Module 1: Properties of Fluid.

Fluid is a substance which deform continously under the action of shear force.

Fluid mechanics is that branch of science which deals with the behaviour of the fluids (liquids or gases) at rest as well as in motion.

Thus this branch of science deals with Static, Kinematics and dynamic aspects of fluid.

The study of fluids at rest is known as fluid statics.

The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics.

If the pressure forces are considered and the fluids is in motion, that branch of science is called fluid dynamics.

Density. → Density or mass density of a fluid is defined as the ratio of the mass of the fluid to its volume.

$$\underset{(Kg/m^3)}{\rho} = \frac{\text{Mass of fluid}}{\text{Vol. of fluid}}$$

Mass per unit volume of a fluid is density.

The density of water is $1 gm/cm^3$ or $1000 Kg/m^3$.

Specific weight or Weight Density.

It is defined as the ratio between the weight of fluid to its volume.

The weight per unit volume of a fluid is also called weight density.

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}}$$

$$= \frac{(\text{mass of fluid}) \times (\text{acceleration due to gravity})}{\text{volume of fluid}}$$

$$= \frac{m \times g}{v}$$

$$\boxed{w = \rho \times g}$$

Specific volume

Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

$$\text{specific volume } (m^3/kg) = \frac{\text{volume of fluid}}{\text{Mass of fluid}}$$

$$= \frac{1}{\frac{\text{mass of fluid}}{\text{vol. of fluid}}} = \frac{1}{\rho}$$

Specific gravity – It is defined as the ratio of the weight density of a fluid to the weight density of a standard fluid.

$$S(\text{liquid}) = \frac{\text{weight density of a fluid}}{\text{weight density of water}}$$

$$S(\text{gases}) = \frac{\text{weight density of a gases}}{\text{weight density of air}}$$

Q. Calculate the specific weight, density and specific gravity of one litre of a liquid which weight 7N.

Given: Volume = 1 litre = $\frac{1}{1000}$ m^3.

weight = 7N.

(i) specific weight (w) = $\frac{\text{Weight of the liquid}}{\text{volume of the liquid}}$

$$= \frac{7N}{\frac{1}{1000}} = 7000 \, N/m^3.$$

(ii) density (ρ) = $\frac{w}{g}$ = $\frac{7000}{9.81}$ = 713.5 kg/m^3

(iii) specific gravity = $\frac{\text{Density of liquid}}{\text{Density of water}}$

$$= \frac{713.5}{1000} = 0.7135$$

Q. Calculate the density, specific weight and specific gravity of one litre of petrol of specific gravity is 0.7.

Given: volume = 1 litre = $\frac{1}{1000}$ m^3.

$S = 0.7$

$s = \frac{\rho}{1000}$

$\Rightarrow \rho = s \times 1000$

$= 0.7 \times 1000$

$= 700\ kg/m^3$.

ii) $w = \rho \times g$.

$= 700 \times 9.81$

$= 6867\ N/m^3$.

iii) $w = \frac{Weight}{volume}$.

$6867 \times 0.001 = W$

$W = 6.867\ N$.

Assignment

1) If $2.5\ m^3$ of a certain oil has a mass of 20 tonnes, find its mass density.

2.) In an experiment, the weight of $2.5\ m^3$ of a certain liquid was found to be $18.75\ KN$. Find the specific weight of the liquid. Also find its density.

3.) Find the specific gravity of an oil whose specific weight is $7.85\ KN/m^3$.

4.) A vessel of $4m^3$ volume contains an oil, which weighs $30.2\ KN$. Determine the specific gravity of the oil.

Solⁿ.

1. $V = 2.5\,m^3$ mass = 20 tonnes.

$\rho = \frac{mass}{vol.} = \frac{20}{2.5} = 0.8\ t/m^3$

$= 800\,kg/m^3$.

2. volume = $2.5\,m^3$ weight = 18.75 KN

$w = \frac{weight}{volume} = \frac{18.75}{2.5} = 7.5\ KN/m^3$

$\rho = \frac{weight}{acceleration} = \frac{18.75}{9.81} = 1.91\ t/m^3$

density$(\rho) = \frac{mass}{vol.} =$

3. $s = \frac{7.85}{9.81} = 0.8$.

4. Given, vol = $4\,m^3$.

weight = 30.2 KN.

specific weight of the oil $= \frac{weight}{volume} = \frac{30.2}{4.}$

$= 7.55\ KN/m^3$

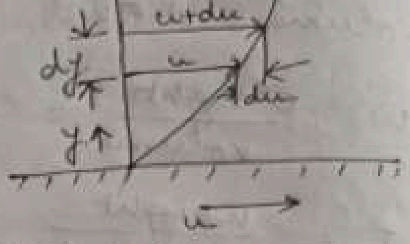
specific gravity of oil $= \frac{\text{specific weight of liquid}}{\text{specific weight of pure water}}$

$= \frac{7.55}{9.81} = 0.77$.

VISCOSITY

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

When two layers of fluid at a dist. dy apart, move one over the other at diff. velocities say u and u+du.



The viscosity together with relative velocity between the fluid causes a shear stress acting between the fluid layers.

The top layers causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to y.

$$\tau \propto \frac{du}{dy}.$$

$$\tau = \mu \frac{du}{dy}.$$

μ = const. of proportionality and is also known as coefficient of dynamic viscosity or viscosity.

$\frac{du}{dy}$ = rate of shear strain or rate of shear deformation.

$$\mu = \frac{\tau}{\frac{du}{dy}}.$$

Kinematic Viscosity.

It is defined as the ratio between the dynamic viscosity and density of the fluid.

$$\nu = \frac{\text{viscosity}}{\text{density}} = \frac{\mu}{\rho}.$$

In MKS and SI, the unit of Kinematic viscosity is metre^2/sec.

In CGS unit, Kinematic viscosity is also known as stroke.

CGS unit — cm^2/sec

$$1 \text{ stroke} = \text{cm}^2/\text{sec} = \left(\frac{1}{100}\right)^2 \text{m}^2/\text{sec}.$$

$$= 10^{-4} \text{ m}^2/\text{sec}$$

Centistroke means $= \frac{1}{100}$ stroke.

Newton's law of viscosity.

It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain.

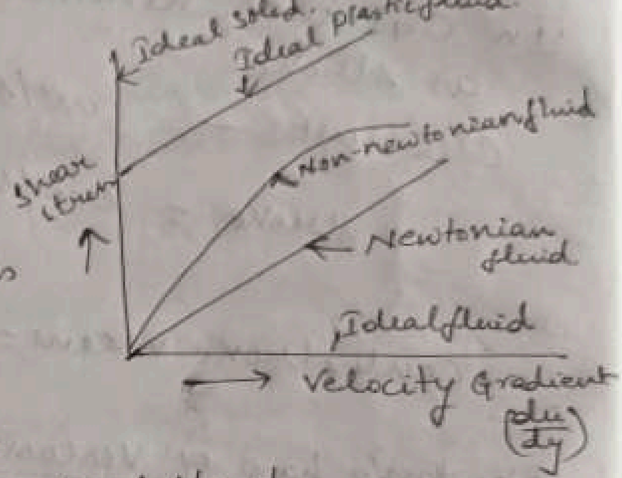
$$\tau = \mu \frac{du}{dy}.$$

Fluids which obey the above relation are known as Newtonian fluids and the fluids which donot obey the above relation are called Non-Newtonian fluids.

Types of Fluids

The fluids are classified into the following five types

1. Ideal fluid
2. Real fluid
3. Newtonian fluid
4. Non-Newtonian fluid
5. Ideal plastic fluid.



Ideal fluid - A fluid which is incompressible and is having no viscosity is known as an ideal fluid.

Real fluid - A fluid which possesses viscosity is known as real fluid.

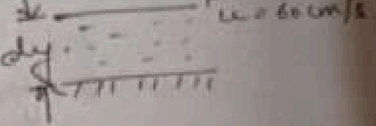
Newtonian fluid - A real fluid is which the shear stress is directly proportional to the rate of shear strain is known as a Newtonian fluid.

Non-Newtonian fluid - A real fluid in which the shear stress is not proportional to the rate of shear stress known as New ~~New~~tonian fluid

Ideal Plastic fluid - A fluid in which the shear stress is more than the yield value and shear stress is proportional to the rati of shear strain is known as ideal plastic fluid

Q. A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m² to maintain this speed. Determine the fluid viscosity between the plates

Given:



$dy = 0.025$ mm.
$= 0.025 \times 10^{-3}$ m.

$u = 60$ cm/s $= 0.6$ m/s.

$F = 2.0$ N/m².

$$\tau = \mu \frac{du}{dy}$$

Change in velocity $= u - 0 = u = 0.60$ m/s.

$$2.0 = \mu \frac{0.60}{0.025 \times 10^{-3}}$$

$$\mu = 8.33 \times 10^{-5} \frac{Ns}{m^2}$$

Q. A flat plate of area 1.5×10^6 mm² is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15 mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity as 1 poise.

Given:

Area $= 1.5 \times 10^6$ mm²
$= 1.5$ m².

$du = 0.4$ m/s.

$dy = 0.15$ mm
$= 0.15 \times 10^{-3}$ m.

$\mu = 1\,\text{poise} = \frac{1}{10}\,\frac{Ns}{m^2}$

$\tau = \mu \frac{du}{dy}$

$= \frac{1}{10} \times \frac{0.4}{.15 \times 10^{-3}} = 266.66\,\frac{N}{m^2}$

(i) Shear force

$F = \tau \times \text{area}$

$= 266.66 \times 1.5$

$= 400\,N.$

(ii) Power req. to maintain the speed at

$= F \times u$

$= 400 \times 0.4 = 160\,W.$ (1 Nm/s = Watt)

Q. Determine the intensity of shear of an oil having viscosity = 1 poise. The oil is used for lubricating the clearance between a shaft of diameter 10 cm and its journal bearing. The clearance is 1.5 mm and the shaft rotates at 150 r.p.m.

Given: $\mu = 1\,\text{poise} = \frac{1}{10}\,\frac{Ns}{m^2}.$

$D = 10\,cm = 0.1\,m.$

Distance between shaft and journal bearing,

$dy = 1.5\,mm$

$= 1.5 \times 10^{-3}\,m.$

Speed of shaft, $N = 150$ r.p.m,

Tangential speed of shaft,

$$u = \frac{\pi D N}{60} = \frac{\pi \times 0.1 \times 150}{60} = 0.785\ m/s.$$

$$\tau = \mu \frac{du}{dy}$$

$$= \frac{1}{10} \times \frac{0.785}{1.5 \times 10^{-3}} = 52.33\ N/m^2.$$

Q. Two horizontal plates are placed 1.25 cm apart, the space between them being filled with oil of viscosity 14 poises. Calculate the shear stress in oil, if upper plate is moved with a velocity of 2.5 m/s

Given:

$dy = 1.25\ cm = 0.0125\ m.$

$\mu = 14\ poise = \frac{14}{10}\ Ns/m^2.$

$u = 2.5\ m/sec.$

$$\tau = \mu \frac{du}{dy}.$$

$$\tau = \frac{14}{10} \times \frac{2.5}{.0125} = 280\ N/m^2.$$

Q. Find the Kinematic viscosity of an oil having density 981 kg/m³. The shear stress at a point in oil is 0.2452 N/m² and velocity gradient at that point is 0.2 per second.

Given: $\rho = 981\ kg/m^3$

$\tau = 0.2452\ N/m^2.$

$\frac{du}{dy} = 0.2/s.$

$$\tau = \mu \frac{du}{dy}$$

$$0.2452 = \mu \times 0.2$$

$$\mu = \frac{0.2452}{0.2} = 1.226 \ Ns/m^2.$$

Kinematic viscosity,

$$\nu = \frac{\mu}{\rho} = \frac{1.226}{981} = .125 \times 10^{-2} \ m^2/sec.$$

Q. Determine the specific gravity of a fluid having viscosity 0.05 poise and kinematic viscosity 0.035 strokes.

Given:

$\mu = 0.05$ poise $= \frac{0.05}{10} \ Ns/m^2$

Kinematic viscosity, $\nu = 0.035$ strokes

$= 0.035 \ cm^2/s$

$= 0.035 \times 10^{-4} \ m^2/s.$

$$\nu = \frac{\mu}{\rho}$$

$$0.035 \times 10^{-4} = \frac{0.05}{10} \times \frac{1}{\rho}$$

$$\rho = \frac{0.05}{10} \times \frac{1}{0.035 \times 10^{-4}} = 1428.5 \ Kg/m^3$$

$\therefore$ sp. gr of liquid $= \frac{\text{Density of liquid}}{\text{Density of water}}$

$$= \frac{1428.5}{1000} = 1.4285 \simeq 1.43$$

Q. Determine the viscosity of a liquid having Kinematic viscosity 6 stokes and specific gravity 1.9

Given: $\nu = 6$ strokes $= 6\,cm^2/s$
$= 6 \times 10^{-4}\,m^2/s$.

sp. gr. of liquid $= 1.9$

Let the viscosity of liquid $= \mu$

sp. gr. of a liquid $= \frac{\text{Density of the liquid}}{\text{Density of water}}$

$1.9 = \frac{\text{Density of liquid}}{1000}$

$\therefore$ Density of liquid $= 1000 \times 1.9$
$= 1900\ kg/m^3$

$\nu = \frac{\mu}{\rho}$

$6 \times 10^{-4} = \frac{\mu}{1900}$

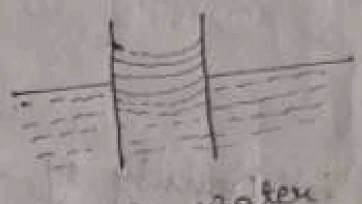
$\mu = 6 \times 10^{-4} \times 1900$
$= 1.14\ Ns/m^2$.

Surface Tension and Capillarity.

The surface tension of a liquid is its property, which enables it to resist tensile stress. It is due to the cohesion between the molecules at the surface of a liquid.

For ex: —

When a glass tube of small diamter is dipped in water, the water rises up in the tube with an upward concave surface.



(a) in water.

But when the same tube is dipped in mercury, the mercury depresses down in the tube with an upward convex surface.

(b) in mercury.

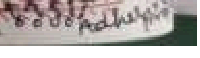
The phenomenon of rise or fall of a liquid in a small diameter tube is known as capillary or Menisus effect.

* This effect is due to the surface tension

Cohesion — cohesion is defined as intermolecular attraction between the molecules of same liquid.

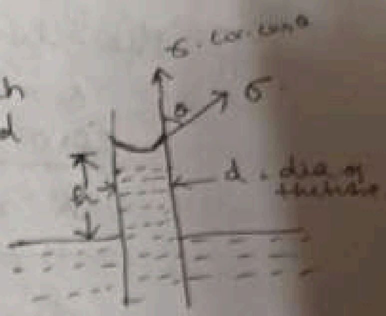
cohesive property.

Adhesion — It is defined as force of attraction between molecules of liquid and the molecules of solid boundary surface in contact with the liquid.



Capillary rise

Wetting liquid → The liquids which wet the surface and make a contact with the surface and the force of attraction is more.
eg: water.



The phenomenon of rising water in the tube of small diameter is called the capillary rise.

(i) weight $= m \times g$

$= \rho \times v \cdot g$

$= \rho \times (\text{Area} \times \text{height}) \times g$

(ρ → density, height → h)

(ii) Tensile force in upward direction.

$= \sigma \times \text{circumference} \times \cos\theta$

$= \sigma \times \pi \cdot d \times \cos\theta$

At equillibrium (weight in downward = Tensile force in upward direcⁿ)

$\rho \cdot (\text{area} \times h) \times g = \sigma \times \pi d \times \cos\theta$

$\rho \left(\frac{\pi}{4} d^2 \times h\right) g = \sigma \pi d \times \cos\theta$

$$h = \frac{\sigma \pi d \cos\theta}{\rho \frac{\pi}{4} d^2 \cdot g}$$

$$\boxed{h = \frac{4\sigma \cos\theta}{\rho d g}}$$

If the liquid, is water than the θ value is zero.

$\theta = 0^\circ$

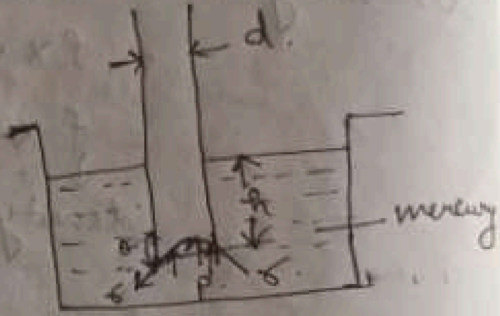
$\cos 0 = 1$

$$\boxed{h = \frac{4\sigma}{\rho \times g \times d}}$$

Capillary fall — It is observed in non-wetting liquid.

In which cohesion force is more i.e. intermolecular force of attraction is more compare to the adhesion force.

eg: mercury.



(i) hydrostatic force.

$= p \times \text{area}$

$= p \times \frac{\pi d^2}{4}$

$= \rho \times g \times h \times \frac{\pi}{4} d^2$

(ii) Tensile force $= \sigma \times \pi d \times \cos\theta$.

In equillibrium.

$$\rho \times g \times h \times \frac{\pi}{4} d^2 = \sigma \times \pi d \cos\theta$$

$$h = \frac{\sigma \pi d \cos\theta}{\rho g \times \frac{\pi}{4} d^2}$$

$$= \frac{4\sigma \cos\theta}{\rho \times g \times d}$$

$\theta = 128^\circ$

Q. Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (i) water and (ii) mercury. Take surface tension $\sigma = 0.0725$ N/m for water and $\sigma = 0.52$ N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact $= 130°$

Given.

Dia of tube $d = 2.5$ mm
$= 2.5 \times 10^{-3}$ m.

~~surface~~

surface tension, σ (water) $= 0.0725$ N/m

σ (mercury) $= 0.52$ N/m

sp. gravity of mercury $= 13.6$

$\therefore$ Density $= 13.6 \times 1000$ kg/m^3.

(i) capillary rise for water

$\theta = 0°$
$\cos\theta = 1$

$$h = \frac{4\sigma\cos\theta}{\rho \times g \times d}$$

$$= \frac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

$= 0.0118$ m.

(ii) For mercury.

$\theta = 130°$

$$h = \frac{4 \times 0.52 \times \cos 130}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

$= -0.004$ m.

The negative sign indicates the capillary depression

Q. Calculate the capillary effect in millimetres in a glass tube of 4mm diameter, when immersed in (i) water and (ii) Mercury. The temperature of the liquid is 20°c and the values of the surface tension of water and mercury at 20°c in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero and that for mercury is 130°. Take density of water at 20°c as equal to 998 Kg/m³.

Given. $d = 4mm = 4 \times 10^{-3}$ m.

(i) capillary effect for water

$\sigma = 0.073575$ N/m.

$\rho = 998$ Kg/m³.

$$h = \frac{4 \times 0.073575 \times \cos 0°}{998 \times 9.81 \times 4 \times 10^{-3}}$$

$= 7.51 \times 10^{-3}$ m.

(ii) capillary effect for mercury.

$\sigma = 0.51$ N/m.

$\rho = 13.6 \times 1000$

$= 13600$ Kg/m³.

$$h = \frac{4 \times 0.51 \times \cos 130}{13600 \times 9.81 \times 4 \times 10^{-3}}$$

$= -2.46 \times 10^{-3}$ m.

Q. The capillary rise in the glass tube is not to exceed 0.2 mm of water. Determine its minimum size, given that surface tension for water in contact with air = 0.0725 N/m.

Given: $h = 0.2\ mm = 0.2 \times 10^{-3}\ m$.

$\sigma = 0.0725\ N/m$.

Let dia of tube = d.

$\theta = 0°$

$\rho = 1000\ kg/m^3$.

$$h = \frac{4\sigma}{\rho \times g \times d} \Rightarrow 0.2 \times 10^{-3} = \frac{4 \times 0.0725}{1000 \times 9.81 \times d}$$

$$\Rightarrow d = \frac{4 \times 0.0725}{1000 \times 9.81 \times 0.2 \times 10^{-3}}$$

$= 0.148\ m.$

Q. Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to restricted to 2mm. Consider surface tension of water in contact with air as 0.073575 N/m.

Given: $h = 2mm = 2 \times 10^{-3}\ m$.

$\sigma = 0.073575\ N/m$.

$\theta = 0°$

$$h = \frac{4\sigma}{\rho \times g \times d} =$$

$$\Rightarrow 2 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}.$$

$\Rightarrow d = 0.015\ m.$

Module 2: Pressure and its Measurement

Fluid pressure at a point

Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction. Then the ratio of dF/dA → is known as intensity of pressure or pressure.

$$P = \frac{dF}{dA}$$

If the force is uniformly distributed over the area (A), then pressure at any point is given by

$$P = \frac{F}{A}.$$

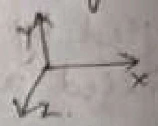
Force or pressure force $F = P \times A$.

The units of pressure are Pascal (PA) or N/m^2 or N/mm^2.

Pressure→ When any fluid is kept in a container then it exerts force on all points of all sides of the vessel.

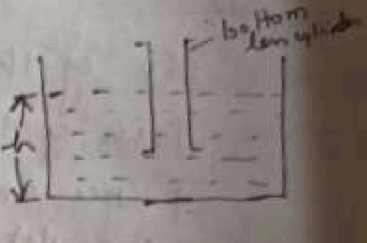
Pascal's Law

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.



$$P_x = P_y = P_z.$$

~~Extern~~ Pressure Head.



In a static liquid, vertical distance from the datum line to the free surface of liquid is known as pressure head.

$$\text{pressure} = \frac{\text{Force}}{\text{Area}} \text{ or } \frac{\text{weight of a liquid in the cylinder}}{\text{Area of the cylinder}}$$

$$= \frac{\text{volume} \times \text{weight density}}{\text{Area of the cylinder}}$$

$\left\{ \text{weight density} = \frac{\text{weight of the liquid}}{\text{volume}} \right.$

$$= \frac{A \times h \times \rho \times g}{A}.$$

ρ = density of water
g = acceleration due to gravity

$$P = \rho \times g \times h.$$

$$\boxed{h = \frac{P}{\rho \times g}.}$$

h = pressure head.

$1\ \text{Pa} = 1\ N/m^2.$
$1\ \text{KPa} = 10^3\ N/m^2.$
$1\ \text{MPa} = 10^6\ N/m^2.$
$1\ \text{GPa} = 10^9\ N/m^2$
$= 10^3\ \text{MPa}$

$1\ \text{bar} = 10^5\ \text{Pa}.$

Thus we know that the intensity of pressure is proportional to the height of water above the base.

Q. A hydraulic press has a ram of 30cm diameter and a plunger of 4.5cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500N.

Given:

Dia of the ram = $D = 30cm = 0.3m$.

Dia of the plunger = $d = 4.5cm = 0.045m$

Force on plunger $F = 500N$.

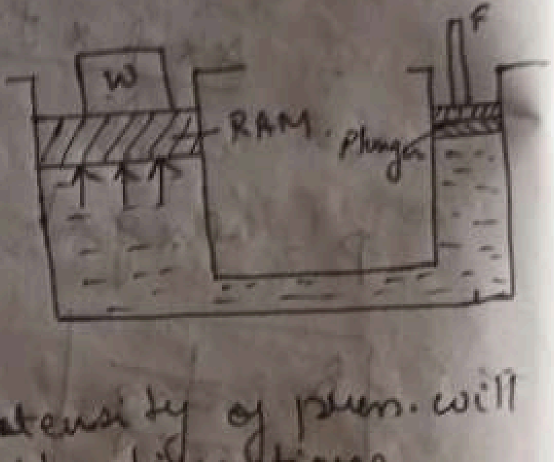
Weight lifted = W

Area of ram, $A = \frac{\pi}{4}D^2$.

$= \frac{\pi}{4} \times (0.3)^2 = 0.07068 \, m^2$.

Area of plunger, $a = \pi/4 \, d^2$.

$= \pi/4 \times (0.045)^2 = 0.00159 \, m^2$.



Pressure intensity due to plunger

$= \frac{Force}{Area}$

$= \frac{500}{0.00159}$.

Due to Pascal's law, the intensity of pres. will be equally transmitted in all directions.
Hence, the pressure intensity at the ram

$= \cancel{\frac{500}{0.00159}}$

$= \frac{W}{Area} = \frac{W}{0.07068}$.

$\frac{W}{0.07068} = \frac{500}{0.00159} \Rightarrow W = 22222N$

$= 22.222 \, kN$.

Atmospheric Pressure.

It is the pressure of the atmosphere on atmospheric air on the earth surface. As air is compressible, hence its density varies with height, temperature and humidity. So, it cannot be calculated like the liquids. But it can be measured by finding the liquid column height which it can support. At the sea level, the pressure exerted by $1 m^2$ cross-sectional area air column is 103 KN. The atmospheric pressure at the sea level is 103 KN/m^2 or (103 K Pa). It can be expressed as 10.3 metres of water in terms of water, in terms of equivalent water column or 760 mm of mercury in terms of equivalent mercury column.

Gauge Pressure.

It is the pressure measured with the help of a pressure measuring instrument in which the atmospheric pressure is taken as datum. The atmospheric pressure on the gauge scale is marked as zero.

This gauge pressure is of two types: (a) Positive gauge pressure and (b) negative gauge pressure.

Positive gauge pressure

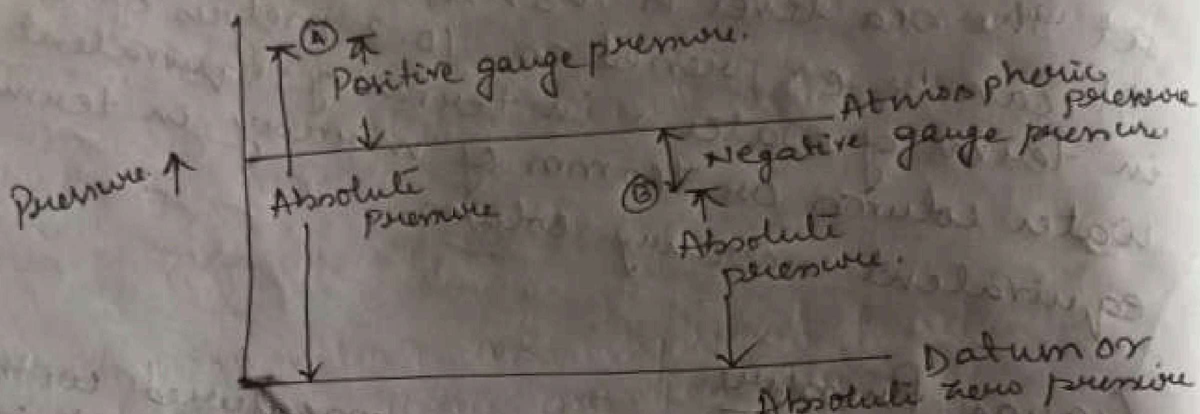
— The value of it is positive and it is above the atmospheric pressure.

Negative Gauge Pressure or Vacuum [illegible]

It has negative value and measured below the atmospheric pressure. Its unit is N/m^2 or Kgf/m^2

Absolute Pressure

It is the pressure equal to the algebric sum of atmospheric and gauge pressure



Let A is a point. The pressure at A from absolute zero pressure is the absolute pressure at A.

Absolute pressure at A = Gauge pressure at A (positive gauge pressure) + Atmospheric pressure.

But Absolute pressure at B = Atmospheric pressure − Gauge pressure at B (negative gauge pressure)

So, absolute pressure can be defined as the total pressure measured from the absolute zero pressure.

$$\boxed{P_{Abs.} = P_{atm} + P_{gauge}}$$

where, P_{abs} = Absolute pressure.
P_{atm} = Atmospheric pressure.
P_{gauge} = Gauge pressure.

Pressure Measurement

It can be measured in 2 ways:

→ By balancing one liquid column where pressure is to be measured with another liquid column. These are called as tube gauges.

→ By balancing the liquid column with spring force or dead weight. These are called as mechanical gauges.

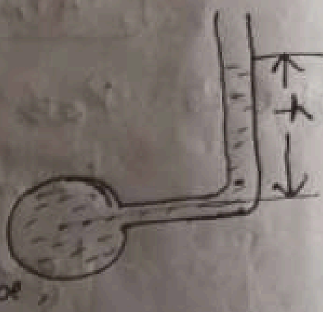
Tube gauges:
These are of two types.

(i) Piezometer.
(ii) Manometer.

Piezometer.

It is a simplest form of instruments used for measuring moderate pressure. It consists of a tube, one end of which is connected to the pipeline in which the pressure is required to be found out.

The other end is open to ... freely without in which the liquid can rise freely without overflow. The height, to which the liquid rises up in the tube, gives the pressure head directly.



The piezometer tube is meant for measuring gauge pressure only as the surface of the liquid, in the tube, is exposed to the atmosphere.

A piezometer tube is not suitable for measuring negative pressure, the air will enter in the pipe through the tube.

Manometer

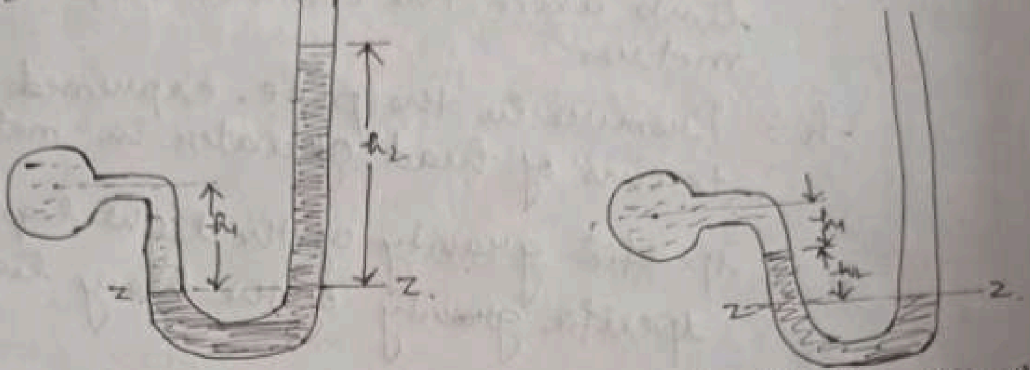
A manometer is an improved form of a piezometer tube. There are few types of manometers:

1. Simple manometer
2. Micromanometer
3. Differential manometer
4. Inverted differential manometer.

Simple Manometer

A simple manometer is a slightly improved form of a piezometer tube for measuring high as well as negative pressure.

A simple manometer, in its simplest form, consists of a tube bent in a U-shape, one end of which is attached to the gauge point and the other end is open to the atmosphere.



(a) positive pressure.

(b) Negative pressure.

The liquid used in the bent tube or simple manometer is generally mercury which is 13.6 times heavier than water. Hence it is suitable for measuring high pressures also.

Now consider a simple manometer connected to a pipe containing a light liquid under a high pressure. The high pressure in the pipe will force the heavy liquid, in the left limb of the U-tube, to move downward. This downward movement of the heavy liquid in the left limb will cause a corresponding rise of the heavy liquid in the right limb.

The horizontal surface, at which the heavy and light liquid meet in the left limb, is known as common surface or datum line. Let Z-Z be the datum line.

Let, h_1 = height of the light liquid in the left limb above the common surface in metres.

h_2 = height of the heavy liquid in the right limb above the common surface in metres.

h = Pressure in the pipe, expressed in terms of head of water in metres.

s_1 = specific gravity of the light liquid

s_2 = specific gravity of the heavy liquid

Pressure in the left limb above the datum line Z - Z. — (i)

$= h + s_1 h_1$ m of water.

Pressure in the right limb above the datum line Z - Z. — (ii)

$= s_2 h_2$ m of water.

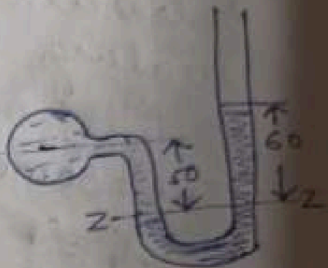
Pressure in both the limbs above the Z-Z datum is equal. therefore, equating the pressure in eq^n (i) & (ii)

$$h + s_1 h_1 = s_2 h_2$$

$$h = (s_2 h_2 - s_1 h_1)$$

Q. A simple manometer containing mercury is used to measure the pressure of water flowing in a pipeline. The mercury level in the open tube is 60 mm higher than that on the left tube. If the height of water in the left tube is 50 mm, determine the pressure in the pipe in terms of head of water.

$h_1 = 50$ mm



pressure head in the left limb above Z-Z

$= h + s_1 h_1$

$= h + (1 \times 50)$

$= h + 50$

pressure head in the right limb above Z-Z.

$= s_2 h_2 = 13.6 \times 60$

$= 816$ mm of water.

$h + 50 = 816.$

$h = 816 - 50$

$= 766$ mm of water.

Negative Pressure

Now the pressure in the left limb above the datum line $= h + s_1 h_1 + s_2 h_2$ m. of water.

Pressure in the right limb above $= 0$.

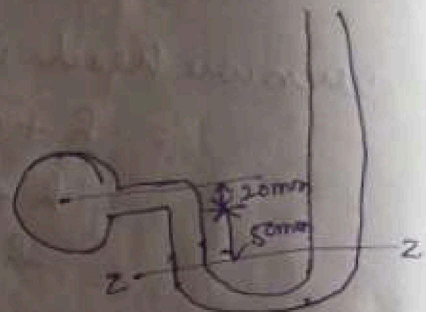
$h + s_1 h_1 + s_2 h_2 = 0.$

$h = -(s_1 h_1 + s_2 h_2)$ m of water.

Q: A simple manometer containing mercury was used to find the negative pressure in the pipe containing water as shown in fig. The right limb of the manometer was open to atmosphere. Find the negative pressure, below atmosphere in the pipe. If the manometer reading are given in the figure.

$h_2 = 50mm$. $h_1 = 20mm$

$s_1 = 1$ $s_2 = 13.6$.



We know, pressure head in the left limb above Z-Z

$= h + s_1 h_1 + s_2 h_2$.

$= h + (1 \times 20) + (13.6 \times 50)$

$= (h + 700)$ mm.

Pressure head in the right limb above Z-Z.

$= 0$.

$h + 700 = 0$.

$h = -700$ mm of water.

$= -7$ m of water

Gauge pressure in the pipe

$p = -gh$.

$= -(9.81 \times 7)$

$= -68.67$ KN/m²

$= -68.67$ KPa

Differential Manometer

A differential manometer is used to measure the difference in pressure between two points in a pipe or in two different pipes. It consists of a u-tube containing a heavy fluid. The two ends of the u-tube are connected to the points whose difference of pressure is to be found..

Differential manometers are of two types:

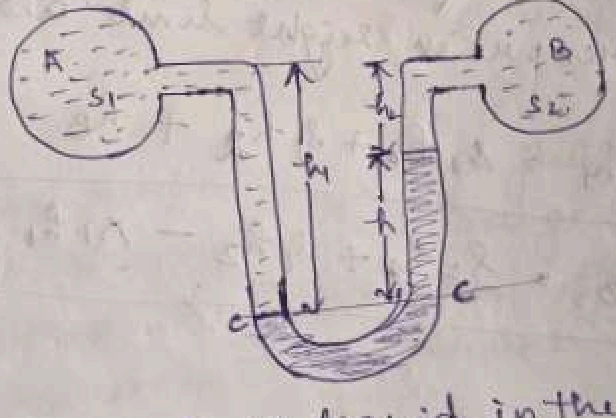
→ U-tube differential manometers.

→ Inverted u-tube differential manometers.

U-tube Differential Manometers

The connected pipes may be at different levels or they may be at the same level.

(a) When the pipes are at different levels.



h_1 = height of light liquid in the left limb above C-C

h_2 = height of light liquid in the right limb above heavy liquid.

h = height of heavy liquid in the right limb above C-C.

ρ_1 = density of liquid at A.
ρ_2 = density of liquid at B
ρ = density of heavy liquid
P_A = pressure at A.
P_B = pressure at B.
Pressure in the left limb above C-C
$= P_A + \omega_1 h_1$

$=$

s_1 = specific gravity of light liquid.
s_2 = specific gravity of heavy liquid.

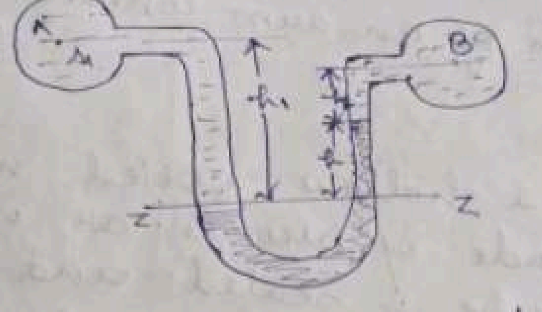
h_A = Pressure head of liquid (light) in pipe A.
h_B = Pressure head of liquid (heavy) in pipe B.

The pressure in left limb above datum C-C.
= The pressure in right limb above datum C-C.

$$h_A + s_1 h_1 = s_2 h + s_1 h_2 + h_B$$

$$\boxed{h_A - h_B = s_2 h + s_1 h_2 - s_1 h_1}$$

(b) When the pipes are at different levels.



h = height of heavy liquid above Z-Z in the right limb.

h_1 = height of light liquid above Z-Z in the left limb

h_2 = height of light liquid in right limb above the heavy liquid.

The pressure in left limb above datum Z-Z = h_A

The pressure in right limb above datum Z-Z.

$$h_A + S_1 h_1 = S_2 h + S_1 h_2 + h_B.$$

Mechanical Gauges

Whenever a very high fluid pressure is to be measured, a mechanical gauge is best suited for this purpose. A mechanical gauge is also used for the measurement of pressure in boilers.

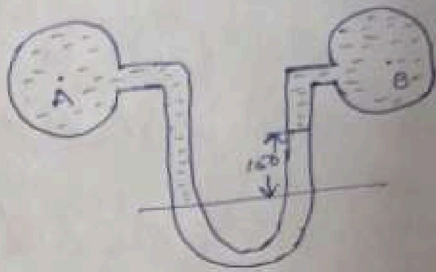
1. Bourdon's tube pressure gauge.

The most common type of pressure gauge is a Bourdon's pressure gauge. It is simple in construction and is generally used for measuring high pressure.

A Bourdon gauge uses a coiled tube, which as it expands due to pressure increases causes a rotation of an arm connected to the tube.

It consists of a hollow coiled metallic tube usually made of bronze or nickle. One end of the tube is sealed and the other end is connected to the pipe whose pressure is to be measured. When the pressure in the hollow tube increases, the tube will tend to uncoil and when the pressure decreases, it will tend to coil more tightly. This movement is transferred through a rack or pinion arrangement connected to a pointer over a calibrated dial, directly giving the pressure of the fluid. This gauge is capable of measuring both positive and negative gauge pressure.

Q. A differential manometer was connected with two points at the same level in a pipe containing liquid of sp. gravity 0.85 as shown in fig. Find the difference of pressure at the two points, if the difference of mercury levels be 150 mm.



$s_1 = 0.85$

$s_2 = 13.6$

$\rho_1 = s_1 \times 1000$
$= 0.85 \times 1000 = 850 \, kg/m^3$.

$\rho_2 = s_2 \times 1000$
$= 13.6 \times 1000 = 13600 \, kg/m^3$

$h = 150 \, mm = 0.15 \, m$.

$h_A - h_B = gh(13600 - 850)$
$= 9.81 \times 0.15 \times 12750$
$= 18761.62 \, N/m^2$.
$= 18.76 \, kN/m^2$.

Q. A differential manometer containing mercury was used to measure the difference of pressures in two pipes containing water as shown in fig. Find the difference of pressure in the pipes, if the manometer reading is 0.8 m.

Module 3: Hydrostatics (Related to equilibrium of liquids and the pressure exerted by liquid at rest).

Hydrostatic Pressure:

It is the pressure exerted by the liquid at rest on any body which is immersed in it. The direction of this pressure is always at right angles with the immersed surface.

Total Pressure.

Total Pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of Pressure

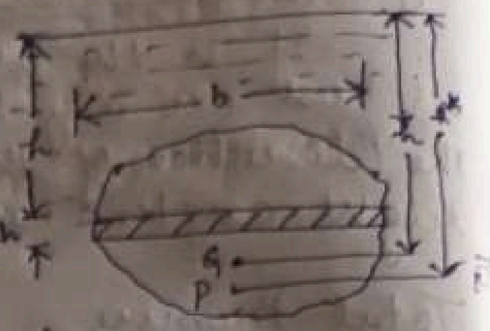
It is defined as the point of application of the total pressure on the surface.

There are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined. The submerged surfaces may be:

→ Vertical plane surface.
→ Horizontal plane surface,
→ Inclined plane surface, and
→ Curved surface.

Vertical plane surface Submerged in Water

The plane vertical surface is immersed in a liquid.



Let, A = Total area of the surface.

$\bar{h}$ = Dist. of C.G of the area from free surface of liquid.

G = Centre of gravity of plane surface.

P = Centre of pressure.

h^* = Distance of centre of pressure from free surface of liquid.

(a) Total Pressure (F).

The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips.

→ The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness dh, width b at a depth of h from the free surface of liquid.

pressure on the strip, $p = \rho g h$.

Area of the strip, $dA = b \times dh$.

Total pressure force on the strip

$$dF = p \times Area.$$
$$= \rho g h \times b \times dh.$$

Total pressure force on the whole surface

$$F = \int dF = \int \rho g h \times b \times dh.$$
$$= \rho g \int \underbrace{b \times h}_{} \times dh.$$
$$= \rho g \int \underbrace{b \times dh}_{\substack{Area.\\ dA.}} \times h$$
$$= \rho g \int dA \times h.$$
$$= \rho g A \bar{h}$$

For water, $\rho = 1000\ Kg/m^3$.

$g = 9.81\ m/s^2$.

(b) Centre of Pressure (h^*)

Centre of Pressure is calculated by using the "Principle of Moments."

Principle of Moments – It states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P at a distance h^* from free surface of the liquid. Hence, moment of the force F about free surface of the liquid $= F \times h^*$ — (i)

Moment of force dF, acting on a strip about free surface of liquid.

$$= dF \times h.$$

$$= \rho \times g h \times b \times dh \times h.$$

Sum of moments of all such forces about free surface of liquid.

$$= \int \rho g h \times b \times dh \times h$$

$$= \rho g \int b \times h \times h dh$$

$$= \rho g \int b h^2 dh. \qquad b\, dh = dA$$

$$= \rho g \int h^2 dA \qquad \int h^2 dA = \int b\, h^2 dA = I_0.$$

I_0 = Moment of inertia of the surface about free surface of the liquid

$\therefore$ Sum of moments about free surface

$$= \rho g I_0. \qquad \text{— (ii)}$$

$$F \times h^* = \rho g I_0$$

$$F = \rho g A \bar{h}.$$

$$\rho g A\bar{h} \times h^* = \rho g I_0.$$

$$h^* = \frac{\rho g I_0}{\rho g A\bar{h}} = \frac{I_0}{A\bar{h}}$$

By the theorem of parallel axis,

$$I_0 = I_G + A \times \bar{h}^2.$$

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}.$$

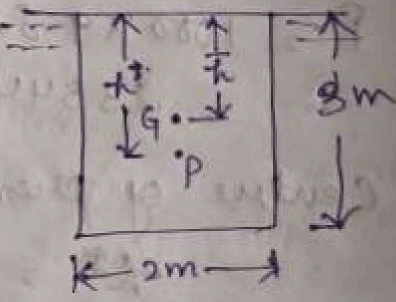
Q. A rectangular plane is 2m wide and 3m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water surface, (b) 2.5m below the free water surface.

Soln Given:
width, $b = 2m$.
depth, $d = 3m$.

(a) Upper edge coincides with water surface.

Total pressure on the surface

$$F = \rho g A\bar{h}$$



$\rho = 1000\ kg/m^3$ $\quad g = 9.81\ m/s^2$

$A = 3 \times 2 = 6 m^2$.

$\bar{h} = \frac{d}{2} = \frac{3}{2} = 1.5 m$.

$\therefore F = 1000 \times 9.81 \times 6 \times 1.5$
$= 88290 N$.

Centre of pressure,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

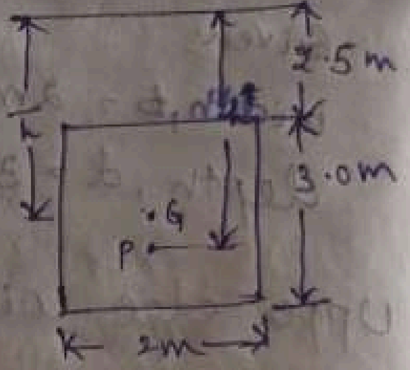
$I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 m^4$.

$$h^* = \frac{4.5}{6 \times 1.5} + 1.5$$
$$= 0.5 + 1.5 = 2.0 m$$

(b) 2.5 m below the free water surface.

Total pressure,

$F = \rho g A \bar{h}$



$\bar{h} = 2.5 + \frac{3}{2} = 4.0 m$.

$F = 1000 \times 9.81 \times 6 \times 4$
$= 235440 N$.

Centre of pressure,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

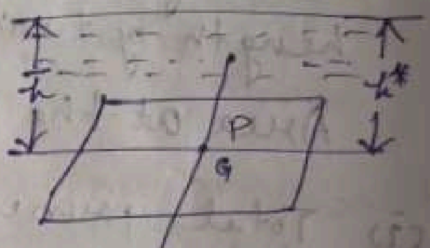
$I_G = 4.5 m^4$.

$$h^* = \frac{4.5}{6 \times 4} + 4.$$

$$= 4.187 \text{ m}.$$

Horizontal plane surface submerged in liquid.

Consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface and equal to, $p = \rho g h$, where h is depth of surface.



Let A = Total area of surface

Total force F, on the surface

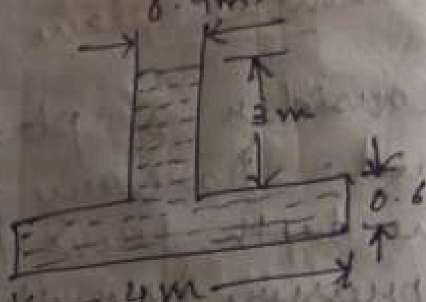
$= p \times \text{Area}$

$= \rho g \times h \times A = \rho g A \bar{h}$.

where, $\bar{h}$ = Depth of CG from free surface of liquid $= h$

h^* = Depth of centre of pressure from free surface $= h$.

Q. Figure shows a tank full of water. ...
(i) Total pressure on the bottom of tank
(ii) Weight of water in the tank
(iii) Hydrostatic paradox between the results of (i) & (ii). Width of tank is 2m.



Given

Depth of water on bottom of tank

$h_1 = 3 + 0.6$
$= 3.6$ m.

Width of tank $= 2$ m.

Length of tank at bottom $= 4$m.

Area at the bottom $A = 4 \times 2 = 8\,m^2$

(i) Total pressure on the bottom of the tank

$F_1 = \rho g A \bar{h}$

$= 1000 \times 9.81 \times 8 \times 3.6$

$= 282528$ N.

(ii) Weight of water in the tank

$= \rho g \times$ volume of tank

$= 1000 \times 9.81 \times [3 \times 0.4 \times 2 + 4 \times 0.6 \times 2]$

$= 70632$ N.

It is observed that the total weight of water in the tank is much less than the total pressure at the bottom of the tank. This is known as Hydrostatic paradox.

3.4. Archimedes Principle.

It states that when a body is immersed fully or partly in a fluid, then the upward ~~tow~~ buoyant force that is exerted on a body is equal to the weight of the fluid that the body displaces.

Bouyancy.

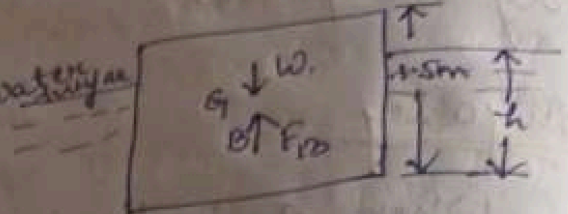
When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of bouyancy or simply bouyancy.

Centre of Bouyancy.

It is defined as the point, through which the force of bouyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

But if the force of buoyancy is less than the weight of the body, it will sink down.

Q. Find the volume of the water displaced and position of centre of buoyancy for a wooden block of width 2.5m and of depth 1.5m, when it floats horizontally in water. The density of wooden block is 650 kg/m³ and its length is 6.0 m.



Given

width = 2.5m.
Depth = 1.5 m.
Length = 6.0m.

volume of the block = $2.5 \times 1.5 \times 6$
$= 22.50\ m^3$.

Density of wood $\rho = 650\ kg/m^3$.

Weight of the block = $\rho \times g \times$ volume.
$= 650 \times 9.81 \times 22.50$.
$= 143471\ N$.

For equilibrium, the weight of water displaced
= weight of the wooden block.
$= 143471\ N$.

volume of water displaced = $\frac{\text{weight of water displaced}}{\text{weight density of water}}$

$= \frac{143471}{1000 \times 9.81} = 14.625\ m^3$.

Position of centre of Buoyancy.

volume of wooden block in water
= volume of water displaced

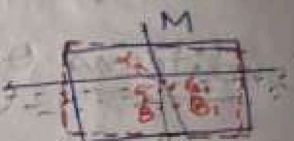
$2.5 \times h \times 6 = 14.625$

$h = 0.975 \text{ m}$

Centre of buoyancy $= \frac{0.975}{2} = 0.4875$ m from base.

Metacentre

Whenever a body, floating in a liquid, is given a small angular displacement, it starts oscillating about some point. This point, about which the body starts oscillating is called metacentre.



Metacentric Height

The distance between the centre of gravity of a floating body and the metacentre is called metacentric height.

The metacentric height of a floating body is a direct measure of its stability.

More the metacentric height of a floating body more it will be stable.

Floatatoon.

Whenever a body is placed in a liquid then two forces i.e. gravitational force and upthrust of liquid acts on the body. Both the forces are opposite in direction to each other. When the gravitational force is less than the upthrust, then the body will float. But if the gravitational force is more than the upthrust of liquid, then the body will sink down in the liquid.

Conditions of equilibrium (Floating Body)-

When a body is floating under ready condition in any liquid then the body is under equilibrium. There are three conditions available for equilibrium.

(a) Stable equilibrium

When a small angular displacement is given to a floating body and then the body returns to its original position, then only the body is under stable equilibrium.

(b) Unstable equilibrium.

When a small angular displacement is given and the body instead of returning to original position moves further away, then the body is under unstable equilibrium.

(c) Neutral equilibrium.

When a small angular displacement is given and the body achieves a new position of rest then the body is under neutral equilibrium.

Chapter 4 Kinematics of Flow

~~4.0 Types of fluid flow~~

A fluid may be in static condition or in motion.
The study of fluid properties when it is in static condition is called fluid static.
and the study of fluid motion without any forces causing the motion is called fluid Kinematics.

4.1 Types of Fluid flow

Fluid flow are classified as:

(i) steady and unsteady flow
(ii) Uniform and Non-uniform flow
(iii) Laminar and Turbulent flow
(iv) Compressible and incompressible flow
(v) Rotational and irrotational flow
(vi) ~~Rea~~ Ideal and real flow
(vii) One, two and three dimensional flow

Steady and Unsteady flow

Steady flow is that type of flow in which fluid parameters (velocity, pressure, density etc) at any point in flow field donot change with time. This means that the fluid particles passing through a fixed point have the same flow parameters like velocity, pressure, surface tension etc $\frac{\partial v}{\partial t} = 0$

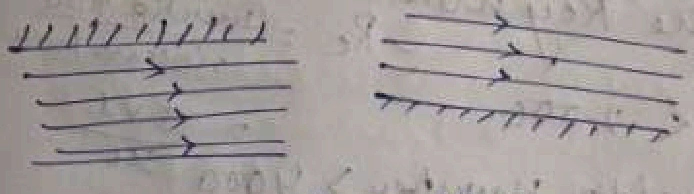
Unsteady flow is that of flow in which fluid parameters at a point changes with time.

$$\frac{\partial v}{\partial t} \neq 0.$$

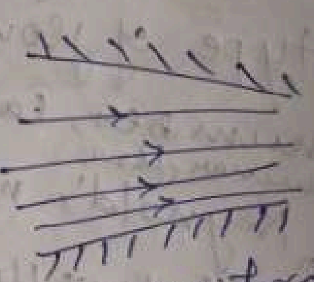
Uniform and non-uniform flow.

Uniform flow is defined as that type of flow in which the velocity of flow of a fluid at any instant doesnot change with respect to space.

Non-uniform flow is defined as that type of flow in which the velocity of flow changes with respect of space at any given time.
It means the velocity of flow is different for different section in the path of flow.



uniform flow.



Non-uniform flow.

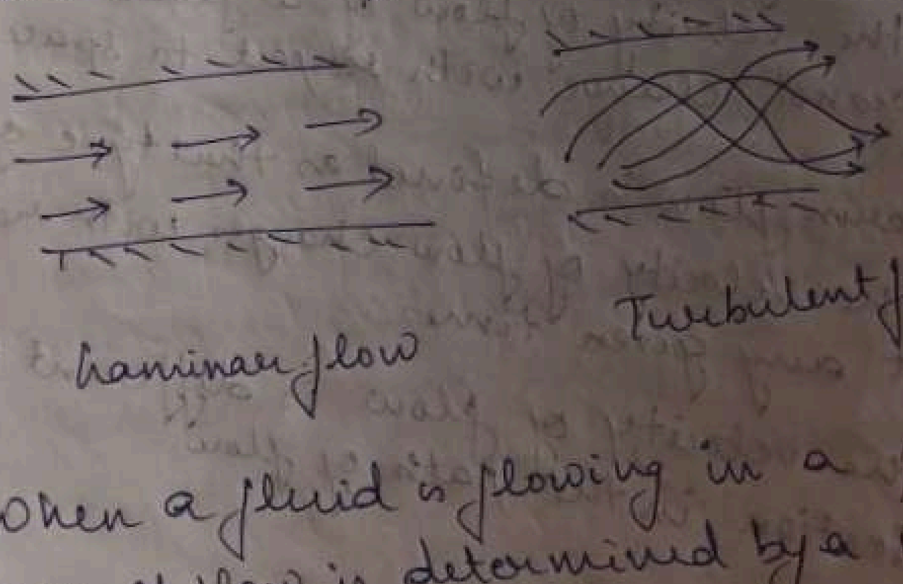
Laminar and Turbulent flow.

Laminar flow is defined as that of type of flow in which each fluid particle has a definite path and paths of individual particles donot cross each other.

Laminar flow is called stream line or viscous flow

~~eo~~

Turbulent flow is defined as that type of flow in which each fluid particle doesnot have a definite path and the paths of individual particles cross each other. It is the flow in which fluid particles move in a zig-zag path.



Laminar flow   Turbulent flow.

When a fluid is flowing in a pipe, the type of flow is determined by a non-dimentional number, called the Reynold's number. (Re)

$$Re = \frac{\text{Inertia force}}{\text{viscous force}} = \frac{\rho v d}{\mu}$$

For laminar flow, Reynold's number < 2000

For turbulent flow, Reynold's number > 4000

Compressible and Incompressible flow

The flow in which the density of fluid changes, due to pressure and temperature variations, from point to point during the flow is called compressible flow.

$$\rho \neq \text{constant}$$
(density)

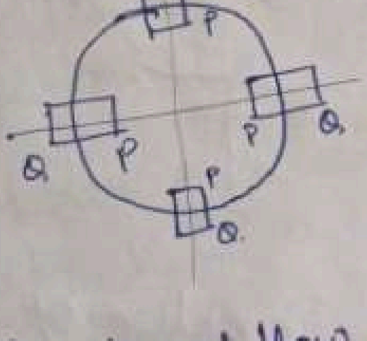
The flow in which the density of fluid does not change during the flow is called incompressible flow.

$$\rho = \text{constant.}$$

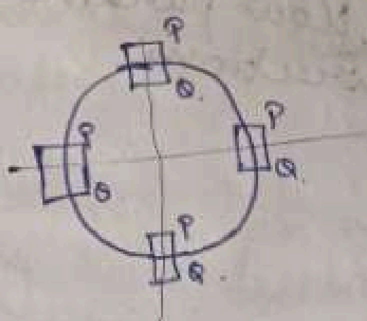
Rotational and Irrotational flow.

Rotational flow is that type of flow in which fluid particles also rotate about their own axes while flowing along a stream line.

Irrotational flow is that type of flow in which fluid particles do not rotate about their own axes while flowing.



Rotational flow.



Irrotational flow.

Ideal and Real flow.

An ideal flow is the flow of a non-viscous fluid. In the ideal flow, no shear stress exists between two adjacent layers or between the fluid layer and boundary, only normal stresses can exists in ideal flows.

Flow of real (viscous) fluids is called real flow. In real flow, shear stress exits between two adjacent fluid layers.

One, Two and Three Dimensional flow.

One dimensional flow is the flow in which parameter (velocity, pressure, density, viscosity and temperature) vary only in one direction.

Two dimensional flow is the flow in which fluid parameters vary along two directions.

Three dimensional flow is the flow in which flow parameters change in all the three directions.

Difference between Laminar flow and Turbulent flow.

Laminar flow	Turbulent flow
1. Fluid particles move in layers, one sliding over another.	Fluid particles move in random and zig-zag manner.
2. In adjacent layers, there is no intermixing of fluid particles.	In adjacent layers, there is intermixing of fluid particles.
3. It occurs in liquids having high viscosity.	It occurs in liquids having low viscosity.
4. For laminar flow, Reynold number is less than 2000.	For turbulent flow, Reynold number is greater than 4000.
5. Rare both in nature & in engg. practice.	In all flows in nature & engg. practice.

Rate of flow (Discharge)

The vol. of a fluid flowing per second through a section of a pipe is called discharge or rate of flow. It is denoted by Q.

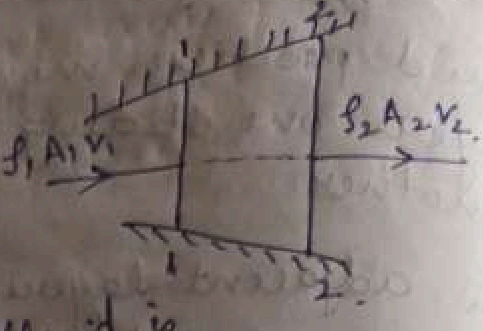
Consider a fluid flowing with an average velocity V in a pipe of cross-sectional area A. The discharge of the fluid is given by

$$Q = \text{Area} \times \text{velocity}$$
$$= A \times V.$$

It S.I units, is m^3/s

4.2 Continuity Equation

An equation which is based on the principle of conservation of mass is called continuity equation.



The mass of a fluid passing through different cross-section of a pipe, and its flow is the same if no fluid is added or removed from the pipe.

Let, A_1 = cross-sectional area at 1-1

ρ_1 = density of fluid at 1-1

V_1 = velocity of fluid at 1-1

A_2 = cross-sectional area at 2-2

ρ_2 = density of fluid at 2-2

V_2 = velocity of fluid at 2-2

Mass of fluid flowing in the pipe through 1-1 per second = Density × Velocity of flow × Area

$= \rho_1 \times V_1 \times A_1$

$= \rho_1 V_1 A_1$

Mass of fluid flowing out through 2-2 per second $= \rho_2 V_2 A_2$

According to the law of conservation of mass, mass of the fluid flowing in at 1-1 = mass of fluid flowing out at 2-2.

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}.$$

This is known as continuity Equation.

For incompressible fluid.

$$\rho_1 = \rho_2$$

$$\therefore \rho_1 A_1 V_1 = A_2 V_2.$$

Q. The water is flowing through a pipe line of 100mm diameter with a velocity of 1.5m/s. Determine the discharge through the pipe in litres/s.

Given: $d = 100mm$

$= 0.1m.$

$V = 1.5 m/s$

$Q = ?$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.1)^2 = 7.854 \times 10^{-3} m^2.$$

$$Q = A \times V.$$

$$= 7.854 \times 10^{-3} \times 1.5$$

$$= 7.854 \times 10^{-3} \times 1.5 \times 1000$$

$$= 11.78 \, l/s.$$

Bernoulli's theorem

Bernoulli's theorem states that in a steady continous flow of a frictionless incompressible fluid, the total energy at any point in the fluid is constant. The total energy is sum of pressure energy, kinetic energy and potential energy.

$$\frac{p}{w} + \frac{v^2}{2g} + z = \text{constant}$$

where, $\frac{p}{w}$ = pressure head.

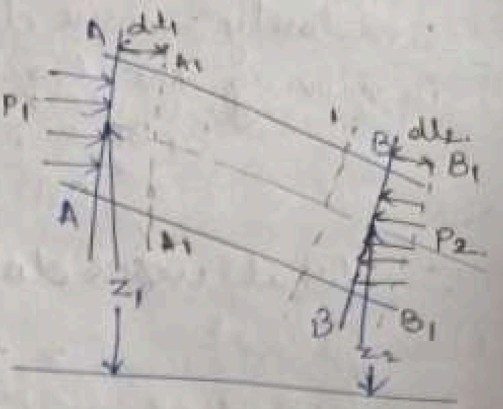
$\frac{v^2}{2g}$ = velocity head.

z = Potential head.

Assumptions

- → The flow is steady. i.e. $\frac{\partial \rho}{\partial t} = 0$
- → The fluid is ideal. i.e. viscosity is zero.
- → The flow is incompressible, i.e. ρ = const.
- → The flow is continous with uniform velocity
- → The flow is one dimentional. i.e. along a streamline.

Proof



Let the pipe be running full and there is a continuity of flow between the two sections.

Let z_1, p_1, v_1, and a_1 be the height above datum, pressure intensity, velocity and cross-sectional area of the pipe respectively at section A-A.

Similarly, z_2, p_2, v_2 and a_2 be the corresponding quantities at section B-B.

Let the liquid between the sections AA and BB moves to position A_1A_1 and B_1B_1 in an infinitely small interval of time.

Let W be the weight of liquid between AA and A_1A_1 or BB and B_1B_1.

As the flow is continous.

$$W = (a_1 dl_1)w = (a_2 dl_2)w.$$

$w = \rho g$ weight of fluid.

$$(a_1 dl_1) w = W \quad \therefore a_1 dl_1 = \frac{W}{w}$$

$$(a_2 dl_2) w = W \quad \therefore a_2 dl_2 = \frac{W}{w}$$

Work done by pressure at AA is moving the liquid to A_1A_1.

$= \text{Force} \times \text{distance}$

$= (p_1 a_1) \times dl_1 = p_1 a_1 dl_1$

Similarly, work done by pressure at BB in moving the liquid to

$B_1B_1' = -p_2 a_2 dl_2$. (Minus sign shows the direction of p_2 is opposite to p_1.)

$\therefore$ Total work done by pressure

$$= p_1 a_1 dl_1 - p_2 a_2 dl_2.$$

$$= p_1 \times \frac{W}{w} - p_2 \times \frac{W}{w}$$

$$= \frac{W}{w}(p_1 - p_2)$$

Loss of potential energy

$$= WZ_1 - WZ_2 = W(Z_1 - Z_2)$$

Gain in kinetic energy

$$= W\frac{V_2^2}{2g} - W\frac{V_1^2}{2g} = \frac{W}{2g}(V_2^2 - V_1^2)$$

$\therefore$ Loss of potential energy + work done by pressure

= Gain in kinetic energy.

$$W(Z_1 - Z_2) + \frac{W}{w}(p_1 - p_2) = \frac{W}{2g}(V_2^2 - V_1^2)$$

$$(Z_1 - Z_2) + \frac{1}{w}(p_1 - p_2) = \frac{1}{2g}(V_2^2 - V_1^2)$$

$$Z_1 - Z_2 + \frac{p_1}{w} - \frac{p_2}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$Z_1 + \frac{V_1^2}{2g} + \frac{p_1}{w} = Z_2 + \frac{V_2^2}{2g} + \frac{p_2}{w}$$

Limitations.

→ The derivation of Bernoulli's equation is based on the assumptions that the velocity is uniform over the cross-section of the pipe. Practically it is not true. The velocity of liquid is maximum at the centre and is zero at the boundary of the pipe.

→ No consideration for loss of energy is made in the equation, but there is bound to be certain loss of energy in liquid flow.

→ Loss of energy due to pipe friction is considered.

Applications.

This theorem is generally applied in hydraulics and applied hydraulics. The other practical applications are venturimeter and pitot tube.

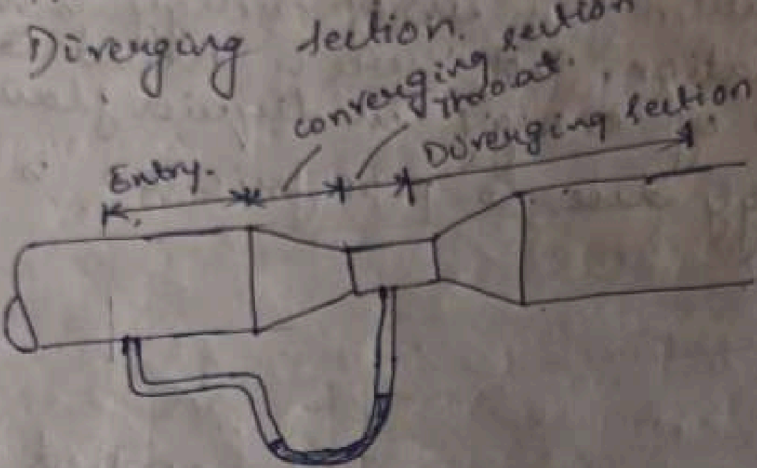
Venturimeter.

It was invented by Clemens Herschel in 1887. This is simple and reliable device used for extensively for measurement of water flow through large diameter pipes and for large flow rates.

Construction

The venturimeter consists of:

(i) cylindrical Entrance section.
(ii) Converging Conical section.
(iii) Throat.
(iv) Diverging section.



cylindrical Entrance section –

This section has a cylindrical pipe whose diameter is similar to the diameter of the pipe to which it is connected.

The length of straight portion should be 5 to 10 pipe diameter from the point where venturimeter is to be connected and in this straight portion there will be no fittings etc. to avoid turbulence.

ii) Converging conical section – In this portion pipe is tappered uniformly. The ~~throat~~ angle of convergence is 21°. The velocity of fluid increases as it passes through the converging section. In this section, the static pressure is less.

iii) Throat – This is a cylindrical section having minimum cross-sectional area. The throat diameter is about 1/4 to 1/2 of the inlet diameter and length equal to its diameter. It is made lined with brass and machined smooth to reduce friction.

iv) Diverging Section – In this section the diameter of the pipe changes from minimum to the diameter of the main pipe. The angle of divergence is 5° to 15°. This angle is kept small so that the flowing fluid has less tendency to seperate out from the boundary.

Advantages.

1. The loss of head due to installation in the pipelines is small. Due to low losses, the co-efficient of discharge is high.
2. Chances of clogging due to sedimentation is small because of smooth surface, so wear & tear is less.
3. This is ideally suited for large flow of waters suspended solids and gases etc.
4. As it is simple and extensively used. ~~[illegible]~~
~~[illegible]~~

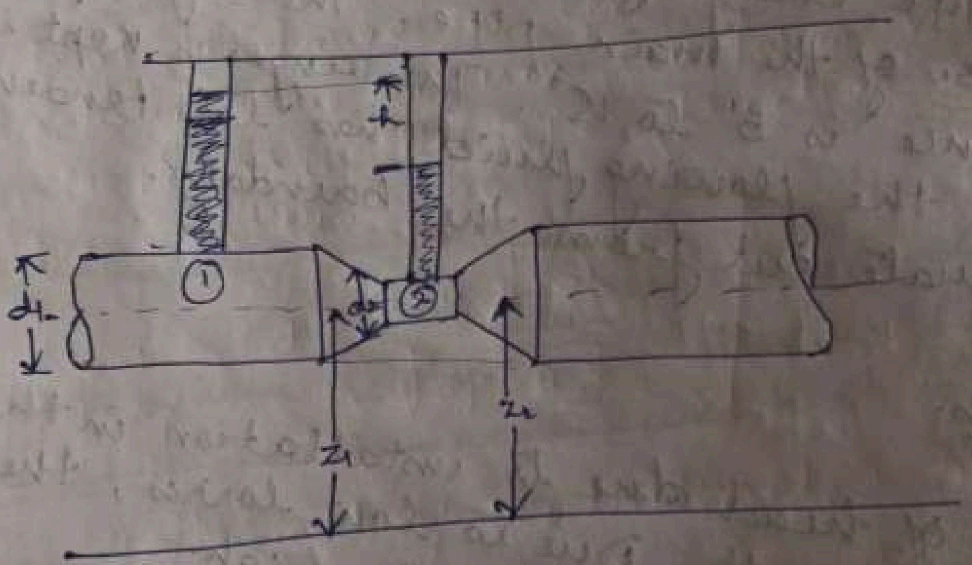
Limitations.

→ The space requirement is more due to its long length.

→ It is quite expensive in installation and replacement.

→ It is not suitable for pipe diameter below 75 mm.

Expression for Rate of flow through Venturimeter.



In section ①.

d_1 = dia of main pipe.

a_1 = cross-sectional area at section 1.

p_1 = press. at section 1.

Z_1 = Datum head at section 1

v_1 = velocity at section 1.

$$a_1 = \frac{\pi}{4} d_1^2.$$

Similarly. for section 2.

i.e, d_2, a_2, p_2, z_2, v_2.

$$z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}$$

$w \rightarrow$ sp. weight of the liquid in the pipe.

As the pipe is horizontal, $z_1 = z_2$.

$$\frac{v_1^2}{2g} + \frac{p_1}{w} = \frac{v_2^2}{2g} + \frac{p_2}{w}$$

$$\underbrace{\frac{p_1}{w} - \frac{p_2}{w}}_{h} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}.$$

$$h = \frac{v_2^2}{2g}\left(1 - \frac{v_1^2}{v_2^2}\right)$$

$h \rightarrow$ diff. in piezometeric head betn 1 and 2

Applying continuity equation at 1 and 2.

$$a_1 v_1 = a_2 v_2.$$

$$v_1 = \frac{a_2 v_2}{a_1}.$$

$$h = \frac{v_2^2}{2g}\left[1 - \frac{1}{v_2^2} \times \frac{a_2^2 v_2^2}{a_1^2}\right]$$

$$= \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$v_2^2 = 2gh \cdot \left(\frac{a_1^2}{a_1^2 - a_2^2}\right)$$

$$v_2 = \frac{a_1\sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

Discharge, $Q = a_2 v_2$

$$= a_2 \times \frac{a_1\sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$= \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

This discharge is under ideal condition, called theoretical discharge. Actual discharge is less than the theoretical discharge due to non-uniform velocity distribution in the pipe, friction and other losses.

$$\boxed{Q_{act} = C_d \times \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}}$$

C_d = coefficient of venturimeter or coefficient of discharge.

Its value is less than 1.0.

$h = y\left(\frac{S_m}{S} - 1\right)$, if $S_m > S$

$= y\left(1 - \frac{S_m}{S}\right)$, if $S > S_m$.

y = diff. in level of manometric liquids in two levels.

S_m = sp. gravity of manometric fluid.

S = " " liquid in the pipe)

Pitot Tube

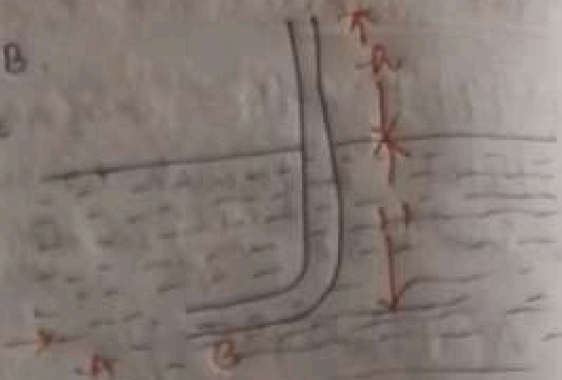
Pitot tube was named in the honour of Mr. Pitot, a french scientist, who used it first time in 1732 in France.

A Pitot tube is a device used to measure the velocity of flow at any point in a pipe or a channel. Pitot tube is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of kinetic energy into pressure energy.

It consists of a glass tube bent at right angle. The lower end known as body is placed in the direction of flow and the other end called the stem. is open to the atmosphere. The liquid rises up in the tube through a height h.

The pressure far away from the tube is known as static pressure and the pressure at the tip of the tube is called stagnation pressure and the point is known as stagnation point. At the stagnation point the fluid is in a state of zero velocity. At this point the velocity head of fluid converts into pressure head. So, rise of liquid in the stem is the sum of the static head and the velocity head.

Consider two points A and B.
The point B is just at the inlet of the pitot tube.
The point A is at some distance from B and is at the same level as that of B.



Point B is known as stagnation point.

Let p_1 = Press. at point A.

p_2 = " " " B.

v_1 = velocity of flow of liquid at A.

v_2 = " " " " " B.

h = Rise of liquid in the stem above the free surface.

H = Depth of stem in the liquid.

Applying Bernoulli's eq^n. at points A and B.

$$z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}.$$

$z_1 = z_2$ (As both A & B are at the same level & $v_2 = 0$)

$$\frac{v_1^2}{2g} + \frac{p_1}{w} = \frac{p_2}{w}.$$

$\frac{p_1}{w}$ = press. head at point A = H (static press.)

$\frac{p_2}{w}$ = pressure head at point B = H+h.
(stagnation pressure head).

$$\frac{v_1^2}{2g} + H = H + h.$$

$$\frac{v_1^2}{2g} = h.$$

$$v_1^2 = 2gh.$$

$$v_1 = \sqrt{2gh}.$$

Actual velocity is given by.

$$v_1 = C_v \sqrt{2gh}.$$

C_v = co-efficient of pitot tube = 0.95

A horizontal venturimeter 300 mm x 150 mm is used to measure the flow rate of water. The differential gauge connected to the inlet and throat shows a reading of 180 mm mercury. Find the rate of flow through the venturimeter. Take Cd = 0.85.

Given d_1 = 300mm = 0.3m. , S_m = 13.6

d_2 = 150mm = 0.15m. , S = 1.

y = 180mm of mercury = 0.18m.

Cd = 0.85.

$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(0.3)^2 = 0.071\, m^2.$

$a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(0.15)^2 = 0.018\, m^2.$

h = difference of press. head.

$h = y\left(\frac{S_m}{S} - 1\right) = 0.18\left(\frac{13.6}{1} - 1\right)$

$= 2.268\, m.$

$$Q = C_d \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$= \frac{0.85 \times 0.071 \times 0.018 \times \sqrt{2 \times 9.8 \times 2.268}}{\sqrt{(0.071)^2 - (0.018)^2}}$$

$= 0.1051\, m^3/s.$

Q. A venturimeter with a 150mm diameter at inlet and 100mm at throat is laid with its axis horizontal and is used for measuring the flow of oil specific gravity 0.9. The oil mercury differential manometer shows a gauge difference of 200mm. Assume coefficient of the metre as 0.98. Calculate the discharge in litres per minute

Given: $d_1 = 150mm = 0.15m$ $y = 200m = 0.2m.$

$d_2 = 100mm = 0.1m.$

sp. gravity of oil, $s = 0.9.$

$Cd = 0.98.$

$a_1 = \frac{\pi}{4}(d_1)^2 = \frac{\pi}{4}(0.15)^2 = 17.67 \times 10^{-3} m^2.$

$a_2 = \frac{\pi}{4}(d_2)^2 = \frac{\pi}{4}(0.1)^2 = 7.854 \times 10^{-3} m^2.$

$h = 0.2\left(\frac{13.6}{0.9} - 1\right)$

$= 2.82 m$ of oil.

$$Q = C_d \cdot \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$= \frac{0.98 \times 17.67 \times 10^{-3} \times 7.854 \times 10^{-3} \times \sqrt{2 \times 9.81 \times 2.82}}{\sqrt{(17.67 \times 10^{-3})^2 - (7.854 \times 10^{-3})^2}}$$

$= 63.9 \times 10^{-3} \; m^3/s.$

Q. A venturimeter has an area ratio of 9 to 1, the larger diameter being 300mm. During the flow, the recorded pressure head in the large section is 6.5 metres and that at the throat 4.25 m. If the metre coefficient = 0.99, compute the discharge through the metre.

$d_1 = 300mm = 0.3m.$

$\frac{a_1}{a_2} = 9:1$ $h_1 = 6.5 m.$ $h_2 = 4.25.$

$Cd = 0.99.$

$$a_1 = \frac{\pi}{4}(d_1)^2 = \frac{\pi}{4}(.3)^2$$
$$= 70.69 \times 10^{-3}\, m^2$$

$$\cancel{a_2 = \frac{\pi}{4}(d_2)^2 = \frac{\pi}{4} \times}$$

$$a_2 = \frac{a_1}{9}$$
$$= \frac{70.69 \times 10^{-3}}{9} = 7.854 \times 10^{-3}\, m^2$$

$$h = h_1 - h_2$$
$$= 6.5 - 4.25 = 2.25\, m$$

$$Q = C_d \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$
$$= \frac{0.99 \times 70.69 \times 10^{-3} \times 7.854 \times 10^{-3} \times \sqrt{2 \times 9.81 \times 2.25}}{\sqrt{(70.69 \times 10^{-3})^2 - (7.854 \times 10^{-3})^2}}$$

$$= 52 \times 10^{-3}\, m^3/s$$

Q. A static pitot tube is used to measure the velocity of flow of water in a pipe line. Determine the velocity of flow at a location where stagnation pressure head and static pressure head are recorded as 3.6 m and 2.4 m of water respectively. Take co-efficient of pitot tube = 0.98.

Given: $h_2 = 3.6\text{ m}$

$h_1 = 2.4\text{ m}$

$C_v = 0.98$

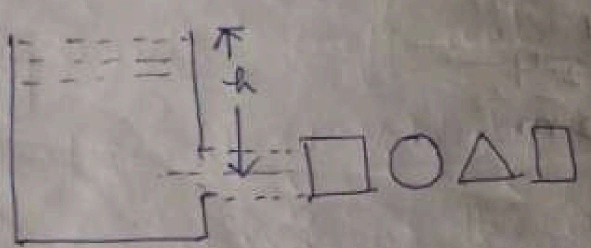
$$h = h_2 - h_1$$
$$= 3.6 - 2.4$$
$$= 1.2\text{ m}$$

$$v_1 = C_v\sqrt{2gh}$$
$$= 0.98\sqrt{2 \times 9.81 \times 1.2} = 4.75\text{ m/s}$$

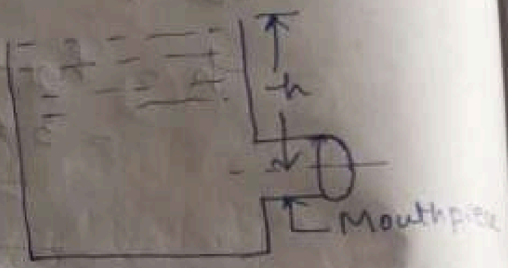
Orifices, notches and weirs.

5.1 Define Orifice.

An orifice is a small opening in the wall or bottom of a tank through which a fluid flows out. The top edge of the opening lies below the level of the fluid in the tank. An orifice is used to measure the discharge of a fluid from a tank or reservoir. The cross-section of the opening may be square, rectangular, triangular or circle.



Geometry of orifice



Mouthpiece fitted in a tank

A mouthpiece is a short length or pipe fitted in a tank containing the fluid. The length of the pipe is usually 2 to 3 times its diameter. Mouthpiece is also used to measure discharge.

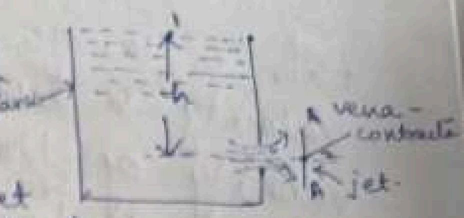
5.2 Flow through Orifice.

Consider a tank fitted with a circular orifice at one side of the tank.

Let h be the head of liquid in the tank above the centre of the orifice.

The liquid flowing out of the tank through the orifice forms a jet of liquid and attains a parallel form approximately at a distance of half the diameter of the orifice.

The point where the stream lines first become parallel is known as vena-contracta.



The cross-sectional area of the jet is minimum at this point and is less than the area of the orifice.

Beyond vena-contracta, the jet diverges and is attracted in the downward direction by gravity.

Consider two points 1 and 2. Point 1 is on the free surface of the liquid and point 2 is on the vena-contracta. Let the flow be steady.

Applying Bernoulli's equation at 1 and 2.

$$\frac{p_1}{w} + \frac{v_1^2}{2g} + h = \frac{p_2}{w} + \frac{v_2^2}{2g} + 0$$

The pressure head at both the points 1 and 2 is atmospheric.

$$\therefore \frac{p_1}{w} = \frac{p_2}{w} = 0.$$

The velocity v_1 and the free surface of liquid is practically zero as compared to velocity (v_2) at vena-contracta.

$$v_1 = 0.$$

$$\therefore h = \frac{v_2^2}{2g}.$$

$$V_2^2 = 2gh.$$

$$V_2 = \sqrt{2gh}.$$

This is the theoretical velocity and actual velocity is less than this velocity. The velocity of flow through an orifice is equal to the velocity of free fall from the surface of the tank. This is known as Torricelli's theorem.

5.3 Hydraulic coefficients

Coefficient of Velocity (cv)

The ratio of the actual velocity of the jet at vena-contracta to the theoretical velocity of the jet is called coefficient of velocity.

$$C_v = \frac{\text{Actual Velocity of jet at vena-Contracta}}{\text{Theoretical velocity}}$$

$$= \frac{v}{V} = \frac{v}{\sqrt{2gh}}$$

The value of C_v – 0.95 to 0.99.

$C_v = 0.98$ for sharp crested orifice

Coefficient of Contraction (C_c)

The ratio of area of jet at vena-contracta to the area of the orifice is known as coefficient of contraction.

$$C_c = \frac{\text{Area of jet at Vena-contracta}}{\text{Area of orifice}}$$

$$= \frac{a_c}{a}$$

The value of C_c varies from 0.61 to 0.69.

$C_c = 0.65$ – sharp crested orifice

Coefficient of Discharge (C_d)

The ratio of actual discharge of the orifice to the theoretical discharge is called coefficient of discharge.

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{Q_a}{Q_t}$$

$$= \frac{Q_a}{a \times \sqrt{2gh}}$$

$C_d = 0.61$ to 0.65. , avg. value – 0.62.

Relation between C_d, C_c and C_v.

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}}$$

$$= \frac{\text{Actual velocity} \times \text{area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$

$$= C_v \times C_c$$

$$\boxed{C_d = C_v \times C_c}$$

Q. The head of water over an orifice of diameter 50 mm is 12 m. Find the actual discharge and actual velocity of jet at vena-contracta. Take $C_d = 0.6$ and $C_v = 0.98$.

Given: $d = 50\text{mm} = 0.05\text{m}$

$h = 12\text{m}$

$C_d = 0.6$

$C_v = 0.98$

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.05)^2 = 0.002\ m^2$$

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}}$$

actual discharge $= Cd \times$ Theoretical discharge

$= Cd \times \sqrt{2gh} \times a$.

$= 0.6 \times \sqrt{2 \times 9.81 \times 12} \times 0.002$.

$= 0.018\ m^3/s$.

$Cv = \dfrac{\text{actual velocity}}{\text{Theoretical velocity}}$

actual velocity $= Cv \times$ theoretical velocity.

$= Cv \times \sqrt{2gh}$.

$= 0.98 \times \sqrt{2 \times 9.81 \times 12}$

$= 15.04\ m/s$.

Q. The head of water over the centre of an orifice of diameter 30mm is 1.5 m. The actual discharge through the orifice is 2.35 l/sec. Find the coefficient of discharge.

$d = 30\ mm = 0.03\ m$.

$h = 1.5\ m$.

$Qa = 2.35\ l/sec. = 0.00235\ m^3/sec$

$a = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(0.030)^2$

$= 0.00071\ m^2$.

Theoretical velocity $= \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.5}$

$= 5.425\ m/s$.

Theoretical discharge,

Q_t = area of orifice × theoretical discharge

= 0.00071 × 5.45

= 0.00385 m^3/s.

C_d = $\frac{\text{Actual discharge}}{\text{Theoretical discharge}}$ = $\frac{Q_a}{Q_t}$

= $\frac{0.00235}{0.00385}$ = 0.61.

5.4 Notches and Weirs

Notch — A notch is an opening in the side of a tank or a vessel in such away that the liquid surface is below the top edge of the opening.

It is a device used to measure the rate of flow of a liquid through a tank or a small channel.

Notch is a very thin structure, usually made of a metallic plate.

Weir -An weir is an obstruction constructed across the channel, which causes the upstream level to rise until the flow occurs.

A weir is a concrete or masonry structure across a stream or an open channel over which the fluid flow occurs.

A weir is a concrete or masonry structure across a stream or an open channel over which the liquid flow occurs. A weir is a notch on a large scale.

Nappe or Vein — The sheet of water flowing through a notch or weir is known as nappe or vein.

Crest or Sill — The bottom edge of a notch or the top edge of a weir over which water flows is known as crest or sill.

Difference between Notch and weir.

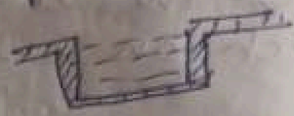
Notch	Weir
→ Water flows through a notch.	Water flows over a weir.
→ Notch is a small structure.	Weir is a large structure.
→ It is generally made of metallic plate.	It is made of concrete or masonry.
→ The edges of notch are thin and sharp.	Crest of a weir is wide in the direction of flow
→ It measures a small discharge from a reservoir.	It measures large discharge from a river or a open channel.

Classification of Notches and Weir.

Notches.

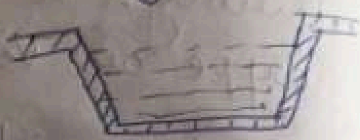
Notches are classified as

(i) On the basis of their shape of opening.



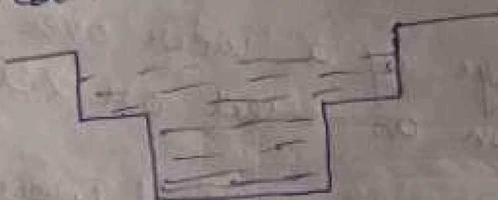
(a) Rectangular notch.

(b) Triangular notch (v-notch)



(c) Trapezoidal notch.

(d) Stepped notch.



Weirs.

i) According to shape:

→ Rectangular Notch.

→ Triangular notch.

→ Trapezoidal notch (cippoletti weir)

ii) According to the shape of crest.

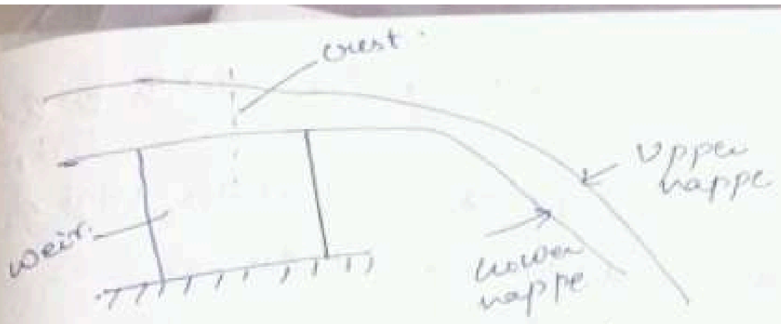
→ Narrow crested weir.

→ Broad crested weir

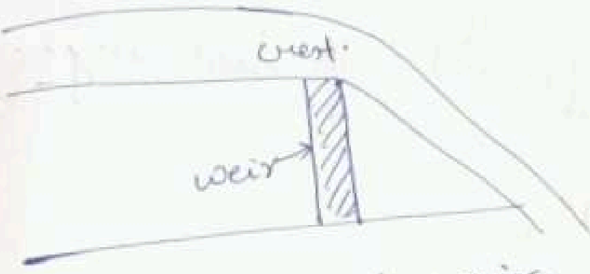
→ Sharp crested weir

→ Ogee crested weir.

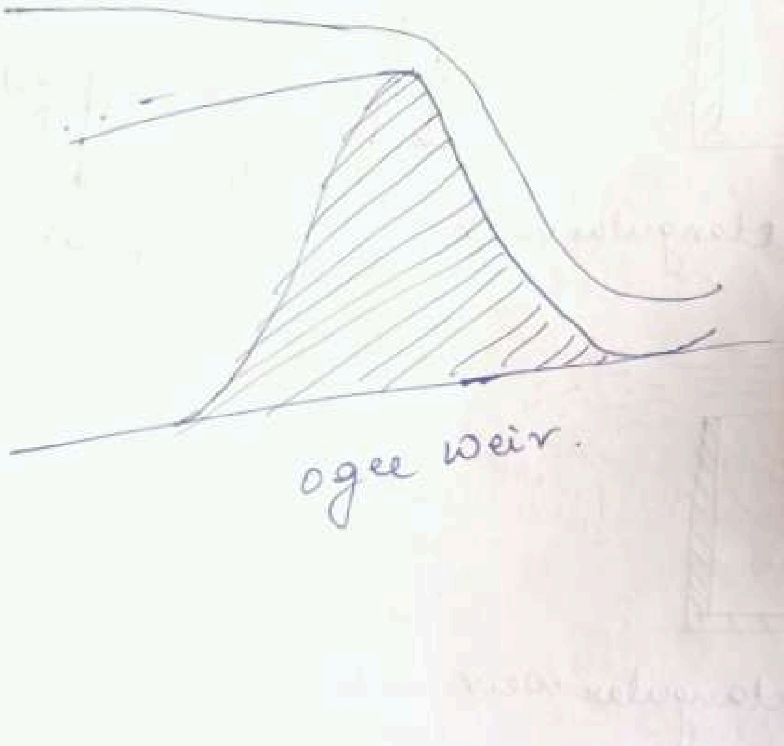
narrow crested weir.



Broad crested weir.



Sharp crested weir.



ogee weir.

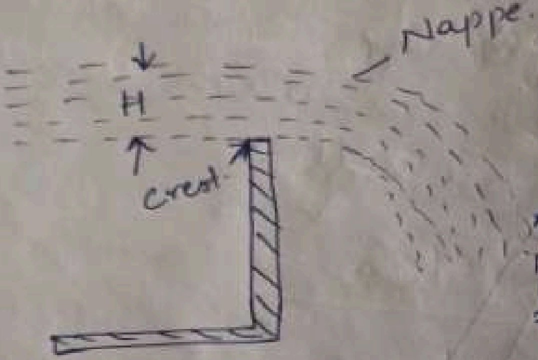
Discharge over a Rectangular Notch or Weir.

Consider a rectangular notch or weir in a channel carrying water.

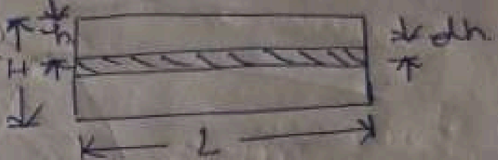
Let L = Length of the notch or weir.

H = Height of water over the crest.

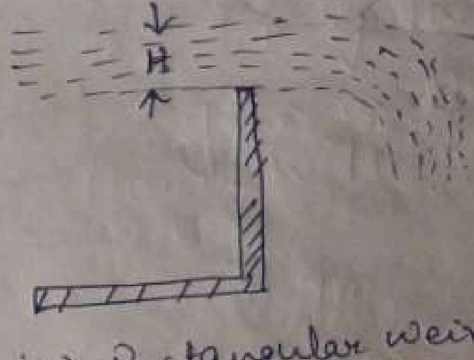
C_d = Coefficient of discharge



(i) Rectangular notch



(ii) Section at crest.



(iii) Rectangular weir.

consider an elementary horizontal strip of water of thickness dh at a depth h from the free surface of water.

$\therefore$ Area of strip of water $= L \cdot dh$

theoretical velocity of water through the strip $= \sqrt{2gh}$.

The discharge through the strip, dQ

$$dQ = C_d \times \text{area of the strip} \times \text{theoretical velocity}$$

$$= C_d \times L \cdot dh \times \sqrt{2gh}.$$

Total discharge,

$$Q = \int_0^H C_d L \sqrt{2gh}\, dh$$

$$= C_d L \sqrt{2g} \int_0^H \sqrt{h}\, dh.$$

$$= C_d L \sqrt{2g} \cdot \left[\frac{h^{3/2}}{3/2}\right]_0^H$$

$$= C_d L \sqrt{2g} \times \frac{2}{3} H^{3/2}.$$

$$= \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}.$$

Q. Find the discharge of water flowing through a rectangular notch of 3m length when the constant head over the notch is 200mm. Take co-efficient of discharge as 0.65.

$L = 3m$

$H = 200mm = 0.2m$

$C_d = 0.65$

$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$

$= \frac{2}{3} \times 0.65 \times 3 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2}$

$= 0.515 \; m^3/s$

Q. A rectangular weir 0.9m wide and 1.2m deep is discharging water from a vessel. The top of the weir is 0.6m below the water surface in the vessel. Calculate discharge through the weir. Take $C_d = 0.6$.

$L = 0.9m$ $\quad h = 1.2m$

$H = 0.6m$

$C_d = 0.6$

$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$

$= 0.7411 \; m^3/s$

A rectangular notch has a discharge of 30 m^3/min, when the head of water is half the length of the notch. Find the length of the notch. Take $C_d = 0.62$

Given

$Q = 30\ m^3/min = \frac{30}{60} = 0.5\ m^3/s.$

$C_d = 0.62.$

Let L be the length of the notch.

H = head of water $= \frac{L}{2}$.

Discharge, $Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}.$

$$0.5 = \frac{2}{3} \times 0.62 \times L \sqrt{2g} \left(\frac{L}{2}\right)^{3/2}.$$

$$\frac{0.5 \times 3 \times 2^{3/2}}{2 \times 0.62 \times \sqrt{2 \times 9.81}} = L \times L^{3/2}.$$

$$\frac{4.243}{5.492} = L^{5/2}.$$

$$\left(\frac{4.243}{5.492}\right)^{2/5} = L.$$

$$L = 0.9\ m.$$

Discharge over a Triangular Notch or Weir.

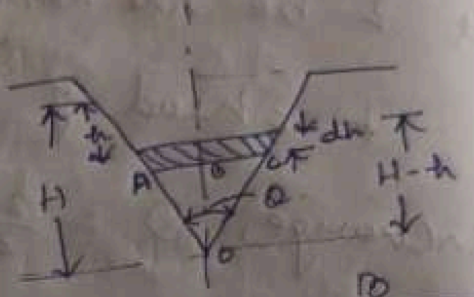
The discharge over a triangular notch is

The discharge over a triangular notch is the apex of the notch

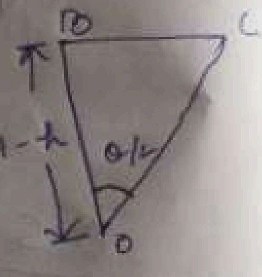
H = head of water above the apex of the notch

θ = angle of notch

C_d = co-efficient of discharge



Let us consider a strip of water of thickness 'dh' at a distance h from the free water surface.



$$\tan\frac{\theta}{2} = \frac{BC}{OB} = \frac{BC}{H-h}$$

$$BC = (H-h)\tan\frac{\theta}{2}.$$

width of strip $AC = 2BC = 2(H-h)\tan\frac{\theta}{2}.$

area of strip $= AC \times dh.$

$$= 2(H-h)\tan\frac{\theta}{2}\cdot dh.$$

Theoretical velocity through this strip $= \sqrt{2gh}.$

discharge, $dQ = C_d \times$ Area of the strip $\times$ theoretical velocity

$$= C_d \times 2(H-h)\tan\frac{\theta}{2}\, dh \times \sqrt{2gh}$$

$$= C_d \times \tan\frac{\theta}{2}\sqrt{2g}\, h^{1/2}(H-h)\,dh.$$

$$dQ = 2C_d \tan\frac{\theta}{2}\sqrt{2g}\,(H h^{1/2} - h^{3/2})\,dh.$$

Total discharge, $Q = \int_0^H 2C_d \tan\frac{\theta}{2}\sqrt{2g}\,(H h^{1/2} - h^{3/2})\,dh$

$$= 2C_d \tan\frac{\theta}{2}\sqrt{2g}\int_0^H (H h^{1/2} - h^{3/2})\,dh.$$

$$= 2C_d \tan\frac{\theta}{2}\sqrt{2g}\left[\frac{H h^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$= 2C_d \tan\frac{\theta}{2}\sqrt{2g}\left[\frac{2}{3}H\cdot h^{3/2} - \frac{2}{5}h^{5/2}\right]_0^H$$

$$= 2C_d \tan\frac{\theta}{2}\sqrt{2g}\left[\frac{2}{3}H\cdot H^{3/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2C_d \tan\frac{\theta}{2}\sqrt{2g}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2C_d \tan\frac{\theta}{2}\sqrt{2g}\left[\frac{4}{15}H^{5/2}\right].$$

$$= \frac{8}{15}C_d \tan\frac{\theta}{2}\sqrt{2g}\,H^{5/2}$$

for a right angled triangular notch.

If $\theta = 90^\circ$;

$$\tan\frac{\theta}{2} = \tan\frac{90}{2} = \tan 45 = 1.$$

$$Q = \frac{8}{15}C_d \times 1 \times \sqrt{2g}\,H^{5/2}.$$

$$= \frac{8}{15}C_d\sqrt{2g}\,H^{5/2}.$$

Assuming $C_d = 0.6$

discharge, $Q = \frac{8}{15} \times 0.6 \times \sqrt{2 \times 9.81} \times H^{5/2}$.

$$\boxed{Q = 1.417\, H^{5/2}}$$

Q. A right angled triangular notch is discharging water under a constant head of 300 mm. What will be the discharge, if C_d for the notch is 0.61.

Given. $\theta = 90^\circ$

$\tan\frac{\theta}{2} = \tan\frac{90}{2} = \tan 45^\circ$

$H = 300$ mm $= 0.3$ m

$C_d = 0.61$.

$$Q = \frac{8}{15} C_d \tan\frac{\theta}{2} \sqrt{2g}\, H^{5/2}.$$

$$= \frac{8}{15} \times 0.61 \times \tan 45^\circ \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$$= 1.441\, (0.3)^{5/2}$$

$$= 0.071\ m^3/s.$$

In a laboratory experiment, 14.8 litres of water was found to be discharged over a right angled V-notch under a head of 160 mm. Determine the co-efficient of discharge for the notch.

$Q = 14.8\ l/s = \frac{14.8}{1000} = 0.0148\ m^3/s$

$\theta = 90^\circ$

$H = 160mm = 0.16m$

$Cd = ?$

$Q = \frac{8}{15} Cd \tan\frac{90}{2} \times \sqrt{2g}(H)^{5/2}$

$0.0148 = \frac{8}{15} \times Cd \times \tan 45 \times \sqrt{2 \times 9.81} \times (0.16)^{5/2}$

$0.0148 = 0.0242\ Cd$

$Cd = \frac{0.0148}{0.0242} = 0.61.$

Q. Water flows over a rectangular weir 800mm wide at a depth of 120 mm and afterwards passes through a triangular right angled weir. Taking Cd for the rectangular and triangular weir as 0.62 and 0.59 respectively, find the depth over the triangular weir.

For rectangular weir

$L = 800mm = 0.8m$

$H = 120 mm = 0.12m.$

$C_d = 0.62.$

Discharge over rectangular weir

$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}.$

$= \frac{2}{3} \times 0.62 \times 0.8 \times \sqrt{2 \times 9.81} \times (0.12)^{3/2}.$

$= 0.061 m^3/s.$

For ~~rectangular~~ triangular weir.

$Q = 0.061 m^3/s.$

$C_d = 0.59$

$\theta = 90^\circ$

$H_1 = ?$

Discharge over triangular weir

$Q = \frac{8}{15} C_d \tan\frac{\theta}{2} \sqrt{2g} H_1^{5/2}.$

$0.061 = \frac{8}{15} \times 0.59 \times \tan\frac{90}{2} \times \sqrt{2 \times 9.81} \times H_1^{5/2}$

$= 1.394 H_1^{5/2}.$

$H_1^{5/2} = \frac{\cancel{1.394} \quad 0.061}{1.394}$

$= 0.044.$

$H_1 = (0.044)^{2/5} = 0.29m$

Questions.

1. Define an orifice and a mouth piece. What is the difference between the two?
2. What do you mean by orifice? What is its function?
3. What is vena- contracta? Draw a sketch showing vena- contracta
4. Derive the expression $C_d = C_v \times C_c$
5. Define the following co-efficients:
(i) coefficient of velocity.
(ii) coefficient of contraction
(iii) coefficient of discharge.

Chapter 6 Flow through Pipes

Pipe - The pipe is defined as the closed conduits used to carry of liquids under pressure. The liquid in a pipe is under pressure if it fills the entire cross-section of the pipe and there is no free surface in the pipe.

Loss of energies in Pipes:

Whenever any liquid or fluid flows through the pipe, then it experience resistance in its flow. Due to the resistance, the velocity and head of liquid are reduced. The losses are of generally two types. like (a) major loss (b) minor loss.

Head losses.

- Major head loss
- Minor head loss.

Major head loss - is due to friction between pipe and fluid. It is calculated by using following formulae:

(a) Darcy weisbach formula
(b) Chezy's formula.

Minor head loss is due to

(i) sudden change in area i.e. expansion or contraction.

(ii) change in direction i.e bend in pipe

(iii) Pipe fittings.

(iv) Any other obstruction in pipe

Darcy - Weisbach formula

The loss of head (or energy) due to friction in pipes is determined by using Darcy.

$$h_f = \frac{4fLv^2}{2gd}$$

where, h_f = loss of head due to friction.

L = length of pipe

d = Diameter of pipe.

v = Mean velocity of flow.

f = Co-efficient of friction, a function of Reynolds number.

Darcy coefficient of friction

$= \frac{16}{Re}$ for $Re < 2000$.

$= \frac{0.079}{Re^{1/4}}$ for Re ranging between 4000 to 10^6.

$$f = 0.005\left(1+\frac{1}{12d}\right),$$ for new cast iron and steel pipes. (smooth pipe & new one)

$$f = 0.01\left(1+\frac{1}{12d}\right),$$ for old cast iron & steel pipes. (rough pipe & old one)

Chezy's formula.

$$V = C\sqrt{mi}$$

where, V = Average velocity of flow through pipe

C = Chezy's const. $= \sqrt{\frac{\rho g}{f'}}$

f' = Frictional resistance per unit area per unit velocity.

m = Hydraulic mean depth $= \frac{A}{P} = \frac{\text{Area of flow}}{\text{Wetted perimeter}}$

i = Hydraulic gradient $= \frac{h_f}{L}$.

h_f = Loss of head due to friction.

L = Length of pipe.

Wetted perimeter of a pipe is the perimeter which remains is contact with the following fluid.

Hydraulic mean depth is the ratio of cross-sectional area of flow to the wetted perimeter of the pipe. It is denoted by m.

$$m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}.$$

Q. Find the head lost due to friction in a pipe of diameter 250 mm and length 60 m through which water is flowing at a velocity of 3.0 m/s, using (i) Darcy formula
(ii) Chezy's formula for which C = 55.
Take ν for water = 0.01 stroke.

Given $d = 250\,mm = 0.25\,m$.

$L = 60\,m$.

$V = 3.0\,m/s$.

$C = 55$

$\nu = 0.01$ stroke $= 0.01 \times 10^{-4}\,m^2/s$.

(i) Darcy's formula for head loss due to friction

$$h_f = \frac{4fLV^2}{2gd}$$

$$Re = \frac{Vd}{\nu} = \frac{3 \times 0.25}{0.01 \times 10^{-4}} = 7.5 \times 10^5$$

$$f = \frac{0.079}{Re^{1/4}}$$

$$= \frac{0.079}{(7.5 \times 10^5)^{1/4}} = 2.68 \times 10^{-3}$$

$$h_f = \frac{4 \times 2.68 \times 10^{-3} \times 60 \times 3^2}{2 \times 9.81 \times 0.25}$$

$$= 1.18\,m.$$

(ii) Chezy's formula for head loss due to friction

$V = C\sqrt{mi}$

$C = 55$, $m = \frac{d}{4} = \frac{0.25}{4} = 0.0625$.

$3 = 55\sqrt{0.0625 \times i}$

$\left(\frac{3}{55}\right)^2 = 0.0625 \times i$

$i = \left(\frac{3}{55}\right)^2 \times \frac{1}{0.0625} = 0.0476$

$i = \frac{h_f}{L}$.

$h_f = L \times i$

$= 0.0476 \times 60$.

$= 2.856$ m.

Q. Find the head lost due to friction in a pipe of diameter 300mm and length 100m through which water is flowing at a rate 3m/sec using Dracy's formula. Take ν for water $= 0.01$ stoke

$d = 300$ mm

$L = 100$ m.

$V = 3$ m/sec

$\nu = 0.01$ stoke $= 0.01 \times 10^{-4}$ m^2/s.

$$Re = \frac{Vd}{\nu} = \frac{3 \times 0.3}{0.01 \times 10^{-4}} = 9 \times 10^{5}.$$

$$f = \frac{0.079}{Re^{1/4}}.$$

$$= \frac{0.079}{(9 \times 10^{5})^{1/4}} = 2.56 \times 10^{-3}$$

Dracy's formula is given by

$$h_f = \frac{4fLV^2}{2gd} = \frac{4 \times 2.56 \times 10^{-3} \times 100 \times 3^2}{2 \times 9.81 \times 0.3}$$

$$= 1.566 \text{ m}.$$

Q. Find the diameter of a pipe of length 2500 m when the rate of flow of water through the pipe is 0.25 m^3/s and head lost due to friction is 5 m. Take C = 50 in Chezy's formula.

$L = 2500 m.$

$Q = 0.25 \, m^3/s.$

$h_f = 5 m$

$C = 50$

Let d be the diameter of the pipe and v be the velocity of flow.

$$A = \pi/4 \, d^2$$

$$m = \frac{d}{4}.$$

$$V = \frac{Q}{A} = \frac{0.25}{\pi/4 \, d^2} = \frac{0.25 \times 4}{\pi d^2}.$$

$$i = \frac{h_f}{L} = \frac{5}{2500} = 0.002.$$

Chezy's formula is given by,

$$V = C\sqrt{mi}$$

$$\frac{0.25 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times 0.002}$$

$$\frac{0.25 \times 4}{\pi d^2 \times 50} = \sqrt{\frac{d}{4} \times 0.002}$$

$$\frac{6.37 \times 10^{-3}}{d^2} = \sqrt{\frac{d}{4} \times 0.002}$$

$$\left(\frac{6.37 \times 10^{-3}}{d^2}\right)^2 = \frac{d}{4} \times 0.002$$

$$\frac{4.06 \times 10^{-5}}{d^4} = \frac{d}{4} \times 0.002.$$

$$\frac{4.06 \times 10^{-5} \times 4}{0.002} = d^5.$$

$$d^5 = 0.0812$$

$$d = (0.0812)^{1/5} = 0.605\ m.$$

$$= 605\ mm.$$

Q. An oil of kinematic viscosity 0.5 strokes flowing through a pipe of diameter 300 mm at the rate of 320 litres per sec. Find the head lost due to friction for a length of 60 m of the pipe.

$\nu = 0.5$ stroke $= 0.5 \times 10^{-4}\ m^2/s$.

$d = 300\ mm = 0.3\ m$

$Q = 320$ litres/s. $= \frac{320}{1000} = 0.32\ m^3/s$

$L = 60\ m$.

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.3)^2 = 0.071\ m^2$.

Let V be the velocity of flow.

$$V = \frac{Q}{A} = \frac{0.32}{0.071} = 4.51\ m/s.$$

$$Re = \frac{Vd}{\nu} = \frac{4.51 \times 0.3}{0.5 \times 10^{-4}} = 27060$$

$$f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(27060)^{1/4}} = 6.15 \times 10^{-3}$$

Head loss due to friction is given by

$$h_f = \frac{4fLV^2}{2gd} = \frac{4 \times 6.15 \times 10^{-3} \times 60 \times (4.51)^2}{2 \times 9.81 \times 0.3}$$

$$= \frac{30.02}{5.89} = 5.1\ m$$

Hydraulic Gradient Line (H.G.L)

Hydraulic gradient line is the line which shows the sum of pressure head $\left(\frac{P}{w}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line.

H.G.L gives the total energy at any point in the flowing fluid less the kinetic energy.

$$H.G.L = \frac{P}{w} + Z$$

Total Energy Line (T.E.L)

Total energy line is the line which shows the total heads i.e. sum of pressure head (P/w), kinetic head ($v^2/2g$) and datum head (z) of a flowing fluid in a pipe with respect to some reference line. TEL gives the total energy at any point in the flowing fluid.

$$T.E.L = \frac{P}{w} + z + \frac{V^2}{2g}$$

The following facts should be kept in mind regarding total energy line and hydraulic gradient line.

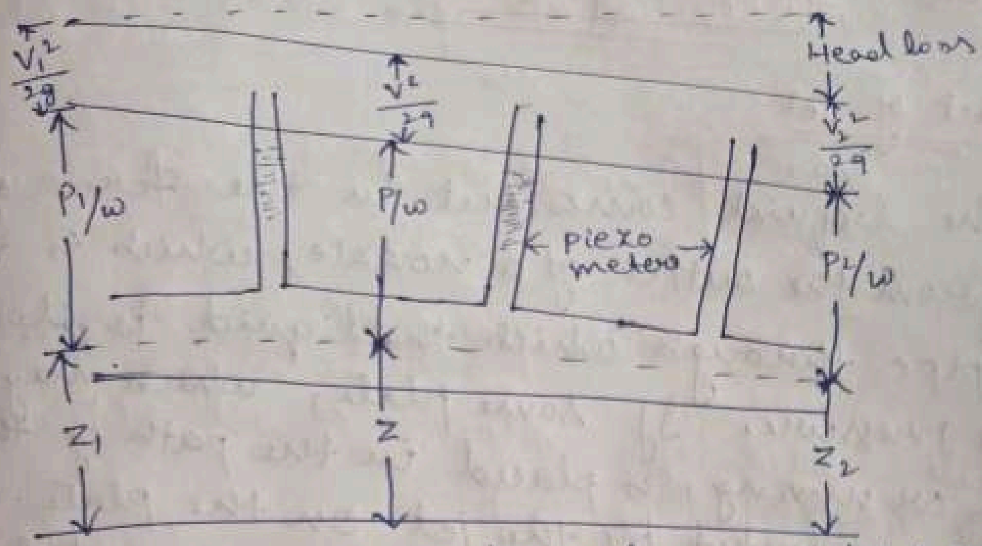
(i) Hydraulic gradient line is always below the total energy line.

(ii) Both the lines coincide when the velocity head is zero.

(iii) The vertical intercept between the two lines is equal to the kinetic head ($\frac{V^2}{2g}$)

(iv) Hydraulic gradient line may rise or fall depending upon the pressure and velocity changes.

(v) Due to different head losses, total energy line always slopes downward in the direction of flow.



Hydraulic gradient line and total energy line.

Chapter 7 - Impact of Jets

Impact of Jet

The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressure. If some plate, which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate. This force is obtained from Newton's second law of motion or from impulse - momentum equation. Thus impact of jet means the force exerted by the jet on a plate which may be stationary or moving.

The force exerted by the jet on a plate

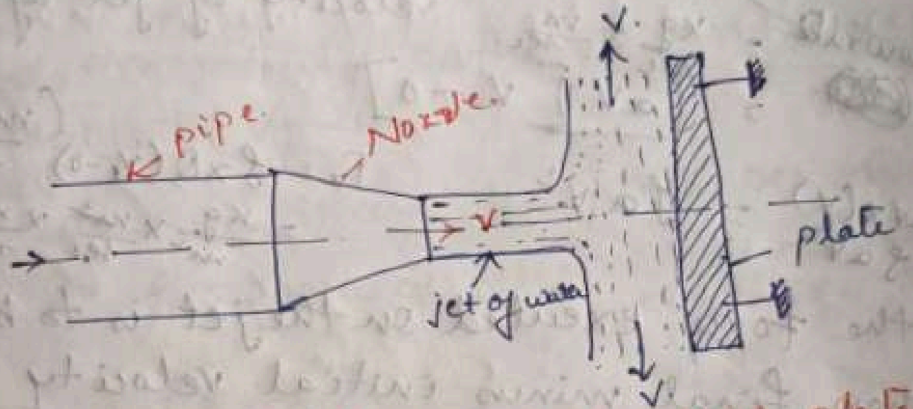
1. Force exerted by the jet on a stationary plate when
   (a) Plate is vertical to the jet
   (b) Plate is inclined to the jet
   (c) Plate is curved.
2. Force exerted by the jet on a moving plate when
   (a) Plate is vertical to the jet
   (b) Plate is inclined to the jet
   (c) Plate is curved.

Force Exerted by the Jet on a Stationary Vertical Plate

OR

Impact of jet on fixed vertical flat plates

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate.



Force exerted by jet on vertical plate

The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking will get deflected through 90°. Hence the component of the velocity of jet, in the direction of jet, after striking will be zero.

The force exerted by the jet on the plane in the direction of jet,

$F_x = ma$ F_x = Rate of change of momentum in the direction of force

$= m\frac{dv}{dt}$

$= m\frac{(v_1 - v_2)}{dt}$ = $\frac{\text{Initial momentum - Final momentum}}{\text{time}}$

$a = \frac{dv}{dt} = \frac{v_1 - v_2}{t_1 - t_2}$

$= \text{mass} \times \dfrac{\text{Initial velocity} - \text{mass} \times \text{final velocity}}{\text{Time}}$

$= \dfrac{\text{mass}}{\text{time}} \left[\text{Initial velocity} - \text{final velocity} \right]$

$= \left(\dfrac{\text{mass}}{\text{sec}}\right) \left[\text{velocity of jet before stricking} - \text{velocity of jet after striking} \right]$

mass flow rate

$= \dfrac{kg}{sec} \times \dfrac{m}{sec}$

$= \rho a V [V - 0]$ $\left(\dfrac{\text{mass}}{\text{sec}} = \rho \times aV\right)$

$= \rho a V^2$

$F_x = \rho \times Q (V_1 - 0)$

$= \dfrac{kg}{m^3} \times \dfrac{m^3}{sec} \times \dfrac{m}{sec}$

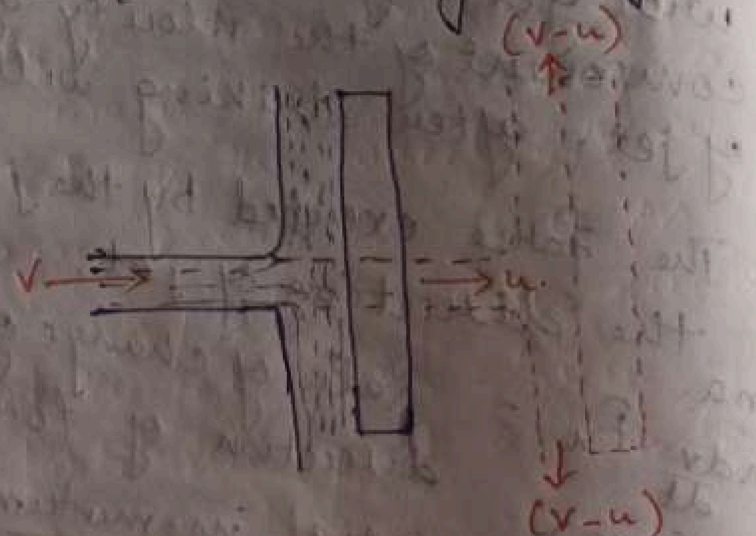
$= \rho \times aV \times (V)$
$= \rho a V^2$

If the force exeuted on the jet is to be calculated, then final minus initial velocity is taken.

*. for stationary plate workdone is zero.

Impact of jet on moving vertical flat plate

Jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.



Jet striking a flat vertical moving plate.

Let V = velocity of the jet.
a = Area of cross-section of the jet
u = Velocity of the flat plate.

The jet does not strike the plate with a velocity V, but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus the velocity of the plate $(V-u)$

Hence relative velocity of the jet with respect to plate $= (V-u)$

Mass of water striking the plate per sec
$= \rho \times$ Area of jet $\times$ velocity with which jet strikes the plate.

$= \rho \times a \times [V-u]$

$\therefore$ Force exerted by the jet on the moving plate in the direction of the jet

F = Mass of water striking the plate per sec $\times$ [Initial velocity with which water strikes − final velocity]

$= \rho a(V-u)[(V-u)-0]$

$= \rho a(V-u)^2$.

The workdone by the jet on the plate, as the plate is moving.

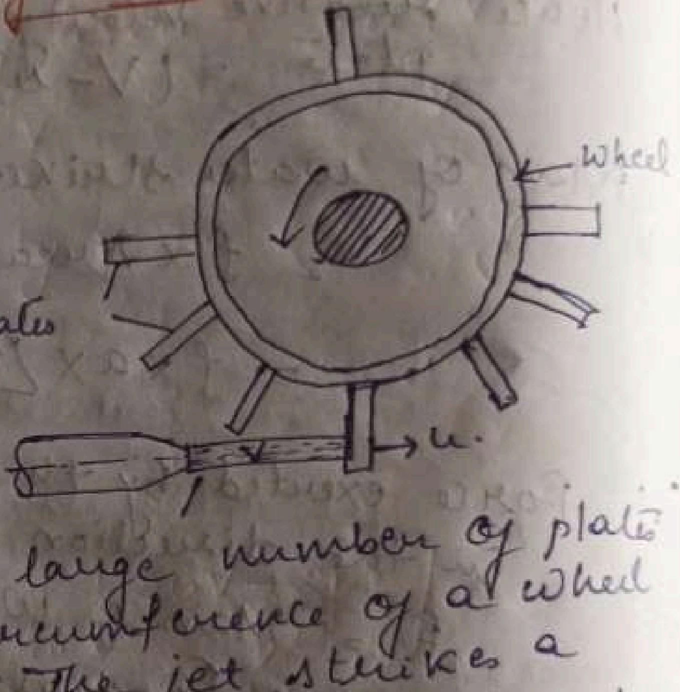
For stationary plate, the work done is zero.

∴ Work done per second by the jet on the plate

$$= \frac{\text{Force} \times \text{Distance in the direction of force}}{\text{Time}}$$

$$= F_x \times u = \rho a (V-u)^2 \times u \quad \left(\frac{Nm}{s}\right)$$

Force Exerted by a jet of water on a series of Vanes



In actual practice, a large number of plates are mounted on the circumference of a wheel at a fixed distance. The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving and the 2nd plate mounted on the wheel appears before the jet which again exerts the force on the 2nd plate. Thus each plate appears successively before the jet and the jet exerts force on each plate. The wheel starts moving at a constant speed.

V = velocity of jet.
d = Diameter of jet
a = cross-sectional area of jet
$= \pi/4 \, d^2$

u = velocity of vane.

The mass of water coming out from the nozzle per second is always in contact with the plates. When all the plates are considered.

Hence mass of water per second striking the series of plates $= \rho a V$

The jet strikes the plate with a velocity $= V-u$

After striking the jet moves tangential to the plate and hence the velocity component in the direction of motion of plate is equal to zero.

∴ The force exerted by the jet in the direction of motion of plate.

F_x = mass per second [Initial velocity − Final velocity]

$= \rho a V[(V-u)-0] = \rho a V(V-u)$

Work done by the jet on the series of plates per second

= Force × Distance per second in the direction of flow

$= F_x \times u$.

$= \rho a V(V-u) \times u$.

Kinetic energy of the jet per second

$$= \frac{1}{2} m V^2$$

$$= \frac{1}{2} (\rho a V) V^2 \quad = \frac{1}{2} \rho a V^3.$$

$$\text{Efficiency} = \eta = \frac{\text{Work done per second}}{\text{Kinetic energy of the jet per second}}$$

$$= \frac{\rho a V (V-u) \times u}{\frac{1}{2} \rho a V^3} = \frac{2u(V-u)}{V^2}$$

For a given jet velocity V, the efficiency will be maximum when

$$\frac{d\eta}{du} = 0.$$

$$\frac{d}{du}\left(\frac{2u[V-u]}{V^2}\right) = 0, \quad \frac{d}{du}\left[\frac{2uV - 2u^2}{V^2}\right] = 0$$

$$\frac{2V - 2 \times 2u}{V^2} = 0.$$

$$2V - 4u = 0.$$

$$V = \frac{4u}{2} \quad \text{or} \quad u = \frac{V}{2}.$$

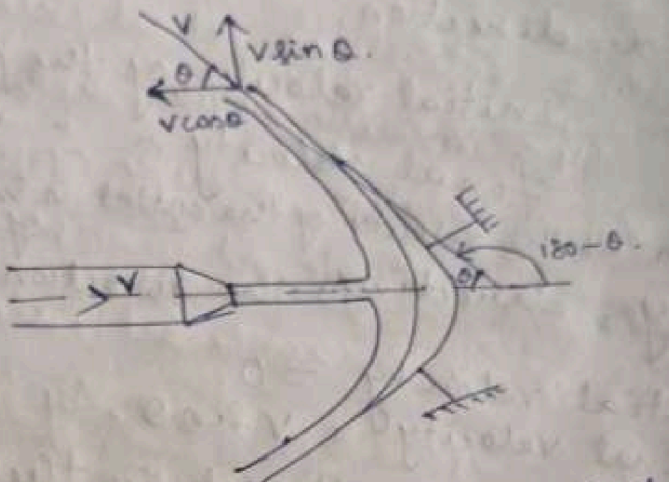
$$\eta_{max} = \frac{2u[2u-u]}{(2u)^2} = \frac{2u \times u}{2u \times 2u} = \frac{1}{2} = 0.5$$

$$= 50\%.$$

Force Exerted by a Jet on Stationary curved plates Or Impact of jet on the fixed curved plate.

---

consider a jet of water strikes a fixed curved plate.



The jet after striking the plate, comes out with the same velocity if the plate is smooth and there is no loss of energy due to impact of the jet, in the tangential direction of the curved plate.

The velocity at Outlet of the plate can be resolved into two components, one in the direction of jet and other perpendicular to the direction of the jet.

Let diameter of jet = d.

Velocity of the jet = V.

Blade angle at inlet = θ.

Blade angle at outlet = θ.

Blade angle

Angle through which the jet is deflected = 180 - θ

a = area of the jet

ρ = density of fluid

m = mass flow rate of jet.

In x- direction,

Initial velocity of the jet = V.

final velocity " " = - V cos θ.

(-) sign as the velocity of the outlet is in the opp direcⁿ of the jet of water coming out from wheel.

In y - direction, the direction ⊥r to the jet

initial velocity = 0.

final velocity = V sin θ.

Force exerted by the jet in the direction of jet,

F_x = mass flow rate × (initial velocity - final velocity)

$$= m[V - (-V\cos\theta)]$$

$$= m[V + V\cos\theta]$$

$$= mV[1 + \cos\theta]$$

$$= \rho aV \cdot V(1 + \cos\theta)$$

$$F_x = \rho aV^2(1 + \cos\theta)$$

Similarly,

Force exerted in y - direction

F_y = Mass flow rate × (initial velocity - final velocity)

$= \rho a V(0 - V \sin\theta)$

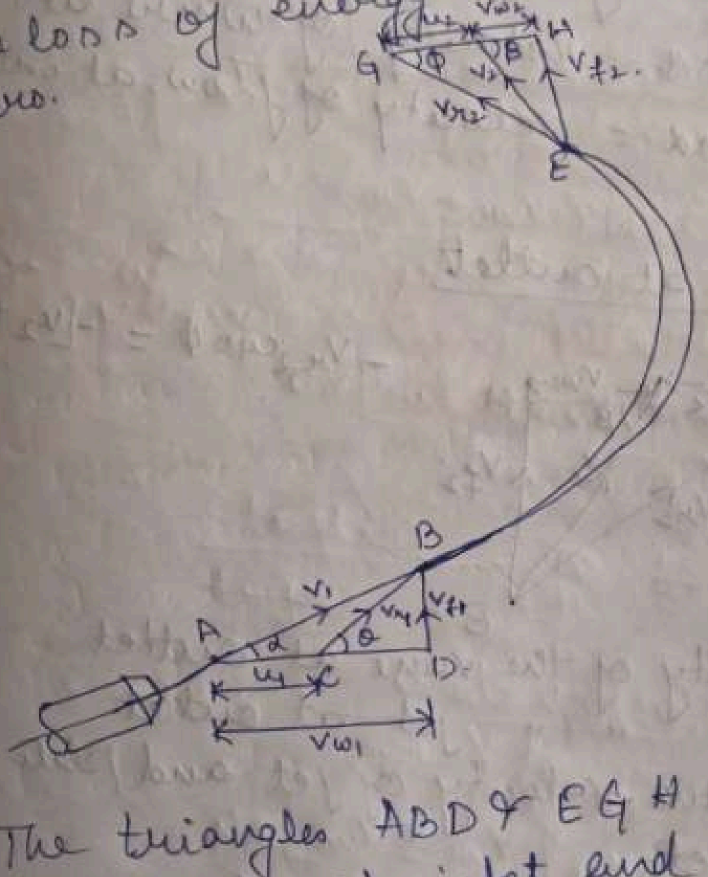
$= -\rho a V^2 \sin\theta$

-ve sign means the force is acting in the downward direction.

The angle of deflection of the jet $= 180 - \theta$

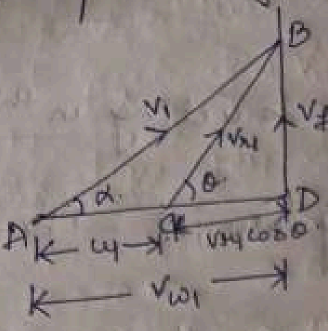
Impact of jet on unsymmetrical vane at one of its tips

A jet of water striking a moving curved plate (also called vane tangentially) at one of its tips. The loss of energy due to impact of the jet will be zero.



The triangles ABD & EGH are called the velocity triangles at inlet and outlet.

Velocity triangles at inlet.



$\boxed{V_{r1}\cos\theta = V_{w1} - u_1}$

$AB = V_1$ = Velocity of jet at inlet

$AC = u_1$ = Velocity of the vane

$CB = V_{r1}$ = Relative velocity of jet and plate at inlet

$\angle BAC = \alpha$ = Angle at which the jet enters the vane at inlet.

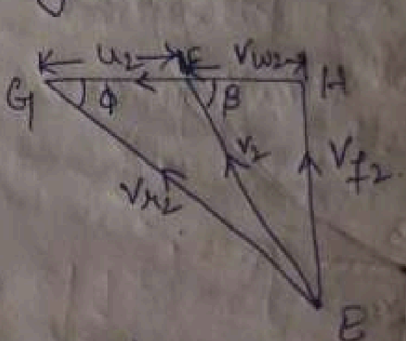
$\angle BCD = \theta$ = vane angle at inlet

$AD = V_{w1} = V_1\cos\alpha$ = Velocity of whirl at inlet

$DB = V_{f1} = V_1\sin\alpha$ = Velocity of flow at inlet.

<u>Velocity triangles at outlet</u>

$-V_{r2}\cos\phi = -[u_2 + V_{w2}]$



$GF = u_2$ = velocity of the vane at outlet.

$EF = V_2$ = " " " jet at outlet.

$EG = V_{r2}$ = Relative velocity of jet and plate at outlet

$\angle EFH = \beta =$ Angle at which the jet leaves the vane
$\angle EGF = \phi =$ Vane angle at outlet.
$FH = V_{w2} = V_2 \cos\beta =$ velocity of whirl at outlet
$EH = V_{f2} = V_2 \sin\beta =$ velocity of flow at outlet.

Force exerted by the jet on the vane in the direction of motion.

$F_x =$ Mass of water striking per sec × [Initial velocity with which jet strikes in the direction of motion − final velocity of jet in the direction of motion]

∴ velocity in the direction of motion $= V_{w1} - u_1$.

velocity at outlet in the direction of motion

$$= -[u_2 + V_{w2}]$$

$$F_x = \rho a V_{r1} [V_{w1} - u_1] - [-u_2 + V_{w2}]$$

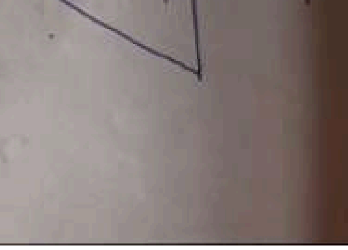
$$= \rho a V_{r1} [V_{w1} - u_1 + u_2 + V_{w2}].$$

If the vane is smooth $u_1 = u_2 = u$.

$$\therefore F_x = \rho a V_{r1} [V_{w1} + V_{w2}] \rightarrow m(V_{w1} + V_{w2})$$

∴ This is true only when angle β is an acute angle.

$\cancel{\beta < 90^\circ}$ $\beta < 90^\circ$



If $\beta = 90^\circ$, $V_{w2} = 0$.

$$F_x = \rho a V_{r1} [V_{w1}]$$

$$\boxed{F_x = m V_{w1}}$$

If β is obtuse angle, $\beta > 90°$ $V_{w_2} = -ve$.

$$F_x = \rho a V_{r_1} [V_{w_1} - V_{w_2}] = m (V_{w} - V_{w_2})$$

Work done per second on the vane by the jet = Force × displacement of vane in the direction of jet.

N m/s.

$$W = F_x \times u.$$

$$= \rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times u. \quad \text{If } \beta < 90 \text{ or } \beta > 90$$

~~Work done per second on the vane by the jet~~

Work done per second per unit weight of fluid striking per second

mass flow rate per kg is

If we ~~divide~~ multiply $g \times \rho a V_{r_1}$ = weight ~~g~~ per sec

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times u}{\text{Weight of fluid striking/sec.}}$$

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times u}{g \times \rho a V_{r_1}} \qquad W = \rho a V_{r_1} \times g.$$

$$W = \frac{1}{g} [V_{w_1} \pm V_{w_2}] \times u.$$

Efficiency of jet, $\eta = \dfrac{\text{output}}{\text{input}}$

$$= \frac{\text{work done}}{\text{Initial K.E.}}$$

$$\eta = \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times u}{\frac{1}{2} m V_1^2}.$$

$$= \frac{\rho a V_{r1} (V_{w1} \pm V_{w2}) \times u}{\frac{1}{2} (\rho a V_1) V_1^2}$$

$$= \frac{2 V_{r1} (V_{w1} \pm V_{w2}) \times u}{V_1^3}$$

Q. A jet of water of 100 mm diameter moving with a velocity of 20 m/s strikes a stationary flat plate. Find normal force on the plate ($\rho_{\text{for water}} = 1000\ kg/m^3$)

Given

dia. of jet = 100 mm = 0.1 m.

$V = 20\ m/sec$

$\rho = 100\ kg/m^3$.

Area of the jet $= \frac{\pi}{4} d^2$

$= \frac{\pi}{4} \times (0.1)^2$

$= 0.00785\ m^2$.

Normal force $= F_n = \rho a V^2$

$= 1000 \times 0.00785 \times (20)^2$

$= 3140\ N$.

Q. A jet of water of diameter 50mm strikes a flat vertical fixed plate with a velocity of 30 m/s. Find the force exerted on the plate by the jet.

$d = 50mm = 0.05 m$.

$V_1 = 30 \, m/s$.

$\rho = 1000 \, Kg/m^3$.

$a = \pi/4 \, d^2 = \pi/4 \times (0.05)^2$.

$= 0.00196 \, m^2$.

$F_n = \rho a V^2$

$= 1000 \times 0.00196 \times (30)^2$

$= 1766.25 \, N$.

Q. If a jet of water discharging 30 lit/sec and strikes vertically plate normally. if jet diameter is 20mm. Find the force exerted on the plate by the jet.

Given

$d = 20mm = 0.2m$.

$Q = 30 \, lit/sec$. $\quad$ 1 lit $= 0.001 m^3$

$= 0.03 \, m^3/sec$.

$\rho = 1000 Kg/m^3$.

$a = \pi/4\, d^2$

$= \frac{\pi}{4} \times (0.2)^2 = 0.00031 m^2$

$F_n = \rho a v^2 = \rho \frac{Q^2}{a}$ $\quad Q = av.$ $\quad av^2 = \frac{Q^2}{a}$

$= 1000 \times \frac{(0.03)^2}{0.00031}$

$= 2866.24 N.$

Q. A jet of water moving with a velocity of 25m/sec. strikes normally on a plate. The jet diameter is 60mm. Determine the force on the plate when it is moving in the direction of jet with a velocity of 5m/sec.

$d = 60mm = 0.06 m.$

$v = 25 m/s.$

$u = 5 m/sec$

$\rho = 1000 kg/m^3.$

$a = \pi/4\, d^2 = \pi/4\,(0.06)^2.$

$= 0.00283 m^3.$

$F_x = \rho a (v-u)^2.$

$= 1000 \times .00283 \times (25-5)^2.$

$= 1132 N.$

Q. A jet of water moving with a velocity of 15 m/sec, strikes normally on a plate. The jet diameter is 50mm. Determine the force on the plate when it is moving in the direction of jet with a velocity of 5 m/sec. Also determine work done and the efficiency by the jet?

Given $V = 15$ m/sec.

$d = 50mm = 0.05m$.

$u = 5$ m/sec.

$\rho = 1000 kg/m^3$.

$area = \pi/4 \, d^2$

$= \pi/4 (0.05)^2$

$= 0.00196 m^2$.

$F_x = \rho a (V-u)^2$.

$= 1000 \times 0.00196 (15-5)^2$

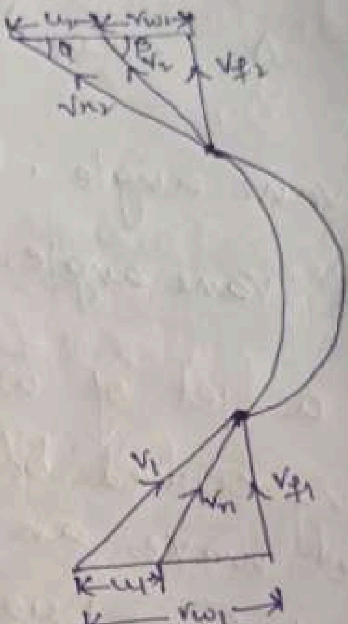
$= 196.25 N$.

work done $= F_x \times u$.

$= 196.25 \times 5$.

$= 981.25$ Nm/sec.

A jet of water with a velocity of 50 m/sec strikes a curved vane which is moving at a velocity of 25 m/sec. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves radially at outlet. Draw the velocity triangles and determine the vane angles at inlet and outlet.



$V_1 = 50$ m/sec.

$u_1 = 25$ m/sec.

$\alpha = 30°$

$V_{w1} = V_1 \cos\alpha$

$= 50 \cos 30$

$= 43.30$ m/sec.

$V_{f1} = V_1 \sin\alpha$

$= 50 \sin 30$

$= \cancel{43.30 \text{ m/sec}}$ 25 m/sec.

$\theta = \tan^{-1}\left[\dfrac{V_{f1}}{V_{w1} - u}\right]$

$= \tan^{-1}\left[\dfrac{25}{43.3 - 25}\right]$

$= \cancel{59.18°}$ 53.79.

$V_{r1} = \dfrac{V_{f1}}{\sin\theta} = \dfrac{25}{\sin 53.79} = 30.98$ m/sec

$V_{r_1} = V_{r_2} \quad u_1 = u_2 = u.$

$\cos\phi = \frac{u}{V_{r_1}}$

$= \frac{20}{30.98}$

$\phi = \cos^{-1}(0.645) = 49.79$

Inlet vane angle, $\theta = 53.79$

Outlet vane angle, $\phi = 49.79$

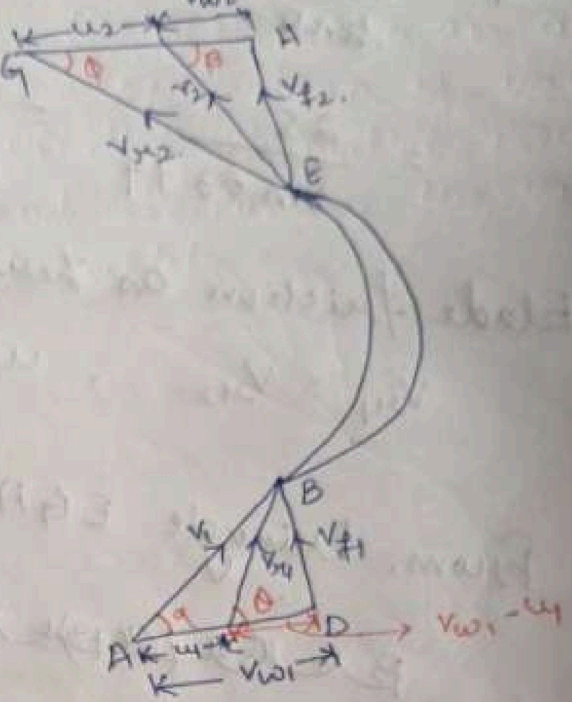
Q. If a jet of water strikes a series of moving curved blade with a velocity of 50m/s. The jet makes an angle at inlet is 30° and leaves at 60°. Draw the velocity triangles and find when the blade velocity is 25 m/sec.

(i) Blade angles

(ii) Work done per kg of water.

(iii) Efficiency.

Sol^n



$V_1 = 50$ m/sec.

$\alpha = 30°$

$\beta = 60°$

$u = 25$ m/sec

$V_{w1} = V_1 \cos\alpha.$

$= 50 \cos 30.$

$= 43.30$ m/sec

$V_{f1} = V_1 \sin\alpha.$

$= 50 \sin 30$

$= 25$ m/sec

$\tan\theta = \dfrac{V_{f1}}{V_{w1} - u_1}$

$= \dfrac{25}{43.3 - 25}$

$= \dfrac{25}{18.3}$

$\theta = \tan^{-1}\left(\dfrac{25}{18.3}\right) = 53.79°$

$$V_{r1} = \frac{V_{f1}}{\sin\theta}$$

$$= \frac{25}{\sin 53.79} = 30.98 \text{ m/sec}$$

blade friction as zero,

$V_{r1} = V_{r2}$, $u_1 = u_2 = u$.

From, triangle EGH.

$\cancel{\beta = 180 - (60+90)} = 60°$

$\angle FEH = 180 - 60 = 120°$

$$\frac{V_{r2}}{\sin\beta} = \frac{u}{\sin(\beta - \phi)}$$

$$\sin(\beta - \phi) = \frac{u \times \sin\beta}{V_{r2}}$$

$$\sin(60 - \phi) = \frac{25 \times \sin 60}{30.98}$$

$$60 - \phi = \sin^{-1}(0.698)$$

$$= 44.35.$$

$$\phi = 60 - 44.35$$

$$= 15.66.$$

$V_{r2} \cos\phi = V_{w2} + u.$

$V_{w2} = V_{r2} \cos\phi - u.$

$= 31 \cos(15.66) - 25.$

$= 4.84 \text{ m/sec}$

workdone, $w = m(V_{w1} + V_{w2})u.$

$\frac{\text{workdone}}{\text{kg}} = (V_{w1} + V_{w2})u$

$= (43.3 + 4.84) \times 25$

$= 1203.5 \text{ J}.$

Energy supplied $= \frac{1}{2} m V_1^2.$

Energy supplied per kg $= \frac{1}{2} V_1^2.$

$= \frac{1}{2} \times (50)^2.$

$= 1250 \text{ J}.$

Efficiency, $\eta = \frac{\text{workdone}}{\text{Energy supplied}}$

$= \frac{1203.5}{1250} = 0.962.$

$= 96.2\%.$