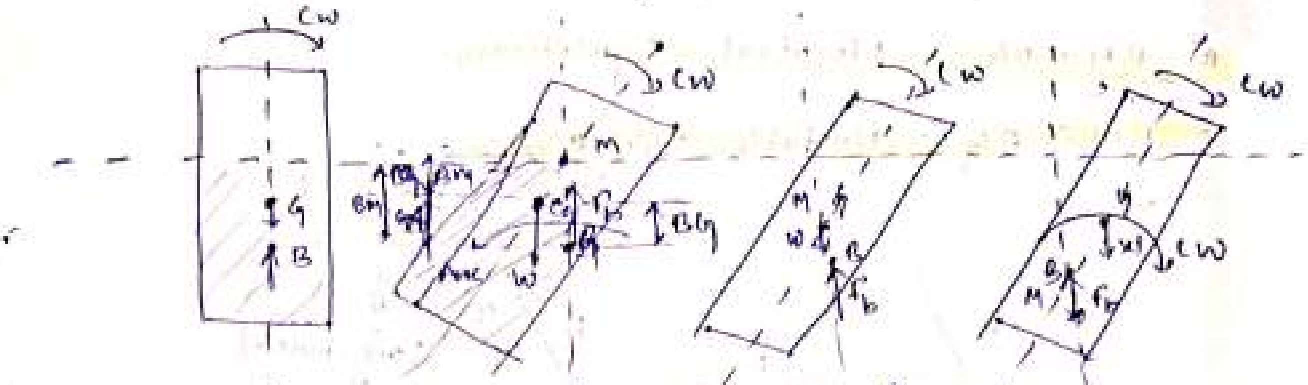
{Stable Equilibrium}

c) "G co-incide with B" :-



{Neutral equilibrium}

Case-II: PARTIALLY SUBMERGED BODY :-



{Stable} {Neutral} {Unstable}

METACENTRE :-

It is the Intersection of neutral axis and new line of action of new buoyancy. It is represented by "M".

* Metacentric Height (GM) :- The distance between centre of Gravity and Meta-centre. is called Metacentric height.

$$\boxed{GM = BM - BG}$$

$$\boxed{BM = \frac{I_{xx}}{V_{fd}}}$$

where

I_{xx} = Area Moment of Inertia Longitudinal axis

V_{fd} = Volume of fluid displaced by body

* Time Period of Oscillation :-

$$\boxed{T = 2\pi\sqrt{\frac{k^2}{g \cdot GM}}}$$

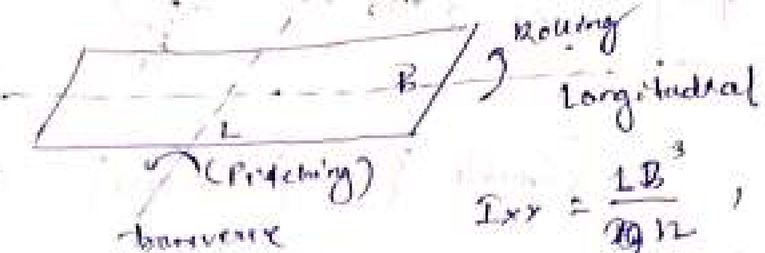
where

k = Radius of Gyration

GM = Metacentric height.

"Metacentre is a point about which a floating body suppose to oscillate", More Metacentric height More stability.

* $BM > BG$: Stable equilibrium.
* $BM = BG$: Neutral Equilibrium.
* $BM < BG$: Unstable equilibrium.



$$I_{xx} = \frac{LB^3}{12}, \quad I_{yy} = \frac{BL^3}{12}$$

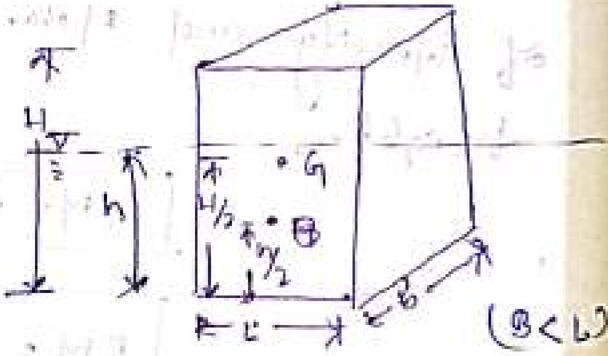
$$\boxed{I_{xx} < I_{yy}}$$

$$\boxed{\text{Torque} = I_{xx} \cdot \alpha}$$

* Rolling is more critical than pitching, we always design for rolling.

Example-1 find metacentric height:-

Sol^n



$(B < L)$

$$GM = BM - BG$$

$$GM = \frac{I_{xx}}{V_{fd}} - BG$$

$$BG = \left[\frac{H}{2} - \frac{h}{2}\right]$$

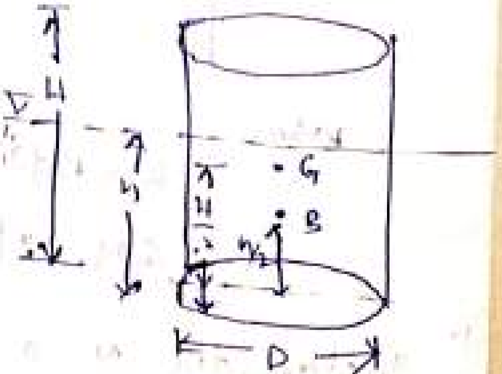
* $V_{fd} = L \times B \times h$

* $I_{xx} = \frac{LB^3}{12}$

$$BM = \frac{I_{xx}}{V_{fd}} = \frac{\frac{LB^3}{12}}{L \times B \times h}$$

$$\boxed{BM = \frac{B^2}{12h}}$$

Examp. 2



$$GM = BM - BG$$

$$BG = \left[\frac{H}{2} - \frac{h}{2}\right]$$

$$BM = \frac{I_{xx}}{V_{fd}} = \frac{\frac{\pi}{64} \times D^4}{\frac{\pi}{4} \times D^2 \times h}$$

$$\boxed{BM = \frac{D^2}{16h}}$$

$V_{fd} = \frac{\pi}{4} \times D^2 \times h$

$I_{xx} = \frac{\pi}{64} D^4$

CH-3. FLUID KINEMATICS

Date- 24/03/2023

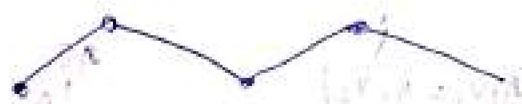
→ Study of fluid motion without considering force.

→ Reason behind the motion not considered.

** There are two approach to study fluid :-

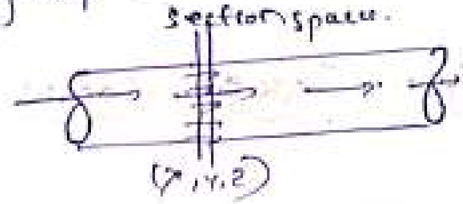
A. LAGRANGIAN APPROACH :- (w.r.t. space)

In this approach a fluid particle is taken and its behavior is analyze at different section with respect to space. Based on fixed mass.



B. EULARIAN APPROACH :- (w.r.t time) :-

In this approach a certain section is taken in space and the behavior of fluid particles passing through see this section is analyzed at different instant. We mainly use this approach in fluid mechanics. It is based on fixed control volume.



TYPE OF FLUID FLOW :-

1. STEADY AND UNSTEADY FLUID FLOW (w.r.t time) :-

A flow is said to be study flow if fluid behaviour ($\rho, v, p, \ldots$ etc) dose not change with time at any section. Otherwise unsteady flow.

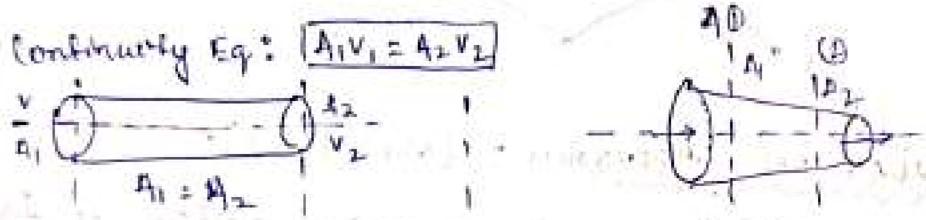
$$\boxed{\frac{\partial v}{\partial t} = \frac{\partial p}{\partial t} = \frac{\partial a}{\partial t} = \frac{\partial t}{\partial t} = 0}$$ Steady flow

$$\boxed{\frac{\partial v}{\partial t} = \frac{\partial p}{\partial t} = \frac{\partial a}{\partial t} = \frac{\partial t}{\partial t} \neq 0}$$ Unsteady flow.

2. UNIFORM AND NON-UNIFORM FLOW (w.r.t Space)(V):-

A flow is said to be uniform, if flow velocity dose not change with space at any instant, otherwise it is non-uniform flow.

Ex:- Continuity Eq: $\boxed{A_1V_1 = A_2V_2}$



$A_1 > A_2$

$(v_1 = v_2)$

$(v_1 \neq v_2)$

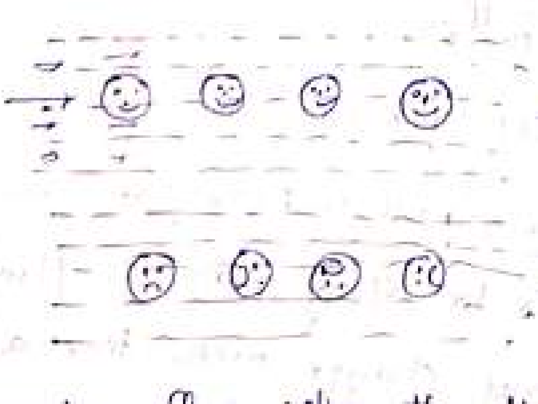
$\left(\frac{\partial v}{\partial s}\right)_c = 0$

(Uniform flow)

$\left(\frac{\partial v}{\partial s}\right)_c \neq 0$

(Non-uniform flow)

3. ROTATIONAL AND IRROTATIONAL FLOW:-



→ Irrotational flow

→ Rotational flow.

→ During flow when the fluid particles rotate about their mass centers then it is rotational flow. otherwise irrotational flow.

Rotation $\xrightarrow{\text{due to}}$ Torque $\xrightarrow{\text{due to}}$ tangential / shear force

↓ due to

$\boxed{\tau \neq 0}$ ⟵ Shear stress

~~Rotation → torque → neither stress~~

No rotation → torque = 0 → f = 0 → $\tau = 0$

There is no relative motion, it means ideal flow is irrotational and this is the condition of irrotational flow. Real flow is Rotational.

4. LAMINAR AND TURBULENT FLOW :-

Laminar flow = Layer-wise flow

During flow when the flow particles moves in layers with one particle sliding over the other. Due to the sliding of layer viscous force dominates so it is also called viscous flow, which generally at low velocity. In this flow the fluid particles move along a well-defined path is called streamlines, so the flow is also called streamline flow.

Turbulent flow :-

During flow when the fluid particle continuously mix from one layer into another resulting in disorganised motion leading to large momentum transfer. It is called turbulent flow.

The Nature of flow is judged by Reynolds number. It is the ratio of inertia force to viscous force. Called Reynolds number

$$\boxed{Re = \frac{\text{Inertia force } (F_i)}{\text{Viscous force } (F_v)}}$$

$Re \rightarrow$ Less (Viscous ~~flo~~ force dominant) ~~[illegible]~~ (Laminar)

$Re \rightarrow$ High (Turbulent flow

Real flow (Rotational) $\rightarrow$ [illegible] Laminar / turbulent.

5. COMPRESSIBLE AND INCOMPRESSIBLE FLOW:-

* Compressible flow $\rightarrow$ ($\rho \neq$ constant)
* Incompressible flow $\rightarrow$ (ρ = constant)

6. 1D, 2-D and 3-D Flow:-

1-D:-

$v = f(x)$

$v = f(y)$

$v = f(z)$

$v = f(r)$

2-D:-

$v = f(x, y)$

$v = f(r, \theta)$

3-D:-

$v = f(x, y, z)$

(Cartesian coordinate)

$v = f(r, \theta, z)$

Note:-

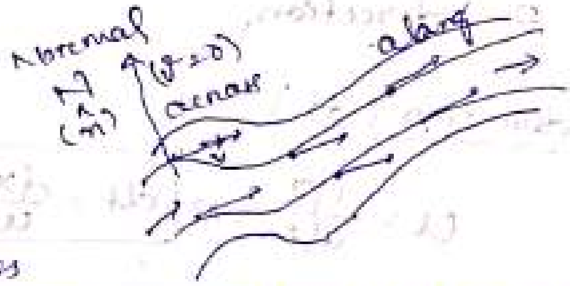
* IDEAL FLOW:-

IDEAL FLOW → non-viscous / frictionless (INVISID) ($\mu = 0$)
→ Irrotational
→ Incompressible.

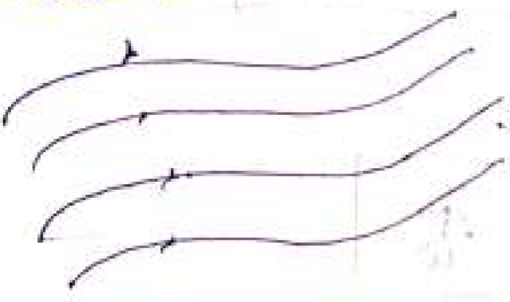
STREAMLINE:-

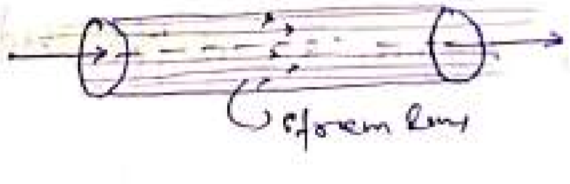
It is an imaginary line drawn through a flowing fluid in space such that ~~the~~ a tangent drawn to it at any point gives velocity of flow at that point at any instant or it is a line tangent to velocity vector at ~~any~~ every point in a flow ~~fre.~~ field.

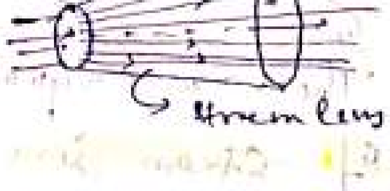
As there is no component of velocity in normal direction ~~of~~ it so flow cannot be possible across a streamline; flow is always along a stream line.



* Stream line never intersect with each other. & parallel to each other.

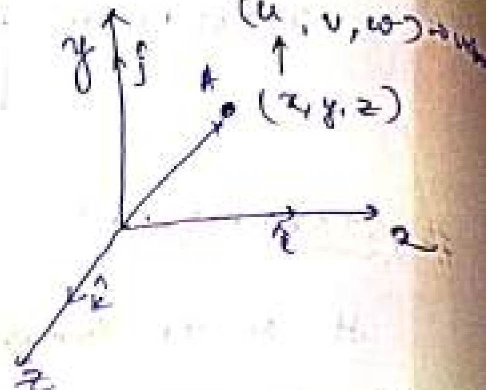






Differential Equation of a St...

Velocity vector ($\vec{V}$) in cartesian co-ordinate system can be written as:



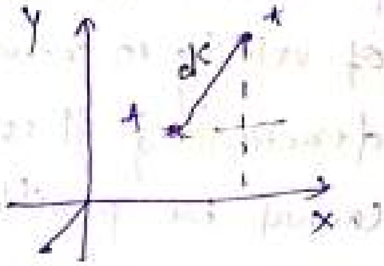
$$\boxed{\vec{V} = \vec{u}\,\hat{i} + \vec{v}\,\hat{j} + \vec{w}\,\hat{k}}$$

$$\boxed{|\vec{V}| = \sqrt{u^2 + v^2 + w^2}}$$ (Magnitude of velocity)

where, u, v, and w are velocity component in x, y, and z - direction.

x- direction (dx) :-

$$u = \frac{dx}{dt} \Rightarrow \boxed{dt = \frac{dx}{u}} \quad -①$$



y- direction (dy) :-

$$v = \frac{dy}{dt} \Rightarrow \boxed{dt = \frac{dy}{v}} \quad -②$$

at time (dt)

z - direction (dz) :-

$$w = \frac{dz}{dt} \Rightarrow \boxed{dt = \frac{dz}{w}} \quad -③$$

from eqⁿ ① ② & ③

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

The above equation is known as differential eqⁿ of stream line.

Example 1

$u = x^2 t$
$v = xyt$ find the eqn of stream line?

Solⁿ

Use from differential eqn of stream line

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{x^{2}\cancel{t}} = \frac{dy}{x \cdot y \cdot \cancel{t}}$$

$$\frac{dx}{x} - \frac{dy}{y} = 0$$

Integrating both side.

$$\int \frac{dx}{x} - \int \frac{dy}{y} = 0$$

$$\ln x - \ln y = C$$

$$\ln\left[\frac{x}{y}\right] = c$$

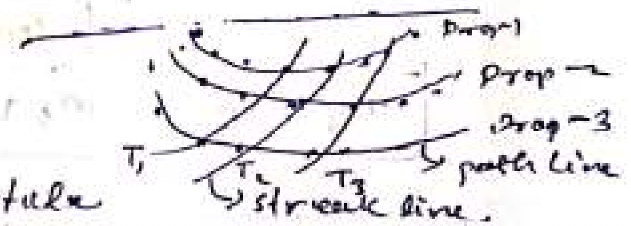
$$\frac{x}{y} = e^{c} \qquad \because e^{c} = k$$

$\boxed{x = ky}$ - eqn of stream line.

07 - 27/03/2023

PATH LINE :-

Actual path ~~Holled~~ followed by fluid particle is called path line.



STREAK LINE :-

Locus of various fluid particle which have passed through a fixed point at any instant of time is called streak line.

* For Steady flow

→ In the case of steady flow path-line and streak line becomes identical.

→ Path line & streakline can intersect but stream line can never be intersect.

CONTINUITY EQUATION :-

Discharge (Q) $= \frac{V}{t} = \frac{A \cdot L}{t}$

↳ volume flow rate.

Steady

volume $= \frac{\pi}{4} D^2 h$

$\boxed{Q = A \cdot V}$

↳ velocity (m/s)

↳ c/s Area (m^2)

↳ Discharge / volume flow rate (m^3/s)

$\frac{V}{t} = Q$

$\frac{m}{t} = \dot{m}$

* Continuity equation is based on "Conservation of Mass".

$\boxed{\dot{m} = \text{Constant}}$ —(1)

$\dot{m} = \frac{m}{t} = \rho \cdot \frac{V}{t}$

$m = \rho \cdot V$

$\frac{V}{t} = Q$

$\boxed{\dot{m} = \rho \cdot Q}$

$\boxed{\dot{m} = \rho \cdot A V}$

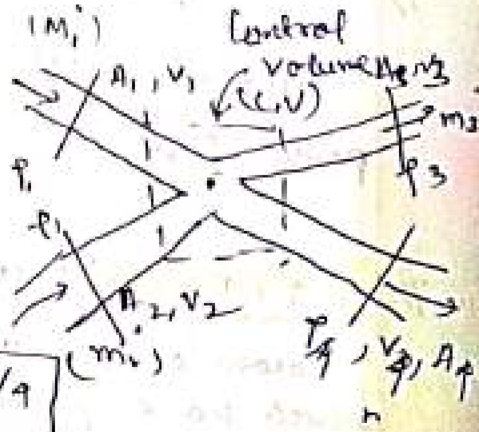
from eqn (1)

$\boxed{\rho \cdot A \cdot V = C}$

↳ continuity eqn.

$\boxed{\dot{m}_1 + \dot{m}_2 = \dot{m}_3 + \dot{m}_4}$

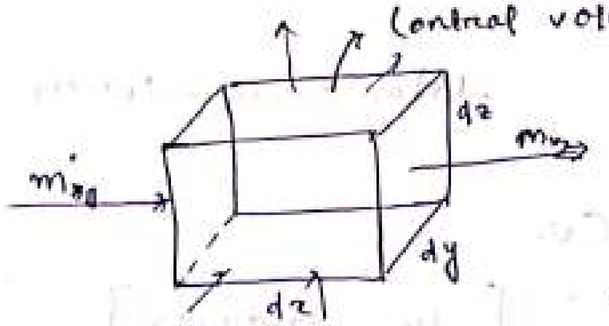
$\boxed{\rho_1 A_1 V_1 + \rho_2 A_2 V_2 = \rho_3 A_3 V_3 + \rho_4 A_4 V_4}$



SSC JE (Mains)

GENERALISED CONTINUITY EQUATION :-

X- Direction :-



Mass enter info (C.V) in x-direction.

$$m_{x_1}^{\cdot} = \rho \cdot u \cdot dA$$

$$m_{1}^{\cdot} = \rho \cdot u \cdot dz \cdot dy \quad \text{— ①}$$

Mass exit from Control volume (C.V) in x direction.

$$m_2^{\cdot} = m_1^{\cdot} + \left(\frac{\partial m_1^{\cdot}}{\partial x} \cdot dx\right)$$

$$m_2^{\cdot} = \rho \cdot u (dy \cdot dz) + \frac{\partial}{\partial x} (\rho \cdot u \cdot dy \cdot dz) dx$$

$$m_2^{\cdot} = \rho u (dy\,dz) + (dy \cdot dz\,dx) \frac{\partial}{\partial x}(\rho \cdot u) \quad \text{— ②}$$

Total accumulation of mass in x-direction:

$$= m_1^{\cdot} - m_2^{\cdot}$$

$$= \rho \cdot u \cdot (dz \cdot dy) - \rho \cdot u \cdot (dy \cdot dz) - (dx\,dy \cdot dz) \frac{\partial}{\partial x}(\rho \cdot u)$$

Total accumulation (in x-direction)

$$= -\frac{\partial}{\partial x} (\rho \cdot u) (dx \cdot dy \cdot dz)$$

similarly.

Total accumulation (in z-direction)

$$= \frac{\partial}{\partial z} (\rho \cdot w) (dx \cdot dy \cdot dz)$$

total accumulation (in y-direction)

$$= \frac{\partial}{\partial y} (\rho \cdot v) (dx\,dy \cdot dz)$$

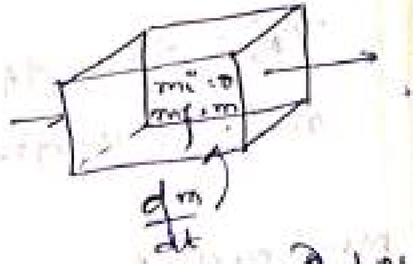
~~Continuity eqn~~ :-

Mass Conservation :-

total accumulation of mass = Net rate of mass transfer inside C.V.

C.V. =

$\because$ [$dV = dx \cdot dy \cdot dz$]



$$\Rightarrow \left[-\frac{\partial}{\partial x}(\rho . u) - \frac{\partial}{\partial y}(\rho . v) - \frac{\partial}{\partial z}(\rho . w)\right] dV = \frac{\partial}{} \frac{dm}{dt}\bigg|_{cv}$$

$$\Rightarrow \left[-\frac{\partial}{\partial x}(\rho . u) - \frac{\partial}{\partial y}(\rho . v) - \frac{\partial}{\partial z}(\rho . w)\right] dV = \frac{\partial}{\partial t}(\rho \, dV)$$

$$\Rightarrow \boxed{\frac{\partial}{\partial x}(\rho . u) + \frac{\partial}{\partial y}(\rho . v) + \frac{\partial}{\partial z}(\rho . w) + \frac{\partial \rho}{\partial t} = 0}$$

↳ Generalised ~~coat~~ continuity equation.

* flow is Steady & Incompressible :-

$$\rho \left\{ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + 0 \right\} = 0$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

* for 2d flow :- (u, v) [w = 0]

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

If nothing is given then use this eqn.

↳ 2D. steady, Incompressible continuity eqn.

ACCELERATION :-

1D · flow :- (~~x-direction~~)

$$v = f(t, s)$$

$$\cancel{dv = \frac{\partial v}{\partial t} \cdot dt + \frac{\partial v}{\partial s} \cdot ds}$$

$$\cancel{\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial s}}$$

$$dv = \frac{\partial v}{\partial t} \, dt + \frac{\partial v}{\partial s} \cdot ds$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} \cdot \frac{dt}{dt} + \frac{\partial v}{\partial s} \left(\frac{ds}{dt}\right) \quad \therefore \left\{ \begin{array}{l} \frac{ds}{dt} = v \\ \frac{dv}{dt} = a \end{array} \right\}$$

$$\boxed{a_{Total} = \frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial s}}$$

where

$\frac{\partial v}{\partial t}$ = ~~Loal~~ Local / Temporal acceleration.

$v \cdot \frac{\partial v}{\partial s}$ = Convective acclaration

$$\boxed{a_{local} = \frac{\partial v}{\partial t}}$$

$$\boxed{a_{convective} = v \cdot \frac{\partial v}{\partial s}}$$

$$\boxed{a_T = a_{loc} + a_{conv}}$$

Acclaration in x-direction :-

$$\boxed{a_T = \frac{\partial u}{\partial t} + u \cdot \frac{\partial u}{\partial x}}$$

Flow steady :-

$\boxed{a_{local} = 0}$, $\boxed{a_{convt} = v \cdot \frac{du}{ds}}$ $\boxed{a_T = v \cdot \frac{\partial v}{\partial s}}$

* ~~Flow~~ is uniform

$\boxed{a_t = \frac{\partial v}{\partial t}}$, $\boxed{a_{conv} = 0}$

$\boxed{a_T = a_{local}}$

* Flow is steady and uniform.

$\boxed{a_t = 0}$, $\boxed{a_{conv} = 0}$

$\boxed{a_T = 0}$

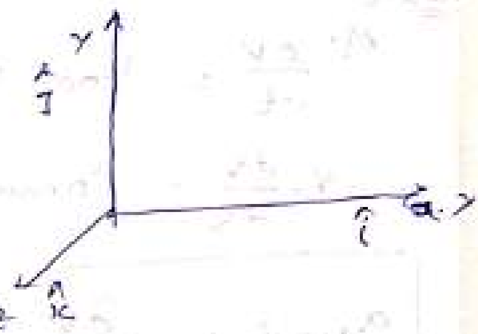
Lec-3 28/05/2020 ~~For 3D flow~~ :-

* Acceleration : (3D-flow) :-

$$\boxed{\bar{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}}$$

$$\boxed{|\bar{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}}$$

↳ magnitude of acceleration.



* 3D-flow in x-direction :-

$$u = f(x, y, z, t)$$

$$du = \frac{\partial u}{\partial x} \cdot dx + \frac{\partial u}{\partial y} \cdot dy + \frac{\partial u}{\partial z} \cdot \partial z + \frac{\partial u}{\partial t} \cdot dt$$

$$\frac{du}{dt} = u \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$\boxed{a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}}$$

Similarly.

$$\boxed{a_y = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}}$$

$$\boxed{a_z = u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}}$$

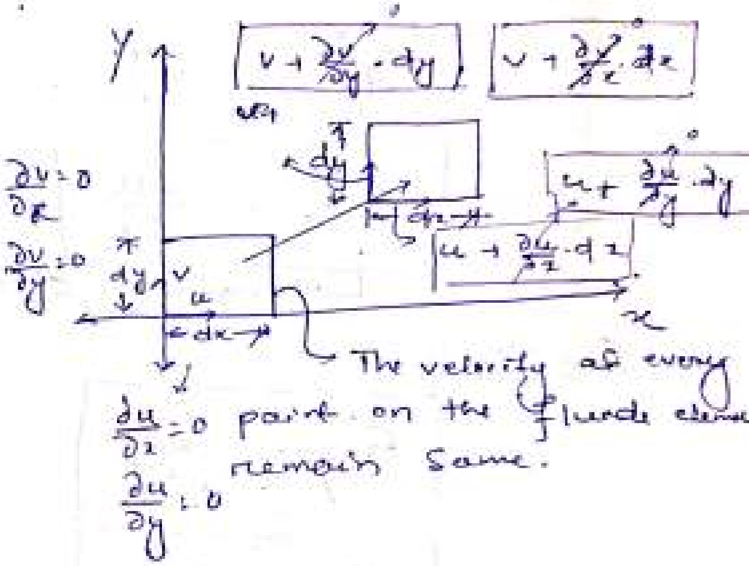
Types of fluid flow :-

1. Translation motion.
2. Linear deformation
3. Angular Deformation
4. Rotation.

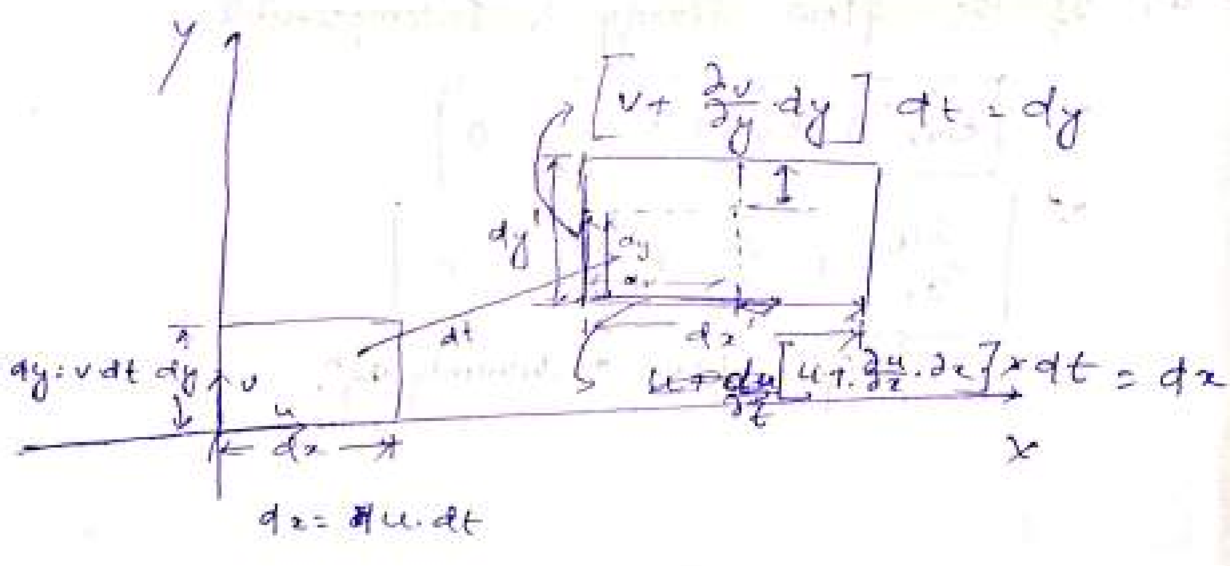
1. Translation Motion :-

Assumption

- 2D flow
- Steady flow
- $u \neq f(x,y)$, $v \neq f(x,y)$ } uniform flow



2. Linear Deformation :-



* Normal strain in x-direction

$$\epsilon_{xx} = \frac{dx' - dx}{dx}$$

$$= \frac{\left[u + \frac{\partial u}{\partial x} \cdot dx\right] dt - u \cdot dt}{\cancel{u \cdot dt}\ dx}$$

$$\epsilon_{xx} = \frac{u \cdot dt + \frac{\partial u}{\partial x} \cdot dx \cdot dt - u \cdot dt}{\cancel{u \cdot dt}\ dx}$$

$$\epsilon_{xx} = \frac{\partial u}{\partial x} \cdot dt$$

$$\frac{\epsilon_{xx}}{dt} = \frac{\partial u}{\partial x}$$

$$\Rightarrow \boxed{\dot{\epsilon}_{xx} = \frac{\partial u}{\partial x}}$$

→ normal strain rate.

Similarly

$$\boxed{\dot{\epsilon}_{yy} = \frac{\partial v}{\partial y}}$$

$$\boxed{\dot{\epsilon}_{zz} = \frac{\partial w}{\partial z}}$$

* If the flow Steady & Incompressible.

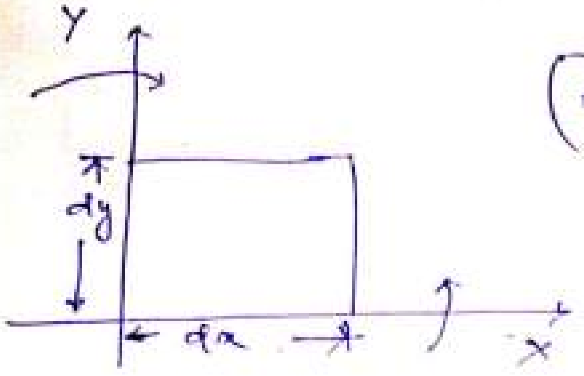
$$\boxed{\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy} + \dot{\epsilon}_{zz} = 0}$$

**

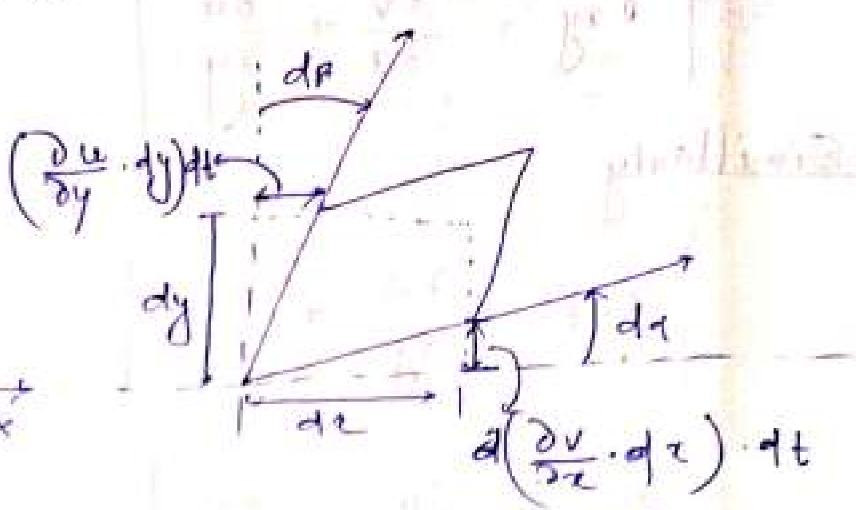
$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

→ Generalised Continuity eqn.

3. Angular Deformation :-



(Initial)



(after t time of flow)

- $\tan(d\alpha) = \dfrac{\frac{\partial v}{\partial x}\cdot dx\cdot dt}{dx}$ | $\tan(d\beta) = \dfrac{\frac{\partial u}{\partial y}\cdot dy\cdot dt}{dy}$

$\tan(d\alpha) \simeq d\alpha$ | $\tan(d\beta) \simeq d\beta$

$d\alpha = \frac{\partial v}{\partial x}\cdot dt$ | $d\beta = \frac{\partial u}{\partial y}\cdot dt$

$\frac{d\alpha}{dt} = \frac{\partial v}{\partial x}$ | $\frac{d\beta}{dt} = \frac{\partial u}{\partial y}$

"Total / Average angular deformation" γ_{xy} :-

$$\gamma_{xy} = \frac{\frac{d\alpha}{dt} + \frac{d\beta}{dt}}{2} = \frac{1}{2}$$

$$\boxed{\gamma_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)}$$

For SSC ~~and~~ or AE/JE

$$\boxed{\gamma_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)} \quad \times$$

Imp

$$\boxed{\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}}$$

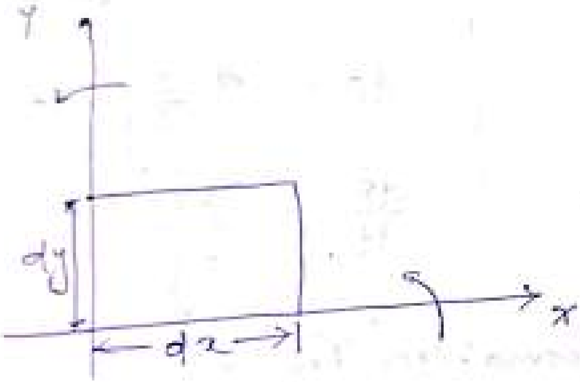
$$\boxed{\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}}$$

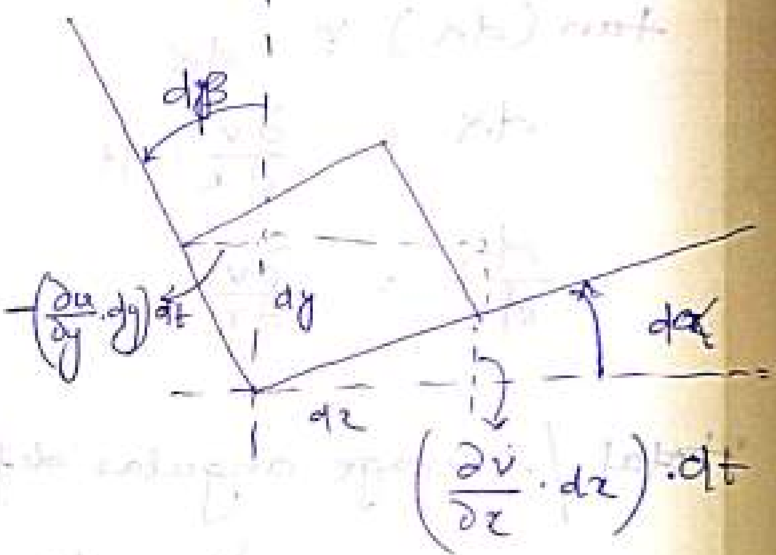
Simillarly

$$\boxed{\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}}$$

$$\boxed{\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}}$$

4. ROTATION :-





$$\tan(d\alpha) = \frac{\frac{\partial v}{\partial x}\cdot \cancel{dx}\cdot dt}{\cancel{dx}}$$

$$\boxed{\frac{d\alpha}{dt} = \frac{\partial v}{\partial x}}$$

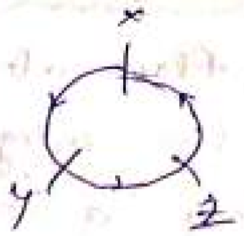
$$\tan(d\beta) = \frac{\frac{\partial u}{\partial y}\cdot \cancel{dy}\cdot dt}{\cancel{dy}}$$

$$\boxed{\frac{d\beta}{dt} = -\frac{\partial u}{\partial y}}$$

* Total/Average Rate of rotation 'ω_z' It the direction of "xy".

$$\omega_z = \frac{\frac{d\alpha}{dt} + \frac{d\beta}{dt}}{2}$$

$$\boxed{\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}$$



$$\boxed{\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)}$$

$$\boxed{\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)}$$

$$\boxed{\overline{\omega} = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}}$$

* Irrotational flow $(\overline{\omega} = 0)$

$$\boxed{\overline{\omega}_x = 0}$$
$$\boxed{\overline{\omega}_y = 0}$$
$$\boxed{\overline{\omega}_z = 0}$$

L-9
29/02/2023

- **VORTICITY (ξ)**

The ability of fluid particle to spin or rotate is called vorticity.

$$\boxed{\xi = 2\overline{\omega} = \text{Curl}(\overline{v})}$$

$$\overline{v} = u\hat{i} + v\hat{j} + w\hat{k} \quad \text{(Given)}.$$

$$\text{Curl}(\overline{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\text{Curl}(\overline{v}) = \hat{i}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right] - \hat{j}\left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right] + \hat{k}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\text{Curl}(\overline{v}) = 2\omega_x\hat{i} + 2\omega_y\hat{j} + 2\omega_z\hat{k}$$

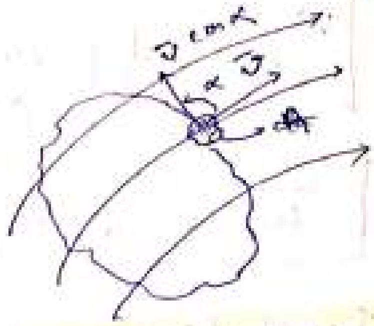
$$\text{Curl}(\overline{v}) = 2(\omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k})$$

$$\text{Curl}(\overline{v}) = 2\overline{\omega}$$

* $\text{Curl}(\overline{v}) = 0$ (flow is Irrotational), $\text{Curl}(\overline{v}) \neq 0$ [flow Rotational

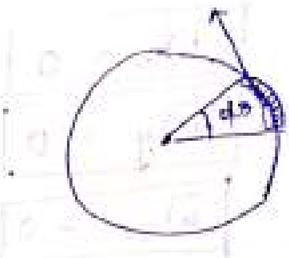
CIRCULATION (Γ)

Flow along a closed curve. Mathematically, it is a line integral of tangential component of velocity along a closed curve.



$$\boxed{\Gamma = \oint \bar{v} \cdot dA}$$

* Taking a circular section



$$\Gamma = \oint v \cdot dA$$

$$\Gamma = \oint_0^{2\pi} r \cdot \omega \times r \cdot d\theta \qquad \bar{v} = r \cdot \omega$$

$$\Gamma = r^2 \omega [\theta]_0^{2\pi}$$

$$\Gamma = 2\pi r^2 \omega$$

$$\Gamma = A\xi \qquad \because 2\omega = \xi, \quad \pi r^2 = A$$

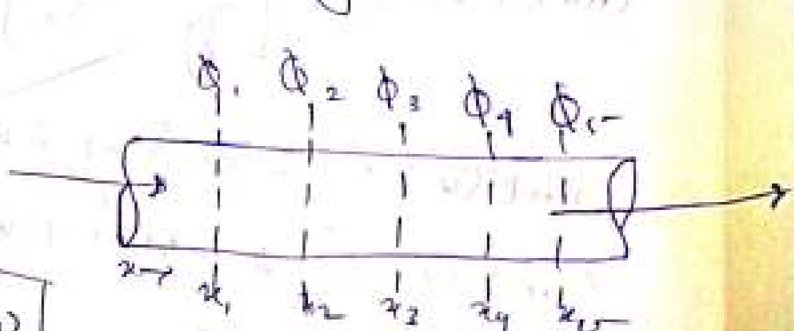
$$\boxed{\xi = \frac{\Gamma}{A}}$$

VELOCITY POTENTIAL FUNCTION (ϕ) :-

It is a scalar function of space and time such that its negative derivative gives the ~~velocity~~ with respect to any direction gives the velocity in the same direction.

$$\boxed{-\frac{\partial \phi}{\partial x} = u}$$

$$\boxed{-\frac{\partial \phi}{\partial y} = v} \qquad \boxed{-\frac{\partial \phi}{\partial z} = w}$$



$$\text{curl}(\bar{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\text{curl}(\bar{v}) = \hat{i}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right] - \hat{j}\left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right] + \hat{k}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\text{curl}(\bar{v}) = \hat{i}\left[\frac{\partial}{\partial y}\left(\frac{-\partial\phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{-\partial\phi}{\partial y}\right)\right] - \hat{j}\left[\frac{\partial}{\partial x}\left(-\frac{\partial\phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(-\frac{\partial\phi}{\partial x}\right)\right]$$

$$+ \hat{k}\left[\frac{\partial}{\partial x}\left(\frac{-\partial\phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial\phi}{\partial x}\right)\right]$$

$$\text{curl}(\bar{v}) = \hat{i}\left[-\frac{\partial^2\phi}{\partial y\cdot\partial z} + \frac{\partial^2\phi}{\partial z\cdot\partial y}\right] + \hat{j}\left[-\frac{\partial^2\phi}{\partial x\cdot\partial z} + \frac{\partial^2\phi}{\partial z\,\partial x}\right]$$

$$+ \hat{k}\left[-\frac{\partial^2\phi}{\partial x\,\partial y} + \frac{\partial^2\phi}{\partial y\,\partial x}\right]$$

$$\boxed{\text{curl}(\bar{v}) = 0}$$

↳ Irrotational flow.

Note:-

* Velocity Potential function is only defined for irrotational flow.

* for 2D - incompressible continuity eq^n:-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(-\frac{\partial\phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial\phi}{\partial y}\right) = 0$$

$$-\left[\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right] = 0$$

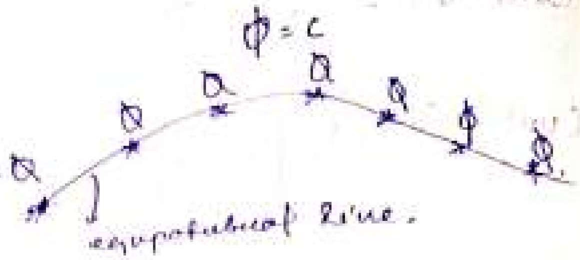
$$\boxed{\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0}$$

↳ Laplace Equation

* Fluid flow is only possible when it satisfy Laplace equation.

EQUIPOTENTIAL LINE:-

* The line joining the points having same potential function is called equipotential line.



$$d\phi = 0$$

$$\phi = f(x, y)$$

$$d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} \cdot dy$$

$$d\phi = -u\,dx - v \cdot dy$$

$$0 = -u \cdot dx - v \cdot dy$$

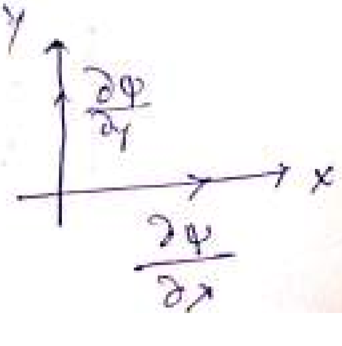
$$u \cdot dx = -v \cdot dy$$

$$\boxed{\text{Slope} = \left.\frac{dy}{dx}\right|_{\phi} = \frac{-u}{v}}$$

STREAM FUNCTION (ψ)

The stream function ψ is defined as a scalar function of space and time such that its partial derivative ~~given at any d~~ with respect to any right angle direction gives velocity component at right angle. It satisfy 2-D continuity equation.

$$\boxed{v = \frac{\partial \psi}{\partial x}} \quad \boxed{u = -\frac{\partial \psi}{\partial y}}$$



* 2D – Continuity eqⁿ.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

LHS

$$= \frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right)$$

$$= -\frac{\partial^2 \psi}{\partial x \cdot \partial y} + \frac{\partial^2 \psi}{\partial x \cdot \partial y}$$

$$= 0 = (RHS)$$

STREAM LINE :-

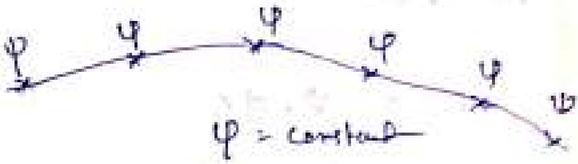
$$\psi = f(x, y)$$

$$d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$$

$$0 = v \cdot dx - u \cdot dy$$

$$v \cdot dx = u \cdot dy$$

$$\boxed{\left.\frac{dy}{dx}\right|_{\psi} = \frac{v}{u}}$$



ψ = constant

$d\psi = 0$

* The line joining the point having same stream function is called stream line.

Note:-

$$\left.\frac{dy}{dx}\right|_{\phi} \cdot \left.\frac{dy}{dx}\right|_{\psi} = -\frac{u}{v} \times \frac{v}{u}$$

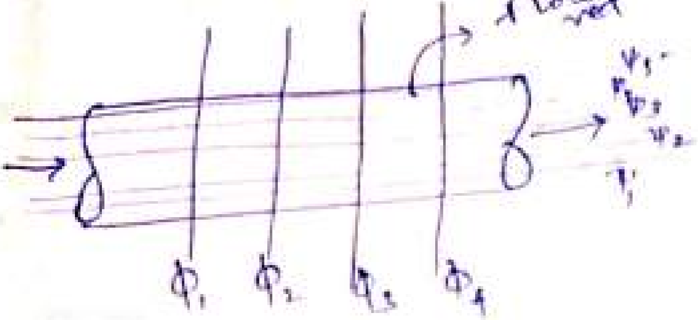
$$\boxed{\left.\frac{dy}{dx}\right|_{\phi} \cdot \left.\frac{dy}{dx}\right|_{\psi} = -1}$$ $\{m_1 \times m_2 = -1\}$

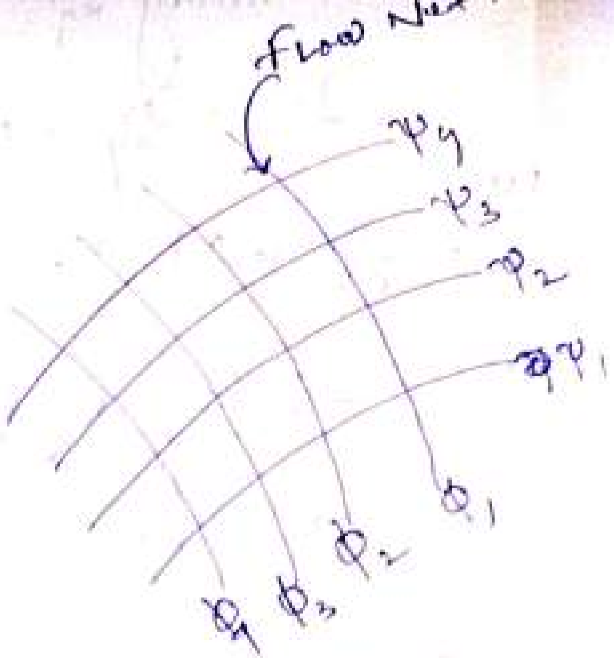
↳ Line ⊥ to each other

Note:

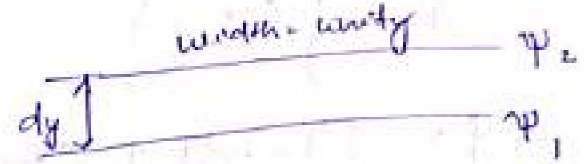
Equipotential Line & Stream line both are perpendicular to each other

Flow Net :-





* 1D - flow :-



$$\psi = f(y) \text{ only}$$

$$\frac{\partial \psi}{\partial y} = \frac{d\psi}{dy}$$

$$dQ = v \cdot dA$$

$$dQ = \int u \cdot dA$$

$$dQ = u \cdot dy \cdot 1$$

$$dQ = -\frac{\partial \psi}{\partial y} \cdot dy$$

$$dQ = -\frac{d\psi}{\cancel{dy}} \cdot \cancel{dy}$$

$$dQ = -d\psi$$

$$Q = -\int_{\psi_1}^{\psi_2} d\psi$$

$$Q = -[\psi]_{\psi_1}^{\psi_2}$$

$$Q = -[\psi_2 - \psi_1]$$

$$\boxed{Q = |\psi_1 - \psi_2|} \quad m^3/s$$

Note:-

from rotation

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

$$\omega_z \neq 0$$

Note:-

Stream function is valid for both rotational and irrotational flow.

Note:-

Proof of velocity potential function is scalar quantity

$$\bar{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\bar{V} = -\frac{\partial \phi}{\partial x}\hat{i} - \frac{\partial \phi}{\partial y}\hat{j} - \frac{\partial \phi}{\partial z}\hat{k}$$

$$= \underbrace{-\phi}_{\text{Scalar function}} \underbrace{\left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right]}_{\text{Del Operator}}$$

CH-4th Lec-1 | FLUID DYNAMICS | Dt- 30th march 2023

ENERGY AND ENERGY EQUATIONS

TYPE OF FORCES :- (Major forces) :-

1. Pressure force (f_P)
2. Gravity force (f_g)
3. Viscous force (f_v)
4. Turbulent force (f_T)

* If we consider f_P, f_g, f_v & f_T then,
  (Reynolds Equation) → Turbulent flow.

* If we consider f_P, f_g, ~~f_T~~ and ~~f_g~~ f_v then,
  (Navier - Stokes Equation) → Laminar Flow.

* Navier - Stokes equation is based on conservation of Momentum.

* If we considered f_P, f_g then,
  (Euler's Equation) → Energy & Energy Equation.

* Euler Equation is based on linear momentum conservation.

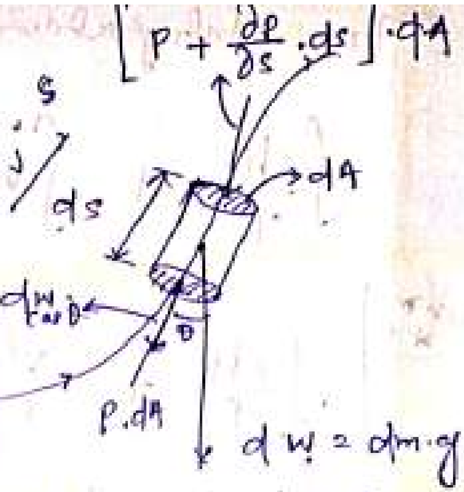
Euler's Eqn :-

Assumption :-

① → flow is Non-viscous.
② → flow is ~~Stea~~ along stream line.
④

Newton's 2nd Law :-

$$\boxed{\Sigma F_s = m \cdot a_s}$$



$$P \cdot dA - dw \cos\theta - \left[P + \frac{\partial P}{\partial s} ds\right] dA$$

$$= \rho \cdot (dA \cdot ds)\left[\frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial s}\right]$$

$$\Rightarrow \cancel{P \cdot dA} - \rho \cdot (dA\, ds)\, g \cos\theta - \cancel{P \cdot dA} - \frac{\partial P}{\partial s} \cdot ds \cdot dA$$

$$= \rho \cdot (dA \cdot ds)\left[\frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial s}\right]$$

$$\Rightarrow \rho(\cancel{dA\, ds})\, g \cos\theta + \frac{\partial P}{\partial s}(\cancel{ds\, dA}) + \rho \cdot (\cancel{dA\, ds})\left[\frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial s}\right] = 0$$

$$\Rightarrow \rho \cdot g \cos\theta + \frac{\partial P}{\partial s} + \rho\left[\frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial s}\right] = 0$$

* Assumption-3 :- Flow is steady. $\left\{ \because \frac{\partial v}{\partial t} = 0 \right.$

$$\Rightarrow \rho \cdot g \cos\theta + \frac{\partial P}{\partial s} + \rho \cdot v \cdot \frac{\partial v}{\partial s} = 0$$

$$\Rightarrow \rho \cdot g \cos\theta + \frac{dP}{ds} + \rho \cdot v \cdot \frac{dv}{ds} = 0$$

$$\Rightarrow \frac{\rho \cdot g\,(ds \cos\theta) + dP + \rho \cdot v \cdot dv}{ds} = 0$$

$$\Rightarrow \rho \cdot g\,(ds \cos\theta) + dP + \rho \cdot v \cdot dv = 0$$

$$\Rightarrow \rho \cdot g\, dz + dP + \rho \cdot v \cdot dv = 0$$

~~Taking integration~~

$$\Rightarrow \frac{dP}{\rho} + v \cdot dv + g \cdot dz = 0$$

Take Integration

$$\int \frac{dp}{\rho} + \int v \cdot dv + \int g \cdot dz = C$$

$$\boxed{\int \frac{dp}{\rho} + \frac{v^2}{2} + gz = C}$$

↳ Euler's Equation.

* This is the Euler's Equation of compressible fl

BERNOULLI'S EQUATION :—

Assumption :-

1. Inviscid flow / Non-viscous.
2. flow along stream line.
3. flow is steady
4. flow is incompressible ($\rho = c$)

∴ from Euler eq^n

$$\frac{p}{\rho} + \frac{v^2}{2} + gz = C$$

dividing 'g' by both side

$$\boxed{\frac{p}{\rho \cdot g} + \frac{v^2}{2g} + z = C}$$

↳ Bernoulli's Equation.

where :-

$\frac{p}{\rho \cdot g}$ = Pressure head.

$\frac{v^2}{2g}$ = Kinetic head

z = Potential / Dat...

Units, Equation

$$\frac{P}{\rho g} = \frac{N}{m^3 \cdot \frac{kg}{m^3} \cdot \frac{m}{s^2}}$$

$$= \frac{N \cdot m}{kg} = \frac{N \cdot m}{W} = \boxed{\frac{Joule}{Weight}} \text{ or } \boxed{J/N} \text{ or } \boxed{J/\text{weight}}$$

or

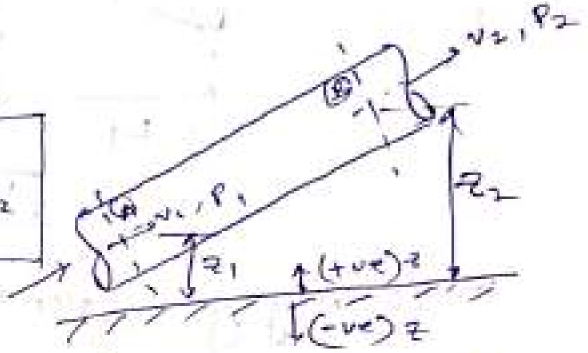
$$\frac{P}{\rho \cdot g} = \frac{N}{m^2 \cdot \frac{kg}{m^3} \cdot m/s^2} = \frac{N \cdot s^2}{m} = \frac{kg \cdot m/s^2 \cdot s^2}{s^2 \cdot kg} = \boxed{m}$$

Note: So the unit are of Bernoulli's eq^n are J/weight, J/N and m.

Ideal flow

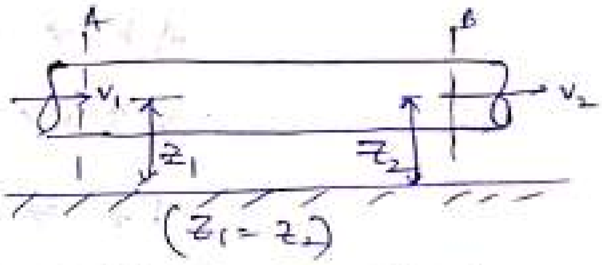
Case-1 $E_1 = E_2$

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2}$$



Case-2 $z_1 = z_2$

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}}$$



Real flow:-

$E_1 = E_2 + h_L$

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L}$$

h_L → Head Loss.

Note:-

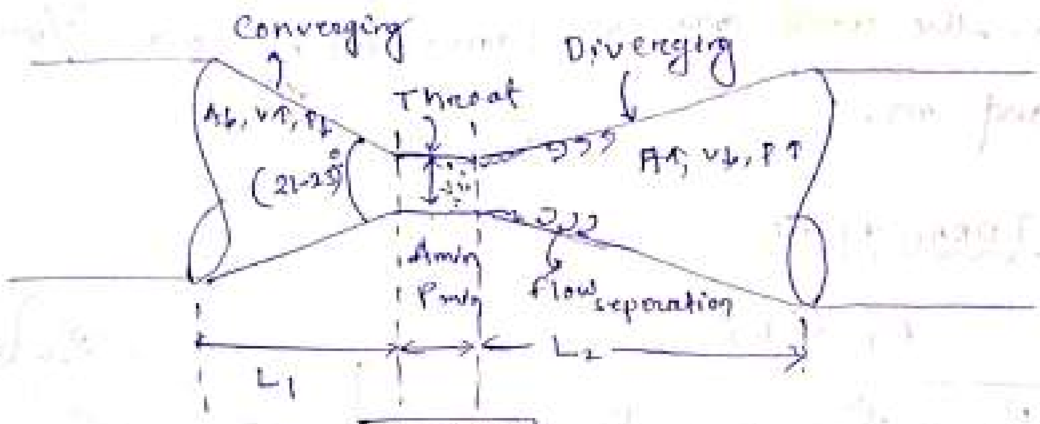
* Bernoulli's Equation is based on Energy conservation.

APPLICATION OF BURNOULLI'S EQUATION :-

(1) Venturimeter (flow rate/Q)
(2) Orifice meter (flow rate/Q)
(3) Pitot Tube (flow velocity)

(1) VENTURIMETER :-

It is a gradually converging and diverging section. It is used to measure flow rate.



$\boxed{L_2 > L_1}$

* **Converging Section :-**

$A \downarrow \Rightarrow A_1V_1 = A_2V_2$

$V \uparrow \Rightarrow$ Bernoulli's Eqn

$P \downarrow \Rightarrow$ Pressure decreases.

* **Throat Section :-** We will get minimum pressure in throat.

If → $P_{min} < P_v$ (vapour pressure)
↳ Then cavitation occurs.

→ The chances of cavitation is maximum at throat.

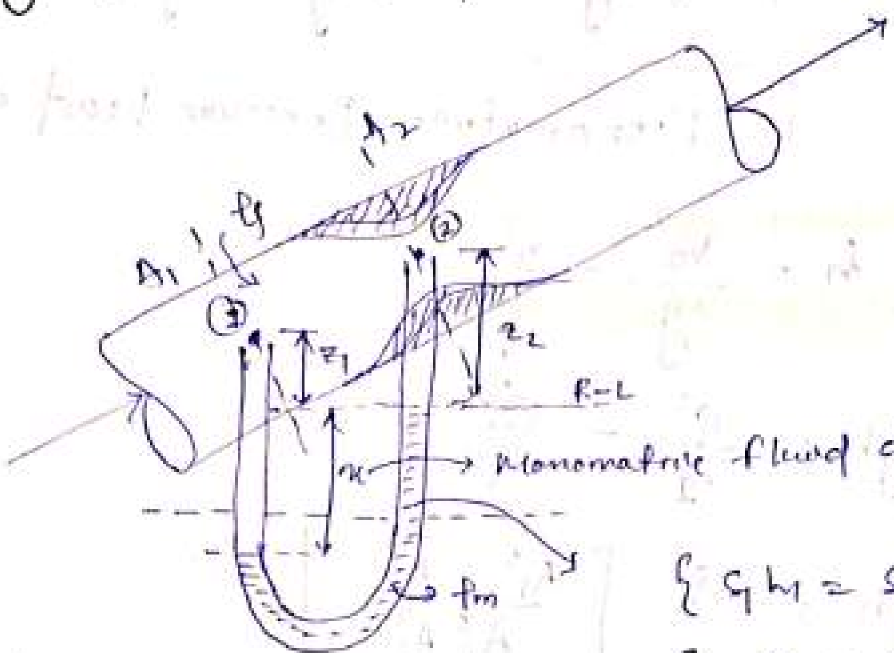
* **Diverging Section :-**

$A \uparrow \Rightarrow V \downarrow \rightarrow P \uparrow$

* The chances of flow seperatation is maximum at the diverging section.

Dt- 3/01/2023

* Discharge Measure in Venturimeter :-



$\{ S_1 h_1 = S_2 h_2 \}$

$S_m \cdot x = S_f \cdot h_2$

$$\boxed{h_2 = \frac{S_m}{S_f} \cdot x}$$

↳ Manometric liquid head in terms of flowing fluid head.

- Venturimeter.
- Manometer.

A_1 = Pipe diameter

V_1 = The Inlet velocity

V_2 = velocity at throat

A_2 = Throat Area.

* Continuity Eqn:-

$$Q = A_1 V_1$$

$$Q = A_2 V_2$$

$$V_1 = \frac{Q}{A_1} \quad , \quad V_2 = \frac{Q}{A_2}$$

\# Bernoulli's Eqn:-

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\left\{ \frac{P}{\rho g} + z = \text{Piezometric Pressure Head} \right\}$$

$$\Rightarrow \underbrace{\left(\frac{P}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right)}_{\text{Piezometric Pressure head difference}} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\Rightarrow 2gh = \frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}$$

$$2gh = Q^2 \left[\frac{A_1^2 - A_2^2}{A_1^2 \cdot A_2^2}\right]$$

$$Q^2 = \frac{A_1^2 \cdot A_2^2 \cdot 2gh}{A_1^2 - A_2^2}$$

$$\boxed{Q_{th} = \frac{A_1 \cdot A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$$

* **Pressure Equation** :-

$$\frac{P_1}{\rho g} + (Z_1 + x) - \frac{S_m}{S_f} \cdot x - Z_2 - \frac{P_2}{\rho g} = 0$$

$$\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = \frac{S_m}{S_f} x - x$$

$$\boxed{h = x\left(\frac{S_m}{S_f} - 1\right)}$$

* for actual condition there are some losses.

$$\boxed{Q_{act} < Q_{th}}$$

$$\boxed{Q_{act} = C_D \cdot Q_{th}}$$

$\hookrightarrow$ co-efficient of Discharge.

$$\boxed{C_D = (0.97 - 0.98)}$$ imp

$$\boxed{C_D = \frac{Q_{act}}{Q_{th}}}$$

** $$\boxed{Q_{act} = \frac{C_D \cdot A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$$

$$\boxed{Q_{act} = \frac{A_1 A_2 \sqrt{2g(h-h_L)}}{\sqrt{A_1^2 - A_2^2}}}$$

Comparing both the equation.

$$\frac{C_D \cdot \cancel{A_1 A_2} \sqrt{\cancel{2g}h}}{\cancel{\sqrt{A_1^2 - A_2^2}}} = \frac{\cancel{A_1 A_2} \sqrt{\cancel{2g}(h-h_L)}}{\cancel{\sqrt{A_1^2 - A_2^2}}}$$

** $$\boxed{C_D = \sqrt{1 - \frac{h_L}{h}}}$$

Note

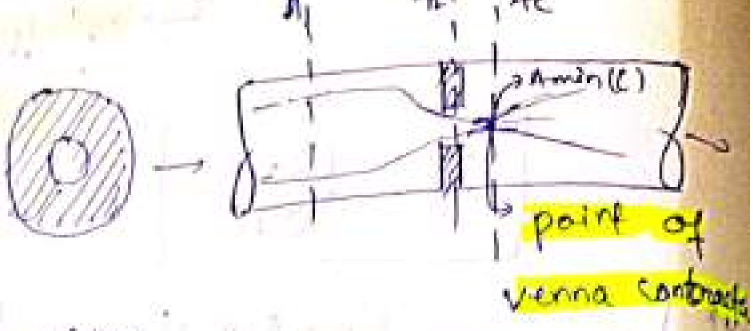
* In venturimeter $Q_{th} \simeq Q_{act}$, because of gradual change in cross-sectional area.

* Venturimeter gives more accurate result

** $$\boxed{\text{Beta Ratio} = \frac{\text{throat dia}(D_2)}{\text{Pipe dia}(D_1)} - \text{Range } (0.3 - 0.75)}$$

2. ORIFICE METER:-

→ It is also used to measure discharge of [?]. It is cheaper at the cost and of Venturimeter. It is cheaper and less accurate.



$$\boxed{C_c = \frac{A_c}{A_1}}$$

↳ Co-efficient of contraction
↳ (0.464 — 0.76)

$$\boxed{A_c < A_2 < A_1}$$

$$\boxed{Q_{act} = C_c \, Q_{TH}}$$

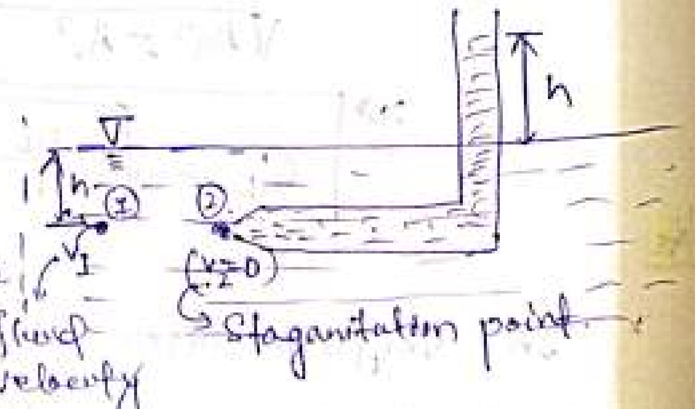
3. PITOT TUBE:-

→ It is a 90° bend tube, used to measure the flow velocity.

Conc.

* **Bernoulli's Eqn:-**

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{Z_2}$$



$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g}}$$

Static pressure head

Dynamic pressure head

Stagnation Pressure head

$$\left\{ \begin{array}{l} P_1 = \rho g h_0 \\ P_2 = \rho g h_0 + \rho g h \end{array} \right.$$

$$\Rightarrow \frac{\cancel{\rho g}\, h_0}{\cancel{\rho g}} + \frac{v_1^2}{2g} = \frac{\cancel{\rho g} h_0 + \rho \cdot g \cdot h}{\cancel{\rho g}}$$

$$\Rightarrow \cancel{h_0} + \frac{v_1^2}{2g} = \cancel{h_0} + h$$

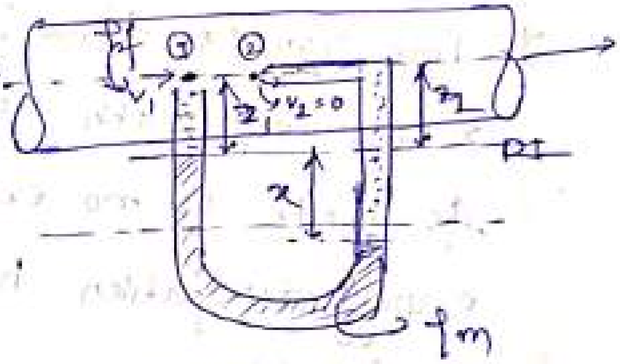
$$\boxed{V_1/_{th} = \sqrt{2gh}}$$

$$\boxed{V_1/_{act} = C_v \cdot V_{th}}$$

C_v → Co-efficient of velocity.

Case-2

Bernoulli's Equation



$$\frac{P_1}{\rho \cdot g} + \frac{v_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho \cdot g} + \overset{0}{\cancel{\frac{v_2^2}{2g}}} + \cancel{Z_2}$$

$$\frac{P}{\rho \cdot g} + \frac{v_1^2}{2g} = \frac{P}{\rho \cdot g}$$

$$\frac{v_1^2}{2g} = \frac{P_2}{\rho \cdot g} - \frac{P_1}{\rho \cdot g} \quad \text{—— (1)}$$

$S_m \cdot x = S_f \cdot h_2$

$$\boxed{h_2 = \frac{S_m}{S_f} \cdot x}$$

$(Z_1 = Z_2)$

* Pressure Equation:-

$$\frac{P_1}{\rho \cdot g} + \cancel{Z_1} + \frac{S_m}{S_f} \cdot x - x - \cancel{Z_2} - \frac{P_2}{\rho \cdot g} = 0$$

$$\frac{P_1}{\rho \cdot g} - \frac{P_2}{\rho \cdot g} = x - \frac{S_m}{S_f} \cdot x$$

$$\frac{P_2}{\rho \cdot g} - \frac{P_1}{\rho \cdot g} = x\left(\frac{S_m}{S_f} - 1\right) \quad \text{—— (2)}$$

Replacing eq^n ② in eq^n ①.

$$\frac{V_1^2}{2g} = x\left(\frac{S_m}{S_f} - 1\right)$$

***

$$\boxed{V = \sqrt{2gx\left(\frac{S_m}{S_f} - 1\right)}}$$

VORTEX FLOW :-

flow along a curved path is called vortex motion.

I. free Vortex flow :- In this flow fluid moves in a curved path due to motion imparted to its earlier. As their is no external torque present, there is no energy dissipation hance Bernoulli's equation is applicable.

$$Torque (T) = F \times R$$

$$\boxed{Torque\ (T) = F \cdot r = 0}$$

$$T = m a \cdot r = 0$$

$$m\frac{dv}{dt} \cdot r = 0$$

$$\cancel{m \cdot v \cdot r} = 0$$

$$m \cdot \frac{dv}{dt} \cdot v = 0$$

$$\frac{d}{dt}(m \cdot v \cdot r) = 0$$

$$d\,(m \cdot v \cdot r) = 0$$

$$m \cdot v\, r = C$$

$$\cancel{m} v \cdot R = \frac{C}{m} = K$$

$$v \cdot R = K$$

$$\boxed{V_\theta = \frac{K}{R}}$$

↳ ~~fact~~ tangential velocity

$$\boxed{V_r = 0}$$

↳ radial velocity

$$V_\theta \propto \frac{1}{r}$$

where

r = Radius Distance.

K = Constant / circulation constant

* $$\text{Vorticity}(\xi_c) = \frac{1}{r}\left[\frac{\partial}{\partial r}(r \cdot v_\theta) - \frac{\partial v_r}{\partial r}\right]$$

$\hookrightarrow$ for polar co-ordinate system.

proof

$$\xi_c = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(\cancel{r} \cdot \frac{K}{\cancel{r}}\right) - 0\right]$$

$$\xi_c = \frac{1}{r} \times 0 \Rightarrow \boxed{\xi_c = 0}$$

$\Rightarrow$ free vortex flow is irrotational flow.

2. FORCED VORTEX FLOW :—

The fluid is continuously rotated under the influence of external torque. As there is continuous energy dissipation, Bernoulli's equation is not applicable.

$\boxed{V_\theta = r \cdot \omega}$ $\boxed{V_r = 0}$

r = Radial distance.

ω = angular velocity (rad/s)

$V_\theta \propto r$

Checking for rotational

$$\xi_c = \left[\frac{1}{r}\left(\frac{\partial}{\partial r}(r \cdot v_\theta) - \frac{\partial v_r}{\partial r}\right)\right]$$

$$\xi_c = \frac{1}{2}\left[\frac{\partial}{\partial r}(r \cdot r \cdot \omega) - 0\right]$$

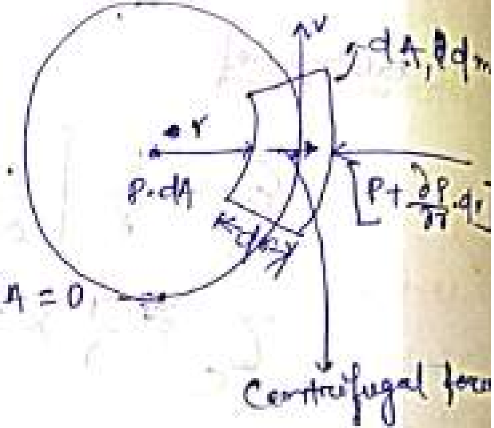
$$\xi_c = \frac{1}{r} \cdot \omega \cdot 2r$$

$$\boxed{\xi_c = 2\omega} \text{ (Rotational flow)}$$

* forced vortex flow is Rotational flow.

Energy Equation for Vortex flow :-

By Applying 2nd law of Newton.



$$\Sigma f_r = m \cdot a_r \quad \left\{ \begin{array}{l} v_r = 0 \\ \therefore a_r = 0 \end{array} \right.$$

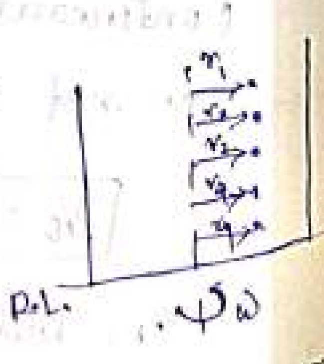
$$\Rightarrow p \cdot dA + \frac{dm \cdot v^2}{r} - \left[p + \frac{\partial p}{\partial r} \cdot dr\right] dA = 0$$

$$\Rightarrow p \cdot dA + \rho (dA \cdot dr) \frac{v^2}{r} - p \cdot dA - \frac{\partial p}{\partial r} dr \cdot dA = 0 \quad \frac{dm \cdot v^2}{r}$$

$$\rho (dA \cdot dr) \frac{v^2}{r} - \frac{\partial p}{\partial r} (dA \cdot dr) = 0$$

$$dA \cdot dA \left[\frac{\rho v^2}{r} - \frac{\partial p}{\partial r}\right] = 0 \qquad \because \left\{ \begin{array}{l} dm = \rho \cdot dV \\ \quad = \rho \cdot dA\,dr \end{array} \right.$$

$$\boxed{\frac{\partial p}{\partial r} = \rho \frac{v^2}{r}}$$



* $$p = f(r, z)$$

$$dp = \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial z} \cdot \partial z$$

$(r_1 = r_2 = r_3 = r_4 = r)$

$$\boxed{dp = \frac{\rho \cdot v^2}{r} \cdot dr - \rho \cdot g \cdot dz} \qquad \boxed{\frac{\partial p}{\partial z} = -\rho \cdot g}$$

↳ energy Eqn for Vortex flow.

value

due to it is the condition of fluid static

Condition 1:

free vortex flow:—

$$\boxed{v = \frac{K}{r}}$$

$$\int_1^2 dp = \int_1^2 \frac{\rho \cdot K^2}{r^2 \cdot r} dr - \int_1^2 \rho g \cdot dz$$

$$(P)_1^2 = \rho K^2 \left[-\frac{1}{2r^2}\right]_1^2 - \rho g [z]_1^2$$

$$P_2 - P_1 = -\frac{\rho K^2}{2}\left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right] - \rho g [z_2 - z_1]$$

$$P_2 - P_1 = \frac{\rho}{2}\left[-\frac{K^2}{r_2^2} + \frac{K^2}{r_1^2}\right] - \rho g z_2 + \rho g z_1$$

$$P_2 - P_1 = -\frac{\rho V_2^2}{2} + \frac{\rho V_1^2}{2} - \rho g z_2 + \rho g z_1$$

$$\frac{P_2}{\rho g} - \frac{P_1}{\rho g} = \frac{-V_2^2}{2g} + \frac{V_1^2}{2g} - z_2 + z_1$$

$$\boxed{\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1}$$

→ Bernoulli's equation is applicable

Condition-2

forced vortex flow:—

$$\boxed{V = r\omega}$$

$$dp = \frac{\rho \cdot v^2}{r} \cdot dr - \rho g \, dz$$

$$\int_1^2 dp = \int_1^2 \frac{\rho}{r} r^2 \omega^2 dr - \int_1^2 \rho g \, dz$$

$$[P]_1^2 = \rho \omega^2 [r^2]_1^2 - \rho g [z]_1^2$$

$$\Rightarrow P_2 - P_1 = \frac{\rho\cdot\omega^2}{2}(r_2^2 - r_1^2) - \rho\cdot g(z_2 - z_1)$$

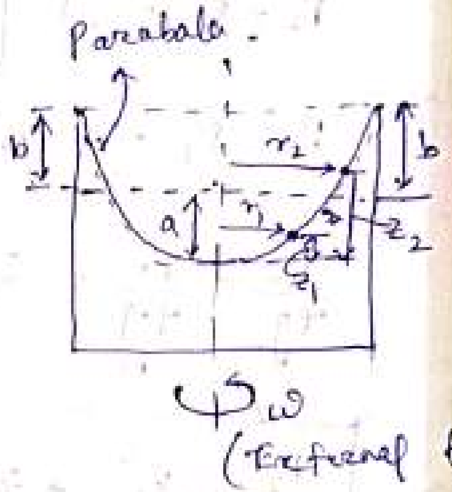
$$\Rightarrow P_2 - P_1 = \frac{\rho}{2}(\omega^2 r_2^2 - \omega^2 r_1^2) - \rho g z_2 + \rho\cdot g z_1$$

$$\Rightarrow P_2 - P_2 = \frac{\rho v_2^2}{2} - \frac{\rho v_1^2}{2} - \rho g z_2 + \rho\cdot g z_1$$

$$\Rightarrow \frac{P_2}{\rho g} - \frac{P_1}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} - z_2 + z_1$$

$$\Rightarrow \boxed{\frac{P_2}{\rho g} - \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\rho g} - \frac{v_1^2}{2g} + z_1}$$

$$\frac{\cancel{P_1}}{\cancel{\rho g}} - \frac{v_1^2}{2g} + z_1 = \frac{\cancel{P_2}}{\cancel{\rho g}} - \frac{v_2^2}{2g} + z_2$$



$\boxed{P_1 = P_2 = P_{atm}}$

$$z_2 - z_1 = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

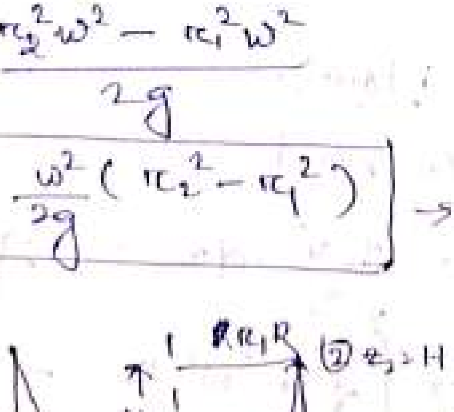
$$z_2 - z_1 = \frac{v_2^2 - v_1^2}{2g}$$

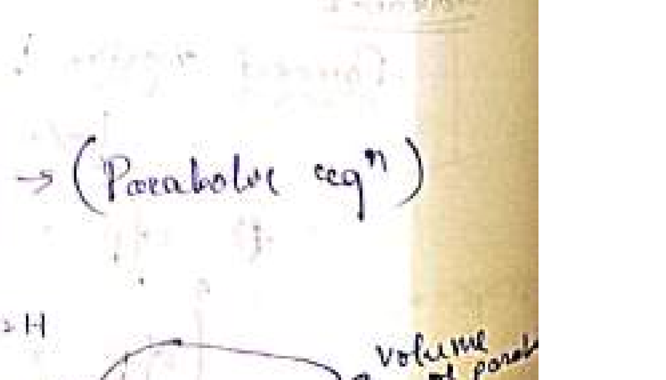
$V_1 = r_1\omega$

$V_2 = r_2\omega$

$$z_2 - z_1 = \frac{r_2^2\omega^2 - r_1^2\omega^2}{2g}$$

$$\boxed{z_2 - z_1 = \frac{\omega^2}{2g}(r_2^2 - r_1^2)} \rightarrow \text{(Parabola eq}^n\text{)}$$





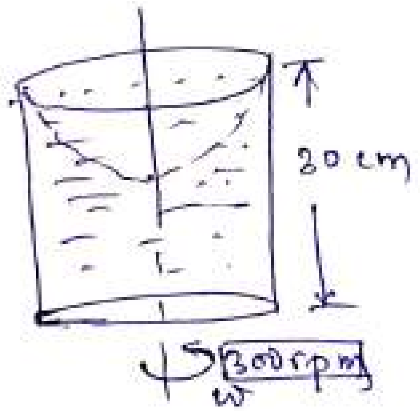
$$H - 0 = \frac{\omega^2}{2g}(R^2 - 0)$$

** $\boxed{H = \frac{\omega^2 R^2}{2g}}$ $\boxed{\text{or } \frac{v^2}{2g}}$ $\because \omega^2 R^2 = v^2$

$\boxed{V_p = \frac{\pi R^2 H}{2}}$ → volume of paraboloid.

Numerical :—

(1) An open cylindrical of 12cm dia. and 20cm. long with water upto top. find volume of water remain, if it is rotated about its axis at a speed of 300 rpm?



Sol^n total volume of fluid inactive

$$V_L = \frac{\pi}{4} D^2 H$$

$$= \frac{\pi}{4} \times (0.12)^2 \times 0.3$$

$$\simeq 0.00339 \, m^3$$

volume loss due to rotation

$$V_p = \frac{\pi R^2 H}{2}.$$

** $\boxed{\omega = \frac{2\pi N}{60}} = \frac{2\pi \times 300}{60} = 10\pi$

$$H = \frac{\omega^2 R^2}{2g} = \frac{(10\pi)^2 (0.06)^2}{2 \times 9.81} = 0.18 \, m$$

$$V_p = \frac{\pi(0.06)^2 \times 0.18}{2} = 0.00101$$

Remaining liquid in cylindrical container

$= V_f - V_p = 0.00339 - 0.00101 = \cancel{0.0028}$

$= 0.00238\ m^3$

CH-5

Dt - 03/10/2021

MOMENTUMTUM

* MOMENTUM & MOMEMTUM EQUATION *

** MOMEMTUM :—

Momentum (M) = $m \cdot v$

from Newton 2nd Law.

$$\Sigma f_x = m \cdot a_x$$

$$\Sigma f_x = m \cdot \frac{dv}{dt}$$

$$\boxed{\Sigma f_x \cdot dt = m \cdot dv}$$

$\Sigma f_x \cdot dt$ → Impulse

$m \cdot dv$ → Change in momentum.

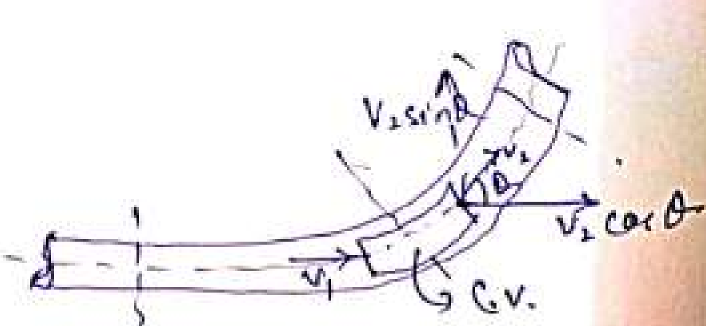
Again

$$\Sigma f_x = \frac{m}{dt} dv$$

$$\boxed{\Sigma f_x = \dot{m} (v_f - v_i)}$$

→ Linear Momentum Equation.

○ Case - study CASE : STUDY



- X - Direction :-

$V_f = V_2 \cos\theta$

$V_i = V_1$

- Y- Direction.

$V_f = V_2 \sin\theta$

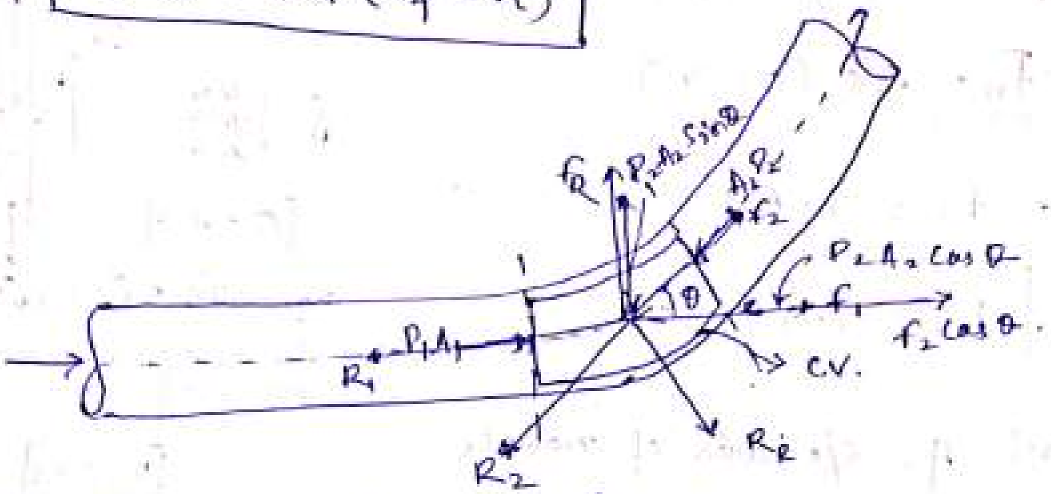
$V_i = 0$

Linear Momentum Equation :-

$$\Sigma F_x = \dot{m}\,(V_f - V_i)$$

**

$$\boxed{\Sigma F_x = \rho \cdot Q\,(V_f - V_i)}$$



$F_1, F_2, F_R \longrightarrow$ forces on fluid

$R_1, R_2, R_R \longrightarrow$ force on pipe.

• for fluid :-

x- Direction :-

$$\Sigma F_x = \rho \cdot Q\,(V_{fx} - V_{ix})$$

$$P_1A_1 - P_2A_2\cos\theta - F_1 + F_2\cos\theta = \rho \cdot Q\,(V_2\cos\theta - V_1) \quad \text{--- (1)}$$

y- Direction :-

$$\Sigma F_y = \rho \cdot Q\,(V_{fy} - V_{iy})$$

$$-P_2A_2\sin\theta + F_2\sin\theta = \rho \cdot Q\,(V_2\sin\theta - 0) \quad \text{--- (2)}$$

**

$$\boxed{F_R = \sqrt{F_1^2 + F_2^2}}$$

IMPACT OF JET

Case-1 : Normal force on a statanary flat Plate :-

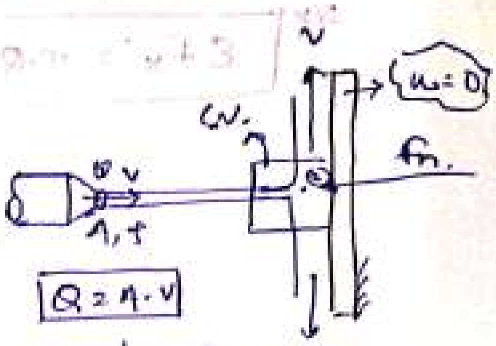
x-derection :-

$$\Sigma f_x = m^{\cdot} (V_{fx} - V_{ix})$$

$$-F_n = \rho . Q (0 - v)$$

$$- f_n = \rho \cdot A . v \times - v$$

$$\boxed{f_n = \rho . A v^2}$$



$\boxed{Q = A . V}$

Bernaulli's Eq^n ($1 \to 2$)

$$\frac{\cancel{P_1}}{\rho . g} + \frac{v_1^2}{2g} + \cancel{z_1} = \frac{\cancel{P_2}}{\rho g} + \frac{v_1^2}{2g} + \cancel{z_2}$$

$$\boxed{v_1 = v_2}$$

where :- A = c/s area of nozzle

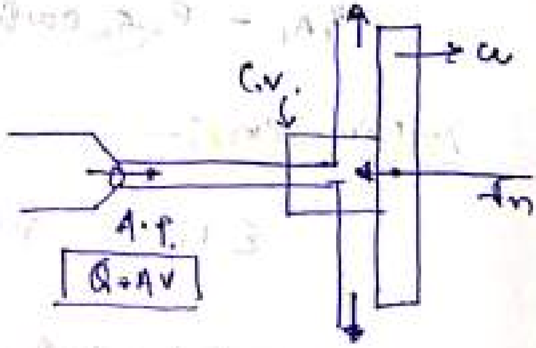
v = velocity of Jet

u = velocity of plate.

$\boxed{\text{Power} = f_n \cdot v}$ ** $\boxed{P = 0}$

Case-1.1 :- f_n on a moving plate.

- If the same plate is moving with velocity u
- (v > u)
- Relative Velocity = (v - u)
- Q = A (v - u) [c.v]



- $\Sigma f_x = m^{\cdot} (V_{fx} - V_{ix})$

$$-f_n = \rho . A (v - u) [0 - (v - u)]$$

***

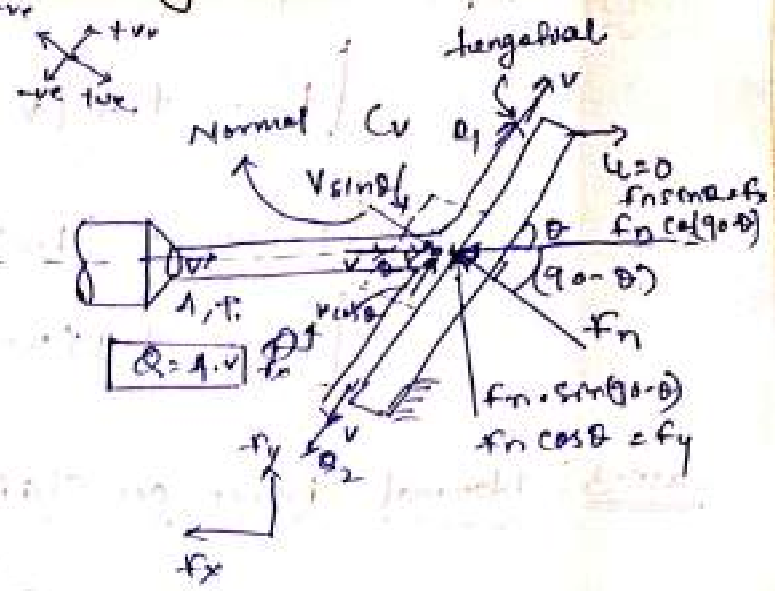
$$\boxed{f_n = \rho . A . (v - u)^2}$$

$$\boxed{\text{Power} = f_n . u}$$

**

$$\boxed{P = \rho A (v - u)^2 \cdot u}$$

Case-2:- Normal force on an stationary Inclined plate:-



- $f_n = \dot{m}\,(V_{fn} - V_{in})$

$$-f_n = \rho\cdot Q\,(0 - V\sin\theta)$$

$$f_n = \rho\cdot A\cdot V\cdot V\sin\theta$$

$$\boxed{f_n = \rho\cdot A V^2 \sin\theta}$$

x-direction:-

$$-f_x = -f_n \sin\theta$$

$$\boxed{f_x = \rho\cdot A V^2 \cdot \sin^2\theta}$$

y-direction:-

$$f_y = f_n \cos\theta$$

$$\boxed{f_y = \rho\cdot A\, V^2 \sin\theta\cdot\cos\theta}$$

* Discharge:-

$$Q = Q_1 + Q_2$$

Momentum eq^n in tengential derection.

$$\Sigma f_t = \dot{m}\,(V_{ft} - V_{it})$$

$$0 = \dot{m}\,V_{ft} - \dot{m}\,V_{it}$$

$$0 = \rho\cdot Q_1 V - \rho\cdot Q_2\cdot V - \rho\, Q\, V\cdot\cos\theta$$

$$\rho V\,(Q_1 - Q_2 - Q\cos\theta) = 0$$

$$Q_1 - Q_2 = (Q_1 + Q_2)\cos\theta$$

$$\frac{(Q_1 - Q_2)}{(Q_1 + Q_2)} = \frac{\cos\theta}{1}$$

Apply Componendo dividendo.

$$\frac{2Q_1}{-2Q_2} = \frac{\cos\theta + 1}{\cos\theta - 1}$$

$$\boxed{\frac{Q_1}{Q_2} = \frac{1+\cos\theta}{1-\cos\theta}}$$

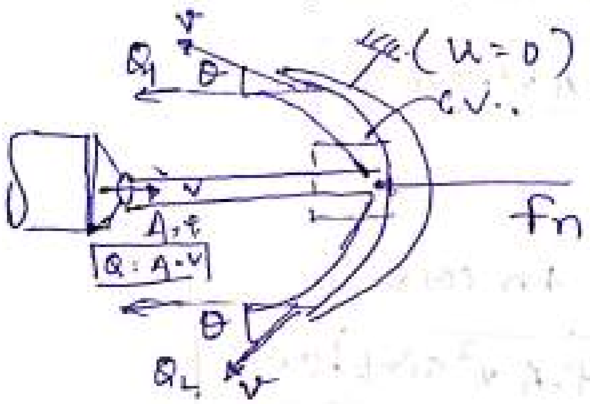
* If the plate is moving with velocity :-

$$\boxed{F_n = \rho \cdot A (V-u)^2 \cdot \sin\theta}$$

Power = $F_n \times u$

$$\boxed{P = \rho \cdot A (V-u)^2 \cdot \sin\theta \cdot u}$$

Case-3 :- Normal force on stationary Curved Plate :-



$$\Sigma F_x = \dot{m} (V_{fx} - V_{ix})$$

$$-F_n = -\rho Q_1 V\cos\theta - \rho Q_2 V\cos\theta - \rho \cdot Q \cdot V$$

$$F_n = \rho \cdot V\cos\theta \underbrace{(Q_1 + Q_2)}_{Q} + \rho \cdot Q V.$$

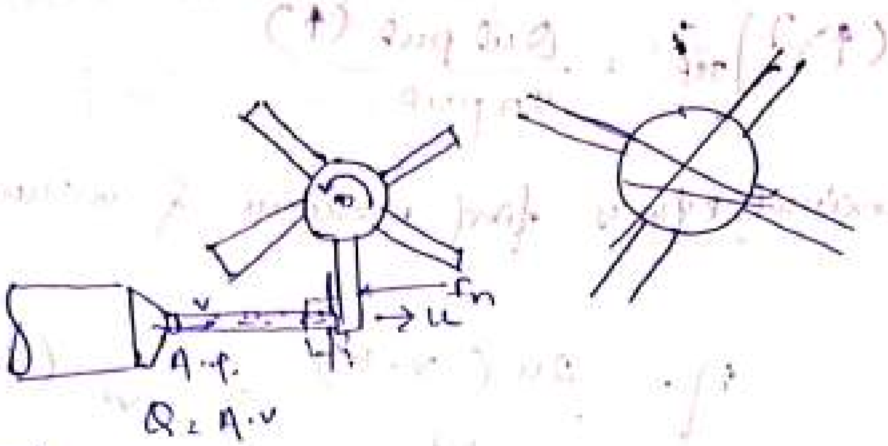
$$\boxed{F_n = \rho V \cdot Q (\cos\theta + 1)}$$

* If plate is moving with velocity u :-

$$\boxed{F_n = \rho A (u-V)^2 (1 + \cos\theta)}$$

$$\boxed{\text{Power} = F_n \cdot u}$$

Case-4: forces on a Series of flat plate mounted on a wheel



$$\boxed{Q = A \cdot v}$$

$$-F_n = \rho \cdot Q\,[0 - (v - u)]$$

$$\boxed{F_n = \rho \cdot A v (v - u)}$$

Power $= F_n \times v$

$$\boxed{\text{Power} = \rho \cdot A \cdot V (V - u) \cdot u}$$

$$K.E_{JET} = \frac{1}{2} m v^2 = \frac{1}{2} \rho . Q . v^2 = \frac{1}{2} \rho . A v^3$$

$$\eta_{jet} = \frac{\text{Output}}{\text{Input}}$$

$$\eta_{jet} = \frac{\cancel{\rho} \cdot \cancel{A} \cdot \cancel{V} (v - u) \cdot u}{\frac{1}{2} \times \cancel{\rho} \cdot \cancel{A} \times v^3}$$

$$\boxed{\eta_{jet} = \frac{2 (V - u) u}{v^2}}$$

Eg:- $v = 20\,m/s$

$u = 10\,m/s$

$$\%\eta_{jet} = \frac{2(20 - 10) \times 10}{(20)^2} \times 100$$

$$\boxed{\%\eta_{jet} = 50\%}$$

* Maximum efficiency:

$$(4)\ \eta_{jet} = \frac{\text{Output}\ (4)}{\text{Input}}$$

For maximum efficiency find maximum & minimum

$$\eta = \frac{2u(v-u)}{v^2} = \frac{2}{v^2}(u \cdot v - u^2)$$

$$\frac{d\eta}{du} = 0$$

$$\frac{d}{du}\left[\frac{2}{v^2}(u \cdot v - u^2)\right] = 0$$

$$\frac{\cancel{2}}{v^2} v - \frac{\cancel{4}\,2}{v^2} \cdot u = 0$$

$$v - 2u = 0$$

$\boxed{v = 2u}$ → for maximum efficiency condition.

* Maximum efficiency of jet with ~~height~~ flat plate turbi

$$\%\ \eta_{max} = \frac{\cancel{2} \times \frac{v}{\cancel{2}}\left(v - \frac{v}{2}\right)}{v^2} \times 100$$

$$\%\ \eta_{max} = \frac{1}{\cancel{v}}\cancel{v}\left(1 - \frac{1}{2}\right) \times 100$$

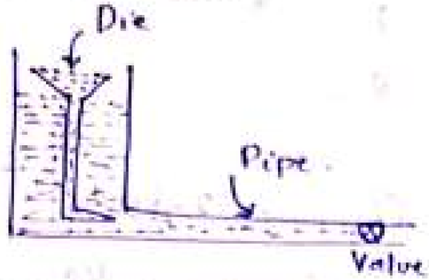
$$\%\ \eta_{max} = \frac{1}{2} \times 100$$

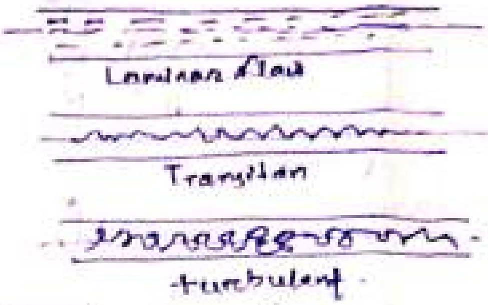
$$\boxed{\%\ \eta_{max} = 50\%}$$

CH-6 : LAMINAR FLOW

Dt - 4th April 2023

Reynolds Experiment :-





Reynolds Number $\boxed{Re = \dfrac{\text{Inertia force}}{\text{Viscous force}}}$

- Inertia force

$$f_i = m \cdot a$$

$$f_i = \rho \cdot V \cdot \frac{v}{t}$$

$$f_i = \rho \cdot l^3 \times \frac{v}{t}$$

$$f_i = \rho \cdot l^2 \cdot \frac{l}{t} \cdot v$$

$$\boxed{f_i = \rho \cdot l^2 v^2}$$

$\because \frac{l}{t} = v$

Viscous force

$$\frac{f_v}{A} = \frac{\mu v}{h}$$

$$f_v = \mu \frac{v}{h} \cdot A$$

$$f_v = \mu \frac{v}{l} \cdot l^2$$

$$\boxed{f_v = \mu v \cdot l}$$

- $Re < 2000$ [Laminar flow]
- $2000 < Re < 4000$ [Transition Flow]
- $Re > 4000$ [Turbulent flow]

$$Re = \frac{I.f}{V.f} = \frac{\rho \cdot l^2 \cdot v^2}{\mu\, v\, l}$$

$$\boxed{Re = \frac{\rho v l}{\mu}}$$

$$\boxed{Re = \frac{v l}{\nu}}$$

$\because \nu = \frac{\mu}{\rho}$

where

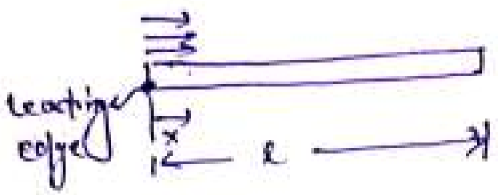
ρ = Density, l = Characteristic length, ν = Kinematic viscosity

v = velocity, μ = Dynamic viscosity

About Characteristic length

Plate :

{ external flow)



Pipe

{ Internal flow

$\boxed{\text{Characteristic length (L) = Hydraulic dia. } (D_H)}$

$$\boxed{D_H = \frac{4\,A_{cs}}{P}}$$

→ c/s Area

→ parameter.

* Hydraulic Diam

* Circular pipe :-

$$D_H = \frac{4 \cdot A_{cs}}{P} = \frac{4 \times \frac{\pi}{4} D^2}{\pi D} = D$$

$$\boxed{D_H = D}$$

$$\boxed{Re = \frac{\rho \cdot V \cdot D}{\mu}}$$

* Rectangular section :-

$$D_H = \frac{4\,A_{cs}}{P} = \frac{4 \times ab}{2(a+b)}$$

$$\boxed{D_H = \frac{2ab}{a+b}}$$

Note :-

Re (Reynolds Number)

$\boxed{Re = \frac{\rho \cdot V x}{\mu}}$ (External flow)

$\boxed{Re = \frac{\rho \cdot V \cdot D_H}{\mu}}$ (Internal flow)

INTERNAL flow :-

- The 1st step in the case of internal flow is to convert non-circular section into circular section.

- Critical Reynolds Number

$$\boxed{Re_c = 2000}$$

↳ $V \rightarrow$ critical velocity

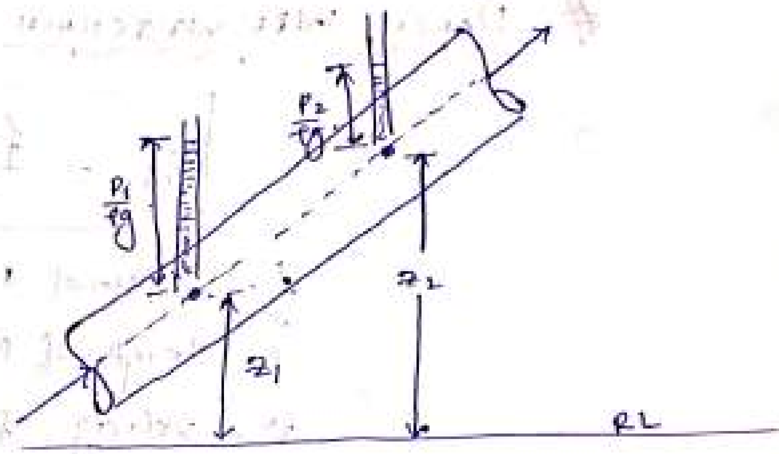
- The velocity at which laminar flow going to convert turbulent flow is known as "critical velocity".

- Continuity eq^n :-

$$Q_1 = Q_2$$

$$A_1 V_1 = A_2 V_2$$

$$\boxed{V_1 = V_2} \quad \because A_1 = A_2$$



- Bernoulli's Equation :-

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

h_L ⟶ (Head Loss)

$$\Rightarrow \underbrace{\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right)}_{\text{piezometric pressure head difference}} = h_L$$

$$\Rightarrow \frac{(P_1 + \rho g z_1)}{\rho g} - \frac{(P_2 + \rho g z_2)}{\rho g} = h_L$$

$$\boxed{\frac{\Delta P}{\rho g} = h_L}$$

[$(P_1 + \rho g z_1) - (P_2 + \rho g z_2)$ = Pressure difference in piezometer (ΔP)]

Note:-

for study and uniform flow, pressure variation is linear and it is ~~decreasing in the direction~~ ~~constant~~ decreasing in the direction of flow to compensate losses.

$$\boxed{\frac{dp}{dx} = -ve}$$

Pressure gradient

(constant pressure drops)

unit

$\frac{dp}{dx} = \frac{N/m^2}{m} = \frac{N}{m^3}$ or $\frac{Pa}{m}$

Darcy Weisbach Equation:-

$$\boxed{h_L = \frac{f \cdot l \cdot v^2}{2gd}}$$

$$\boxed{h_L = \frac{4f' l v^2}{2dg}}$$

f = frictional flow factor

l = length of pipe

v = velocity of flow

d = dia of pipe

f' = friction co-efficient

$$\boxed{f = 4f'}$$

In terms of discharge

$$h_L = \frac{f l \cdot Q^2}{2gd \times \frac{\pi^2}{4^2} d^4}$$

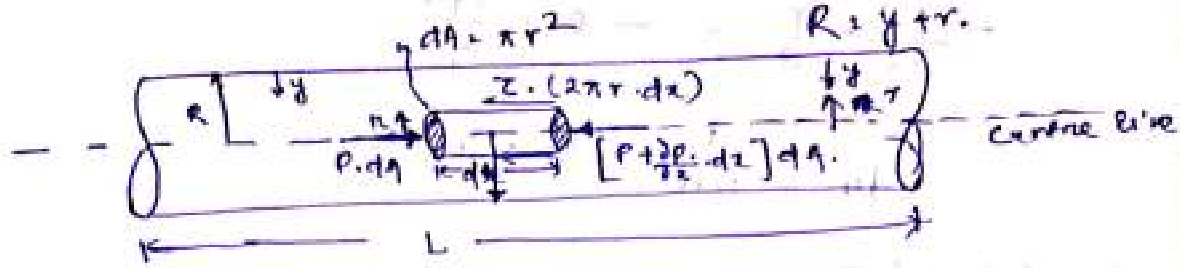
$$\boxed{h_L = \frac{f l \cdot Q^2}{12.1 d^5}}$$

$$\boxed{h_L = \frac{\Delta P}{\rho g} = \frac{f l \cdot v^2}{2gd} = \frac{f \cdot l \cdot Q^2}{12.1 \cdot d^5}}$$

(Darcy Weisbach Equation)

→ This equation is valid for both laminar and turbulent flow.

LAMINAR FLOW THROUGH PIPE :- { Re < 2000 }



- for a study and uniform flow :- { Assumption }
- Newton's 2nd Law :

$$\Sigma F_x = m \cdot a_x$$

$$P \cdot dA - \tau(2\pi r \cdot dx) - \left[P + \frac{\partial p}{\partial x} \cdot dx\right] dA = 0$$

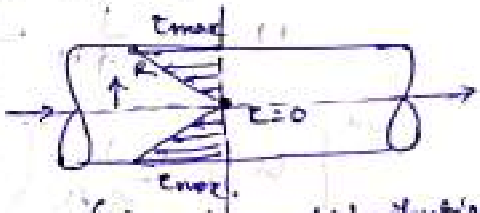
$$\Rightarrow \cancel{P \cdot dA} - \tau(2\pi r\, dx) - \cancel{P \cdot dA} - \frac{\partial p}{\partial x} \cdot dx \cdot dA = 0$$

$$\Rightarrow \tau(2\cancel{\pi}\cancel{r} \cdot \cancel{dx}) = -\frac{\partial p}{\partial x} \cdot (\cancel{dx} \cdot \cancel{\pi} r^2) = 0$$

$$\boxed{\tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}}$$ (for small element)

$$\boxed{\tau = \frac{\Delta P}{L} \cdot \frac{r}{2}}$$

$\boxed{\tau \propto r}$
↳ Linear variation.



(Shear stress distribution)

* Shear stress variation is linear.

$$\tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2} \quad \text{—— (1)}$$

* from newton Law of viscosity

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dr} \quad \text{—— (2)}$$

[R = r + y]
0 = dr + dy
dy = −dr

Compare eqⁿ ① & ②

$$-\frac{\partial p}{\partial x}\cdot\frac{r}{2} = -\mu\cdot\frac{du}{dr}$$

$$du = \frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}\cdot r\cdot dr$$

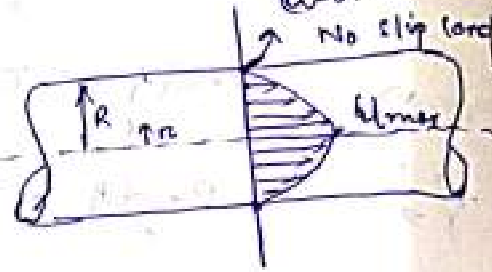
by integrating

$$\int du = \int \frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}\cdot r\cdot dr$$

$$u = \frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{r^2}{2} + C$$

* Boundary Condition.

r @ $r = R$, $u = 0$



$$0 = \frac{1}{4\mu}\frac{\partial p}{\partial x}\cdot R^2 + C$$

$$C = -\frac{1}{4\mu}\cdot\frac{\partial p}{\partial x}\cdot R^2$$

$$u = \frac{1}{4\mu}\cdot\frac{\partial p}{\partial x}\cdot r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x}R^2$$

** $\boxed{u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)[R^2 - r^2]}$ → Parabolically change so, (parabolic Distribution)

* If $r = R \rightarrow u = 0$
* If $r = 0 \rightarrow u =$ ~~max~~ u_{max}.

$$\boxed{u_{max} = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\cdot R^2}$$

$$\frac{u}{u_{max}} = \frac{\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2)}{\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)R^2}$$

$$\frac{u}{u_{max}} = \frac{R^2 - r^2}{R^2} = \boxed{\frac{u}{u_{max}} = 1 - \frac{r^2}{R^2}}$$ *****

$$\tau = -\mu \frac{\partial u}{\partial r}$$

$\because$

$$u = u_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$\frac{du}{dr} = u_{max}\left(0 - \frac{2r}{R^2}\right) = -\frac{2r \cdot u_{max}}{R^2}$$

$$\tau = -\mu \cdot -\frac{2r \cdot u_{max}}{R^2}$$

** $$\boxed{\tau = \frac{2\mu\, u_{max} \cdot r}{R^2}}$$

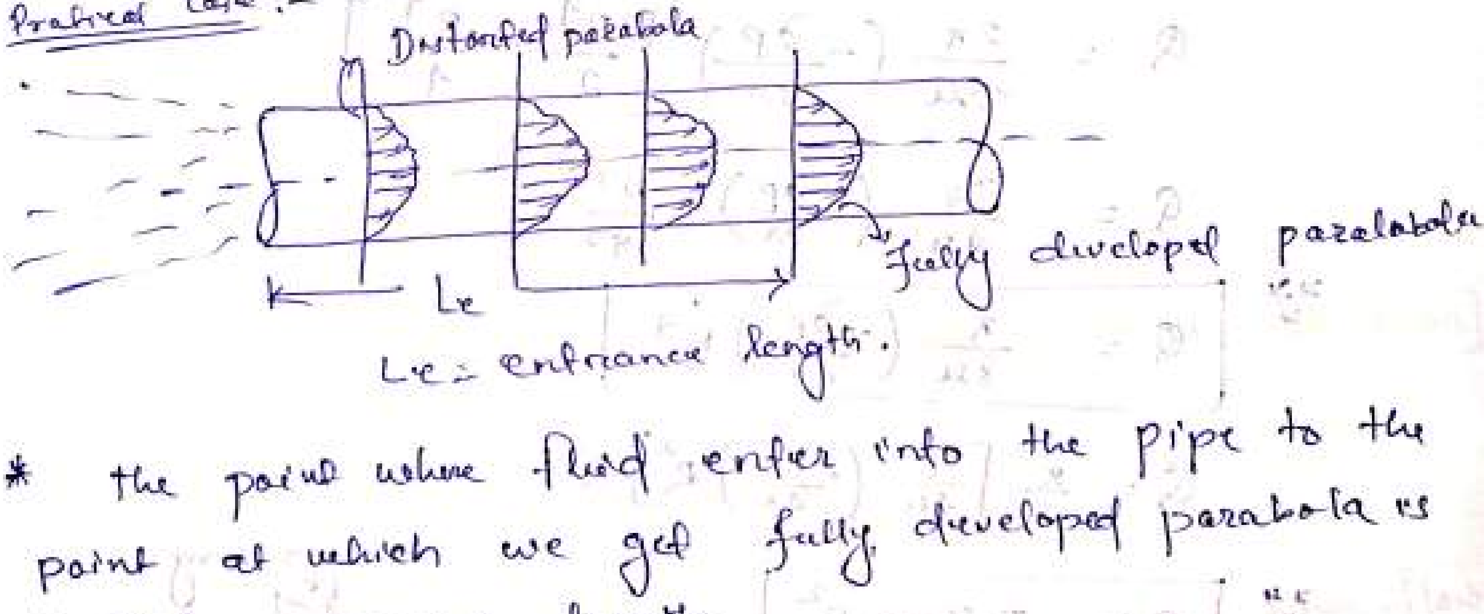
- If $r = R \longrightarrow \tau = \tau_{max}$

$$\tau_{max} = \frac{2\mu\, u_{max} \cdot R}{R^2}$$

*** $$\boxed{\tau_{max} = \frac{2\mu\, u_{max}}{R}}$$

~~Practical analysis~~ :-

Practical Case :-

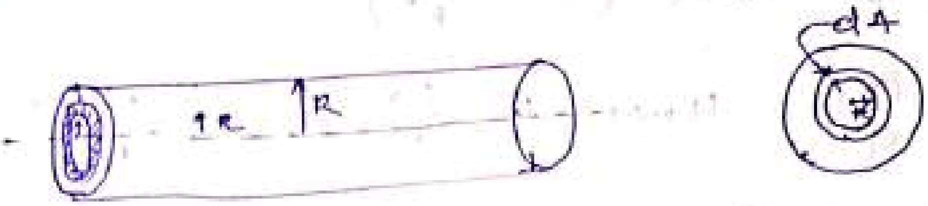


L_e = entrance length.

* the point where fluid enter into the pipe to the point at which we get fully developed parabola is called entrance length.

L-2 : Dt:- 5th 05/04/2023 (Laminar flow)

Discharge :-



$dA = 2\pi r dr$

R = pipe dia.

$Q = \int_0^R u \cdot dA$

$\therefore \left\{ \begin{array}{l} A = \pi r^2 \\ \frac{dA}{dr} = 2\pi r \\ dA = 2\pi r dr \end{array} \right.$

$Q = \int_0^R \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \cdot (R^2 - r^2) 2\pi r dr$

$Q = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) 2\pi \int_0^R (R^2 - r^2) \cdot r \cdot dr$

$Q = \frac{2\pi}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \int_0^R (R^2 r - r^3) \cdot dr$

$Q = \frac{2\pi}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R$

$Q = \frac{2\pi}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$

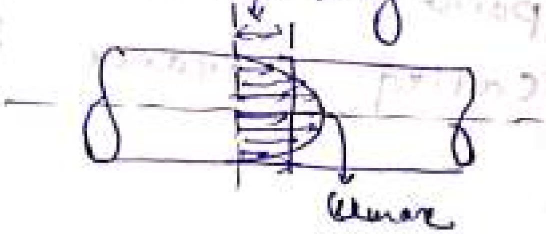
$Q = \frac{2\pi}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \cdot \frac{R^4}{\cancel{4}2}$

*** $\boxed{Q = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^4}$

$Q = \frac{\pi}{2}\left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) R^2\right] R^2$

*** $\boxed{Q = \frac{\pi\, u_{max} R^2}{2}}$ — (1)

$\left\{ \boxed{Q = A \cdot V_{avg}} \right\}$ — (2)



from eqⁿ ① & ②

$$A \cdot V_{avg} = \frac{\pi U_{max} R^2}{2}$$

$$\pi R^2 V_{avg} = \frac{\pi U_{max} R^2}{2}$$

$$\boxed{\frac{U_{max}}{V_{avg}} = 2}$$

* Pressure drop through a pipe of length "L".

$$\tau = \frac{r}{2}\left(-\frac{\partial p}{\partial x}\right)$$

$$\boxed{\tau = \frac{r}{2} \cdot \frac{\Delta p}{L}}$$

Pressure Drop (−ve)

$$V_{avg} = \frac{1}{2} U_{max}$$

$$V_{avg} = \frac{1}{2} \times \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \cdot R^2$$

$$\frac{8\mu V_{avg}}{R^2} = \frac{\Delta P}{L}$$

$$\Delta P = \frac{8\mu V_{avg} \cdot L}{\left(\frac{D}{2}\right)^2}$$ ∵ {D = dia of pipe}

$$\boxed{\Delta P = \frac{32\mu \cdot V_{avg} \cdot L}{D^2}}$$ [Hagen Poiseuille Equation]

$$h_L = \frac{\Delta P}{\rho g}$$

$$\boxed{h_L = \frac{32\mu V_{avg} \cdot L}{\rho \cdot g \cdot D^2}}$$ only valid for laminar flow.

Note: – Capillary tube viscometer utilizes Hagen Poiseuille eqⁿ to find the viscosity of unknown fluid.

$$h_L = \frac{32\mu \cdot V_{avg} \cdot L}{\rho g \cdot D^2}$$

$$f' = \frac{f}{4} = \frac{16}{R_e}$$

$$\frac{fLV^2}{2gD} = \frac{32\mu \cdot V_{avg} \cdot L}{\rho \cdot g \cdot D^2}$$

$$f = \frac{64\mu}{\rho \cdot D \cdot V_{avg}} \qquad R_e = \frac{\rho \cdot v D}{\mu}$$

** $\boxed{f = \frac{64}{R_e}}$ ** [Moody's Equation]

* In the case of laminar flow friction factor is inversely proportional to Reynold's number.

SHEAR VELOCITY :-

$$\sqrt{\frac{\tau_0}{\rho}} \rightarrow \sqrt{\frac{N/m^2}{kg/m^3}} = \sqrt{\frac{\frac{kg \cdot m}{s^2} \cdot m^2}{kg \cdot m^3}}$$

$$\boxed{v^* = \sqrt{\frac{\tau_0}{\rho}}}$$

**

$$\tau = \frac{r}{2} \cdot \frac{\Delta P}{L}$$

$$\tau = \frac{r}{2L} \cdot \rho g h_L$$

$$\tau = \frac{D}{4L} \times \rho \cdot g \cdot \frac{f \cdot l V^2}{2gD}$$

$$\frac{\tau}{\rho} = \frac{f}{8} \cdot V_{avg}^2$$

$$\sqrt{\frac{\tau}{\rho}} = \sqrt{\frac{f}{8}} \cdot V_{avg}$$

* Laminar flow Through pipe.-

- $$\boxed{\tau = \frac{r}{2}\left(-\frac{\partial p}{\partial x}\right)}$$

- $$\boxed{f' = \frac{f}{4} = \frac{16}{R_e}}$$

- $$\boxed{u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2)}$$

- $$\boxed{U_{max} = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)R^2}$$

* $$\boxed{\frac{u}{U_{max}} = \left[1 - \frac{r^2}{R^2}\right]}$$

- $$\boxed{\tau = \frac{2\mu r \cdot U_{max}}{R^2}}$$

* $$\boxed{\tau_0 = \frac{2\mu U_{max}}{R}}$$

* $$\boxed{Q = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)\cdot R^4}$$

* $$\boxed{Q = \frac{\pi}{2} u_{max} \cdot R^2}$$

* $$\boxed{\frac{U_{max}}{V_{avg}} = 2}$$

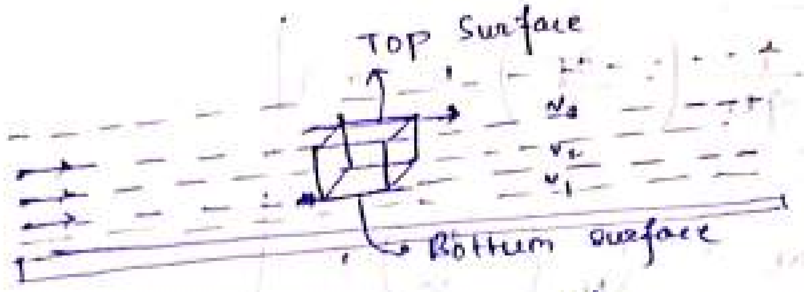
* $$\boxed{\Delta P = \frac{32\mu \cdot V_{avg} \cdot L}{D^2}}$$

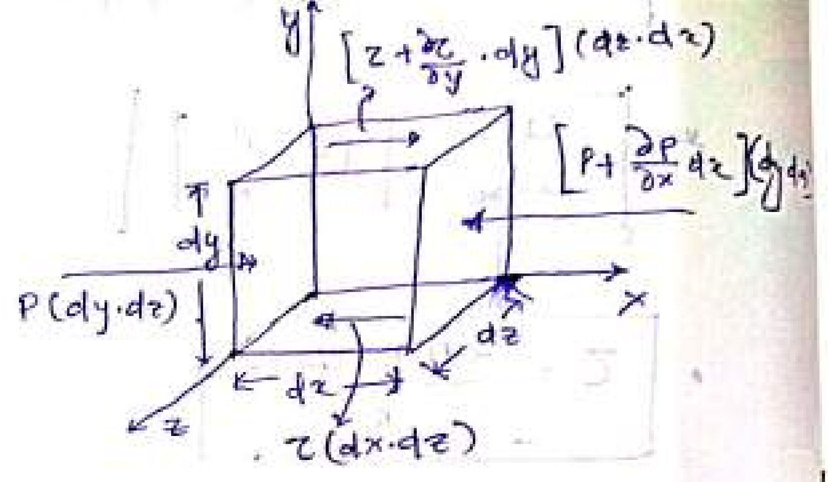
* $$\boxed{f = \frac{64}{Re}}$$

* $$\boxed{V^* = \sqrt{\frac{\tau}{\rho}} = \sqrt{\frac{f}{8}} \cdot V_{avg}}$$

Dt- 06/04/...

Navier- Stoke's Equation :-





• from Newton 2nd Law :-

$$\boxed{\Sigma f_x = m a_x}$$

$$\Rightarrow p(dy \cdot dz) + \left[\tau + \frac{\partial \tau}{\partial y} \cdot dy\right](dx \cdot dz) - \tau(dx \cdot dz) - \left[p + \frac{\partial p}{\partial x} \cdot dx\right](dy \cdot dz) = \rho(dx \cdot dy \cdot dz)\left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}\right]$$

$$\Rightarrow \cancel{p(dy \cdot dz)} + \cancel{\tau(dx \cdot dz)} + \frac{\partial \tau}{\partial y}(dx \cdot dy \cdot dz) - \cancel{\tau(dx \cdot dz)} - \cancel{p(dy \cdot dz)} - \frac{\partial p}{\partial x} \cdot (dx \cdot dy\, dz) = \rho(dx \cdot dy \cdot dz)\left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}\right]$$

$$\Rightarrow \frac{\partial \tau}{\partial y} - \frac{\partial p}{\partial x} = \rho \cdot \left[u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}\right]$$

~~• for Steady flow $\left[\frac{\partial u}{\partial t} = 0\right]$~~

$$\cancel{u \frac{\partial u}{\partial x}} + v \frac{\partial u}{\partial y}$$

* for steady flow $\left[\frac{\partial u}{\partial t} = 0\right]$

$$\boxed{u \cdot \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + \frac{1}{\rho} \cdot \frac{\partial \tau}{\partial y}}$$

↳ Navier Stoke's Equation.

* from Newton's law of viscosity

$\cancel{\tau = \mu \frac{\partial y}{d}}$

$\cancel{\tau = \mu \frac{du}{y}}$

$$\tau = \mu \frac{du}{dy}$$

$$\boxed{\frac{\partial \tau}{\partial y} = \mu \frac{d^2 u}{dy^2}} \quad \text{— (2)}$$

* $$\boxed{u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \frac{d^2 u}{dy^2}}$$

LAMINAR FLOW b/w TWO FIXED PARALLEL PLATE :-

Assumption :-

- Study flow
- Uniform flow

* Considered 2 fixed parallel plate of length (L), width (b)

* for study and uniform flow. [change w.r.t space = 0]

$$-\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \frac{d^2 u}{dy^2} = 0$$

$$\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} = \frac{\mu}{\rho} \cdot \frac{d^2 y}{dy^2}$$

$$\frac{\partial p}{\partial x} = \mu \frac{d^2 u}{dy^2}$$

$$\int d^2u = \int \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right) dy$$ (Integrate eqⁿ)

$$\int du = \int \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right) y\,dy + C_1$$ (Integrate Eqⁿ)

$$u = \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right)\cdot\frac{y^2}{2} + C_1 y + C_2$$

* Boundary Condition

- at $y = 0$, $u = 0$ | at $y = h$, $u = 0$

1st Boundary Condition:

$$\boxed{C_2 = 0}$$

- 2nd Boundary condition

$$0 = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\cdot h^2 + C_1 h.$$

$$\boxed{C_1 = \frac{-1}{2\mu}\left(\frac{\partial p}{\partial x}\right) h.}$$

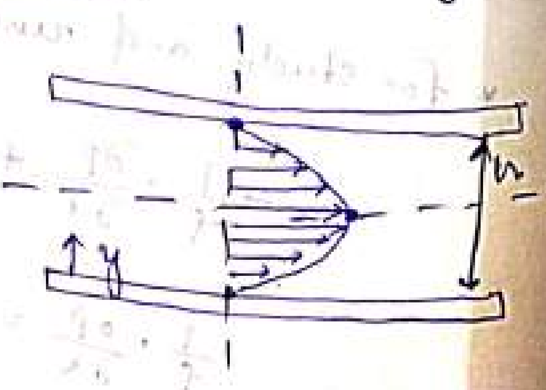
$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) y^2 - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) h\cdot y$$

$$\boxed{u = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left[h\cdot y - y^2\right]}$$ (Local velocity)

$$\frac{du}{dy} = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(h - 2y) = 0$$

$$h - 2y = 0$$

$$\boxed{y = \frac{h}{2}}$$



$$u_{max} = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left[h\cdot\frac{h}{2} - \frac{h^2}{4}\right]$$

$$u_{max} = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left(\frac{2h^2 - h}{4}\right)$$

$$\boxed{u_{max} = \frac{1}{8\mu}\cdot\left(-\frac{\partial p}{\partial x}\right)\cdot h^2}$$ (Max^m Value)

* Shear Stress :-

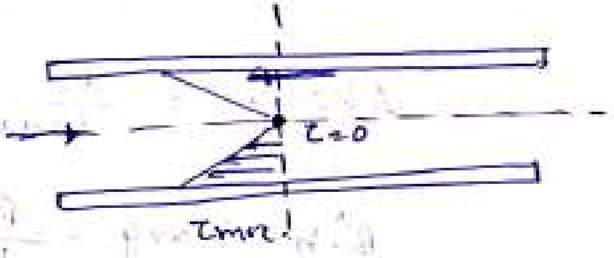
$\cancel{\tau = \mu \frac{dy}{}}$

$$\tau = \mu \frac{du}{dy}$$

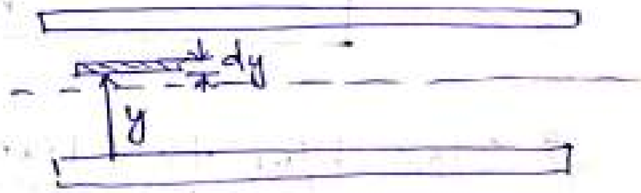
$$\tau = \mu\cdot\frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\frac{d}{dy}\left[h y - y^2\right]$$

$$\boxed{\tau = \frac{1}{2}\left(-\frac{\partial p}{\partial x}\right)(h - 2y)}$$ ** (Shear Stress)

$$\boxed{\tau_0 = \frac{1}{2}\left(-\frac{\partial p}{\partial x}\right)\cdot h}$$



* Discharge (Q) :-



$$dQ = u\cdot dA$$

$$Q = \int_0^h u \times B\,dy$$

$$Q = \int_0^h \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(hy - y^2)\,B\cdot dy$$

$$Q = \frac{B}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\int_0^h (hy - y^2)\,dy$$

$$Q = \frac{B}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h$$

$$Q = \frac{B}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left[\frac{h^3}{2} - \frac{h^3}{3}\right]$$

$$\boxed{Q = \frac{B}{12\mu}\left(-\frac{\partial p}{\partial x}\right)h^3}$$ ** (Discharge)

$$A \cdot V_{avg} = \frac{B}{6}\left[\frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)h^2 \frac{1}{4}\right]h \times \frac{4}{4}$$

$$A \cdot V_{avg} = \frac{B}{6_3} \cdot U_{max} \cdot h \cdot 4^2$$

$$h \cdot B \cdot V_{avg} = \frac{2}{3} B \cdot h\, U_{max}$$

$$\boxed{\frac{U_{max}}{V_{avg}} = \frac{3}{2}}$$ ** [Relation b/w U_{max} - V_{avg}]

* Pressure Drop :-

$$Q = \frac{B}{12\mu}\left(-\frac{\partial p}{\partial x}\right) \cdot h^3$$

$$A V_{avg} = \frac{B}{12\mu}\left(\frac{\Delta P}{L}\right) \times h^3$$

$$B \cdot h \cdot V_{avg} = \frac{B}{12 \cdot \mu}\left(\frac{\Delta P}{L}\right) \cdot h^3$$

$$\boxed{\Delta P = \frac{12\,\mu\, V_{avg} \cdot L}{h^2}}$$ ** (pressure drop)

Flow b/w two fixed parallel plate (All formula) :-

- $$\boxed{u = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(hy - y^2)}$$
- $$\boxed{U_{max} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) \cdot h^2}$$
- $$\boxed{\tau = \frac{1}{2}\left(-\frac{\partial p}{\partial x}\right)(h - 2y)}$$

- $\boxed{\tau_0 = \frac{1}{2}\left(-\frac{\partial p}{\partial x}\right)\cdot h}$

- $\boxed{Q = \frac{B}{12\mu}\left(-\frac{\partial p}{\partial x}\right)\cdot h^3}$

- ~~Umax~~ $\boxed{\frac{U_{max}}{V_{avg}} = \frac{3}{2}}$

- $\boxed{\Delta P = \frac{12\cdot\mu\cdot V\cdot L}{h^2}}$

CORRECTION FACTOR :—

$\Sigma F = \dot{m}(V_f - V_i)$ → average Velocity [Momentum Correction factor (β)]

$Q = A\cdot V$ → average velocity

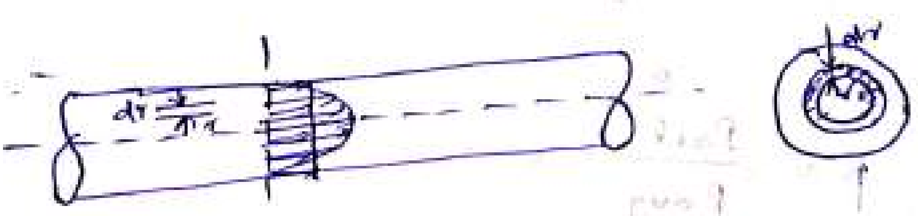
$\Delta \dot{KE} = \frac{1}{2}\dot{m}(V_f^2 - V_i^2)$ [Kinetic Energy Correction factor (α)]
→ average velocity

- $\boxed{\dot{P}_{act} = \beta\cdot \dot{P}_{avg}}$

- $\boxed{\Delta \dot{KE}_{act} = \alpha\cdot \Delta \dot{KE}_{avg}}$

L-3
Dt- 07/02/2023

① MOMENTUM CORRECTION FACTOR (β) :—



- $\frac{U_{max}}{V_{avg}} = 2$

→ Momentum correction factor is defined as the ratio of rate of momentum based on actual velocity to the rate of momentum based on average velocity.

$$\dot{P}_{act} = \beta \cdot \dot{P}_{avg}$$

$$\boxed{\beta = \frac{\dot{P}_{act}}{\dot{P}_{avg}}}$$

* Avg- Rate of momentum :-

$$\dot{P}_{avg} = \dot{m} \cdot V_{avg}$$

$$\dot{P}_{avg} = \rho \cdot Q \cdot V_{avg}$$

$$\dot{P}_{avg} = \rho \cdot A\, V_{avg}^2$$

$$\boxed{\dot{P}_{avg} = \rho \cdot \pi R^2 \cdot V_{avg}^2}$$

* Actual rate of moment for a small strip :-

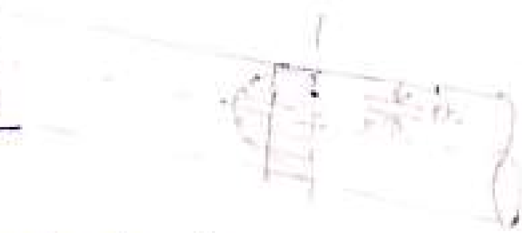
$$\dot{P}_{act} = \dot{m} \times u$$

$$\dot{P}_{act} = \rho \cdot Q \cdot u$$

$$\dot{P}_{act} = \rho \cdot 2\pi r \cdot dr \cdot u^2$$

for total pipe :-

$$\dot{P}_{act} = \int_0^R \rho \cdot 2\pi r \cdot dr \cdot u^2$$



$$\beta = \frac{\dot{P}_{act}}{\dot{P}_{avg}}$$

$$\beta = \frac{\int_0^R \rho\, 2\pi r \cdot dr \cdot u^2}{\rho \cdot \pi R^2 V_{avg}^2}$$

$$\boxed{\beta = \frac{2}{R^2 \cdot V_{avg}^2} \cdot \int_0^R u^2 \cdot r \cdot dr}$$ **

In terms of Area :—

$$P_{avg} = m \cdot v = \rho \cdot Q \cdot v = \rho A V^2$$

$$P_{act} = m u = \rho \cdot Q \cdot v = \int_0^R \rho \cdot dA \cdot u^2$$

$$\beta = \frac{\int_0^R \rho \cdot u^2 \cdot dA}{\rho \cdot A V^2}$$

$$\boxed{\beta = \frac{1}{A v^2} \int_0^R u^2 \cdot dA}$$ **

• Laminar flow through pipe

$$u = u_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$\beta = \frac{2}{R^2 v^2} \int_0^R u^2_{max} \left(1 - \frac{r^2}{R^2}\right)^2 \cdot r \cdot dr.$$

$$\beta = \frac{2}{R^2} \cdot 2^2 \int_0^R \left[1 + \frac{r^4}{R^4} - \frac{2r^2}{R^2}\right] r\, dr.$$

$$\beta = \frac{8}{R^2} \int_0^R \left[r + \frac{r^5}{R^4} - \frac{2r^3}{R^2}\right] dr$$

$$= \frac{8}{R^2}\left[\frac{r^2}{2} + \frac{r^6}{6R^4} - \frac{2 r^4}{4 R^2}\right]_0^R$$

$$= \frac{8}{R^2} \cdot \left[\frac{R^2}{2} + \frac{R^6}{6R^4} - \frac{R^4}{2R^2}\right]$$

$$= \frac{8}{R^2} \times R^2 \left[\frac{1}{2} + \frac{1}{6} - \frac{1}{2}\right]$$

$$\beta = \frac{8}{6} \Rightarrow \boxed{\beta = 1.33}$$

2. Kinetic Energy Correction factor(α) :-

$$d\,\dot{KE}_{avg} = \frac{1}{2} m \cdot v^2$$

$$= \frac{1}{2} \rho \cdot Q \cdot v^2$$

$$d\,\dot{KE}_{avg} = \frac{1}{2} \cdot \rho \cdot A \cdot v^3 \quad \text{——— (1)}$$

$$d\,\dot{KE}_{act} = \frac{1}{2} m \cdot u^2$$

$$= \frac{1}{2} \cdot \rho \cdot Q\, u^2$$

$$d\,\dot{KE}_{act} = \int \frac{1}{2} \rho \cdot A u^3 \cdot dA \quad \text{——— (2)}$$

- Kinetic Energy correction factor is the ratio of actual rate of K.E. based on actual velocity to the rate of K.E based on avg velocity.

$$\dot{KE}_{act} = \alpha\, \dot{KE}_{avg}$$

$$\boxed{\alpha = \frac{\dot{KE}_{act}}{\dot{KE}_{avg}}}$$

$$\alpha = \frac{\int \frac{1}{2} \cdot \rho \cdot u^3 \cdot dA}{\frac{1}{2} \rho \cdot v^2 \cdot dA}$$

$$\boxed{\alpha = \frac{1}{Av^3} \int u^3 \cdot dA}$$

$$\boxed{\alpha = 2}$$

$$\boxed{\text{Laminar flow}}$$

$$\boxed{\beta = 1.33} \qquad \boxed{\alpha = 2}$$

CH-6: TURBULENT flow

Dt- 8 Apr- 2023

- Darcy weisbach eqⁿ :-

$$\boxed{h_L = \frac{\Delta P}{\rho g} = \frac{f L V^2}{2gd} = \frac{f L Q^2}{12.1\, d^5}}$$

- Shear Velocity :-

$$\boxed{V^* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f}{8}} \cdot V_{avg}}$$

- Shear Stress :-

$$\boxed{\tau_0 = \frac{\Delta P}{L} \times \frac{R}{2}}$$

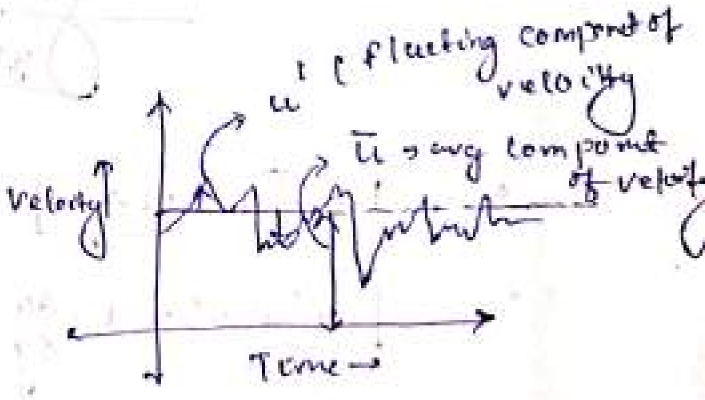
- $R_e > 4000$ [Turbulent flow]

$R_e = \frac{\text{Inertia force}}{\text{Viscous force}}$

- Velocity ↑ ⇒ Laminar flow ⟶ Turbulent flow.
- Because of mixing of fluid, Turbulent flow is a unsteady flow.

$\bar{u}$ → average component of velocity.

u' → flucting component of velocity



$$\boxed{u = \bar{u} + u'}$$ **

* Shear stress :-

$$\boxed{\tau = \tau_{Laminar} + \tau_{Turbulent}}$$ **

$$\boxed{\tau = \mu \frac{du}{dy} + \eta \frac{du}{dy}}$$ → J. Boussinesq Eqⁿ.

$\mu \rightarrow$ Dynamic viscosity

$\eta \rightarrow$ Eddy viscosity

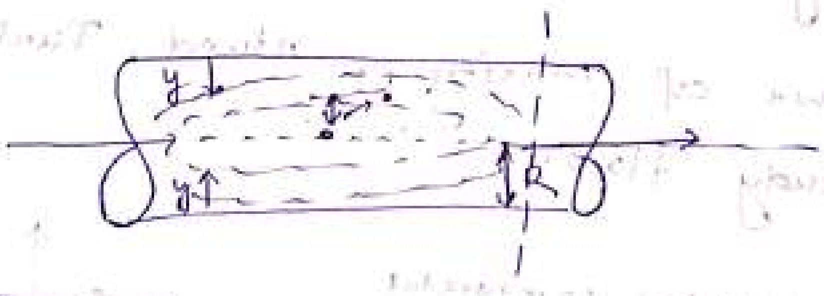
Reynold's Equation :—

$$\boxed{\tau = \rho \cdot u' \cdot v'}$$

where u' & v' are the fluctuating component of velocity in x and y direction.

PRANDTL MIXING LENGTH :—

- Distance between two layer in ~~the~~ ~~directio~~. transverse direction such that particle from one layer could reach another layer and mix ~~up~~ in another layer in such a way that momentum of fluid particle ~~a~~ ~~constant~~ ~~in~~ x will remain same in x-direction.



- $\Rightarrow$ $\boxed{l = 0.4y}$

- $\boxed{u' = v' = l \cdot \frac{du}{dy}}$

from Reylond's Eqⁿ

$$\tau = \rho \cdot u' \cdot v'$$

$$\boxed{\tau = \rho\, l^2 \cdot \left(\frac{du}{dy}\right)^2}$$

$$\frac{\tau}{\rho} = l^2 \left(\frac{du}{dy}\right)^2$$

take underroot both side

$$\sqrt{\frac{\tau}{\rho}} = l \cdot \frac{du}{dy}$$

$$V^* = l \cdot \frac{du}{dy}$$

$$du = \frac{V^*}{0.4y} \cdot dy$$

$$\int du = \int 2.5\, V^* \cdot \frac{1}{y} \cdot dy$$

$$\boxed{u = 2.5\, V^* . \ln(y) + C}$$

at $y = R$, $u = u_{max}$ (Boundary Cond^n)

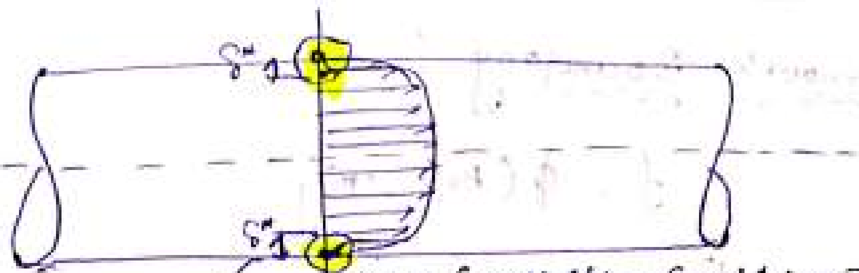
$$u_{max} = 2.5\, V^* \ln(R) + C$$

$$C = u_{max} - 2.5\, V^* \ln(R)$$

$$u = 2.5\, v^* . \ln(y) + u_{max} - 2.5\, v^* \ln(R)$$

$$u - u_{max} = 2.5\, v^* \left[\ln y - \ln R\right]$$

**
$$\boxed{\frac{u - u_{max}}{v^*} = 2.5\, v^* \ln\left(\frac{y}{R}\right)}$$
[Logarithmic Velocity profile]



$u = 0$ (No slip Condition)

Laminar sub-layer Thickness (δ^*)

**
$$\boxed{\delta^* = \frac{11.6\, \nu}{V^*}}$$
[ν: kinematic viscosity
V^*: shear velocity.]

* Turbulent flow velocity profile ...
Laminar flow.

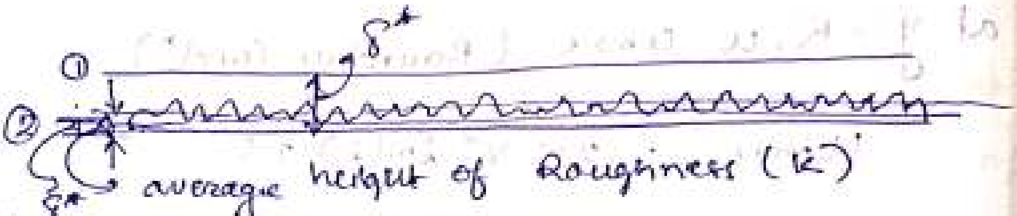
* ~~Laminar~~ $\boxed{U_{max}|_L > U_{max}|_T}$

* $\boxed{h_L|_T > h_L|_L}$

* $\boxed{\Delta P|_T > \Delta P|_L}$

* $\boxed{\beta = 1.2,\ \alpha = 4/3 = 1.33}$

Hydrodynamic Smooth & Rough Surface :-



* Smooth boundary/surface

* $\delta^* > K$

* $\boxed{\frac{K}{\delta^*} \leq 0.25}$

* Rough boundary/surface.

* $\delta^* < K$

$\boxed{\frac{K}{\delta^*} > 6}$

* Smooth Boundary :-

$f = \phi(R_e)$ only

$$\boxed{f = \frac{0.316}{R_e^{1/4}} \text{ or } \frac{0.316}{R^{0.25}}}$$

* Rough Boundary :-

$$f = \phi(K) \text{ only}$$

$\because$ R = Radius of pipe

$$\boxed{\frac{1}{\sqrt{f}} = 2.\log_{10}\left(\frac{R}{K}\right) + 1.74}$$

* for turbulent flow,

$$\boxed{\frac{U_{max}}{V_{avg}} = 1.43\sqrt{f} + 1}$$

CH-4

FLOW THROUGH PIPE

LOSSES IN PIPE

Losses.

$\boxed{\text{Losses}}$

Major Losses (>95%)

- Due to friction

Minor Losses (<5%)

- Sudden expansion
- Sudden contraction
- Pipe bend
- ~~Wall~~ valve in a pipe

① Major Losses :-

- Darcy Weisbach Equation

$$\boxed{h_L = \frac{\Delta P}{\rho g} = \frac{f l v^2}{2 g d} = \frac{f l Q^2}{12.1 d^5}}$$

- Chezy's formula :-

$$v = c\sqrt{mi}$$

$$Q = A \cdot V$$

$$\boxed{Q = A \cdot C\sqrt{mi}}$$

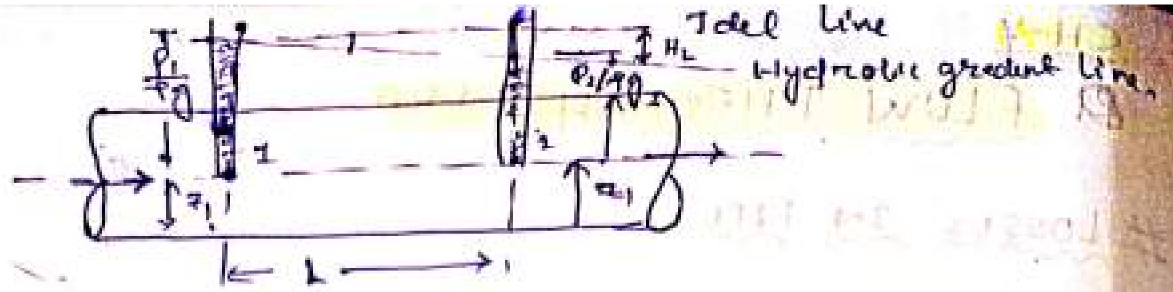
where

A - Cross section area of flow.

C → Chezy's Constant

$m = \frac{A}{P}$ (Hydraulic Mean depth, hydraulic radius) } A - Area of C/S, P → Parimeter.

i = Hydraulic slop.



- $\boxed{i = \frac{h_L}{L}}$

$$\boxed{E_1 = E_2 + h_L}$$

- $m = \frac{A}{P} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4}$

from chezy's formula

$$V = C\sqrt{mi}$$

$$V^2 = c^2\, m.i$$

$$V^2 = c^2 \frac{D}{4} \cdot \frac{h_L}{L}$$

$$\cancel{V^2} = c^2 \frac{\cancel{D}}{4} \cdot \frac{1}{\cancel{L}} \times \frac{f\cancel{L}\cancel{V^2}}{2g\cancel{D}}$$

$$1 = \frac{c^2}{4} \times \frac{f}{2g}$$

$$c^2 = \frac{8g}{f}$$

$$\boxed{c = \sqrt{\frac{8g}{f}}}$$ Chezy's Constant.

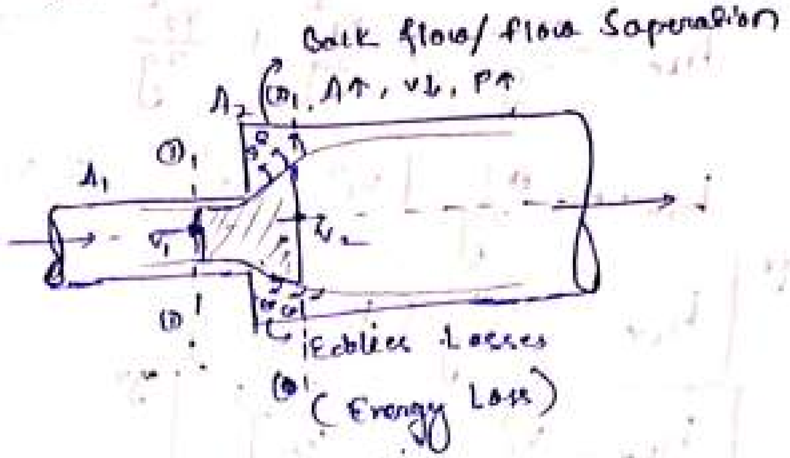
unit

$$C = \sqrt{L/s^2} = \sqrt{LT^{-2}} = [M^0 L^{1/2} T^{-1}]$$

(dimensional formula of of chezy's formula)

2. MINOR LOSSES :-

A. Sudden Expansion Loss :-



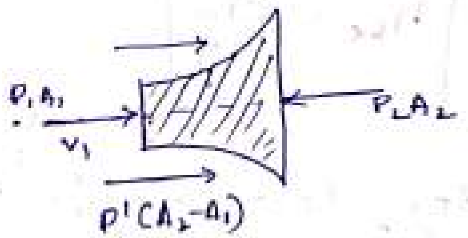
* Bernoulli's Eqⁿ

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + \cancel{z_2} + h_{Le}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g} + h_{Le} \quad \text{——— (1)}$$

* Linear Momentum Eqⁿ :-

$$\Sigma f_x = m^{\circ}(V_f - v_i)$$



$$\Rightarrow P_1A_1 + P_2(A_2 - A_1) - P_2A_2 = \rho Q(v_2 - v_1) \qquad \because \boxed{P' = P_2}$$

$$\cancel{P_1A_1} + \cancel{P_2A_2} - P_2A_1 - \cancel{P_2A_2} = \rho Q(v_2 - v_1)$$

$$A_2(P_1 - P_2) = \rho A_2 v_2 (v_2 - v_1) \qquad \boxed{\because Q = A_1v_1 = A_2v_2}$$

$$P_1 - P_2 = \rho v_2(v_2 - v_1) \quad \text{—— (2)}$$

from eqⁿ (1)

$$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g} + h_{Le}$$

$$\frac{\cancel{\rho} v_2(v_2 - v_1)}{\cancel{\rho} g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} + h_{Le}$$

$$\frac{v_2^2}{g} - \frac{v_2^2}{g} + \frac{v_2^2}{2g} + \frac{v_1^2}{2g} = h_{Le}$$

$$h_{Le} = -\frac{v_1 v_2}{g} + \frac{v_1^2}{2g} + \frac{v_2^2}{2g}$$

$$h_{Le} = \frac{1}{2g}\left[v_1^2 + v_2^2 - 2 v_1 v_2\right]$$

Imp **

$$\boxed{h_{Le} = \frac{1}{2g}(v_1 - v_2)^2}$$

$$\boxed{h_{Le} = \frac{v_1^2}{2g}\left[1 - \frac{v_2}{v_1}\right]^2}$$

$v_1 A_1 = v_2 A_2$

$\therefore \left[\frac{v_1}{v_2} = \frac{A_2}{A_1}\right]$

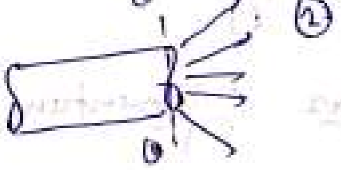
**

$$\boxed{h_{Le} = \frac{v_1^2}{2g}\left[1 - \frac{A_1}{A_2}\right]^2}$$

Case-I

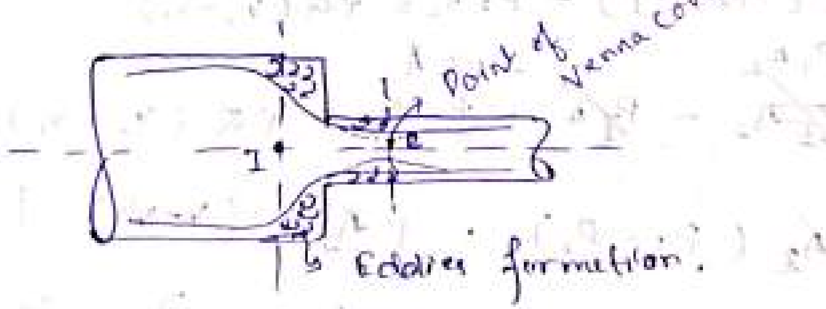
fluid exit from pipe.

$$\boxed{h_{Le} = \frac{v_1^2}{2g}}$$



$A_2 \to \infty$

B. Sudden Contraction :-



$$\boxed{h_{Lc} = \frac{(v_1 - v_2)^2}{2g}}$$

$\therefore \left\{ C_c = \frac{A_c}{A_1} \right.$

$$h_{Lc} = \frac{(v_c - v_1)^2}{2g}$$

$A_1 v_1 = A_c v_c$

$$h_{Lc} = \frac{v_1^2}{2g}\left[\frac{v_c}{v_1} - 1\right]^2$$

$\frac{v_c}{v_1} = \frac{A_1}{A_c}$

$$\boxed{h_{Lc} = \frac{V_1^2}{2g}\left(\frac{A_1}{A_c} - 1\right)^2}$$

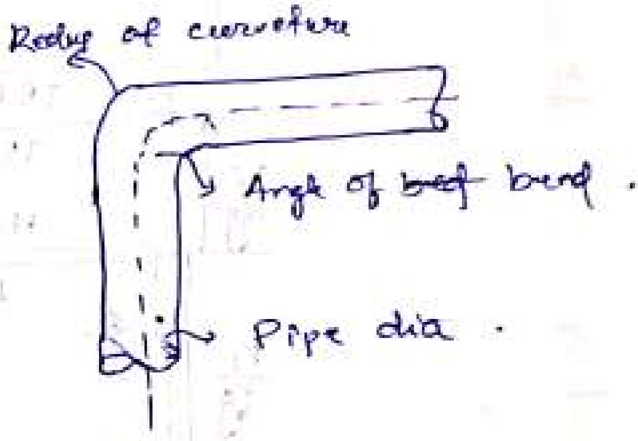
$$\boxed{h_{Lc} = \frac{V_1^2}{2g}\left(\frac{1}{C_c} - 1\right)^2}$$

If C_c is not given, $\left(\frac{1}{C_c} - 1\right)^2 = 0.5$

$$\boxed{h_{Lc} = 0.5 \cdot \frac{V_1^2}{2g}}$$

C. Bend Pipe :-

$$\boxed{h_{Lb} = K_b \cdot \frac{V_1^2}{2g}}$$



K_b = Co-efficient of bend

- It depends on:

a) Pipe diameter

b) Radius of curvature.

c) Angle of bend.

Energy Lines:-

① **Hydraulic Gradient Line (H.G.L):-**

The line joining piezometric head is called Hydraulic Gradient Line (H.G.L)

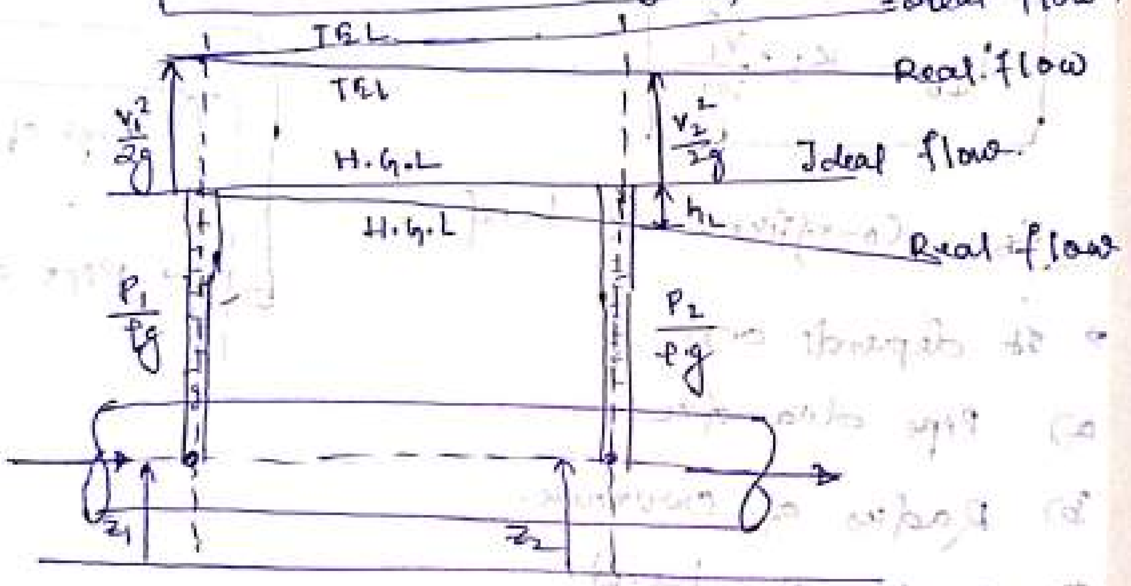
② **Total Energy Line (T.E.L)**

The line of joining total energy head is called T.E.L.

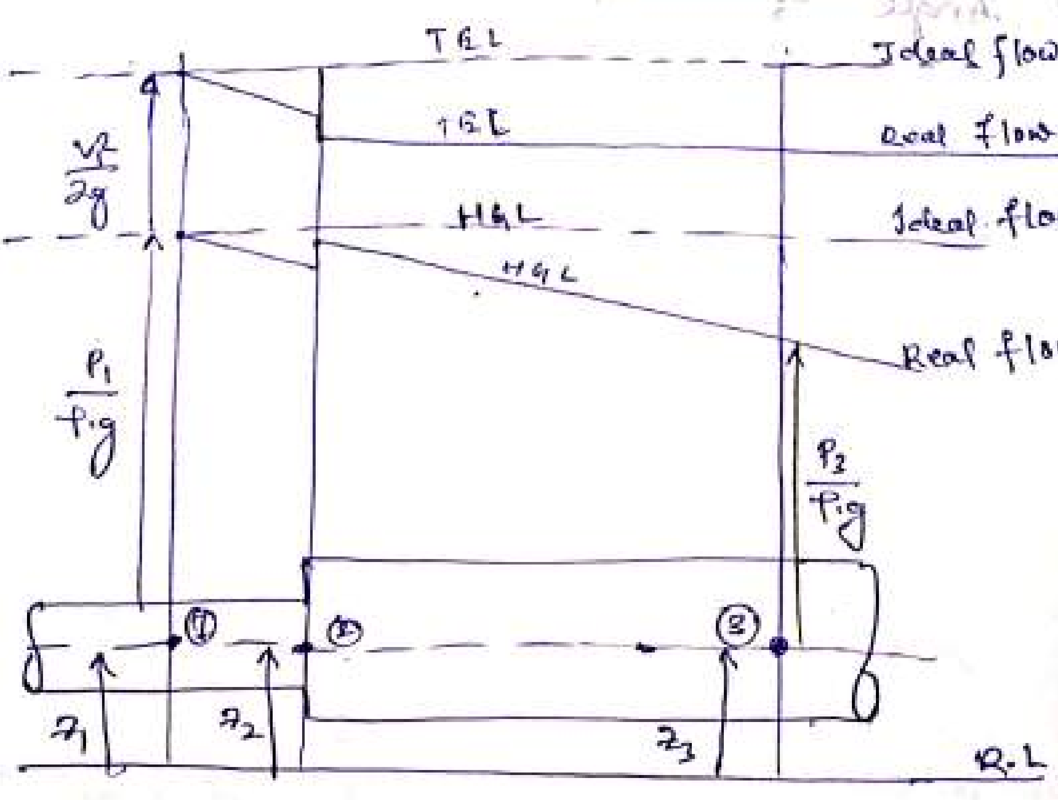
$$\boxed{H.G.L = z + \frac{p}{\rho g}}$$

$$\boxed{T.E.L = z + \frac{p}{\rho g} + \frac{v^2}{2g}}$$

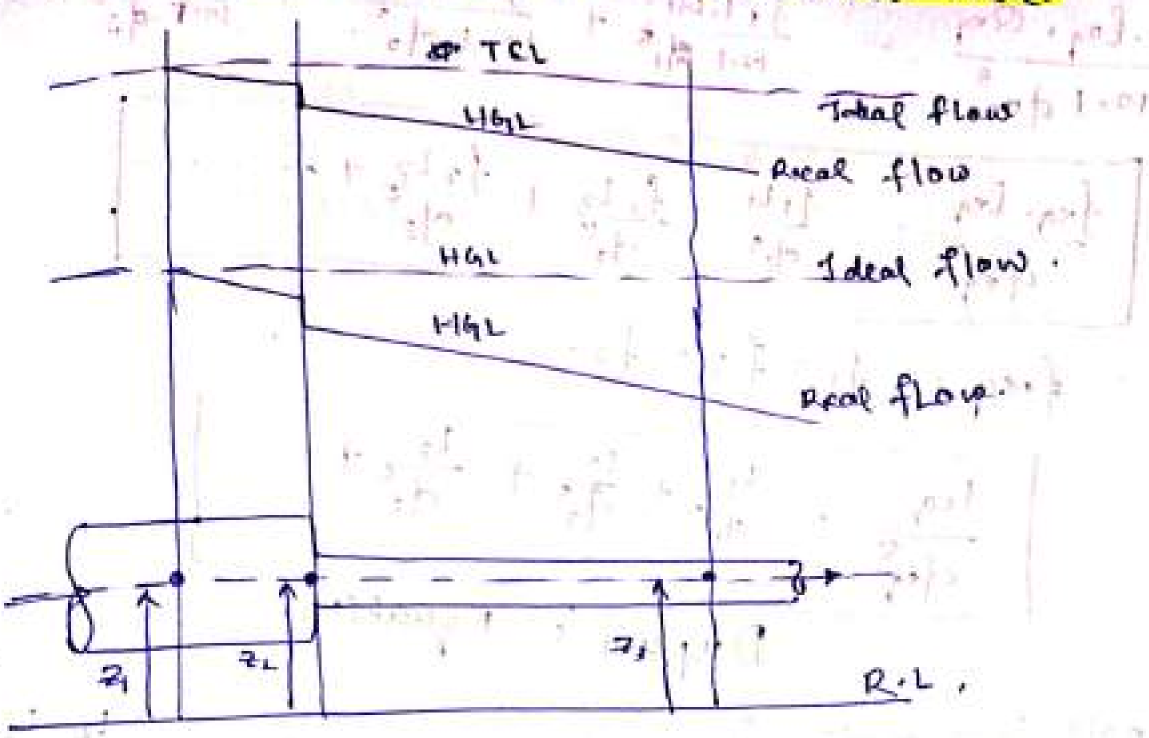
Case-1



Case-2



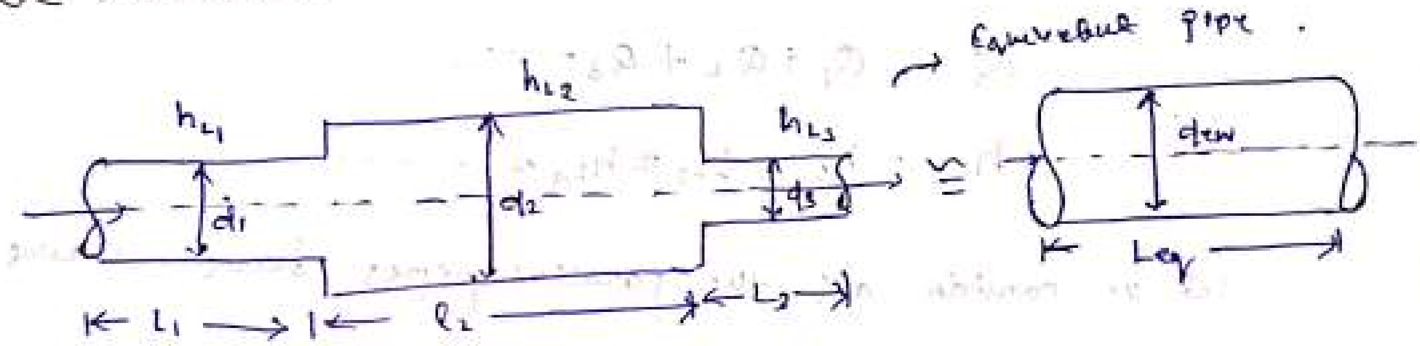
H.G.L can rise but T.E.L can never raise.



Date- 11/04/2023

Multy – Pipe Connection System:–

1] Serise Connection:–



$Q = Q_1 = Q_2 = Q_3 \ldots\ldots$, $\boxed{Q = \text{Constant}}$

$$h_L = \frac{fLV^2}{2gd}$$

$$h_{LT} = h_{L_1} + h_{L_2} + h_{L_3} + \ldots\ldots$$

Compair both the cases

$$h_L /_{\text{Equivalent pipe}} = h_{L_1} + h_{L_2} + h_{L_3} + \ldots$$

$$\frac{f_{eq} \cdot l_{eq} \cdot Q_{eq}^2}{12.1\, d_{eq}^5} = \frac{f_1 L_1 Q_1^2}{12.1\, d_1^5} + \frac{f_2 \cdot l_2 \cdot Q_2^2}{12.1\, d_2^5} + \frac{f_3 l_3 Q_3^2}{12.1\, d_3^5} + \dots$$

$$\boxed{\frac{f_{eq} \cdot l_{eq}}{d_{eq}^5} = \frac{f_1 l_1}{d_1^5} + \frac{f_2 l_2}{d_2^5} + \frac{f_3 l_3}{d_3^5} + \dots}$$

If $f_{eq} = f_1 = f_2 = f_3 \dots\dots$

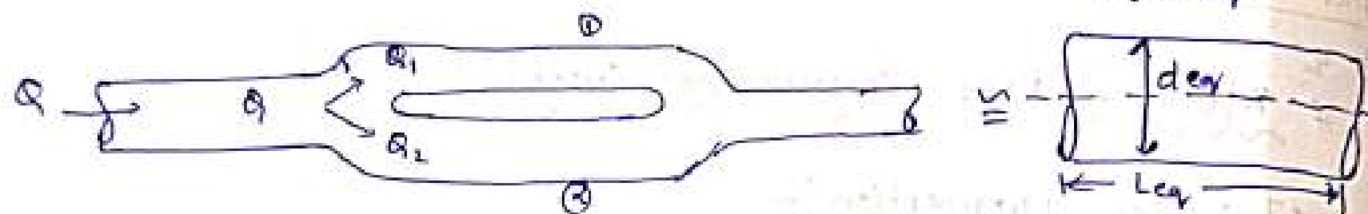
$$\boxed{\frac{l_{eq}}{d_{eq}^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5} + \dots}$$

Dupuit's Equation.

2. Parallel Combination :-

$h_L = h_{Leq}$

$Q = Q_{eq}$



$$Q = Q_1 + Q_2 + Q_3 \dots$$

$$h_{L_1} = h_{L_2} = h_{L_3} = h_{L_4} = \dots\dots$$

Let us consider all the parallel pipes have same d, L & f

$$Q_1 = Q_2 = Q_3 = \dots = \frac{Q}{n}$$ n = no. of pipe

$\boxed{l_1 = l_2 = l_3 = L}$ $\boxed{d_1 = d_2 = d_3 = d}$

$$h_{Leq} = h_{L_1}$$

$$\cancel{\frac{f_1 Q_{eq}^2}{12.1}} \quad \frac{\cancel{f_{eq}} \cdot l_{eq} \cdot Q_{eq}^2}{12.1\, d_{eq}^5} = \frac{\cancel{f_1} L_1 Q_1^2}{12.1\, d_1^5}$$

$$\frac{l_{eq}\, Q_{eq}^2}{d_{eq}^5} = \frac{l \cdot \left(\frac{Q}{n}\right)^2}{d^5}$$

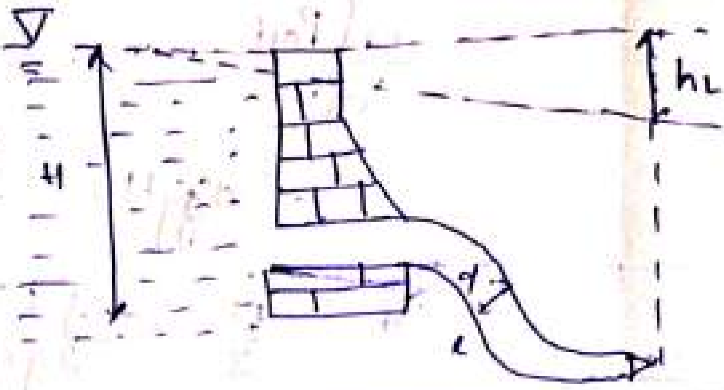
$$\boxed{\frac{L_{eq}}{d_{eq}^5} = \frac{l}{n^2 d^5}}$$

POWER TRANSMISSION THROUGH PIPE :-

$$Power = \frac{Energy}{t}$$

$$= \frac{m \cdot g \cdot H}{t}$$

$$= \dot{m} \cdot g \cdot H$$



$$\boxed{P_{th} = \rho Q \cdot g H}$$

$$\boxed{P_{act} = \rho Q g (H - h_L)}$$

$$\boxed{\dot{m} = \frac{M}{t} = \frac{\rho \cdot V}{t} = \rho \cdot Q}$$

$$\boxed{P_{Loss} = \rho \cdot Q g h_L}$$

$$\boxed{h_L = \frac{f l v^2}{2gd}}$$

Pipe Efficiency :-

$$\eta_p = \frac{P_{act}}{P_{th}} =$$

$$\eta_p = \frac{\rho Q \cdot g (H - h_L)}{\rho \cdot Q \cdot g \cdot H}$$

$$\boxed{\% \eta_p = \left[1 - \frac{h_L}{H}\right] \times 100}$$ **

$$\cancel{\eta_p = 1 - \frac{f l v^2}{2}}$$

$$P_{act} = \rho \cdot Q \cdot g H - \rho \cdot Q \cdot g h_L$$

$$P_{act} = \rho \cdot Q g H - \rho \cdot Q g \cdot \frac{f l v^2}{2gd}$$

$$P_{act} = \rho A v \cdot g \cdot H - \frac{\rho f l A v^3}{2d}$$

for maximum power transmission.

$$\frac{dP_{act}}{dv} = \rho.A.gH - 3v^2.\frac{\rho.f l A}{2g} = 0$$

$$\rho.A.gH = \frac{3v^2.\rho.f.lA}{2g}$$

$$\frac{H}{3} = \frac{f l v^2}{2gd}$$

$$\frac{H}{3} = h_L$$

$$\boxed{h_L = \frac{H}{3}}$$

$$\% \text{ of Maximum } \eta = \left(1 - \frac{h_L}{H}\right) \times 100$$

$$\%\eta_{max} = \left(1 - \frac{\cancel{H}}{3\cancel{H}}\right) \times 100$$

$$\boxed{\%\eta_{max} = 66.67\%}$$

- To maintain flow, $\boxed{P = \rho.Qg(H + h_L)}$

Actual maximum

$$P_{act} = \rho.Qg(H - h_L)$$

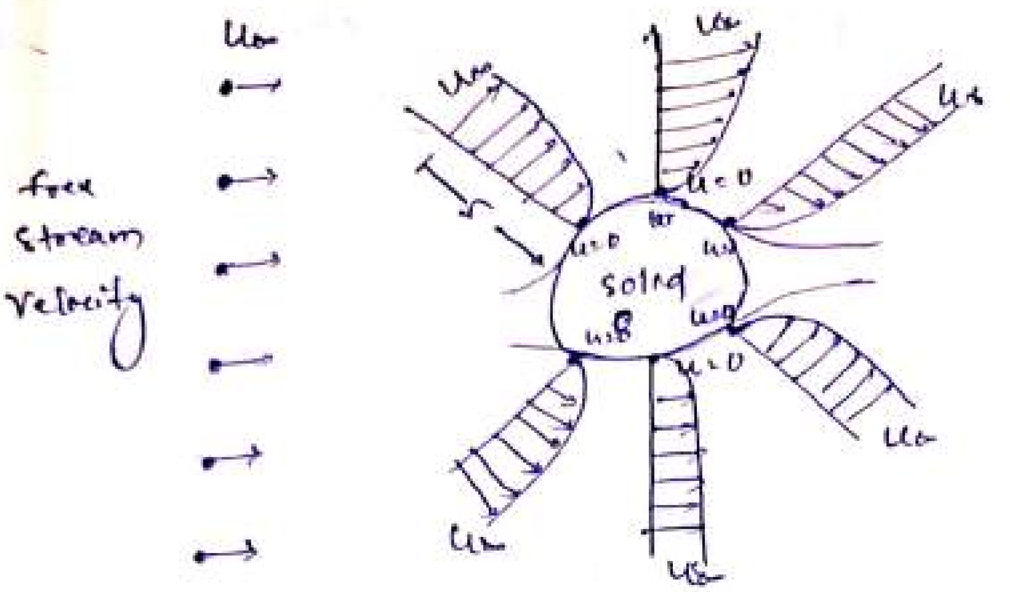
$$P_{act}/_{max} = \rho.gQ\left(H - \frac{H}{3}\right)$$

**

$$\boxed{P_{act}/_{max} = \frac{2}{3}\rho.gQH}$$

CH8 : BOUNDARY LAYER THEORY.

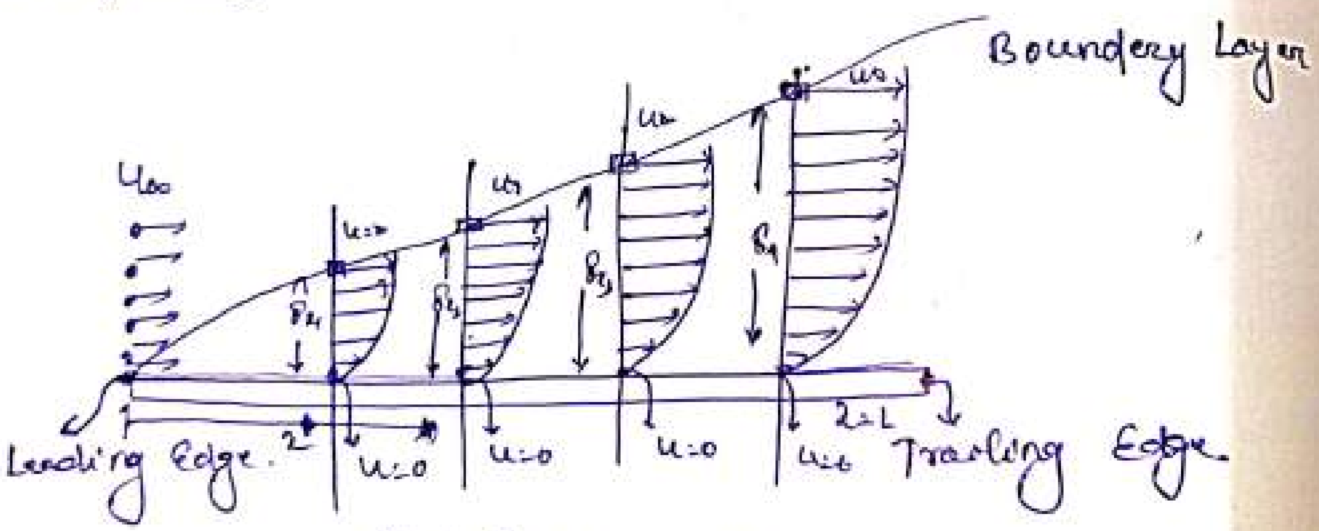
BOUNDARY LAYER FORMATION
Prandtl (1904)



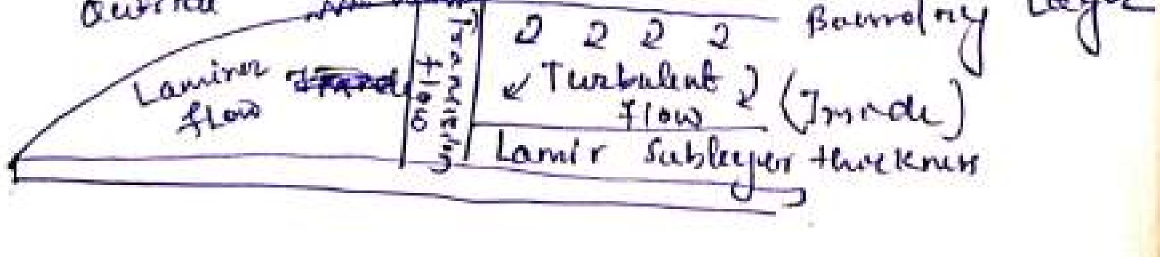
Near Boundary → $\frac{du}{dy}$ (maximum)

↓

Bernoulli's Equation not applicable ← $\tau \neq 0$ ← Relative Motion



$\delta_{x_1} < \delta_{x_2} < \delta_{x_3} < \delta_{x_4} \dots$



Boundary Layer

Inside B.L (Region)

$\frac{du}{dy} \neq 0$, $\tau \neq 0$

- viscous flow
- Bernoulli's Eq^n is not applicable.

Out side (B.L Region)

$\frac{du}{dy} = 0$, $\tau = 0$

- flow. Inviscid
- Bernoulli's equation is applicable.

- $Re < 5 \times 10^5$ [laminar]
- $Re > 5 \times 10^5$ [Turbulent]

* Boundary Layer thickness :- (δ) :-

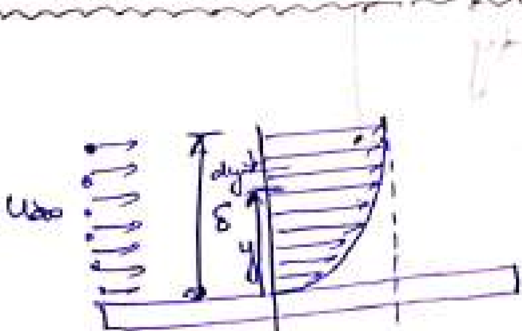
→ It is a distance from free surface in y-direction to the point where local velocity u will reach 99% of the free stream velocity (u_∞).

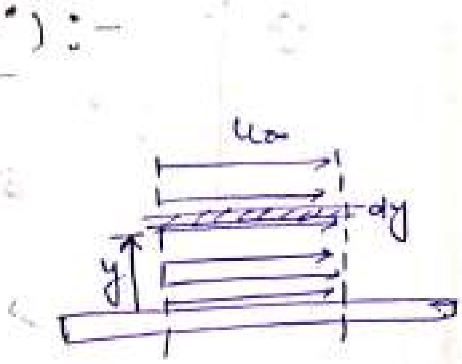
$$\boxed{u = 99\% \; u_\infty}$$

$$\boxed{\delta \propto \sqrt{x}}$$

Dt- 12/04/2023

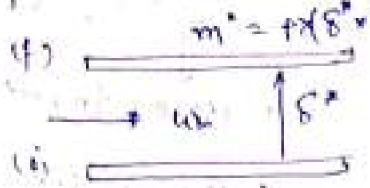
*1. DISPLACEMENT THICKNESS (δ^*) :-

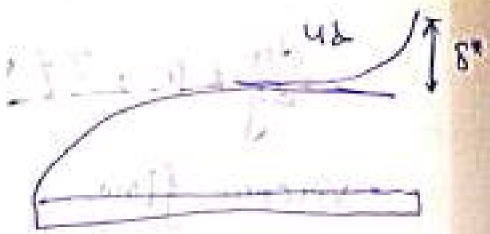




$\left\{ \begin{array}{l} \dot{m} = \rho . Q \\ \dot{m} = \rho (A) . v \;\; \text{velocity} \end{array} \right\}$

It is a distance, by which surface boundary should be displaced in y-direction in order to compensate the reduction in mass influx due to boundary layer formation.





* Mass flow rate with B.L.

$$\dot{m}_1 = \rho . A \times . V$$

$$\dot{m}_1 = \int_0^{\delta} \rho . (dy \times 1)\, u$$

* Mass flow rate without B.L

$$\dot{m}_2 = \rho . A\, u_\infty$$

$$\dot{m}_2 = \int_0^{\delta} \rho . (dy \times 1)\, u_\infty$$

* Reduction in mass flow rate $= \dot{m}_2 - \dot{m}_1$

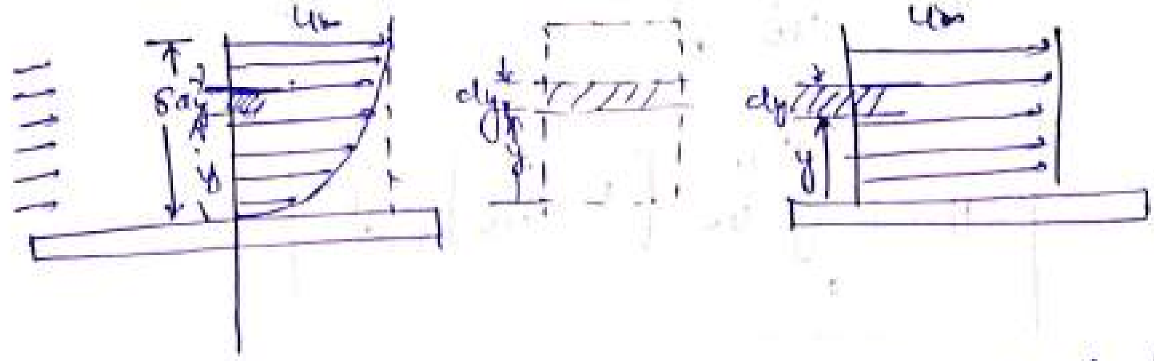
$$\rho . (\delta^* \times 1) \times u_\infty = \int_0^{\delta} \rho . (dy \times 1)\, u_\infty - \int_0^{\delta} \rho (dy \times 1)\, u$$

$$\cancel{\rho} (\delta^* \times 1) \times u_\infty = \cancel{\rho} \int_0^{\delta} (u_\infty - u)\, dy$$

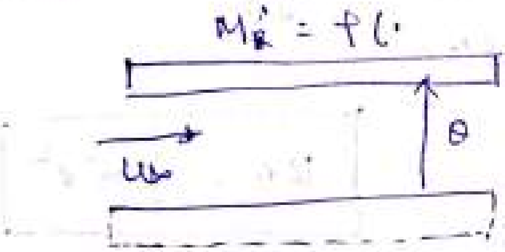
**

$$\boxed{\delta^* = \int_0^{\delta} \left[1 - \frac{u}{u_\infty}\right] dy}$$

(2) MOMENTUM THICKNESS (θ) :—



- It is the distance by which surface of boundary should be displaced in y-direction in order to compensate the reduction in momentum flux due to boundary layer formation.



$$dm = \int_0^\delta \rho \cdot dy\, u$$

$$\boxed{M^{\cdot} = m \times v}$$

momentum

Momentum flux with B.L

$$M_1^{\cdot} = dm^{\cdot} \times u$$

$$M_1^{\cdot} = \int_0^\delta \rho \cdot (dy \times 1)\, u^2$$

Momentum flux without B.L.

$$M_2^{\cdot} = \int_0^\delta \rho \cdot (dy \times 1) \cdot u \cdot U_\infty$$

Reduction in momentum flux = $M_2 - M_1$

$$\rho\,(\theta \times 1) \cdot U_\infty^2 = \int_0^\delta \rho \cdot (dy \times 1)\, u \cdot U_\infty - \int_0^\delta \rho \cdot (dy \times 1) u^2$$

$$\therefore \theta = \frac{1}{u_\infty} \int_0^{\delta} (u u_\infty - u^2)\, dy$$

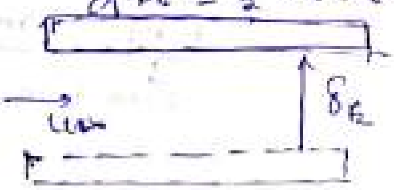
$$\boxed{\theta = \int_0^{\delta} \frac{u}{u_\infty}\left[1 - \frac{u}{u_\infty}\right] dy}$$

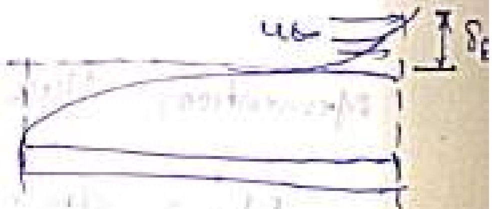
ENERGY THICKNESS (δ_E) :-

It is a distance, by which surface boundary should be displaced in y-direction in order to compensate the reduction in Energy flux due to the boundary layer formation.

$$\boxed{KE = \frac{1}{2} m v^2}$$

$$d\,KE' = \frac{1}{2}\left[\rho \cdot (\delta_E \times 1) u_\infty\right] u_\infty^2$$





Energy flux with boundary Layer

$$E_1 = \frac{1}{2}\int_0^{\delta}\left[\rho \cdot (dy \times 1) u\right] u^2$$

Energy flux without B.L.

$$E_2 = \frac{1}{2}\int_0^{\delta}\left[\rho \cdot (dy \times 1)\, u \cdot u_\infty\right] u_\infty^2$$

Reduction in Energy flux $= E_2 - E_1$

$$\frac{1}{2}\rho \cdot \delta_E\, u_\infty^3 = \frac{1}{2}\int_0^{\delta} \rho \cdot u \cdot u_\infty^2\, dy - \frac{1}{2}\int_0^{\delta} \rho \cdot u^3\, dy$$

$$\delta_E = \frac{1}{U_\infty^3}\left[\int_0^\delta u(U_\infty^2 - u^2)\,dy\right]$$

~~$\delta_E = \frac{u}{U_\infty}\int_0^\delta$~~

$$\boxed{\delta_E = \int_0^\delta \frac{u}{U_\infty}\left[1-\left(\frac{u}{U_\infty}\right)^2\right]dy}$$

~~Shape factor (H) = δ^*~~

$$\boxed{\text{Shape factor } (H) = \frac{\delta^*}{\theta}}$$

Von-Karman Momentum Integral Equation:-

Assumptions:-

① It is used for 2D co. steady incompressible flow.

② No pressure gradient in flow direction

③ Plate is parallel to the flow.

$$\boxed{\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx}}$$

Von-Karman Momentum Integral Eqn.

Where

τ_0 = Wall shear stress

θ = Momentum thickness

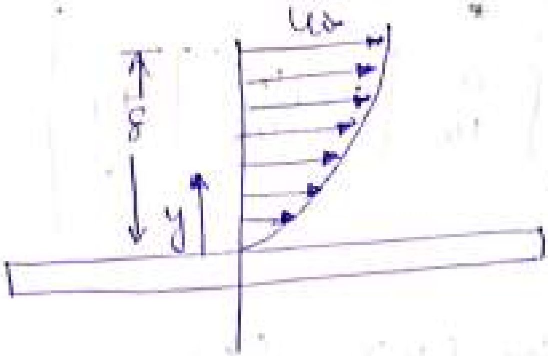
x = Distance from leading edge.

Note

* It is obtained by mass and momentum conservation.

Velocity Profile over a Flate Plate-

A] Laminar B.L. Region :- [$Re \leq 5\times10^5$]



- Boundary Condition :-

① $y = 0$
$u = 0$

② $y = \delta$
$u = u_\infty$

③ $y = \delta$
$\frac{du}{dy} = 0$

④ $y = 0$
$\frac{du}{dy} \Rightarrow$ nonlinear
$\frac{d^2u}{dy^2} = 0$

Let,

$$\boxed{u = A + By + Cy^2 + Dy^3}$$

① at $y = 0$, $u = 0$

$$0 = A + 0 + 0 + 0$$

$$\boxed{A = 0}$$

③ at $y = \delta$, $\frac{du}{dy} = 0$

$$\frac{du}{dy} = 0 + B + 2yC + 3y^2 D.$$

$$\cancel{B + 2yC + 3D\delta^2}$$

$$B + 2\delta C + 3\delta^2 D = 0 \quad \text{——— (1)}$$

(1) at $y=0$, $\frac{d^2u}{dy^2}=0$

$$\frac{d^2u}{dy^2} = 2C + 6Dy$$

$$0 = 2C + 0$$

$$\boxed{C = 0}$$

from eq^n - (1)

$$B + 3D\delta^2 = 0 \quad \text{——— (2)}$$

$$\boxed{u = By + Dy^3}$$

at $y=\delta$, $u = u_\infty$

$$u_\infty = B\delta + D\delta^3 \quad \text{——— (3)}$$

from eq^n - 2

$$B = -3D\delta^2$$

from eq^n - 3

$$u_\infty = (-3D\delta^2)\,\delta + D\delta^3$$

$$u_\infty = D[-2\delta^3]$$

$$\boxed{D = \frac{-u_\infty}{2\delta^3}}$$

$$\boxed{B = \frac{3u_\infty}{2\delta}}$$

$$u = A + By + cy^2 + Dy^3$$

$$u = 0 + \frac{3}{2}\frac{u_\infty}{\delta} \times y + 0 - \frac{u}{2\delta^3} y^3$$

$$\boxed{\frac{u}{u_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3}$$ Blasius Eq^n for Laminar flow.

$$\boxed{\frac{u}{u_\infty} = \frac{3}{2}\eta - \frac{1}{2}\eta^3}$$ $\frac{y}{\delta} = \eta$

↳ 3rd order non Linear Ordinary differential Equation.

putting value of $\frac{u}{u_\infty}$ in Reduction momentum flux eq^n.

$$\boxed{\theta = \frac{39}{280}\delta}$$

Boundary Layer Thickness :-

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \times \left.\frac{du}{dy}\right|_{y=0}$$

$$\tau = \mu \cdot \frac{3u_\infty}{2\delta} \quad \text{—(1)}$$

$$\left\{ u = u_\infty\left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]\right.$$

$$\frac{du}{dy} = u_\infty\left[\frac{3}{2\delta} - \frac{1}{2\delta^3}3y^2\right]$$

$$\left.\frac{du}{dy}\right|_{wall} = \frac{3u_\infty}{2\delta}$$

Compare eq^n (1) with Von-Karman.

$$\tau_0 = \rho . u_\infty^2 \times \frac{d\theta}{dx} \quad \text{—(2)}$$

from eq^n - (1) & (2).

$$\mu \times \frac{3u_\infty}{2\delta} = \rho . u_\infty^2 \times \frac{d}{dx}\left(\frac{39}{280}\delta\right)$$

$$\frac{3\mu}{2\delta} = \frac{39}{280}\rho\cdot\frac{d(\delta)}{dx}u_\infty$$

$$\delta\cdot d(\delta) = \frac{140}{13}\cdot\frac{\mu}{\rho u_\infty}\cdot dx$$

$$\int_0^{\delta x}\delta\cdot d\delta = \int_0^x\frac{140}{13}\cdot\frac{\mu}{\rho\cdot u_\infty}\cdot dx$$

$$\left[\frac{\delta^2}{2}\right]_0^{\delta x} = \frac{140}{13}\times\frac{\mu}{\rho\cdot u_\infty}[x]_0^x$$

$$\frac{\delta_x^2}{2} = \frac{140}{13}\frac{\mu x}{\rho\cdot u_\infty}\times\frac{x}{x}$$

Local Reynolds no.

$$\left[Re_x = \frac{\rho u_\infty\cdot x}{\mu}\right]$$

$$\delta_x = \frac{140\times 2}{13}\times\frac{x^2}{Re_x}$$

$$\delta_x = \sqrt{\frac{140\times 2}{13}\times\frac{x^2}{Re_x}}$$

$$\boxed{\delta_x = \frac{4.64\cdot x}{\sqrt{Re_x}}}$$

[Boundary Layer thickness Eq^n]

$$\delta_x = \frac{4.64x}{\sqrt{Re_x}}$$

$$\delta_x = \frac{4.64\cdot x}{\sqrt{\frac{\rho\cdot u_\infty x}{\mu}}}$$

$$\delta_x \propto \frac{x}{\sqrt{x}} = \sqrt{x}$$

$$\boxed{\delta_x \propto \sqrt{x}}$$

Problem Example:-

For a Laminar flow over a flat plate the thickness of boundary layer at a distance 'x' from leading edge is found to be 5 mm. the thickness of boundary layer at a distance from leading Edge which is double than previous — ?

Sol^n

$x_2 = 2x$

$x_1 = x$

$\delta_1 = 5\,mm$, $\delta_2 = ?$

$\delta \propto \sqrt{x}$

$\left.\begin{array}{l}\delta_1 \propto \sqrt{x_1} \\ \delta_2 \propto \sqrt{x_2}\end{array}\right] \frac{\delta_2}{\delta_1} = \sqrt{\frac{x_2}{x_1}}$

$\frac{\delta_2}{5} = \sqrt{\frac{2x}{x}}$

$\delta_2 = 5 \times \sqrt{2} = 5\sqrt{2}$ mm

Q.2 ~~wrt~~ Increa in flow velocity of fluid boundary layer thickness will.

(a) decrease

b) increase

c) no change

d) Increase then decrease.

Q.3 The given velocity profile

$$\boxed{\frac{u}{u_\infty} = \frac{y}{\delta}}$$

the ratio of displacement thickness to nominal thickness

Sol^n

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy$$

$$= \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

$$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta = \left[\delta - \frac{\delta^2}{2\delta}\right]$$

$$\delta^* = \left(\delta - \frac{\delta}{2}\right) = \frac{\delta}{2} \Rightarrow \boxed{\frac{\delta^*}{\delta} = \frac{1}{2}}$$

Ans

SHEAR STRESS (τ_0) :-

$$\boxed{\delta_x = \frac{4.64x}{\sqrt{Re_x}}}$$

$$\tau_w = \frac{3}{2}\frac{\mu U_\infty}{\delta}$$

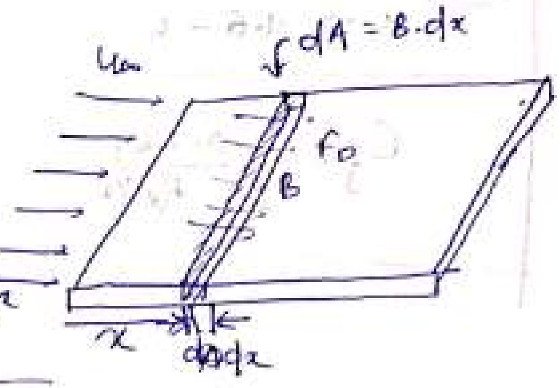
$$\tau_0 = \frac{3}{2}\,\mu \cdot U_\infty \frac{\sqrt{Re_x}}{4.64x}$$

$$\boxed{\tau = \frac{0.323 \cdot \mu U_\infty}{x} \cdot \sqrt{Re_x}} \;**$$

$$\tau \propto \frac{\sqrt{x}}{x}$$

$$\boxed{\tau \propto \frac{1}{\sqrt{x}}} \;**$$

Drag force :-



$$F_D = \int \tau_0 \times dA$$

$$F_D = \int_0^x \frac{0.323 \cdot \mu U_\infty}{x} \sqrt{Re_x}\, B\, dx$$

$$F_D = 0.323\, \mu U_\infty \cdot B \int_0^x \frac{\sqrt{\frac{\rho U_\infty x}{\mu}}}{x}\, dx$$

$$F_D = 0.323\, \mu U_\infty \cdot B \sqrt{\frac{\rho U_\infty}{\mu}} \int_0^x \frac{\sqrt{x}}{x} \cdot dx$$

$$F_D = 0.323 \cdot \mu U_\infty \cdot B \sqrt{\frac{\rho U_\infty}{\mu}} \int_0^x \frac{1}{\sqrt{x}} \cdot dx$$

$$F_D = 0.323\, \mu U_\infty \cdot B \sqrt{\frac{\rho \cdot U_\infty}{\mu}} \left[2\sqrt{x}\right]_0^x$$

$$F_D = 0.323\, \mu U_\infty \cdot B \sqrt{\frac{\rho \cdot U_\infty}{\mu}}\; 2\sqrt{x}$$

$$** \boxed{F_D = 0.646\, \mu U_\infty \cdot B \sqrt{Re_x}}$$

Turblent flow Velocity Profile :-

$$\boxed{\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7}} \rightarrow 1/7^{th} \text{ Power Law.}$$

* Local Drag co-efficient : $\boxed{C_{fx} = \frac{0.646}{\sqrt{Re_x}}}$

* If velocity profile - not given then we use emperical formulas.

For

Laminar flow	Turbulent flow
$\delta_x = \frac{Kx}{\sqrt{Re_x}}$ or $\frac{K \cdot x}{Re_x^{1/2}}$ where $K \rightarrow 4.6 - 5$	$\delta_x = \frac{0.376 \cdot x}{(Re_x)^{1/5}}$
$C_{fx} = \frac{0.664}{Re^{1/2}}$	$C_{fx} = \frac{0.059}{Re^{1/5}}$

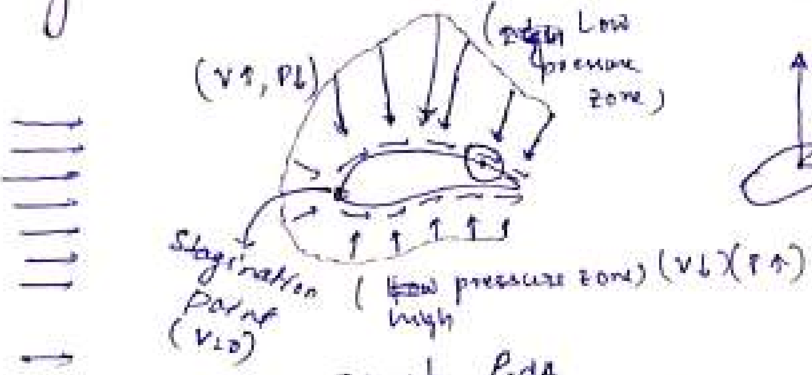
Dt. 13/04/2023

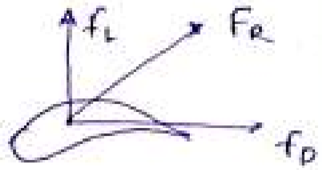
* **Drag force : (F_D) :-**

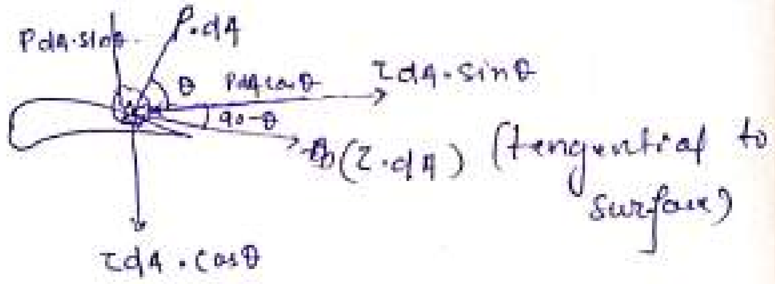
The force a flowing fluid exert on a body in the direction of flow is called drag force. (F_D).

* **Lift force (F_L) :-**

The force a ~~flowin~~ flowing fluid exert on the body in normal direction is called lift force. (F_L).







$$\boxed{F_D = \int_A \tau \cdot dA \cdot \sin\theta - P \cdot dA \cdot \cos\theta}$$

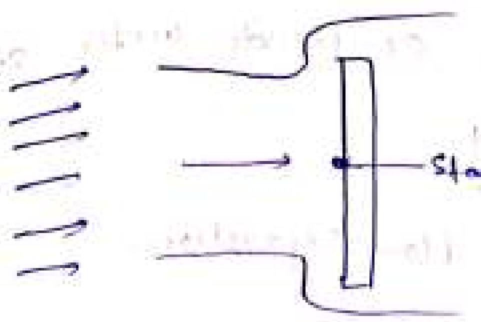
$$\boxed{F_L = \int_A - P \cdot dA \cdot \sin\theta - \tau \cdot dA \cdot \cos\theta}$$

Special Cases :-

Case-1 : Flat plate :-

{ Drag force is depending upon well shear stress

Case-1 Vertical flat plate



Drag force is depend on pressure force

* Drag and lift forces depend on:-

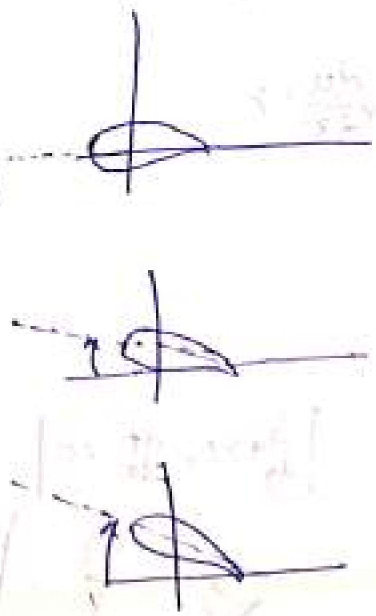
1. fluid density (ρ)
2. flow velocity (V)
3. Shape, size and ~~area~~ orientation of the body.

$$\boxed{C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot U_\infty^2\cdot A}} \qquad \boxed{C_L = \frac{F_L}{\frac{1}{2}\cdot\rho\cdot U_\infty^2\cdot A}}$$

↳ Drag co-efficient ↳ lift co-efficient.

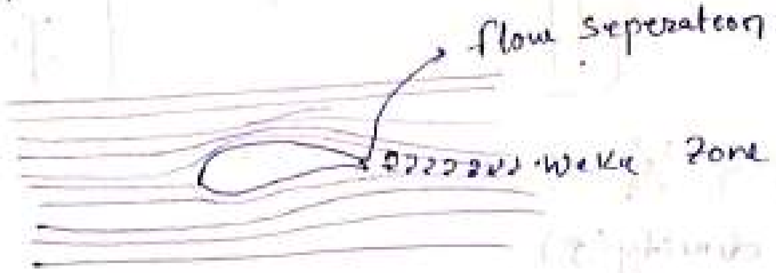
ANGLE OF ATTACK:-

→ If we are increasing angle of attack specially more then 15° then the lift force will reduce.



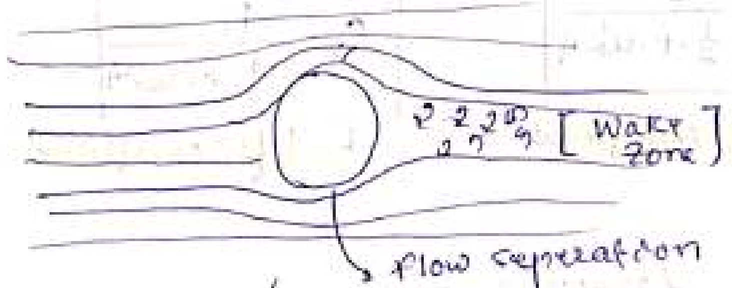
Reduction of Drag by Stream Lined Body :-

A body whose surface co-incide with stream line is called stream lined body.

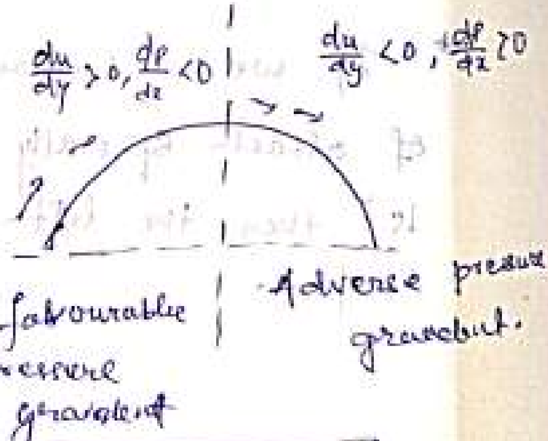


Bluff Body :-

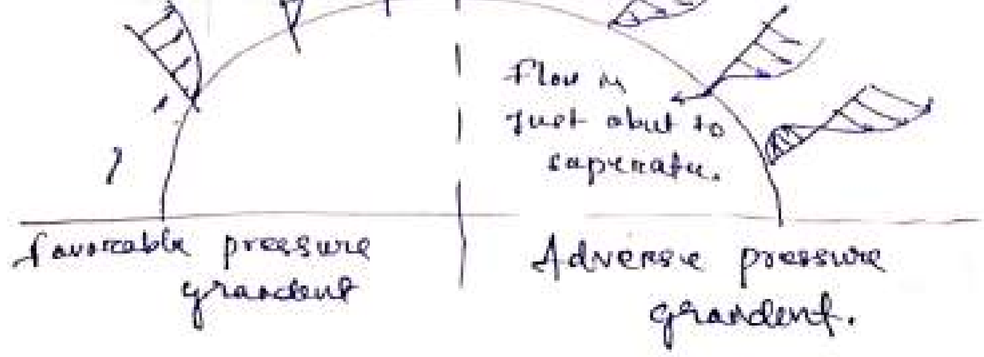
The body whose surface does not co-inside with Stream line is called Bluff Body.



Flow seperation :-



$\boxed{\frac{du}{dy} > 0,\ \frac{dP}{dx} < 0}$ $\boxed{\frac{du}{dy} < 0,\ \frac{dP}{dx} > 0}$



Drag on Sphere :—

$$\boxed{C_D = \frac{24}{Re}}$$

$C_D = \frac{24}{Re}$	$C_D = \frac{22.2}{Re}$
$C_D = \frac{20.4}{Re}$	$C_D = \frac{13.6}{Re}$

[Out of box data]

$$f_D = C_D \times \frac{1}{2} \rho u_\infty^2 \cdot A$$

$$f_D = \frac{24}{Re} \times \frac{1}{2} \cdot \rho \cdot u_\infty^2 \times \frac{\pi}{4} D^2$$

$$f_D = \frac{24\mu}{\rho \cdot u_\infty D} \times \frac{1}{2} \rho \cdot u_\infty^2 \times \frac{\pi}{4} \cdot D^2$$

$$\boxed{f_D = 3\pi \mu \cdot u_\infty \cdot D}$$

↳ Stoke's Law.

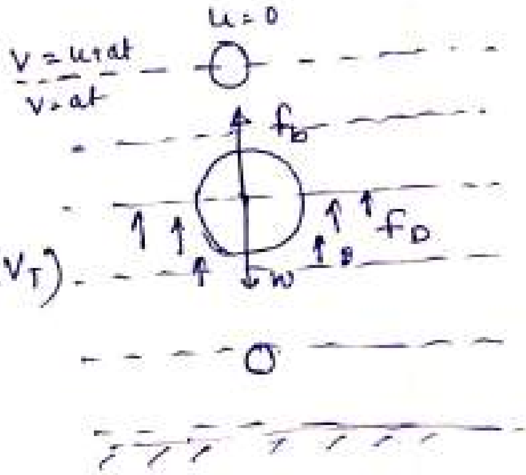
Terminal Velocity (V_T) :—

Equilibrium :

$$\boxed{F_D + f_b = W}$$

↳ V = constant

↳ ~~Terminal~~ Terminal velocity (V_T).



If we neglect buoyancy.

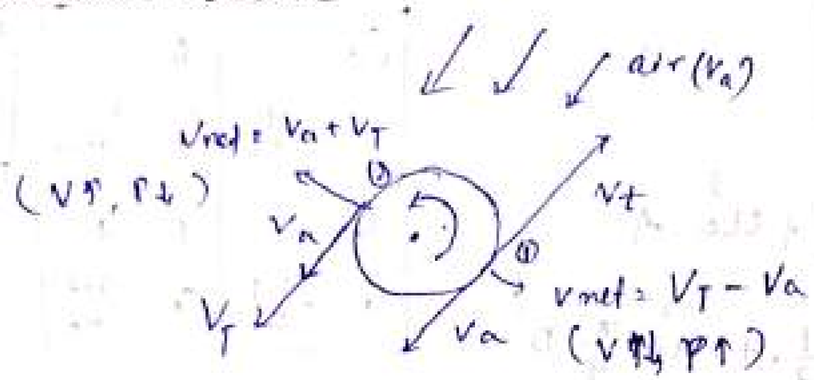
$$f_D = W$$

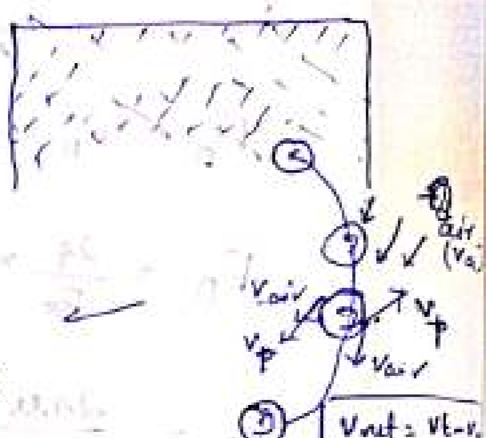
$$\boxed{3\pi \mu V_T \cdot D = m \cdot g}$$

$$\boxed{3\pi V_T \mu \cdot D = \rho_b \cdot V_b \cdot g}$$

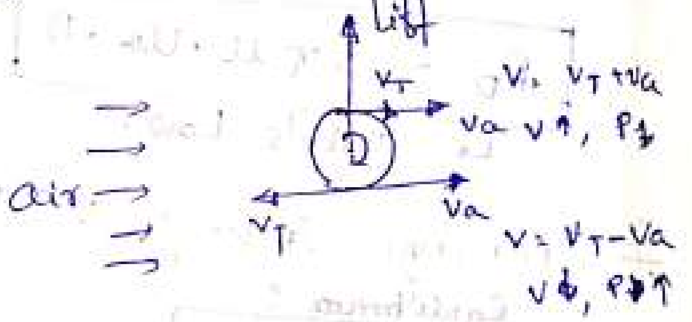
→ When water droplets balanced by all forces (f_D, f_b, W) and reach to terminal velocity. After that the fall of water drop let due to Newton 1st Law of motion.

Magnus Effect :—





* Production of lift due to rotation of body is called Magnus Effect.



CH-~~10~~ 10

NOTCH & WEIR

DT- 14/04/2023

→ It is used to measure discharge in an open channel.

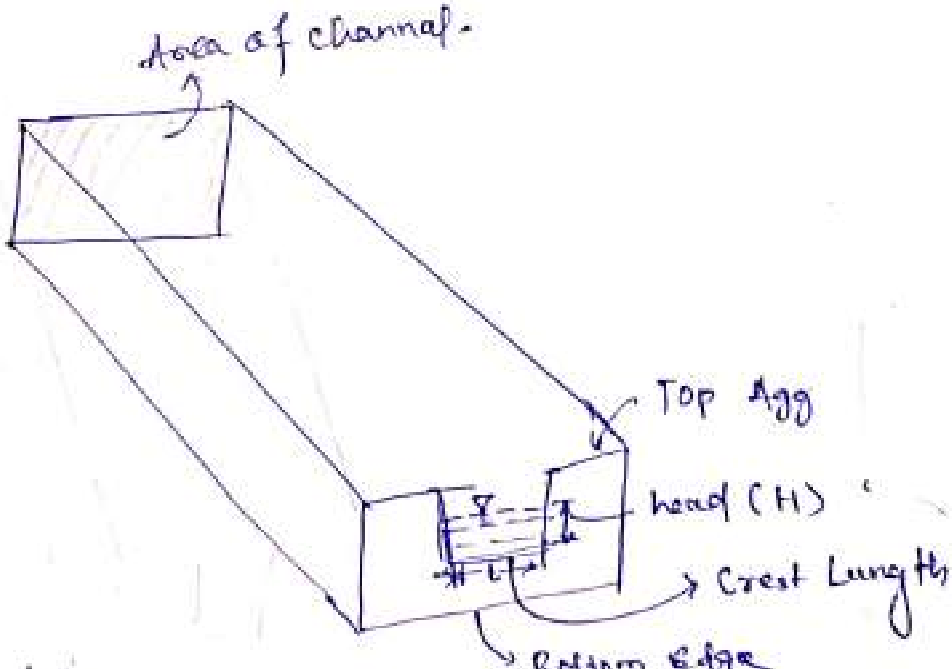
* Notch:-

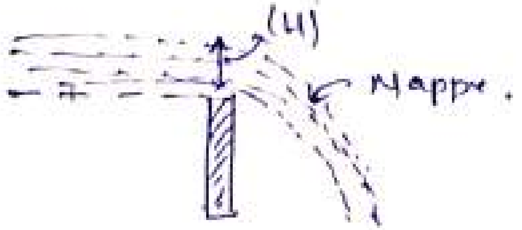
→ It is an opening in side of the tank in such a way that liquid surface in the channel is below the top Edge of opening.

→ It is used for measuring the rate of flow of a liquid through a small channel or tank.

* Weir:-

→ It is a concrete or masonry structure used to measure flow rate in a large channel.



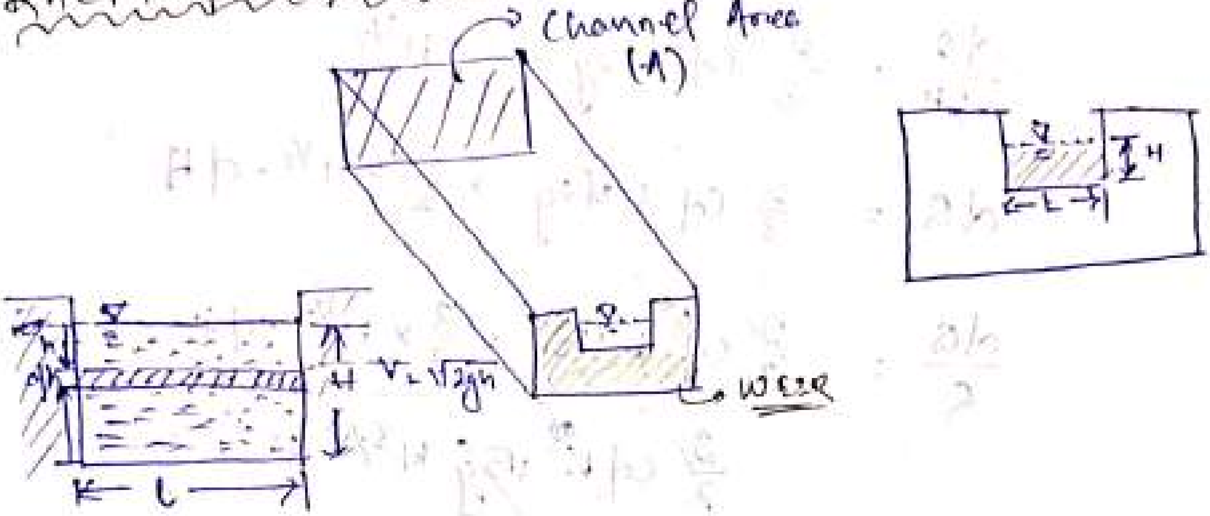


NAPPE :-

the layer of fluid flowing above the weir is called nappe/ Vain .

TYPE OF WEIR :-

1. RACTANGULAR WEIR :-



H = head of water above the crest

L = Length of the crest .

Q_{th} = Theoritical discharge.

Q_{act} = Actual discharge.

C_D = Co-efficent of discharge.

* from continuity Eq^n

$$\boxed{Q = A \cdot V}$$

$$dQ = dA \cdot V$$

$$dQ = \sqrt{2gh} \times L \times dh$$

$$Q = \int_0^H L \cdot \sqrt{2g} \cdot \sqrt{h} \cdot dh$$

$$Q = \sqrt{2g} \cdot L \left[\frac{H^{3/2}}{3/2} \right]_0^H$$

$$Q = \sqrt{2g} \cdot L \frac{2}{3} \left[H^{3/2} \right]$$

$$\boxed{Q_{th} = \frac{2}{3} \cdot L\sqrt{2g} \; H^{3/2}}$$ $$\boxed{Q_{act} = \frac{2}{3} C_D \cdot L \cdot \sqrt{2g} \cdot H^{3/2}}$$

* Error in discharge due to error in ~~xxx~~ measurement of head :

Differentiate Q wrt H.

$$\frac{dQ}{dH} = \frac{2}{3} \cdot C_d \cdot L \cdot \sqrt{2g} \times \frac{3}{2} H^{3/2 - 1}$$

$$\frac{dQ}{dH} = \frac{2}{3} C_d \sqrt{2g} \cdot \frac{3}{2} H^{1/2}$$

$$dQ = \frac{2}{3} C_d \, L \sqrt{2g} \times \frac{3}{2} \times H^{1/2} \cdot dH$$

$$\frac{dQ}{Q} = \frac{\cancel{\frac{2}{3}} \cancel{C_d} \cancel{L} \cancel{\sqrt{2g}} \times \frac{3}{2} \times H^{1/2} \cdot dH}{\cancel{\frac{2}{3}} \cancel{C_d} \cancel{L} \cancel{\sqrt{2g}} \, H^{3/2}}$$

$$\boxed{\frac{dQ}{Q} = \frac{3}{2} \cdot \frac{dH}{H}}$$

$$\boxed{\% \text{ Error in discharge} = \frac{dQ}{Q} \times 100}$$

1-2

VELOCITY OF APROACH :-

$$\boxed{\frac{V_a^2}{2g} = h_a}$$

$$\boxed{\text{Velocity of aproach } (V_a) = \frac{\text{Total discharge over notch}}{\text{Channal Area.}}}$$

→ Velocity of aproach may be defined as the velocity with which the fluid aproach or reach the weir before it passes over it.

$$Q_d = \int_{h_a}^{H + h_a} L \cdot dh \times \sqrt{2gh} \; \sqrt{2g} \times \sqrt{h} \times C_d.$$

$$\boxed{Q_{act} = \frac{2}{3} C_d \cdot L \cdot \sqrt{2g}\left[(H+h_a)^{3/2} + h_a^{3/2}\right]}$$

- So many scientist & researchers did lot of experiment and created experimental result for rectangular weir.

[A] Franci's formula/Eq^n :-

It is the one of the most commonly used formula for computing the discharge over sharp are narrow crested weir with or without end contraction.

- when velocity of aprach is not considerd :-

$$\boxed{Q = 1.84(L - 0.1nH)H^{3/2}}$$

n = no. of end contraction.

- when velocity of approch is considerd :-

$$\boxed{Q = 1.84(1 - 0.1nH)\left[H^{3/2} - h_a^{3/2}\right]}$$

[B] Bezin's formula :-

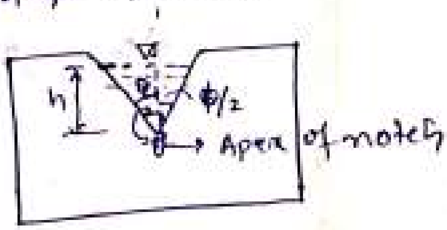
$$\boxed{Q = m\sqrt{2g} \cdot H^{3/2} \cdot L}$$

$$\boxed{m = 0.405 + \frac{0.003}{H}}$$

→ Bezin's Co-efficients.

TRIANGULAR NOTCH :-

θ → Angle of notch



$AB = AC + CB$

$AC = CB$

$\tan\frac{\theta}{2} = \frac{AC}{H-h}$

$AC = (H-h) \cdot \tan\frac{\theta}{2}$

$$dQ = dA \times v$$

$$dQ = A(x dh \times \sqrt{2gh}$$

$$dQ = (H-h)\tan\frac{\theta}{2} \cdot dh \sqrt{2g}\sqrt{h}$$

$$Q = \tan\frac{\theta}{2}\sqrt{2g}\int_0^H [H\sqrt{h} - h^{3/2}]\, dh.$$

$$Q = \tan\frac{\theta}{2}\sqrt{2g}\left[H \cdot \frac{h^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H.$$

$$Q = \tan\frac{\theta}{2} \cdot \sqrt{2g}\left[H \cdot \frac{H^{3/2}}{3/2} - \frac{H^{5/2}}{5/2}\right]$$

$$Q = \frac{4}{15}\tan\frac{\theta}{2} \cdot \sqrt{2g} \cdot H^{5/2}$$

$$Q_{th} = 2Q$$

$$\boxed{Q_{th} = \frac{8}{15}\tan\frac{\theta}{2} \cdot \sqrt{2g} \cdot H^{5/2}}$$

$$\boxed{Q_{act} = \frac{8}{15} \cdot C_d \cdot \tan\frac{\theta}{2} \cdot \sqrt{2g} \cdot H^{5/2}}$$

* Error in Discharge :-

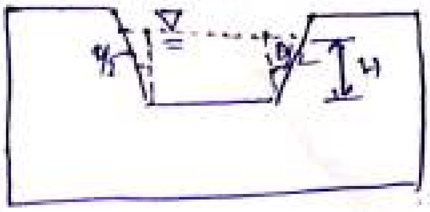
$$\frac{dQ}{dH} = \frac{8}{15} \cdot C_d \cdot \tan\frac{\theta}{2} \times \sqrt{2g} \cdot \frac{5}{2} H^{3/2}$$

$$\frac{dQ}{Q} = \frac{\cancel{\frac{8}{15}} \cdot \cancel{C_d} \cdot \cancel{\tan\frac{\theta}{2}} \cdot \cancel{\sqrt{2g}} \cdot \frac{5}{2} H^{3/2}}{\cancel{\frac{8}{15}} \times \cancel{\tan\frac{\theta}{2}} \cdot \cancel{C_d} \cdot \cancel{\sqrt{2g}} \cdot H^{5/2}}$$

$$\frac{dQ}{Q} = \quad \boxed{\frac{dQ}{Q} = \frac{5}{2} \cdot \frac{dH}{H}} \rightarrow \text{error in discharge.}$$

TRAPIZOIDAL WEIR :-

** $\boxed{Q_{Trapizoidal} = Q_{triangular} + Q_{rectangular}}$



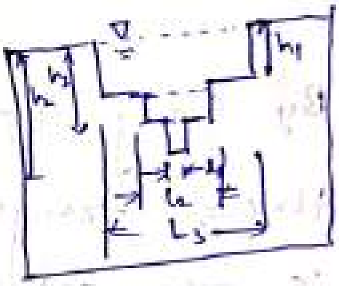
** $$\boxed{Q_{Trapizoidal} = \frac{8}{15} Cd_1 \cdot \tan\frac{\theta}{2} \cdot \sqrt{2g} \cdot H^{5/2} + \frac{2}{3} Cd_2 \sqrt{2g} \cdot L\, H^{3/2}}$$

DISCHARGE THROUGH STAPPED NOTCH :-

$$Q_1 = \frac{2}{3} Cd_1 \sqrt{2g} \cdot l_1 \cdot H_1^{3/2}$$

$$Q_2 = \frac{2}{3} \cdot Cd_2 \cdot \sqrt{2g} \cdot l_2 \left[H_2^{3/2} - H_1^{3/2} \right]$$

$$Q_3 = \frac{2}{3} Cd_3 \sqrt{2g} \cdot l_3 \cdot \left[H_3^{3/2} - H_2^{3/2} \right]$$



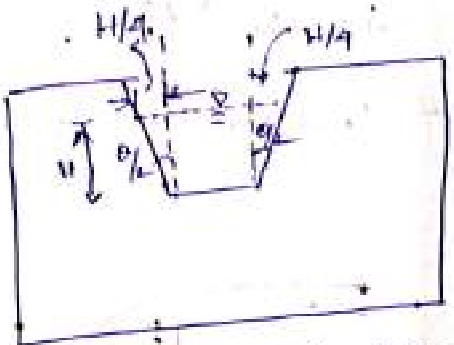
$$\boxed{Q_{stapped} = Q_1 + Q_2 + Q_3 \ldots\ldots}$$

CIPPOLLETTI WEIR :-

- It is a special type of trapizoidal weir having 1 horizontal and 4 vertical slop.

$$\tan\frac{\theta}{2} = \frac{H/4}{H}$$

** $\boxed{\tan\frac{\theta}{2} = \frac{1}{4}}$ $\boxed{\frac{\theta}{2} = 14.03}$



** Discharge through rectangular weir with 2 end contraction.

$$\boxed{Q_{Rec} = \frac{2}{3} Cd \sqrt{2g} \cdot (L - 0.2H)\, H^{3/2}}$$

$$Q_{net} = \frac{2}{3} \cdot C_d \sqrt{2g} \cdot L \cdot H^{3/2} - \underbrace{\frac{2}{3} \cdot C_d \cdot \sqrt{2g} \cdot 0.2\, H^{5/2}}_{\text{Loss}}$$

$$\therefore \quad \frac{2}{3} \cdot C_d \cdot \sqrt{2g} \cdot 0.2 \cdot H^{5/2} = \frac{8}{15} \cdot C_d \sqrt{2g} \cdot H^{5/2} \cdot \tan\frac{\theta}{2}$$

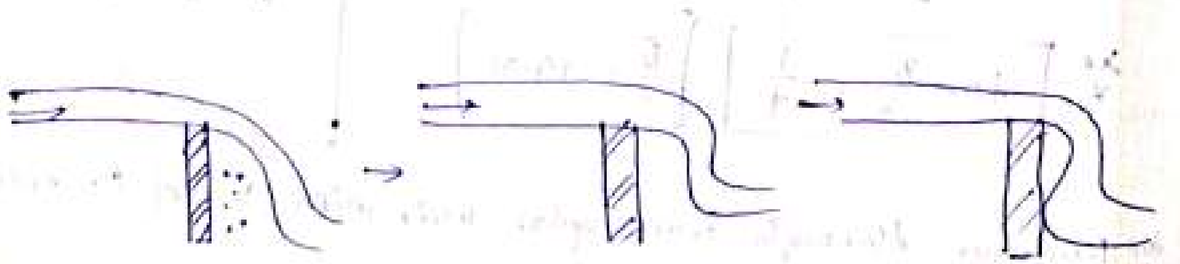
$$\frac{1}{30} = \frac{8}{15} \cdot \tan\theta/2$$

$$\boxed{\tan\frac{\theta}{2} = \frac{1}{4}}$$

→ By giving slop to the side and increase in discharge through triangular notch is obtained. If side slop is not given then the weir would be rectangular weir, and due to end contraction discharge ~~would~~ would would decrease.

→ The decrease in discharge must be compensated by side slop in such a way that the discharge comming through triangular notch will be equal to the loss.

VENTILATION OF WEAR :-



Ventilation Holes.

Depressed Nappe [(7-8% more] discharge

Clinging Nappe [(25-39)% more] discharge

CH – 17 : DIMENSIONAL AND MODEL ANALYSIS.

DIMENSIONAL ANALYSIS :–

→ It is a method of Dimensions.

→ It is a mathematical technique used in research world to design and to do model testing.

* Fundamental Quantities :–

M	L	T	t	Candela	A	mole
Mass	Length	Time	temp.	Luminous Intensity	Current	Amount of substance.

Derived Quantity :–

velocity (v)	m/s or [LT^{-1}]
acclaration (a)	m/s^2 & [LT^{-2}]
Viscosity (μ)	kg/ms & [$ML^{-1}T^{-1}$]
force (F)	$kg\,m/s^2$ & [MLT^{-2}]

* There are 2 methods to do dimensional analysis:
  1) Rayleigh Method [maximum no. of variable is 4]
  2) Buckingam – π Theorem. [we can apply more than 4 Variable.

1) RAYLEIGH METHOD :–

* Use for less no. of variable.

Eg:– Drag force on a submerged sphere depend. on fluid velocity 'μ', flow velocity u, and diameter of sphere 'D', Develope the expression from R.M.

Sol^n : $f_D \propto f(\mu, u_\infty, D)$ $f_D = k \cdot f(\mu, u_\infty, D)$

$f_D = k \cdot f(\mu, u_\infty \cdot D)$

$f_D = k [\mu]^a [u_\infty]^b \cdot [D]^c$

$[M L T^{-2}] = k [M L^{-1} T^{-1}]^a [L T^{-1}]^b [L]^c$

$\boxed{1 = a}$

$1 = b + c - a$
$1 = b + c - 1$ $\Big\}$ $b + c = 2$, $c = 1$

$\boxed{b + c = 2}$ —— (1)

$-2 = -a - b$

$-2 = -1 - b$

$\boxed{b = 1}$

where $k = 3\pi$

$\boxed{f_D = k \cdot \mu \cdot u_\infty \cdot D}$ → Stoke law.

2. BUKINGUM - π THEOROM :-

- for more then 4 no. of variable.

Step-1: Total variable involved = m.

Step-2: fundamental Quantity involved = n.
(M, L, T, θ, I, mole, t) = 3 (M, L, T)

Step-3: Number of π terms = $m - n$

Step-4: Select repeating variable = n
- fluid property (ρ, μ, σ, etc)
- flow property (v, a, ω, etc)
- Geometric property. (L, A, V, etc)

Eg:- The resistance force F on a ship is a function of its length (L), velocity (v), acceleration (g), and fluid property like density (ρ) & viscosity (μ). Find the Relation in dimensionless form.

sol $f = \phi(L, v, g, \rho, \mu)$

$m = 6$

$n = 3$ (M, L, T)

no. of π-term $= m - n$
$= 6 - 3 = 3$ (Ans)

MODEL ANALYSIS :-

#: FORCES ON A FLUID :-

* Inertia force :-

$f_i = m.a$

$f_i = \rho . \underset{\text{volume.}}{V} . a$

$f_i = \rho . L^3 . \frac{L}{T^2}$

$\boxed{f_i = \rho . L^2 . v^2}$ → velocity

* Viscous force (f_v) :-

$\frac{F}{A} = \frac{\mu v}{H}$

$f_v = \frac{\mu v}{H} . A$

$f_v = \frac{\mu . v . L^2}{L}$

$\boxed{f_v = \mu\, v . L}$

* Gravity force (f_g) :-

$f_g = m . g$

$f_g = \rho . V . g \Rightarrow \boxed{f_g = \rho . L^3 . g}$

$\cancel{f_g = P \cdot L^2}$

* Pressure force (f_p) :-

$$f_p = P \times A$$

$$\boxed{f_p = P \cdot L^2}$$

* Surface tension force (f_s) :-

$$\sigma = \frac{f_s}{l}$$

$$\boxed{f_s = \sigma \cdot l}$$

* Electric force (f_e) :-

$$\boxed{f_e = K \cdot L^2}$$

↳ bulk modulus of electricity.

DIMENSIONLESS NUMBER :-

① Reynolds Number (Re) :-

$$\boxed{Re = \frac{\text{Inertia force}}{\text{viscous force}} = \frac{f_i}{f_v}}$$

② Euler Number (E_u) :-

$$\boxed{E_u = \sqrt{\frac{\text{Inertia force}}{\text{Pressure force}}}}$$

③ Froude Number (f_r) :-

$$\boxed{f_r = \sqrt{\frac{\text{Inertia force}}{\text{Gravity force}}}}$$

④ Waber Number (W) :-

$$\boxed{W = \sqrt{\frac{\text{Inertia force}}{\text{Surface Tension force}}}}$$

(5) Mach Of Number (M_a) :-

$$\boxed{M_a = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}}}$$

MODEL ANALYSIS :-

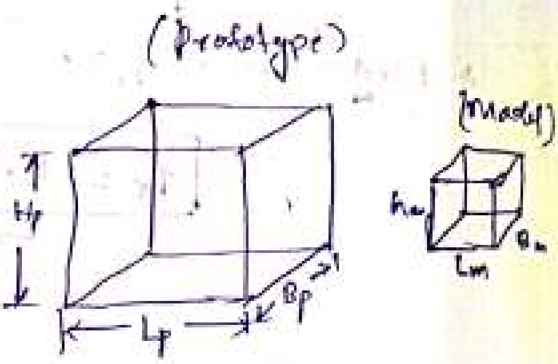
* **Model** :- Replica of actual system.

* **Prototype** :- Actual System.

Various type of Similarities :-

(1) **Geometric Similarities** :-

* If the ratio of corresponding dimensions Geometric dimensions is constant then it is called Geometric similarity.



$$\boxed{\frac{L_p}{L_m} = \frac{H_p}{H_m} = \frac{B_p}{B_m} = \text{Constant } (L_r)}$$

L_r — Geometric scale ratio.

$$\boxed{\frac{A_p}{A_m} = L_r^2}$$

$$\boxed{\frac{V_p}{V_m} = L_r^3}$$

(2) Kinematic Similarity :-

$$\boxed{\frac{v_p}{v_m} = \text{Constan}(V_r)} \rightarrow \text{velocity ratio.}$$

$$\boxed{\frac{a_p}{a_m} = \text{Const.}(a_r)} \rightarrow \text{acceleration ratio.}$$

(3) Dynamic Similarity :-

$$\boxed{\frac{f_{ip}}{f_{im}} = \frac{f_{vp}}{f_{vm}} = \frac{f_{p_p}}{f_{pm}} = \text{Constant!}}$$

$$\boxed{\frac{f_{ip}}{f_{vp}} = \frac{f_{im}}{f_{vm}}}$$

$$\boxed{Re_p = Re_m}$$

Note:- If dynamic similarity is existing then Kinematic and geometric similarity will also ~~exist~~ exist.

* Reynolds Model Law:-

$$\boxed{Re_m = Re_p}$$

* Pipe flow
* Submerged body
* Boundary Layer formation.
* flow through structure.

* Euler's Model Law:-

$$\boxed{Eu_m = Eu_p}$$

* Cavitation.
* Water Hammer.

* Froude Model law:-

$$\boxed{Fr_m = Fr_p}$$

* flow through nozzle, orifice.
* free surface flow.
* Spill way.

* Weber Model law :-

$\boxed{W_m = W_p}$

* Capillarity
* Free surface flow.

* Mach Model law :-

$\boxed{M_{am} = M_{ap}}$

* Projectile motion
* Compressible flow.