LECTURE NOTES

ON

HYDRAULICS & IRRIGATION ENGG(TH2)

FOR

DIPLOMA IN CIVIL ENGINEERING

(4^{TH} SEMESTER STUDENTS)

AS PER SCTE&VT SYLLABUS

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Fluid Mechanics

Introduction to fluid and fluid mechanics

Fluid Mechanics

Fluid – Any Substance which can flow is called fluid

→ Finite Mass
→ Occupies Space
→ Tangible

} Substance

Flow – Relative change of Position w.r.t time

Matter

→ Solid
→ Liquid
→ Gas

Liquid and Gas → Fluid

Fluid

→ have mass
→ Fluid does not have any certain shape
→ It occupies shape of vessel
→ which can flow under it's own weight.

Microscopic Approach

→ Fluid motion or rest study in molecular level
→ each & individual particles is ~~det~~ analyse Separately
→

Mechanics – It is study of Force and Effect of Force.

Fluid Mechanics

Study of Effect of force on fluid.

→ Fluid at Rest (Fluid statics)
→ Fluid in Motion (Fluid Dynamics)

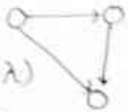
Macroscopic Approach

→ Overall behaviour of fluid is studied.
→ Continuous distribution of Mass
↳ "There are no voids between molecules" (Assumption)

Knudsen Number (K_n)

$$\boxed{K_n = \frac{\text{Mean Free Path}}{\text{Characteristics length}} = \frac{\lambda}{L}}$$

(λ)



Mean free Path

No-Slip cond^n

$\boxed{Kn < 0.01}$ → Continuous distribut^n of mass or Continuum

↓

Macroscopic approach

$\boxed{0.01 < Kn < 10}$ → Slip flow

$\boxed{Kn > 10}$ → Free Molecular flow → Microscopic approach

Force on a Fluid

① Surface Force - Act on Surface - Internal or External force

② Body Force (Gravitational Forces)
↳ Through out the mass of the fluid

↳ Ex- Atmospheric Force

↳ Ex- Gravitational Pull

Mass → unit of quanity weight → Measure of a gravitational force exerted by a given mass

Properties of Fluid

① Density or Mass Density (ρ)

Mass per unit volume.

$$\boxed{\rho = \frac{m}{V}}$$

$\rho\uparrow \rightarrow m\uparrow$ (compact) Dense material

$\rho\downarrow \rightarrow m\downarrow$

* SI unit of Density

$$\boxed{\rho \Rightarrow kg/m^3}$$

* CGS unit of Density

$$\boxed{\rho = gm/cm^3 \text{ or } gm/cc}$$

NOTE

Density of water change w.r.t time. because of hydrogen bond inside the water

Maximum $\boxed{\rho_{water} = 1000\ kg/m^3 \text{ at } 4^\circ C}$

(II) Weight Density (γ)

$$\boxed{\gamma = \frac{\text{weight}}{\text{Volume}}}$$

*SI unit - N/m^3

$$\gamma = \frac{W}{V} = \frac{m.g}{V} = \rho g$$

$$\boxed{\gamma = \rho . g}$$

(iii) Specific Gravity or Relative Density (S)

$$\boxed{S = \frac{\text{Density of fluid}}{\text{Density of standard fluid}}}$$

(STP → Standard temperature & Pressure)

For liquid

$$\boxed{S_L = \frac{\text{Density of liquid}}{\text{Density of water at } 4^\circ C}}$$

For Gases

$$\boxed{S_G = \frac{\text{Density of Gas}}{\text{Density of air at STP}}}$$

$$\boxed{\begin{aligned} \rho_{water} &= 1000\ kg/m^3 \quad (4^\circ C) \\ \rho_{air} &= 1.21\ kg/m^3 \text{ at STP} \end{aligned}}$$

Ex. S.G of Mercury 13.6

$\rho_{Hg} = ?$

$(SG) = \frac{\rho_{Hg}}{\rho_w} \Rightarrow 13.6 = \frac{\rho_{Hg}}{1000}$

$$\boxed{\rho_{Hg} = 13600\ kg/m^3}$$

(iv) Surface tension (σ)

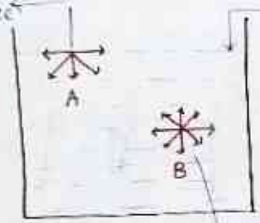
Free Surface of liquid behaves like a stretched membrane and it tries to minimise its area upto maximum possible extent. This property is known as "Surface Tension"

$$\boxed{\sigma = \frac{\text{Force}}{\text{length}}} \ (N/m)$$

Asymmetrical Pull Force

$\Sigma F \neq 0$

(Inward Pull)



Free Surface

→ It is a Surface with constant Normal Stress

→ Zero shear stress (No External force in tangential Direction)

Pull Force Symmetrical ($\Sigma F = 0$, Equilibrium cond^n)

Surface Energy

Work done in change in Free Surface (contracting)

$$\boxed{W = \sigma . x}$$

Surface Tension (σ) $= \frac{\text{Force}}{\text{length}} \times \frac{\Delta L}{\Delta L}$

$\Rightarrow \frac{Nm}{m^2} = \frac{Joule}{m^2}$

σ = Surface energy per unit area.

$$\boxed{\sigma \Rightarrow N/m \text{ or } \frac{Joule}{m^2}}$$

* It occurs only in liquid because the distance is confined

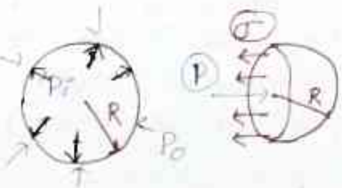
Surface tension in different Cases

① Water Droplet

$P \rightarrow$ Excess Pressure

$P_i \rightarrow$ Inside Pressure

$P_o \rightarrow$ Outside Pressure



$\boxed{P = P_i - P_o}$

* No. of Free Surface = 1

For eqlb^n $\Rightarrow \Sigma F = 0$

$$P \cdot \pi R^2 = \sigma \times 2\pi R$$

$\boxed{P = \frac{2\sigma}{R}}$ $\boxed{P = \frac{4\sigma}{D}}$

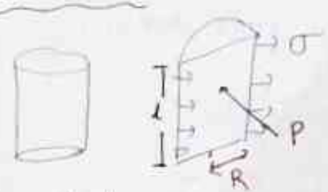
② Soap Bubble

* No. of free Surface = 2

$$P\pi R^2 = \sigma \times 2 \times (2\pi R)$$

$\boxed{P = \frac{4\sigma}{R}}$ $\boxed{P = \frac{8\sigma}{D}}$

③ Water Jet



$$2\sigma \times l = P \times 2R \times l$$

$\boxed{P = \frac{\sigma}{R}}$

$\boxed{P = \frac{2\sigma}{D}}$

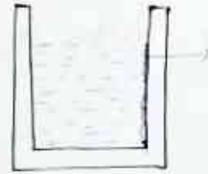
* No. of free Surface = 1

There are 2 types of Force

→ Adhesive Force - Force acting between 2 surface (or between molecule of two surface) diff

→ Cohesive Force

↳ Force acting betn molecule of same medium



Wetting or Non wetting Liquids.

$\boxed{\text{Adhesion Force} >> \text{Cohesion Force}}$

→ Liquid will wet the solid boundary

→ wetting liquid → Ex- H_2O- glass interface

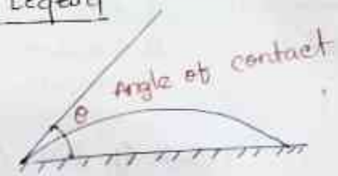
$\boxed{\text{Cohesive Force} >> \text{Adhesion Force}}$

→ Non wetting liquid

→ Ex- Hg - glass interface

ⓘ For wetting Liquid

Ex- water Glass

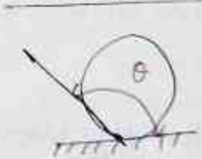


$\boxed{\theta < 90^\circ \text{ or } \pi/2}$

$\Rightarrow \boxed{\text{Adhesive Force} > \text{Cohesive Force}}$

ⓘⓘ For Non-wetting Liquid (Ex- Mercury-Glass)

$\boxed{\theta > 90^\circ \text{ or } \pi/2}$

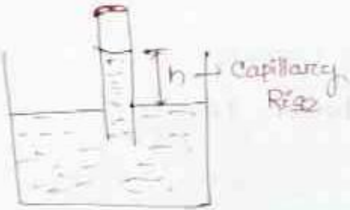


$\boxed{\text{Cohesive Force} > \text{Adhesive Force}}$

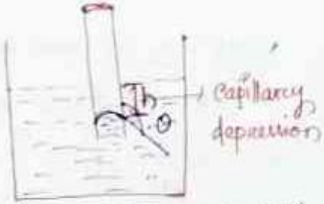
(V) Capillarity

It is defined as phenomenon of Rise or Fall of a liquid surface in a small tube relative to adjacent level of liquid when tube is held vertical.

* due to adhesion & Cohesion

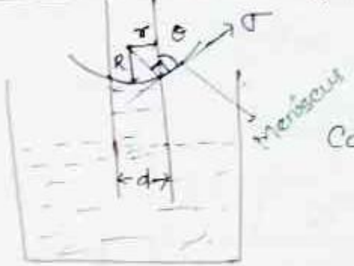


(Wetting liquid)
(Rise)



(Nonwetting Liquid)
(depression)

Formula For Capillarity



r → Radius of capillary tube
R → Radius of Curvature of Meniscus

Capillary Rise / Depression

$$\boxed{h = \frac{4\sigma\cos\theta}{\rho g d}}$$

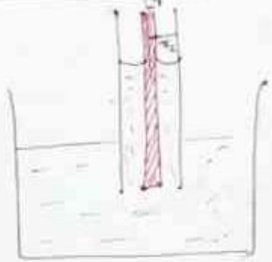
σ → Surface tension (N/m)
ρ → Density of liquid (kg/m³)
d → Diameter of capillary tube (m)

h → +ve (Rise)
h → -ve (Depression)

$\boxed{\theta = 0}$ For water - Glass

$\boxed{\theta = 128°c}$ For Hg - Glass

Annular Capillarity



$$\boxed{h = \frac{2\sigma\cos\theta}{\rho g(r_2 - r_1)}}$$

(VI) Viscosity

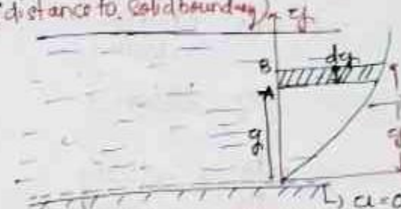
"Resistance to Flow of a Fluid."

Reasons of viscosity :-

1. Cohesive Force (intermolecular attraction)
2. Intermolecular momentum Transfer
   - According to kinetic theory of gases it consist of kinetic energy.
   - it has particular mass and velocity so it give momentum of that molecule.
   - when it collide it will transfer the momentum and create intermolecular friction which result to viscosity.

$(\text{Cohesive Force})_{Gas} << (\text{Cohesive Force})_{Liquid}$.

$(\text{Molecular momentum})_{Gas} >> (\text{Molecular momentum})_{liquid}$

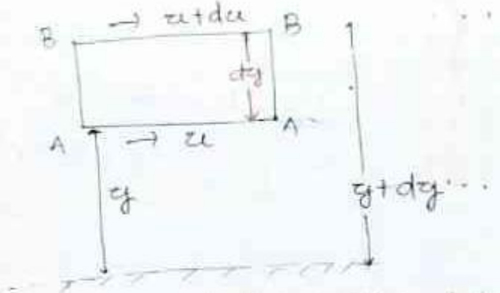


$\boxed{Kn < 0.01}$

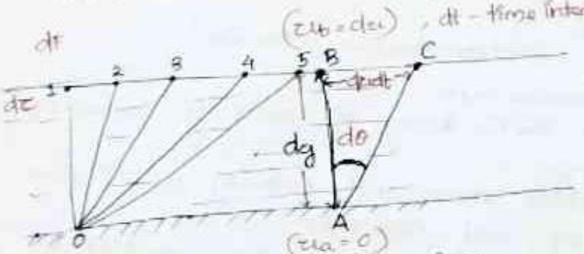
Velocity Profile or distribution

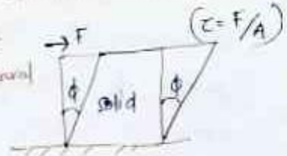
→ u (Velocity of fluid)

Adhesive Force & van der Waals Forces



→ If there is a relative motion between 2 adjacent layer of fluid they + a resistance to flow that is known as viscosity.





For solid

$$\boxed{\tau \propto E}$$

hooke's law

Newton's Law of Viscosity

Shear Stress ∝ Rate of shear strain

$$\boxed{\tan d\theta = \frac{du.dt}{dy}}$$

As dθ is very small, so $\boxed{\tan d\theta = d\theta}$

$$d\theta = \frac{du\,dt}{dy}.$$

$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

$\frac{d\theta}{dt}$ → Rate of shear strain

du - change in velocity

dy - Distance.

$\frac{du}{dy}$ → velocity gradient.

Hence, Rate of shear strain = Velocity Gradient

$$\tau \propto \frac{d\theta}{dt}$$

$$\tau \propto \frac{du}{dy} \quad \left(\because \frac{d\theta}{dt} = \frac{du}{dy}\right)$$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

μ → Dynamic Viscosity of the fluid.

Newton's Law of viscosity

⇒ Shear stress in a fluid is directly Proportional to rate of shear strain

μ → Dynamic Viscosity

SI unit

$$\tau = \mu \frac{du}{dy}.$$

$$(N/m^2) = \mu \frac{m}{s\,m}$$

$$\mu = \frac{N}{m^2} \times \frac{m\,sec}{m}$$

$$\boxed{\mu \Rightarrow \frac{NS}{m^2}} \text{ or } \boxed{PaS}$$

CGS

$$\tau\left(\frac{dyne}{cm^2}\right) = \mu \frac{du}{dy}\left(\frac{cm/s}{cm}\right)$$

$$\mu = \frac{dyne \cdot s}{cm^2}$$

$$\boxed{\frac{dyne.s}{cm^2} = \text{Poise}}$$

$$1 \text{ Poise} = \frac{1}{10} \frac{NS}{m^2} \text{ or PaS}$$

or

$$1 \text{ PaS} = 10 \text{ Poise}$$

* Fluid is a substance which deforms continuously under action of a very small shear force or shear stress.

Kinematic Viscosity (ν) - nu

$$\boxed{\nu = \frac{\mu}{\rho}} \quad (m^2/s) \quad (cm^2/s)$$

→ It is the ability of fluid to diffuse a disturbance in molecular momentum.

$$\boxed{1\ cm^2/s = 1\ \text{stokes}}$$

$$\boxed{1\ \text{stoke} = 10^{-4}\ m^2/s}$$

Effect of Temperature on Dynamic Viscosity

(i) Liquid (T↑, K.E↑, Attraction Force↓, Cohesive Force↓)

→ μ of liquid decreases with increase in temperature

$$\boxed{\mu = \frac{\mu_0}{1+\alpha T+\beta T^2}} \qquad \mu \propto \frac{1}{T}$$

μ_0 → Dynamic viscosity of liquid at 0°C

α, β → Constant ($\alpha > \beta$)

(ii) Gases

T↑, Molecule momentum↑, Frictional Force↑, μ↑

→ μ of gases increases with respect to temperature

$$\boxed{\mu = \mu_0 + \alpha T + \beta T^2} \qquad (\alpha > \beta)$$

Rheological Behaviour of Fluid

Branch of science which deal with behaviour of fluid.

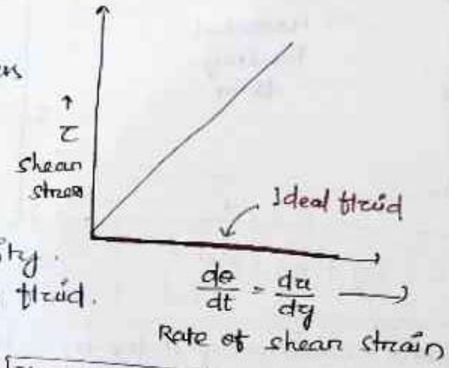
(i) Ideal fluid:

→ Inviscide (No viscosity ($\mu=0$)) → Newton's law: $\tau = \mu \frac{du}{dy}$ → $\boxed{\tau = 0}$ → No shear stress

→ Incompressible

→ No shear stress

→ Constant Normal stress



(ii) Newtonian Fluid:

→ Fluid which obey's Newton Law of viscosity is known as Newtonian fluid.

$$\boxed{\tau = \mu \frac{du}{dy}} \qquad \boxed{y = mx + c} \ (c \to 0)$$

$y \to \tau$, $m \to \mu$, $x \to \frac{du}{dy}$

(iii) Non-Newtonian Fluid

→ doesn't obey newton's Law of viscosity.

$$\boxed{\tau = m\left(\frac{du}{dy}\right)^n}$$

where, m → Consistancy index

n → Flow Index

if $\boxed{n > 1}$

→ Dillantent Fluid

Shear thicking fluids

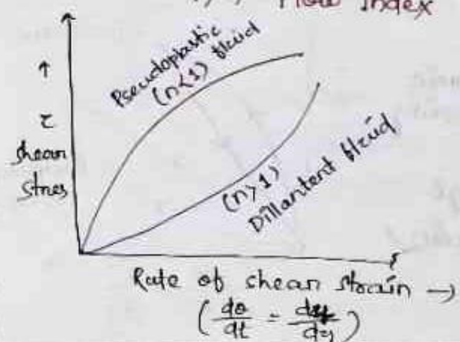
Ex- Starch Suspension

$\boxed{n < 1}$

Pseudoplastic fluid

shear thining fluid

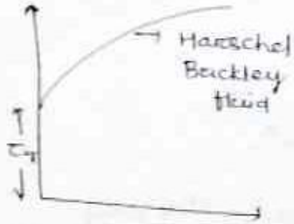
Ex- Blood

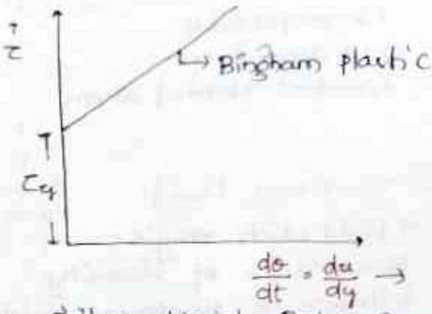


(iv) Bingham Plastic & Harschel Buckley Fluid

$$\boxed{\tau = \tau_y + m\left(\frac{du}{dy}\right)^n}$$

where, $\tau_y \rightarrow$ yield shear stress





$$\boxed{\tau = \tau_y + m\left(\frac{du}{dy}\right)^n}$$

where - Consistency Index $\rightarrow m$
Flow Index $\rightarrow n$

→ It a fluid Behave like newtonian fluid.

$\boxed{m = \mu}$ $\boxed{n = 1}$

Ex - Raisin Paste

Ex - Toothpaste.

Thixotropic & Rheopetic Fluid

Viscosity - Time dependent

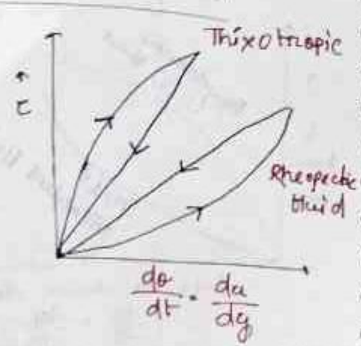
$$\boxed{\text{Apparent Viscosity} = \frac{\text{Shear stress}}{\text{Rate of shear strain}}}$$

Newtonian Fluid

Apparent Viscosity = Dynamic viscosity

Non-Newtonian Fluid

Apparent viscosity will change w.r.t Rate of shear strain



Example

Thixotropic - Water & Bentonite clay

↳ Apparent viscosity decreases with respect to time.

Rheopectic Fluid - Printer ink.

↳ Apparent viscosity increases w.r.t time.

Thermodynamic Properties of Fluid

① Compressibility → Compress + ability (B)

→ ease of compression

→ Volumetric strain per compressive stress

$\boxed{dv/v}$ $\boxed{B = \frac{-dv/v}{dp}}$ Pressure on the fluid

* during compression the volume of the material decreases

Let V_f = final volume
V = initial volume

$\boxed{dv = V_f - V}$ $(V_f < V)$
↳ it will be negative

Mass (m) = Mass density (ρ) x volume (V)

$m = \rho V$

Law of Conservation of Mass.

m = Constant

ρV = Constant.

⇒ $\rho dv + v \cdot d\rho = 0$ (by differentiating)

⇒ $\rho\, dv = -v d\rho$

$\rho dv = -v d\rho$

$$\boxed{\frac{-dv}{V} = \frac{d\rho}{\rho}}$$

we know,

$$\boxed{\text{compressibility } (\beta) = \frac{-dv}{V} \Big/ dp}$$

$\Rightarrow \beta = \frac{d\rho/\rho}{dp} = \frac{1}{\rho}\frac{d\rho}{dp}$

$$\boxed{\beta = f(\rho)} \qquad \boxed{\beta = \frac{1}{\rho}\frac{d\rho}{dp}}$$

(i) $\rho = f(P)$

$\frac{d\rho}{dp} \neq 0$

$\boxed{\beta \neq 0}$

→ Compressible material

(ii) $\rho \neq f(P)$

$\frac{d\rho}{dp} = 0$

$\boxed{\beta = 0}$

→ Incompressible fluid

(ii) Bulk Modulus (K)

$$\boxed{K = \frac{\text{Compressive stress or Bulk stress}}{\text{volumetric strain or Bulk strain}}}$$

$$\boxed{K = \frac{1}{\beta}} \qquad \beta = \frac{1}{\rho}\frac{d\rho}{dp}$$

$\Rightarrow \boxed{K = \frac{\rho dp}{d\rho}}$

K → Bulk Modulus of fluid or elasticity

Isothermal Compressibility (β_{iso})

→ Temperature is constant throughout the Process

$$\boxed{T = \text{Constant}}$$

$PV = mRT$ → Ideal Gas eq^n

$\Rightarrow P = \frac{mRT}{V}$

$\Rightarrow \boxed{P = \rho RT}$

$\Rightarrow \rho = \frac{P}{RT}$

Differentiating w.r.t. P

$$\boxed{\frac{d\rho}{dP} = \frac{1}{RT}} \quad \text{— (I)}$$

$$\boxed{\beta = \frac{1}{\rho}\frac{d\rho}{dP}} \quad \text{— (II)}$$

from eq^n (I) & (II)

$\beta = \frac{1}{\rho RT}$

$$\boxed{\beta_{iso} = \frac{1}{P}}$$

where, R =

* For Isothermal Process

$K_{iso} = \frac{1}{\beta_{iso}} = \frac{1}{1/P}$

$\Rightarrow \boxed{K_{iso} = P}$

Adiabatic Compressibility (β_{adia})

$PV^{\gamma} = C$

$P\left(\frac{m}{\rho}\right)^{\gamma} = C$

$P\left(\frac{m^{\gamma}}{\rho^{\gamma}}\right) = C$

m^{γ} → constant

$\frac{P}{\rho^{\gamma}} = \text{Constant}$

where γ =

$P \times \rho^{-\gamma} = C$

Differentiating

$P \times (-\gamma)\rho^{-\gamma-1} \cdot d\rho + \rho^{-\gamma} dp = 0$

$-P\gamma \frac{\rho^{-\gamma}}{\rho} \times d\rho + \rho^{-\gamma} dp = 0$

$-\frac{P\gamma \, d\rho}{\rho} + dp = 0$

$-P\gamma \frac{d\rho}{\rho} = -dp$

$$\boxed{\frac{d\rho}{dp} = \frac{\rho}{p\gamma}}$$

$$\beta = \frac{1}{\rho}\frac{d\rho}{dp}$$

$$\beta = \frac{1}{\rho}\frac{\rho}{p\gamma}$$

$$\Rightarrow \boxed{\beta_{adi} = \frac{1}{p\gamma}}$$

$$\boxed{K_{adi} = p\gamma}$$

What is the bulk modulus of elasticity of a liquid which is compressed in a cylinder from a volume of 0.0125 m³ at 80 N/cm² Pressure to 0.0124 m³ at 150 N/cm² Pressure

Sol^n

$$K = \frac{dp}{-dV/V} \qquad \boxed{K = \frac{1}{\beta}}$$

$$K = \frac{150 - 80}{\frac{(0.0124 - 0.0125)}{0.0125}}$$

$$\boxed{K = 8.75 \ N/cm^2}$$

(iii) Vapour Pressure

Pressure exerted by vapour of a Liquid on the Surface of the Liquid.

* When the liquid vapourise due to heat. The vapour exert some pressure due to it's kinetic energy which is known as vapour pressure

→ It is difficult to measure

* Volatile Material

Material in which very less energy is required to change it's phase from the surface (molecules)

→ The surface molecules take energy from the inside molecules in the form of heat

Evaporation

Vapour Pressure at Equilibrium (Saturation Pressure & Saturation Temperature

where condensation = Evaporation

* if. VP < Pressure acting on surface of liquid

$\boxed{\text{Evaporation}}$ → Surface Phenomenon

* if. VP = Pressure acting on the surface of Liquid

$\boxed{\text{Boiling}}$ → Special condition $\boxed{P_{atm} = VP}$

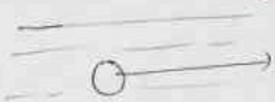
Vapourisation :-

→ Phase convertion Process

→ It occurs throughout the mass

→ M_{Liquid} kg of liquid converted to M kg of Gas or M kg of vapour.

Cavitation



Pressurised Vapour in bubble

$\boxed{\text{Pressure at a Point in liquid} < V.P}$

* When the high speed bubble collide with any material generally in pump impeller there impeachment creat in the material (less area more force).

The material corroded from the applied force which is called ~~Corro~~ erosion.

* To assure the reduction in cavitation problem then we shall maintain a particular pressure. which is known as NPSH (Net positive Suction Head).
* Cavitation is main reason behind the erosion on impeller blade.

Module - 2 Fluid Statics

→ Study of fluid at rest and Study of fluid in rigid body motion

$$\boxed{\begin{aligned} u &\neq f(x, y, z) \\ u &= 0 \quad \text{w.r.t time} \end{aligned}}$$

$\boxed{V = 0}$

* When the container is at rest

Relative velocity of fluid particles

$\boxed{v - v = 0}$

→ Rigid body motion of fluid

$\boxed{V \neq 0}$

* When the container is moving

Fluid at Rest

$$u \neq f(x, y, z)$$

$\frac{du}{dy}$ (velocity gradient) $= 0$ in y direction

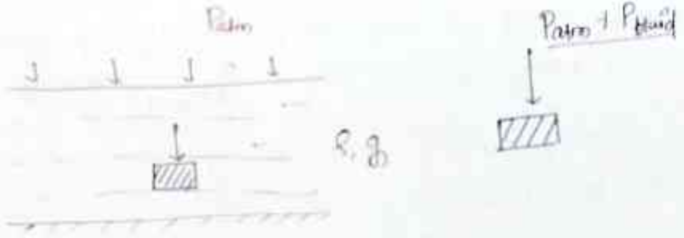
$$\frac{du}{dx} = 0, \quad \frac{du}{dz} = 0$$

Shear stress in any direction

$$\boxed{\tau_x = \mu \frac{du}{dx} = 0}, \quad \boxed{\tau_y = \mu \frac{du}{dy} = 0}, \quad \boxed{\tau_z = \mu \frac{du}{dz} = 0}$$

⇒ Shear stress in each plane will zero (or) There is no shear stress acting on fluid

* When fluid is at rest there is only Normal stress acting on the fluid.



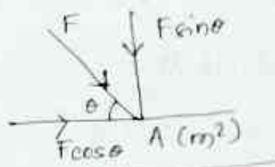
Normal Stress = Pressure acting on fluid

Pressure

Normal Component of force per unit area

$$\boxed{P = \frac{F_{\perp}}{A}}$$

$$\boxed{\text{Pressure} = \frac{F_{\perp}}{|A|}}$$



$$\boxed{P = \frac{F\sin\theta}{A}}$$

$F_{\perp}$ → Normal components of force

$|A|$ → Magnitude of Area

* Pressure is a scalar quantity
* unit of Pressure — N/m^2 or Pascal (Pa)

$\boxed{1\ N/m^2 = 1\ Pa}$ — SI unit

$\frac{Dyne}{cm^2}$ — C.G.S unit

→ 1 bar = 10^5 Pa
→ 1 KPa = 1000 Pa
→ 1 bar = 100 KPa

1 Torr = 1 mm of Hg

1 Atm Pressure = 760 mm of Hg

1 torr = $\frac{1}{760}$ × Atm Pressure

1 Atm Pressure = 101.3 KPa
= 1.013 bar

1 Torr = 0.13328×10^3 Pa = 133.28 Pa

1 PSI = 6894.76 Pa

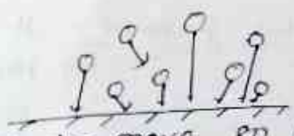
1 Pa = 0.000145 PSI

1 PSI = 1 Pound Per Square inch

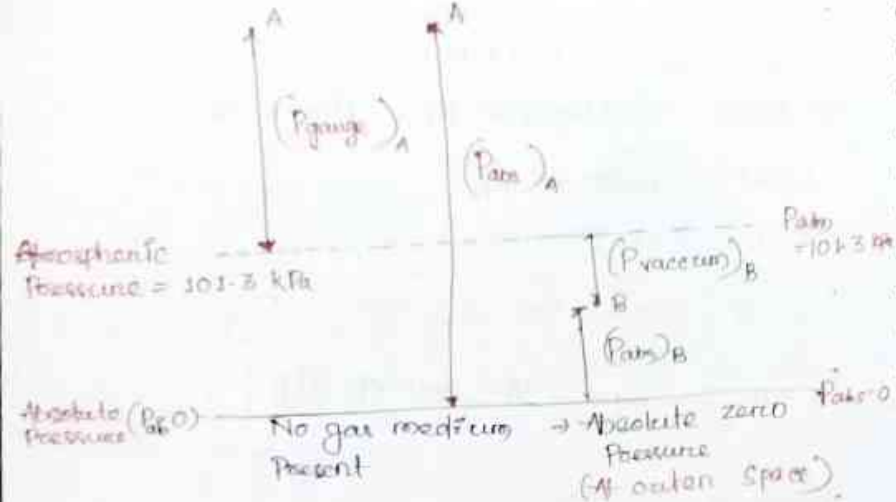
1 atm Pressure = 760 mm of Hg

Types of Pressure

(i) Absolute Pressure
(ii) Gauge Pressure
(iii) Vaccum Pressure



* The fluid Particles are free to move so during motion when the Particles collide with any surface then the momentum of Particles will change.
* According to Newton's 2nd law of Motion the impact of force is equal to the rate of change of momentum. So the particles create force when it collide with the surface at that Particular area.
* The intensity of force per unit area is called Pressure.
* No. of molecules of a fluid collide with any surface the collision between them create Pressure. According to kinetic theory of gases.



Absolute Pressure - It is the value of Pressure measured above the absolute zero pressure.

Gauge Pressure - It is the value of Pressure measured above the atmospheric Pressure.

Vacuum Pressure - It is the Pressure which is less than (P_{atm}) or atmospheric Pressure.

$$\boxed{(P_{vacuum})_B = P_{atm} - (P_{abs})_B}$$

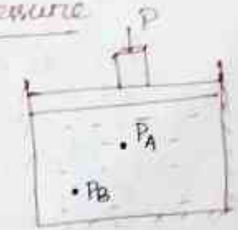
Pascal's Law

Pressure at a point in a fluid at rest is same in all direction.

$$\boxed{\text{Normal stress} = \text{Pressure at A}}$$

$$(P_x = P_y = P_z)_{at\ A} = (\sigma_x = \sigma_y = \sigma_z)$$

Law of Transmissibility of Pressure



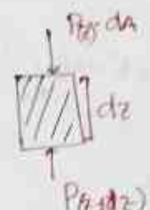
$$P_A = (P_{fluid})_A + P$$

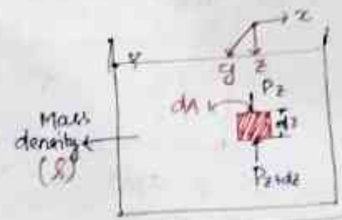
$$P_B = (P_{fluid})_B + P$$

Mechanical advantage

Hydraulic Jack, JCB machine, digger machine

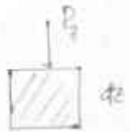
Hydrostatic Law





$$\boxed{dv = dA \cdot dz}$$

$$\boxed{P_{(z+dz)} = P_z + \frac{\partial}{\partial z} P_z \cdot dz} \rightarrow \text{Acc. to Taylor's series}$$



$$\boxed{P_{z+dz} = P_z + \frac{\partial P_z}{\partial z}\, dz}$$

$\sum F_z = 0$

$$\left(P_z\, dA + \rho g\, dA \cdot dz\right) - \left(P_z + \frac{\partial P_z}{\partial z} \cdot dz\right) \cdot dA = 0$$

$$\Rightarrow \cancel{P_z\, dA} + \rho g\, dA \cdot dz - P_z\, dA - \frac{\partial P_z}{\partial z} \cdot dz \cdot dA = 0$$

$$-\frac{\partial P_z}{\partial z}\, dz\, dA = -\rho g\, dA \cdot dz$$

$$\boxed{\frac{\partial P_z}{\partial z} = \rho g = \gamma}$$

* Rate of change in pressure or change in pressure along depth (z-direction) is equal to the weight density of the fluid.

$$\boxed{\frac{\partial P_z}{\partial z} = -\gamma}$$

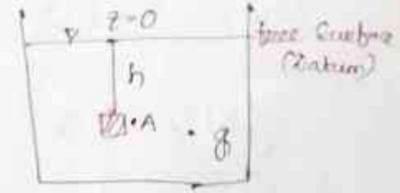
when z axis is taken vertically upward.

$$\boxed{P = f(z) \neq f(x, y)}$$

so,

$$\boxed{\frac{\partial P_z}{\partial z} = \frac{dP_z}{dz}}$$

$$\frac{dp}{dz} = \gamma$$



$$\int dp = \int \gamma\, dz$$

$$\int_{P_{atm}}^{P_a} dP = \int_0^h (\gamma\, dz)$$

$$P_a - P_{atm} = \gamma h - 0$$

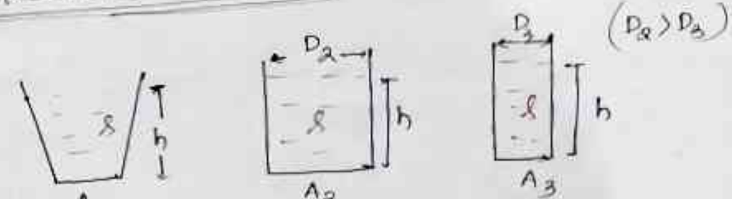
$$P_a = P_{atm} + \gamma h$$

$$\boxed{P_a = P_{atm} + \rho g h}$$ Absolute Pressure at (A)

* Pressure at free surface (P_{atm}) gauge $= 0$

✓ $$\boxed{P_a = \rho g h}$$ → Gauge Pressure ($P_{atm} = 0$)

Hydrostatic Paradox



$P = \rho g h$ $\quad$ $P = \rho g h$ $\quad$ $P = \rho g h$

Pressure Force at the bottom (F)

$\boxed{F_1 = P A_1}$ $\quad$ $\boxed{F_2 = P A_2}$ $\quad$ $\boxed{F_3 = P A_3}$

$$\boxed{F_1 \neq F_2 \neq F_3}$$

Flow of Fluid from Rest

$h_2 > h_1$

$\boxed{P_B > P_A}$

* Fluid is not flowing here because the force applied by the fluid in a closed container is canceled out by reaction created by the container.
* So unbalanced force carried by the wall thats why the fluid is not flowing.

$P_A > P_{atm}$

* Fluid at rest can flow from High Pressure to low Pressure.

Numerical on Properties of Fluid.

Q) A liquid at 20°C has a Relative density of 0.8 and a kinematic viscosity of 2.3 centistokes. Determine it's unit weight & Dynamic viscosity of fluid in Pa.S

Weight density / unit weight (γ)

$\boxed{\gamma = \rho g}$

Specific gravity = Relative density = $\dfrac{\rho_{liquid}}{\rho_{water}}$

$\boxed{\rho_{water} = 1000\ kg/m^3}$

$\Rightarrow 0.8 = \dfrac{\rho_{liquid}}{1000}$

$\Rightarrow \rho_{liq} = 0.8 \times 1000 = 800\ kg/m^3$

$\gamma_{liq} = \rho_{liq} \times g = 800 \times 9.81 =$

(i) $\boxed{\gamma_{liq} = 7868\ N/m^3}$

$\boxed{1\ \text{centistokes} = 10^{-6}\ m^2/s}$

(ii) Dynamic viscosity (μ)

$\mu = \rho \nu$

$\mu = 800 \times 2.3 \times 10^{-6}\ m^2/s$

$\boxed{\mu = 1.84 \times 10^{-3}\ \frac{Ns}{m^2} \text{ or } Pa\,S}$

Q) Calculate the workdone in blowing a soap bubble of diameter 12 cm. Assume the surface tension of soap solution = 0.040 N/m

Given Data

$\sigma = 0.040$ N/m

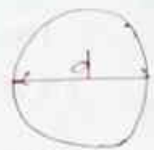
dia of soap bubble $(d) = 12\,cm = 0.12\,m$

Surface Energy: Workdone in changing the surface area of liquid or gas

W = Surface tension x change in surface area

$W = \sigma\,(N/m) \times \Delta A\,(m^2)$

$\boxed{W = \sigma \Delta A}$ N.m



Surface Area of Soap bubble

$A = 4\pi r^2$

$\boxed{A = 2 \times 4\pi r^2}$

(Surface area = 2)

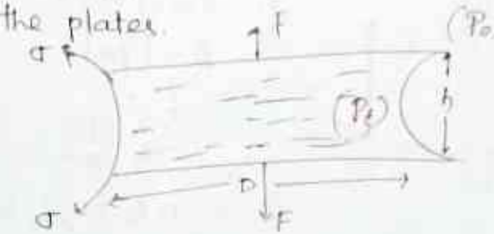
$\sigma = 0.040$ N/m

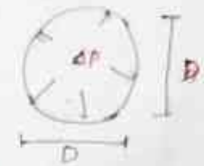
$W = \sigma \times \Delta A$

$W = 0.040 \times 8\pi \left(\frac{0.12}{2}\right)^2$

$\boxed{W = 3.619 \times 10^{-3}}$ N.m or Joule

Q) A very small quantity of liquid having a surface tension σ forms a circular spot of diameter D between two glass plate separated by a small distance 'h' obtain the expression for force required to pull the plates.





Solⁿ Excess Pressure in the Liquid

$\boxed{\Delta P \text{ or } P = P_i - P_o}$

Force due to Pressure = Force due to Surface Tension

$\pi D h \times \Delta P = 2\pi D \times \sigma$

$\boxed{\Delta P = \frac{2\sigma}{h}}$

So, The force required to separated the plates:

$F = \Delta P \times \frac{\pi}{4} \times D^2$

$F = \frac{2\sigma}{h} \times \frac{\pi}{4} \times D^2$

$\boxed{F = \frac{\pi D^2 \sigma^2}{2h}}$

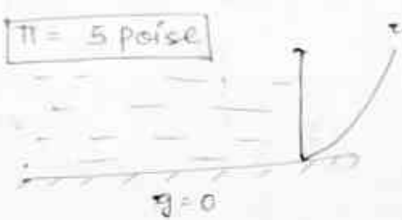
Q) Velocity distribution of a liquid is given as

$$u = 5 \sin(5\pi y)$$

for $y \leq 0.1$ m

Compute the shear stress at wall

Take $\boxed{\mu = 5 \text{ poise}}$



$\mu = 5$ poise

$= \frac{5}{10} \frac{NS}{m^2}$

$= 0.5 \ NS/m^2$

$= 0.5$ Pa s.

$$\boxed{\tau = \mu \frac{du}{dy}}$$

$$\tau = 0.5 \times \frac{d}{dy} \left(5 \sin(5\pi y)\right)$$

$$\tau = 0.5 \times 5 \times 5\pi \cos(5\pi y)$$

At wall $y = 0$.

$$\tau_0 = 0.5 \times 5 \times 5\pi \cos(0)$$

$$\tau_0 = 0.5 \times 5 \times 5\pi \times 1$$

$$\boxed{\tau_0 = 39.269 \ N/m^2}$$

Q) If the velocity distribution of a plate is given by $u = \frac{2}{3} y - y^2$, where y is distance from solid Boundary in meters. u is velocity in m/s. Determine shear stress at $y = 0$ m & $y = 0.15$ m

Take Dynamic viscosity as $\mu = 8.63$ poise

Given Data

$\mu = 8.63$ poise $= 0.863 \ \frac{NS}{m^2}$

velocity distribution

$$\boxed{u = \frac{2}{3} y - y^2}$$

To find

Shear stress (τ) at $y = 0$ m & $y = 0.15$ m

$$\boxed{\tau = \mu \frac{du}{dy}} \qquad \tau = \mu \frac{d}{dy}\left(\frac{2}{3} y - y^2\right)$$

$$\boxed{\tau = \mu \times \left[\frac{2}{3} - 2y\right]}$$

(i) $y = 0$

$$\tau = 0.863 \times \left[\frac{2}{3} - 2 \times 0\right]$$

$$\tau = 0.863 \times \frac{2}{3}$$

$$\boxed{\tau = 0.5753 \ N/m^2}$$ Ans

(ii) $y = 0.15$ m

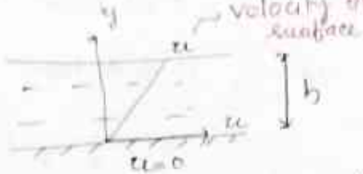
$$\tau = 0.863 \times \left[\frac{2}{3} - 2 \times 0.15\right]$$

$$\tau = 0.863 \times \left[\frac{2}{3} - 0.3\right]$$

$$\boxed{\tau = 0.3164 \ N/m^2}$$ Ans

Linearization of Newton's law of viscosity.

$$\boxed{\tau = \mu \frac{du}{dy}}$$



If h is in order of milimeters then we can consider the curve as linear.

As pe Newton's law of viscosity

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{u-0}{h-0}$$

$$\boxed{\tau = \mu \frac{u}{h}}$$ → Linearization

$v \rightarrow$

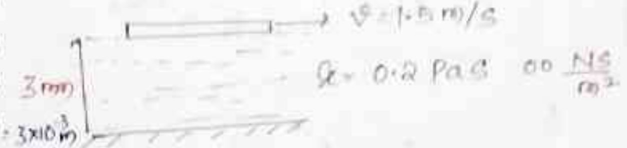
$$\boxed{u = v}$$

velocity of fluid element = velocity of solid boundary.

* For No slip condition so the velocity of fluid adjacent to the plate is equal to velocity of plate.

* For linearization $\boxed{\frac{du}{dy} = \text{constant}}$

(Q) The space between two parallel plates kept 3 mm apart is filled with an oil of dynamic viscosity 0.2 Pa s what is the shear stress at lower fixed plate. If the upper one is moved with a velocity of 1.5 m/s.



$$\boxed{\frac{du}{dy} = \text{constant}}$$

we know, $\tau = \mu \frac{u}{h} = 0.2 \times \frac{1.5}{3 \times 10^{-3}} = 500 \times 0.2$

$$\boxed{\tau = 100 \text{ N/m}^2}$$

$\therefore \tau = 100 \text{ N/m}^2 \rightarrow$ shear stress in the fixed plate.

$\Rightarrow \frac{du}{dy} \neq f(y)$, $\mu \neq f(y)$

hence, $\tau \neq f(y)$

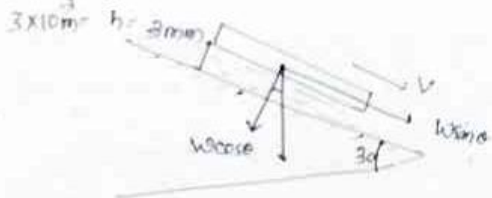
* so τ value is same throughout the section

(Q) A 90 N rectangular solid block slides down a 30° inclined plane. The plane is lubricated by a 3 mm thick film of oil of relative density 0.9 and viscosity of 8 poise. If the contact area is 0.3 m². Estimate the terminal velocity of the block.

For terminal velocity

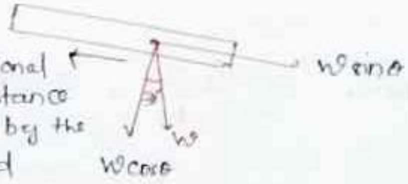
Net force exerted on the body (ΣF) in the direction of motion is equal to zero.

$\boxed{\Sigma F = 0}$ → in direction of motion
(for Terminal velocity)



$\mu = 8$ Poise

$\boxed{\mu = 0.8 \ \frac{Ns}{m^2}}$



$\boxed{\text{Skin drag} = \tau A}$ =

In direction of Motion

For terminal velocity $\Sigma F = 0$.

$W\sin\theta - F = 0$

$\boxed{90 \sin(30°) - \tau A = 0}$ ⟶ eqⁿ ①

τ from Newton's Law of viscosity

$$\tau = \mu \frac{v}{h} = 0.8 \times \frac{v}{3\times10^{-3}}$$

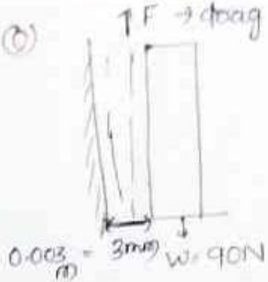
$\boxed{\tau = 266.67\, v}$ N/m² ⟶ eqⁿ ②

by Putting the value of τ in eqⁿ ①

$90 \sin 30° - 266.67\, v \times 0.3 = 0$

$\boxed{v = 0.5625 \text{ m/s}}$

(i)



μ = 0.8 Poise = 0.08 $\frac{Ns}{m^2}$

Area of Contact A = 0.8 m²

Weight of Plate W = 90N

Find the Terminal velocity.

Solⁿ $\Sigma F = 0$

$W - F = 0$

$W - \tau A = 0$

$90 - \mu \frac{du}{dy} \cdot A = 0$

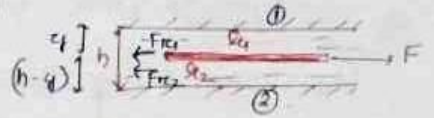
$90 - 0.08 \times \frac{v}{0.003} \times 0.8 = 0$

Terminal velocity - $\boxed{v = 4.21875 \text{ m/s}}$ $\boxed{v = 4.21875 \text{ m/s}}$

(ii) A thin plate is placed between Flat surface at a distance h cm apart such that the viscosity of liquid on top & bottom of plate is μ_1 & μ_2 respectively.
Determine position of thin plate such that minimum resistance to uniform motion will be minimum (Assume h is very small)

Resistance Force

$\boxed{F_r = F_{r_1} + F_{r_2}}$



Resistive Force

$F_{r_1} = \tau_1 \times A$

$F_{r_1} = \left(\mu_1 \times \frac{v}{y}\right) \times A$

$F_{r_2} = \tau_2 \times A$

$F_{r_1} = \left(\mu_2 \times \frac{v}{(h-y)}\right) \times A$

$$\boxed{F_r = vA\left(\frac{\mu_1}{y} + \frac{\mu_2}{(h-y)}\right)}$$

For minimum value of F.

$$\frac{df}{dy} = 0$$

$$\frac{d}{dy}\left[VA\left(\frac{\mu_1}{y} + \frac{\mu_2}{h-y}\right)\right] = 0$$

$$\frac{d}{dy} \quad VA\left[-\frac{\mu_1}{y^2} + \frac{\mu_2}{(h-y)^2}\right] = 0$$

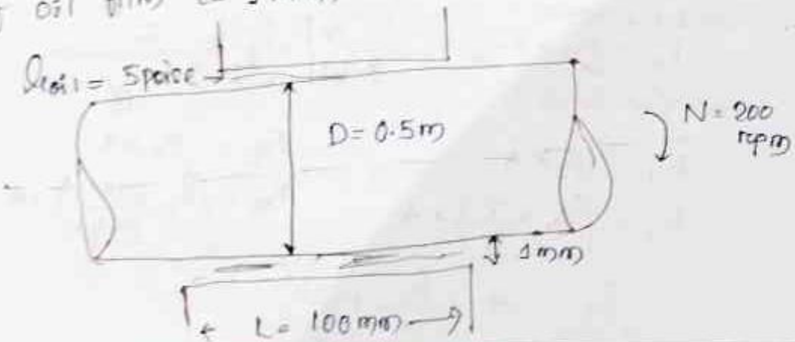
$$\frac{-\mu_2}{(h-y)^2} = \frac{\mu_1}{y^2}$$

$$\left(\frac{y}{h-y}\right)^2 = \frac{\mu_1}{\mu_2}$$

$$\boxed{\frac{y}{h-y} = \sqrt{\mu_1/\mu_2}}$$

Numerical Based on Newton's Law of Viscosity:

(c) An oil of viscosity 5 poise is used for lubrication between shaft & sleeve the diameter of shaft is 0.5m and it rotates at 200 rpm Calculate the power lost in friction for a sleeve length of 100 mm. The thickness of oil film is 1 mm.



Given Data-

N = 200 rpm

(L) length sleev = 100mm

(D) Diameter of shaft = 0.5m

(h) Thickness of film = 1mm = 1×10^{-3} m

(μ) Dynamic Viscosity = 5poise

$$= \frac{5}{10}\ \frac{Ns}{m^2} = 0.5\ \frac{Ns}{m^2}$$

Sol^n

Power Lost

Power Lost → $\boxed{P = \frac{2\pi NT}{60}}$

T = Torque

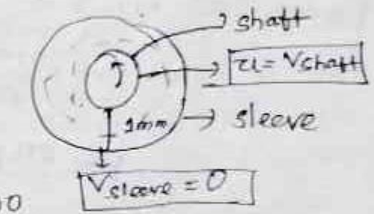
$\boxed{T = \text{Force} \times \text{Radius}}$

$\boxed{\text{Skin Friction Force (F)} = \text{shear stress } (\tau) \times \text{Area (A)}}$

(A) cylinder Area = πdL

$$F = \tau \times A$$

$$\boxed{\tau = \mu \frac{u}{h}}$$



Circumferential velocity

$$V_{shaft} = \frac{\pi DN}{60} = \frac{\pi \times 0.5 \times 200}{60}$$

$$\boxed{V_{shaft} = 5.236\ m/s}$$

τ = shear stress

$$\tau = \mu \frac{u}{h} = \mu \frac{V_{shaft}}{h}$$

$$= 0.5 \times \frac{5.236}{1 \times 10^{-3}}$$

$$\boxed{\tau = 2618\ N/m^2}$$ → shear stress @ shaft

Skin friction (F) = τA

$$= 2618 \times \pi \times 0.5 \times 0.1$$

$$\boxed{F = 411.234\ N}$$

Frictional Torque (T) = $F \times$ radius

$$= 411.234 \times \frac{0.5}{2}$$

$$\boxed{T = 102.81\ Nm}$$

Power Lost in friction (P)

$$P = \frac{2\pi N T}{60}$$

$$P = \frac{2\pi \times 200 \times 102.81}{60}$$

$$\boxed{P = 2153.25\ W}$$

$$\boxed{P = 2.153\ kW}$$

Pressure Measurement

- Manometers
  - Simple Manometers
  - Differential Manometers
- Mechanical Gauges
  - → Bourdon tube
  - → Pressure transducer
  - → Blow Pressure gauge
  - → Diaphragm " "

Simple Manometers

→ It measure the Pressure at a particular point

Types

* Piezometer
* U-tube manometer
* Single column or well type or micromanometer
* Inclined U-tube manometer

Differential Manometers

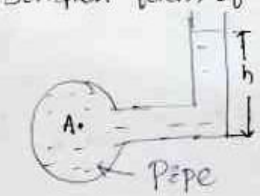
→ It measure the difference of 2 Points

Types

* Differential U-tube manometer
* Inverted U-tube Manometer
* Inclined U-tube Differential manometer

(1) Piezometer

→ Simplest form of Manometer.



* if $\boxed{P_a > P_{atm}}$ then the inside fluid comes to piezometer tube

Drawbacks

→ It can measure positive Pressure only.

→ So, it can't measure vaccume Pressure.

$$\boxed{P_a = \rho g h}$$

ρ → density of fluid in Pipe.

→ It can not measure high Pressure

$$P_a = \rho g h \Rightarrow h \propto P_a \Rightarrow h \propto \frac{1}{\rho g} \propto \frac{1}{\gamma}$$

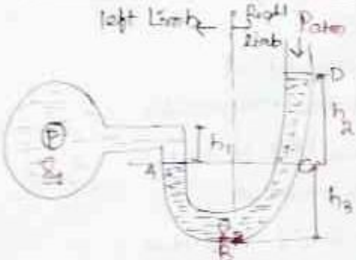
→ It can't measured gas

→ it only measure moderate and very low Pressure

→ Pressure of light liquid can not be measured because very long column needs a very manometric tube

$$\boxed{h \propto \frac{1}{\rho}}$$

(b) U-tube Manometer



$$\boxed{\rho_{\text{Manometric fluid}} > \rho_{\text{fluid}}}$$

(i) High Pressure can be measure

(ii) U-tube can measure Positive as well as Negative pressure

ρ_1 → density of fluid

ρ_2 → density of Manometric fluid

* Free surface to downward — Positive
* towards free surface — negative.

As Per Law of Hydrostatic

$$\boxed{P + \rho_1 g h_1 + \rho_2 g h_3 - \rho_2 g h_3 - \rho_2 g h_2 + P_{atm} = 0}$$

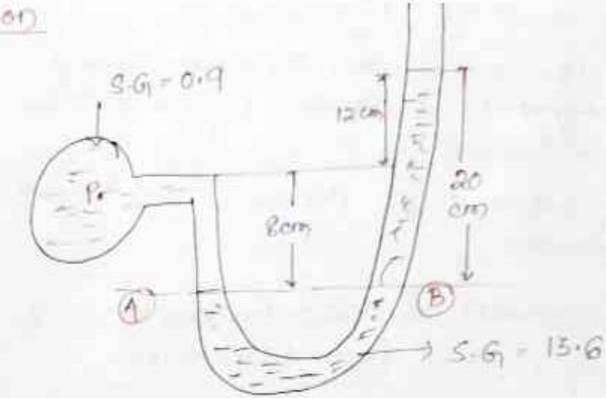
Rule

* Take the lower level of manometric fluid as reference

$$\boxed{(P_{atm})_{gauge} = 0}$$

* if Patm is considered than we get Absolute Pressure
* if Patm is not considered than we get gauge Pr.

Question



$\rho_1 = 0.9 \times 1000 \ kg/m^3$

$= 900 \ kg/m^3$

$\rho_2 = 13.6 \times 1000 \ kg/m^3$

$= 13600 \ kg/m^3$

$$P + \rho_1 \times g \times \frac{8}{100} - \rho_2 \times g_2 \times \frac{20}{100} = 0$$

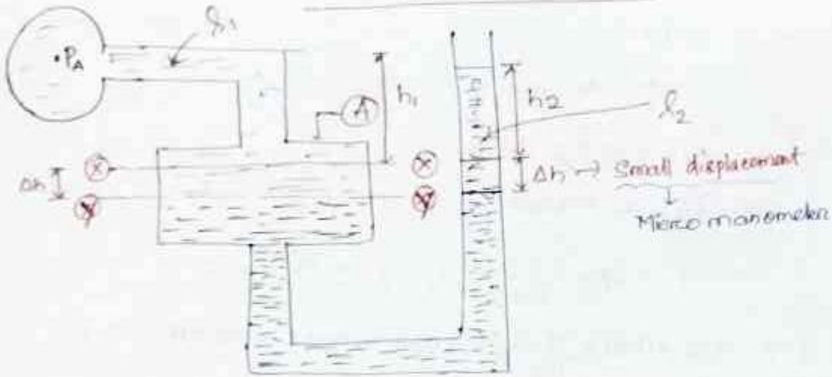
$$\Rightarrow P + 900 \times 9.81 \times \frac{8}{100} - 13600 \times 9.81 \times 20 \times 10^{-2} = 0$$

$$\boxed{P = 25977 \ N/m^2}$$

Advantages of U-tube Manometer

* U-tube Manometer can measure positive as well Negative Pressure (vaccum Pressure)
* Easy to install
* can measure high Pressure as compare to Piezometer.

(5) Single column or Micro manometer or well type Manometer



A = Area of c/s of Reservoir/well

a = Area of c/s of Manometer of Pipe

volume or displaced vol^m of Limb
= Rise of vol^m in Right limb.

$$A \times \Delta h = a \cdot h_2$$

$$\boxed{\Delta h = \frac{a h_2}{A}}$$

Manometric eq^n

$$P_a + \rho_1 g (h_1 + \Delta h) = \rho_2 g (h_2 + \Delta h)$$

$$P_a = \rho_2 g (h_2 + \Delta h) - \rho_1 g (h_1 + \Delta h)$$

$$P_a = \rho_2 g h_2 + \rho_2 g \Delta h - \rho_1 g h_1 - \rho_1 g \Delta h$$

$$P_a = \rho_2 g \Delta h - \rho_1 g \Delta h + \rho_2 g h_2 - \rho_1 g h_1$$

$$P_a = \Delta h g (\rho_2 - \rho_1) + \rho_2 g h_2 - \rho_1 g h_1$$

$$\boxed{P_a = \frac{a h_2 g}{A} (\rho_2 - \rho_1) + \rho_2 g h_2 - \rho_1 g h_1}$$

$a <<< A$

$\frac{a}{A}$ → negligible.

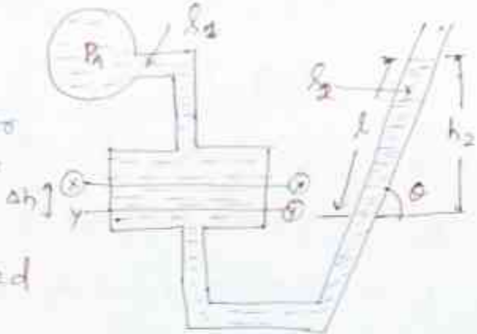
$$\Rightarrow \boxed{P_a = \rho_2 g h_2 - \rho_1 g h_1}$$

Advantages

* Micro manometer can measure positive as well as negative Pressure (vaccum Pressure).
* Easy to install
* It is more sensitive than the U-tube manometer.

Inclined Single Column Manometer

$$\boxed{\sin\theta = \frac{h_2}{l}}$$



A → Area of the Reservoir
a → Area of Manometer tube.

* Right limb is inclined at (θ) degree

$$\boxed{h_2 = l\sin\theta}$$

$$\boxed{P_A = \frac{ah_2}{A}\cdot g(\rho_2-\rho_1) + \rho_2 g h_2 - \rho_1 g h_1}$$

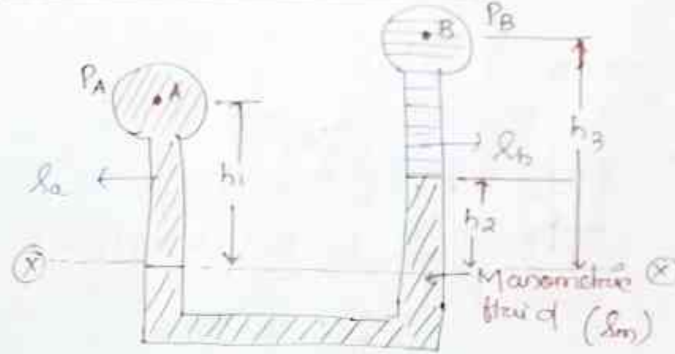
$$\boxed{P_A = \rho_2 g h_2 - \rho_1 g h_1}$$

Put $h_2 = l\sin\theta$.

Differential Manometer

* A differential Manometer is a device which is used to measure the pressure difference between two point.
  (i) U-tube differential Manometer
  (ii) Inverted U-tube differential Manometer
  (iii) Micromanometer of differential type.

① U-tube Differential Manometer



$$\boxed{\rho_m > \rho_a, \rho_b}$$

* When the pressure difference is high or moderate

$$\boxed{\text{Pressure Difference} = P_A - P_B}$$

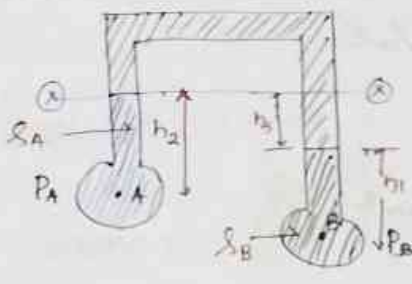
As we know $h = \frac{1}{\gamma}$

→ Column of manometric fluid (h_2) will be less if Pressure difference is low & we couldn't get accuracy.

* Pressure at X-X plane

$$\boxed{\text{Pressure at left limb} = \text{Pressure in right limb}}$$

② Inverted U-tube differential Manometers

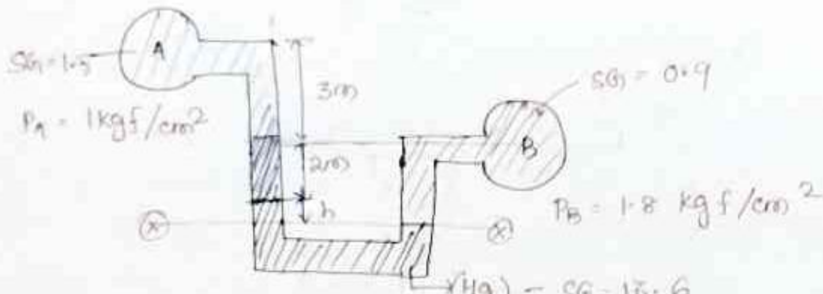


If ($P_A - P_B$) is very small

$$\boxed{\rho_m < \rho_A, \rho_B}$$

* ρ_m → Air is also used
* Upper level is used as reference axis.

Question



$\rho_m = 13.6 \times 1000 = 13600\ kg/m^3$

Sol^n using Pascal's Law

Pressure in the left limb = Pressure in the right limb

$\rho_A = 1.5 \times 1000 = 1500\ kg/m^3$

$\rho_B = 0.9 \times 1000 = 900\ kg/m^3$

$$\boxed{P_A + \rho_A g \times (3+2) + \rho_m g h = P_B + \rho_B g (2+h)}$$

$P_A = 1\ kgf/cm^2$

$1cm = 10^{-2}\ m$

$1cm^2 = 10^{-4}\ m^2$

$P_A = \frac{1\ kgf}{10^{-4} m^2} = 10^4\ kgf/m^2$

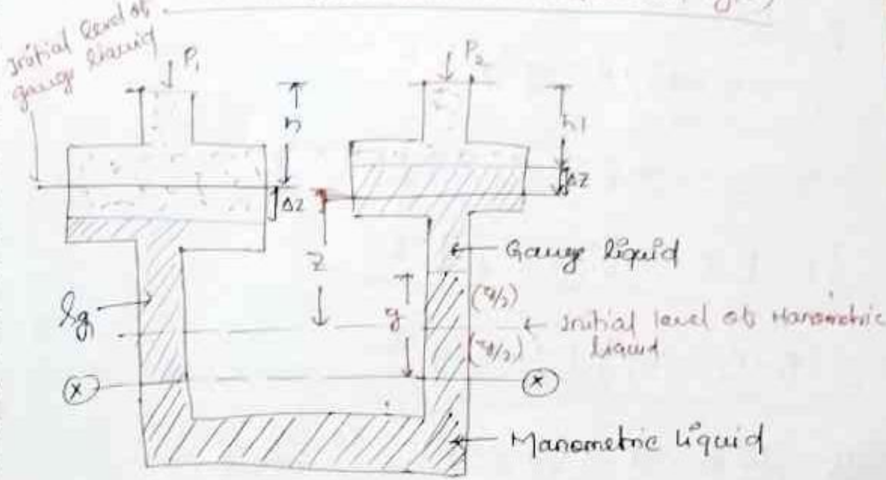
$\boxed{P_A = 10^4 \times 9.81}\ N/m^2$

$\boxed{P_B = 1.8 \times 10^4 \times 9.81}\ N/m^2$

$h = 0.181\ m$

$\boxed{h = 18.1\ cm}$

③ Micro manometer (differential type)



C/S Area - a

Pressure above xx in left limb = Pressure above xx in right limb

$$P_1 + \rho_w g (h + \Delta z) + \rho_g g (z - \Delta z + y/2)$$
$$= P_2 + \rho_w g (h_1 - \Delta z) + \rho_g g (z + \Delta z - y/2) + \rho_m g y$$

$$\Rightarrow \boxed{P_1 - P_2 = \rho_m g y + 2 \rho_g g \Delta z - 2 \rho_g g \left(\frac{y}{2}\right) - 2 \rho_w g \Delta z}$$

Continuity eq^n in gauge liquid

vol^m displaced from the well = vol^m displaced of gauge liquid moved in tube.

$\Rightarrow A \times \Delta z = a\, y/2$

$\Rightarrow \boxed{\Delta z = \frac{a}{A} \frac{y}{2}}$

if $a <<<< A$

⇒ Δz will be negligible.

$$P_1 - P_2 = \rho_m g y - 2 \times \rho_g \times g \times \frac{y}{2}$$

$$\boxed{P_1 - P_2 = \rho_m g y - \rho_g g y}$$

$$\Rightarrow \boxed{P_1 - P_2 = g y (\rho_m - \rho_g)}$$

if $\frac{a}{A}$ is not negligible. $\Delta z = \frac{a}{A}\frac{y}{2}$

We have the Formula

$$P_1 - P_2 = \rho_m g y + 2\rho_g g \Delta z - 2\rho_g g \left(\frac{y}{2}\right) - 2\rho_w g \Delta z$$

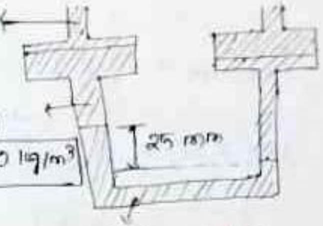
by putting $\Delta z = \frac{a}{A}\frac{y}{2}$, we get

$$\boxed{P_1 - P_2 = g y \left\{ \rho_m - \rho_g\left(1 - \frac{a}{A}\right) - \rho_w\left(\frac{a}{A}\right) \right\}}$$

Q) ρ_{basin} = 0.75 = ρ_{gauge}.

ρ_m =

Q) A Macromanometer is used to measure Pressure difference between two points. The basin partially filled with liquid of SG = 0.75 & lower portion (U tube) is filled with mercury of SG = 13.6. The diameter of basin is 20 times higher than that of the U-tube. Find the Pressure diff. if the reading is 25 mm & Liquid in pipe has ~~SG = 0~~ specific wt. of 0.475 N/m³



$0.75 = (SG)_g$

$\boxed{\rho_g = 750\ kg/m^3}$

$(SG)_m = 13.6$

$\boxed{\rho_m = 13600\ kg/m^3}$

Sol^n

let D = Dia of Basin

d = Dia of tube

$$\boxed{D = 20d}$$

$$\frac{a}{A} = \frac{d^2}{(20d)^2} = \frac{d^2}{400d^2}$$

$$\boxed{\frac{a}{A} = \frac{1}{400}}$$

Pressure difference

$$P_1 - P_2 = (\rho_m - \rho_g)\, g y$$

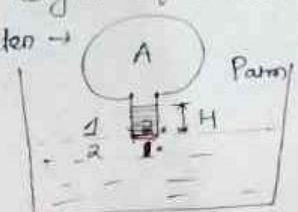
$$\Rightarrow P_1 - P_2 = (13600 - 750)\ 9.81 \times \frac{25}{1000}$$

$$\Rightarrow P_1 - P_2 = 3151.4625\ N/m^2$$

$$\Rightarrow \boxed{P_1 - P_2 = 3.151\ KPa}$$

Q) For given Figure, if the Pressure of gas in bulb A is 50 cm of Hg vaccum & P_{atm} = 76 cm of Hg. what will be height of column (H) of Hg.

Barometer →



P_{atm} = 760 mm of Hg

P_A = 500 mm of Hg

$\boxed{\rho_m g H = P_{atm}}$

Absolute Pressure at point ① = Absolute Pressure at Point ②

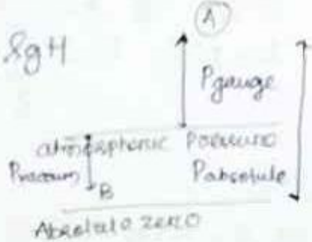
$P_{atm} = P_{atm} - P_{vacuum} + \rho g H$

$P_{vacuum} = \rho g H$

$\rho g \times 50 = \rho g H$

$\boxed{H = 50 \text{ cm}}$

height of column



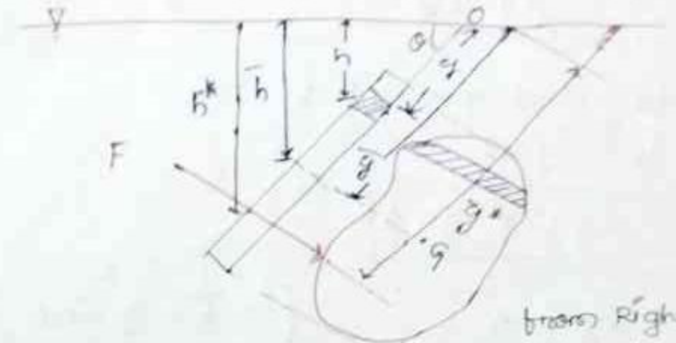
$\boxed{\rho_m = 13600 \text{ kg/m}^3}$

$P_v = \rho g (h)$

50 mm of Hg

Centre of Pressure

Hydrostatic force on a plane



from Right angle Triangle

$\boxed{\frac{h}{y} = \sin\theta}$

$\boxed{\frac{\bar{h}}{\bar{y}} = \sin\theta}$

ρ = Density of fluid

θ = inclination of plane with the Free Surface

y = Distance of element from 'O'

$\bar{y}$ = Distance of Centroid of plane from 'O'

h = depth of element from free surface.

$\bar{h}$ = Depth of centroid of the plane from free surface.

Force on Element

$dF = P \cdot dA$

$\Rightarrow \boxed{dF = \rho g h \, dA}$ → Pressure force

$\rightarrow \boxed{dF = \rho g (y \sin\theta) \, dA}$ $(\because h = y \sin\theta)$

Total Pressure force on the element

$$F = \int dF$$

$$\Rightarrow F = \int \rho g\, y \sin\theta \cdot dA$$

$$\Rightarrow F = \rho g \int y \sin\theta\, dA = \rho g \sin\theta \int y \cdot dA$$

$\int y \cdot dA$ = First moment of Area

$$\boxed{\int y \cdot dA = \bar{y} \cdot A}$$

$$\boxed{F = \rho g \sin\theta\, \bar{y} \cdot A}$$

$$\boxed{F = \rho g \bar{h} \cdot A} \qquad \left[\because \bar{h} = \bar{y} \sin\theta\right]$$

Total Force acting on the plane.

$$\boxed{F = \rho g \bar{h} A}$$

NOTE

Varignon Theorem:

The moment of Resultant force about a point is equal to the sum of moment of all forces about that point

$$\boxed{F \cdot y = \Sigma F_n y_n}$$

So moment of Total Force (F) about 'O' = Moment of all forces acting on plane about 'O'

$$F \times y^* = \int dF \cdot y$$

$$\Rightarrow F \times y^* = \int dF \cdot y$$

$$\Rightarrow F \times y^* = \int (\rho g\, y \sin\theta\, dA)\, y$$

$$\Rightarrow \rho g \bar{h} A\, y^* = \rho g \sin\theta \int y^2\, dA$$

$\int y^2 dA$ = Second moment of Area or moment of Inertia of a plane

$$\boxed{\int y^2\, dA = I_0} \rightarrow$$ moment of inertia of plane about point 'O'

$$\Rightarrow \rho g \bar{h} A\, y^* = \rho g \sin\theta\, I_0$$

$$\Rightarrow \bar{h} A\, y^* = \sin\theta\, I_0$$

$$\boxed{I_0 = I_G + A \cdot \bar{y}^2} \rightarrow$$ From Parallel axis theorem

I_G → M.O.I of plane about its centroid

$\bar{y}$ = C.G from Free Surface 'O'

$$\Rightarrow \bar{h} A\, y^* = (I_G + A\bar{y}^2) \sin\theta$$

h^* → The Depth of Point of application of total pressure Force (or) Centre of Pressure

$$\Rightarrow \bar{h} \cdot A \frac{h^*}{\sin\theta} = I_G \sin\theta + A\bar{y}^2 \sin\theta \qquad \boxed{\frac{h^*}{y^*} = \sin\theta}$$

$$\Rightarrow h^* = \frac{I_G \sin^2\theta + A(\bar{y}\sin\theta)^2}{\bar{h} \cdot A} \longrightarrow \bar{h}$$

$$\Rightarrow h^* = \frac{I_G \sin^2\theta}{\bar{h} \cdot A} + \frac{A \times \bar{h}^2}{\bar{h} \times A}$$

$$\Rightarrow \boxed{h^* = \frac{I_G \sin^2\theta}{\bar{h} \cdot A} + \bar{h}}$$

↳ Centre of Pressure

So, For any Inclined plane

$$\boxed{h^* = \frac{I_G \sin^2\theta}{\bar{h} \cdot A} + \bar{h}}$$

I_G - moment of inertia of the plane about its centroid

I_G is a non zero value ($I_G > 0$)

hence $\boxed{h^* > \bar{h}}$ The centre of Pressure is below the centroid of Plane

* $A \rightarrow$ Area of plane immersed in fluid

Case I Plane is vertical ($\theta = 90°$)

$$h^* = \frac{I_G \sin^2(90°)}{\bar{h} A} + \bar{h}$$

$$\boxed{h^* = \frac{I_G}{\bar{h} A} + \bar{h}}$$

→ Centre of Pressure for a vertical plane

$$\boxed{h^* > \bar{h}}$$

→ Centre of Pressure (C.O.P) lies below the centroid of the plane

Case II Plane is Horizontal ($\theta = 0$)

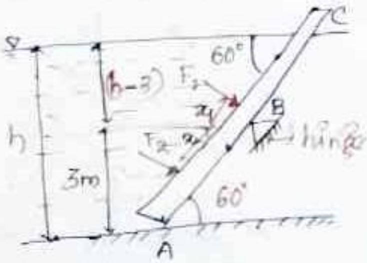
$$h^* = \frac{I_G \times 0}{\bar{h} \cdot A} + \bar{h}$$

$$\boxed{h^* = \bar{h}}$$

⇒ The depth of centroid & centre of Pressure is same for a horizontal surface.

Question

A Gate supporting water is shown in figure. Find the height of water so that the Gate tips about the hinge. Take width of gate as unity.



F is at a distance x from hinge 'B'

$F_x = \sum F_i x_i$

$\boxed{F_x = F_1 x_1 - F_2 x_2}$

if $F_x, > F_2 x_2$ — The gate will tip about hinge

Limiting Case

$\boxed{F_1 x_1 = F_2 x_2}$

$F.x = F_1 x_1 - F_1 x_1$

$\boxed{F.x = 0}$

$F \neq 0$

$\boxed{x = 0}$

When gate is just about to tip the resultant force will act at the Hinge

Given data

Width of plane = 1

To find $h = ?$

$\boxed{h^* = h - 3}$ —— ①

$\boxed{h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}}$ — ⑪

$\boxed{\bar{h} = h/2}$

$A \rightarrow$ Area of plane immersed in fluid

length of plate immersed in fluid.

$$AC = \frac{h}{\sin 60^\circ} = \frac{2h}{\sqrt{3}}$$

Area of the plane $(A) = A.C \times 1$

$\boxed{A = 2h/\sqrt{3}}$

$$I_G = \frac{BD^3}{12} = \frac{1 \times \left(\frac{2h}{\sqrt{3}}\right)^3}{12} = \frac{\frac{8h^3}{3\sqrt{3}}}{12} = \frac{2h^3}{9\sqrt{3}}$$

$\boxed{I_G = \frac{2h^3}{9\sqrt{3}}}$

$$h^* = \frac{\frac{2h^3}{9\sqrt{3}} \sin^2 60}{\frac{2h}{\sqrt{3}} \times \frac{h}{2}} + \frac{h}{2}$$

$\boxed{h^* = \frac{2h}{3}}$

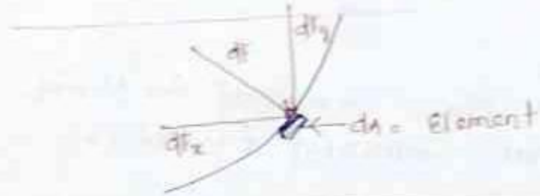
$\frac{2h}{3} = h - 3$ (from eq^n ①&⑪)

$2h = 3h - 9$

$\boxed{h = 9m}$

∴ The Gate will start tiping when the water filled upto height 9m.

Force on Curved Surface



Total force = $\boxed{F = \sqrt{F_x^2 + F_y^2}}$

Direction of Force

$$\boxed{\theta = \tan^{-1}\left(F_y / F_x\right)}$$

$F_x = \int \rho g h \, dA \sin\theta$ = Force due to Pressure on the Projected area on vertical plane

$F_y = \int \rho g h \, dA \cos\theta$ = weight of liquid supported by the surface

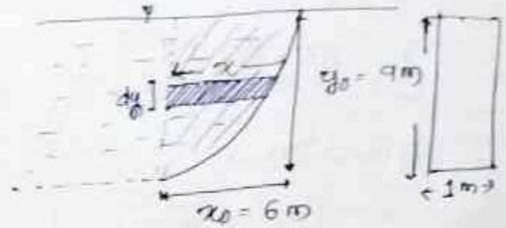
F_x = Pressure × Projected Area

$\boxed{F_x = \rho g \bar{h} A}$ → $\bar{h}$ centroid of the projected plane

Q) A Dam has parabolic shape. See figure $\boxed{y = y_0 \left(\frac{x}{x_0}\right)^2}$ Take Density of water as $1000 \, kg/m^3$. Find the Resultant force and its direction.

$y = 9\left(\frac{x}{6}\right)^2 = \frac{9x^2}{36}$

$\boxed{x = 2\sqrt{y}}$



① Horizontal components of Force

$\boxed{F_x = \rho g \bar{h} \cdot A}$ let width = 1

$A = 9 \times 1 = 9m^2$

$\bar{h} = 4.5 \, m$

$F_x = 1000 \times 9.81 \times 4.5 \times 9$

$\boxed{F_x = 397305 \, N}$

F_y = Weight of water supported by the surface

$F_y = \rho g \times vol^m$ of water supported by the surface.

$F_y = \rho g \times \int_0^9 x \, dy$

$F_y = 1000 \times 9.8 \int_0^9 2\sqrt{y} \, dy$ $(x = 2\sqrt{y})$

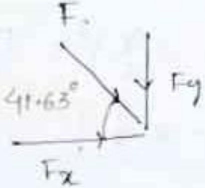
$\boxed{F_y = 353160 \, N}$

① Magnitude of Resultant Force

$$F = \sqrt{F_x^2 + F_y^2}$$

$$F = \sqrt{397305^2 + 353160^2}$$

$$\boxed{F = 531576 \text{ N}}$$



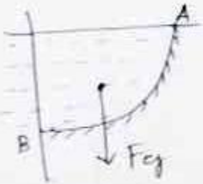
② Direction

$$\theta = \tan^{-1}\left(\frac{F_x}{F_y}\right)$$

$$\theta = \tan^{-1}\left(\frac{397305}{353160}\right)$$

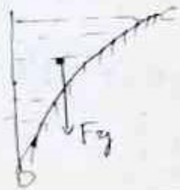
$$\boxed{\theta = 41.63°}$$

— x —

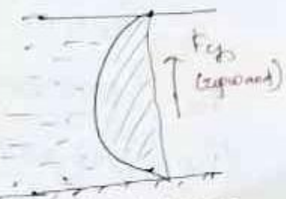


→ AB Surface is supporting the water

F_y = wt of water supported by AB



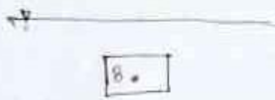
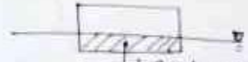
→ Force on CD will be equal to the weight of water supported by CD



Vertical force (F_y) =

(i) wt. of Imaginary liquid supported by the surface

(ii) F_y = wt of liquid displaced by the surface

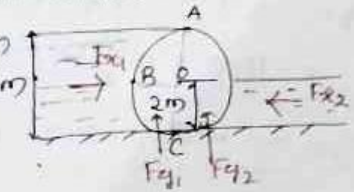
Submerged Body	Floating Body
 Submerged body	 ↳ Centre of Gravity of Floating Body displaced liquid
→ centre of buoyancy is fixed (doesnot change)	→ Centre of Gravity of displaced

→ Force on Curved Surface

(Q)

A cylindrical Gate has diameter & 2m long has water on its both sides

Find magnitude & direction of the force acting on cylinder.



Given

length = 2m

Projected Area on Left side = $4 \times 2 = 8 \, m^2$

Projected Area on Right side = $2 \times 2 = 4 \, m^2$

① Force on Left side

F_{x1} = Force on Projected area (A)

$$F_{x1} = \rho g \bar{h} A = 1000 \times 9.81 \times \frac{4}{2} \times 8$$

$$\boxed{F_{x1} = 156960 \text{ N}}$$

$F_{y_1} = \rho g \times$ (displaced vol^m)

$= 1000 \times 9.81 \times \left[\frac{1}{2} \times \frac{\pi}{4} \times 4^2 \times 2\right]$

$\hookrightarrow \frac{1}{2}(\pi r^2 h)$

$\boxed{F_{y_1} = 397305\ N}$

(ii) Force on Right side of Gate

F_{x_2} = Force on Projected area 'OC'

$F_{x_2} = \rho g \bar{h} A$

$= 1000 \times 9.81 \times \frac{2}{2} \times (2 \times 2)$

$\boxed{F_{x_2} = 39240\ N}$

$F_{y_2} = \rho g \times$ (displaced vol^m)

$= 1000 \times 9.81 \times \left(\frac{1}{4} \times \frac{\pi}{4} \times 4^2 \times 2\right)$

$\boxed{F_{y_2} = 61638.047\ N}$

$F_x = F_{x_1} - F_{x_2}$

$\boxed{F_x = 117720\ N}$

$F_y = F_{y_1} + F_{y_2}$

$\boxed{F_y = 184914\ N}$

Magnitude

$F = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{117720^2 + 184914^2}$

$\boxed{F = 219205.8}$ N Ans

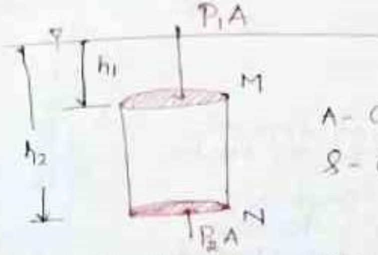
Direction

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{184914}{117720}\right)$

$\boxed{\theta = 57.518^\circ}$ Ans

Buoyancy

When a body is wholly or partially immersed in a fluid, the hydrostatic lift due to the Net vertical components of hydrostatic pressure force experienced by body is called Buoyant Force & the phenomenon is called Buoyancy.



A - C/S of element

ρ - density of liquid

Hydrostatic Pressure on M

$\boxed{P_1 = \rho g h_1}$

Hydrostatic Pressure on N

$\boxed{P_2 = \rho g h_2}$

$\boxed{\Sigma F = 0}$

Buoyant Force $(F_B) = (P_2 - P_1) A$

$= (\rho g h_2 - \rho g h_1) A$

$\boxed{F_B = \rho g A (h_2 - h_1)}$

$h_2 - h_1 \rightarrow$ length of the cylindrical element

$A \times (h_2 - h_1)$ = volume of "

$\boxed{F_B = \rho g \forall}$

Volume of the immersed body = displaced vol^m of fluid

$\forall \rightarrow$ volume of displaced fluid

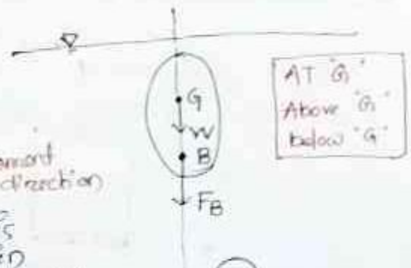
Centre of Buoyancy

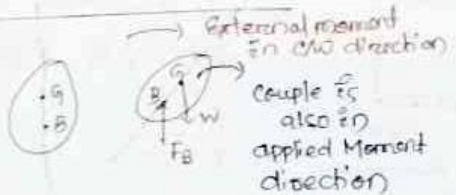
→ The Point at which Buoyant force acts is known as Centre of Buoyancy.

→ Centre of Buoyancy is Denoted by 'B'

Case - 1

B is Below 'G'

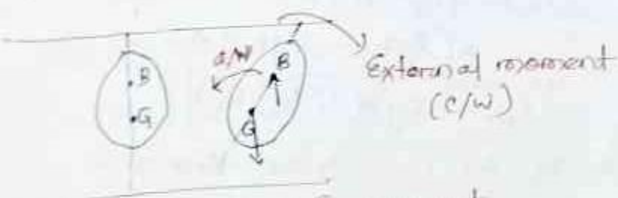




⇒ unstable Equilibrium →

Case-2

B is Above 'G'



⇒ ACW → Restoring Moment

⇒ Stable equilibrium →

Case-3

B is at G ⇒ Neutral equilibrium

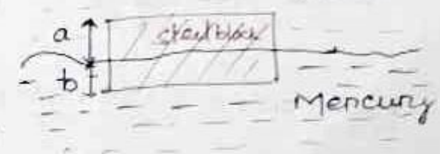
Stability Condition

B above G → Stable eqlbm

B below G → unstable eqlbm

B at G → Neutral eqlbm

Q A Block of steel floats at the mercury water interface what will be the ratio of a & b. Take SG of steel block as 7.85.



Object density << Liquid density

→ Float

↳ As per Archimedes

* The steel block will Float if total Buoyant Force is equal to the weight of Block

F_{B_1} = Buoyant Force offered by water

$$\boxed{F_{B_1} = \rho \times g \times (A \cdot a)}$$

F_{B_2} = Buoyant Force offered by Hg

$$\boxed{F_{B_2} = \rho_{Hg} \times g \times (A \cdot b)}$$

A → c/s area

Weight of block = $W = \rho_s \times g \times A \cdot (a+b)$

$$F_{B_1} + F_{B_2} = W$$

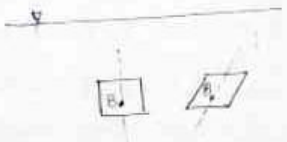
$$\rho g\,(A \cdot a) + \rho_{Hg}\, g\,(A \cdot b) = \rho_s\, g\, A \cdot (a+b)$$

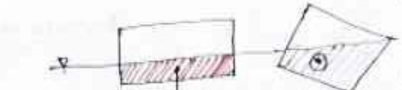
$$a + \frac{\rho_{Hg}}{\rho} b = \frac{\rho_s}{\rho}(a+b) \Rightarrow a + 13.6b = 7.85(a+b)$$

$$\boxed{\frac{a}{b} = 0.839}$$

Floatation

Submerged Body	Floating Body





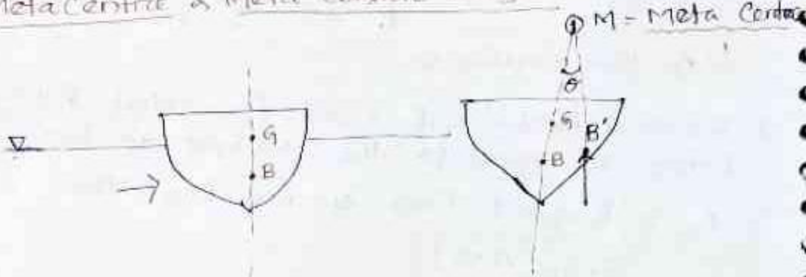
centre of gravity of displaced liquid is CB

Floating Body

Submerged Body
→ Centre of Buoyancy does not change

→ Center of buoyancy will shift

MetaCentre & Meta Centric Height

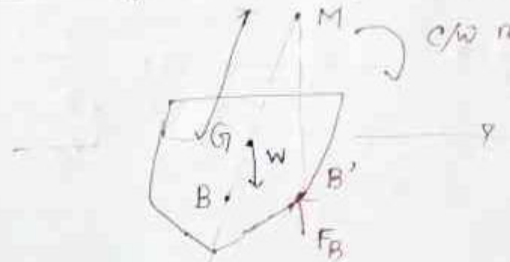


B' → New Center of Buoyancy

Meta Centre:-
It is a point of intersection between the line of action of vertical Buoyant force and the vertical axis of the object

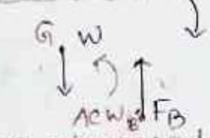
→ It is a imaginary point where the floating object about to oscillate.

Stability of floating body



GM → Metacentric Height
→ Distance between Centre of Gravity & Metacentre

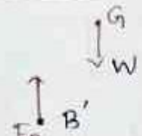
① M is above G

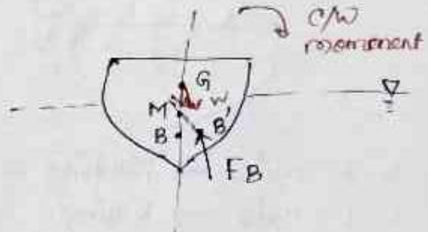


(Restoring momment)

→ $\boxed{\text{Stable Equilibrium}}$

② M is below G



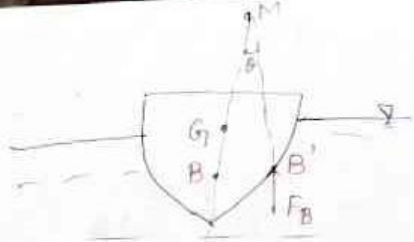


→ $\boxed{\text{Unstable Equilibrium}}$

③ When M is at G
This is the condion of Neutral Equilibrium.

Meta Centric Height

Distance Between centre of gravity & Meta centre.



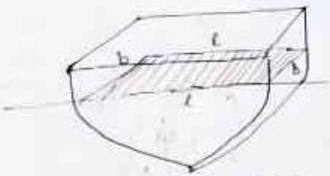
$GM = BM - BG$

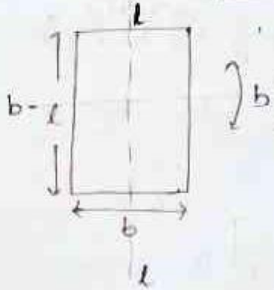
$$\boxed{GM = \frac{I}{\forall} - BG}$$

I - Moment of Inertia of surface which is intersecting the free surface

$\forall$ = Displaced volume of Liquid

Rolling





Pitching

b-b axis → Pitching

l - l axis → Rolling

$$\boxed{I_{ll} = \frac{lb^3}{12}}$$

Rolling - longitudnal axis

$$\boxed{I_{bb} = \frac{bl^3}{12}}$$

Pitching → lateral axis

$$\boxed{b < l}$$

so, $I_{ll} < I_{bb}$

$$\boxed{(GM)_l = \frac{I_{ll}}{\forall} - BG}$$

$$\boxed{(GM)_b = \frac{I_{bb}}{\forall} - BG}$$

$(GM)_b > (GM)_l$

$$\boxed{GM = \frac{I}{\forall} - BG}$$

I → least Moment of Inertia (More unstable)

so, I_{ll} is considered. $\boxed{I_{ll} < I_{bb}}$

Time Period of Oscilation

$$\boxed{T = 2\pi\sqrt{\frac{(K)^2}{g\,(GM)}}}$$

$$\boxed{K^2 = \frac{I}{A}}$$

where,

T → Time Period

K → Radius of Gyration (constant) $\left(K = \sqrt{I/A}\right)$

GM → Meta Centric height

Radius of Gyration

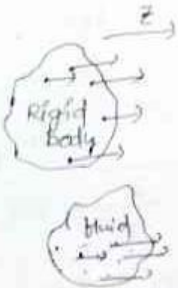
A radius of gyration in general is the distance from the centre of mass of a body at which the whole mass ~~of a body~~ could be concentrated without changing its moment of rotational Inertia about an axis through the centre of Mass.

$$\boxed{T \propto \frac{1}{\sqrt{(GM)}}}$$

Rigid Body — Hypothetical body (No deformation)

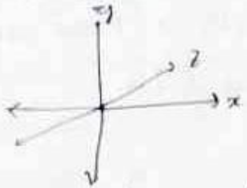
Rigid Body Motion

Translation motion | Rotational motion



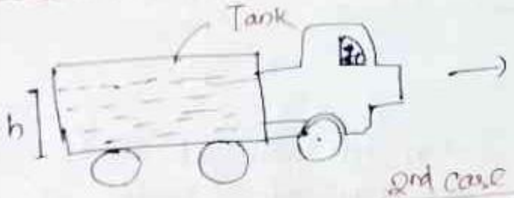
velocity of each Particle is same.

Translation Motion



- in a particular axis when a body is moving back & fro motion is called translation
- → when a body revolve about a particular axis is called Rotation.

Acceleration in Horizontal direction



1st case

Tank is at Rest

$v = 0$

2nd case

Tank is in motion

$v = v$

→ when we accelerate a body the velocity is change from the Rest to a particular body velocity. in a closed container.

Newton's 2nd Law

→ The Rate of change of momentum is equal to the impulse of force.

(OR)

→ The Net Force acting on a body is equal to the Product of Mass & acceleration

$$\boxed{F = m.a}$$

$a \neq 0$, then there will a resultant force acting in the direction of Motion

* Hydrostatic Law

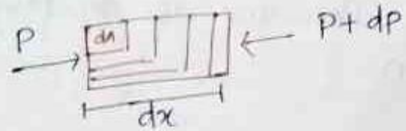
if fluid is at absolute rest, then Pressure vary along the depth.

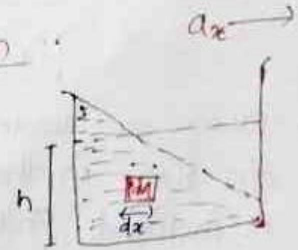
$$\boxed{\frac{dp}{dz} = -\gamma}$$ in z direction

$$\boxed{\frac{dp}{dx} = \frac{dp}{dy} = 0}$$ → No motion

Rigid Body Motion

(1) Horizontal direction





Newton's 2nd Law of Motion

$$\boxed{\Sigma F - ma = 0}$$

$(F = ma)$

$\boxed{F = ma}$

$P \cdot dA - (P + dP)\,dA = m\,a_x$

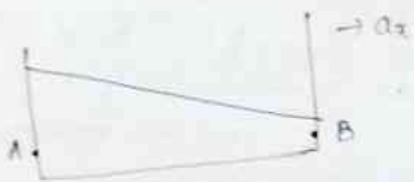
$P \cdot dA - P \cdot dA - dP \cdot dA = dA \cdot dx \cdot \rho \cdot a_x$ $(\because m = V\rho)$

$(\because V = dA \cdot dx)$

$-dP \cdot dA = dA \cdot dx\, \rho\, a_x$

$$\boxed{\frac{dP}{dx} = -\rho\, a_x}$$

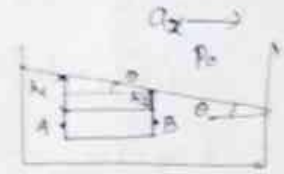
$\therefore$ (-ve) significant of Pressure is decreasing along direction of acceleration.

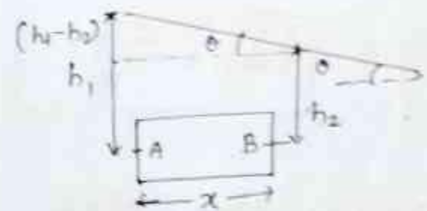


$\boxed{P_A > P_B}$

* when we accelerate the fluid in horizontal direction in that case the free surface is inclined. And the Pressure is decreasing along the acceleration

Slope of Free Surface of a liquid accelerated in horizontal direction :-





ρ - Density of fluid.

Pressure at A - P_A

Pressure at B - P_B

$P_0 \to$ atmospheric Pressure

$$\boxed{\frac{dP}{dx} = -\rho\, a_x}$$

$dP = -\rho a_x\, dx$

Integrating both side

$$\int_{P_A}^{P_B} dP = \int_0^x -\rho a_x\, dx$$

$[P_B - P_A] = -\rho a_x x$

$[P_A - P_B] = \rho a_x x$ —— eq^n ①

$\boxed{P_A = P_0 + \rho g h_1}$ → absolute pressure of Point A

$\boxed{P_B = P_0 + \rho g h_2}$ → " Point B

From eq^n ①

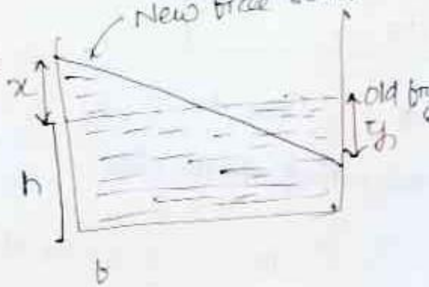
$(P_0 + \rho g h_1) - (P_0 + \rho g h_2) = \rho a_x \cdot x$

$$(P_0 + \rho g h_1) - (P_0 + \rho g h_2) = \rho a_x x$$

$$\rho g (h_1 - h_2) = \rho a_x x$$

$$\boxed{\frac{h_1 - h_2}{x} = \frac{a_x}{g}} \qquad \boxed{\tan\theta = \frac{h_1 - h_2}{x}}$$

$$\boxed{\tan\theta = \frac{a_x}{g}}$$ → slope of free surface

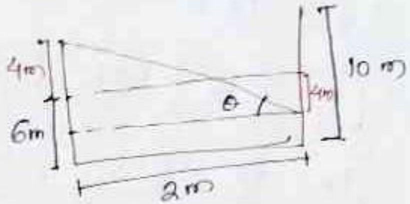


Fluid is Incompressible

The volume of liquid (fluid) will remain constant

$$\boxed{x = y}$$

Q) Find the maximum value of acceleration such that the water will not spill away from the vessel



$a_x = ?$

$x = y = 4\,m$

$x = y$

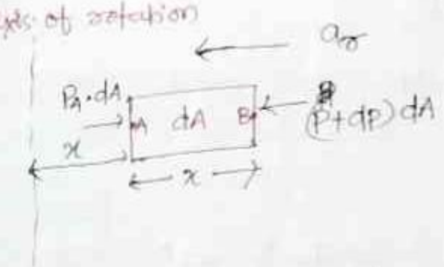
$\tan\theta = \frac{4+4}{2} = 4$

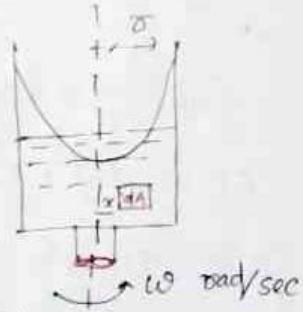
If x > 4m
⇒ spill away.

$\tan\theta = \frac{a_x}{g} = 4$

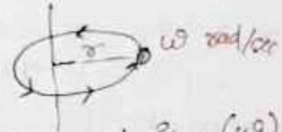
$\Rightarrow a_x = 4 \times 9.81 \Rightarrow \boxed{a_x = 39.24\ m/s^2}$

Pressure Difference in Rotating Fluid



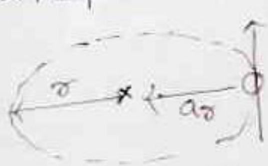


* change in pressure along the radius so the surface (−) change obtain a new shape (∪)
* when an object is rotating along an axis then there are 2 components of acceleration



Angular velocity (ω)
→ Linear velocity (v)

$$\boxed{v = \omega \cdot r}$$



a_T → Tangential component

along acceleration (+ve)

a_r → centripetal component

Newton's 2nd Law

$$\boxed{a_c = x \cdot \omega^2}$$

$$F = m a$$

$$(P + dP)\cdot dA - P\cdot dA = m a_c$$

$$P\,dA + dP\,dA - P\,dA = m a_c.$$

$$dP\,dA = \rho \times (dA \times dx) \cdot x\,\omega^2$$

$$\boxed{dP = \rho \cdot dx\, x\, \omega^2}$$

$$\boxed{\frac{dp}{dx} = \rho x \omega^2}$$

$$\int dp = \int \rho x \omega^2 . dx$$

$$\Delta P = \rho \omega^2 . \frac{x^2}{2}$$

$$\boxed{\Delta P = \frac{\rho \omega^2 x^2}{2}}$$ → Parabolic eqn

* In case of rotation as we get parabolic eqn so the free surface take the parabolic shape.

Fluid Kinematics

Static

A static body does not react to any force, Impulse, or collision and does not move.

Dynamic

A dynamic body reacts to forces, impulses, collisions & any other world event.

Kinematic

A Kinematic body is something hybrid between a static & a dynamic body. Kinematics bodies do not react to Forces.

Mechanics
- Statics
- Dynamics
  - Kinematics
  - Kinetics

Statics

It is the branch of Mechanics which deals with the analysis of loads & their effect on a system when its acceleration is zero or it is static equilibrium with the environment (relative motion is zero). Statics usually represent the analysis of loads on the stationary rigid body. The load on the system are force, torque or momentum.

Dynamics

Study of bodies when it is in motion. It is further divided into kinematics & kinetics.

Kinematics

It is the branch of mechanics which deals with the study of relative motion between the various components of system or machine. Kinematics describe the motion of an object without considering the forces that cause the motion.

- kinematics interested in pure motion only the system is considered as point body.
- It is used to learn about the properties such as displacement, velocity, acceleration & their variation with time.

Ex - study of the motion of transmission pulley.

Kinetics

The branch of mechanics which concerned about the relationship between the motion of the system and the cause (forces & torque acting on them) of the motion.

Fluid Kinematics

- It is the study of fluid motion regardless of the cause of motion
- 3 parameters
  - Displacement
  - Velocity
  - Acceleration
- 2 approach to study Fluid kinematics or mechanics

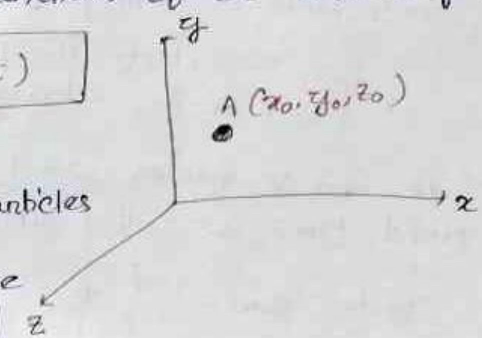
Lagrangian Approach
- Identity of Particle
- observe the motion through out a particular time

Eulerian Approach
- Particular instance or location we will take picture of particles.
- Instantaneous pic

Lagrangian Approach

- In this approach each fluid particle is observed w.r.t time
- The kinematic behaviour of the fluid particle will be the function of its identity.

$$\boxed{\vec{S} = f(x_0, y_0, z_0, t)}$$

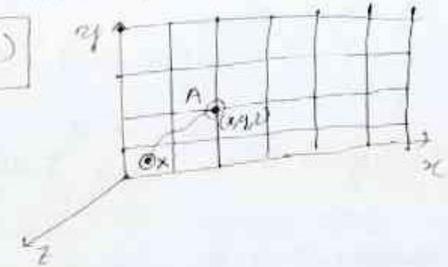


Problem
- To study the each particles is very difficult
- Differential eq^n s are difficult / complicated to solve.

Eulerian Approach

Defines a frame of reference & entire flow field is described with space coordinate.

$$\boxed{\vec{S} = f(x,y,z,t)}$$



Dimensions of flow

① one dimensional flow Ex- Pipe
② Two dimensional flow
③ three dimensional flow

Types of flow

① steady flow & unsteady flow

→ If R & V is invariant w.r.t time cont such flow is called steady flow

R → The set of fluid properties
V → V is the velocity of fluid particles

→ If R & V varies w.r.t time such a fluid flow is called unsteady flow

Steady flow → $\boxed{\left.\frac{\partial R}{\partial t}\right|_{space} = 0}$ $\boxed{\left.\frac{\partial V}{\partial t}\right|_{space} = 0}$

unsteady flow $\boxed{\left.\frac{\partial R}{\partial t}\right|_{space} \neq 0}$ $\boxed{\left.\frac{\partial V}{\partial t}\right|_{space} \neq 0}$

② Uniform & Non-uniform flow

If the R & V is invariant w.r.t space such flow is called uniform flow

$\boxed{\left.\frac{\partial R}{\partial space}\right|_{time} = 0}$ $\boxed{\left.\frac{\partial V}{\partial space}\right|_{space} = 0}$

If the R & V of fluid varies w.r.t space for a given instance of time such flow is known as Non uniform flow.

$\boxed{\left.\frac{\partial P}{\partial space}\right|_{time} \neq 0}$ $\boxed{\left.\frac{\partial V}{\partial space}\right|_{time} = 0}$

③ Steady & uniform flow →

$\boxed{\left.\frac{\partial R}{\partial t}\right|_{space} = 0}$ $\boxed{\frac{\partial R}{\partial space} = 0}$

$\boxed{\frac{\partial V}{\partial t}, \frac{\partial V}{\partial space} = 0}$

④ Non-uniform & unsteady flow

$\boxed{\frac{\partial R}{\partial t}, \frac{\partial R}{\partial space} \neq 0 \qquad \frac{\partial V}{\partial t}, \frac{\partial R}{\partial space} \neq 0}$

(5) Compressible & Incompressible flow

$$\boxed{\frac{\partial \rho}{\partial P} = 0} \longrightarrow \text{Incompressible flow}$$

~~$\rho = f(P)$~~

→ ρ (density) is constant & invariant w.r.t change in Pressure

→ density of fluid does not change w.r.t Pressure this flow is called Incompressible flow

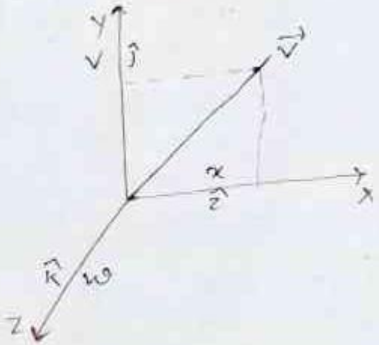
$$\boxed{\frac{\partial \rho}{\partial P} \neq 0} \qquad \boxed{\rho = f(P)}$$

↳ Compressible flow

→ density (ρ) is the function of Pressure which varies w.r.t time is called Compressible flow.

Acceleration of Fluid Particles

$$\boxed{\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}}$$



$\left.\begin{matrix} \to u \\ \to v \\ \to w \end{matrix}\right\} f(x, y, z, t)$

→ Scalar components of velocity.

Change in velocity

$$\boxed{d\vec{V} = \frac{\partial \vec{V}}{\partial x}dx + \frac{\partial \vec{V}}{\partial y}dy + \frac{\partial \vec{V}}{\partial z}dz + \frac{\partial V}{\partial t}dt}$$

acceleration - change in velocity w.r.t time.

$$\vec{a} = \frac{d\vec{V}}{dt}$$

$$\vec{a} = \frac{\partial \vec{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy}{dt} + \frac{\partial \vec{V}}{\partial z}\frac{dz}{dt} + \frac{\partial V}{\partial t}\frac{dt}{dt}$$

$$\boxed{\vec{a} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\text{change of velocity} \to \text{w.r.t space coordinate or direction}} + \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{w.r.t time}}}$$

change of velocity → w.r.t space coordinate or direction → **Convective Acceleration**

w.r.t time → **temporal (or) Local acceleration**

acceleration along x-direction (components)

$$\boxed{\vec{a}_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}}$$

$$\boxed{a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}}$$

$$\boxed{a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}}$$

Total Acceleration of fluid Particles

$$\boxed{\vec{a} = \sqrt{a_x^2 + a_y^2 + a_z^2}}$$

Acceleration for diff. type of flow

① Steady flow

$\boxed{\text{Total acceleration} = \text{Convective acceleration}}$

* $\boxed{\text{Local acceleration} = 0}$

$$\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t} = 0$$

② Uniform flow

$\boxed{\text{Total acceleration} = \text{local acceleration}}$

* Convective acceleration = 0

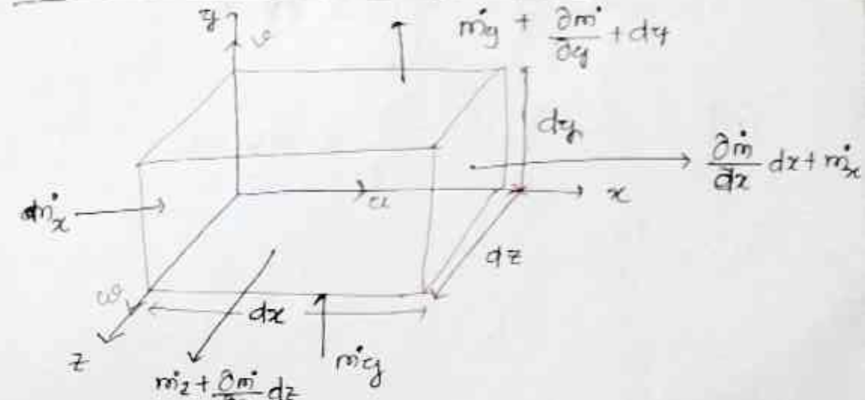
$$\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial w}{\partial z} = 0 \text{ etc.}$$

③ Steady & uniform acceleration

$\boxed{\text{Total acceleration} = 0}$

Continuity Equation

General form of continuity equation:



$\boxed{\dot{m}_{in} - \dot{m}_{out} = \dot{m}_{st}}$ → Conservation of mass.

ρ → density of fluid.

$$(\dot{m}_{in} - \dot{m}_{out})_x = \dot{m}_x - \left(\dot{m}_x + \frac{\partial \dot{m}_x}{\partial x} \cdot dx\right)$$

$$= -\frac{\partial \dot{m}_x}{\partial x} dx$$

Mass flow Rate – Desity x Area of c/s along flow x velocity along the dired^n of flow

$$(\dot{m}_{in} - \dot{m}_{out})_x = -\frac{\partial \rho \cdot (dy\, dz) u}{\partial x} dx$$

$$= -\frac{\partial}{\partial x} \rho \cdot (dx\, dy\, dz) u$$

$$\boxed{(\dot{m}_{in} - \dot{m}_{out})_x = -\frac{\partial}{\partial x} \rho\, dv \cdot u}$$

Similarly for y & z direction

$$(\dot m_{in} - \dot m_{out})_y = -\frac{\partial}{\partial y}\rho \, dV \, v$$

$$(\dot m_{in} - \dot m_{out})_z = -\frac{\partial}{\partial z}\rho \, dV \, w$$

$$\dot m_{st} = \frac{\partial m_{cv}}{\partial t}$$

$$= \frac{\partial}{\partial t}(\rho \, dV)$$

$$\boxed{\dot m_{st} = \frac{\partial \rho}{\partial t}\cdot dV} \quad —— ①$$

From the conservation of Mass

$(\dot m_{in} - \dot m_{out})$ = x directⁿ + y directⁿ + z directⁿ

$$= -\left(\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w\right)\cdot dV$$

$\hookrightarrow$ ②

From eqⁿ ① & eqⁿ ②

$$\frac{\partial \rho}{\partial t}\cdot dV = -\left(\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w\right)\cdot dV$$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w = 0}$$

↳ General continuity eqⁿ.

① For steady flow

$\rho \neq f(t)$

$\rho = f(x, y, z)$

$$\boxed{\frac{\partial \rho}{\partial t} = 0}$$

$$\boxed{\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w = 0}$$

② For Incompressible & steady flow (3-D flow)

ρ = const $\rho \neq f(t, x, y, z)$

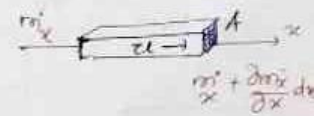
$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0} \rightarrow (\rho = \text{constant})$$

– 3-D flow

③ For 2-D flow

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0} \rightarrow \text{Steady \& Incompressible}$$

④ For 1-D flow



$$\boxed{\dot m_{in} - \dot m_{out} = \dot m_{st}}$$

$$\dot m_x - \left(\dot m_x + \frac{\partial \dot m_x}{\partial x}dx\right) = \frac{\partial}{\partial t}(\rho \, dV)$$

steady flow

$$= dV \, \frac{\partial \rho}{\partial t}^{=0} \quad (\rho \neq f(t))$$

$$\Rightarrow -\frac{\partial \dot m_x}{\partial x}dx = 0$$

$$\Rightarrow -\frac{\partial}{\partial x}(\rho \cdot A \cdot u) = 0$$

$\boxed{u = V}$ $(v, w = 0)$

By integrating the eqⁿ

V → velocity

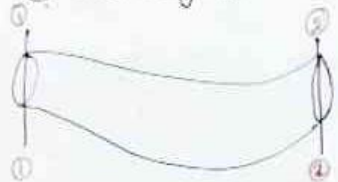
$$\boxed{\rho A V = \text{constant}}$$

$\boxed{\rho AV = \text{constant}}$ → Valid for compressible as well as incompressible

→ Continuity eq^n for

(i) 1-Dimensional flow

(ii) Steady flow.



Mass flow rate → $\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$

For incompressible fluid

ρ = constant

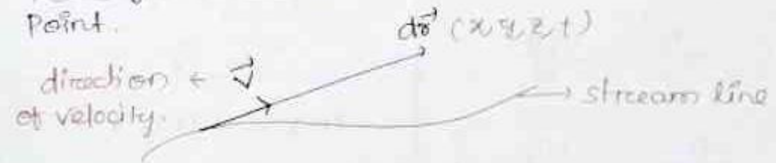
Volume flow rate → $\boxed{A_1 V_1 = A_2 V_2}$

$\boxed{AV = \text{constant}}$

Continuity eq^n for → For 1-D, steady & incompressible flow

Stream line

It is an imaginary line drawn in the flow field such that the tangent drawn at any point on this line represents the direction of velocity vector of the fluid particle at that point.



$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

From above definition $d\vec{r}$ & $\vec{V}$ are colinear

$$\boxed{\theta = 0^\circ}$$

$$\vec{V} \times d\vec{r} = V \cdot dr \sin\theta$$

$$\vec{V} \times d\vec{r} = 0 \quad (0\hat{i} + 0\hat{j} + 0\hat{k})$$

After solving the above eq^n, we will get

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

→ The relation for stream line.

→ The stream line eq^n

* We can determine the stream line eq^n from Eularian Approach of fluid kinematics. (instantaneous location of flow field in which we draw a imaginary curve & the tangent of that curve denotes fluid vector of velocity of fluid).

Q) A 2D incompressible flow field is given by

$$\boxed{\vec{V} = 3x\hat{i} - 3y\hat{j}}$$

Find the eq^n of stream line passing through (1, 1)

$u = 3x$

$v = -3y$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{3x} = \frac{-dy}{3y}$$

$$\Rightarrow \frac{dx}{x} = \frac{-dy}{y}$$

Integrating both side

$\ln x = -\ln y + \ln C$

$\ln x + \ln y = \ln C$

$\ln (x \cdot y) = \ln C$

$$\boxed{x \cdot y = C}$$

$(x, y) \rightarrow (1, 1)$

$$\boxed{C = 1}$$

eq^n of stream line

$x y = 1$

$$\boxed{y = 1/x}$$

NOTE:
Two streamlines cannot intersect each other.

② Path line

→ It is based on langragian approach of fluid mechanics.

→ The locous of a fluid particle in the flow phase

→ Path followed by fluid particle.

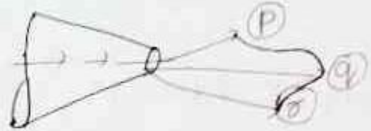
* Two path lines can intersect each other.

* A path line can intersect itself

(It can form a loop)

③ Streak lines:

Δt → time interval for observation



→ streak line is a imaginary line which is passing through the points which have passed through the same point at a given interval of time.

→ no of Particles – instantaneous location after a certain time interval. the line joining the Particle point is streak lines.

0 sec – 10 sec – 20 sec like wise

Imp

If the flow is steady – Stream line, Path line & streak line coincide for steady flow (Same)

Q) Which of the following set of eq^n represents possible case of two dimensional incompressible flow.

① $u = x + y$, $v = x - y$

② $u = x + 2y$, $v = x^2 - y^2$

we know,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

① $\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x+y) = 1 + 0 = 1$

$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(x - y) = 0 - 1 = -1$

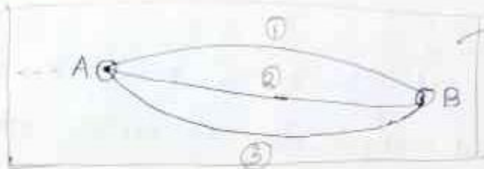
$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 1 - 1 = 0.}$$ Satisfied.

Potential Function

If vector field is conservative than the velocity vector is equal to gradient of scalar funct^n. This scalar function is called potential function.

-> Potential function exists for a conservative velocity vector field.

$$\boxed{\vec{V} = \nabla\phi}$$ -> Potential funct^n ; Gradient



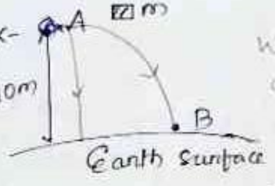
-> Conservative field

-> Line integral is same for all path (for velocity vector)

$$\boxed{\nabla \times \vec{V} = 0}$$ -> Curl of vector $\vec{V}$

-> For conservative field

Ex-



Work done - same
gravity force - same in all path

$$\boxed{\vec{F} = \nabla \cdot \phi}$$ ϕ -> scalar funct^n

$$\boxed{\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}}$$ ∇ -> not a vector

-> Hamiltonian operator or Del operator (used for linear vector Differential)

-> ∇ is not a vector

-> It neither has magnitude nor direction

ϕ exists when $\boxed{\nabla \times \vec{V} = 0}$

$$\boxed{\phi = f(x, y, z, t)}$$

$$\boxed{-\frac{\partial \phi}{\partial x} = u}, \quad \boxed{-\frac{\partial \phi}{\partial y} = v}, \quad \boxed{-\frac{\partial \phi}{\partial z} = w}$$

Potential function

It is function of space & time in a given place such that its negative derivative w.r.t space give the velocity components along that direction.

Imp.

$\boxed{\nabla \times \vec{V} = 0}$ → curl of $\vec{V}$

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix}$$

↓

$\boxed{\text{Vorticity}}$ → It represents the relative (Ω) motion bet^n 2 fluid particles.

$\boxed{\omega = \frac{1}{2}\Omega}$

$\boxed{\Omega = 2\omega}$

→ Rotational Components (ω) is equal to ½ of vorticity (Ω)

→ If ϕ exists, $\nabla \times \vec{V} = 0$, $\Omega = 0$, $\omega = 0$

→ if ϕ exists, then the flow must be irrotational.

$\Omega = 2\omega$

$\omega = \frac{1}{2}\Omega$

$\boxed{\omega = \frac{1}{2}\nabla \times \vec{V}}$

$$\omega = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix}$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{\partial v}{\partial z}\right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$\boxed{\omega_x = \omega_y = \omega_z = 0}$ → flow is irrotational flow

$$\boxed{\begin{aligned} u &= -\frac{\partial \phi}{\partial x} \\ v &= -\frac{\partial \phi}{\partial y} \\ w &= -\frac{\partial \phi}{\partial z} \end{aligned}}$$

Continuity eq^n

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

$$-\left\{\frac{\partial}{\partial x}\frac{\partial \phi}{\partial x} + \frac{\partial}{\partial y}\frac{\partial \phi}{\partial y} + \frac{\partial}{\partial z}\frac{\partial \phi}{\partial z}\right) = 0$$

$$\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0}$$

← Continuity eq^n for steady, 3D, incompressible flow.

NOTE

If ϕ satisfies the continuity eq^n then flow field is irrotational

$$\nabla \cdot \vec{V} = 0$$

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(u\hat{i} + v\hat{j} + w\hat{k}\right) = 0$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

$$\boxed{\nabla \cdot \nabla = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}$$

↳ Laplacian operator

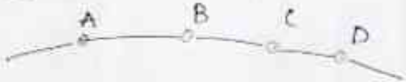
$\boxed{\nabla^2 \phi = 0}$ → Laplace equation

→ The potential funct^n must satisfy laplace eq^n.

Equipotential Line

↳ Along

$\boxed{\phi = f(x, y).}$



→ Line joining same value of potential funct^n (ϕ)

→ ϕ is constant along equipotential line

$$\boxed{d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy.}$$

$$\partial \phi = -u\,dx + -v\,dy.$$

For an equipotential line

$$d\phi = 0$$

So, $-u\,dx - v\,dy = 0$

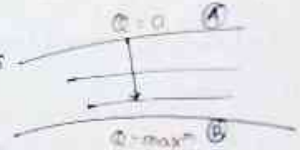
$$-v\,dy = u\,dx$$

$\boxed{\frac{dy}{dx} = \frac{-u}{v}}$ → Slope of equipotential line.

Stream function (ψ)

This function is defined as function of space & time such that it always satisfy continuity eq^n.

$\boxed{u = \frac{-d\psi}{\partial y}}$ $\quad$ $\boxed{v = \frac{\partial \psi}{\partial x}}$



As we moving towards (B) the discharge is increasing according to the distance covered, so there is a relation bet^n discharge & distance and this function is stream funct^n (ψ) along the stream line.

NOTE

→ ψ exists for Rotational as well as irrotational flow

$\boxed{\nabla^2 \psi = 0}$ Irrotational flow

↳ Laplace eq^n

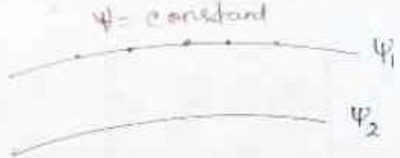
$\boxed{\nabla^2 \psi \neq 0}$ Rotational flow

* if $\boxed{\nabla^2 \phi \neq 0}$, then we can't say that flow is rotational. but if $\boxed{\nabla^2 \psi \neq 0}$ then we can say flow is rotational.

Equi stream function line

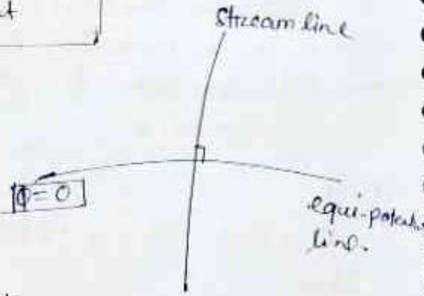
or

Stream line



→ The ψ value is constant through out the stream line.

$$\boxed{\begin{aligned} d\psi &= 0 \\ \psi &= \text{constant} \end{aligned}}$$



$\psi = f(x, y)$

$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$

$d\psi = v\,dx - u\,dy$

$d\psi = 0 = v\,dx - u\,dy$

⇒ $v\,dx = u\,dy$

⇒ $\boxed{\frac{dy}{dx} = \frac{v}{u}}$ → slope of equi stream funct^n line.

$\boxed{\frac{dy}{dx} = -\frac{u}{v}}$ → equipotential line

Imp

Equipotential line ($\phi = 0$) and equi stream function line is orthogonal (90°) to each other.

Ch-4- Fluid Dynamics

→ Study of motion of fluid flow along with the force causing the flow

↳ Newton's 2nd Law

$\boxed{F = ma}$ — Law of Net Force

↳ Resultant or Net force

→ Let us consider fluid is flowing in x-direction

$$\boxed{F_x = m \cdot a_x}$$

Forces exerts in fluid

1. F_g - Gravity force
2. F_P - Pressure force
3. F_v - Viscous force
4. F_T - Force due to turbulance
5. F_c - Force due to compressibility

Resultant force along x-direction $\boxed{F_x = (F_x)_g + (F_x)_p + (F_x)_v + (F_x)_T + (F_x)_c}$

1. If we neglect force due to compressibility (F_c)

$$\boxed{F_x = (F_x)_g + (F_x)_p + (F_x)_v + (F_x)_T}$$

↳ Reynold's equation

2. If we neglect F_c & F_T. ~~are~~

$\boxed{F_x = (F_x)_g + (F_x)_p + (F_x)_v}$ → Navier stokes eq^n

(iii) If we considered F_g & F_P

$$\boxed{F_x = (F_x)_g + (F_x)_P} \rightarrow \text{Euler's equation}$$

Euler's Eqn of Motion

→ Resultant force will be equal to the sum of gravity force and Pressure force.

* after taking a stream line solving the eqn we get

Euler's eqn of motion.

$$\boxed{\frac{dP}{\rho} + g\,dz + v\,dv = 0}$$

Important point for Euler's eqn

(i) → As we negleted (F_v), so This eqn is valid for non viscous or inviscid fluid flow.

→ F_c is also neglected. so this eqn is valid for incompressible fluid

Hence, Valid for $\boxed{\text{Ideal flow}}$ → Inviscid & incompressible

(ii) $\boxed{\frac{\partial v}{\partial t} = 0}$ for steady

→ It is valid for steady flows.

(iii) F_T is neglected

→ Flow is Laminar or / stream line flow

→ Inviscid, incompressible, steady & Laminar or laminar flow.

Bernoulli's Equation

-Assumptions

(i) Ideal fluid flow (viscosity = 0)

(ii) Incompressible fluid (ρ - constant)

(iii) Steady flow (flow parameters doesn't change w.r.t time)

$\frac{dv}{dt} = 0$, $\frac{da}{dt} = 0$ (velocity & acceleration)

(iv) Irrotational flow (doesn't rotate about it's axis)

Derivation:-

Euler's equation:

$$\boxed{\frac{dP}{\rho} + g\,dz + v\,dv = 0} \rightarrow \text{Ideal fluid}$$

Integrating Euler's eqn

$$\int \frac{dP}{\rho} + \int g\,dz + \int v\,dv = \text{Constant}$$

$$\frac{P}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\boxed{\frac{P}{\rho g} + z + \frac{v^2}{2g} = \text{Constant}}$$

$\frac{P}{\rho g}$ = Pressure Energy or Pressure head

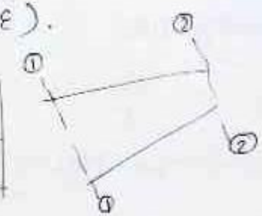
$\frac{v^2}{2g}$ = Kinetic energy or kinetic head

Z = Potential energy or potential head

Definition

In steady & Ideal flow total energy at any point constant where, total energy consists of (PE + KE + PoE).

$$\boxed{\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2}$$



For Ideal fluid

Viscosity = 0 (Ideal fluid)

* but in real Practice (viscosity $\neq 0$) for Real fluid.
* In Real fluid there is a friction force for which loss occur

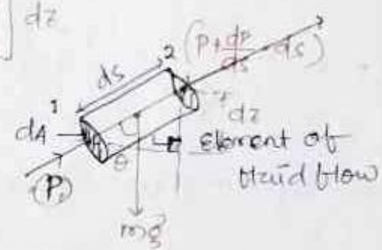
$$\boxed{\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 + h_L}$$

$h_L \to$ friction loss (head loss)

* friction loss is considered for Real fluid

Derivation of Euler's Eqⁿ of Motion

$$\boxed{\rho = \frac{m}{V}}$$



Mass

$m = \rho V$

$$\boxed{m = \rho A \times l}$$

For the element

$$\boxed{m = \rho \times dA \times ds}$$

$$\boxed{mg \to \rho g\, dA \cdot ds}$$

Pressure

$$\boxed{P = \frac{F}{A}} \qquad F = P \cdot A$$

$$\boxed{F_1 = PdA} \quad \text{(for the element)}$$

$\frac{dP}{ds} \to$ change in Pressure per unit length.

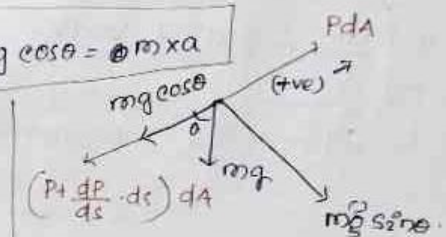
Total length $\to$ (ds)

$\frac{dP}{ds} \times ds$

$$\boxed{F_2 = \left[P + \left(\frac{dP}{ds} \cdot ds\right)\right] \times A}$$

FBD

$$\boxed{PdA - \left(P + \frac{dP}{ds} \cdot ds\right)dA - mg\cos\theta = m \times a}$$



$\cos\theta = \frac{dz}{ds}$

$a = \frac{dv}{dt} \qquad v = f(s,t)$

Chain Rule.

$$\frac{df(s,t)}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt}$$

$\Rightarrow a = \frac{dv}{ds} \times \frac{ds}{dt} + \cancel{\frac{dv}{dt}} \times \cancel{\frac{dt}{dt}}$ $\left(\frac{dv}{dt} = 0\right)$ Steady flow.

$\boxed{a = v \frac{dv}{ds}}$ $\boxed{\cos\theta = \frac{dz}{ds}}$

After replacing the above 2

$$P\,dA - \left(P + \frac{dP}{ds} ds\right) dA - \rho g\, dA\, ds \frac{dz}{ds} = \rho\, dA\, ds\, v\frac{dv}{ds}$$

$$P\,dA - P\,dA - \frac{dP}{ds} ds\, dA - \rho g\, dA\, dz = \rho\, dA\, v\, dv$$

deviding eq^n by $(\rho\, dA\, ds)$

$$-\frac{dP}{ds} \cdot \frac{1}{\rho} - g\frac{dz}{ds} = v\frac{dv}{ds}$$

$$\Rightarrow \frac{dP}{\rho ds} - g\frac{dz}{ds} - v\frac{dv}{ds} = 0$$

$$\Rightarrow \boxed{\frac{dP}{\rho} + g\, dz + v\, dv = 0}$$

→ Euler's eq^n of motion.

Application of Bernoulli's Equation

1. Venturimeter
2. Orifice
3. Pitot tube

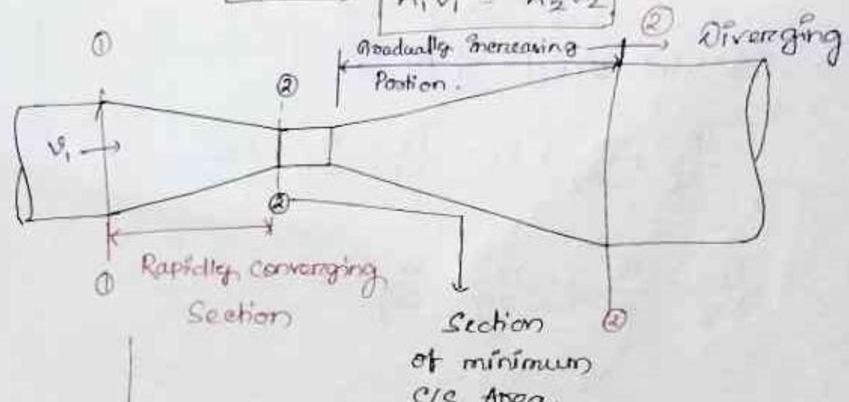
1. Venturimeter

→ It is used to measure flow Rate (Discharge)

→ Venturimeter is a short pipe consisting of two conical parts with a short portion of minimum cross-sectional area.

Incompressible fluid $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

→ $\boxed{A_1 V_1 = A_2 V_2}$



→ To reduce the loss of Energy.

In Converging Portion

(i) Velocity increases along the converging portion.

$$\boxed{A_1V_1 = A_2V_2}$$

* $\boxed{A_2 < A_1}$

$$V_2 = \frac{A_1}{A_2} V_1$$

$\Rightarrow \boxed{V_2 > V_1}$

(ii) The Pressure decreases along the converging section from Bernoulli's equation. $(P_1 > P_2)$

→ For a horizontal venturimeter.

$$\boxed{Z_1 = Z_2}$$

For ideal fluid $\boxed{h_L = 0}$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}.$$

$$V_2 > V_1$$

$$\frac{V_2^2}{2g} > \frac{V_1^2}{2g}.$$

RHS > 0.

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} > 0$$

$$\frac{P_1}{\rho g} > \frac{P_2}{\rho g}$$

$$\boxed{P_1 > P_2}$$

Theoretical Discharge

$$\boxed{Q_T = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2g\Delta h\left(\frac{\rho_m}{\rho} - 1\right)}}$$

For Discharge (Q) m^3/s

$$\boxed{Q = AV = A_2V_2}$$

$Q_{act} < Q_T$

Coefficient of Discharge :

$$\boxed{C_d = \frac{Q_{act}}{Q_T}}$$

Actual Discharge

if $\rho_m > \rho$

$$\boxed{Q_{act} = C_d \times \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2g\Delta h\left(\frac{\rho_m}{\rho} - 1\right)}}$$

if $\rho > \rho_m$

$$\boxed{Q_{act} = C_d \times \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2g\Delta h\left(1 - \frac{\rho_m}{\rho}\right)}}$$

$\rho \to$ fluid in Venturimeter

$\rho_m \to$ density of Manometricfluid.

Difference in static head or Piezometric head.

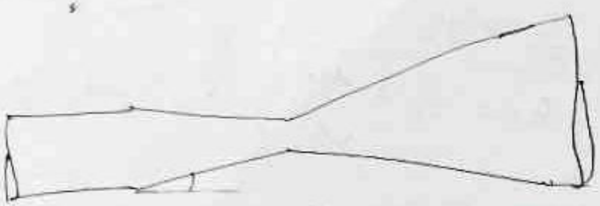
$$\boxed{h_1^* - h_2^* \text{ or } h_1 - h_2 = \Delta h \left(\frac{\rho_m}{\rho} - 1\right)}$$

$\Delta h \to$ Manometric Reading

In terms of Specific Gravity (SG)

$$\boxed{h_1 - h_2 \text{ or } h_1^* - h_2^* = \Delta h \left(\frac{(SG)_m}{SG} - 1\right)}$$ $(SG)_m > (SG)$

$$\boxed{h_1 - h_2 \text{ or } h_1^* - h_2^* = \Delta h \left(1 - \frac{(SG)_m}{(SG)}\right)}$$

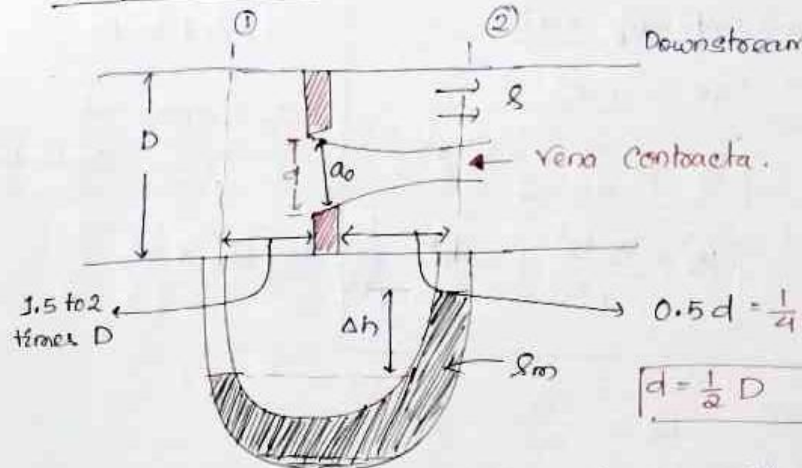


* Angle of Converging section = $18° - 20°$

* Angle of Diverging Section = $6° - 7°$

  ↳ Semicone angle of diverging section $\leq 7°$

Orifice Meter



$\boxed{d = \frac{1}{2}D}$

→ Minimum cross section at Downstream side is called – vena contracta.

→ Use of Orifice is to measure discharge (flow Rate)

By applying Bernoulli's Eqn

$$\boxed{h_1^* - h_2^* = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}}$$

h^* = Differential head $(h_1^* - h_2^*)$

$$\boxed{h^* = \frac{V_2^2 - V_1^2}{2g}}$$

$$\boxed{v_2^2 = v_1^2 + 2gh^*} \quad \text{—(I)}$$

$$\boxed{h^* = \Delta h\left(\frac{\rho_m}{\rho} - 1\right)}$$

From continuity eqⁿ

$$a_1 v_1 = a_2 v_2$$

$$\boxed{v_1 = \frac{a_2 v_2}{a_1}} \quad \text{—(II)}$$

$$\boxed{v_2^2 = \frac{a_2^2 v_2^2}{a_1^2} + 2gh^*} \quad \text{(III)}$$

Let

$a_0 \to$ Area of c/s of orifice

$C_c \to$ coefficient of contracta

$a_2 \to$ area of vena-contracta.

$$\boxed{a_2 < a_0}$$

so, $C_c = \dfrac{a_2}{a_0}$

$$\boxed{a_2 = C_c a_0} \quad \text{—(IV)}$$

From eqⁿ (II) & (IV)

$$v_1 = \frac{C_c a_0 v_2}{a_1}$$

$$\boxed{v_2 = \sqrt{\frac{C_c^2 a_0^2 v_2^2}{a_1^2} + 2gh^*}}$$

on Simplification

$$\boxed{v_2 = \frac{\sqrt{2gh^*}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}}$$

Discharge

$$\boxed{Q = a_2 v_2}$$

$$\boxed{C_c = C_d \times \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}}$$

Discharge through Orifice :-

Imp

$$\boxed{Q = C_d \frac{a_1 a_0}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh^*}}$$

$$\boxed{h^* = \Delta h\left(\frac{(SG)_m}{(SG)} - 1\right)} \quad \boxed{\rho_m > \rho}$$

$$\boxed{h^* = \Delta h\left(1 - \frac{\rho_m}{\rho}\right)} \quad \rho_m < \rho$$

Q) $d_0 = 15$ cm, $D_p = 30$ cm, $\Delta h = 50$ cm of Hg

$S.G_{oil} = 0.9$, $C_d = 0.64$, $Q = ?$, $S.G_{Hg} = 13.6$

$$Q = 0.64 \times \frac{\left(\frac{\pi}{4} \times 30^2\right)\left(\frac{\pi}{4} \times 15^2\right)}{\sqrt{\left(\frac{\pi}{4} \times 30^2\right) - \left(\frac{\pi}{4} \times 15\right)^2}} \times \sqrt{2 \times 9.81 \times 705.55}$$

$$h^* = 50 \times \left[\frac{13.6}{0.9} - 1\right]$$

$$h^* = 705.55 \text{ cm}$$

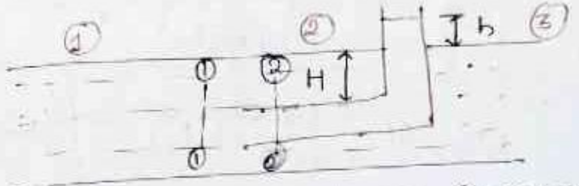
$$\boxed{Q = 137414.25 \text{ cm}^3/\text{s}}$$

(3) Pitot tube

Pitot tube is a device used to measure the fluid flow measurement invented by Henri Pitot, in 18^{th} century.

The pitot tube also called a differential pressure measuring device.

It is widely use to measure the airspeed of aircrafts, speed boat speed & for fluid flow measurement in industrial application.

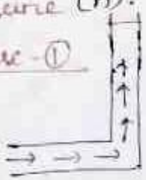


* It convert kinetic energy head to pressure energy head.

→ When the water particle entre the tube the velocity convert into dynamic pressure (h).

$$\boxed{h + H = \text{stagnation head}}$$

Phase - ①



→ In phase ① the fluid flow inside the tubes & it goes upward.

→ Initially since it dynamic the fluid goes to a higher level.

Phase - ②

→ The fluid under the influence of gravity goes down.

→ one side water is rushing inside

Phase-3

→ The Force due to the water going up is equal to the force due to water going down.

→ during this particles of fluid which is are in section ② they are at rest because of static equilibrium.

H → Static head (constant velocity)

h → Dynamic head (velocity change.

Bernoull's Principle. at section ① & ②

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + \cancel{z_2}$$

For Ideal fluid → $\boxed{h_L = 0}$

$$\boxed{z_1 = z_2,\ v_2 = 0}$$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g}.$$

$$P_1 + \rho \frac{v_1^2}{2} = P_2 = P_{stag}.$$

$$\boxed{P_{stag} = P_1 + \rho \frac{v_1^2}{2}}$$

→ Dynamic Pressure (h)

Static Pressure (H)

Stagnation Pressure - When the pressure at a point where the fluid is brought to complete stop ($v_2 = 0$)

Momentum Equation

→ Based on conservation of momentum

Newton's 2nd Law → law of Net Force

$$\boxed{F = ma} \qquad \boxed{a = \frac{dv}{dt}}$$

$\Rightarrow F = m \cdot \frac{dv}{dt}$

$\Rightarrow F = \frac{d(mv)}{dt}$ $\qquad m \cdot v \rightarrow$ momentum

$\Rightarrow \boxed{F\,dt = d(mv)}$ $\qquad \frac{d(mv)}{dt} \rightarrow$ Rate of change of momentum.

↳ Impulse

Momentum Theorem → $F\,dt = d\,mv$

$F \cdot dt$ → impulse of force for a small time interval.

$$\boxed{\text{Impulse} = \text{change in momentum}}$$

↳ Impulse momentum theorem

$F\,dt = d(mv)$

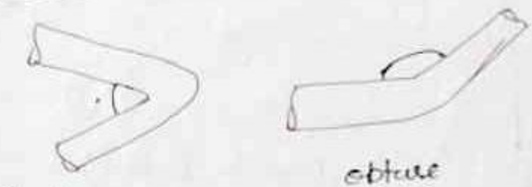
$F = \frac{d(mv)}{dt}$

$$\boxed{\text{Net force} = \text{change in momentum}}$$

→ Applicable for Rigid body as well as for Newtonian fluid.

Application of Momentum Equation

① Pipe Bend

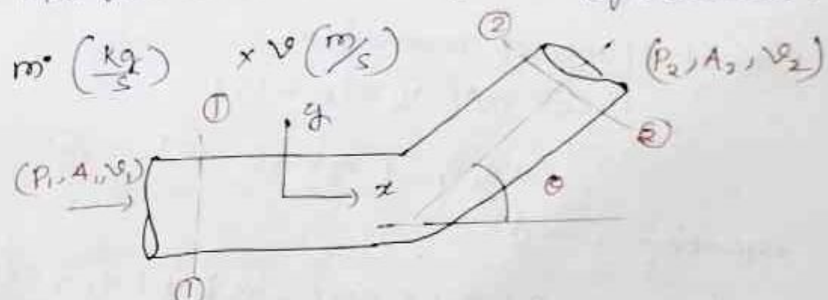


Acute $\qquad$ obtuse

$$\boxed{\Sigma F_{ext} = \text{Rate of change in momentum}}$$

Mass flow Rate × velocity = Rate of change of momentum.

$\dot{m}\left(\frac{kg}{s}\right) \times v\left(\frac{m}{s}\right)$



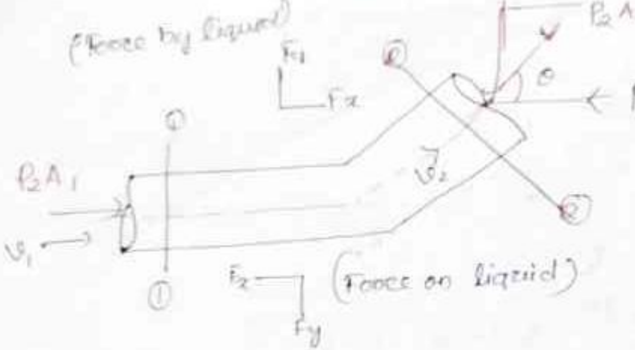
* Liquid is incompressible } Ideal fluid
* Inviscid

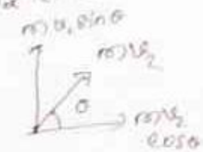
(F_x) → Force exerted by liquid on nozzle or Pipe bend in x-direction

$(-F_x)$ → Force exerted by nozzle by Pipe bend on liquid

(F_y) → Force exerted by liquid on Pipe bend.

$(-F_y)$ → Force exerted by Pipe bend on liquid.





Force in x-direction

Net force in x-direction

$= P_1A_1 - P_2A_2\cos\theta - F_x$ — (i)

Rate of change of momentum

$= \dot m(v_2\cos\theta - v_1)$

$= \rho A_1 v_1 (v_2\cos\theta - v_1)$ — (ii)

equating (i) & (ii)

$$\boxed{F_x = -\rho A_1 v_1 (v_2\cos\theta - v_1) + P_1A_1 - P_2A_1\cos\theta}$$

Like wise (section 1 $v_y = 0$)

Net force in y-direction

$= F_y = 0 - P_2A_2\sin\theta - F_y$

Rate of change in momentum (momentum$_y$ at (1) $= 0$)

$= \dot m v_2\sin\theta - 0$

equating above eqn

$$\boxed{F_y = P_2A_2\sin\theta - \dot m v_2\sin\theta}$$

Resultant Force

$= \sqrt{F_x^2 + F_y^2}$

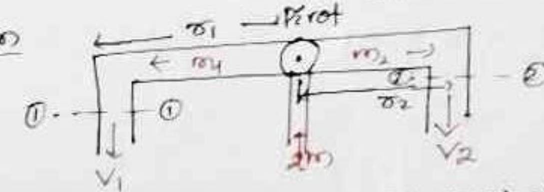
$\boxed{\tan\theta = \frac{F_y}{F_x}}$ → Inclination or slope.

Moment of momentum Equation

Sprinkler

momentum

$\boxed{A_1 = A_2}$



Momentum @ section (1)

$= mv_1$

$= mv_1r_1$

@ section (2)

$= mv_2$

$= mv_2r_2$

(+ve) (-ve)

Net torque = Rate of change of moment of momentum

= momentum × distance.

@ section (1)

Moment of momentum

$= mv_1r_1$

@ section (2)

$= mv_2r_2$

Resulting torque = Net torque = T

$T = \dot m(v_2r_2 - v_1r_2)$

$$\boxed{T = \rho A_1 V_1 (v_2 r_2 - v_1 r_2)} \quad (\dot{m}_1 = \dot{m}_2 \times \dot{m}_3)$$

In sprinkler when $(A_1 = A_2)$

* if $A_1 \neq A_2$ then mass flow rate at both outlets or ends will not be same.

$$(\dot{m}_1 \neq \dot{m}_2)$$

Ch-5 Fluid Flow

① Ideal Fluid Flow

* A fluid which is incompressible & Inviscid such a fluid is called Ideal fluid

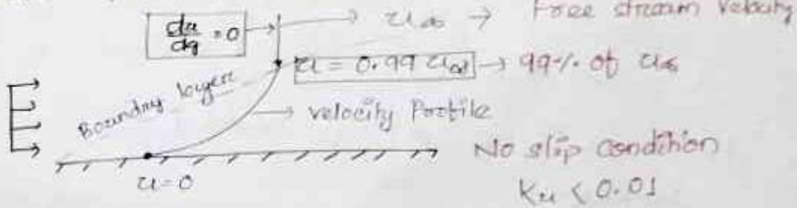
ρ = constant → Incompressible
$\mu = 0$ → Inviscid

* As per Newton's law

$$\boxed{\tau = \mu \frac{du}{dy}}$$

$\boxed{\tau = 0}$ ⟶ for Ideal fluid flow

* doesn't exist in nature.



above boundary layer - $\boxed{\frac{du}{dy} = 0}$

Line joining $\boxed{u = 0.99\,u_\infty}$

* For Real fluid above boundary layer

$$\tau = \mu \frac{du}{dy}$$

$$\boxed{\tau = 0}$$

For an Ideal fluid flow →

① $\tau = 0$, There will be no shear stress

$\boxed{\begin{matrix}\tau_P = 0 \\ \tau_B = 0\end{matrix}}$

→ Particle & Boundary shear stress will be zero

→ There will be only normal stress acting of the fluid

Normal stress at a point = Pressure at that point

② 

F – Force acting on the body / solid due to fluid

→ Force will be acting normal (90°) to the surface.

→ $\underbrace{\nabla \cdot \vec{V}}_{\text{vorticity}} = 0$, continuity eq^n for an Ideal fluid

→ $\nabla \times \vec{V} = 0$. Conservative velocity vector fluid

Curl of $\vec{V} = 0$

$\boxed{\vec{V} = \nabla \cdot \phi}$ → Scalar fun^n (potential function)

Potential flow Theory

The branch of mathematics which focus on finding "ϕ"

Necessity cond^n for Irrotational flow

then $\boxed{\nabla \times \vec{V} = 0}$

where, $u = \dfrac{-\partial \phi}{\partial x}$, $w = \dfrac{\partial \phi}{\partial z}$, $v = -\dfrac{\partial \phi}{\partial y}$

* $\boxed{\nabla^2 \phi = 0}$ → Laplace eq^n

→ flow is incompressible
→ flow is steady
→ flow is irrotational

} $\boxed{\text{Potential flow}}$

Important Cases for Potential flow

① uniform flow

→ R, Set of fluid Properties
→ $\vec{V}$, velocity of flow

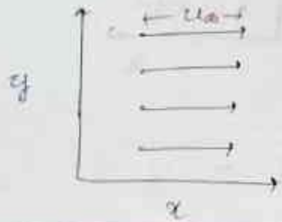
} not function of space

$\boxed{\dfrac{\partial \vec{V}}{\partial space} = 0 \qquad \dfrac{\partial R}{\partial space} = 0}$

ⓘ uniform flow in y-direction

$u = f(x)$, $\boxed{u = \dfrac{-\partial \phi}{\partial x} = -\dfrac{\partial \psi}{\partial y}}$

ϕ → Potential fun^n , ψ = Stream function.



$u = \dfrac{\partial \psi}{\partial y}$

Stream function

$$\boxed{u = u_\infty}$$

$$\frac{\partial \psi}{\partial y} = u \;,\quad \frac{\partial \psi}{\partial y} = u_\infty$$

$$\partial \psi = u_\infty dy$$

by integrating :

$$\boxed{\psi = u_\infty y + C_1}$$

If $y = 0$, $\psi = 0$, $C_1 = 0$

$$\boxed{\psi = u_\infty y}$$

Stream Line

$$\boxed{\psi = \text{const}}$$

Potential funⁿ

$$\frac{\partial \phi}{\partial x} = u$$

$$\frac{\partial \phi}{\partial x} = u_\infty$$

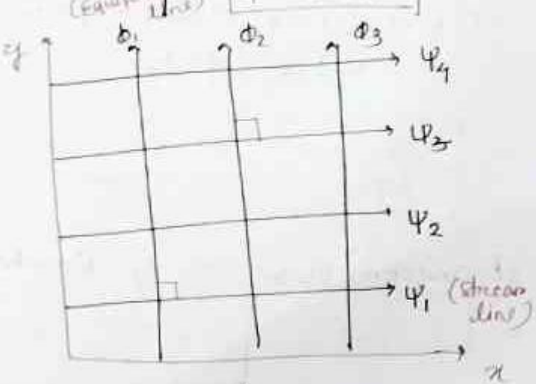
$$\partial \phi = u_\infty x + C_2$$

$$\phi = 0,\ x = 0,\ C_2 = 0$$

$$\boxed{\phi = u_\infty x}$$

EquiPotential Line

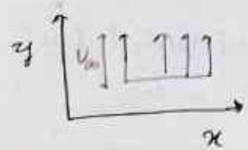
$$\boxed{\phi = \text{const}}$$



(6) Uniform flow along x-direction

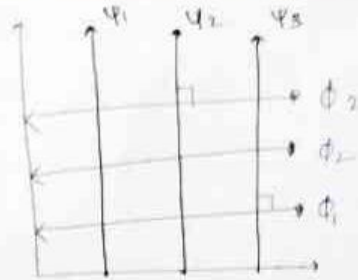
$$v = f(y)$$

$$\boxed{v = -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}}$$



By appling integration, we get

$$\boxed{\psi = V_\infty x} \qquad \boxed{\phi = -V_\infty y}$$



$$\boxed{\phi = -V_\infty y}$$

Flow Net

It is grid like diagram when we plot stream line & Equipotential line for a given fluid flow.

Source flow and Sink flow