Plant Location

chapter-1

The selection of location for the set up of the plant is an important decision as a large investment is made in building plant & machinery which can not be changed very often. So before selection of plant location a long range of forecast is needed by considering the factors like company's expansion plan & policy, diversification plan for the products, changing market conditions, changing sources of raw materials & many other factors.

Purpose of location study is to find an optimum location, which will result in the greatest advantage to the orgn.

Plant location problem →

1) First stage → Selection of a region:
It refers to selection of a particular geographical zone by considering the factors like nearness to market & sources of raw material, basic infrastructure facilities available, climatic condns, & taxation and laws.

2) Second stage → Selection of a community:
It refers to the selection of a specific locality within the selected region. The Influencing factors are availability of labour, community attitude, social structure & service facility. Ex- Urban area, Rural area & semi-Urban area.

3) Third stage → Selection of a particular site:
It refers to selection of specific site within the community. The influencing factors are cost of land, availability & suitability of land.

Advantages of Urban, Suburban & Rural locations:→

1) Urban area →

Advantages

1. Excellent communication network
2. Good transportation facilities for material & people
3. Availability of skilled & trained manpower
4. Excellent sourcing facilities
5. Good educational, recreational & medical facilities.
6. Availability of service of consultants, training institutes & trainers.

Disadvantages

1 High cost of land compared to rural area.
2 Sufficient land is not available for expansion.
3 Labour cost is high
4. management labour relations are much influenced by union activities.
5. High labour turnover.

Rural area →

Advantages

1. cheaper availability of site.
2. " labour cost.
3. Good Industrial relation
4 Scope for expansion & diversification
5 No slums & environmental pollution

Disadvantages

1 poor transpotation network.
2 No good communication facilities
3. far away from market.
4 No educational, medical & recreational facilities.
5. Sourcing of components & materials should be from outside

Suburban area →

Advantage

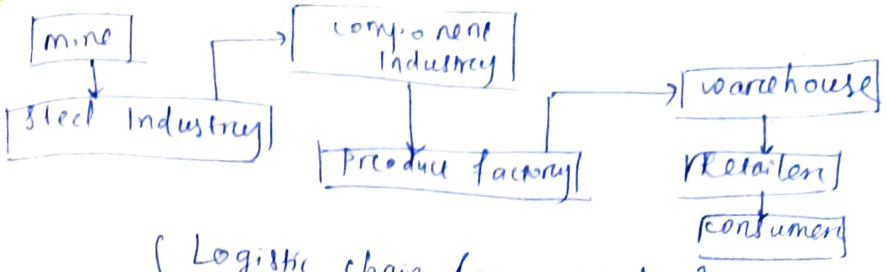
1. Land is available at cheaper rate compared to urban location
2. Infrastructure facilities are developed by promotional agencies.
3. easy availability of skilled manpower.
4 Educational, medical facilities are available.

Disadvantage

1. Due to frequent arrival of people from rural area, it will become crowdy & become urban in short period.
2 High mobility of workers so higher labour turn over
3. Govt incentive & subsides to promote industry

Importance of location :→

1> Competition
2> cost
3> Indirect benefit



[Logistic chain for a product]

Factors Influencing plant location :→

1) Proximity to market → Every firm aims to provide service to its customer at the right time & at reasonable price so location of the facilities should be close to the market, which reduces cost of transportation

2) Supply of raw material →
For ~~uninterrupt~~ uninterrupted ~~supply~~ production, supply of raw material with required qualities is essential in time Nearness to source of raw material is advisable.

3) Transport facility →
For timely supply of raw material & finished goods, good transportation facility from & to the location of the plant is essential. Medium of transport → physical transportation, air, road, rail, water & pipeline

4) Infrastructure availability →
Basic infrastructure facilities like power, water & waste disposal etc are essential

5) Labour & wages →
Securing adequate no. of labours with skill should be considered. Productivity of labour has to be measured. Prevailing wage pattern, cost of living & industrial relation & bargaining power of unions are considered for plant location.

6) Law & taxation →
Policies of state Govt & local bodies concerning labour laws, building codes, safety etc.

7) Suitability of land & climate →
Geology of the area with climatic cond^n (humidity, temp^n etc are considered as it affects human performance

8) Supporting industries & services →
To subcontract some of the company's work to other pla

[illegible] minimize [illegible]

[illegible] handling [illegible] is supposed [illegible]

Solid [illegible]

Availability of facilities for handling, recreational, commercial, medical facilities etc.

Single facility location problem: Euclidean distance / rectilinear distance

A set of existing facilities with their coordinates and the movement of material from a new facility to all these existing facilities are known. The objective is to determine optimal location for the new facility.

Q. Consider the location of the new plant which will supply raw materials to a set of existing plants in a group of companies. Suppose there are 5 existing plants which have a material movement relationship with the new plant. Let the existing plants have locations of (400, 200), (800, 500), (1100, 800), (200, 900) & (1300, 300). Suppose that the no. of tonnes of material transported per year from the new plant to various existing plants are 450, 1200, 300, 800 & 1500 respectively. Determine the optimum location of new plant.

Sol. Let (x, y) be the coordinates of new plant.

Computing X coordinate

Existing plant i	X coordinate in ascending order	weight in tonnes	cumulative weight
4	200	800	800
1	400	450	1250 ← [illegible]
2	800	1200	2450
3	1100	300	2750
5	1300	1500	4250
		Total	4250 tonnes

The median location corresponding to cumulative weight is $\frac{4250}{2} = 2125$

This lies between 1 & 2. Take the upper bound.

Corresponding X coordinate = 800

Optimum Y-coordinate

Existing plant	Y-coordinate	weight	cummulative weight
1	200	450	450
5	300	1500	1950
2	500	1200	3150
3	800	300	3450
4	900	800	4250

total 4250 tons

median location on Y-axis corresponding to cummulative weight $4250/2 = 2125$ is 500.

so, optimal Y-cordinate = 500.

optimal (X, Y) = (800, 500)

Q.2 A group of company has plants in 6 different locations, whose co-ordinates in Km are (100, 200), (200, 200), (200, 500) (300, 500), (400, 300) & (500, 100). The company wants to locate a centralized raw material warehouse, from which the quantity of materials transported to the plants 1, 2, 3, 4, 5 & 6 in tones are 1000, 1200, 800, 2000, 1800 & 900 resp. Find the optimal location for the warehouse.

Sol: Given data

plant	1	2	3	4	5	6
coordinates	100, 200	200, 200	200, 500	300, 500	400, 300	500, 100
weight (tons)	1000	1200	800	2000	1800	900

X-coordinate

existing facility	X-coordi	weight	Σ weight
1	100	1000	1000
2	200	1200	2200
3	200	800	3000 ← 3850
4	300	2000	5000
5	400	1800	6800
6	500	900	7700

Σ weight = 7700

median $= \frac{7700}{2} = 3850$

optimal X = 300

Optimum y. coordinate

existing facility	y-coordn	weight	Σ weight
6	100	900	900
1	200	1000	1900
2	200	1200	3100
5	300	1800	4900 ←
3	500	800	5700
4	500	2000	7700

Σ weight = 7700

median = $\frac{7700}{2}$ = 3850

corresponding y = 300

optimal (x, y) of warehouse = (300, 300)

Minimax location problem → (rectilinear distance)

Let us consider there are m existing facilities which are located at (a_1, b_1), (a_2, b_2), (a_3, b_3) .... (a_i, b_i) ..., (a_m, b_m) in X-Y plane so, (a_i, b_i) is the location of ith existing facility in the X-Y plane. Objective is to locate the new emergency facility at (x, y) such that the max^m distance from the new emergency facility to any of the existing facilities. is minimized

$f_i(x, y)$ = rectilinear distance betn new & existing facility i.

$= |x - a_i| + |y - b_i|$

Assume that there are m different shops existing in an industry. The objective of locating fire fightening equipment is such that it can reach the spot quickly i.e max^m distance travel from the base location is min^m.

Procedure

Step-1: find C_1, C_2, C_3, C_4 & C_5 using formulas

$$C_1 = \min_{1 \le i \le m} (a_i + b_i)$$

$$C_2 = \max_{1 \le i \le m} (a_i + b_i)$$

$$C_3 = \min_{1 \le i \le m} (-a_i + b_i)$$

$$C_4 = \max_{1 \le i \le m} (-a_i + b_i)$$

$$C_5 = \max_{1 \le i \le m} (C_2 - C_1, C_4 - C_3)$$

Step-2 : Find the points P_1 & P_2 using the following formula

$$P_1 = \left[\frac{1}{2}(C_1 - C_3),\ \frac{1}{2}(C_1 + C_3 + C_5)\right]$$

$$P_2 = \left[\frac{1}{2}(C_2 - C_4),\ \frac{1}{2}(C_2 + C_4 - C_5)\right]$$

} Two coordinates of the location

Step-3 : Any point (x, y) on the line segment joining points P_1 & P_2 is a minimax location that minimized $f_{max}(x, y)$

Q In a foundry, there are 7 shops whose coordinates are summarized in the following table

SL No	Existing facility	coordinates of centroid (a_i, b_i)
1	Sand plant	10, 20
2	moulding shop	30, 40
3	pattern "	10, 120
4	melting "	10, 60
5	Fetling "	30, 100
6	Gauging "	30, 140
7	Annealing "	20, 190

Step 1

$C_1 = \min(10+20,\ 30+40,\ 10+120,\ 10+60,\ 30+100,\ 30+140,\ 20+190)$

$= \min(30, 70, 130, 70, 130, 170, 210)$

$= 30.$

$C_2 = \max(30, 70, 130, 70, 130, 170, 210)$

$= 210$

$C_3 = \min(-10+20,\ -30+40,\ -10+120,\ -10+60,\ -30+100,\ -30+140,\ -20+190)$

$= \min(10, 10, 110, 50, 70, 110, 170)$

$= 10$

$C_4 = \max(10, 10, 110, 50, 70, 110, 170)$

$= 170.$

$C_5 = \max(210-30,\ \{170-10\})$

$= \max(180, 160)$

$= 180$

Step 2

$P_1 = \left[\frac{1}{2}(C_1 - C_3),\ \frac{1}{2}(C_1 + C_3 + C_5)\right]$

$= \frac{1}{2}(30-10),\ \frac{1}{2}(30+10+180)$

$= (10, 110)$

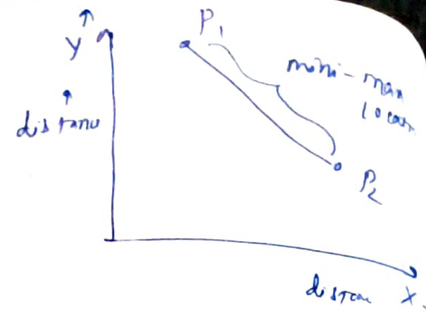
$P_2 = \left[\frac{1}{2}(C_2 - C_4),\ \frac{1}{2}(C_2 + C_4 - C_5)\right]$

$= \frac{1}{2}(210-170),\ \frac{1}{2}(210+170-180)$

$= (20, 100)$

Step 3

Any point (x, y) on the line segment joining points $(10, 110)$ & $(20, 100)$ is a minimax location for the fire fighting equipment.



Gravity location problem → (Euclidean distance)

Objective is to locate the single new facility such that the total material handling cost based on the squared euclidean distance is minimized.

If (p, q) & (r, s) are the coordinates of two locations, then the squared euclidean distance betn these two locations is $= (p-r)^2 + (q-s)^2$.

Formulae

Sum of the weighted squared euclidean distance

$$f(x, y) = \sum_{i=1}^{m} w_i \left[(x-a_i)^2 + (y-b_i)^2 \right]$$

$$x = \frac{\sum_{i=1}^{m} w_i a_i}{\sum_{i=1}^{m} w_i} \qquad y = \frac{\sum_{i=1}^{m} w_i b_i}{\sum_{i=1}^{m} w_i}$$

(x, y) are the optimal coordinates for the new facility. The soln of this problem is called as centroid or centre of gravity.

Q There are 5 existing facilities which are to be served by a single new facility. The details of the existing facilities are given below.

existing facilities (i)	1	2	3	4	5
coordinates (a_i, b_i)	5, 10	20, 5	15, 20	30, 25	25, 5
Loads/year (w_i)	100	300	200	300	100

$$x = \frac{\sum_{i=1}^{5} w_i a_i}{\sum_{i=1}^{5} w_i} = \frac{(5\times100) + (20\times300) + (15\times200) + (30\times300) + (25\times100)}{100 + 300 + 200 + 300 + 100}$$

$= 21$

$$y = \frac{\sum_{i=1}^{5} w_i b_i}{\sum_{i=1}^{5} w_i} = \frac{(10\times100) + (5\times300) + (20\times200) + (25\times300) + (5\times100)}{100 + 300 + 200 + 300 + 100}$$

$= 14.5$

optimal $(x, y) = (21, 14.5)$

PLANT LAYOUT :→

Plant layout is a plan of an optimum arrangement of facilities including personnel, operating equipment, storage space, material handling equipment & all other supporting services along with the design of best structure to contain all these facilities.

Objective of plant lay out →

→ minimize investment in equipment.
→ " overall production time.
→ Utilize existing space most effectively.
→ provide for employee convenience, safety & comfort
→ maintain flexibility of arrangement & operation
→ minimize material-handling cost
→ facilitate the manufacturing process
→ " organisational structure

1.3 Principles of plant layout →

1. Principle of integration → A good layout is one that integrates men, materials, machines & supporting services & others for optimum utilization of resources.
2. Principle of min^m distance → It refers to min^m travel of man & materials & as far as possible straight line movement is preferred.
3. Principle of cubic space utilization → It should utilize horizontal & vertical space. Along with the floor space, height has also to be utilised effectively.
4. Principle of flow → The materials should move in forward dirn towards completion stage i.e. without backtracking.
5. Principle of max^m flexibility → Future requirements should be considered while designing the present layout
6. Principle of safety, security & satisfaction → It should give safety & satisfaction to workers. safeguards to plant & machinery against fire, theft etc
7. principle of min^m handling → Material handling should be min^m

Advantages of plant layout →

1. Advantages to workers - provide better working condn
2. " " management - ↓ labour cost & ↑ productivity.
3. " manufacture - ↓ movement among work centres
4. " production control - produce cycle completion within stipulated time

Factors influencing plant Layout →

1. Type of production -
2. Production system -
3. Scale of production -
4. Availability of total area -
5. Arrangement of material handling system -
6. Type of building -
7. Future expansion plan.
8. Type of production facilities

Types of production →

Accn to volume & standardisation of production, production systems are of 3 types
i) Job type production
ii) Batch "
iii) continuous "

i) Job type production

It is the manufacturing of one or few quantities of products designed & produced as per the specifications of the customers within the prefixed time & cost ie.

Production → High variety & low volume.

Characteristics

1. High variety, low volume.
2. General purpose machines & equipments to perform variety of operation.
3. manufacturing cycle time is more.
4. High skilled workforce required.
5. Highly competent & qualified supervisors are required
6. Very large work-in-process inventory.
7. Flexible material handling system.
8. Difficulty in planning, scheduling & coordinating the production of numerous components of wide variety.

ii) Batch production →

characterised by manufacture of limited number of products produced at regular intervals & stocked at warehouses awaiting sales.

Ex- pharmaceutical industry, chemical industry assembly shops etc. electrical goods, computer software
seasonal products

characteristics

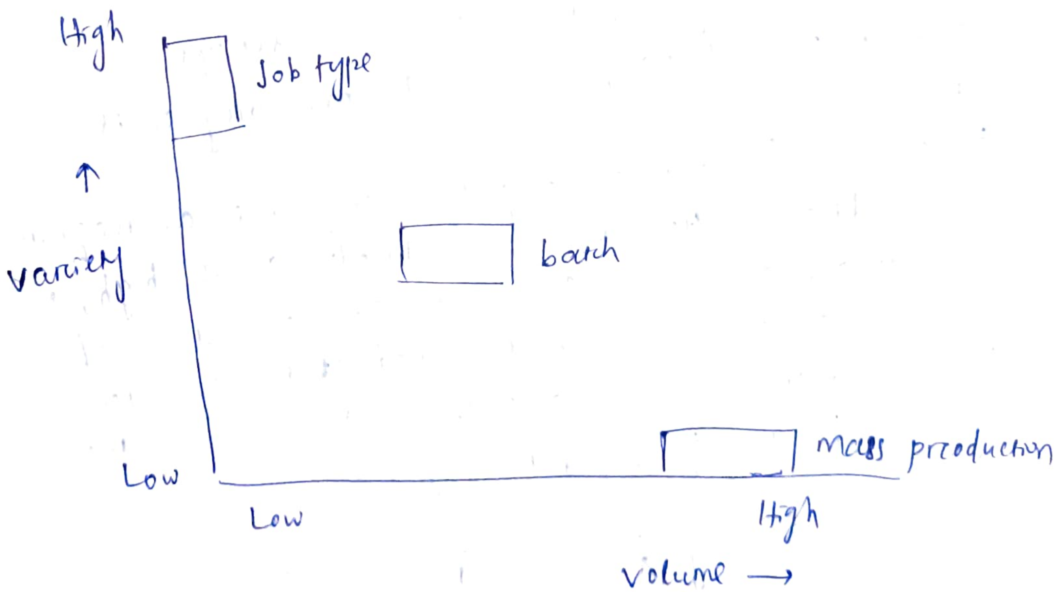
1. short production runs
2. more no. of set-ups.
3. The workers are expected to posess skill in one particular manufacturing operⁿ
4. Amount of supervision required is less than job type.
5. Plant & machineries are flexible.
6. manufacturing cycle time is comparatively lower
7. Large work-in-process inventory.
8. Flexible material handing system

iii) Continuous / repetitive / mass production

Characterised by high volume & low variety. It manufactures several standard products & stocked in the warehouses as finished goods awaiting for despatch. Ex- plastic goods, Manufacture & assembly shops of automobiles etc.

Characteristics

1. Flow of material is continuous
2. Special purpose machines are used.
3. material handing system is mechanised.
4. Relatively lower skilled persons can manage work
5. Shorter cycle time.
6. work-In-process is comparatively low
7. Higher inventory of raw materials
8. Less flexibility of equipment & machines.



[Types of manufacturing systems]

1.4 Types of Layout :→

1. ~~Process / Functional layout→~~

- It is of ~~4~~ 5 types
  - i) process layout
  - ii) product "
  - iii) ~~Group~~ / combination layout
  - iv) Fixed position "
  - v) Group layout

1. Process / Functional Layout→

Suitable for job shops. All machines performing similar type of operations are grouped at one location in process layout e.g. all lathes, milling, drilling m/cs etc.

Here the arrangement of facilities are grouped together according to their functions.

Advantages

1. Flexibility of equipment & personnel.
2. Higher utilisation of production facilities.
3. Lower investment as less no. of machines.
4. Variety of job makes the job challenging & interesting.
5. Greater flexibility in distribution of work to workers & machineries.

disadvantages

1. material handling cost is high.
2. Production planning & control systems are complex.
3. more space is required.
4. Higher skill is required.
5. Lower productivity due to large no. of set-ups.
6. Process time is prolonged which ↑ inventory costs.

ii) Product layout / Line layout→

Here machines are arranged in sequence as required by the product. Here volume of production is high, so facilities are arranged such that efficient flow of materials can be done which will reduces the cost per unit. Special purpose machines are used to perform the required function quickly & reliably.

Ex.

Turning oper^n → Milling → Drilling → Assembly → Inspection → package / despatch

Advantage

1. Reduced material handling cost.
2. cycle time is short due to continuous flow of materials
3. Simplified production, planning & control.
4. small work-in-process inventory required
5. Unskilled workers can learn & manage the production

Disadvantage

1. Lack of flexibility.
2. Large capital investment.
3. special purpose m/c required
4. Dependence of whole activity on each part. Breakdown of any m/c in the sequence will stop production process.

iii) Combination / ~~Group~~ layout →

It is a mixed or hybrid layout of process & product layout Ex- Automobile manufacture unit.

Different parts / components are prepared by process layout & the final assembly is done by product layout.

Advantage

1. Effective m/c operation
2. High productivity.
3. overall production time ↓
4. " cost ↓.

Disadvantage

may not be feasible for all situations. If product mix is completely dissimilar then we can't opt for this.

iv) Fixed position layout →

→ Also called project type layout.

→ Here material, or major components remain in a fixed location & tools, machinery, men & other materials are brought to this location.

→ It is suitable when one or ~~more~~ few pieces of identical heavy products are to be manufactured & when the assembly consists of large no. of heavy parts, the cost of transportation of these parts is very high

Advantage

1. Greater flexibility
2. Lower capital investment is lower.
3. Helps in job enlargement & upgrades the skills of the operators
4. The workers identify themselves with a product in which they take interest & pride in doing the job

v) Group layout →

→ Group technology is the analysis & comparisions of items with similar characteristics to group them into families

→ It can be used to develop a hybrid between pure process layout & product layout.

→ Useful for companies producing variety of parts in small batches.

→ GT involves 2 basic steps. First step is to determine component families or groups & second step is to arrange plant equipments for particular family of components.

Advantage

1. Reduces production planning time.
2. " set up time.

Comparision between product & process layout →

Product layout	Process layout
1. A sequence of facilities as per processing requirement of products	1. All similar facilities are grouped together.
2. High volume, few products	2. High volume high variety
3. Standadised, stable rate of o/p products.	3. Diversified ~~or~~ products.
4 work flow is straight line. same sequence of oper for all product.	4. variable flow for each product type.
5. material handling. Flow is systematic & predictable & can be automated easily.	5. cann't be automated as flow depends upon product type.
6. High turnover of raw materials & WIP inventory	6. Low turnover of both raw material & WIP.
7 Break down of any m/c stops production.	7. can tolerate break down.
8 Low flexibility	8. High
9. Production control simple.	9. complex
10 Efficient space utilisation.	10 Low
11. product cost - high fixed cost & low variable cost.	11. Low FC & high VC

Symptoms of bad layout →

1. Long material flow lines & backtracking
2. poor utilisation of space.
3. Long production cycle
4. Excessive handling of materials.
5. more frequent accidents.
6. Difficult to supervise & control.
7. Spoilage of products during handling.
8. congestion for movement of materials & men.

Procedure of plant layout →

1. collecting the detailed information about product, process etc & recording data systematically.
2. Analysing the data by using various techniques & analysis
3. Selecting the general flow pattern for materials
4. Designing the individual work stations.
5. Assembling the individual layout into total layout in accordance with the general flow pattern & the building facilities.
6. coordinate the plan with plan for handling materials.
7. completing the plant layout.
8. converting the plant layout into floor plans that is to be used by the plant engg. for instalation of equipment

Tools & techniques of plant layout →

For a good layout, various data regarding processes, sequence of operation, material, flow sequence, space requirements, activities & their relationships are required.

Tools & techniques used for analysis are

i) process chart (operⁿ process chart, flow process chart)
ii) Travel chart
iii) Diagrams
iv) Relationship chart (REL)
v) Scale models

Chapter-2 Operations Research

Operation research is the scientific method of providing executive with an analytical & objective basis for decisions.

Scope of operation research →

OK is useful in the following fields

i) Agriculture
ii) Finance
iii) Industry
iv) marketing
v) Personnel management
vi) Production "
vii) Research & development
viii) Military operations

phases/stages of operation research →

1) Formulating the problem
2) Constructing a mathematical model
3) Deriving the solution from the model
4) Testing the model & its solution
5) Controlling the solution
6) Implementation

Characteristics of model in OR →

model in OK is a simplified representation of an operation. It should possess the following characteristics

i) The no. of variables used should be as few as possible
ii) " assumptions " "
iii) It should be easy & economical to construct
iv) It should not take much time in construction for any problem.
v) It should be adaptable to parametric type of treatment.

Applications of OR →

1. It provides a logical & systematic approach to the problem.
2. It allows modification of mathematical solution.
3. It suggests all the alternative solutions for the same problem.
4. It facilitates improved quality of decision
5. It leads to optimum use of resources
6. It gives scope for new research & improvement in the system
7. It prepares future managers by improving their

Knowledge & skill

8. It indicates the scope as well as limitation of a problem.

Linear Programming Problem (LPP) →

LPP deals with the optimization (maximization or minimization) of a function of variables called as objective function. It consists of a set of linear equalities / inequalities called as constraints.

LPP is a mathematical technique which involves allocation of limited resources in an optimal manner.

Formulation of LPP →

Step-1: To write down the decision variables of the problem.

Step-2: To formulate the objective function to be optimized as a linear function of the decision variables.

Step-3: To formulate the other conditions of the problem such as resource limitations, market constraints, interrelations between variables etc as linear inequations or equations in term of decision variables.

Step-4: To add non-negative constraints from the considerations.

So, a LPP consist of
i) Objective function
ii) set of constraints
iii) non-negative restrictions

General formulation of LPP →

To find the values of n decision variables $x_1, x_2, \dots x_n$ to maximize or minimize the objective function

$$Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

to satisfy m constraints

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1 \text{ (or } \ge)$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le b_2 \text{ (or } \ge)$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le b_m \text{ (or } \ge)$$

& $x_1, x_2, \dots x_n \ge 0$

Matrix form of LPP →

Min or Max $Z = Cx$ → objective funⁿ

S.t $Ax \; (\leq = \geq) \; b$ → constraint eqⁿ

$b \geq 0, \; x \geq 0$ → non-negative restriction

where $x = (x_1, x_2, \ldots x_n)$

$C = (c_1, c_2, \ldots c_n)$

$$b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}_{m \times 1} \qquad A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n}$$

Q [2] A manufacturer produces two types of models M_1 & M_2. Each model of the type M_1 requires 4 hours of grinding & 2 hours of polishing where as each model of type M_2 requires 2 hours of grinding & 5 hours of polishing. The manufacturer has 2 grinders & 3 polishers. Each grinder works 40 hours a week & each polisher works for 60 hours a week. Profit on M_1 model is ₹ 3 & on model M_2 is ₹ 4. Whatever is produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models so that he may make maxᵐ profit in a week.

Solⁿ Decision variables: Let x_1 & x_2 be the no. of units of M_1 & M_2 models respectively.

Objective function: Since profit we have to maximize

$$\text{Max } Z = 3x_1 + 4x_2$$

Constraints: There are 2 constraints – one for grinding & other for polishing.

No. of hours available for each grinder for one week = 40. As there are 2 grinders, so manufacture does not have more than $40 \times 2 = 80$ hours for grinding. M_1 requires 4 hours of grinding & M_2 require 2 hours of grinding.

So, $4x_1 + 2x_2 \leq 80$

Since 3 polishers are there. So time availability is less than $3 \times 60 = 180$ hours. M_1 requires 2 hours of polishing & M_2 requires 5 hours of polishing.

So, $2x_1 + 5x_2 \leq 180$

Finally we have

$$\max Z = 3x_1 + 4x_2$$
$$4x_1 + 2x_2 \le 80$$
$$2x_1 + 5x_2 \le 180$$
$$x_1, x_2 \ge 0$$

2) A company manufacturers two products A & B. These products are processed in the same machine. It takes 10 minutes to process one unit of product A & 2 minutes for each unit of product B & the machine operates for a max^m of 35 hours in a week. Product A requires 1kg & B requires 0.5 kg of raw material per unit, the supply of which is 600kg per week. Market constraint on product B is known to be min^m of 800 units every week. Product A costs ₹5/unit & sold at ₹10. Product B costs ₹6/unit & sold at ₹8. ~~Determine the no. of units of A & B per week to maximize the profit~~. Formulate the LPP.

Sol^n Let x_1 & x_2 be the no. of products A & B respectively.

objective function

For product A, cost/unit = ₹5 & selling price/unit = 10

so, profit = 10 − 5 = ₹5/unit

for product B, cost/unit = 6 & S.P/unit = 8

So, profit = 8 − 6 = ₹2/unit

$$\max Z = 5x_1 + 2x_2$$

Constraints

Time requirement constraint $\quad 10x_1 + 2x_2 \le (35 \times 60)$

$\Rightarrow 10x_1 + 2x_2 \le 2100$

Raw material " $\Rightarrow x_1 + 0.5x_2 \le 600$

Market demand of B $\Rightarrow x_2 \ge 800$

So, finally

$$\max Z = 5x_1 + 2x_2$$
$$10x_1 + 2x_2 \le 2100$$
$$x_1 + 0.5x_2 \le 600$$
$$x_2 \ge 800$$
$$x_1, x_2 \ge 0$$

[1]

3) A person requires 10, 12 & 12 units of chemicals A, B & C respectively for his garden. A liquid product contains 5, 2 & 1 units of A, B & C resp per jar. A dry product contains 1, 2 & 4 units of A, B, C per carton. If the liquid product is sold for ₹3 per jar & dry product at ₹2 per carton, how many units of each product should be purchased in order to minimize the cost & meet the requirement.

Sol^n Min Z → Let x_1 & x_2 be the no. of units of Liquid & dry products.

$$Min\ Z = 3x_1 + 2x_2$$

As there are 3 chemicals, so we have 3 constraints for each

$$5x_1 + x_2 \geq 10$$
$$2x_1 + 2x_2 \geq 12$$
$$x_1 + 4x_2 \geq 12$$
$$x_1, x_2 \geq 0.$$

4) A paper mill produces two grades of paper namely X & Y. Owing to raw material restrictions, it cannot produce more than 400 tons of grade X & 300 tons of grade Y in a week. There are 160 production hours in a week. It requires 0.2 & 0.4 hours to produce a ton of products X & Y resp. with corresponding profits of ₹200 & ₹500 per ton. Formulate the LPP to maximize profit.

Sol^n Let x_1, x_2 → no. of units of two grades X & Y resp.

$$Max\ Z = 200x_1 + 500x_2$$

s.t.

$$x_1 \leq 400$$
$$x_2 \leq 300$$
$$0.2x_1 + 0.4x_2 \leq 160$$
$$x_1, x_2 \geq 0.$$

5) A firm manufacturers 3 products A, B & C. The profits are ₹3, ₹2 & ₹4 resp. The firm has 2 machines & given below is the required processing time in minutes for each machine on each product

Machines	Product wise processing time (min)		
	A	B	C
M_1	4	3	5
M_2	3	2	4

M/c M_1 & M_2 have 2000 & 2500 machines minutes resp. The firm must manufacture 100 units of A's, 200 units of B's & 50 units of C's but not more than 150 units of A's. Set up an LPP to maximize the profit.

Sol^n Let x_1, x_2, x_3 be the no. of units of product A, B & C resp.

$$Max\ Z = 3x_1 + 2x_2 + 4x_3.$$

For M_1, $4x_1 + 3x_2 + 5x_3 \leq 2000$

For M_2, $3x_1 + 2x_2 + 4x_3 \leq 2500$

&

$$100 \leq x_1 \leq 150$$
$$200 \leq x_2 \geq 0$$
$$50 \leq x_3 \geq 0$$

$x_1, x_2, x_3 \geq 0.$

Graphical method of Solution

Procedure

Step-1: Consider each inequality constraint as eqn.

Step-2: Plot each eqn on graph, such that it will geometrically represent straight line.

Step-3: Mark the region.

If inequality is $\leq$ then mark the region below the straight line in 1st quadrant only.

If " $\geq$ then mark the region above the straight line in 1st quadrant only.

The common region satisfying all the constraints is called feasible region.

Step-4:

Q-1 Solve the LPP by graphical method.

$\min Z = 20x_1 + 10x_2$

s.t $x_1 + 2x_2 \leq 40$

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \geq 60$

$x_1, x_2 \geq 0$

Sol^n

$x_1 + 2x_2 = 40$ A (0, 20), B (40, 0)

$3x_1 + x_2 = 30$ C (0, 30), D (10, 0)

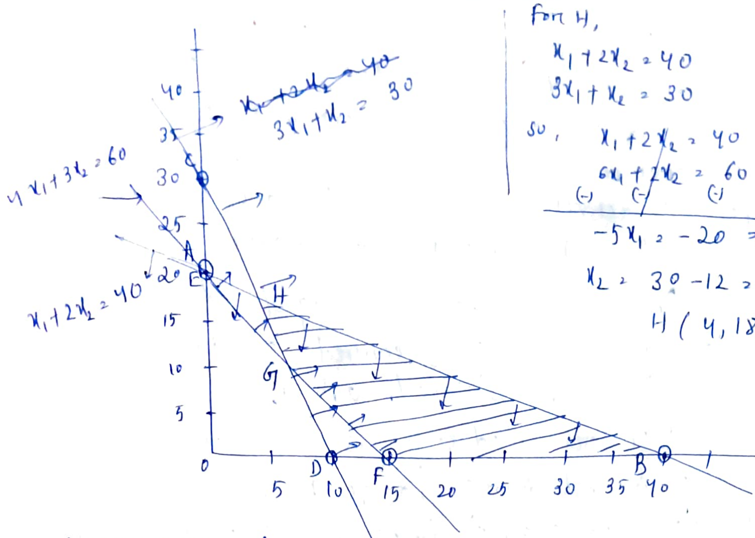
$4x_1 + 3x_2 = 60$ E (0, 20), F (15, 0)

For G, $4x_1 + 3x_2 = 60$
$3x_1 + x_2 = 30$

$$4x_1 + 3x_2 = 60$$
$$\underset{(-)}{9x_1} \underset{(-)}{+ 3x_2} = \underset{(-)}{90}$$
$$-5x_1 = -30 \Rightarrow x_1 = 6$$

So, $x_2 = 30 - 18 = 12$

So, G (6, 12)



For H,

$x_1 + 2x_2 = 40$
$3x_1 + x_2 = 30$

so, $x_1 + 2x_2 = 40$
$6x_1 + 2x_2 = 60$
(-) (-) (-)

$-5x_1 = -20 \Rightarrow x_1 = 4$

$x_2 = 30 - 12 = 18$

H (4, 18)

Corner points of feasible region		min $Z = 20x_1 + 10x_2$
F (15, 0)	→	300
B (40, 0)	→	800
G (6, 12)	→	260
H (4, 18)	→	240 (min^m)

So, soln is $x_1 = 4$
$x_2 = 18$

Q.2 Solve the LPP by graphical method

max $Z = 5x_1 + 7x_2$

s.t $x_1 + x_2 \le 4$

$3x_1 + 8x_2 \le 24$

$10x_1 + 7x_2 \le 35$

$x_1, x_2 \ge 0$.

Project Scheduling with CPM & PERT:→

A project is any task which has definable beginning & definable end & requires investment & expenditure of one or more resources each separate but interrelated & interdependent activities which must be accomplished to achieve the objectives for which the task or project was instituted.

Project has to be managed effectively through proper planning, scheduling & control as project requires a heavy investment & associated with risk & uncertainties.

Network scheduling is a technique used for planning & scheduling large projects in the fields of construction, maintenance, fabrication & any other areas.

There are two basic planning & control techniques.

1) CPM (Critical Path Method)
2) PERT (Programme Evaluation & Review Technique)

Objectives of network analysis→

1) Used for planning, scheduling & controlling of projects
2) Minimisation of total project cost & time
3) Effective utilisation of resources & minimisation of effective resources.
4) Minimisation of delays & interruption during project implementation.

Application of Network Analysis (CPM & PERT)→

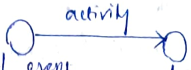
1. Research & development project.
2. Equipment maintenance & overhauling.
3. Construction projects (building, bridges, dams)
4. Setting up new industries.
5. Planning & launching of new products.
6. Design of plants, machines & systems
7. Shifting the manufacturing location from one location to another.
8. Control of production in large job shops.
9. Market penetration programs
10. Organisation of big programs, conferences etc.

Basic concepts in Network →

1) **Network** → It is a graphical representation of project & consist of series of activities arranged in a logical sequence to show the inter-relationship between the activities.

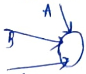
2) **Activities** → It is a physically identifiable part of the project, which consumes time & resources. Each activity has a definite start & end. It is represented by an arrow (→).

3) **Event** → It represents the start or the completion of an activity. The beginning & end points of an activity are events.



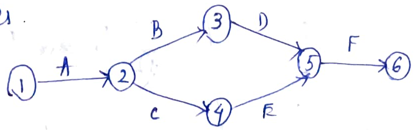
Ex- Matching a component is an activity.
- Start matching is event
- m/c completed is event

→ **merge event** → when no. of activities terminate into a single event or node.



→ **burst event** → when no. of activities may emanate from a single event or node.

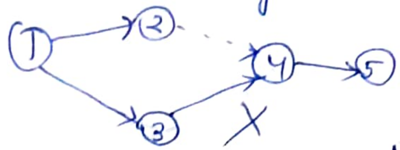
4) **Predecessor & successor activities** →
All the activities which must be completed before starting the activity under consideration are called predecessor activity.
All the activities which have to follow the activity under consideration are called successor activities.

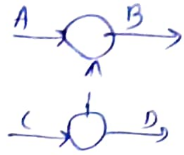


Activity	immediate predecessor
A	-
B	A
C	A
D	B
E	C
F	D, E

5) **Path** → An unbroken chain of activities between two events is called a path. e.g.
A → B → D → F
A → C → E → F

6) Dummy activity → An activity which depicts the dependency or relationship over the other but does not consume time or resources. It is used to maintain a logical sequence.

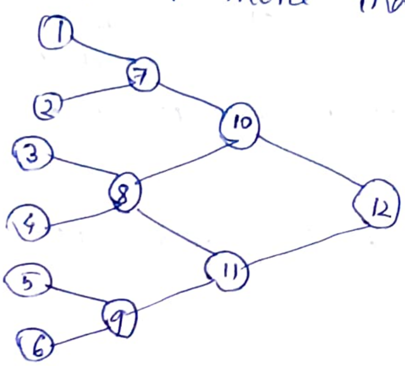




Here activity B preceded by A & C
D " C

Numbering of events (Fulkerson's Rule) →

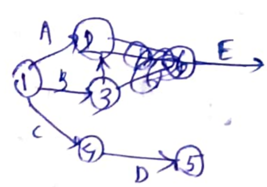
1. An event having no preceeding activity is called initial event & is numbered 1.
2. The numbering of events is done from left to right in case of successive events & from top to bottom in case of more than one initial events



3. Continue the process till all the events are numbered. The last event will be at higher number.

Q-2 Construct the network from the infn given

Activity	immediate predecessor	activity time (weeks)
A	–	6
B	–	10
C	–	14
D	C	6
E	A, B	14
F	E, D	6
G	D	4
H	F, G	4



Soln