Module 1:

Engineering Mechanics.

The subject of Engineering mechanics is that branch of Applied Science, which deals with the laws and principles of mechanics, along with their applications to engineering problems.

The subject of Engineering Mechanics may be divided into the following two main groups:

1. Statics   2. Dynamics.

Statics.

It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies at rest.

Dynamics

It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies in motion. The subject of Dynamics may be further sub-divided into the following two branches:

1. Kinetics   2. Kinematics.

Kinetics.

It is the branch of dynamics, which deals with the bodies in motion due to the application of forces.

Kinematics.

It is that branch of dynamics, which deals with the bodies in motion, without any references to the forces which are responsible for the motion.

Fundamental Units

In Engg. Mechanics, all the physical ~~fundamental~~ quantities are expressed in terms of three fundamental quantities.

1. length
2. mass
3. time

The units are also expressed in other units known as derived units.

eg: unit of area, velocity, acceleration, pressure etc.

Systems of Unit.

There are only four systems of units, which are commonly used. These are known as

1. CGS units — centimeter gram sec
2. FPS units — Foot kilogram-sec
3. MKS units — meter kilogram sec
4. SI units — International system of units *

Fundamental SI units

Quantities	Fundamental Unit	Symbol
Length.	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Luminous Intensity	Candela	cd
Temperature	Kelvin	K.

(*Most accepted unit system all over the world)

S.I Derived Units

Quantities	Derived Unit	Symbol.
Force	Newton	N
Moment	Newton - meter	Nm
Work done	Joule	J
Velocity	Meter per second	m/s
Pressure	Pa or Newton per sq. meter	Pa or N/m^2.

Mass and Weight

→ Mass of a body is the total quantity of matter contained in the body (m)

→ Weight of a body is the force with which the body is attracted towards the centre of the earth.

$\boxed{W = m \times g}$

Mass	Weight
→ Mass is the total quantity of matter contained in a body.	Weight of a body is the forces with which the body is attracted towards the centre of the earth.
→ Mass is a scalar quantity because it has only magnitude and no direction.	Weight is a vector quantity, because it has both magnitude and direction.
→ Mass of a body remains the same at all places.	Weight of body varies from place to place (due to acceleration due to gravity.
→ Mass resists motion in a body.	Weight produces motion in a body.
→ Mass can be measured by an ordinary balance.	It can be measured by a spring balance.

→ Mass of a body can never be zero. | weight of a body can be zero.

Rigid Body and Elastic Body

→ A body is said to be rigid if it does not undergo deformation whatever force may be applied to the body.

→ A body is said to be elastic if it undergoes deformation under the action of force. All bodies are more or less elastic.

Scalar and Vector.

All physical quantities can be divided into scalar and vector quantity.

Scalar quantity is that physical quantity which has only magnitude and no direction.

for ex- length, mass, energy.

Vector quantity is that physical quantity which has both magnitude and direction

for ex- force, velocity etc.

Force

Force is that which changes or tends to change the state of rest of uniform motion of a body along a straight line.

OR

The force is defined as an agent which produces or tends to produce, destroys or tends to destroy motion. It has a magnitude and direction.

Force = mass × acceleration. (Newton's 2nd Law)

Units of force

In SI unit → Newton (N)

One Newton is that force which acting on a mass of one kilogram produces in it an acceleration of one m/sec^2.

$$1 N = 1 Kg \times 1 m/s^2.$$

$$1 N = 1 Kg\, m/s^2.$$

In CGS unit → 1) Dyne or 2) Gram force.

$$1 \text{ dyne} = 1 g \times 1 cm/s^2$$

$$= 1 g\, cm/s^2.$$

$$1 \text{ dyne} = 10^{-5} N.$$

$$1 N = 10^5 \text{ dyne}.$$

MKS system → Kg f.

$$1 \, Kgf = 1 \, Kg \times 9.81 \, m/sec^2$$
$$= 9.81 \, Kg \, m/sec^2$$

Representation.

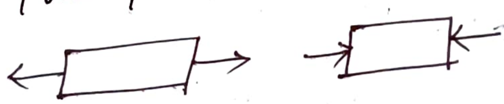
Force is a vector quantity, the sign for a force is ⟶ or ⟵

Characteristics of a force.

→ Magnitude of the force (i.e. 100 N, 50 N, 20KN etc)

→ The direction of the line, along which the force acts. (upward, downward etc).

→ Nature of the force. (pull or push)



→ The point at which the force acts on the body.

Effects of a force

A force may produce the following effects in a body, on which it acts:

→ It may change the motion of the body.

(If a body at rest → motion

If a body in motion → accelerate or decelerate)

→ It may retard the forward motion of a body.
(If a body is motion → decelerate or in equilibrium).

→ A force can change the direction of a moving object.

→ A force can change the shape and size of an object.

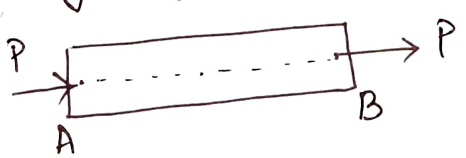
→ It may give rise to the internal stresses in the body, on which it acts.

Principle of Physical Independence of Forces.

It states, "If a number of forces are simultaneously acting on a particle, then the resultant of these forces will have the same effect as produced by all the forces.

Principle of Transmissibility of forces.

It states that, "If a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body."

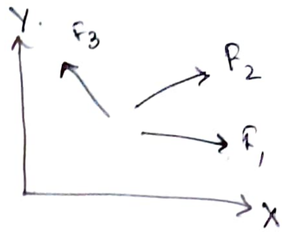


System of Forces.

When two or more forces act on a body, they are called to form a system of forces.

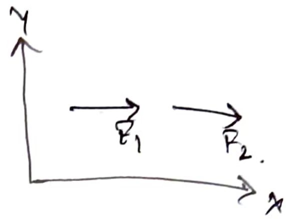
1. Coplanar forces

The forces whose line of action lie on the same plane,



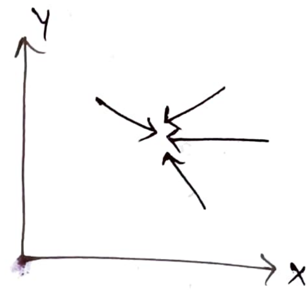
2. Collinear forces.

The forces, whose line of action lie on the same ~~place~~ line.



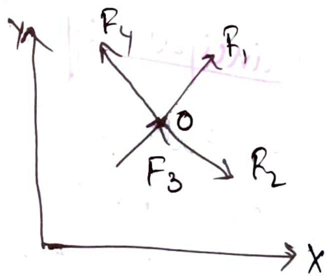
3. Concurrent forces.

The forces which meet alt a single point.



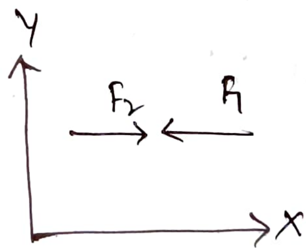
4. Co-planner Concurrent.

The forces which meet at one point and also their line of action lies in one plane.



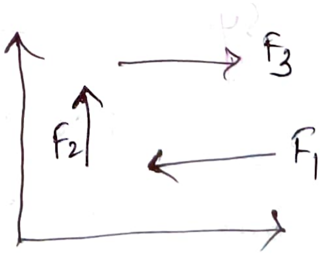
5. Collinear concurrent

The forces whose line of action meet at one point.



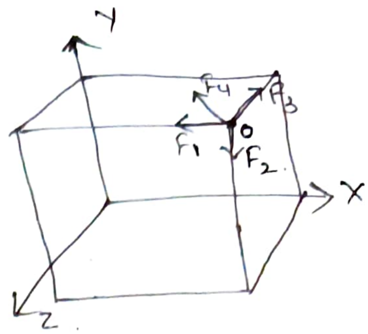
6. Coplanner non-Concurrent

The forces whose line of action donot meet at a point.



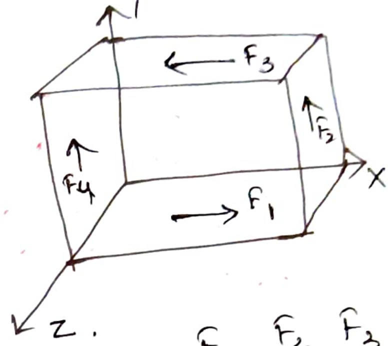
7. Non coplanner concurrent.

The forces whose line of action meet at one point but donot lie in same plane.



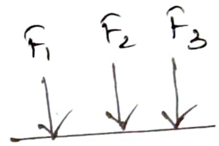
8. Non coplanner non concurrent.

The forces whose line of action donot lie on the same plane and they donot meet at one point



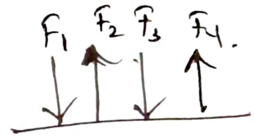
9. Parallel forces.

The forces whose line of action are || to each other in same direcn



10. Non-parallel forces.

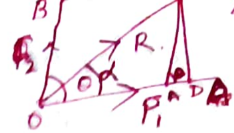
The forces whose line of action || to each other but directed in opposite direction.



Parallelogram Law of Forces.

It states "If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram their resultant may be represented in magnitude and direction by the diagonal of the parallelogram which passes through their point of intersection."

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$



$\tan\alpha = \dfrac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$.

$\triangle OCD$

$R = OC$

$= \sqrt{OD^2 + CD^2}$

$= \sqrt{(OA+AD)^2 + \ldots}$

F_1 and F_2 = Forces whose resultant is req.

θ = Angle between the forces F_1 and F_2.

α = Angle which the resultant force makes with one of the forces.

$\triangle ACD$

$AD = AC\cos\theta$

$= F_2\cos\theta$

$CD = AC\sin\theta$

$= F_2\sin\theta$

$R = F_1^2 + F_2^2 + 2F_1 \ldots$

$\triangle OCD$.

$\tan\alpha = \dfrac{CD}{OD}$

$= \dfrac{CD}{OA+AD}$

$= \dfrac{F_2\sin\theta}{F_1+F_2\cos\theta}$

Q. Two forces of 100N and 150N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°?

Given: $F_1 = 100N$ $F_2 = 150N$

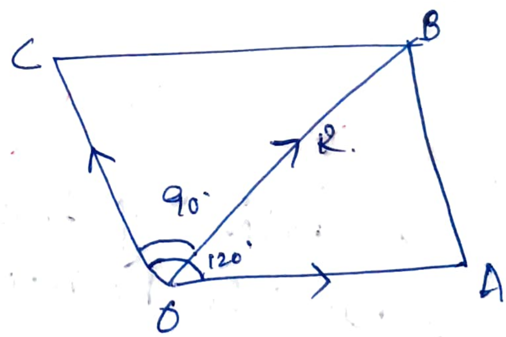
$\theta = 45°$.

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(100)^2 + (150)^2 + 2\times100\times150\cos 45}$$

$= 232$ N.

Q. Two forces act at an angle of 120°. The bigger force is of 40N and the resultant is perpendicular to the smaller one. Find the smaller force.



$\angle AOC = 120°$

$F_1 = 40N$.

(bigger force)

$F_2 = ?$ N.

(smaller force)

$\angle AOB$, $\alpha = 120 - 90$

$= 30°$

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

$$\tan 30 = \frac{F_2 \sin 120}{40 + F_2 \cos 120}$$

$$0.577 = \frac{F_2 \sin 60}{40 + F_2(-\cos 60)}$$

$$F_2 = 20 \text{ N}$$

Q. Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60°, their resultant is $\sqrt{13}$ N.

Given: two forces F_1 and F_2.

First of all, consider the two forces acting at right angles.

When the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{F_1^2 + F_2^2}$$

$$10 = F_1^2 + F_2^2$$

When the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60}$$

$$13 = F_1^2 + F_2^2 + 2F_1F_2 \times 0.5$$

$$F_1F_2 = 13 - 10 = 3.$$

$$(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2.$$

$$= 10 + 6 = 16.$$

$$F_1 + F_2 = \sqrt{16} = 4. \quad \text{— (i)}$$

$$(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2$$
$$= 10 - 6 = 4.$$
$$F_1 - F_2 = \sqrt{4} = 2. \quad \text{— (ii)}$$

$$F_1 + F_2 = 4$$
$$F_1 - F_2 = 2$$

$$2F_2 = 2.$$
$$F_2 = 1\,N.$$

$$F_1 + F_2 = 4.$$
$$F_1 = 4 - 1$$
$$= 3\,N.$$

Resolution of a Force

The process of splitting up the given force into a number of components without changing its effect on the body is called resolution of a force.

Principle of Resolution

It states, "The algebraic sum of the resolved parts of a no. of forces, in a given direction, is equal to the resolved part of their ~~resolution~~ resultant in the same direction."

Method of Resolution for the Resultant force

1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components. (i.e. ΣH)

2. Resolve all the forces vertically and find the algebraic sum of all the vertical components. (i.e. ΣV)

3. The Resultant R of the given forces will be given by the equation:

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

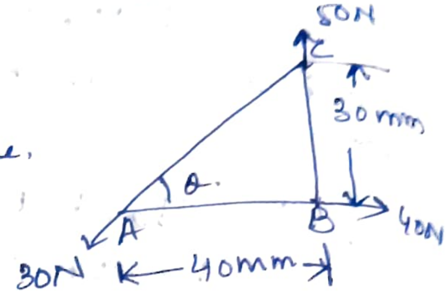
4. The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

Note: When ΣV is +ve, the resultant makes an angle between 0° and 180°.
When ΣV is −ve, the resultant makes an angle between 180° and 360°.

When ΣH is +ve, the resultant makes an angle between 0° to 90° or 270° to 360°.
When ΣH is −ve, the resultant makes an angle between 90° to 270°.

Q. A triangle ABC has its side AB = 40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40 N, 50N and 30N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces.



From the geometry of the figure.

$\triangle ABC$

side $AC = \sqrt{(40)^2 + (30)^2}$

$= 50$ mm.

$\sin\theta = 30/50 = 0.6$.

$\cos\theta = 40/50 = 0.8$.

Resolving all the forces horizontally, i.e. along AB.

$$\Sigma H = 40 - 30\cos\theta$$
$$= 40 - (30 \times 0.8)$$
$$= 16\ N.$$

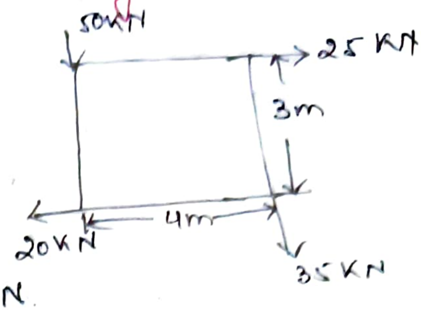
Now, resolving all the forces vertically, i.e. along BC.

$$\Sigma V = 50 - 30\sin\theta.$$
$$= 50 - (30 \times 0.6)$$
$$= 32\ N$$

Magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(16)^2 + (32)^2} = 35.8\ N$$

Q. A system of forces are acting at the corners of a rectangular block as shown in fig 2.4.



Resolving forces horizontally,

$\Sigma H = 25 - 20 = 5\,KN$

Resolving forces vertically,

$\Sigma V = (-50) + (-35) = -85\,KN.$

∴ Magnitude of the resultant force

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(5)^2 + (-85)^2} = 85.15\,KN.$

Direction of the resultant force

Let θ = Angle which the resultant force makes with the horizontal.

$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-85}{5} = -17.$

$\theta = 86.6^\circ$

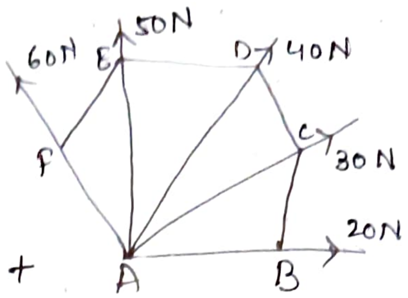
Since ΣH is positive and ΣV is negative, therefore resultant lies between 270° and 360°.

Therefore actual angle of the resultant force

$= 360 - 86.6 = 273.4^\circ$

Q. The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.

Resolving all the forces horizontally,



$\Sigma H = 20\cos 0^\circ + 30\cos 30 + 40\cos 60 + 50\cos 90 + 60\cos 120^\circ$ N

$= 36.0$ N

Resolving all the forces vertically,

$\Sigma V = 20\sin 0 + 30\sin 30 + 40\sin 60 + 50\sin 90 + 60\sin 120^\circ$ N

$= 151.6$ N.

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(36.0)^2 + (151.6)^2} = 155.8$ N.

$\tan\theta = \dfrac{\Sigma V}{\Sigma H} = \dfrac{151.6}{36.0} = 4.211$

$\theta = 76.6$

Q. The following forces act at a point:

i) 20 N inclined at 30° towards North of East,

ii) 25 N towards North,

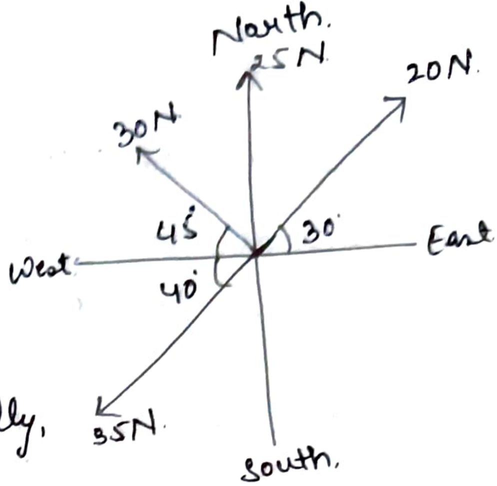
iii) 30 N towards North West and

iv) 35 N inclined at 40° towards South of West,

Find the magnitude and direction of the resultant force.

Resolving all the forces horizontally,

$\Sigma H = 20\cos 30^\circ + 25\cos 90 +$
$30\cos 135 + 35\cos 220$



$= -30.7\,N.$

Resolving all the force vertically,

$\Sigma V = 20\sin 30 + 25\sin 90 +$
$30\sin 135 + 35\sin 220^\circ$

$= 33.7\,N.$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(-30.7)^2 + (33.7)^2}$

$= 45.6\,N.$

$\tan\theta = \dfrac{\Sigma V}{\Sigma H} = \dfrac{33.7}{-30.7} = -1.098.$

$\theta = 47.7.$

Since ΣH is -ve and ΣV is +ve, therefore resultant lies between 90° and 180°. Thus actual angle of the resultant $= 180 - 47.7 = \underline{\underline{132.3^\circ}}$.

Laws for the Resultant force.

The resultant force of a given system of forces.

1. Triangle law of forces.
2. Polygon law of forces.

Triangle Law of forces.

It states that, "If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order, their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order.

Polygon Law of Forces.

It states that, "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order, then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."

Q. Find the magnitude and direction of the resultant of the concurrent forces of 8N, 12N, 15N and 20N making angles of 30°, 70°, 120°, 125° and 155° respectively with a fixed line.

Resolving all the forces horizontally,

$$\Sigma H = 8\cos 30 + 12\cos 70 + 15\cos 120 + 20\cos 155$$

$$= -14.59.$$

Resolving all the forces vertically,

$$\Sigma V = 8\sin 30 + 12\sin 70 + 15\sin 120 + 20\sin 155.$$

$$= 36.71.$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}.$$

$\theta = 111.7^\circ$.

$$= 39.50.$$

Moment of Force (Turning Effect or Rotational Effect)

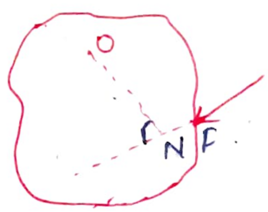
It is the turning effect produced by a force, on the body on which it acts.

The moment of a force is equal to the product of the force and the ⊥r dist. of the point, about which the moment is req. and the line of action of the force.

Moment (M) = P × l.

P = Force acting on the body.
l = ⊥r dist. between the point, about which the moment is req. and the line of action of the force.

Let a force F act on a body which is hinged at O.



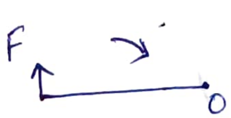
Then Moment of P about the point O in the body is = F × ON

Unit of Moment – Nm

Types of Moments.

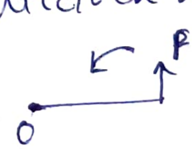
1. Clockwise moments

(-ve)



(-ve)

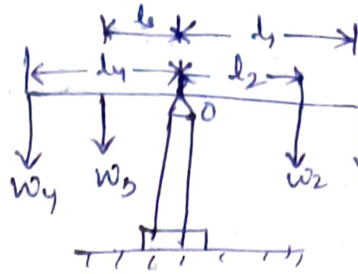
2. Anticlockwise moments.



(+ve)

(+ve)

due to W_1
Moment about O
$= W_1 \times l_1$ ↻



Moment due to W_2 about O $= W_2 \times l_2$.

Moment due to W_3 about O $= W_3 \times l_3$ ↺

" " " W_4 " " $= W_4 \times l_4$ ↺

Hence algebric sum of the Moments of W_1, W_2, W_3, W_4 about O $= -W_1 l_1 - W_2 l_2 + W_3 l_3 + W_4 l_4$

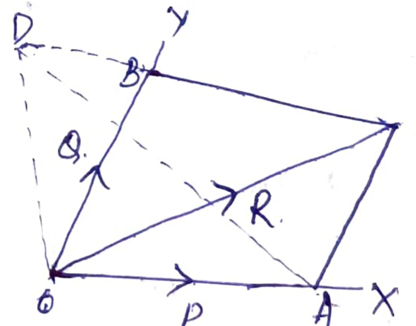
VARIGNON'S THEOREM OR LAW OF MOMENTS

Varignon's theorem states that the algebric sum of the moment, two forces about any point in their plane is equal to the moment of the resultant about the same point.

Proof:

Case (i) When the forces are concurrent.

Let P and Q be any two forces acting at a point O along lines OX and OY respectively.



Let D be any point in their plane.

Line DC is ∥ to OX to meet OY

OA & OB are the two adjacent sides of parallelogram OACB is completed. & OC is joined.
Let R be the resultant of forces P and Q.
Then a/c to the theorem of parallelogram of forces, R is represented in magnitude and direction by the diagonal OC.

The ~~moments of P, Q~~,

$P = 2 \times \text{area } \Delta AOD.$

$Q = 2 \times \text{area of } \Delta OBD.$

$R = 2 \times \text{area of } \Delta OCD$

Since the point D is outside $\angle AOB$, the moments of P, Q and R about D are all anticlockwise and hence these moments are treated as +ve.

∴ The algebraic sum of the moments of P and Q about

$\Delta AOD = \Delta OBC.$

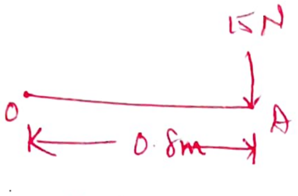
$\Delta AOC = \Delta OBC.$

$$D = 2\Delta AOD + 2\Delta OBD$$
$$= 2(\Delta AOD + \Delta OBD)$$
$$= 2(\Delta OBC + \Delta OBD).$$
$$= 2\Delta CD.$$
$$= \text{Moment of } R \text{ about } D.$$

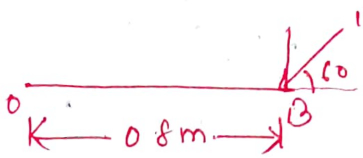
Now, the algebraic sum of the forces P and Q about

$$D = 2\Delta AOD - 2\Delta OBD.$$
$$= 2(\Delta AOD - \Delta OBD)$$
$$= 2(\Delta AOC - \Delta OBD)$$
$$= 2(\Delta OBC - \Delta OBD)$$
$$= 2\Delta OCD.$$
$$= \text{Moment of } R \text{ about } D.$$

Q. A force of 15N is applied perpendicular to the edge of a door 0.8 m wide as shown in fig (a). Find the moment of the force about the hinge.
If this force is applied at an angle 60° to the edge of the same door, as shown in fig. (b), find the moment of this force.



(a)



Soln.

$P = 15N.$

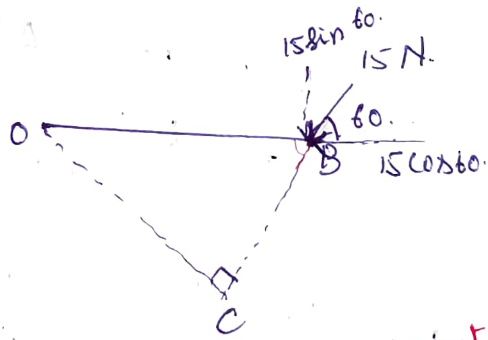
$l = 0.8m.$

Moment of the force about the hinge

$= P \times l.$

$= 15 \times 0.8$

$= 12.0\ N \cdot m.$



$OC = OB \sin 60°$

$= 0.8 \times .866$

$= 0.693 m$

Moment $= P \times l.$

$= 15 \times 0.693$

$= 10.4\ N \cdot m.$

OR.
vertical component of the force.
$15 \sin 60 = 13.0N$

In second case.

$15 \sin 60 = 15 \times 0.866 = 13.0\,N$

Moment $= \cancel{15} \; 13 \times 0.8$

$= 10.4\,Nm$.

Since the dist. between the horizontal component of the force ($15 \cos 60$) and the hinges is 0, therefore, moment of horizontal component of the force about the hinge is also zero.

Classification of Parallel Forces.

The parallel forces are classified into two categories.

→ Like parallel forces.
→ Unlike parallel forces.

Like parallel forces.

The forces, whose line of action are parallel to each other and all of them act in the same direction.

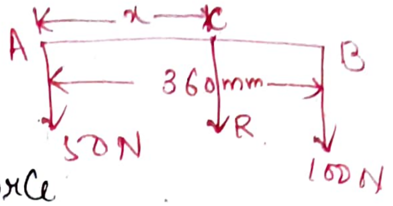
$$\uparrow \uparrow \uparrow \uparrow$$

Unlike parallel forces.

The forces, whose line of action are parallel to each other and all of them do not act in the same directions.

$$\uparrow \downarrow \uparrow \downarrow$$

Q. Two like parallel forces of 50 N and 100 act at the ends of a rod 360 mm long. Fin the magnitude of the resultant force and th point where it acts.



Since the given forces are like and parallel, therefore magnitude of the resultant force

$$R = 50 + 100$$
$$= 150\ N.$$

Let x = Distance between the line of action of the resultant force (R).

$$50 \times x = 100(360 - x)$$
$$150x = 36000$$
$$x = \frac{36000}{150} = 240\ mm.$$

Q. A beam 3m long weighing 400 N is suspended in a horizontal position by two vertical springs each of which can withstand a maximum tension of 350 N only. How far a body of 200N weight be placed on the beam, so that one of the strings may just break.