

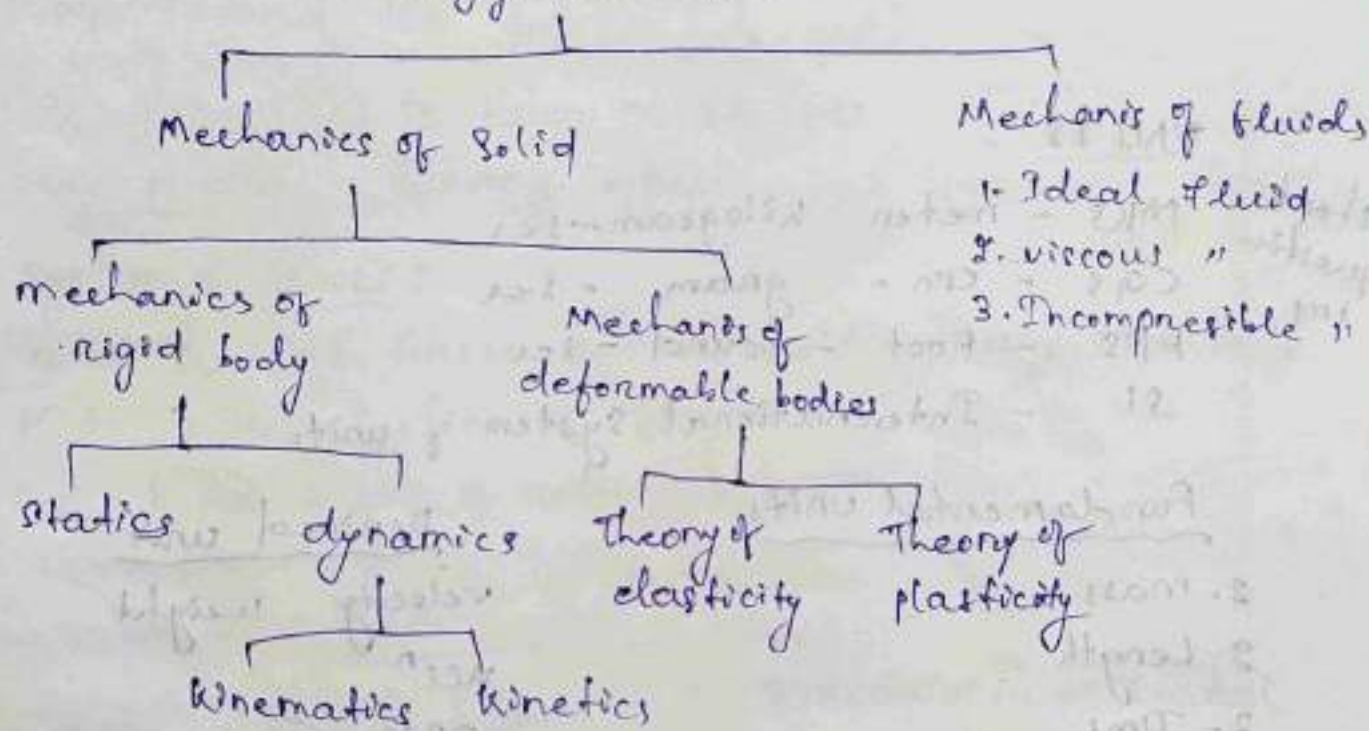
16.11.20

CHAPTER - 1

FUNDAMENTALS OF ENGG MECHANICS

Mechanics may be defined as the branch of Science which deals with the study of effect of forces on material bodies at state of rest or state of motion.

Engg. Mechanics



Statics → It is the branch of mechanics which deal with the forces & their effects on body at rest.

Dynamics → It is the branch of mechanics which deal with the forces & their effects on the body when it is in motion.

Rigid body → A body is said to be rigid if relative position of any two particles in it do not change after application of forces.

Deformable body → If relative positions of particles change after application of forces.



Kinematics → Dynamics dealing with the problems without considering the forces causing the motion of the body is called kinematics.

Kinetics → Dynamics dealing with the problem considering the forces causing the motion of the body is.

Body → An object is an identifiable collection of matter.

UNITS:-

short
question
imp

MKS - meter kilogram - sec

CGS - cm - gram - sec

FPS - Foot - pound - sec

SI - International System of units

Fundamental units

1. mass
2. Length
3. Time
4. Electric current
5. Luminous Intensity
6. Thermodynamic temp
7. Amount of substance

Derived unit

velocity weight

accⁿ
area

Volume

pressure

1.2 Force :- (F)

→ Force is defined as an external agent which change or tends to change the state of rest or state of uniform motion of the body, when it is applied on it.

→ Force is a vector quantity.

→ From Newton's 2nd law $F = ma$

unit of $F = \text{Newton}$

$$1 \text{ N} = 1 \text{ kg} \times 1 \text{ m/s}^2 = 1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

$$1 \text{ N} = 10^5 \text{ dyne}$$

16.11.20 Characteristics of a force:- many question

- magnitude
- point of application
- Line of action
- direction

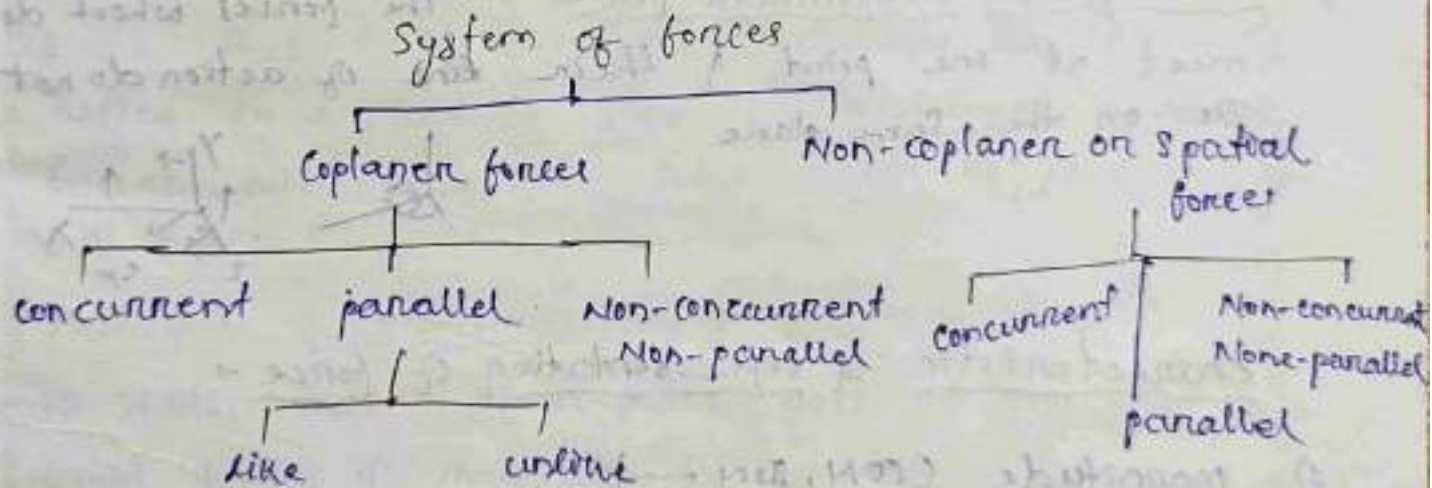
Effect of force:-

- may change the state of the body
- " " " dirⁿ of motion of the body
- May retard the force (de-ac^t)
- May give rise to internal stresses
- May produce turning effect

15 System of forces:-

17.11.20 When several forces act simultaneously on a body, it is called system of forces.

Acc^d to posⁿ of line of action of forces

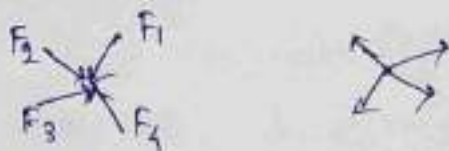


imp 1) Coplanar forces → The forces, whose lines of action lie on the same plane

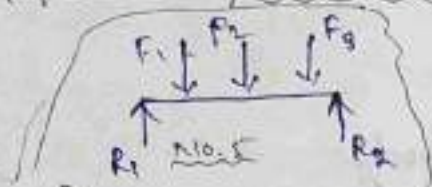
imp 2) Collinear forces → The forces, whose lines of action lie on the same line



3) Concurrent forces → The forces, which meet on one point.

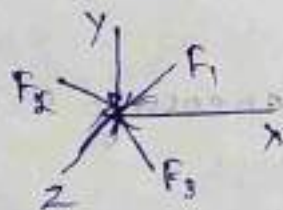


4) Coplanar Concurrent forces → The forces which meet at one point & their lines of action also lie on the same plane.

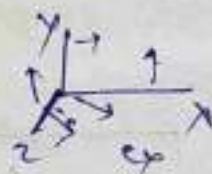


5) Coplanar non-concurrent forces → The forces, which do not meet at one point, but their line of action lie on the same plane.

6) Non-Coplanar concurrent forces → The forces, which meet at one point, but their line of action do not lie on the same plane.



7) Non-Coplanar Non concurrent forces → The forces which do not meet at one point & their line of action do not lie on the same plane.



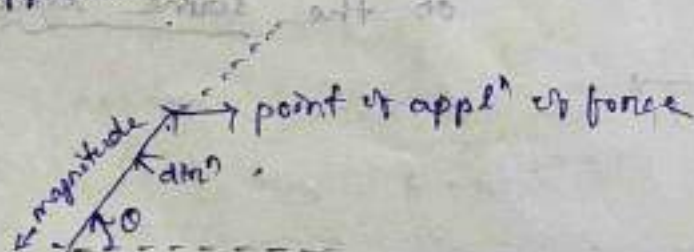
characteristic & representation of force →

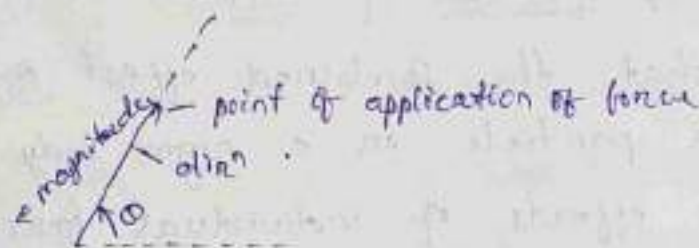
1) magnitude (10N, 20N, ...)

2) Dirⁿ of the force / line of action of the force (30°, 45°)

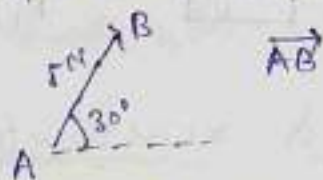
3) Nature of the force (push or pull)

4) Point of application

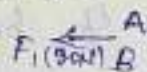




It can be represented by 1) ~~vector~~ vector representation



2) Bow's notation
writing 2 capital letters one on either side of force



* Particle $\rightarrow$ Defined as a body of infinitely small volume & is considered to be concentrated point.

Principle of physical independence of forces :-

states that "if a no. of forces are simultaneously acting on a particle then the resultant of these forces will have the same effect as produced by all forces."

imp Principle of transmissibility of forces :-

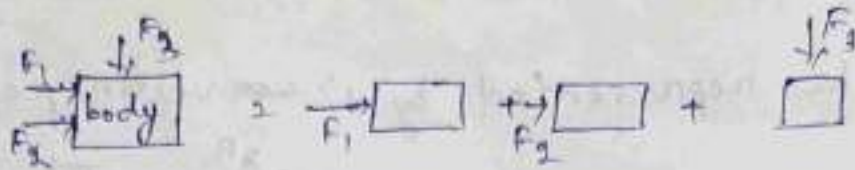
It states that "if a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action."

imp Resultant force $\rightarrow$

if a no. of forces are acting simultaneously on a particle then a single force can be calculated which will replace them. i.e. will produce same effect as produced by all the given forces. This single force is called resultant force.

Principle of superposition of forces

It states that the combined effect of force system acting on a particle or a rigid body is equal to the sum of effects of individual forces.



Action & reaction forces

According to Newton's 3rd law of motion "For every action there is an equal & opposite reaction"

Ex hitting of ball on bat

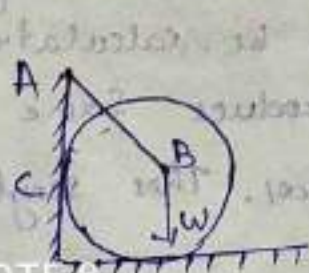
During catching ball ~~from~~ from a height
swimmer swimming

A person pushes against a wall (Action force). & wall exerts equal & opposite force against the person (reaction force)

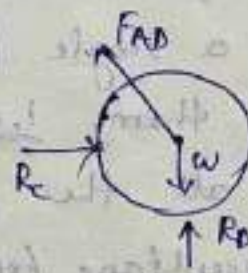
Free Body diagram (FBD)

The diagram of the isolated portion of the structure showing net effects of different types of forces on that portion is called FBD of the portion.

Structure consists of more than one element, portions & supports. In FBD each portion can be isolated individually from the structure & effects of all supports, react can be shown on that particular portion of the structure.



Given diagram



FBD of the sphere

1.3 Resolution of a force :-

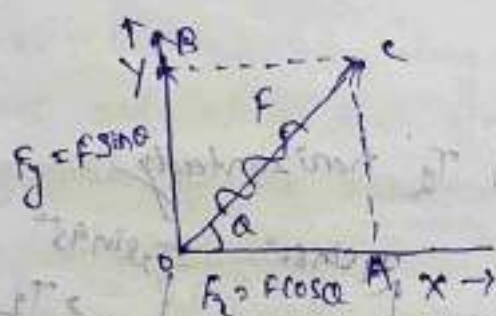
- The process of splitting up of the given force into two components without changing its effect on the body is called resolution of force.

The force which is split into two parts is called as resolved force & the parts are called component force or resolves.

* Generally force is resolved into

- Mutually perpendicular components
- Non perpendicular components

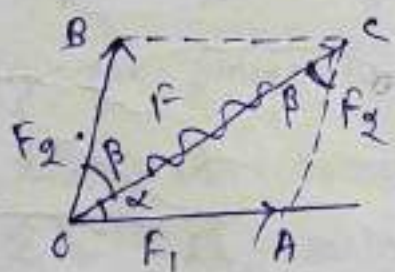
1) Mutually $\perp$ Components



$$F_x = OA = F \cos \alpha$$

$$F_y = OB = F \sin \alpha$$

2) Non $\perp$ components



Anc applying

From sine rule,

$$\frac{OA}{\sin \beta} = \frac{OC}{\sin(180 - (\alpha + \beta))} = \frac{AC}{\sin \alpha}$$

$$\Rightarrow \frac{F_1}{\sin \beta} = \frac{F}{\sin(\alpha + \beta)} = \frac{F_2}{\sin \alpha}$$

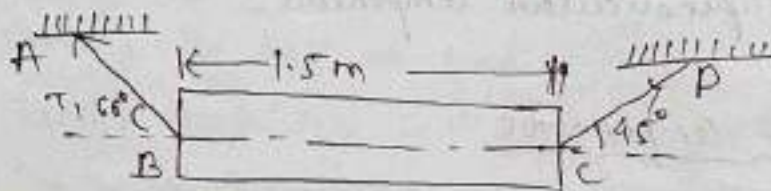
$$\Rightarrow F_1 = \frac{F \sin \beta}{\sin(\alpha + \beta)} \quad \& \quad F_2 = \frac{F \sin \alpha}{\sin(\alpha + \beta)}$$

Principle of resolution of forces →

It states that "The algebraic sum of the resolved parts of a number of forces, in a given dirⁿ, is equal to the resolved parts of their resultant in the same direction."

* In general, force are resolved in vertical & horizontal dirⁿ.

Q A m/c component 1.5m long & weight 1000N is supported by two ropes AB & CD as shown in fig. Calculate the tension T_1 & T_2 in the ropes AB & CD.



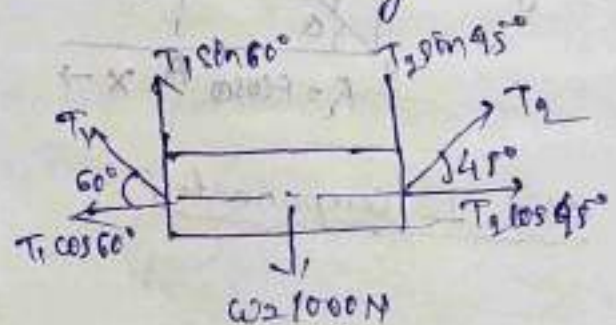
Solⁿ

weight (w) = 1000N

Resolving the forces T_1 & T_2 horizontally

$$T_1 \cos 60^\circ = T_2 \cos 45^\circ$$

$$\Rightarrow \frac{T_1}{\frac{1}{2}} = \frac{T_2}{\frac{1}{\sqrt{2}}} \Rightarrow \frac{T_1}{T_2} = \sqrt{2}$$



Resolving the forces vertically

$$T_1 \sin 60^\circ + T_2 \sin 45^\circ = 1000$$

$$\Rightarrow T_1 \frac{\sqrt{3}}{2} + T_2 \frac{1}{\sqrt{2}} = 1000$$

$$\Rightarrow 1.93 T_2 = 1000$$

$$\Rightarrow T_2 = 518.1 \text{ N}$$

$$\Rightarrow T_1 = \sqrt{2} T_2 = 732.6 \text{ N}$$

Method of resolution of forces (to find resultant of forces) :-

1. Resolve all the forces horizontally & find algebraic sum of all the horizontal component (i.e. ΣF_H)
2. Resolve all the forces vertically & find the algebraic sum of all the vertical components (i.e. ΣF_V)
3. Resultant force (R)

$$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2}$$

It will inclined at an angle θ with horizontal.

$$\tan \theta = \frac{\Sigma F_V}{\Sigma F_H}$$

Q-2 A triangle ABC has its side AB = 40 mm along the x-axis & side BC = 30 mm along the y-axis, 3 forces of 40 N, 50 N & 30 N act along the side AB, BC & CA respectively. Determine magnitude of the resultant of such system of forces.

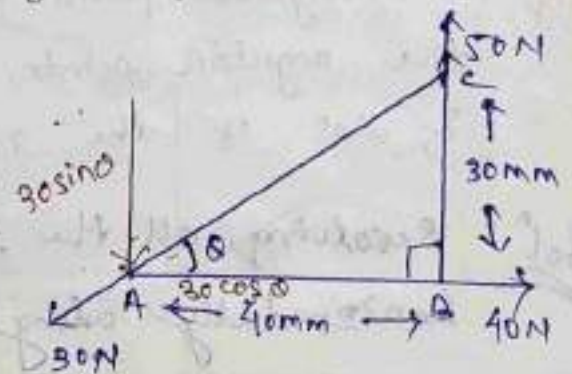
Solⁿ

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{40^2 + 30^2} = 50 \text{ mm}$$

$$\sin \theta = \frac{P}{h} = \frac{BC}{AC} = \frac{30}{50} = 0.6$$

$$\cos \theta = \frac{b}{h} = \frac{AB}{AC} = \frac{40}{50} = 0.8$$



Resolving the forces horizontally (along AB) & their sum is

$$\Sigma F_H = 40 - 30 \cos \theta = 40 - 30(0.8) = 16 \text{ N}$$

$$\Sigma F_V = 50 - 30 \sin \theta = 50 - 30(0.6) = 32 \text{ N}$$

$$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2}$$

$$= \sqrt{16^2 + 32^2} = 35.8 \text{ N}$$

$$\theta = \tan^{-1} \frac{P}{b} \\ = \tan^{-1} \left(\frac{32}{16} \right)$$

$$\tan \theta = \frac{\Sigma F_V}{\Sigma F_H} \Rightarrow \theta = \tan^{-1} \frac{32}{16}$$

Q.3 A system of forces are acting at the corners of a rectangular block as shown as fig. Determine the magnitude & dirⁿ of the resultant force.

Solⁿ $\Sigma F_H = 25 - 20 = 5 \text{ kN}$

$\Sigma F_V = -50 - 35 = -85 \text{ kN}$

$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2}$

$= \sqrt{5^2 + (-85)^2}$

$= 85.15 \text{ kN}$

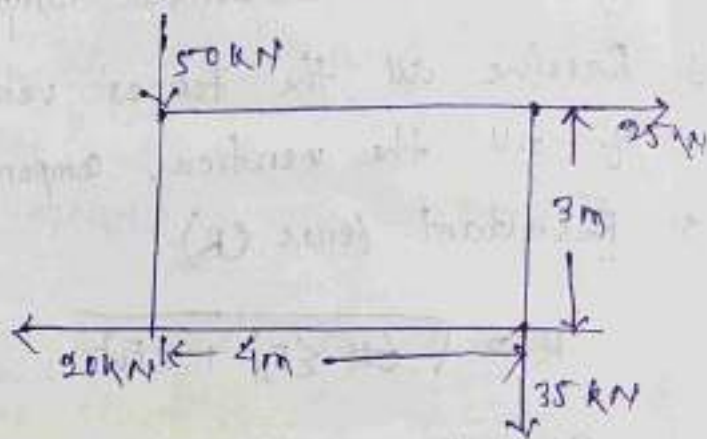
Now $\tan \theta = \frac{\Sigma F_V}{\Sigma F_H} = \frac{-85}{5} = -17$

$\Rightarrow \theta = \tan^{-1}(-17) = 86.6^\circ$

$\theta = 360 - 86.6 = 273.4^\circ$

ΣF_V is -ve

R will lie in 4th quadrant
i.e. 270° to 360°



Q.4 The forces 20N, 30N, 40N, 50N & 60N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude & dirⁿ of the resultant force.

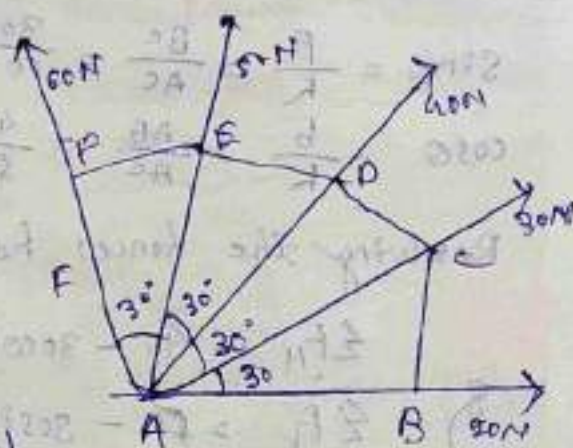
Solⁿ Resolving all the forces horizontally along AB

$\Sigma H \Sigma F_H = 20\text{N} + 30\cos 30^\circ + 40\cos 60^\circ$

$+ 50\cos 90^\circ + 60\cos 120^\circ$

$= 20 + 30(0.866) + 40(0.5)$

$+ 0 - 60(0.3)$



$\frac{1}{2} \times 20 = 0 \Rightarrow 36$

$\left(\frac{0.2}{15}\right) \times 20 =$

$\frac{0.22}{21}$

11.328

$120^\circ = 0^\circ$

$\frac{0.22}{21}$

120°

120°

120°

120°

120°

Resolving all the forces vertically along AE

$$\begin{aligned}\sum F_v &= 20 \sin 0^\circ + 30 \sin 30^\circ + 40 \sin 60^\circ + 50 \sin 90^\circ + 60 \sin 120^\circ \\ &= 0 + 30(0.5) + 40(0.866) + 50 + 60(0.866) \\ &= 151.6 \text{ N}\end{aligned}$$

$$R = \sqrt{\sum F_H^2 + \sum F_v^2}$$

$$= \sqrt{36^2 + 151.6^2} = 155.8 \text{ N}$$

$$\theta = \tan^{-1} \frac{\sum F_v}{\sum F_H} = \tan^{-1} \frac{151.6}{36} = 76.6^\circ$$

Q.5 The following forces act at a point

i) 20N inclined at 30° towards North-east

ii) 25N towards North

iii) 30N towards ~~west~~ west

iv) 35N inclined at 40° towards south-west



Solⁿ Resolving all the forces horizontally & vertically

$$\begin{aligned}\sum F_H &= 20 \cos 30^\circ - 30 - 35 \cos 40^\circ \\ &= -30.7 \text{ N}\end{aligned}$$

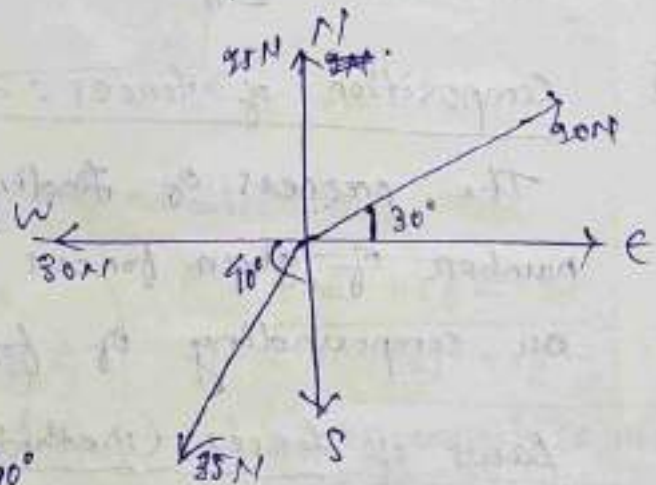
$$\sum F_v = 20 \sin 30^\circ + 25 - 35 \sin 40^\circ$$

$$= 33.7 \text{ N}$$

$$R = \sqrt{\sum F_H^2 + \sum F_v^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6 \text{ N}$$

$$\theta = \tan^{-1} \frac{\sum F_v}{\sum F_H} = \tan^{-1} \frac{33.7}{-30.7} = 47.7^\circ$$

As $\sum F_H$ is -ve, so R will be in 2nd quadrant $\theta = 180^\circ - 47.7^\circ = 132.3^\circ$



Q.6 A horizontal line $PQRS$ is 12m long, where $PQ = QR = RS = 4m$. Forces of 1000N, 1500N, 1000N & 500N act at P, Q, R & S respectively with downward dirⁿ. The lines of action of these forces make angle of 90° , 60° , 45° & 30° respectively with PS. Find the magnitude, dirⁿ?

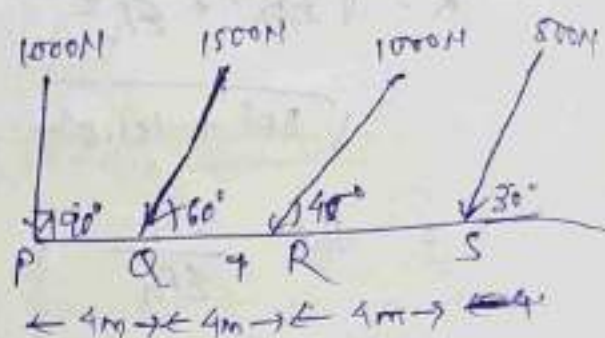
Solⁿ Resolving the forces in horizontally & vertical dirⁿ

$$\begin{aligned}\Sigma F_H &= -1000 \cos 90^\circ - 1500 \cos 60^\circ \\ &\quad - 1000 \cos 45^\circ - 500 \cos 30^\circ \\ &= -1890 \text{ N}\end{aligned}$$

$$\begin{aligned}\Sigma F_V &= -1000 - 1500 \sin 60^\circ - 1000 \sin 45^\circ - 500 \sin 30^\circ \\ &= -3956 \text{ N}\end{aligned}$$

$$R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2} = \sqrt{(-1890)^2 + (-3956)^2} = 3765 \text{ N}$$

$$\alpha = \tan^{-1} \frac{\Sigma F_V}{\Sigma F_H} = \tan^{-1} \frac{-3956}{-1890} = 59.8^\circ$$



1.6

Composition of forces :-

The process of finding the resultant force of a number of given forces is called composition of forces or compounding of forces.

Laws of forces (Methods of composition of forces)

The resultant force of a given system of forces can be found by

1) Parallelogram law of forces

2) Triangle " " " "

3) Polygon " " " "

Methods used to find resultant forces:-

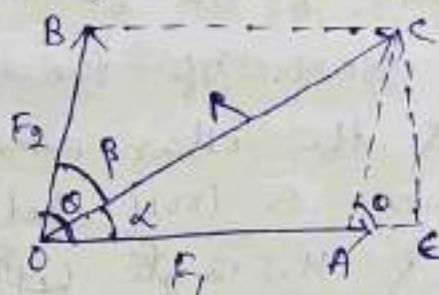
- 1) Analytical method
- 2) Graphical method (vector method)

Analytical method:-

- 1) Parallelogram law of forces
- 2) Method of resolution of forces

Parallelogram law of forces:-

It states that, "If two forces are acting simultaneously on a particle; are represented in magnitude & dirⁿ by the two adjacent sides of a parallelogram, then their resultant can be represented in magnitude & dirⁿ by the diagonal of the parallelogram which passes through their point of intersection"



Consider two forces F_1 & F_2 acting at point O

R = Resultant of F_1 & F_2

θ = Angle betⁿ force F_1 & F_2

α = " " " " F_1 & R

β = " " " " F_2 & R

$$R = \sqrt{OE^2 + CE^2}$$

$$= \sqrt{(OA + AE)^2 + CE^2}$$

$$= \sqrt{(F_1 + F_2 \cos \theta)^2 + (F_2 \sin \theta)^2}$$

$$= \sqrt{(F_1 + F_2 \cos \theta)^2 + (F_2 \sin \theta)^2}$$

$$= \sqrt{F_1^2 + 2F_1F_2 \cos \theta + F_2^2 \cos^2 \theta + F_2^2 \sin^2 \theta}$$

$$= \sqrt{F_1^2 + 2F_1F_2 \cos \theta + F_2^2}$$

Now according to parallelogram

law of cosines $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$

$$\tan \alpha = \frac{CE}{OE} = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

Particular cases

- i) when $\theta = 0^\circ$, $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2} = \sqrt{(F_1 + F_2)^2} = F_1 + F_2$
ii) when $\theta = 90^\circ$, $R = \sqrt{F_1^2 + F_2^2 + 0} = \sqrt{F_1^2 + F_2^2}$
iii) when $\theta = 180^\circ$, $R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2} = \sqrt{(F_1 - F_2)^2} = F_1 - F_2$

Q-1 Two forces of 1000 N & 150 N are acting simultaneously at a point find the resultant, if angle betⁿ the two forces is 45°

Solⁿ $F_1 = 1000 \text{ N}$ & $F_2 = 150 \text{ N}$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

$$= \sqrt{(1000)^2 + 150^2 + 2(1000)(150) \cos 45^\circ}$$

$$= 232 \text{ N}$$

Q-2 The resultant of two forces one of which is double the other is 160 N. If the dirⁿ of the larger force is reversed & the other remains unaltered, the resultant reduces to 180 N. Determine the magnitude of the forces & the angle betⁿ the forces.

Solⁿ Let $F_1 = F$ & $F_2 = 2F$

S. $F_2 = 2F$

Case I $R = \sqrt{F^2 + F_2^2 + 2F_1F_2 \cos \theta} = 160$

$$\Rightarrow \sqrt{F^2 + (2F)^2 + 2(F)(2F) \cos \theta} = 160$$

$$\Rightarrow \sqrt{F^2 + 4F^2 + 4F^2 \cos \theta} = 160$$

$$\Rightarrow 5F^2 + 4F^2 \cos \theta = 160^2 \quad \text{--- (1)}$$

Case II

$$R = \sqrt{F^2 + (-F)^2 + 2(F)(-F) \cos \theta} = 180$$

$$= \sqrt{F^2 + F^2 - 2F^2 \cos \theta} = 180$$

$$= 2F^2 - 2F^2 \cos \theta = 180^2 \quad \text{--- (2)}$$

From eqn (1) & (2) $F = 100 \text{ N}$ So $F_1 = 100 \text{ N}$

$$\text{So } F_2 = 2F = 200 \text{ N}$$

$$\text{So } \cos \theta = 0.44 \Rightarrow \theta = 63.9^\circ$$

Q.2 Two forces act at an angle of 130° . The bigger force is of 40 N & the resultant is $\perp$ to the smaller one. Find the smaller force.

Solⁿ

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

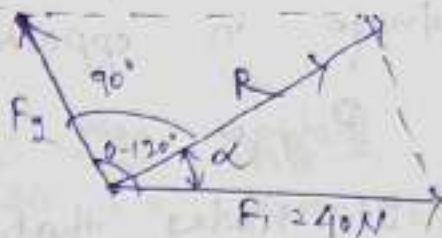
$$= \sqrt{40^2 + F_2^2 + 2(40)F_2 \cos 130^\circ}$$

$$\alpha = 130^\circ - 90^\circ = 40^\circ$$

$$\tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$\tan 40^\circ \Rightarrow \frac{F_2 \sin 130^\circ}{40 + F_2 \cos 130^\circ}$$

$$F_2 = 20$$



Q.4 Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10} \text{ N}$. But if they act at 60° their resultant is $\sqrt{13} \text{ N}$.

Solⁿ

$$\sqrt{F_1^2 + F_2^2} = \sqrt{10} \Rightarrow F_1^2 + F_2^2 = 10$$

$$\text{So } \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos 60^\circ} = \sqrt{13}$$

$$\Rightarrow F_1^2 + F_2^2 + F_1F_2 = 13$$

$$\Rightarrow 10 + F_1F_2 = 13 \Rightarrow F_1F_2 = 3$$

we know that $(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2$

$$\Rightarrow (F_1 + F_2)^2 = 10 + (2 \times 3) = 16 \Rightarrow F_1 + F_2 = 4 \quad \text{--- (1)}$$

Again $(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2$

$$= 10 - (2 \times 3) = 4 \Rightarrow F_1 - F_2 = 2 \quad \text{--- (2)}$$

From eqn (1) & (2) $2F_1 = 6 \Rightarrow F_1 = 3 \text{ N}$

$$\text{So } F_2 = 4 - 3 = 1 \text{ N}$$

Triangle law of forces - (imp)

It states that "if two forces are acting simultaneously on a particle are represented in magnitude & dirⁿ by the two sides of a triangle taken in order; then their resultant can be represented in magnitude & dirⁿ by the 3rd side of the triangle taken in opposite order."

Polygon law of forces - (imp)

It states that "if a no. of forces are acting simultaneously on a particle, are represented in magnitude & dirⁿ by the side of a polygon taken in order, then their resultant can be represented in magnitude & dirⁿ, by the the closing side of the polygon taken in opposite order."

Graphical method to find resultant force:-

Step 1 Construction of space diagram (position diagram)

Here various forces with its magnitude & dirⁿ are shown

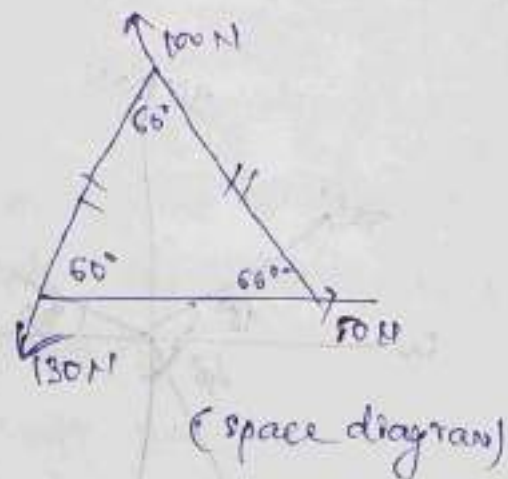
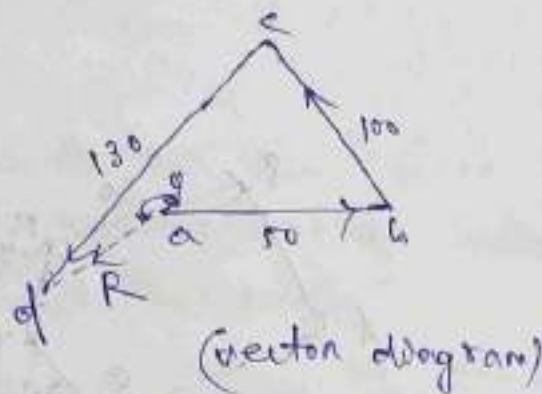
Step 2 Construction of vector diagram (force system)

Here start from a point & go on adding all forces vectorially one by one (by considering dirⁿ) with some suitable scale.

Now closing side of polygon, taken in opposite order will show resultant force.

Q.1 A particle is acted upon 3 forces equal to 50N, 100N & 130N along the 3 side of an equilateral Δ taken in order find graphically the magnitude & dirⁿ of the resultant force.

8/1

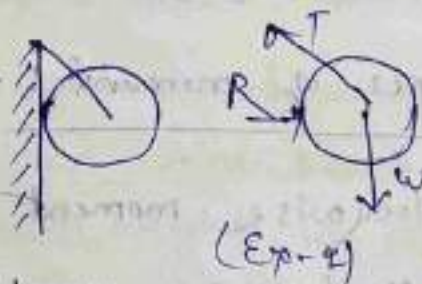
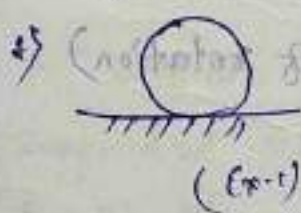


Procedure

- Take reference point 'a' with suitable scale place at 20 cm (draw parallel to 50 N force in space diagram)
- Draw $bc = 100\text{ N}$ with same scale.
- Draw $cd = 130\text{ N}$ " " " "
- Join ad measure & convert to scale $R = 70\text{ N}$ with $\alpha = 53^\circ$

FBD

Diagram in which body under consideration is freed from all contact surface & is shown with all forces on it (including self weight & reaction from other contact surface) is called FBD.

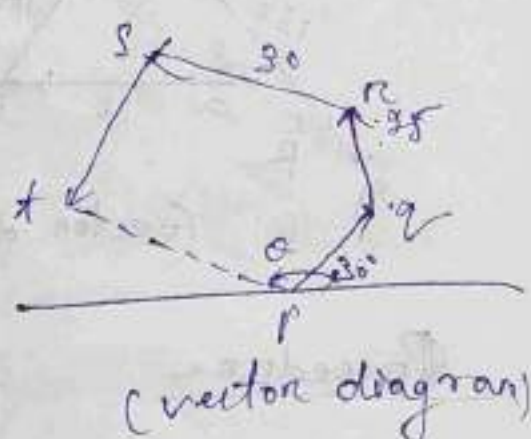
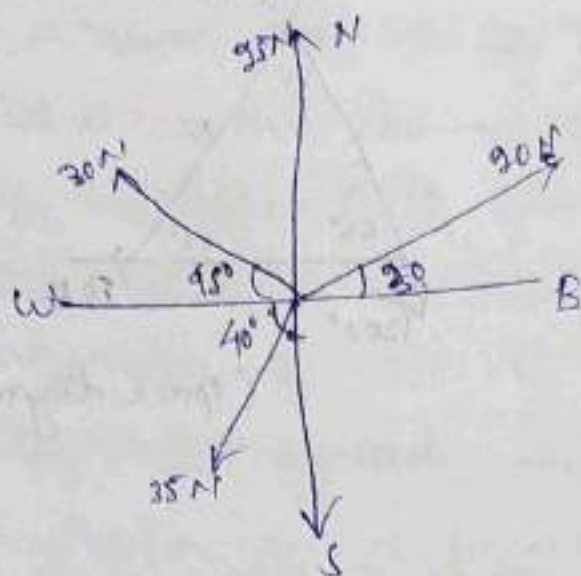


Q.2 The following forces are act at a point

- i) 20 N inclined at 30° towards North of East
- ii) 25 N towards North
- iii) 30 N towards North (west)
- iv) 35 N inclined at 40° towards South of west

Find magnitude & dirⁿ of the resultant force.

Soln



(Space diagram)

1.4 Moment of force (M)

It is the turning effect produced by a force, on the body on which it acts.

The moment of a force is equal to the product of the force & the $\perp$ distance of the point, about which the moment is required.

Mathematically, $M = F \times L$

Unit of moment $\rightarrow$ Nm

Types of moment $\rightarrow$ (Acc to dir of rotation)

1) clockwise moment $\rightarrow$

2) Anticlockwise moment $\rightarrow$

Sign convention

$\downarrow (+)$ & $\uparrow (-)$ on $\downarrow (+)$ & $\uparrow (-)$

Laws of moment :-

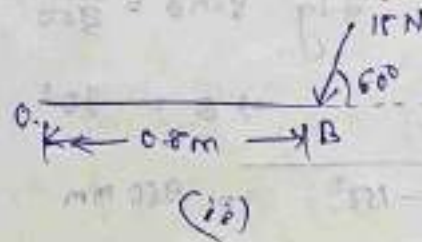
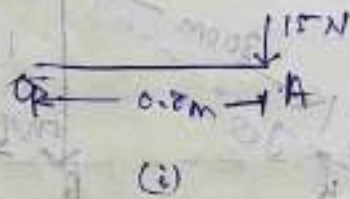
It states that, "If a body is in rotational eqⁿ under the action of a no. of forces, the sum of the clockwise moments of the force about any point is equal to the anti-clockwise moment of the ~~the~~ force about the same point."

$$\sum \downarrow M = \sum \uparrow M$$

Varignon's theorem of moments :- (Imp)

It states that, "If a number of co-planar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to their resultant force about the same point."

Q. Find moment about 'O' of the following figure.



Sol

From fig. (i) $M = 15 \times 0.8 = 12 \text{ N-m}$

" " (ii) $M = 15 \sin 60^\circ \times 0.8 = 13 \times 0.8 = 10.4 \text{ N-m}$

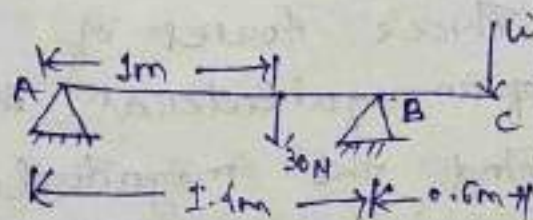
Q-2 A uniform plank ABC of weight 30 N & 2 m long is supported at one end A & at point B 1.4 m from A as shown in fig. Find max^m weight W, that can be placed at C so that the plank does not topple.

Sol

$$\sum M_B = 0$$

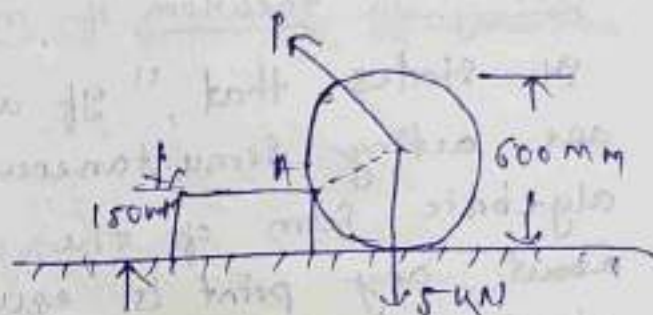
$$\Rightarrow 30 \times 0.4 = W \times 0.6$$

$$\Rightarrow W = 20 \text{ N}$$



Q.3

A uniform wheel of 600mm dia. weighing 5kN rests against a rigid rectangular block of 150mm height as shown in fig. Find the least pull, through the Centre of the wheel required just to turn the wheel over the corner A of the block. Also find the reaction on the block. Take all the surfaces to be smooth.



For the least pull, it must be applied normal to AB

From the fig $\sin \theta = \frac{150}{300}$

$\Rightarrow \theta = 30^\circ$

$AB = \sqrt{300^2 - 150^2} = 260 \text{ mm}$

$\sum M_A = 0$

$\Rightarrow 5(260) = P \times 300 \Rightarrow P = \frac{1300}{300} = 4.33 \text{ kN}$

$R = R_{\text{reaction on the block}}$

Resolving the forces horizontally

$R \cos 30^\circ = P \sin 30^\circ$

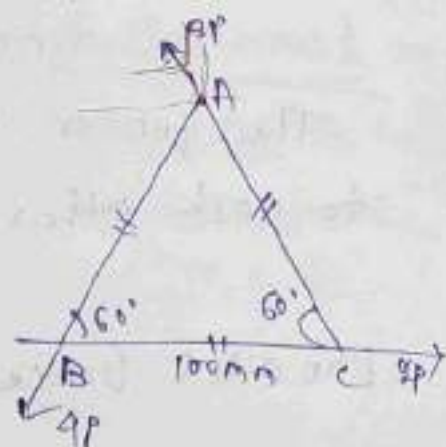
$\Rightarrow R = 2.5 \text{ kN}$

Q.4 Three forces of 2p, 3p, & 4p act along the 3 sides of an equilateral triangle of side 100mm taken in order. Find the magnitude & position of the resultant forces.

Solⁿ Resolving all forces

$$\begin{aligned}\sum F_H &= 3p + 3p \cos 120^\circ - 4p \cos 60^\circ \\ &= 3p - \frac{3}{2}p - 2p = -1.5p\end{aligned}$$

$$\begin{aligned}\sum F_V &= 3p \sin 60^\circ - 4p \sin 60^\circ \\ &= -0.866p\end{aligned}$$



$$R = \sqrt{\sum F_H^2 + \sum F_V^2} = \sqrt{(-1.5p)^2 + (-0.866p)^2} = 1.732p$$

posⁿ of resultant

Let x = distance betⁿ B & line of action of resultant using varignon's theorem

$$\sum M_B = 0$$

$$1.732p \cdot x = 3p \times 100 \sin 60^\circ$$

$$\Rightarrow x = 150 \text{ mm}$$

Q.5 Four forces equal to $p, 3p, 3p$ & $4p$ are respectively acting along the four sides of square ABCD taken in order. Find the magnitude, dirⁿ & posⁿ of the resultant force.

Solⁿ Resolving all forces horizontally & vertically

$$\sum F_H = p - 3p = -2p$$

$$\sum F_V = 3p - 4p = -p$$



$$R = \sqrt{\sum F_H^2 + \sum F_V^2} = \sqrt{(-2p)^2 + (-p)^2} = 2.236p$$

$$\alpha = \tan^{-1} \frac{\sum F_V}{\sum F_H} = \tan^{-1} \frac{-p}{-2p} = 45^\circ$$

As $\sum F_H$ & $\sum F_V$ both are -ve, so lie in 3rd quadrant

$$\alpha \text{ of resultant} = 180^\circ + 45^\circ = 225^\circ$$

Posⁿ Let x = distance betⁿ A & line of action of resultant forces applying varignon's law

$$\begin{aligned}2.236p \cdot x &= (3p \times a) + (3p \times a) \\ 2.236p \cdot x &= 5p \times a \Rightarrow x = \frac{5a}{2.236}\end{aligned}$$

Parallel Forces

The forces whose lines of action are parallel to each other are called parallel forces.

Parallel forces $\begin{cases} \text{like i.e. } \uparrow \uparrow \uparrow \uparrow \text{ (line of act same about all forces)} \\ \text{unlike i.e. } \downarrow \downarrow \uparrow \uparrow \text{ (not same)} \end{cases}$

Methods to find magnitude & posⁿ of the resultant of parallel forces $\rightarrow$

- 1) Analytical method
- 2) Graphical method
- 3) Analytical method

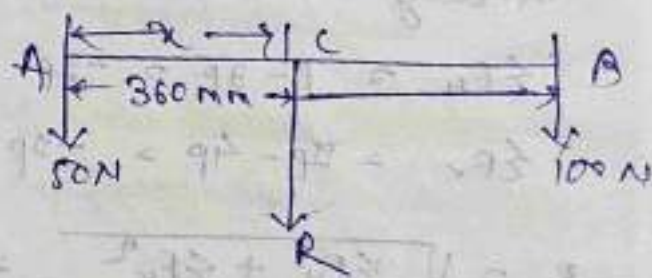
Here $\sum M_A = \sum M$ about a point

Q Two like parallel forces of 50 N & 100 N act at the ends of a rod 360 mm long. Find magnitude of resultant forces & the point where it acts.

Solⁿ As force are like & parallel $R = 50 + 100 = 150 \text{ N}$

Let x = distance betⁿ

line of action of R & point A



Now, $\sum M_A = 0$ or $\sum M_A = \sum M$ about C

$$50 \times 360 = 100 \times (360 - x)$$

$$x = 240 \text{ mm}$$

Q Two unlike parallel forces of magnitude 400 N & 100 N are acting in such a way that their lines of action are 150 mm apart. Determine the magnitude of the resultant force & the point at which it acts.

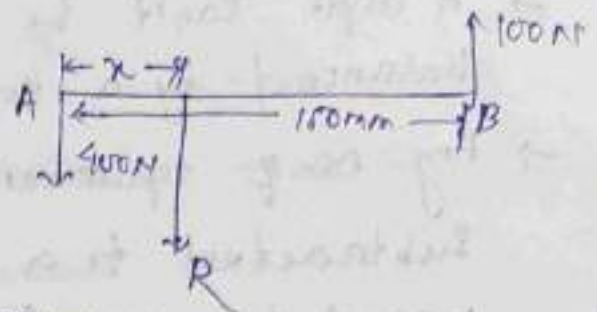
Solⁿ $R = 100 - 400 = -300 \text{ N}$

$\sum M_A = \sum M_B$ about A

$Rx = 100 \times 150$

$\Rightarrow (-300 \times x) = 100 \times 150$

$\Rightarrow x = \frac{-15000}{-300} = -50 \text{ mm}$



COUPLE (Cmp)

A pair of two equal & unlike parallel forces (i.e. force equal in magnitude, with line of action parallel to each other & acting in opposite dirⁿ) is called couple.

Arm of couple

$\perp$ distance betⁿ the lines of action of two equal & opposite parallel forces.

Moment of couple

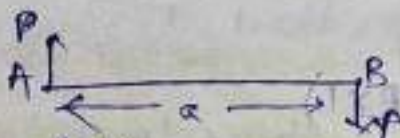
It is the product of either force & $\perp$ distance i.e. arm of couple.

$M = P \times A$

$\hookrightarrow \text{N} \cdot \text{m (unit)}$

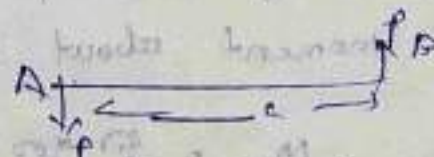


Types of couple



(Clockwise couple)

A couple whose tendency is to rotate the body in $\curvearrowright$ dirⁿ



(Anti-clockwise couple)

A couple whose tendency is to rotate the body in $\curvearrowleft$ dirⁿ

Characteristics of couple :- (imp)

- Algebraic sum of the forces, constituting the couple is zero
- " " " moment, if the forces forming the couple about any point is same & equal to moment of couple if self.
- A couple can't be balanced by a single force but can be balanced by a couple of opposite sense.
- Any no. of coplanar couple can be added or subtracted to a resultant single couple, whose magnitude will be equal to the algebraic sum of the moments of all the compound couples.

Q A square ABCD has forces acting along its sides as shown in fig. Find the value of P & Q, if the system reduces to a couple. Also find magnitude of the couple, if the side of the square is 1m.

Solⁿ

values of P & Q

To reduce the system into a couple, the resultant force in horizontal & vertical dirⁿ must be zero.

$$\sum F_H = 0 \Rightarrow 100 - 100 \cos 45^\circ - P = 0, P = 29.3 \text{ N}$$

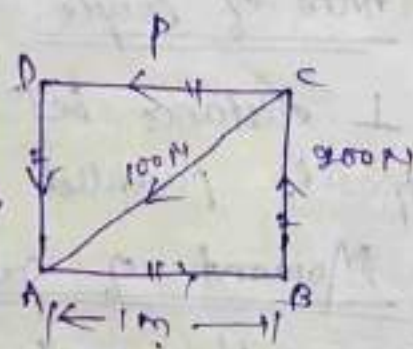
$$\sum F_V = 0 \Rightarrow 900 - Q - 100 \sin 45^\circ = 0 \Rightarrow Q = 929.3 \text{ N}$$

magnitude of the couple

Moment of the couple is equal to the algebraic sum of the moment about any point.

$$M_A = 900 \times 1 + (P \times 1)$$

$$= 900 + 29.3 = 929.3 \text{ N-m}$$



EQUILIBRIUM

If the resultant of all the forces, acting on a stationary ~~rest~~ rigid body is zero, the body is in equilibrium.

For a moving body to be in eqⁿ state, all resultant force as well as resultant moment should be zero.

Difference between resultant & equilibrant →

When body is acted upon by a no. of forces, then the single force which will produce the same effect as produced by the system of forces is called resultant force.

If a force equal, opposite & collinear with the resultant force is acting on the body, then the body will come to eqⁿ state. This force is called equilibrant.
Conditions & eqⁿs of static equilibrium :-

If a no. of coplanar concurrent forces act on a body the body is said to be at rest or in eqⁿ, if the resultant of all the forces acting on the body is zero.
Condⁿs of eqⁿ are

$$\sum F_H = 0 \quad \& \quad \sum F_V = 0$$

Methods for eqⁿ of coplanar forces :-

- 1) Analytical method
- 2) Graphical method
- 3) Analytical method :-

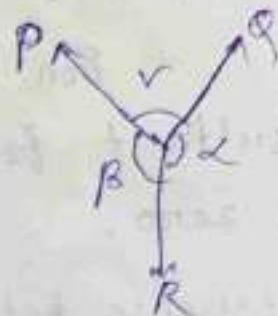
It is studied by Lami's theorem.

LAMI'S THEOREM:-

It states that, "If three coplanar forces acting at a point are in eqn then each force is proportional to the sine of the angle between the other two."

Mathematically

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$



Proof

Consider 3 coplanar forces P, Q & R acting at point O.

α, β & γ are the opposite angles of 3 forces respectively.

Complete the ~~par~~ parallelogram OACB.

Resultant of forces P & Q can be given by diagonal OC.



As forces P, Q & R are in eqn resultant of P & Q will be in line with OP & equal to R, but in opposite dirn.

From the geometry $BC = P$ & $AC = Q$

$$\angle AOC = 180^\circ - \beta$$

$$\angle CAO = 180^\circ - (\angle AOC + \angle ACO)$$

$$= 180^\circ - (180^\circ - \beta + 180^\circ - \gamma)$$

$$= 180^\circ - 180^\circ + \beta - 180^\circ + \gamma$$

$$= \alpha + \beta - 180^\circ$$

$$\Rightarrow \alpha + \beta + \gamma - 180^\circ = 360^\circ$$

$$\Rightarrow \angle CAO + \gamma = 180^\circ$$

$$\Rightarrow \angle CAO = 180^\circ - \gamma$$

Again $\alpha + \beta + \gamma = 360^\circ$

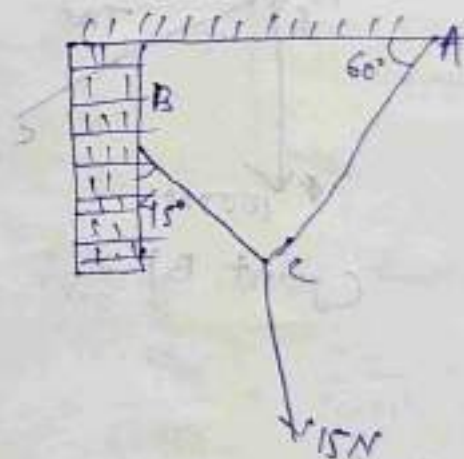
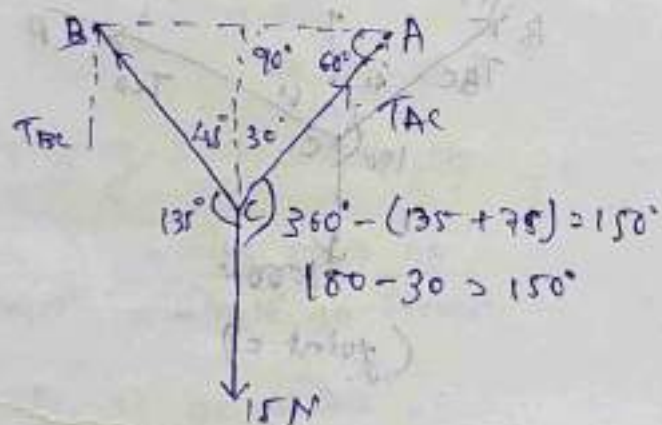
from triangle AOC

$$\frac{OA}{\sin \angle ACO} = \frac{AC}{\sin \angle AOC} = \frac{OC}{\sin \angle CAO}$$

$$\Rightarrow \frac{OA}{\sin(180^\circ - \alpha)} = \frac{AC}{\sin(180^\circ - \beta)} = \frac{OC}{\sin(180^\circ - \gamma)}$$

$$\Rightarrow \left[\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma} \right] \text{ (proved)}$$

Q-1 An electrical light fixture weighing 15N hangs from a point C, by two strings AC & BC. The string AC is inclined at 60° to the horizontal & BC at 45° to the horizontal as shown in fig. Using Lami's theorem, determine the forces in strings AC & BC.



By applying Lami's theorem at point C

$$\frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin 135^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

$$\Rightarrow T_{AC} = \frac{15 \sin 135^\circ}{\sin 75^\circ} = 10.98 \text{ N}$$

$$\& \Rightarrow T_{BC} = \frac{15 \sin 150^\circ}{\sin 75^\circ} = 7.76 \text{ N}$$

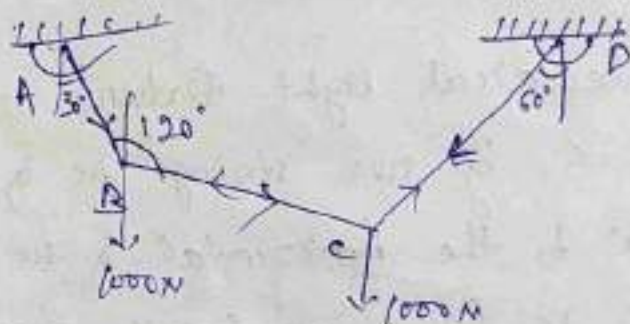
Q-2

A string ABCD, attached to fixed points A & D has two equal weights of 1000N attached to it at B & C. The weights rest with the portions AB & CD inclined at angles as shown in fig. Find the tensions in the portions AB, BC & CD of the string, if the inclination of the portion BC with the vertical is 120° .

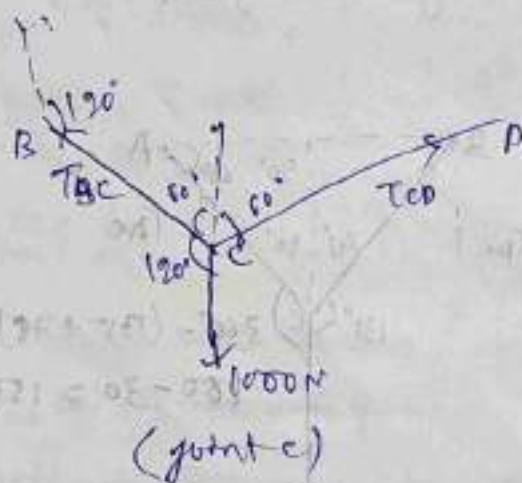
T_{AB} = Tension in string AB

T_{BC} = " " " " BC

T_{CD} = " " " " CD



Sol



Applying Lami's theorem at point B

$$\frac{1000}{\sin 120^\circ} = \frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

$$\Rightarrow T_{AB} = \frac{1000 \sin 60^\circ}{\sin 150^\circ} = 1732\text{N}$$

$$\& T_{BC} = \frac{1000 \sin 150^\circ}{\sin 120^\circ} = 1000\text{N}$$

Applying Lami's theorem at point C

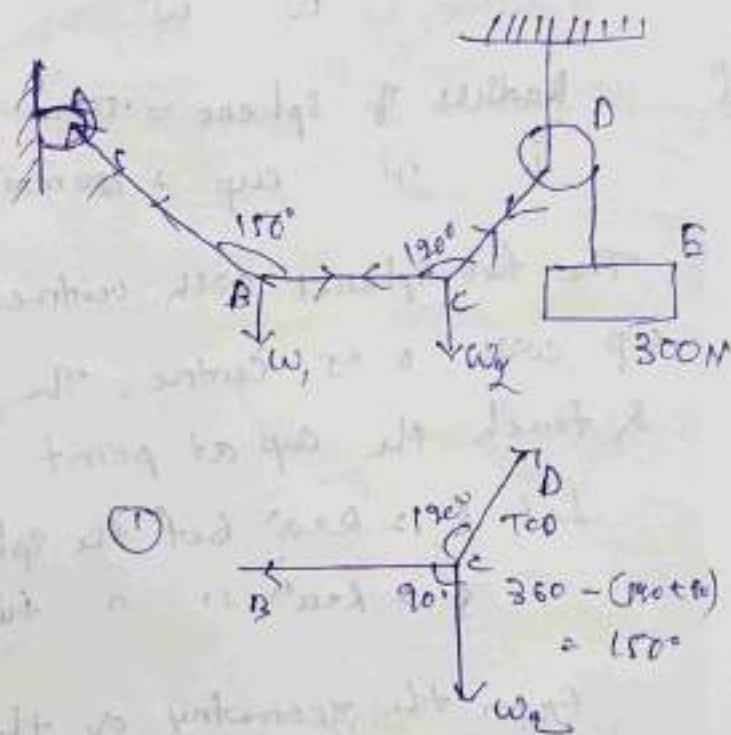
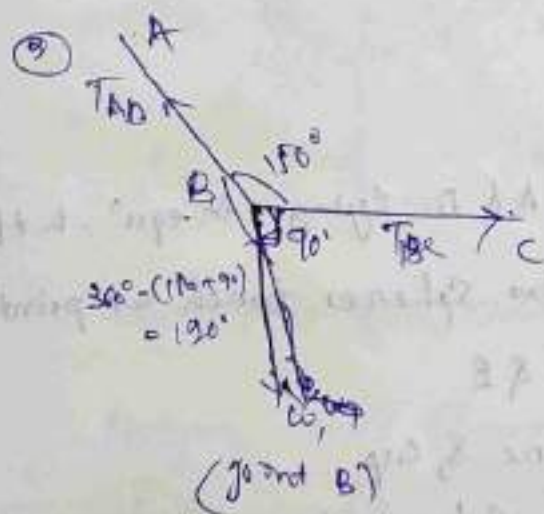
$$\frac{1000}{\sin 120^\circ} = \frac{T_{CD}}{\sin 150^\circ} = \frac{T_{BC}}{\sin 120^\circ}$$

$$\Rightarrow T_{CD} = \frac{1000 \sin 150^\circ}{\sin 120^\circ} = 1000\text{N}$$

Q.3

A light string ABCE whose extremity A is fixed, has weights w_1 & w_2 attached to it at B & C. It passes round a small smooth peg at D carrying a weight of 300 N at the free end E as shown in fig. If in the eq^l posⁿ BC is horizontal & AB & CD makes 150° & 130° with BC, find
 i) Tensions in the portion AB, BC & CD of the string
 ii) magnitudes of w_1 & w_2

Soln



Applying Lami's theorem

$$\frac{w_1}{\sin 150^\circ} = \frac{T_{AB}}{\sin 90^\circ} = \frac{T_{BC}}{\sin 90^\circ}$$

$$\Rightarrow \frac{w_1}{\sin 150^\circ} = \frac{150}{\sin 90^\circ}$$

$$\Rightarrow w_1 = 86.6 \text{ N}$$

~~2.6~~

Applying Lami's theorem

$$\frac{w_2}{\sin 130^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{T_{CD}}{\sin 90^\circ} = \frac{300}{\sin 90^\circ}$$

$$\Rightarrow w_2 = \frac{300 \sin 130^\circ}{\sin 90^\circ} = 300 \sin 130^\circ = 259.8 \text{ N}$$

$$\& T_{BC} = 300 \sin 150^\circ = 150 \text{ N}$$

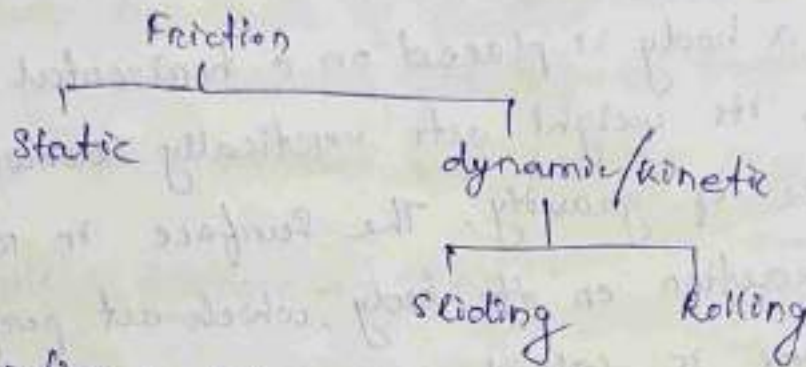
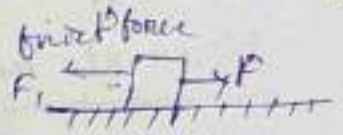
Q.4

Two equal heavy spheres of 10 mm radius are eq^l within a smooth cup of 180 mm radius. show that the reⁿ between the cup & of one sphere is smaller than that between the two spheres.

FRICTION

The opposing force which act in opposite direction of the movement of the body is called force of friction or simply friction. It always acts between two surface in contact.

Types of friction :-



- ✓ Static friction :- It is the friction experienced by a body when it is at rest or when body tends to move.
- ✓ Dynamic friction :- It is the friction experienced by a body when it is in motion.
- ✓ Sliding friction :- It is the friction experienced by a body when it is slide over another body.
- ✓ Rolling friction :- It is the friction experienced by a body when it rolls over another body.

Limiting friction :-

when a small force is applied to a body, it does not move due to frictional force as the applied force is balanced by frictional force. But when the applied force exceeds a limit (beyond which frictional force cannot increase), the force of friction cannot balance the applied force & the body move in the direction of applied force.

The maximum value of frictional force, when the body begins to slide over another body surface when external force is applied is called limiting friction.

when applied force $<$ limiting friction (body remains rest)

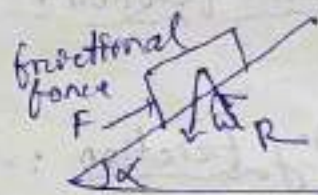
when applied force $>$ limiting friction (body moves)

Normal reaction (R)

When a body is placed on a horizontal or inclined surface is in eqn, its weight acts vertically downward, through its centre of gravity. The surface in return, exerts an upward reaction on the body, which act perpendicular to the plane is called normal reaction.

Angle of friction:

It is the maxⁿ angle of inclination when the body just starts sliding down the plane



or Angle of inclined plane, at which a body just begins to slide down the plane is called angle of friction.

α = angle made by normal react with vertical.

Angle of friction is the angle which the resultant of force of limiting friction & normal reaction makes with the normal react.

Let mass m kept on horizontal plane pulled by a force P . when body is just about to slide. Limiting frictional force act ~~on~~ on opposite side R be the normal react of weight



Let OC = Resultant betⁿ R & P
makes angle Q with R

From ΔOAC , $\tan Q = \frac{OA}{AC} = \frac{P}{R}$

Coefficient of friction (μ)

It is defined as the ratio between limiting friction to the normal reaction between the two bodies.

$$\text{mathematically } \mu = \frac{F}{R} = \tan \phi$$

$$\Rightarrow \boxed{F = \mu R}$$

when ϕ = Angle of friction

F = Limiting friction

R = Normal reaction betⁿ two bodies.

Angle of repose (θ)

Consider a block of weight w resting on an inclined plane which makes an angle θ with the horizontal when θ is very small, the block will rest on the plane. If θ increase gradually, a stage will come when the block start to slide. That angle is called angle of repose.



$$\sum F_v = 0$$

$$\Rightarrow R = w \cos \theta \quad (1)$$

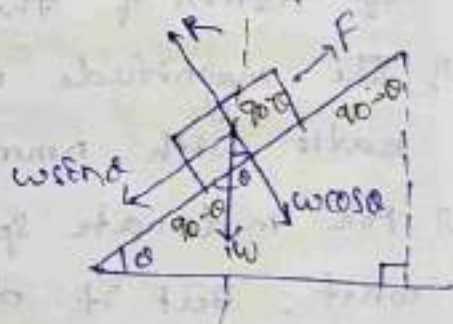
$$\sum F_H = 0 \Rightarrow F = w \sin \theta \quad (2)$$

$$\text{Dividing (2) by (1)} \rightarrow \frac{F}{R} = \frac{w \sin \theta}{w \cos \theta} = \tan \theta$$

$$\Rightarrow \tan \theta = \frac{F}{R}$$

$$\Rightarrow \tan \theta = \tan \phi$$

$$\Rightarrow \boxed{\theta = \phi}$$



Angle of friction = Angle of repose

Laws of friction →

Imp Laws of static friction →

- (i) The force of friction always act opposite to the dirⁿ of applied force.
- (ii) The magnitude of force of friction is exactly equal to the applied force, which tends to move the body.
- (iii) The magnitude of limiting friction maintains a constant ratio to the normal reaⁿ between the two surface

$$\frac{\text{Limiting friction}}{\text{Normal rea}^n} = \frac{F}{R} = \text{const}$$

- (iv) The force of friction is independent of the area of contact between the two surfaces.
- (v) The force of friction depends upon the surface roughness.

Laws of dynamic friction →

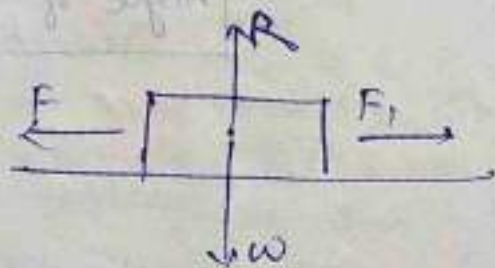
- (i) The force of friction always act opposite to the dirⁿ of motion of the body.
- (ii) The magnitude of kinetic friction maintains a constant ratio with normal reaⁿ betⁿ two surface.
- (iii) For moderate speeds, the force of friction remain remain const. But it decrease slightly with the increase of speed.

Equilibrium of a body on a rough horizontal plane →

For equn of the body

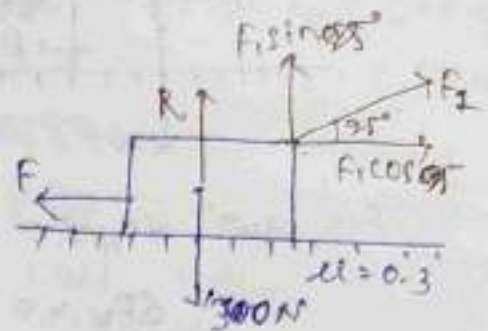
$$F = F_1 \Rightarrow F = \mu R$$

$$\& W = R$$



Q-1 A body of weight 300N is lying on a rough horizontal plane having $\mu = 0.3$. Find the magnitude of the force which can move the body, which acting at an angle of 35° with the horizontal.

Solⁿ Given $W = 300\text{ N}$ $\theta = 35^\circ$
 $\mu = 0.3$
 $F_1 = ?$



For eqnⁿ $\sum F_H = 0 \Rightarrow F = F_1 \cos 35^\circ$

$\sum F_V = 0 \Rightarrow R + F_1 \sin 35^\circ = 300$

$\Rightarrow R = 300 - F_1 \sin 35^\circ$

We know that $F = \mu R$

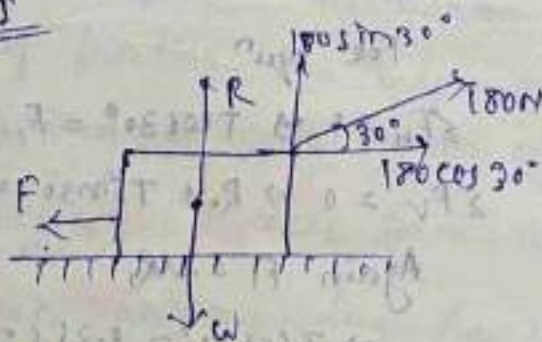
$\Rightarrow F_1 \cos 35^\circ = 0.3 (300 - F_1 \sin 35^\circ)$

$\Rightarrow F_1 = 87.1\text{ N}$

Q-2 A body, resting on a rough horizontal plane, required a pull of 180N inclined at 30° to the plane, just move it. It was found that a push of 990N inclined at 30° to the plane just moved the body. Determine the weight of the body & coefficient of friction.

Solⁿ Given $P = 180\text{ N}$ $P = 990\text{ N}$
 $\theta = 30^\circ$ $\theta = 30^\circ$
 $W = ?$ $W = ?$
 $\mu = ?$ $\mu = ?$

Case 1



(FBD of for case 1)

For eqnⁿ $\sum F_H = 0 \Rightarrow F = 180 \cos 30^\circ$

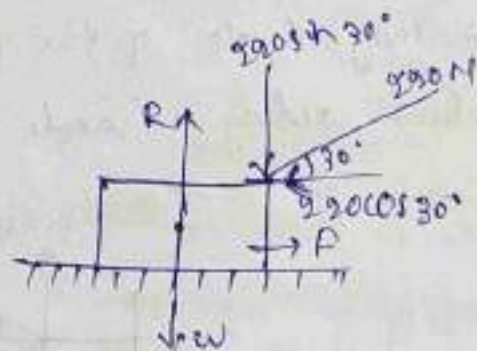
$\sum F_V = 0 \Rightarrow R + 180 \sin 30^\circ = W$

$\Rightarrow R = W - 180 \sin 30^\circ$

We know that $F = \mu R$

$\Rightarrow 180 \cos 30^\circ = \mu (W - 180 \sin 30^\circ)$

Case 2



For eqn $\sum F_H = 0 \Rightarrow F = 990 \cos 30^\circ$

$\sum F_V = 0 \Rightarrow R = W + 990 \sin 30^\circ$

Again we know that $F = \mu R$

$\Rightarrow 990 \cos 30^\circ = \mu (W + 990 \sin 30^\circ) \quad \text{--- (1)}$

Now eqn (1) $\div$ eqn (1) $\Rightarrow \frac{180}{220} = \frac{W - 180 \sin 30^\circ}{W + 990 \sin 30^\circ}$

$\Rightarrow W = 990 \text{ N}$

Putting the value of W in eqn (1)

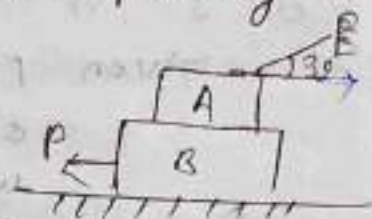
$180 \cos 30^\circ = \mu (990 - 180 \sin 30^\circ) \Rightarrow \mu = 0.173 \text{ (Ans)}$

Q.3

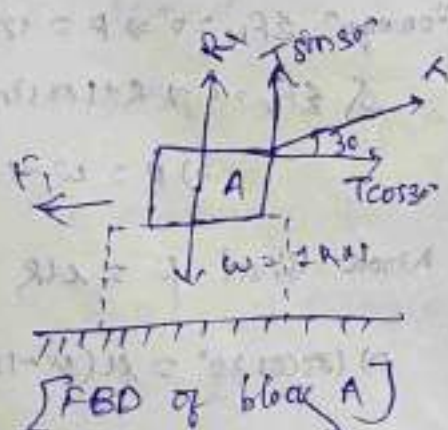
Two blocks A & B of weights 1kN & 2kN respectively are in eqn position as shown in the fig.

If coeff. of friction between the two blocks as well as block B & floor

is 0.3 Find the force (P) required to move the block B.



Sol



For eqn

$\sum F_H = 0 \Rightarrow T \cos 30^\circ = F_1$

$\sum F_V = 0 \Rightarrow R_1 + T \sin 30^\circ = W$

Again $F_1 = \mu R_1$

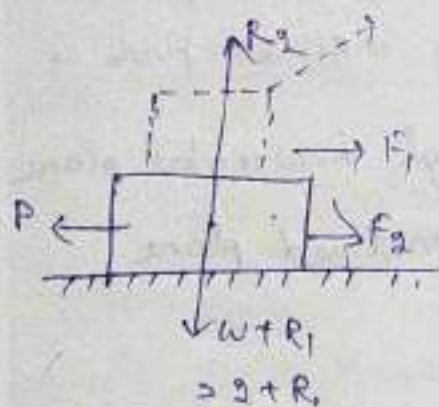
$\Rightarrow T \cos 30^\circ = 0.3 (1 - T \sin 30^\circ)$

$\Rightarrow T = 0.395$

Now $R_1 = W - T \sin 30^\circ$

$F_1 = 1 - T \sin 30^\circ = 0.05 \text{ kN}$

$$\text{So, } F_1 = \mu R_1 = 0.3 \times 0.85 = 0.255 \text{ KN}$$



[FBD of block B]

For eqnⁿ

$$\sum F_H = 0 \Rightarrow P = F_1 + F_2$$

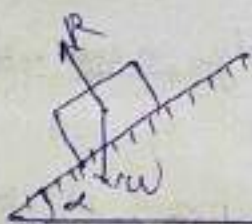
$$\Delta \sum F_V = 0 \Rightarrow R_2 = 2 + R_1 = R_2 = 2 + 0.85 = 2.85 \text{ KN}$$

$$\text{Again } F_2 = \mu R_2 = 0.3 \times 2.85 = 0.855 \text{ KN}$$

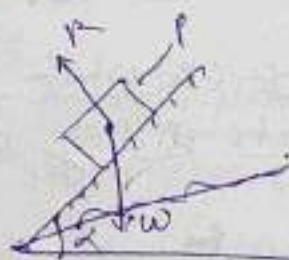
$$\begin{aligned} \text{So } P &= F_1 + F_2 \\ &= 0.255 + 0.855 \\ &= 1.11 \text{ KN (Ans)} \end{aligned}$$

Equilibrium of a body on a rough inclined plane:

Consider a body of weight w , lying on a rough plane inclined at α with the horizontal.



Angle of inclination (α)
less than angle of friction (ϕ)



Angle of inclination more
than angle of friction

If inclination of plane with horizontal $<$ angle of friction, the body will be automatically in eqnⁿ. So to move the body either in upward or downward dirⁿ, force is required to be applied.

If the inclination of plane with horizontal $>$ angle of friction, the body will move down. So an upward force is required to oppose motion of the body in downward dirⁿ.

Majorly 3 cases for movement of body & there

- Force acting along the inclined plane
- Force acting horizontally
- Force acting at some angle with the inclined plane

Equilibrium of a body on a rough inclined plane
Subjected to a force acting along the inclined plane

Consider a body resting on a rough horizontal plane
Subjected to force acting along the inclined plane

Let W = weight of the body

α = Angle of inclined plane with horizontal

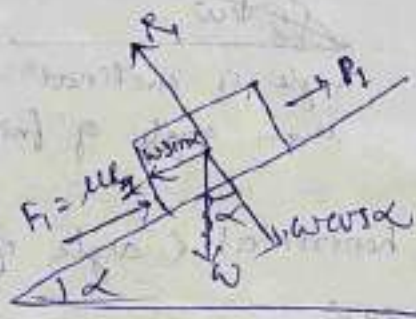
R = Normal Reaction

μ = coeff. of friction

ϕ = Angle of friction $\mu = \tan \phi$

If force is not applied, the body will move downward
There will be two cases

Case 1 Min^m force (P_1) which will keep the body in equl. when
it starts sliding downwards:-



F_1 will act in upward dirⁿ
as body will always move
in $\downarrow$ direction if P_1 is not
applied to it.

For equl $\Sigma F_H = 0 \Rightarrow P_1 + F_1 = W \sin \alpha$

$\Rightarrow P_1 = W \sin \alpha - F_1 \quad \text{--- (i)}$

$\Sigma F_V = 0 \Rightarrow R = W \cos \alpha \quad \text{--- (ii)}$

Now equl (i) $\Rightarrow P_1 = W \sin \alpha - \mu (W \cos \alpha)$

$= W (\sin \alpha - \mu \cos \alpha)$

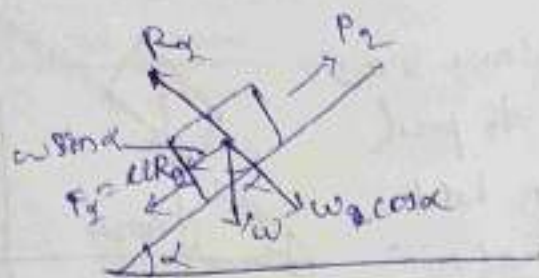
$= W (\sin \alpha - \tan \phi \cos \alpha)$

$\Rightarrow P_1 \cos \phi = W (\sin \alpha \cos \phi - \sin \phi \cos \alpha)$ ($\because$ multiplying both sides by $\cos \phi$)

$\Rightarrow P_1 \cos \phi = W \sin (\alpha - \phi)$

$\Rightarrow P_1 = \frac{W \sin (\alpha - \phi)}{\cos \phi}$

case B Max^m force (P_2) which will keep the body in equⁿ. when it is about to slide upward.



For equⁿ $\sum F_v = 0 \Rightarrow R_2 = w \cos \alpha$

& $\sum F_H = 0 \Rightarrow P_2 = w \sin \alpha + \mu R_2$

$\Rightarrow P_2 = w \sin \alpha + \mu w \cos \alpha$

$\Rightarrow P_2 = w (\sin \alpha + \mu \cos \alpha)$

By multiplying both sides of equⁿ by $\cos \phi$

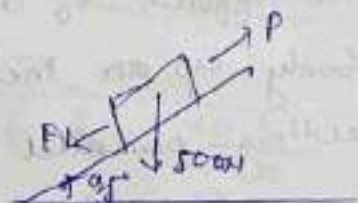
$P_2 \cos \phi = w (\sin \alpha \cos \phi + \mu \cos \alpha \cos \phi)$

$\Rightarrow P_2 \cos \phi = w \sin(\alpha + \phi)$

$\Rightarrow P_2 = w \frac{\sin(\alpha + \phi)}{\cos \phi}$

Solve A body of weight 500N is lying on a rough plane inclined at an angle of 35° with the horizontal. It is supported by an effort (P) parallel to the plane as shown in fig. determine the min^m & max^m values of P for which the equⁿ can exist. If the angle of friction is 20° .

Solⁿ
 Given: $w = 500\text{N}$
 $\alpha = 35^\circ$
 $P_{\text{min}} = ?$
 $P_{\text{max}} = ?$
 $\phi = 20^\circ$

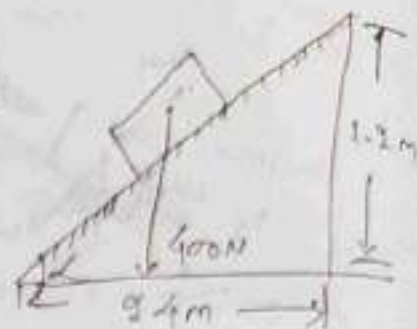


$P_{\text{min}} = \frac{w \sin(\alpha - \phi)}{\cos \phi} = \frac{500 \sin(35^\circ - 20^\circ)}{\cos 20^\circ} = 46.4\text{N}$

$P_{\text{max}} = \frac{w \sin(\alpha + \phi)}{\cos \phi} = \frac{500 \sin(35^\circ + 20^\circ)}{\cos 20^\circ} = 376.2\text{N}$

Q An inclined plane as shown in figure, is used to uniformly slowly a body weighing 400 N from a truck 1.2 height into the ground.

At bottom underside of body & plane is 0.3 . State whether it is necessary to push the body down the plane or hold it back from sliding down. What min force is required parallel to the plane for this purpose.



Soln $\tan \alpha = \frac{1.2}{0.9} \Rightarrow \alpha = 36.5^\circ$

For eqn $\sum F_H = 0 \Rightarrow P = 400 \sin 36.5^\circ + R$

$\sum F_V = 0 \Rightarrow R = 400 \cos 36.5^\circ$
 $= 357.9\text{ N}$

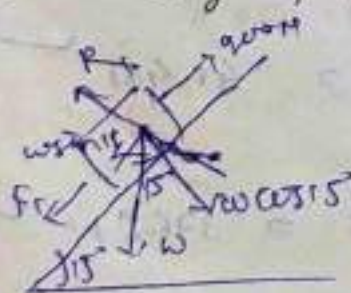


Now $P = 400 \sin 36.5^\circ + R$

$= 400 \sin 36.5^\circ + (0.3 \times 357.9) = 71.2\text{ N}$

Q An effort of 900 N is required just to move a ~~certain~~ ^{certain} body up an inclined plane of angle 20° to 35° the force acting parallel to the plane. If the angle of inclination of the plane is made 20° the effort require again applied parallel to the plane, is found to be 900 N . Find the weight of the body W .

Soln



$\sum F_V = R = W \cos 15^\circ$

Given $P_1 = 900\text{ N}$ $P_2 = 900\text{ N}$

$\alpha_1 = 15^\circ$ $\alpha_2 = 30^\circ$

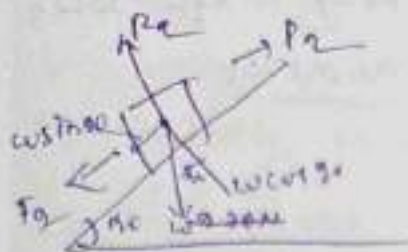
For eqn $\sum F_H = 0$

$\Rightarrow F + W \sin 15^\circ = 900$

$\Rightarrow R + W \sin 15^\circ = 900$

$\Rightarrow W \cos 15^\circ + W \sin 15^\circ = 900$

$\Rightarrow W (\cos 15^\circ + \sin 15^\circ) = 900 \rightarrow$



For eqn $\sum F_H = 0$

to

$$\Rightarrow F_f + W \sin 30 = 250$$

$$\sum F_V = 0$$

$$\Rightarrow R_f = W \cos 30$$

$$\Rightarrow W \cos 30 + W \sin 30 = 250$$

$$\Rightarrow W (\cos 30 + \sin 30) = 250 \quad \text{--- (1)}$$

$$\frac{\text{eqn (1)}}{\text{eqn (2)}} \Rightarrow \frac{W \cos 30 + W \sin 30}{W \cos 15 + W \sin 15} = \frac{250}{9.8}$$

$$\Rightarrow \mu > 0.957$$

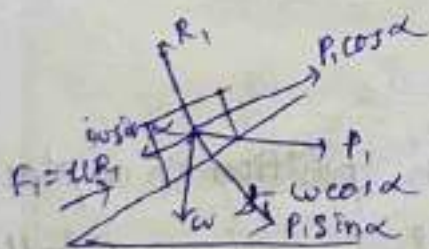
& Now eqn (1) becomes

$$W (0.957 \cos 15 + \sin 15) = 250$$

$$\Rightarrow W = 399.2 \text{ N (Ans)}$$

Equilibrium of a body on a rough inclined plane subjected to a force acting horizontally :-

1) Min force (P) which will keep the body in eqn when it is at the point of sliding downwards.



body at the point of sliding downwards

$$(1 + \mu)$$

Again $\mu = \tan \phi = \frac{\sin \phi}{\cos \phi}$

$$\text{So } P = \frac{W (\sin \alpha - \frac{\sin \phi}{\cos \phi} \cos \alpha)}{(\cos \alpha + \frac{\sin \phi}{\cos \phi} \sin \alpha)}$$

$$\Rightarrow \frac{W \sin \alpha \cos \phi - \sin \phi \cos \alpha}{\cos \alpha \cos \phi + \sin \alpha \sin \phi}$$

$$= \frac{W (\sin (\alpha - \phi))}{\cos (\alpha - \phi)}$$

$$= W \tan (\alpha - \phi)$$

$$P = W \tan (\alpha - \phi)$$

From eqn $\sum F_H = 0 \Rightarrow P \cos \alpha + \mu R_f = W \sin \alpha$

& $\sum F_V = 0 \Rightarrow R_f = W \cos \alpha + P \sin \alpha$

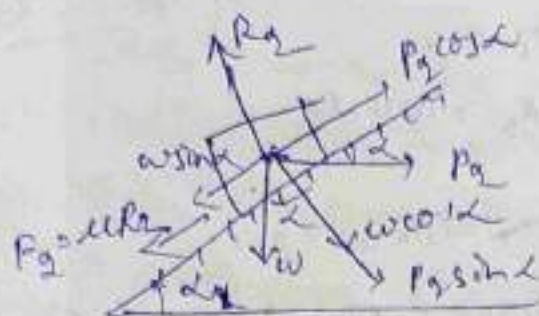
Now $P \cos \alpha + \mu (W \cos \alpha + P \sin \alpha) = W \sin \alpha$

$$\Rightarrow P \cos \alpha = W \sin \alpha - \mu W \cos \alpha - \mu P \sin \alpha$$

$$\Rightarrow P (\cos \alpha + \mu \sin \alpha) = W (\sin \alpha - \mu \cos \alpha)$$

$$\Rightarrow P = \frac{W (\sin \alpha - \mu \cos \alpha)}{\cos \alpha + \mu \sin \alpha}$$

2) Max^m Force (P_2) which will keep the body in equl, when it is at the point of sliding upwards ->



For eq^m $\sum F_v = 0 \Rightarrow R_2 = W \cos \alpha + P_2 \sin \alpha$

of $\sum F_H = 0 \Rightarrow P_2 \cos \alpha = \mu R_2 + W \sin \alpha$

$\Rightarrow P_2 \cos \alpha = \mu (W \cos \alpha + P_2 \sin \alpha) + W \sin \alpha$

$\Rightarrow P_2 (\cos \alpha - \mu \sin \alpha) = W \sin \alpha + \mu W \cos \alpha$

$\Rightarrow P_2 = \frac{W \sin \alpha + \mu W \cos \alpha}{\cos \alpha - \mu \sin \alpha}$

Again $\mu = \tan \alpha = \frac{\sin \alpha}{\cos \alpha}$

So $P_2 = \frac{W (\sin \alpha \cdot \cos \alpha + \sin \alpha \cdot \cos \alpha)}{\cos \alpha \cdot \cos \alpha - \sin \alpha \cdot \sin \alpha}$

$= W \frac{\sin(\alpha + \alpha)}{\cos(\alpha + \alpha)} = W \tan(\alpha + \alpha)$

$P_2 = W \tan(\alpha + \alpha)$

Q.1 An object of weight 100N is kept in position on a plane inclined at 30° to the horizontal by a horizontally applied force (F). If $\mu = 0.25$, determine min^m force (F).

Solⁿ

$F = W \tan(\alpha - \phi)$

$= 100 \tan(30 - 14)$

$= 28.67 \text{ N}$

Q-2 A load of 1.5 kN resting on an inclined rough plane, can be moved up the plane by a force of 2 kN applied horizontally or by a force of 1.25 kN applied parallel to the plane. Find the inclination of the plane α .

Solⁿ From (1) $P = w \tan(\alpha + \phi)$
 $\Rightarrow 2 = 1.5 \tan(\alpha + \phi)$
 $\Rightarrow \alpha + \phi = 53.1^\circ$



From (2) $P = w \frac{\sin(\alpha + \phi)}{\cos \phi}$

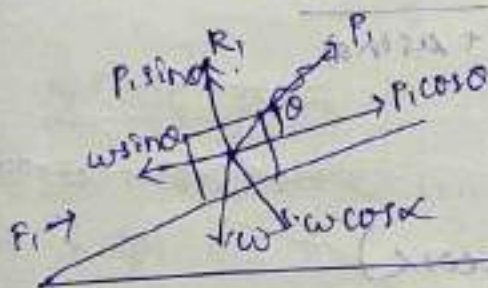
$$\Rightarrow 1.25 = 1.5 \frac{\sin 53.1}{\cos \phi} \Rightarrow \cos \phi = 0.96 \Rightarrow \phi = 16.3^\circ$$

$$\alpha = 53.1^\circ - 16.3^\circ = 36.8^\circ$$

$$\text{Again } \mu = \tan \phi = \tan 16.3^\circ = 0.292$$

Equilibrium of a body on a rough inclined plane subjected to a force acting at some angle with the inclined plane

1) Min^m force (P) which will keep the body in equ^m when it is at the point of sliding downwards.



For equ^m $\sum F_v = 0 \Rightarrow R + P \sin \theta = w \cos \alpha \rightarrow R = w \cos \alpha - P \sin \theta$

& $\sum F_H = 0 \Rightarrow \mu R + P \cos \theta = w \sin \alpha$

$$\Rightarrow \mu (w \cos \alpha - P \sin \theta) + P \cos \theta = w \sin \alpha$$

$$\Rightarrow P (\cos \theta - \mu \sin \theta) = w \sin \alpha - \mu w \cos \alpha$$

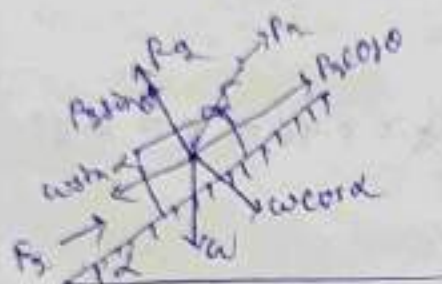
$$\Rightarrow P = \frac{w (\sin \alpha - \mu \cos \alpha)}{(\cos \theta - \mu \sin \theta)}$$

$$\text{Again } \mu = \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\text{Now, } P_1 = \frac{w(\sin \alpha \cos \theta - \sin \theta \cdot \cos \alpha)}{\cos \theta \cos \alpha - \sin \theta \cdot \sin \alpha}$$

$$\Rightarrow \boxed{P_1 = \frac{w \sin(\alpha - \theta)}{\cos(\theta + \alpha)}}$$

Q. Max^m force (P_2) which will keep the body in eqn when it is at the point of sliding upwards.



$$\text{For eqn } \sum F_v = 0 \Rightarrow P_2 + P_2 \sin \theta = w \cos \alpha$$

$$\& \sum F_H = 0 \Rightarrow P_2 \cos \theta = w \sin \alpha + \mu P_2$$

$$\Rightarrow P_2 \cos \theta = w \sin \alpha + \mu (w \cos \alpha - P_2 \sin \theta)$$

$$\Rightarrow P_2 (\cos \theta + \mu \sin \theta) = w \sin \alpha + \mu w \cos \alpha$$

$$\Rightarrow P_2 = \frac{w(\sin \alpha + \mu \cos \alpha)}{(\cos \theta + \mu \sin \theta)}$$

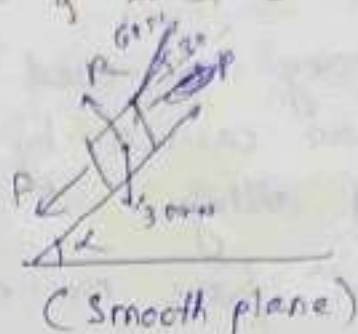
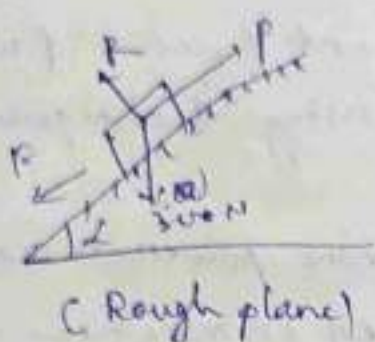
$$\text{Again } \mu = \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$P_2 = \frac{w(\sin \alpha \cdot \cos \theta + \sin \theta \cdot \cos \alpha)}{(\cos \theta \cdot \cos \theta + \sin \theta \cdot \sin \theta)}$$

$$\boxed{P_2 = \frac{w \sin(\alpha + \theta)}{\cos(\theta - \alpha)}}$$

Q Find the force required to move a load of 300N up a rough plane, the force being applied parallel to the plane. The inclination of the plane is such that when the same load is kept on a perfectly smooth plane inclined at the same angle, a force of 60N applied at an inclination of 30° to the plane, keeps the same load in eqn. Coeffⁿ rough plane & load is 0.3.

Solⁿ



For Smooth plane $\mu = 0 \Rightarrow \theta = 0$

$$P = \frac{W \sin(\alpha + \theta)}{\cos(\theta - \theta)} \Rightarrow 60 = \frac{300 \sin \alpha}{\cos 30^\circ} \Rightarrow \alpha = 10^\circ$$

For rough plane

$$P = \frac{W \sin(\alpha + \theta)}{\cos \theta} = \frac{300 \sin(10 + 16.7^\circ)}{\cos 16.7^\circ} = 140.7 \text{ N}$$

Advantage of friction

- It is responsible for many types of motion.
- It helps us to walk on the ground/floor.
- Brakes are applied in automobile to stop or reduce its motion by using friction principle.
- Asteroids are burnt in the atmosphere before reaching earth due to friction.
- It helps in generation of heat when we rub our hands.
- We can write on paper or board.
- We can not fix nail in the wall or wood if there is no friction (as it holds the nail).
- Dragging of atmosphere with earth is possible.

Disadvantage of friction

- It produces unnecessary heat which causes wastage of energy.
- The force of friction acts in opposite direction of motion. So it slows down the motion of moving objects.
- Frictional forces are caused due to friction between two surfaces.
- A lot of money are used to prevent friction & wear & tear caused by it by using techniques like greasing & oiling.
- Due to friction, we have to exert more power in machines.
- " " noise is produced in machines.
- " " engine of automobile consumes more fuel.

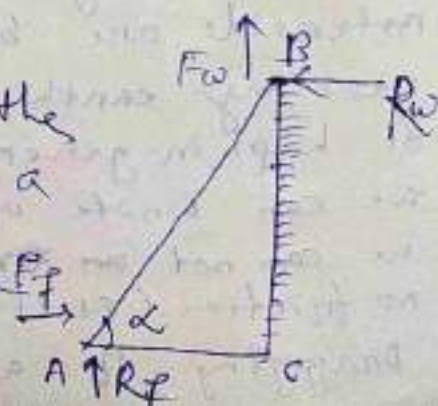
Application of friction :-

- It is used in 1) Ladder friction
- 2) Wedge
- 3) Screw

Ladder friction :-

It is a device used for climbing on roofs or walls. It consists of 2 long uprights of wood, iron or rope connected by a number of cross-pieces called rungs or steps.

Consider a ladder AB resting on the rough ground & leaning against a wall. At the upper end of the ladder, it tends to slip downwards.



So frictional force (F_f) will act in upward dirⁿ.

As lower end of the ladder tends to move left.
So frictional force (F_f) will act along right.

The system to be in eqⁿ, algebraic sum of all horizontal & vertical components of forces will be zero.

$$\therefore \sum F_H = 0 \quad \& \quad \sum F_V = 0$$

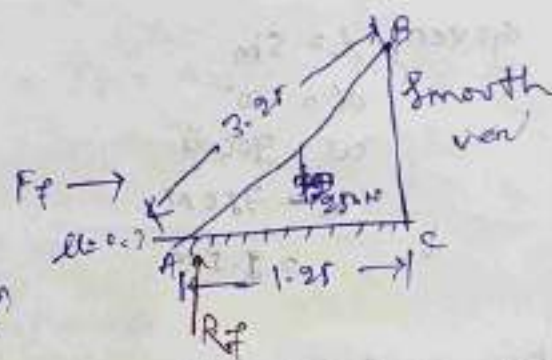
- Q. A uniform ladder of length 3.25 m & weight 250 N is placed against a smooth vertical wall with its lower end 1.95 m from the wall. A betⁿ ladder and floor is 0.3. what is the frictional force acting on the ladder at the point of contact betⁿ ladder & the floor? Show that the ladder will remain in eqⁿ in the posⁿ.

$$L = 3.25 \text{ m}$$

$$W = 250 \text{ N}$$

$$AC = 1.95 \text{ m}$$

$$h_f = 0.3$$



$F_f \rightarrow$ frictional force acting betⁿ ladder & floor

$R_f \rightarrow$ Normal reactⁿ at the floor

As wall is smooth, so no frictional force acting on wall $R_f = 250 \text{ N}$

$$\text{From the fig. } BC = \sqrt{AB^2 - AC^2} = \sqrt{(3.25)^2 - (1.95)^2} = 2.6 \text{ m}$$

$$\text{Now, } \sum M_B = 0$$

$$\Rightarrow +ve R_f (1.95) - F_f (3) - 250 \left(\frac{1.95}{2} \right) = 0$$

$$\Rightarrow 1.95 R_f = 3 F_f + (0.925 \times 250)$$

$$\Rightarrow 1.95 \times 250 = 3 F_f + (0.925 \times 250)$$

$$\Rightarrow F_f = 52.1 \text{ N}$$

For eqn of the ladder

Max^m force of friction available at the point of contact betⁿ ladder & floor is $\mu R_f = 0.3 \times 900 = 270 \text{ N}$. As the amount of force of friction available at the point of contact (270 N) is more than the force of friction required for eqn (52.1 N) ladder will remain in equilibrium condⁿ.

Q A ladder 5m long rests on a horizontal ground & leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900 N & acts at its middle. The ladder is at the point of sliding when a man weighing 750 N stands on a rung 1.5 m from the bottom of the ladder. Find μ betⁿ ladder & floor.

Solⁿ

Given: $L = 5 \text{ m}$
 $\alpha = 70^\circ$
 $W = 900 \text{ N}$
 $W_m = 750 \text{ N}$
 $x = 1.5 \text{ m}$

For eqn $\Sigma F_r = 0$

$$\Rightarrow R_f = 750 + 900$$

$$= 1650 \text{ N}$$

$$F_f = \mu R_f = \mu (1650)$$

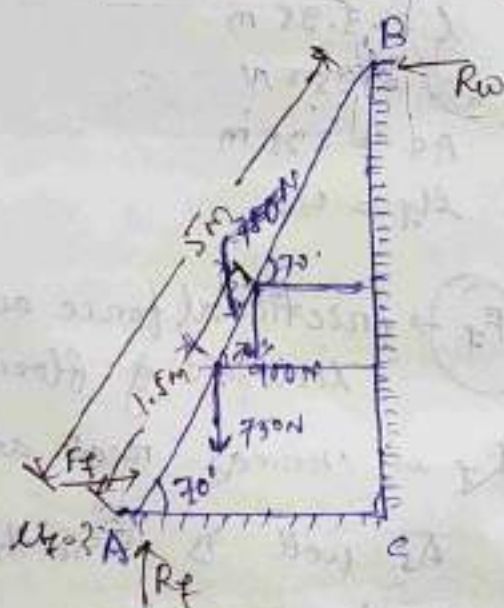
Now taking $\Sigma M_B = 0$

$$\Rightarrow R_f \times 5 \cos 70^\circ = 900 (2.5 \cos 70^\circ) + 750 (3.5 \cos 70^\circ) + F_f \times 5 \sin 70^\circ$$

$$\Rightarrow R_f \times 5 \cos 70^\circ = (900 \times 2.5 \cos 70^\circ) + (750 \times 3.5 \cos 70^\circ) + \mu (1650 \times 5 \sin 70^\circ)$$

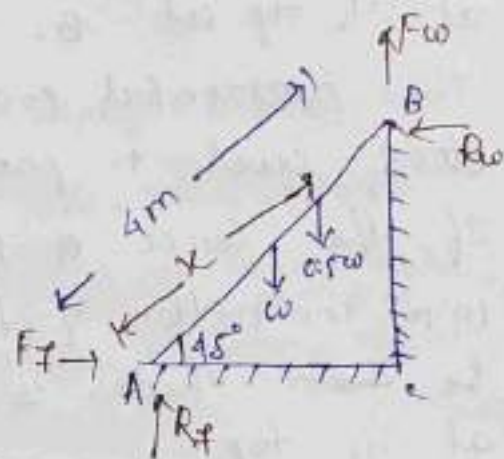
$$\Rightarrow 1650 \times 5 \cos 70^\circ =$$

$$\Rightarrow \mu = 0.15$$



Q A uniform ladder of 4m length rests against a vertical wall with which it makes an angle of 45° . If betⁿ ladder and wall is 0.4 & betⁿ ladder & floor is 0.5 of a man whose weight is $\frac{1}{2}$ that of ladder ascends it, how high will it be when ladder slips

Solⁿ Given $L = 4m$
 $\alpha = 45^\circ$
 $\mu_w = 0.4$
 $\mu_f = 0.5$



when ladder is at its point of slipping w = weight of ladder

R_f = Normal reactⁿ at floor

weight of man = $\frac{w}{2} = 0.5w$

Frictional force at the floor = $F_f = \mu_f R_f = 0.5 R_f$

" " " " " = $F_w = \mu_w R_w = 0.4 R_w$

For eqnⁿ $\sum F_v = 0$

$$\Rightarrow R_f + F_w = w + 0.5w$$

$$\Rightarrow R_f + F_w = 1.5w \quad \text{--- (1)}$$

$$\sum F_H = 0$$

$$\Rightarrow F_f = R_w$$

$$\Rightarrow 0.5 R_f = R_w \Rightarrow R_f = 2 R_w$$

Now eqnⁿ (1) become $2 R_w + F_w = 1.5w$

$$\Rightarrow 2 R_w + 0.4 R_w = 1.5w \Rightarrow R_w = 0.625w$$

$$F_w = 0.4 \times 0.625w = 0.25w$$

Now taking $\sum M_A = 0$

$$\Rightarrow F_w (4 \cos 45^\circ) + R_w (4 \sin 45^\circ) = w (2 \cos 45^\circ) + 0.5w (x \cos 45^\circ)$$

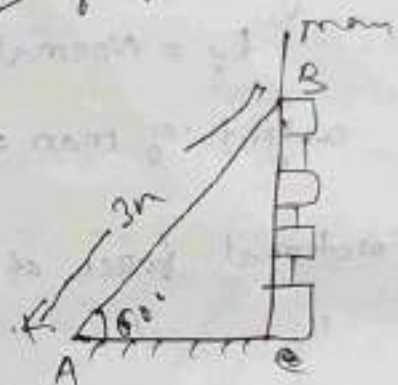
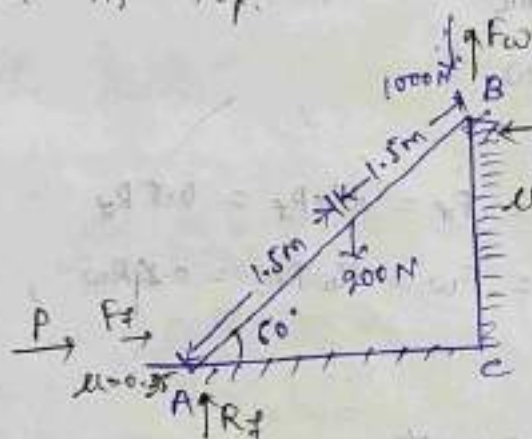
$$\Rightarrow 0.25w (4 \cos 45^\circ) + (0.625w \times 4 \sin 45^\circ) = 2w \cos 45^\circ + 0.5w x \cos 45^\circ$$

$$\Rightarrow 0.25 \times 4 \cos 45^\circ + (0.625 \times 4 \sin 45^\circ) = 2 \cos 45^\circ + 0.5 x \cos 45^\circ$$

$$\Rightarrow x = 3m$$

Q A uniform ladder 3m long weights 200N. It is placed against a wall making an angle of 60° with the floor as shown in fig. If coeff of friction of ladder is 0.25 & coeff of floor & ladder is 0.35. The ladder is subjected to its own weights, has a man support a mass of 1000N at its top at B. calculate.

- i) The horizontal force P to be applied to ladder at the floor level to prevent slipping.
- ii) If the force P is not applied, what should be the min^m inclination of the ladder with the horizontal so that there is no slipping of it with the man at its top.



For eqnⁿ $\sum F_H = 0 \Rightarrow P + F_f = R_w$

& $\sum F_V = 0 \Rightarrow R_f + F_w = 200 + 1000 = 1200\text{N}$

Now $\sum M_A = 0$

$\Rightarrow 200(1.5 \cos 60^\circ) + 1000(3 \cos 60^\circ) = F_w 3 \cos 60^\circ + R_w 3 \sin 60^\circ$

$\Rightarrow F_w + R_w \tan 60^\circ = 1100$

Again $F_w = \mu R_w = 0.25 R_w$

$\therefore 0.25 R_w + R_w \tan 60^\circ = 1100$

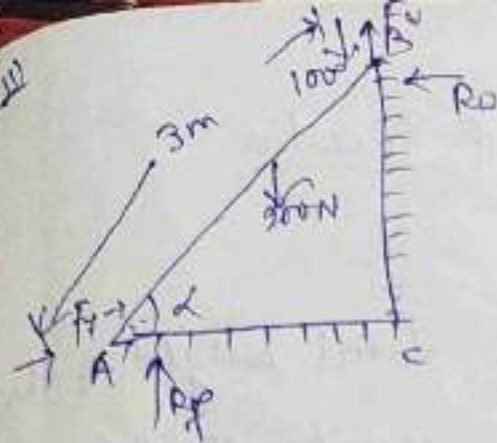
$\therefore R_w = 555\text{N}$

$F_w = 0.25 \times 555 = 138.7\text{N}$

Now $R_f + F_w = 1200 \Rightarrow R_f = 1200 - 138.7 = 1061.3\text{N}$

$F_f = \mu R_f = 0.35 \times 1061.3 = 371.5\text{N}$

Now $P + F_f = R_w \Rightarrow P + 371.5 = 555 \Rightarrow P = 183.5\text{N}$



For $\sum F_H = 0 \Rightarrow R_w = F_f = \mu_f R_f = 0.35 R_f$

$\& \sum F_V = 0 \Rightarrow R_f + F_w = 1000 + 900 = 1900 \text{ N}$

Again $F_w = \mu_w R_w = 0.25 (0.35 R_f) = 0.09 R_f$

$R_f + 0.09 R_f = 1900$

$\Rightarrow R_f = 1101 \text{ N}$

$R_w = 0.35 R_f = 0.35 \times 1101 = 385.4 \text{ N}$

$F_w = 0.09 R_f = 0.09 \times 1101 = 99.1 \text{ N}$

Now $\sum M_A = 0$

$\Rightarrow F_w 3 \cos \alpha + R_w 3 \sin \alpha = 900 (3.5 \cos \alpha) + 1000 (3 \cos \alpha)$

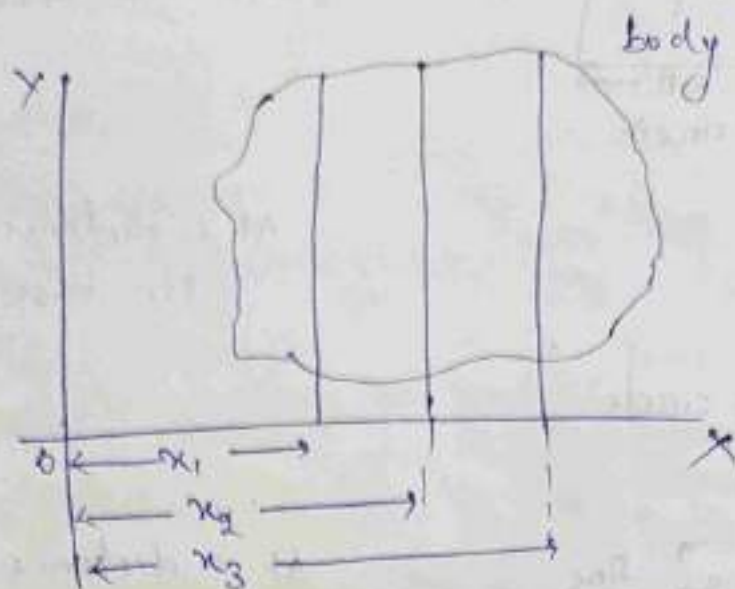
$\Rightarrow 99.1 \times 3 \cos \alpha + 385.4 \times 3 \sin \alpha = 0$

Dividing $\Rightarrow \frac{99.1 \times 3 \cos \alpha}{3 \cos \alpha} + \frac{385.4 \times 3 \sin \alpha}{3 \cos \alpha} = 0$

$\Rightarrow \tan \alpha = \frac{99.1}{385.4} \Rightarrow \alpha = 68.9^\circ$



ii) C.G on centroid by moments $\rightarrow$



Consider a body of mass M & its C.G has to be found. Now dividing the body into nos. of small masses $m_1, m_2, m_3, \dots$ at $(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots$ are the coordinates of the C.G from the fixed point O.

Let $\bar{x} = x$ -coordinates of C.G of the body
 $\bar{y} = y$ - " " " " "

Now from the principle of moments

$$M\bar{x} = m_1x_1 + m_2x_2 + m_3x_3 + \dots$$
$$\Rightarrow \bar{x} = \frac{\sum m_i x_i}{M}$$
$$\bar{y} = \frac{\sum m_i y_i}{M} \quad \text{where } M = m_1 + m_2 + m_3 + \dots$$

Axis of reference $\rightarrow$

C.G of a body is always calculated with respect to some assumed axis called as axis of reference.

Generally the lowest line of the fig. is taken as reference line for $\bar{y}$ & left line of the fig. is taken as reference line for calculating $\bar{x}$.

Centroid on C.G. of plane figure.

In Composite Figures

Let $\bar{x}$ & $\bar{y}$ are the coordinates of C.G. w.r.t some reference axis.

$$\text{then } \bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \dots}{a_1 + a_2 + a_3 + \dots}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + \dots}{a_1 + a_2 + a_3 + \dots}$$

where $a_1, a_2, a_3, \dots \rightarrow$ Areas of ^{divided} fig. from main fig.

$x_1, x_2, x_3, \dots \rightarrow$ x-coordinates of divided fig. from reference axis

$y_1, y_2, y_3, \dots \rightarrow$ y-coordinates " " "

C.G. of Symmetrical Sections

As the figure is symmetrical about one axis, so we have to calculate either $\bar{x}$ or $\bar{y}$ here C.G. will lie on the axis of symmetry.

Q.1 Find the C.G. of a 100mm x 150mm x 30mm T-section.

Solⁿ The section is symmetrical about y-axis (bisecting the web).

Now Dividing the section into

□ ABCD & □ EFGH & taking FG line as reference

□ ABCD

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2$$

$$y_1 = 150 - \frac{30}{2} = 135 \text{ mm}$$

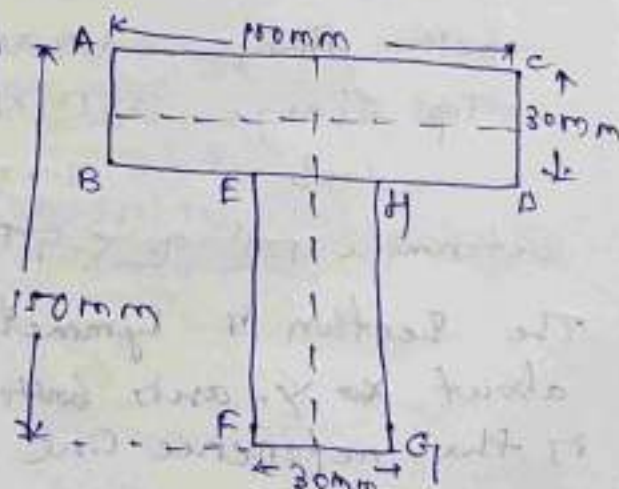
□ EFGH

$$a_2 = 30 \times 30 = 900 \text{ mm}^2$$

$$y_2 = \frac{30}{2} = 15 \text{ mm}$$

distance of C.G. of the T-section from FG line

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{(3000 \times 135) + (900 \times 15)}{3000 + 900} = 94.1 \text{ mm (Ans)}$$



Q-2 Find the C.G. of a channel section 100mm x 50mm x 15mm

Solⁿ The section is symmetrical about X-axis

□ ABCD

$$a_1 = 15 \times 50 = 750 \text{ mm}^2$$

$$x_1 = 50/2 = 25 \text{ mm}$$

□ BGEF

$$a_2 = 70 \times 15 = 1050 \text{ mm}^2$$

$$x_2 = 15/2 = 7.5 \text{ mm}$$

□ EJIH

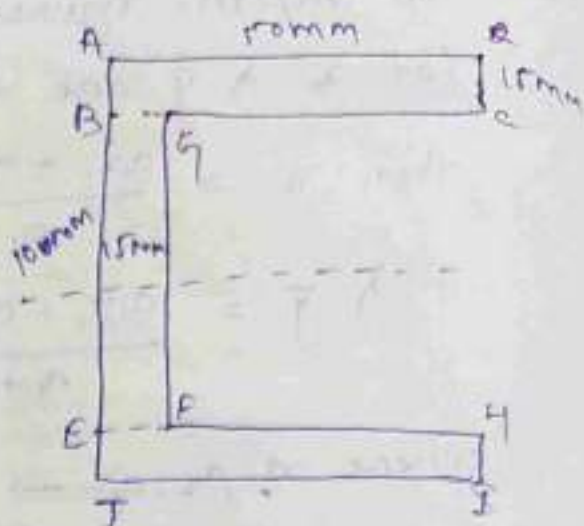
$$a_3 = 50 \times 15 = 750 \text{ mm}^2$$

$$x_3 = 15/2 = 7.5 \text{ mm}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$\frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 7.5)}{750 + 1050 + 750}$$

$$= 17.8 \text{ mm}$$



Q-3 An I section has the following dimensions in mm

bottom flange = 300 x 100

Top flange = 150 x 50

web = 300 x 50

Determine posⁿ of C.G. of the section

Solⁿ The section is symmetrical about the Y-axis bottom line is the reference line.

i) Bottom flange

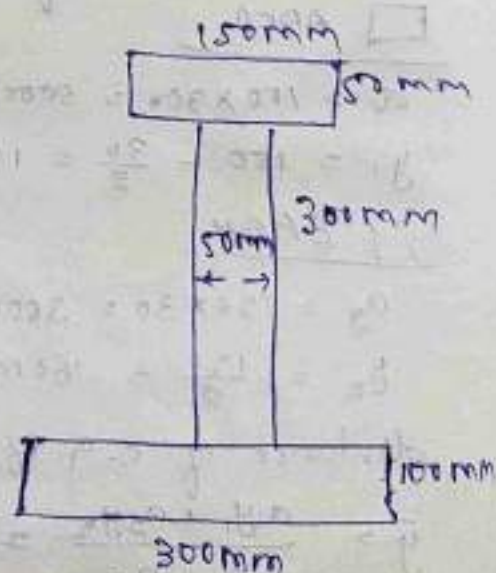
$$a_1 = 300 \times 100 = 30000 \text{ mm}^2$$

$$y_1 = 100/2 = 50 \text{ mm}$$

ii) web

$$a_2 = 300 \times 50 = 15000 \text{ mm}^2$$

$$y_2 = 50 + \frac{300}{2} = 90 \text{ mm}$$



vii) Top Flange

$$a_3 = 150 \times 50 = 7500 \text{ mm}^2$$

$$y_3 = 100 + 300 + 50/2 = 425 \text{ mm}$$

$$\text{Now } \bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$\bar{y} = \frac{(20000 \times 80) + (15000 \times 250) + (7500 \times 425)}{30000 + 15000 + 7500} = 160.7 \text{ mm}$$

C.G of unsymmetrical sections: →

when the section is not symmetrical about any axis i.e x-x, or y-y then both $\bar{x}$ & $\bar{y}$ have to be found out.

Q-1 Find the centroid of an unequal angle section $100 \times 80 \times 20 \text{ mm}$

Soln □ ①

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2$$

$$x_1 = \frac{30}{2} = 15 \text{ mm}$$

$$y_1 = 100/2 = 50 \text{ mm}$$

□ ②

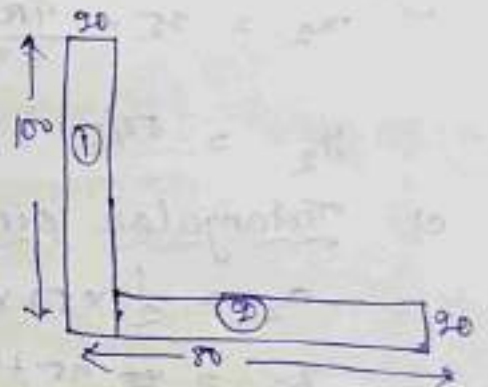
$$a_2 = 60 \times 20 = 1200 \text{ mm}^2$$

$$x_2 = \frac{20}{2} = 10 \text{ mm}$$

$$y_2 = 20/2 = 10 \text{ mm}$$

$$\text{Now } \bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(3000 \times 15) + (1200 \times 10)}{3000 + 1200} = 13.5 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 \times 50) + (1200 \times 10)}{3000 + 1200} = 25 \text{ mm}$$



Q.2
V. Imp

A uniform lamina as shown in the fig. consists of a rectangle a circle & a triangle. Find C.G. the lamina. All dimensions are in mm.

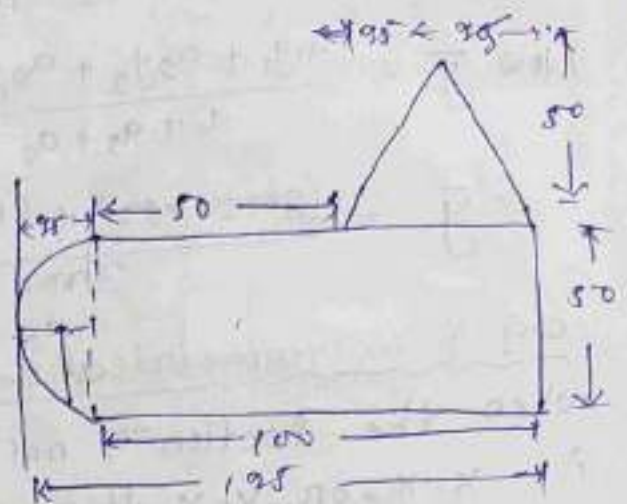
Solⁿ Section is not symmetrical about any axis.

a) rectangle portion

$$a_1 = 100 \times 50 = 5000 \text{ mm}^2$$

$$x_1 = 35 + \frac{100}{2} = 75 \text{ mm}$$

$$y_1 = 50/2 = 25 \text{ mm}$$



b) Semi-circular portion

$$a_2 = \frac{\pi R^2}{2} = \frac{\pi \times 25^2}{2} = 982 \text{ mm}^2$$

$$x_2 = 25 - \frac{4R}{3\pi} = 25 - \frac{4 \times 25}{3\pi} = 14.4 \text{ mm}$$

$$y_2 = 50/2 = 25 \text{ mm}$$

c) Triangular portion

$$a_3 = \frac{1}{2} \times 50 \times 50 = 1250 \text{ mm}^2$$

$$x_3 = 35 + 50 + \frac{50}{3} = 101 \text{ mm}$$

$$y_3 = 50 + \frac{50}{3} = 66.7 \text{ mm}$$

$$\text{Now } \bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 101)}{5000 + 982 + 1250}$$

$$= 71.7 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(5000 \times 25) + (982 \times 25) + (1250 \times 66.7)}{5000 + 982 + 1250}$$

$$= 32.2 \text{ mm}$$

C.G. of sections with cut out holes: -

- 1) A Semicircle of 90mm radius is cut out from a trapezium as shown in the fig. Find out C.G.

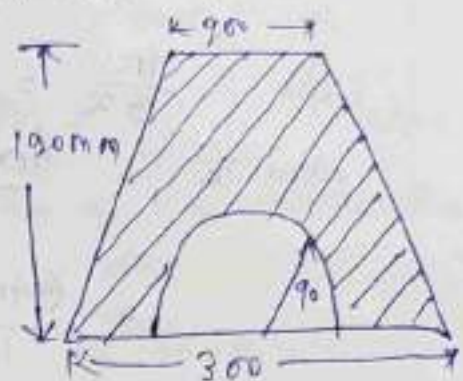
Trapezium Symmetrical about Y-Y axis

$$a_1 = \frac{1}{2} (900 + 300) \times 180$$

$$= 30000 \text{ mm}^2$$

$$y_1 = \frac{1}{3} \left(\frac{a+b}{a+b} \right)$$

$$= \frac{180}{3} \left[\frac{300 + 2(900)}{300 + 900} \right] = 50 \text{ mm}$$



Semi-circle

$$a_2 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \pi \times 90^2 = 4050\pi \text{ mm}^2$$

$$y_2 = \frac{4R}{3\pi} = \frac{4 \times 90}{3\pi} = \frac{120}{\pi} \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{(30000 \times 50) - (4050\pi \times \frac{120}{\pi})}{30000 - 4050\pi} = 69.1 \text{ mm}$$

- Q.2 A Semi-circular circular arc is removed from a trapezium as shown in the figure. (all dimensions are in mm) Find C.G.

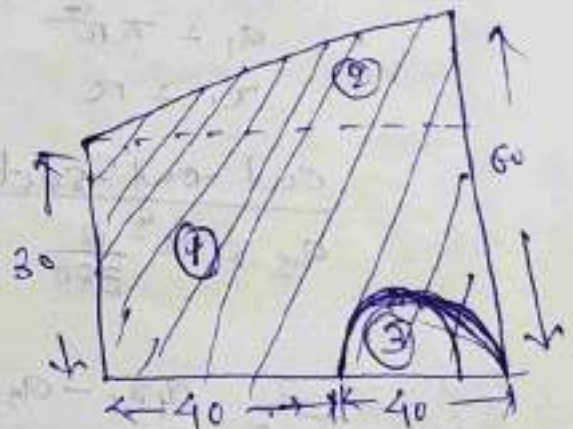
Section is not symmetrical about any axis.

- i) Rectangle

$$a_1 = 30 \times 80 = 2400 \text{ mm}^2$$

$$x_1 = 80/2 = 40 \text{ mm}$$

$$y_1 = 30/2 = 15 \text{ mm}$$



- ii) Triangle

$$a_2 = \frac{1}{2} \times 80 \times 30 = 1200 \text{ mm}^2$$

$$x_2 = \frac{2 \times (40 + 40)}{3} = 53.3 \text{ mm}$$

$$y_2 = 30 + \frac{30}{2} = 45 \text{ mm}$$

m) Semi-circle

$$a_3 = \frac{\pi r^2}{2} = \frac{\pi 90^2}{2} = 638.3 \text{ mm}^2$$

$$x_3 = 40 + \frac{40}{2} = 60 \text{ mm}$$

$$y_3 = \frac{4r}{3\pi} = \frac{4 \times 90}{3\pi} = 8.5 \text{ mm}$$

$$\text{Now } \bar{x} = \frac{a_1 x_1 + a_2 x_2 - a_3 x_3}{a_1 + a_2 - a_3}$$

$$= \frac{(9400 \times 40) + (1900 \times 17.8) - (638.3 \times 60)}{9400 + 1900 - 638.3}$$

$$= 41.1 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 - a_3 y_3}{a_1 + a_2 - a_3}$$

$$= \frac{(9400 \times 15) + (1900 \times 40) - (638.3 \times 8.5)}{9400 + 1900 - 638.3}$$

$$= 36.5 \text{ mm}$$

Q. A circular Sector of angle 60° is cut from the circle of radius r . Find c.g.

Soln Symmetrical about x-axis

main-circle

$$a_1 = \pi r^2$$

$$x_1 = r$$

cut-out-sector

$$a_2 = \pi r^2 \frac{\theta}{360} = \frac{\pi r^2 \times 60}{360} = \frac{\pi r^2}{6}$$

$$\text{Now } \bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$= \frac{(\pi r^2 \times r) - \left[\frac{\pi r^2}{6} \times \left(r + \frac{9r}{\pi} \right) \right]}{\pi r^2 - \frac{\pi r^2}{6}}$$



MS of a rectangular section!

Consider a rectangular section

Let b = width of the section

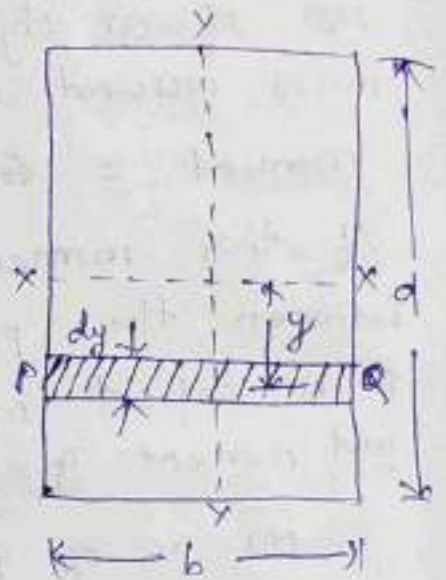
d = depth of the section

Now considering a small strip PQ of thickness dy at a distance of y from x-x axis.

Area of strip = $b \cdot dy$

MS of the strip about

$$\begin{aligned} \text{x-x axis} &= (b \cdot dy) \times y^2 \\ &= b y^2 dy \end{aligned}$$



Now MS of the whole section will be

$$I_{xx} = \int_{-d/2}^{d/2} b y^2 dy = b \left[\frac{y^3}{3} \right]_{-d/2}^{d/2} = \frac{b}{3} \left[\frac{y d^3}{8} + \frac{d^3}{8} \right]$$

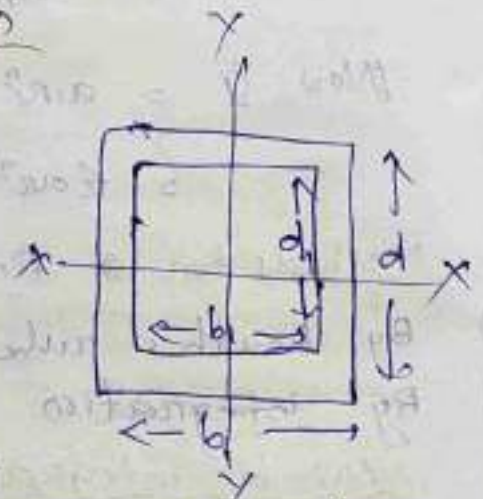
$$\begin{aligned} \text{Similarly } I_{yy} &= \frac{d b^3}{12} \\ I_{xx} &= \frac{b d^3}{12} \end{aligned}$$

MS of hollow rectangular section

MS of hollow rectangular section about xx axis

$$I_{xx} = \frac{b d^3}{12} - \frac{b d_1^3}{12}$$

$$\& I_{yy} = \frac{d b^3}{12} - \frac{d b_1^3}{12}$$



Q Find MS of rectangular sectⁿ 30mm wide & 40mm deep about x-x & y-y axis

$$b = 30, d = 40$$

Solⁿ

$$I_{xx} = \frac{b d^3}{12} = \frac{30 \times 40^3}{12} = 160 \times 10^3 \text{ mm}^4$$

$$I_{yy} = \frac{d b^3}{12} = \frac{40 \times 30^3}{12} = 90 \times 10^3 \text{ mm}^4$$

Q Find MS of hollow rectangular section about its CG centroid b & d are 60 mm & 80 mm respectively & internal b & d are 30 & 40 mm
 given $b = 60 \text{ mm}$, $b_1 = 30 \text{ mm}$
 $d = 80 \text{ mm}$, $d_1 = 40 \text{ mm}$

$$I_{xx} = \frac{bd^3}{12} - \frac{b_1d_1^3}{12} = \frac{60 \times 80^3}{12} - \frac{30 \times 40^3}{12} = 3400 \times 10^3 \text{ mm}^4$$

$$I_{yy} = \frac{db^3}{12} - \frac{d_1b_1^3}{12} = \frac{80 \times 60^3}{12} - \frac{40 \times 30^3}{12} = 1350 \times 10^3 \text{ mm}^4$$

Perpendicular axis theorem:-

State that if I_{xx} & I_{yy} be the MS of a plane section about two $\perp$ axes meeting at O, MS I_{zz} about the axis $z \perp$ to the plane & passing through the intersection of $x-x$ & $y-y$ is given by $I_{zz} = I_{xx} + I_{yy}$

MS of a circular section

Considering a circle of radius r with centre as O

Considering an elementary ring of radius x at and of thickness dx

$$dA = 2\pi x dx$$

MS of the ring about $x-x$ or $y-y$ axis

$$= \text{Area} \times (\text{distance})^2$$

$$= 2\pi x dx \times x^2$$

$$= 2\pi x^3 dx$$

$$\text{Now MS of the whole section} = I_{zz} = \int_0^r 2\pi x^3 dx$$

$$= 2\pi \left[\frac{x^4}{4} \right]_0^r = \frac{2\pi}{4} (r^4 - 0^4) = \frac{\pi}{2} r^4$$

$$\text{As } r = \frac{d}{2}$$

$$I_{zz} = \frac{\pi}{2} \left(\frac{d}{2} \right)^4 = \frac{\pi d^4}{2 \times 16} = \frac{\pi d^4}{32}$$

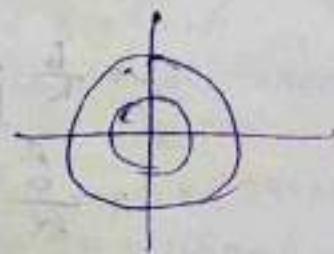
From $\perp$ axis theorem $I_{zz} = I_{xx} + I_{yy}$

$$\text{So } I_{xx} = I_{yy} = \frac{\pi d^4}{64}$$

MS of hollow circular section:-

$$I_{xx} = \frac{\pi}{64} (D^4 - d^4)$$

$$I_{yy} = \frac{\pi}{64} (D^4 - d^4)$$



$$= \frac{b}{k} \left(\frac{6h^4 + 3h^4 - 8h^4}{12} \right) = \frac{b}{k} \frac{h^4}{12} = \frac{bh^3}{12}$$

distance between C.G. of A & base is $= \frac{h}{3}$

So M.I. of the A section about an axis through its C.G. & parallel to x-x axis.

$$I_G = I_{AC} - ah^2$$

$$I_G = \frac{bh^3}{12} - \left(\frac{bh}{2} \right) \left(\frac{h}{3} \right)^2 = \frac{bh^3}{36}$$

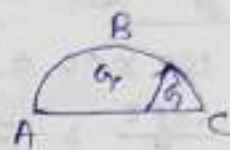
$$\begin{aligned} I_{xx} &= I_G + ah^2 \\ \Rightarrow I_G &= I_{xx} - ah^2 \end{aligned}$$

M.I. of Semi-circular section $\rightarrow$

$$I = \frac{1}{8} \times \frac{\pi}{64} d^4$$

M.I. of Δ about its base

$$A = \frac{\pi R^2}{2} \quad h = \frac{4R}{3\pi}$$



M.I. of Δ about C.G. $= I_G = I_{AC} - ah^2$

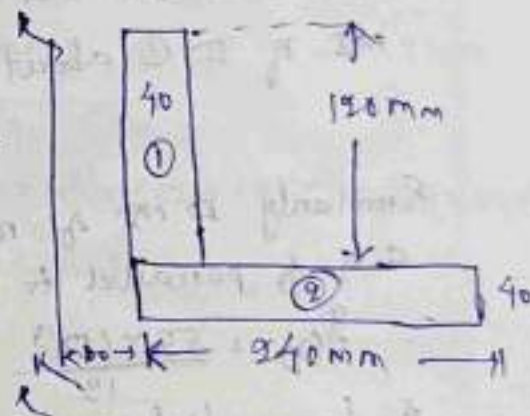
$$= \frac{\pi}{32} d^4 - \frac{\pi R^2}{2} \times \left(\frac{4R}{3\pi} \right)^2$$

Q calculate M.I. of the given fig. about x-x axis

Dividing it into 2 figures

We need to find C.G. of the 2 areas as M.I. will be found about x-x axis.

$$I_{G1} = \frac{40^3 \times 190}{12} = 640 \times 10^3 \text{ mm}^4$$



Distance betⁿ C.G. section ① & x-x $= h_1 = 190 + \frac{40}{2} = 210 \text{ mm}$

M.I. of sectⁿ ① about x-x $= I_{G1} + ah_1^2$

$$I_{G1} = \frac{40 \times 240^3}{12} = 46.08 \times 10^6 \text{ mm}^4$$

$$= (640 \times 10^3) + (190 \times 40 \times 190^2)$$

$$= 69.76 \times 10^6 \text{ mm}^4$$

Distance betⁿ C.G. of section ② & x-x $= h_2 = 190 + \frac{240}{2} = 210 \text{ mm}$

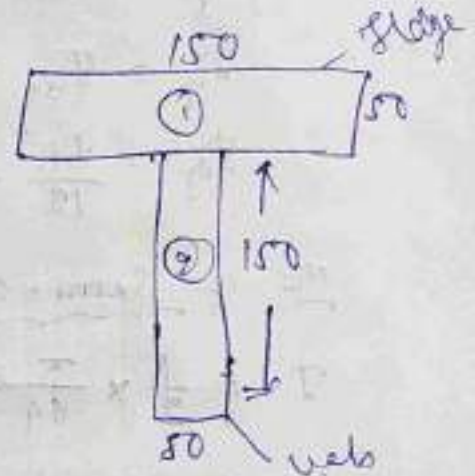
$$\begin{aligned} \text{M.I. of section ② about x-x} &= I_{G2} + ah_2^2 \\ &= (46.08 \times 10^6) + (240 \times 40 \times 210^2) \\ &= 50.72 \times 10^6 \text{ mm}^4 \end{aligned}$$

MT of the whole area about $x-x$

$$I_{xx} = (89.76 \times 10^6) + (510.72 \times 10^6) = 500.48 \times 10^6 \text{ mm}^4$$

Q-2 Find MT of a T-section with flange as $150 \text{ mm} \times 50 \text{ mm}$ & web of $150 \times 50 \text{ mm}$ about $x-x$ & $y-y$ axes through the G.G. of the section.

Solⁿ Rectangle ① (Symmetrical about $y-y$ axis)



$$a_1 = 150 \times 50 = 7500 \text{ mm}^2$$

$$y_1 = 150 + \frac{50}{2} = 175 \text{ mm}$$

Rectangle ②

$$a_2 = 150 \times 50 = 7500 \text{ mm}^2$$

$$y_2 = \frac{150}{2} = 75 \text{ mm}$$

$$e, \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500} = 125 \text{ mm}$$

MT about $x-x$ axis

$$I_{xx} = \frac{150 \times 50^3}{12} = 2.5625 \times 10^6 \text{ mm}^4$$

h_1 = distance betⁿ C.G. of $\square ①$ & $x-x$ axis = $175 - 125 = 50 \text{ mm}$

MT of $\square ①$ about $x-x$ axis = $I_{xx} + a h_1^2$

$$= (2.5625 \times 10^6) + (7500 \times 50^2)$$

$$= 30.1325 \times 10^6 \text{ mm}^4$$

Similarly MT of rectangle ② about an axis through G.G. & parallel to $x-x$ axis

$$I_{xx} = \frac{50 \times 150^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

h_2 = distance betⁿ G.G. of $\square ②$ & $x-x$ axis

$$= 125 - 75 = 50 \text{ mm}$$

MT of rectangle ② about $x-x$ axis $I_{xx} + a h_2^2$

$$= (14.0625 \times 10^6) + (7500 \times 50^2)$$

$$= 30.1325 \times 10^6 \text{ mm}^4$$

MT of $\square ①$ about $y-y$ axis = $\frac{50 (150^3)}{12} = 14.0625 \times 10^6 \text{ mm}^4$

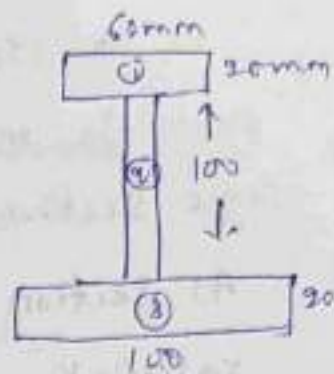
$$I_{yy} = \frac{150 \times 50^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

MT of whole section about y-y axis.

$$I_{yy} = (41.0625 \times 10^6) + (1.5625 \times 10^6) = 15.625 \times 10^6 \text{ mm}^4$$

An I-section is made up of 3 rectangles as shown in fig. Find the MT of the section about horizontal axis passing through the C.G. of the section.

Given section is symmetrical about y-axis.



□ ①

$$a_1 = 60 \times 20 = 1200 \text{ mm}^2$$

$$y_1 = 130 + \frac{20}{2} = 130 \text{ mm}$$

□ ②

$$a_2 = 100 \times 20 = 2000 \text{ mm}^2$$

$$y_2 = 20 + \frac{100}{2} = 70 \text{ mm}$$

□ ③

$$a_3 = 100 \times 20 = 2000 \text{ mm}^2$$

$$y_3 = 20/2 = 10 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000} = 60.8 \text{ mm}$$

$$\text{MT of } \square ① \text{ about its C.G. } \parallel \text{ to } x-x \text{ axis} = I_{g1} = \frac{60 \times 20^3}{12} = 40 \times 10^3 \text{ mm}^4$$

$$h_1 = \text{distance bet}^n \text{ C.G. of } \square ① \text{ \& } x-x \text{ axis} = 130 - 60.8 = 69.2 \text{ mm}$$

$$\begin{aligned} \text{MT of } \square ① \text{ about } x-x \text{ axis} &= I_{g1} + a_1 h_1^2 \\ &= (40 \times 10^3) + (1200 \times 69.2^2) = 57.76 \times 10^6 \text{ mm}^4 \end{aligned}$$

Similarly for $\square ②$ about C.G. $\parallel$ to axis.

$$I_{g2} = \frac{20 \times 100^3}{12} = 1666.7 \times 10^3 \text{ mm}^4$$

$$h_2 = 70 - 60.8 = 9.2 \text{ mm}$$

$$\begin{aligned} \text{MT of } \square ② \text{ about } x-x \text{ axis} &= I_{g2} + a_2 h_2^2 \\ &= (1666.7 \times 10^3) + (2000 \times 9.2^2) \\ &= 1836 \times 10^3 \text{ mm}^4 \end{aligned}$$

MT of $\square ③$ about axis through C.G. $\parallel$ to x-x axis.

$$I_{g3} = \frac{100 \times 20^3}{12} = 66.7 \times 10^3 \text{ mm}^4$$

$$h_3 = 60.8 - 10 = 50.8 \text{ mm}$$

$$\text{MT of } \square ③ \text{ about C.G. } \parallel \text{ to } x-x \text{ axis} = I_{g3} + a_3 h_3^2$$

$$= (66.7 \times 10^3) + (3600 \times 50.8^2) = 5928 \times 10^3 \text{ mm}^4$$

Now MR of the section about x-x axis.

$$I_{xx} = I_1 + I_2 + I_3$$

$$= (5786 \times 10^3) + (1836 \times 10^3) + (5928 \times 10^3)$$

$$= 19850 \times 10^3 \text{ mm}^4$$

Q
2018

Find the MR about the Centroidal x-x & y-y axes of the angle section shown.

Solⁿ

As section is unsymmetrical

calculation of C.G

□ ①

$$a_1 = 90 \times 100 = 9000 \text{ mm}^2$$

$$x_1 = 90/2 = 45 \text{ mm}$$

$$y_1 = 100/2 = 50 \text{ mm}$$

□ ②

$$a_2 = 60 \times 90 = 5400 \text{ mm}^2$$

$$x_2 = 90 + \frac{60}{2} = 120 \text{ mm}$$

$$y_2 = 90/2 = 45 \text{ mm}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(9000 \times 45) + (5400 \times 120)}{9000 + 5400}$$

$$= \frac{20,000 + 60,000}{9900} = 25 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(9000 \times 50) + (5400 \times 45)}{9000 + 5400} = 35 \text{ mm}$$

MR calculation about centroidal x-x axis

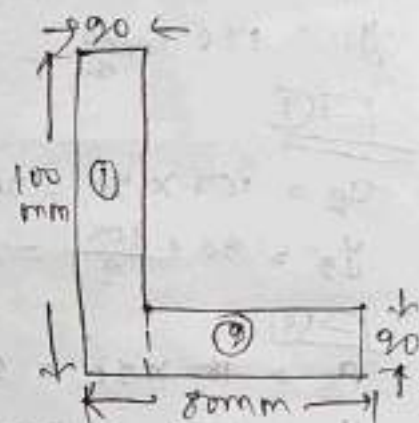
MR of □ ① about axis passing through C.G & || to x-x axis $I_{g1} = \frac{90 \times 100^3}{12} = 1.667 \times 10^6 \text{ mm}^4$

Distance bet axis passing through C.G of □ ① &

x-x axis $= h_1 = 50 - 35 = 15 \text{ mm}$

MR of □ ② about x-x axis $= I_{g2} + a_2 h_1^2$

$$= (1.667 \times 10^6) + (5400 \times 15^2) = 2.117 \times 10^6 \text{ mm}^4$$



Moment of $\square \textcircled{2}$ about C.G. = $\frac{60 \times 90^3}{12} = 0.04 \times 10^6 \text{ mm}^4$

Distance betⁿ C.G. of $\square \textcircled{2}$ & x-x axis = $h_2 = 35 - 10 = 25 \text{ mm}$

Moment of $\square \textcircled{2}$ about x-x axis = $I_{G2} + a_2 h_2^2$

= $(0.04 \times 10^6) + (1900 \times 25^2) = 0.39 \times 10^6 \text{ mm}^4$

Now Moment about x-x axis = $M_{x1} + M_{x2}$

= $(9.117 \times 10^6) + (0.39 \times 10^6)$

= $2.907 \times 10^6 \text{ mm}^4$

Moment calculation about Centroidal y-y axis

Moment of $\square \textcircled{1}$ about axis passing through C.G. & || to y-y axis

$I_{G1} = \frac{100 \times 20^3}{12} = 0.067 \times 10^6 \text{ mm}^4$

Distance betⁿ C.G. of $\square \textcircled{1}$ & y-y axis = $h_1 = 35 - 10 = 15 \text{ mm}$

Moment of $\square \textcircled{1}$ about y-y axis = $I_{G1} + a_1 h_1^2$

= $(0.067 \times 10^6) + (3000 \times 15^2) = 0.517 \times 10^6 \text{ mm}^4$

Similarly Moment of $\square \textcircled{2}$ about axis passing through C.G. & || to y-y axis

$I_{G2} = \frac{30 \times 60^3}{12} = 0.36 \times 10^6 \text{ mm}^4$

Distance betⁿ C.G. of $\square \textcircled{2}$ & y-y axis = $h_2 = 50 - 35 = 15 \text{ mm}$

Moment of $\square \textcircled{2}$ about y-y axis = $I_{G2} + a_2 h_2^2$

= $(0.36 \times 10^6) + (1900 \times 15^2) = 1.1 \times 10^6 \text{ mm}^4$

Moment of L section = $I_1 + I_2$

= $(0.517 \times 10^6) + (1.1 \times 10^6) = 1.629 \times 10^6 \text{ mm}^4$

END CHAPTER-4

SIMPLE MACHINES

Simple m/c can be defined as a device, which enables us to do some work at some point or to overcome some resistance, when an effort or force is applied to it at some other convenient point.

By use of simple m/c, heavy work can be ~~lifted~~ lifted with the help of less efforts.

Compound m/c →

It consists of a number of simple m/cs which enables us to do some useful work at a faster speed or with less effort as compared to simple m/c.

Lifting m/c →

It is a device, which enables us to lift a heavy load (w) by applying comparatively smaller effort (p).

Mechanical Advantage → $(M.A = \frac{w}{p})$

It is defined as the ratio between weight lifted (w) to the efforts applied (p) & is always expressed in pure number.

Mathematically, $M.A = \frac{w}{p}$

Input of a m/c →

It is the work done on the m/c.

In a lifting m/c, it is measured by the product of effort & the distance through which it is moved.

Output of m/c

It is the actual work done by the m/c

In a lifting m/c, it is measured by the product of the weight lifted & the distance through which it has been lifted.

Efficiency of a m/c →

Ratio betⁿ output to input of a m/c,
generally expressed in %.

$$\eta = \frac{o/p}{i/p} \times 100$$

Ideal m/c →

If efficiency of a m/c is 100%, i.e. $o/p = i/p$, the m/c is called as a perfect or an ideal m/c.

Velocity ratio →

Defined as the ratio betⁿ distance moved by effort (y) to the distance moved by the load (x) & is always expressed in pure numbers.

$$\text{Mathematically } V.R = \frac{y}{x}$$

Relation between η , M.A & V.R of a lifting m/c →

Let W = load lifted by the m/c

P = Effort required to lift the load

Y = Distance moved by the effort, in lifting the load

X = " " " " the load

$$M.A = \frac{W}{P} \quad \& \quad V.R = \frac{y}{x}$$

I/P of a m/c = effort applied $\times$ distance through which the effort has moved

$$= P \times y$$

O/P of a m/c = load lifted $\times$ distance through which load has been lifted

$$= W \times x$$

$$\text{Now } \boxed{\eta = \frac{O/P}{I/P} = \frac{Wx}{Py} = \frac{W}{P} \cdot \frac{x}{y} = \frac{M.A}{V.R}}$$

when $M.A = V.R$, then $\eta = 100\%$.

Q In a certain weight lifting m/c, a weight of 1kN is lifted by an effort of 25 N. while the weight moves up by 100 mm, the point of application of effort moves by 8 m. find M.A., V.R & η of a m/c.

Solⁿ

$$W = 1 \text{ kN} = 1000 \text{ N}$$

$$P = 25 \text{ N}$$

$$x = 100 \text{ mm} = 0.1 \text{ m}$$

$$y = 8 \text{ m}$$

$$M.A = \frac{W}{P} = \frac{1000}{25} = 40$$

$$V.R = \frac{y}{x} = \frac{8}{0.1} = 80$$

$$\eta = \frac{M.A}{V.R} = \frac{40}{80} = 0.5 = 50\%$$

CHAPTER-5 (starting on 1st page)

Reversibility of a machine:

Here machine can do some work in the reversed direction after effort is removed.

Condition for reversibility of a m/c:

Let w = Load lifted by the m/c

P = Effort required to lift the load.

y = distance moved by the effort.

x = distance moved by the load.

$$\text{I/P of the m/c} = P \times y$$

$$\text{O/P of the m/c} = w \times x$$

$$\text{m/c fraction} = \text{I/P} - \text{O/P} = (P \times y) - (w \times x)$$

For reversibility, $\text{I/P} > \text{O/P}$

$$\Rightarrow w \times x > (P \times y) - (w \times x)$$

$$\Rightarrow 2(w \times x) > (P \times y)$$

$$\Rightarrow \frac{w \times x}{P \times y} > \frac{1}{2}$$

$$\Rightarrow \frac{w/P}{y/x} > \frac{1}{2}$$

$$\Rightarrow \frac{\text{M.A.}}{\text{V.R.}} > \frac{1}{2}$$

$$\Rightarrow \eta > \frac{1}{2}$$

$$\Rightarrow \eta > 0.5$$

$$\Rightarrow \boxed{\eta > 50\%}$$

(non) Self locking machine or Non-Reversible m/c:

The m/c which can not work in reversed direction when effort is removed.

$$\text{Here } \boxed{\eta < 50\%}$$

Q A certain weight lifting m/c of V.R 30 can lift a load of 1500N with the help of 135N effort. Determine if the m/c is reversible.

Solⁿ Given V.R = 30
W = 1500N
P = 135N

$$M.A = \frac{W}{P} = \frac{1500}{135} = 12$$

$$\eta = \frac{M.A}{V.R} = \frac{12}{30} = 0.4 = 40\%$$

→ m/c is non-reversible.

Q In a lifting m/c, whose V.R is 50, an effort of 100N is required to lift a load of 4kN. Is the m/c reversible? If so, what effort should be applied so that the m/c is at the point of reversing.

Solⁿ Given V.R = 50
P = 100N
W = 4kN = 4000N

$$1) M.A = \frac{W}{P} = \frac{4000}{100} = 40$$

$$\eta = \frac{M.A}{V.R} = \frac{40}{50} = 0.8 = 80\% > 50\%$$

→ Reversible machine.

ii) The m/c will be at the point of reversing at $\eta = 50\%$.

Let P_1 = Effort required to lift a load of 4000N when m/c is at the point of reversing.

$$M.A = \frac{W}{P_1} = \frac{4000}{P_1}$$

$$\eta = \frac{M.A}{V.R} \Rightarrow 0.5 = \frac{4000/P_1}{50} = \frac{80}{P_1}$$

$$\Rightarrow P_1 = \frac{80}{0.5} = 160N \text{ (Ans)}$$

Friction in a machine :

Let P = Actual effort required to lift load

P' = Ideal " " "

W = Actual load to be lifted.

W' = Ideal " " "

$$\eta = \frac{M.A}{V.R} = \frac{W/P}{V.R} \quad \& \quad V.R = \frac{W}{P} \quad (\text{For Ideal m/c } \eta = 1)$$

$$\text{For actual m/c } V.R = \frac{W}{P'} \Rightarrow P' = \frac{W}{V.R}$$

$$\frac{P - P'}{P} = P - \frac{W}{V.R}$$

$W' - W$ is the friction

$$\text{Finally } \boxed{F_{\text{load}} = (P \times V.R) - W}$$

Q In a certain m/c, an effort of 100 N is just able to lift a load of 840 N. Calculate η & Friction both on effort & load side, if V.R of m/c is 10.

Soln Given data $P = 100 \text{ N}$
 $W = 840 \text{ N}$
 $V.R = 10$

$$i) M.A = \frac{W}{P} = \frac{840}{100} = 8.4$$

$$\eta = \frac{M.A}{V.R} = \frac{8.4}{10} = 0.84 = 84\%$$

ii) Friction of m/c

Friction of the m/c in terms of effort

$$F_{\text{effort}} = P - \frac{W}{V.R} = 100 - \frac{840}{10} = 16 \text{ N}$$

Friction of the m/c in terms of load

$$F_{\text{load}} = (P \times V.R) - W = (100 \times 10) - 840 = 160 \text{ N}$$

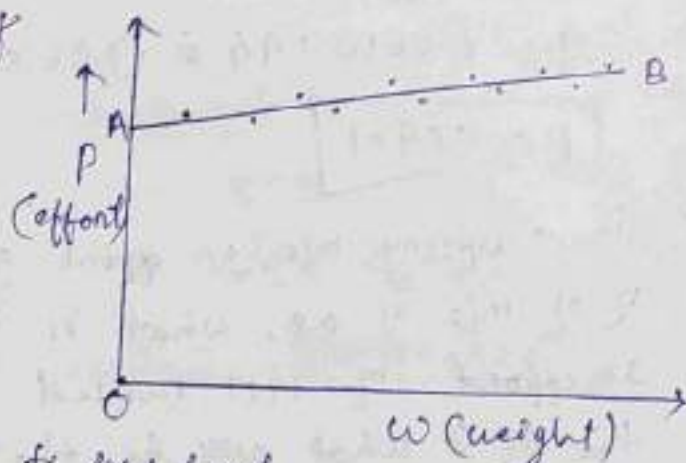
Laws of Machine:

It is defined as the relationship between the effort applied and the load lifted.

OA $\rightarrow$ represent amount of friction offered by m/c

mathematically, law of lifting m/c relation is

$$P = mW + C$$



where P = effort required to lift load

m = const. called as coeff of friction

$\rightarrow$ Slope of line AB

W = load lifted

C = const. to represent m/c friction i.e. OA

Q.1 What load can be lifted by an effort of 190N. If the V.R is 18 & η of the m/c at this load is 50%. Determine the law of m/c, if it is observed that an effort of 300N is required to lift a load of 2600N & find the effort required to turn the m/c at a load of 3.1kN.

Sol Given that: $P = 190N$ i) $W = ?$
 $V.R = 18$ ii) law of m/c = ?
 $\eta = 50\% = 0.5$ iii) P for $W = 3.1kN$

$$i) M.A = \frac{W}{P} = \frac{W}{190} \quad \text{--- (1)}$$

$$\eta = \frac{M.A}{V.R} \Rightarrow 0.5 = \frac{W}{190 \times 18} \Rightarrow \frac{W}{180} \Rightarrow W = 1996N$$

ii) Here $P = 300N$, $W = 2600N$

law of m/c is $P = mW + C$

$$190 = m \times 1996 + C \quad \text{--- (i)}$$

$$300 = m \times 2600 + C \quad \text{--- (ii)}$$

$$\text{Eqn (i) - (ii)} \Rightarrow 80 = 1304m \Rightarrow m = \frac{80}{1304} = 0.06$$

Now Eqn (ii) becomes $300 = (0.06 \times 2600) + C$

$$\Rightarrow C = 44$$

$$\frac{190}{18} = \frac{W}{180} \Rightarrow W = 1996$$

$$1996$$

$$30, P = m\omega + c$$

$$\Rightarrow P = 0.06\omega + 44$$

(m) putting $\omega = 3.5 \text{ kN}$

$$P = 0.06\omega + 44 = (0.06 \times 3.5 \times 1000) + 44 = 254 \text{ N}$$

$$\boxed{P = 254 \text{ N}}$$

Q-2 In a lifting m/c, an effort of 40 N raised a load of 1000 N . If η of m/c is 0.5 , what is the V.R. of the m/c, an effort of 74 N raised a load of 2000 N , what is now the η ? what will be the effort required to raise a load of 5 kN ?

Sol Given data i) $P = 40 \text{ N}$

$$\omega = 1 \text{ kN} = 1000 \text{ N}$$

$$\eta = 0.5$$

$$\text{V.R.} = ?$$

$$\text{ii) } P = 74 \text{ N}$$

$$\omega = 2000 \text{ N}$$

$$\eta = ?$$

$$\text{iii) } P = ?$$

$$\omega = 5 \text{ kN}$$

$$\text{i) } M.A = \frac{\omega}{P} = \frac{1000}{40} = 25$$

$$\eta = \frac{M.A}{\text{V.R.}} \Rightarrow 0.5 = \frac{25}{\text{V.R.}} \Rightarrow \text{V.R.} = \frac{25}{0.5} = 50 \text{ Att.}$$

$$\text{ii) } M.A = \frac{\omega}{P} = \frac{2000}{74} = 27$$

$$\eta = \frac{M.A}{\text{V.R.}} = \frac{27}{50} = 0.54 = 54\%$$

iii) from law of m/c η from two above condⁿs: $40 = m \times 1000 + c$ (i)

$$74 = m \times 2000 + c$$
 (ii)

$$\text{Eqn (ii) - (i)} \Rightarrow 34 = 1000m \Rightarrow m = 0.034$$

$$\text{Now Eqn (i) becomes } 40 = (0.034 \times 1000) + c$$

$$\Rightarrow c = 6$$

$$\text{Now } P = 0.034\omega + 6$$

$$\text{For } \omega = 5 \text{ kN} = 5000 \text{ N}, P = (0.034 \times 5000) + 6$$

$$= 176 \text{ N}$$

Q what load will be lifted by an effort of 12 N , if the V.R. is 18 & η of the m/c at this load is 60% .

If the m/c has a const friction resistance, determine the law of m/c & find effort required to run the m/c at

i) no load ii) a load of 900 N

Given data $P = 12 \text{ N}$

$$V.R = 18$$

$$\eta = 60\% = 0.6$$

$$W = ?$$

$$M.A = \frac{W}{P} = \frac{W}{12}$$

$$\eta = \frac{M.A}{V.R} \Rightarrow 0.6 = \frac{W}{12 \times 18} \Rightarrow W = 0.6 \times 216 = 129.6 \text{ N (Ans)}$$

Law of m/c

$$\text{Effort lost in friction} = F_{\text{effort}} = P - \frac{W}{M.A} = 12 - \frac{129.6}{18} = 4.8 \text{ N}$$

4.8 N is the const frictional resistance offered by the m/c.

Now putting $P = 12$ & $C = 4.8$

$$P = mW + C$$

$$\Rightarrow 12 = (m \times 129.6) + 4.8 \Rightarrow m = \frac{1}{8}$$

$$\text{So, Now } P = \frac{1}{8} W + 4.8$$

i) When $W = 0$, $P = 4.8$

ii) when $W = 900$, $P = \left(\frac{1}{8} \times 900\right) + 4.8 = 84.8 \text{ N}$

Max^m M.A of a lifting m/c - 7

$$M.A = \frac{W}{P}$$

for max^m M.A, $P = mW + C$

$$\text{So max^m M.A} = \frac{W}{mW + C} = \frac{1}{m + \frac{C}{W}} = \frac{1}{m} \quad \left(\because \text{neglecting } \frac{C}{W}\right)$$

Max^m η of a lifting m/c - 7

$$\eta = \frac{M.A}{V.R} = \frac{W/P}{V.R} = \frac{W}{P \times V.R}$$

$$\eta_{\text{max}} = \frac{W}{(m + \frac{C}{W}) \times V.R} = \frac{1}{m \times V.R} \quad \left(\because \text{neglecting } \frac{C}{W}\right)$$

Transmission of power by gear drive:-

Gear can be defined as a pulley or wheel having projection on its rim called as teeth.

In gear drive the problem of slipping (in case of belt drive, rope drive, chain drive), is reduced so it has max^m power transmission efficiency. So it is a positive drive. It is used when distance between driver & driven shaft is less.

Advantages of gear drive: -

- Transmits exact velocity ratio.
- High η of power transmission
- Has compact structure
- Can transmit large power
- Has a reliable service.

Disadvantages -

- Manufacturing of toothed wheels requires special tools & equipments.
- Any error in teeth machinery causes noise & vibration during operation.
- Any defect in one wheel damages the whole set up

Pitch: -

Centre to centre distance between two consecutive teeth is called pitch.

$$\text{Mathematically, } p = \frac{\pi d}{T}$$

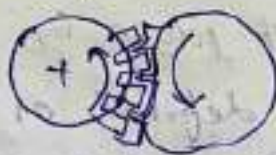
where d - dia of pitch circle

T - No. of teeth

* For meshing, two gears on wheels should have same pitch

Types of gear: -

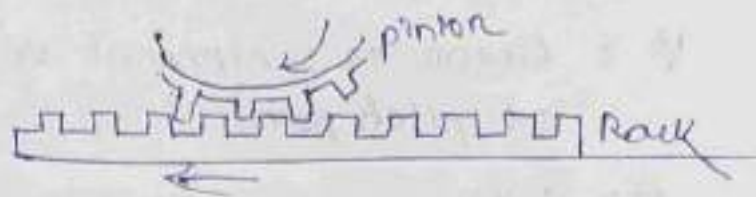
- 1) External gear - Here gears of two shafts mesh externally with each other.



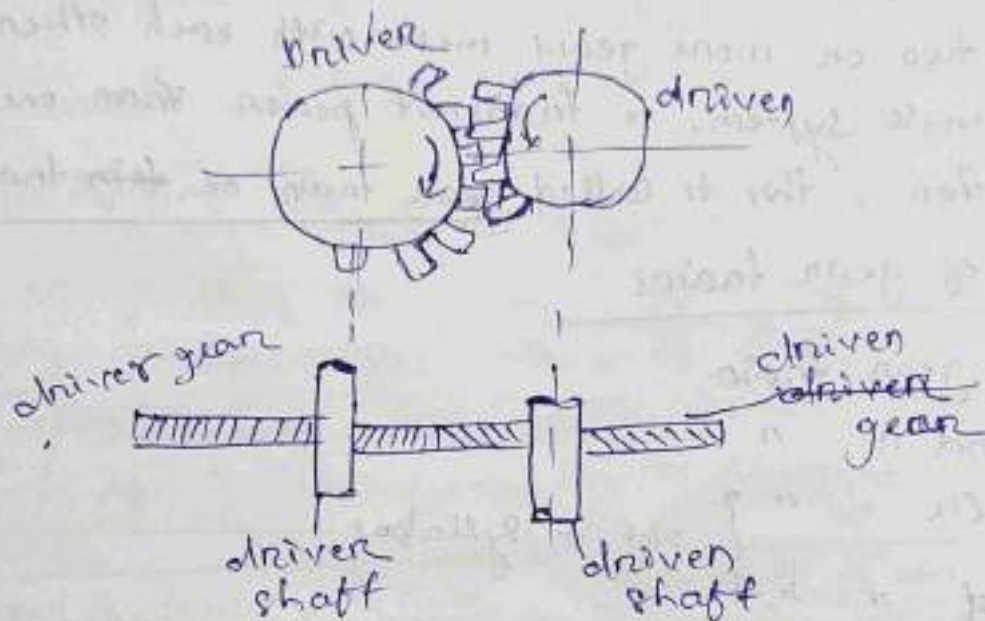
- 2) Internal gear - Here gears of two shafts mesh internally.



3) Rack & pinion $\rightarrow$ Here gear of a shaft meshes internally or externally with the gear in a straight line



Simple gear drive -



When both shafts are connected by meshing of two gears power will be transmitted from driver shaft to driver gear then to driven gear & finally to driven shaft
* Gears are always mounted on the shaft.

velocity ratio of simple gear drive $\rightarrow$

It is the ratio betⁿ speed of driver to driven member
Let $N_1, N_2 \rightarrow$ Speed of driver & driven respectively

$T_1, T_2 \rightarrow$ No of the teeth driver & driven respectively

$d_1, d_2 \rightarrow$ dia for — " " " "

pitch $p = \frac{\pi d_1}{T_1}$ for driver, $p = \frac{\pi d_2}{T_2}$ for driven

for meshing of two gears $\frac{\pi d_1}{T_1} = \frac{\pi d_2}{T_2} \Rightarrow \frac{d_1}{d_2} = \frac{T_1}{T_2}$

$$V.R = \frac{N_1}{N_2} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

power transmitted by a simple gear:-

let F = Tangential force by the driver
(i.e. pressure betⁿ the teeth)

V = linear or peripheral velocity of the driver
at pitch point.

$$P = F \times V$$

Gear Train →

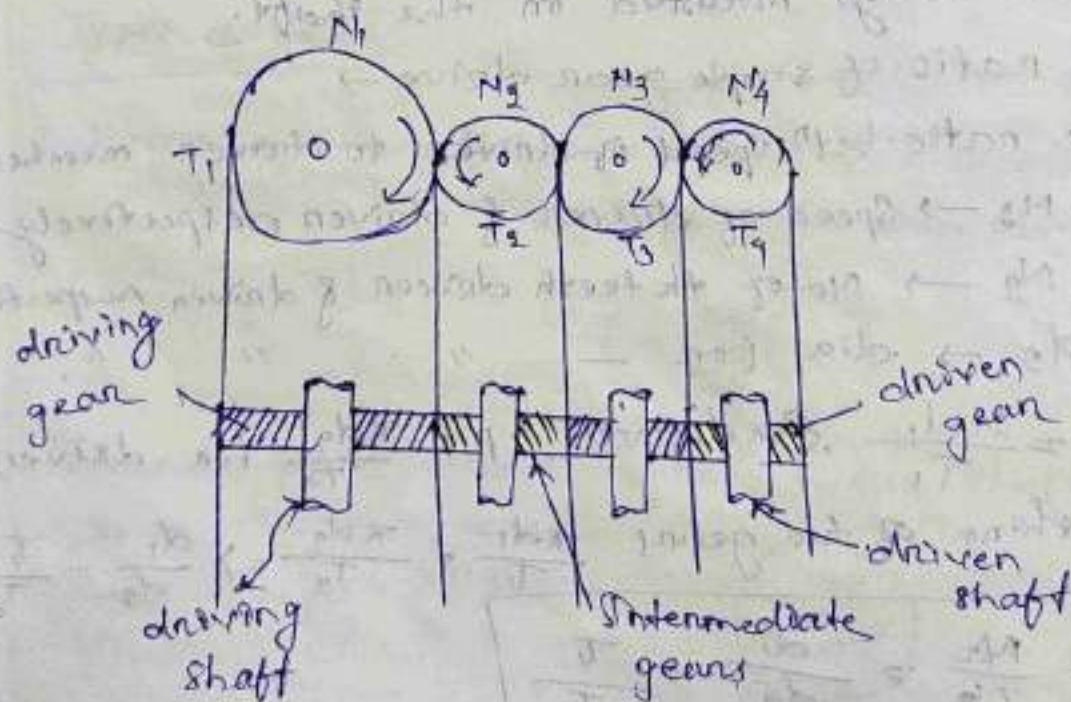
When two or more gears mesh with each other to operate as a single system, to transmit power from one shaft to another, this is called gear train or train of wheels.

Types of gear trains

- 1) Simple gear train
 - 2) Compound " "
 - 3) Epicyclic " "
 - 4) Reverted " "
- } Not in Syllabus

1) Simple gear train:

If one gear is located in each shaft then the gear train is called simple gear train.



Let N_1, T_1 = Speed & No of teeth of driving gear respectively.

• $N_2, N_3, T_5, T_3 = 11$ " " " " " Intermediate gear "

$N_0, T_0 = \dots$ driven $\dots$

As gear 1 & 2 are meshed:

$$\frac{N_b}{N_e} = \frac{T_1}{T_2} \quad \text{--- (6)}$$

Similarly gear 2 & 3 are meshed so, $\frac{\omega_2}{\omega_3} = \frac{T_3}{T_2}$ — (3)

$$150 \frac{\gamma \ln}{M_3} = \frac{T_3}{T_4} \quad \text{--- (2)}$$

Multiplying eqn (1), (2) & (3)

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} = \frac{T_1}{T_2} \times \frac{T_2}{T_3} \times \frac{T_3}{T_4}$$

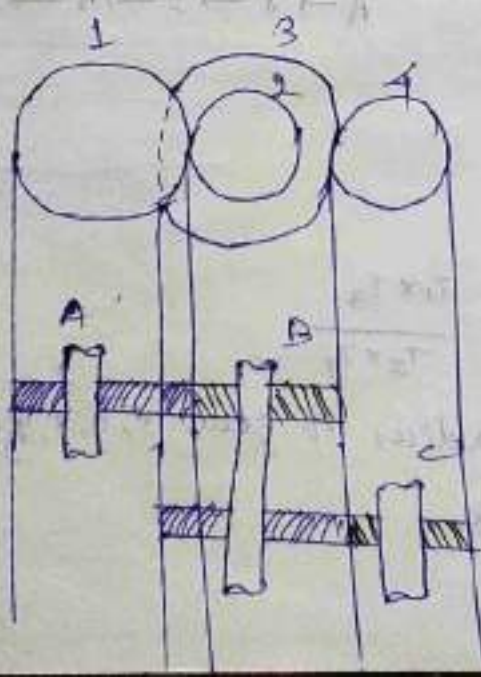
$$\Rightarrow \frac{n_1}{n_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \frac{\text{Speed of train or follower}}{\text{Speed of driver}} = \frac{\text{No. of teeth of driver}}{\text{No. of teeth of driven}}$$

* Compound gear train

• If more than one gear is located on one of the shafts in a gear train then it is called compound gear train.

- It is used to achieve higher speed ratios in small space.



Seq. sequence of power transmission

$$A \rightarrow 1 \rightarrow 2 \rightarrow B \rightarrow 3 \rightarrow 4 \rightarrow C$$

Here intermediate gears play role in power transmission

Let $N_1, T_1 \rightarrow$ Speed & No. of teeth of driver

$N_2, T_2 / N_3, T_3 \rightarrow$ " " " of Intermediate gears

$N_4, T_4 \rightarrow$ " " " of driven gears

Gear 1 & 2 are meshed. So, $\frac{N_2}{N_1} = \frac{T_1}{T_2} \quad \text{--- (1)}$

Similarly gear 3 & 4 are meshed, So, $\frac{N_4}{N_3} = \frac{T_3}{T_4} \quad \text{--- (2)}$

Eqn (1) $\times$ Eqn (2)

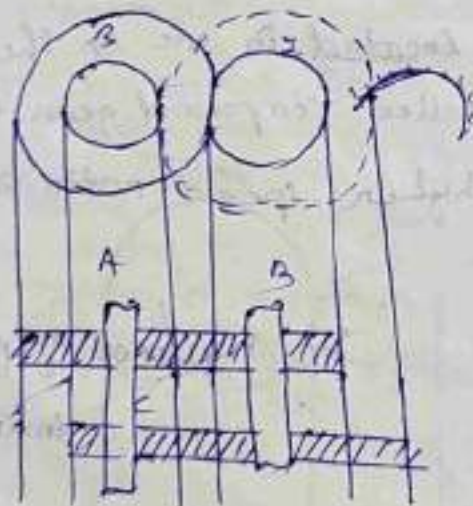
$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

But $N_2 = N_3$ So $\frac{N_4}{N_1} = \frac{T_1 \times T_3}{T_2 \times T_4}$

$\rightarrow$ Speed ratio = $\frac{\text{Product of No. of teeth in driving gears}}{\text{Product of No. of teeth in driven gears}}$

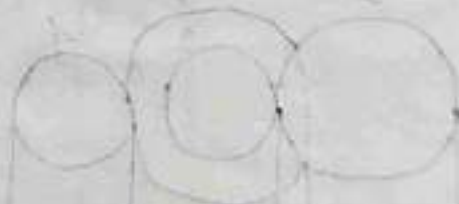
5) Reversed gear train. (Not in Syllabus)

It is a special type of compound gear train where, the driving gear & driven gear are co-axial.



Sequence of power transmission

$A \rightarrow 1 \rightarrow 2 \rightarrow B \rightarrow 3 \rightarrow 4 \rightarrow X$



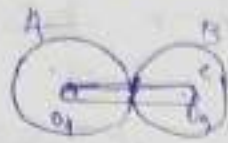
$$\text{Speed ratio} = \frac{N_4}{N_1} = \frac{T_1 \times T_3}{T_2 \times T_4}$$

Let $r_1, r_2, r_3, r_4 \rightarrow$ radius of gear 1, 2, 3, & 4 respectively

Hence $r_1 + r_2 = r_3 + r_4$

Epi-cyclic gear train

Here one of the wheel is fixed & the other makes revolution about the fixed wheel with the help of an ^{arm} ~~rod~~ it is also called as sub-planet gear train.



Here wheel A & arm c has common axis at O_1 , about which they can rotate.

The wheel B meshes with wheel A & has its own axis at O_2 . Arm c is fixed. Now wheels A & B form a simple gear train but if wheel A is fixed & the arm is rotate, the system form an ~~ep~~ epi-cyclic gear train.

Study of Simple Machines: -

- Here application of principles of lifting machine has to be discussed
- Main aim is to increase the M.A of the m/c so that heavier load can be lifted by applying smaller effort
- Some examples of simple lifting m/c are

1. Simple wheel & axle.

2. Differential wheel & axle

3. Worm & worm wheel

4. Single purchase crab winches

5. Double "

6. pulleys

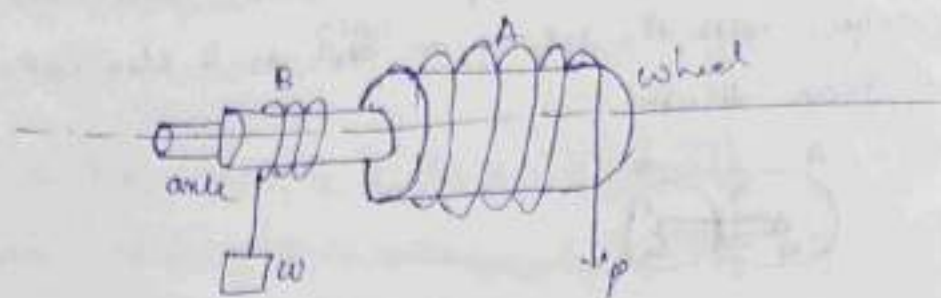
7. Simple screw jack

8. Differential "

$$\frac{W}{P} = \frac{\text{load}}{\text{effort}} = \text{M.A.}$$

$$\frac{\text{M.A.}}{P} = \text{P.A.}$$

§ Simple wheel and axle: -



Here wheel A & axle B are keyed with same shaft. A string is wound round the axle B which carries load W to be lifted.

A second string is wound round the wheel A, in opposite dirⁿ to that of string on B.

Let D = Diameter of the effort wheel

d = " " " " Load axle

W = load to be lifted

P = Effort required

One end of string is lifted to wheel & other end is free where effort is applied.

As both strings are wound oppositely, so downward motion of effort P will rise the load W .

Both wheel & axle are keyed to same shaft.

Displacement of effort in one revolution of wheel A = πD

Displacement of load in one revolution = πd

$$V.R = \frac{\text{Distance moved by effort}}{\text{" " " " load}} = \frac{\pi D}{\pi d} = \frac{D}{d}$$

$$\therefore M.A = \frac{\text{load}}{\text{effort}} = \frac{W}{P}$$

$$\therefore \eta = \frac{M.A}{V.R}$$

A simple wheel & axle has wheel & axle diameters of 30mm & 30mm respectively. Find efficiency of the machine. If it can lift a load of 900N by an effort of 100N.

solⁿ Given data $D = 30\text{mm}$ $W = 900\text{N}$
 $d = 30\text{mm}$ $P = 100\text{N}$
 $\eta = ?$

$$V.R = \frac{D}{d} = \frac{30}{30} = 10$$

$$M.A = \frac{W}{P} = \frac{900}{100} = 9$$

$$\text{So } \eta = \frac{M.A}{V.R} = \frac{9}{10} = 0.9 = 90\%$$

Q A drum weighing 60N & holding 450N of water is to be raised from a pulley by means of wheel & axle. The axle is 100mm in dia & the wheel is 500mm in dia. If force of 150N is to be applied to the wheel, find.

i) M.A ii) V.R iii) η

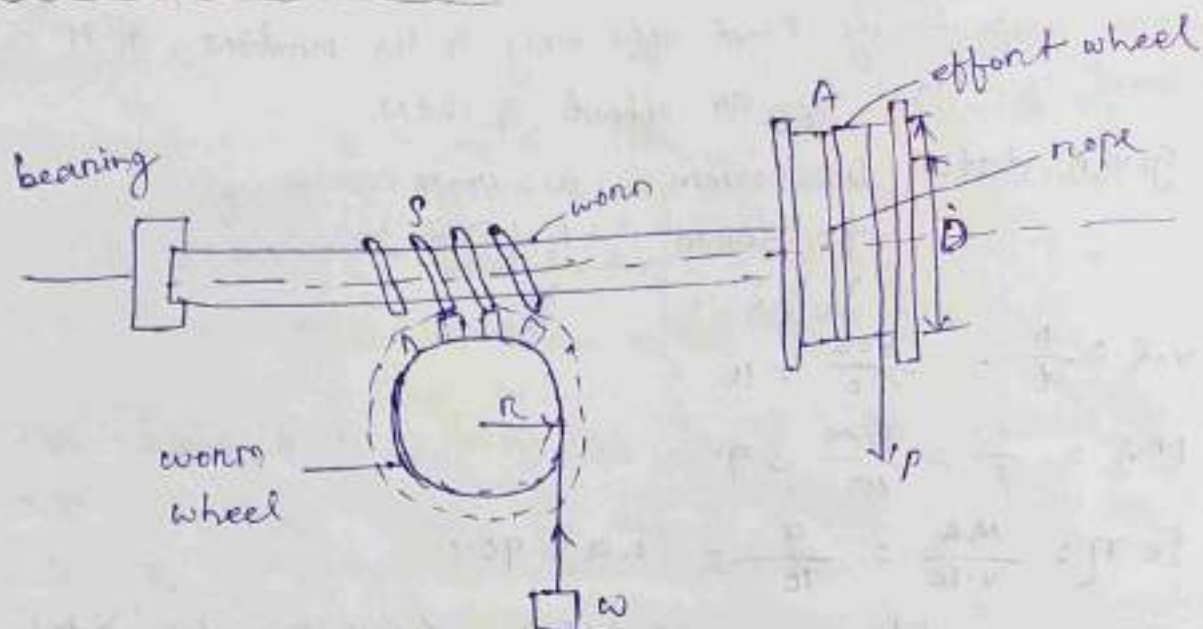
solⁿ Given data Total load to be lifted $W = 60 + 450 = 510\text{N}$
 $d = 100\text{mm}$
 $D = 500\text{mm}$
 $P = 150\text{N}$

$$M.A = \frac{W}{P} = \frac{510}{150} = 3.4$$

$$V.R = \frac{D}{d} = \frac{500}{100} = 5$$

$$\eta = \frac{M.A}{V.R} = \frac{3.4}{5} = 0.68 = 68\%$$

9) Worm and worm wheel :-



It consists of square threaded screws (known as worms) of toothed wheel (called as worm wheel) geared with each other. wheel A is attached to the worm, over which a rope passes. A load drum is mounted on the worm wheel.

Let D = Diameter of the effort wheel

R = radius of the load drum

W = load to be lifted

P = Effort required to lift load

T = No. of teeth on the worm wheel

Distance moved by the effort in one revolution of the wheel = πD

If worm is single threaded (i.e. - one revolution of wheel A pushes screw S by one teeth). then load drum will move through = $\frac{1}{T}$ revolution

Now distance moved by load $\frac{\pi R}{T}$

$$V.R = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= \frac{\pi D}{\pi R/T} = \frac{DT}{2R}$$

$$\therefore \frac{W}{P} \times R = \frac{DT}{2R} \quad \& \quad R = \frac{MT}{V.R}$$

* If worm is double threaded (i.e. one revolution of worm wheel, pushes ~~to~~ two teeth of screw)

$$\text{Here } V.R = \frac{DT}{2 \times 2\pi R} = \frac{DT}{4\pi R}$$

* If worm is ~~n~~ threaded then $V.R = \frac{DT}{2\pi n R}$

* A worm & worm wheel with 40 teeth on the worm wheel has effort of 300mm dia & load drum of 100mm dia. Find the efficiency of the machine. If it can lift a load of 180N with an effort of 24N.

Given data No. of teeth on worm wheel = $T = 40$

Dia of effort wheel = $D = 300\text{mm}$

Dia of load drum = $d = 100\text{mm} \Rightarrow R = 50\text{mm}$

$$W = 180\text{N}$$

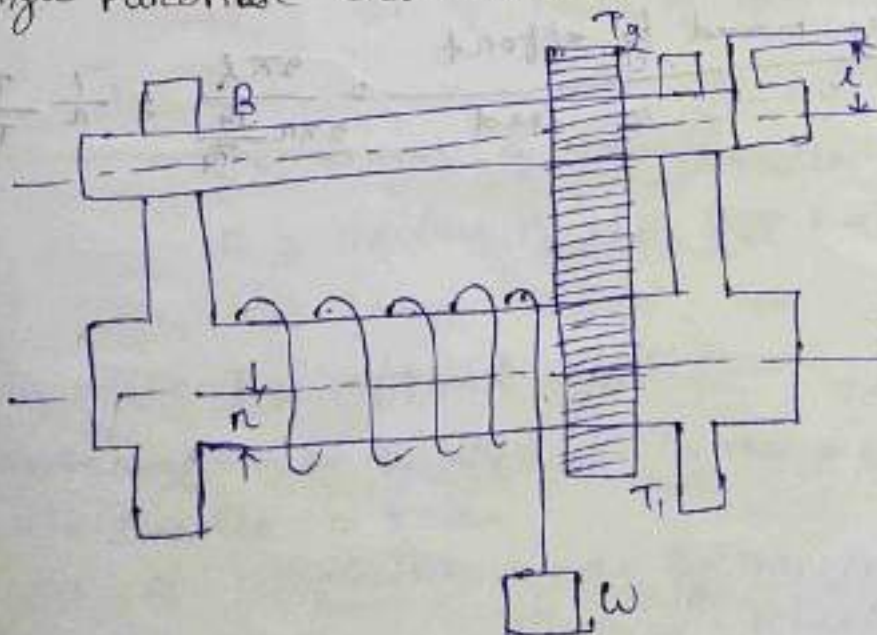
$$P = 24\text{N}$$

$$V.R = \frac{DT}{2\pi R} = \frac{300 \times 40}{2 \times 50} = 120$$

$$M.A = \frac{W}{P} = \frac{180}{24} = 7.5$$

$$\eta = \frac{M.A}{V.R} = \frac{7.5}{120} = 0.0625 = 6.25\%$$

Single Purchase crab winch:-



It consist of a rope fixed to the drum, which is wound around it.

At the free end of the rope, load is attached. A toothed wheel 'A' is rigidly mounted on load drum. Another toothed wheel 'B' called as pinion is geared with toothed wheel A as shown in fig.

Effort is applied at the end of the handle to rotate it.

Let T_1 = No. of teeth on main gear A

T_2 = " " " " " pinion gear B

l = length of the handle

r = radius of load drum

W = load lifted

P = effort applied.

Distance moved by effort in one revolution of handle = $2\pi l$

No. of revolution made by pinion B = 1

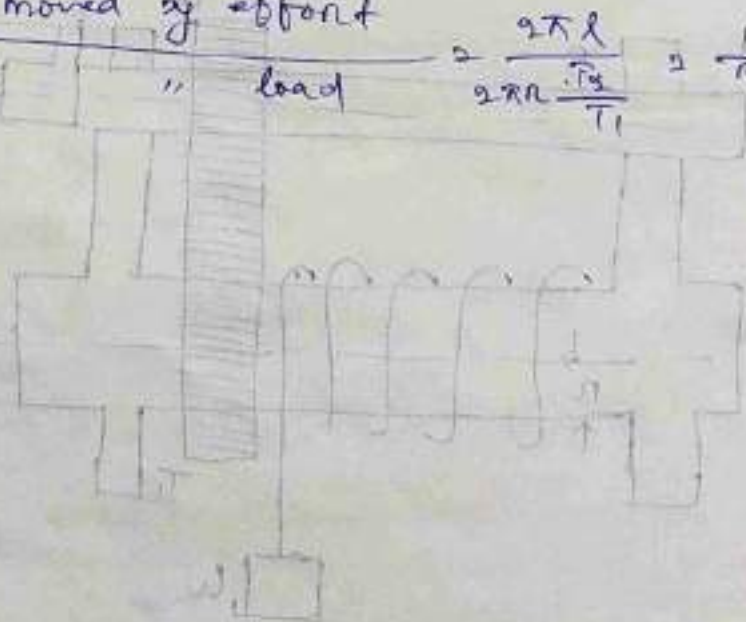
No. of revolution made by wheel A = $\frac{T_2}{T_1}$

Distance moved by the load = $2\pi r \times \frac{T_2}{T_1}$

$$V.R = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{2\pi l}{2\pi r \cdot \frac{T_2}{T_1}} = \frac{1}{r} \cdot \frac{T_1}{T_2}$$

$$M.A = \frac{W}{P}$$

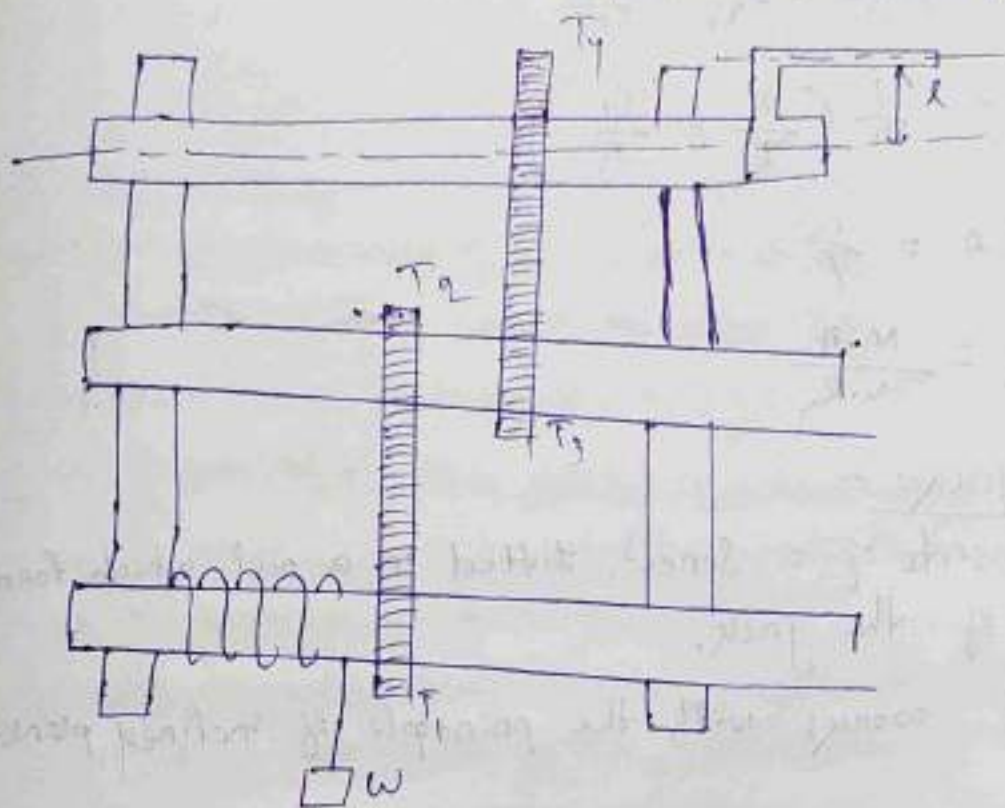
$$\eta = \frac{M.A}{V.R}$$



Double Puncher Crab winch:-

It is similar to ~~single~~ Single Puncher crab wheel winch. Here velocity ratio is intensified with the help of one more spur wheel & a pinion.

Here there are 2 spur wheels & 2 pinions.



Here Spur wheel of teeth T_2 is in mesh with pinion of teeth T_1 .

& Spur wheel of teeth T_4 is in mesh with pinion of teeth T_3 .

Let $T_1, T_3 =$ No. of teeth of spur wheels

$T_2, T_4 =$ " " " " pinions

$L =$ length of the handle

$R =$ radius of the load drum

$W =$ load lifted

$P =$ effort applied

Distance moved by effort in one revolution of the handle $= 2\pi L$

No. of revolution made by pinion 1 $= 1$

wheel 3 $= \frac{T_1}{T_2}$

pinion 2 $= \frac{T_4}{T_3}$

wheel 1 $= \frac{T_2}{T_1} \times \frac{T_4}{T_3}$

Distance moved by the load = $2\pi R \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}$

$$V.R = \frac{\text{Distance moved by effort}}{\text{Distance moved by Load}}$$

$$= \frac{2\pi R}{2\pi R \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}}$$

$$= \frac{1}{n} \left(\frac{T_1}{T_2} \times \frac{T_4}{T_3} \right)$$

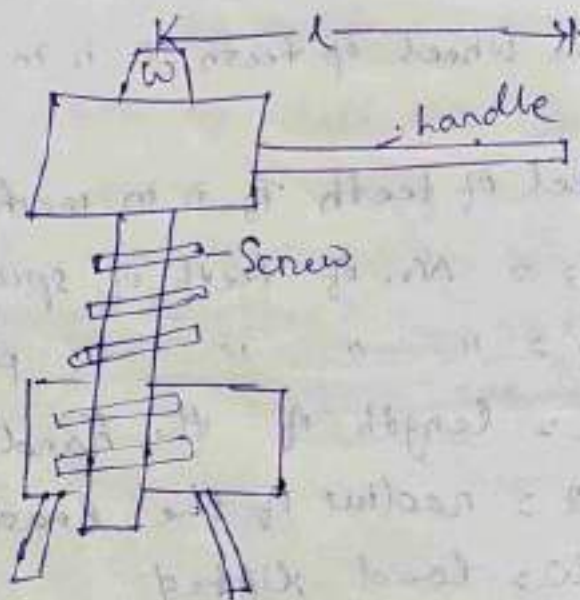
$$\text{Now } M.A = \frac{W}{P}$$

$$\eta = \frac{M.A}{V.R}$$

Screw jack:-

It consists of a screw, fitted in a nut, which forms the body of the jack.

It works with the principle of inclined plane.



Simple screw jack, which is rotated by the application of an effort at the end of the lever, for lifting the load.

Let l = length of effort arm

p = pitch of the screw

W = load lifted

P = effort applied

Distance moved by the effort in one revolution of screw = $2\pi l$

Distance moved by the load $= p$

$$V.R = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= \frac{2\pi l}{p}$$

$$M.A = \frac{W}{P}$$

$$\eta = \frac{M.A}{V.R}$$

Q. A screw jack has a thread of 10mm pitch. What effort applied at the end of a handle of 400mm long will be required to lift a load of 2kN, if the η at this load is 45%.

Sol

Given data $p = 10\text{mm}$

$$l = 400\text{mm}$$

$$W = 2\text{kN} = 2000\text{N}$$

$$\eta = 45\% = 0.45$$

$$V.R = \frac{2\pi l}{p} = \frac{2\pi \times 400}{10} = 251.2$$

$$M.A = \frac{W}{P} = \frac{2000}{P}$$

$$\eta = \frac{M.A}{V.R}$$

$$\Rightarrow 0.45 = \frac{2000}{P \times 251.2}$$

$$\Rightarrow P = \frac{2000}{0.45 \times 251.2} = 15.7\text{ N} \quad \underline{\text{Ans}}$$



Dynamics is the study of body in motion. Dynamics is based on three fundamental laws of motion proposed by Sir Isaac Newton in 1680.

Newton's Law's of motion

1) 1st Law

It states that, "Every body continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force."

It is also called as law of inertia.

Ex - 1) A body at rest want to be in rest until some external force is applied.

2) A moving body want to move in uniform velocity along the same dirⁿ if any external force is not used.

2) 2nd Law

It states that, "The rate of change of momentum is directly proportional to the impressed force & takes place in the same dirⁿ in which the force acts."

It is also called as law of dynamics.

It consist of two parts.

a) A body can possess acceleration only when some force is applied on it.

b) The force applied on the body is proportional to the product of mass of the body & the acceleration produced in it i.e. $F \propto ma$

$$\boxed{\text{Force} = \text{mass} \times \text{acc}^n}$$

$$\begin{aligned} v &= s/t \\ a &= v/t \end{aligned}$$

Rate of change of momentum

$$\frac{m(v-u)}{t} = \frac{m(v-u)}{t} = ma$$

$u \rightarrow$ Initial velocity

$v \rightarrow$ Final "

$m \rightarrow$ mass of the body

$t \rightarrow$ time taken in sec

Accⁿ to Newton's 2nd law of motion

$$F = ma$$

* 1 Newton is defined as the force acting on 1kg mass of a body & it produces acceleration of 1 m/s^2

Q1 Determine the force, which can move a body of mass 100kg with an accⁿ of 2.5 m/s^2

Solⁿ Given $m = 100 \text{ kg}$, $a = 2.5 \text{ m/s}^2$

$$\text{So } F = ma = 100 \times 2.5 = 250 \text{ N.}$$

Q2 A body has 50kg mass on the earth. Find its weight

- a) on the earth where $g = 9.8 \text{ m/s}^2$
- b) on the moon where $g = 1.7 \text{ m/s}^2$
- c) on the sun where $g = 270 \text{ m/s}^2$

Solⁿ Given that $m = 50 \text{ kg}$

$$\text{a) } F = mg = 50 \times 9.8 = 490 \text{ N}$$

$$\text{b) } F = mg = 50 \times 1.7 = 85 \text{ N}$$

$$\text{c) } F = mg = 50 \times 270 = 13500 \text{ N.}$$

Q3 A body of mass 7.5kg is moving with a velocity of 1.2 m/s . If a force of 15 N is applied on the body determine its velocity after 9s.

Solⁿ Given $m = 7.5 \text{ kg}$, $F = 15 \text{ N}$, $t = 9 \text{ s}$

$$u = 1.2 \text{ m/s}, \quad v = ?$$

$$\text{Accⁿ } a = \frac{F}{m} \quad [\because F = ma]$$

$$= \frac{15}{7.5} = 2 \text{ m/s}^2$$

$$\text{Again } v = u + at$$

$$= 1.2 + (2 \times 9) = 19.2 \text{ m/s}$$

Three important relatⁿ of law of motion are

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2$$

$$v^2 - u^2 = 2as$$

u → initial velocity

v → final "

s → displacement of the body

a → accⁿ of the body

t → time period.

Q.4 A vehicle of mass 500 kg is moving with a velocity of 35 m/s. A force of 900 N acts on it for 2 minutes. Find the velocity of the vehicle.

a) when the force acts in the direction of motion.

b) " " " " in the opposite direction of the motion.

Soln

Given $m = 500 \text{ kg}$
 $u = 35 \text{ m/s}$

$F = 900 \text{ N}$

$t = 2 \text{ min} = 120 \text{ sec}$

a) we know that $F = ma \Rightarrow a = F/m = \frac{900}{500} = 1.8 \text{ m/s}^2$

$v = u + at$

$$= 35 + (1.8 \times 120) = 54.6 \text{ m/s}$$

Here $a = 1.8 \text{ m/s}^2$

b) v at 120 sec will be $= v = u + at$

$$= 35 + (-1.8 \times 120) = -28.6 \text{ m/s}$$

acc is -ve as vehicle is moving in opposite direction to the direction of motion of the vehicle.

Q.5 A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a velocity of 15 m/s. How long the body will take to stop?

Soln

Given $F = 50 \text{ N}$, $u = 15 \text{ m/s}$, $v = 0$

$m = 20 \text{ kg}$, $t = ?$

$$F = ma \Rightarrow a = F/m = \frac{50}{20} = 2.5 \text{ m/s}^2$$

again $v = u + at$

$$\Rightarrow 0 = 15 + (-2.5 \times t) \Rightarrow t = \frac{15}{2.5} = 6 \text{ sec}$$

Q.6 A car of mass 9.5 tonnes moves on a level road under the action of 1 kN force. The time taken by the car to increase its velocity from 36 kmph to 54 kmph.

Soln

Given $m = 9.5 \times 1000 = 9500 \text{ kg}$

$F = 1 \text{ kN} = 1000 \text{ N}$

$u = 36 \text{ kmph} = \frac{36 \times 1000}{60 \times 60} \text{ m/s}$

$v = 54 \text{ kmph} = \frac{54 \times 1000}{60 \times 60} \text{ m/s}$

$$F = ma \Rightarrow a = F/m \Rightarrow a = \frac{1000}{9500} = 0.105 \text{ m/s}^2$$

$$\Rightarrow \frac{54 \times 1000}{60 \times 60} = \left(\frac{36 \times 1000}{60 \times 60} \right) + 0.105t \Rightarrow t = 12.5 \text{ sec}$$

Q7 A multiple unit electric train has 800 tonnes mass. The resistance to motion is 100N per tonne of the train mass. If the electric motors can provide 300kN tractive force how long does it take to accelerate the train to a speed of 90km/hr from rest.

sol Given $m = 800 \text{ tonnes} = 800 \times 1000 = 800000 \text{ kg}$

Resistance to motion = $100 \text{ N/tonne} = 100 \times 800 = 80000 \text{ N} = 80 \text{ kN}$

Tractive force = 300 kN

$v = 90 \text{ km/hr}$

$u = 0$ (at body is initially at rest)

$t = ?$

Net force available to move the train =

$$F = \text{Tractive force} - \text{Resistance to motion} \\ = 300 - 80 = 190 \text{ kN}$$

$$\text{Again } F = ma \Rightarrow a = F/m = \frac{190}{800} = 0.15 \text{ m/s}^2$$

Again we know $v = u + at$

$$\Rightarrow \frac{90 \times 1000}{60 \times 60} = 0 + (0.15 \times t)$$

$$\Rightarrow t = 166.7 \text{ sec.}$$

Q8 A man of mass 60kg jumps downwards into a swimming pool from a tower of height 90m. He was found to go down in water by 2m & then started rising. Find the average resistance of water. Neglect air resistance.

sol Given mass = $m = 60 \text{ kg}$

height of tower = $s = 90 \text{ m}$

Considering motion of man from top of tower to water surface.

Here $u = 0$

distance = $s = 90 \text{ m}$

We know that $v^2 = u^2 + 2as$

$$\Rightarrow v^2 = 0^2 + (2 \times 9.81 \times 90) = 392$$

$$\Rightarrow v = \sqrt{392} = 19.8 \text{ m/s}$$

Considering motion of man from water surface to the point on water, he started rising

Here $u = 19.8 \text{ m/s}$, $v = 0$, $s = 2 \text{ m}$

$a =$ retardation due to water resistance.

We know $\Rightarrow v^2 - u^2 = 2as$

$\Rightarrow v^2 = u^2 + 2as$

$\Rightarrow 0 = 19.8^2 - (2a \times 2) \Rightarrow a = 98 \text{ m/s}^2$

Avg. resistance of water $\Rightarrow F = ma = 60 \times 98 = 5880 \text{ N}$.

Q.9 At a certain instant, a body of mass 10 kg , falling freely under the force of gravity, was found to be falling at the rate of 20 m/s , what force will stop the body

i) in 2 sec .

ii) in 2 m .

Soln

Given $m = 10 \text{ kg}$

$F = ?$

$u = 20 \text{ m/s}$

i) $t = 2 \text{ sec}$

$v = 0$

ii) $s = 2 \text{ m}$

i) $v = u + at$

$0 = 20 + (a \times 2) \Rightarrow a = -10 \text{ m/s}^2$

A body is falling under force of gravity i.e. $a = 9.8 \text{ m/s}^2$

So the applied force must be able to produce an accⁿ

of $10 + 9.8 = 19.8 \text{ m/s}^2$

$\therefore$ Force required to stop the body $\Rightarrow F = ma$

$= 10 \times 19.8 = 198 \text{ N}$

ii) $v^2 - u^2 = 2as$

$0 - 20^2 = 2a \times 2$

$\Rightarrow a = -100 \text{ m/s}^2$

The applied force must be able to produce an accⁿ

of $100 + 9.8 = 109.8 \text{ m/s}^2$

So, force required to stop the body $\Rightarrow F = ma$

$= 10 \times 109.8 = 1098 \text{ N}$

Q.10 A body of mass 10 kg is moving over a smooth surface, whose eqn of motion is $s = t^2 + 2t^2$, where s is in meter & t is in sec. Find the magnitude of force responsible for the motion.

Given $m = 10 \text{ kg}$
 $s = t^2 + 2t^2$

$$v = \frac{ds}{dt} = 2t + 2(2t) = 5t + 4t$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(5t + 4t) \\ = 4$$

$$\text{So, } F = ma = 10 \times 4 = 40 \text{ N} \quad \underline{\text{Ans}}$$

D'Alembert's Principle & Dynamic equilibrium:

It states that "if a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction & the line of action may be found out by the method of graphic statics."

From Newton's 2nd law of motion, $\Sigma F = ma$ (eqn of dynamics)

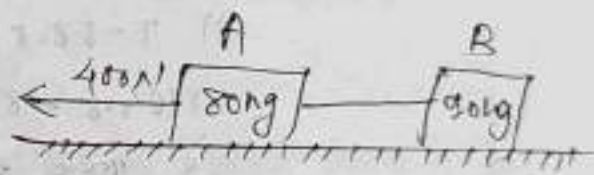
$$\Rightarrow \Sigma F + ma = 0 \text{ (eqn of statics)}$$

$\Sigma F + ma = 0$ is also called as the eqn of dynamic equilibrium

D'Alembert's principle is used for solving questions on dynamics which involve force & acceleration.

- Q Two bodies A & B of mass 80kg & 30kg are connected by a thread & move along a rough horizontal plane under the action of a force 400N applied to the first body of mass 80kg as shown in fig.

$\mu = 0.3$ Determine the accⁿ of two bodies & the tension in the thread, using D'Alembert's principle.

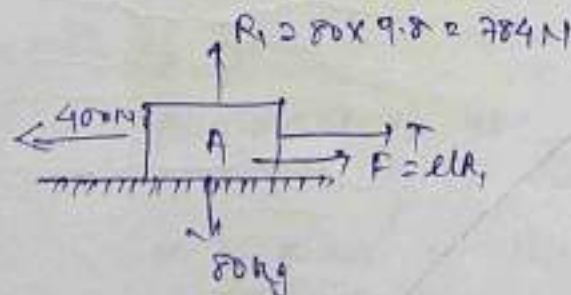


Solⁿ

Given data $m_A = 80\text{kg}$ $F = 400\text{N}$
 $m_B = 30\text{kg}$ $\mu = 0.3$

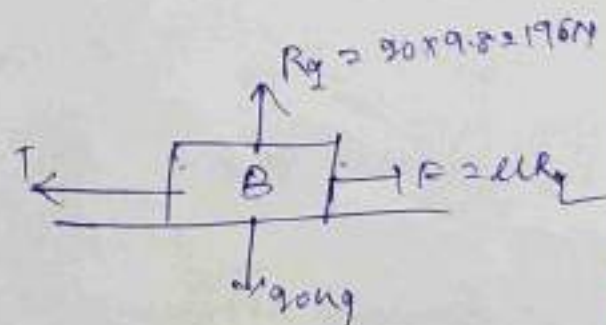
Let $a \rightarrow$ acceleration of the bodies

$T \rightarrow$ Tension in the thread



[FBD of body A]

Fig. 1



[FBD of body B]

Fig. 2

From fig-1 For equl of body A, horizontal force

$$400 = \mu R_1 + T$$

Let P_1 = Net resultant horizontal force

$$P_1 = 400 - T - \mu R_1$$

$$\therefore = 400 - T - (0.3 \times 784) = 164.8 - T \text{ (acting towards left)}$$

Again force causing accⁿ to the body A $= m_A a = 80a$

Accⁿ to D'Alembert's principle

$$P_1 - m_A a = 0$$

$$\Rightarrow 164.8 - T - 80a = 0$$

$$\Rightarrow T = 164.8 - 80a$$

Now, from fig-2 For Net horizontal force $= P_2$ (let)

$$P_2 = T - F_2 = T - \mu R_2 = T - (0.3 \times 196) = T - 58.8$$

Again force causing accⁿ to the body B $= m_B a = 90a$

Accⁿ to D'Alembert's principle

$$P - m_B a = 0$$

$$\Rightarrow T - 58.8 - 90a = 0$$

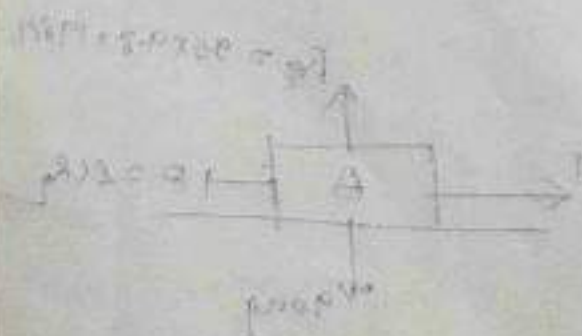
$$\Rightarrow 164.8 - 80a - 58.8 - 90a = 0$$

$$\Rightarrow 100a = 106$$

$$\Rightarrow a = 1.06 \text{ m/s}^2$$

Now, $T = 164.8 - 80a$

$$= 164.8 - (80 \times 1.06) = 80 \text{ N (Ans)}$$



Newton's Third Law of Motion:

It states that "To every action there is always an equal & opposite reaction."

Ex 1) when a bullet is fired from a gun

(The bullet moves with velocity v , the action & in reaction gun gives unpleasant to the man holding the gun.)

ii) when swimmer tries to swim, he pushes the water backwards & the area of water pushes the swimmer forward.

iii) motion of boat.

Laws of Conservation of momentum →

It states that "The momentum is conserved in a system, in which the external force is zero."

Mathematically, Initial momentum = Final momentum

$$\Rightarrow mv = mv$$

Ex - 1) Recoil of gun → when a bullet is fired from a gun, the opposite area of the bullet is called recoil of gun.

2) Motion of a boat → when a boat boy pushes the water back with sticks, it in turn sets the boat in motion.

Recoil of gun →

Let M = mass of gun

V = velocity of the gun with which it recoils

m = mass of the bullet

v = velocity of the bullet after explosion

Momentum of the bullet after explosion = mv

Momentum of the gun = MV

Acc^o to law of conservation of momentum $\Rightarrow \boxed{MV = mv}$

Q.1 A m/c gun of mass 35kg fires a bullet of mass 30gm with a velocity of 950m/s. Find the velocity with which the m/c gun will recoil.

Solⁿ Given $m = 30\text{kg}$
 $m = 30\text{gm} = 0.03\text{kg}$

$$u = 950\text{m/s}$$

$$V = ?$$

$$\text{Again } MV = mv$$

$$\Rightarrow 35 \times V = 0.03 \times 950 \Rightarrow V = \frac{7.5}{35} = 0.21\text{m/s}$$

Q.2 A bullet of mass 20g is fired horizontally with a velocity of 300m/s, from a gun carried in a carriage. The combined mass of gun & carriage is 100kg. The resistance to sliding of the carriage over the ice on which it rests is 90N. Find

- velocity with which the gun will recoil
- distance, in which it comes to rest
- time taken to do so

Solⁿ Given $m = 20\text{g} = 0.02\text{kg}$ $M = 100\text{kg}$
 $u = 300\text{m/s}$ $F = 90\text{N}$

a) we know that $MV = mv$

$$\Rightarrow 100 \times V = 0.02 \times 300$$

$$\Rightarrow V = 0.06\text{m/s}$$

b) Here $u = 0.06\text{m/s}$ & $v = 0$

Resisting force to sliding of carriage (F)

$$F = Ma \Rightarrow 90 = 100 \times a \Rightarrow a = 0.9\text{m/s}^2$$

$$\text{Again } v^2 - u^2 = 2as$$

$$\Rightarrow 0 - 0.06^2 = 2 \times 0.9 \times s$$

$$\Rightarrow s = \frac{0.0036}{1.8} = 0.002\text{m} = 2\text{mm}$$

c) we know that $v = u + at$

$$\Rightarrow 0 = 0.06 + 0.9t$$

$$\Rightarrow t = \frac{0.06}{0.9} = 0.066\text{sec}$$

Motion of a boat →

Boat man always pushes water back with the help of stick, which in return sets the boat in motion.

If the boat is at rest & the boat man runs on it & dives off into the water, the boat will also move backward.

Let M = mass of boat

V = velocity of boat

m = mass of the boat man

v = velocity " " "

According to law of conservation of momentum $MV = mv$

- Q Two men, standing on a floating boat, run in succession, along its length, with a speed of 4.2 m/s relative to the boat & dive off from the end. The weight of each man is 80 kg & that of the boat is 400 kg . If the boat was initially at rest, find the final velocity of the boat. Neglect water friction.

Soln Given data $v = 4.2 \text{ m/s}$

$m = 80 \text{ kg}$ (each)

$M = 400 \text{ kg}$

When first man dives off the boat, it will give some momentum to the boat as well as the 2nd man (who is still standing on the boat)

When the 2nd man also dives off the boat, it will also give some momentum to the boat.

So the total momentum gained by the boat is equal to the momentum given by the 1st man + momentum given by 2nd man to the boat.

Final momentum of the boat = 400v

Momentum given by the 1st man to the boat = 80×4.2

$$= 336 \text{ kg} \cdot \text{m/s}$$

" " " 2nd man "

$$= 80 \times 4.2$$

$$= 336 \text{ kg} \cdot \text{m/s}$$

Final momentum = momentum by 1st man + momentum by 2nd man

$$= 326 + 336 = 672 \text{ kg-m/s}$$

$$\text{Now } 400v = 672$$

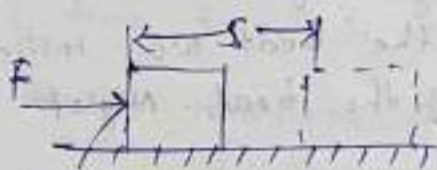
$$\Rightarrow v = 1.68 \text{ m/s}$$

WORK ; POWER & ENERGY

WORK (W) $\rightarrow$ when force acts on a body & the body undergoes some displacement, the work is said to be done.

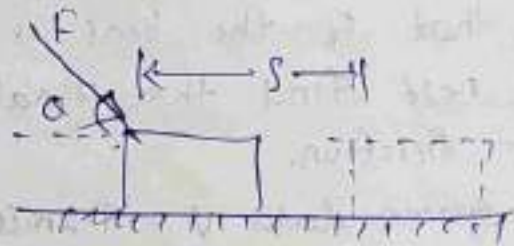
Mathematically $W = \text{Force} \times \text{displacement}$

$$W = F \times S$$



Body moving in dirⁿ of applied force

$$\text{Here } W = FS$$



Body not moving in dirⁿ of applied force

$$\text{Here } W = FScos\theta$$

Unit

$$W = F \times S = \text{N} \times \text{m} = \text{Joule}$$

$$1 \text{ kg} \cdot 1 \text{ kJ} = 1000 \text{ J} = 1000 \text{ N-m}$$

Q. A horse pulling a cart exerts a steady horizontal pull of 300 N & works at the rate of 4.5 inches. How much work is done by horse in 5 minutes.



ENERGY

Defⁿ The capacity to do work is called energy. It is of different type like mechanical, electrical, chemical, heat, light energy.

UNIT - $\text{N} \cdot \text{m} = \text{Joule}$

MECHANICAL ENERGY

→ The capacity to do mechanical work is called mechanical energy. It is of 2 types.

① Potential ② Kinetic

① POTENTIAL ENERGY

→ It is the energy possessed by a body due to its position.

ex A body raised to some height above the ground level. It can do work by falling on earth surface.

ex Compressed air, it can do work by expansion.

ex Compressed spring. It can do work in recovery to its original shape.

Let $m \rightarrow$ mass of the body

$h \rightarrow$ height above the ground level.

Now work done = $w = \text{weight} \times \text{distance}$

$$= (m \times g) \times h \quad (g = 9.81)$$

$W_{PE} = mgh$

→ This work done is stored in the body as PE

Q A man of mass 50 kg dives vertically downward into a swimming pool from a tower of height 90 m. He was found to go down in water by 9 m. & then started rising. Find the average resistance of the water. Neglect the air resistance.

Ex 1

Given $m = 60 \text{ kg}$
 $h = 20 \text{ m}$

$$\begin{aligned}\text{P.E of the man before jumping} &= m \times g \times h \\ &= 60 \times 9.81 \times 20 \\ &= 11760 \text{ N-m}\end{aligned}$$

workdone by the average resistance

$$\begin{aligned}&= \text{average resistance of water} \times \text{depth of water} \\ &= F \times 2 = 2F\end{aligned}$$

As the total potential energy of the man is used to workdone

$$\begin{aligned}\therefore \text{Now equating } 2F &= 11760 \\ \Rightarrow F &= \frac{11760}{2} = 5880 \text{ N}\end{aligned}$$

KINETIC ENERGY (KE)

It is the energy possessed by the body due to its motion. Let $m \rightarrow$ mass of the body
 $v \rightarrow$ velocity of the body

$$\text{Now the expression for } \boxed{\text{K.E} = \frac{1}{2}mv^2}$$

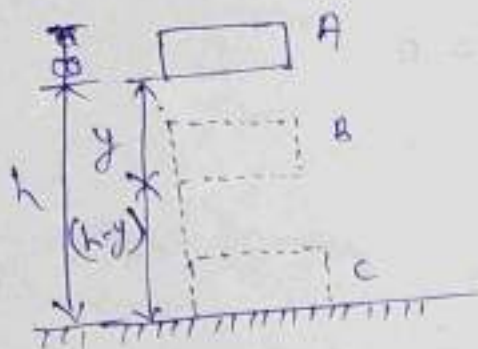
Transformation of energy

Consider a body just dropped on the ground from position A.

Let $m \rightarrow$ mass of the body

$h \rightarrow$ height " " "

B & C are the position of the body when it falls.



Position A

The body is at rest, so velocity $v = 0$. Now potential energy $= mgh$

$$P.E = mgh$$

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m(0)^2 = 0$$

$$\begin{aligned} \text{Total energy at A} &= P.E + K.E \\ &= mgh + 0 = mgh \end{aligned}$$

Position B

$$\text{Here } P.E = mg(h-y)$$

Here body has moved to distance y so velocity of the body at B $= v^2 - u^2 = 2as$
 $\Rightarrow v^2 - 0 = 2gy$
 $\Rightarrow v = \sqrt{2gy}$

$$\begin{aligned} \text{Now } K.E &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}m(\sqrt{2gy})^2 \\ &= \frac{1}{2}m2gy = mgy \end{aligned}$$

$$\begin{aligned} \text{Now Total energy} &= P.E + K.E \\ &= mgh - mgy + mgy \\ &= mgh \end{aligned}$$

Position c

Here $P.E = mgh = mg \times 0 = 0$

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m(\sqrt{2gh})^2$$

$$= \frac{1}{2}m \cdot 2gh$$

$$= mgh$$

Now Total energy = P.E + K.E

$$= 0 + mgh$$

$$= mgh$$

So In all position, Total energy is constant, i.e. mgh .

This is called law of conservation of energy.

Law of conservation of energy

It states that, "Energy can neither be created nor be destroyed." It can only be transfer from one form to another."

$$P.E = mgh$$

$$K.E = \frac{1}{2}mv^2$$

$$P.E = mgh$$

$$K.E = \frac{1}{2}mv^2$$

$$P.E = mgh$$

$$K.E = \frac{1}{2}mv^2$$

$$P.E = mgh$$

$$K.E = \frac{1}{2}mv^2$$

Summary

Statement

Impulse (J)

It is defined as the product of force and the time for which it is acting on a body.

Mathematically $J = F \times t$

UNIT = N·Sec

From Newton's 2nd law $F = ma$

$$\Rightarrow F = m \times \left(\frac{v-u}{t} \right)$$

$$\Rightarrow Ft = m(v-u)$$

~~$\Rightarrow J = m \times \text{change in velocity}$~~
momentum

momentum = mv

$$\Rightarrow Ft = (mv - mu)$$

change in momentum

$$J = mv - mu$$

Impulse - momentum equation

This eqn is used for solving dynamic problem involving Force, velocity and time.

Q A bullet of mass 0.045 kg leaves the barrel of gun with a velocity of 400 m/s. Suppose the force lasted for 0.0025 sec. Find (1) Impulse (2) Average force

Take $g = 9.8 \text{ m/s}^2$

Given

$$m = 0.045 \text{ kg}$$

$$u = 0 \text{ m/s}$$

$$t = 0.0025 \text{ sec}$$

$$v = 400 \text{ m/s}$$

$$t = 0.0025 \text{ sec}$$

$$g = 9.8 \text{ m/s}^2$$

$$mv = 0.045 \times 400$$

$$\frac{18}{0.0025}$$

$$\frac{18 \times 10000}{25}$$

$$\begin{array}{r} 0.045 \\ \times 400 \\ \hline 18.000 \\ 180.000 \\ \hline 18.000 \end{array}$$

26.02.21

From $J = m(v-u)$

$$= 0.045(400-0)$$

$$= 18 \text{ kg-m/s}$$

$$\text{Average force (F)} = m \left(\frac{v-u}{t} \right)$$

$$= 0.045 \times \left(\frac{400-0}{0.0025} \right)$$

$$= 7200 \text{ N}$$

NEW
CHAPTER

COLLISION of ELASTIC BODIES

- The property of ~~body~~ bodies by virtue of which they rebound (raise ~~up~~ up after straining), after ~~the~~ impact (hit) is called elasticity.
- The body which ~~to~~ rebound to a greater height is called more elastic than the body which rebound to a lesser height.
- If a body does not rebound at all after its impact it is called in-elastic body.

Collision phenomenon

When two elastic body collided

- ① The bodies immediately after collision come to rest
- ii) The bodies tend to compress each other
- iii) The two bodies attempt to regain its original shape due to their elasticity. This process of regaining the original shape is called restitution

26.03.21
The time taken by two bodies in compression after the instant of collision is called time of compression and time for which restitution takes place is called time of restitution.

Time of collision = 2 × Time of compression + time of restitution

Law of conservation of momentum for collision

- It states that, "The total momentum of two bodies remains constant after their collision or any other mutual action."

Mathematically - $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

Here $m_1, u_1, v_1 \rightarrow$ mass, initial velocity, final velocity respectively of the 1st body.

$m_2, u_2, v_2 \rightarrow$ mass, initial velocity, final velocity respectively of the 2nd body.

summary

Newton's laws of collision for elastic bodies

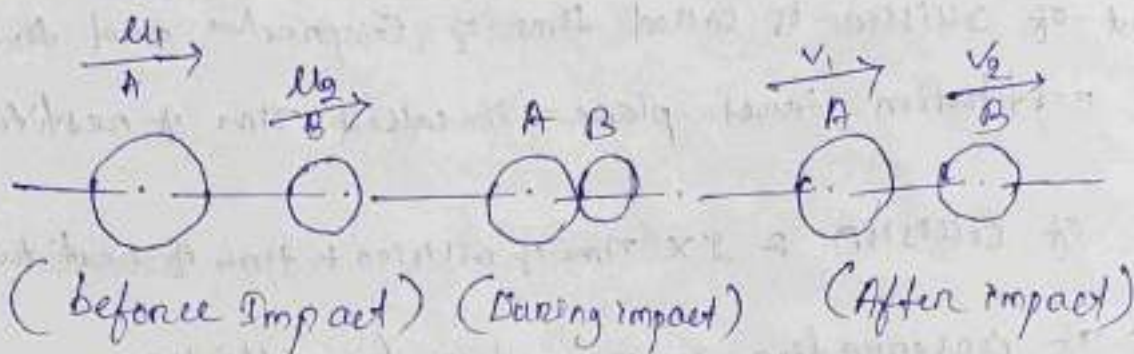
- It states that, "When two moving bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach."

Mathematically $v_2 - v_1 = e(u_1 - u_2)$

where $u, v \rightarrow$ initial, final velocity respectively of the 1st body

$u_2, v_2 \rightarrow$ initial, final velocity respectively of the 2nd body

$e \rightarrow$ Constant of proportionality



- Consider two bodies A & B having direct impact.

Impact is upward $u_1 > u_2$

velocity of approach = $u_1 - u_2$

After impact, separation of two bodies will take place if $v_2 > v_1$

So velocity of separation = $v_2 - v_1$

According to Newton's law of collision

velocity of separation = $e \times$ velocity of approach

$$v_2 - v_1 = e(u_1 - u_2)$$

value of $e = 0 < e < 1$

coefficient of restitution

If $e = 0$, then two bodies are inelastic. I mark should.

If $e = 1$, then two bodies are perfectly elastic.

09.05.21

Type of collision.

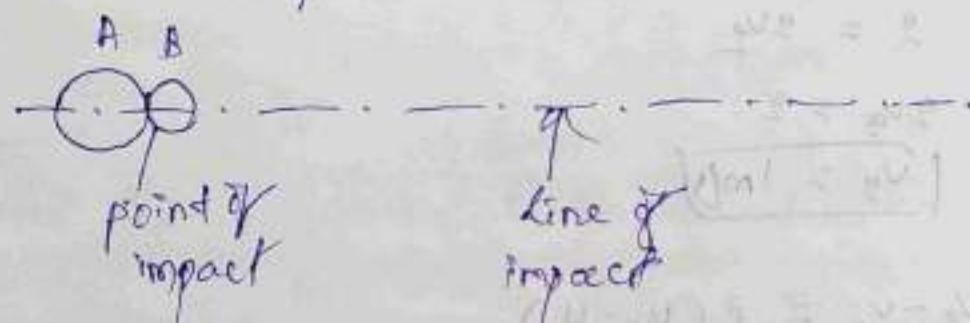
when two bodies collide with one another, they are said to have impact. It is of 2 types

① Direct impact

② Indirect impact. (Not in syllabus)

Direct collision of two bodies

If the two bodies, before impact are moving along the line of impact, the collision is called Direct impact.



Let $m_1, m_2 \rightarrow$ Mass of body A & B respectively
 $u_1, u_2 \rightarrow$ initial velocity body " "
 $v_1, v_2 \rightarrow$ final " " " "

From law conservation of momentum.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Q A ball of mass 1 kg moving with a velocity of 2 m/s impinges directly on a ball of mass 2 kg at rest. The 1st ball after impinging comes to rest. Find the velocity of the second ball after the impact find coefficient of restitution

Given $m_1 = 1\text{kg}$ $u_1 = 2\text{m/s}$

$$v_1 = 0\text{m/s}$$

$$m_2 = 2\text{kg}$$

$$u_2 = 0$$

~~$$m_1 v_1 =$$~~

~~$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$~~

~~$$1 \times 0 + 2 \times 0 = 1 \times 0 + 2 \times v_2$$~~

From Law of conservation of momentum $1 \times 2 + 2 \times 0 =$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$1 \times 2 + 2 \times 0 = 1 \times 0 + 2 \times v_2$$

$$2 = 2v_2$$

$$2v_2 = 2$$

$$v_2 = 1\text{m/s}$$

$$v_2 - v_1 = e(u_2 - u_1)$$

$$e = \frac{v_2 - v_1}{u_2 - u_1}$$

$$= \frac{1 - 0}{0 + 2 - 0}$$

$$e = \frac{1}{2} = 0.5$$

Q A ball over-take another ball of twice its own mass and moving with $\frac{1}{3}$ of its own velocity. If coefficient of restitution betⁿ the two balls is 0.75, so that show that the 1st ball will come to rest after impact.

$m_1 = M$ $m_2 = 2M$ $u_1 = \frac{1}{3}u$ $u_2 = 0$ $A = 2B$
 $e = 0.75$ $e = \frac{v_2 - v_1}{u_1 - u_2}$
 $e = \frac{v_2 - v_1}{\frac{1}{3}u - 0} = 0.75$ $v_2 - v_1 = e(u_1 - u_2)$
 $-v_1 = e(u_1 - u_2) - v_2$
 $e = \frac{v_2 - v_1}{\frac{1}{3}u - 0} = 0.75$
 $e =$

from law of conservatⁿ of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$M u + 2M \cdot \frac{u}{3} = M v_1 + 2M v_2$$

from
Now law of collision of elastic bodies

$$v_2 - v_1 = e(u_1 - u_2)$$

$$\Rightarrow v_2 - v_1 = 0.75(u - \frac{u}{3})$$

$$= 0.75(\frac{2u}{3})$$

$$= 0.5u$$

$$\Rightarrow v_2 = 0.5u + v_1$$

$$\frac{9}{7}Mu = Mv_1 + 2Mv_2$$

$$\frac{9}{7}u = v_1 + 2v_2$$

$$\frac{9}{7}u = v_1 + 2(\frac{9}{14}u + v_1)$$

$$\frac{9}{7}u = v_1 + 2v_1 + \frac{9}{7}u$$

putting value of v_2 in eqn ①

25
25
25

$u(\frac{1}{3} - (2 \times \frac{9}{7})) = 3v_1$
 $= v_1^2 \frac{u(\frac{1}{3} - \frac{18}{7})}{3}$
 $= v_1^2 \frac{u(\frac{1}{3} - \frac{18}{7})}{3}$
 $v_1 = 0$

Q Three perfectly elastic ~~ball~~ ^{bodies} A, B, C, of mass 2 kg , 4 kg & 7 kg move in the same dirⁿ with velocity of 4 m/s , 1 m/s & 0.25 m/s respectively. If the ball A impinges with the ball B, which in turn impinges with the ball C, prove that the ~~of~~ A & B will be brought to rest by the impact.

Mass of $m_A = 2\text{ kg}$ $u_1 = 4\text{ m/s}$ $v_1 = v_2 = 0$
 $m_B = 4\text{ kg}$ $u_2 = 1\text{ m/s}$
 $m_C = 7\text{ kg}$ $u_3 = 0.25\text{ m/s}$
 $e = 1$

From law of conservatⁿ of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow (2 \times 4) + (4 \times 1) = 2v_1 + 4v_2$$

$$2 \Rightarrow 8 + 4 = 2v_1 + 4v_2$$

$$2 \Rightarrow 12 = 2(v_1 + 2v_2)$$

$$2 \Rightarrow 6 v_1 + 2v_2 = 6 \quad \text{--- (I)}$$

As ball A impact with ball B, from Law of Collision of elastic body

$$v_2 - v_1 = e(u_1 - u_2)$$

$$v_2 - v_1 = 1(4 - 1)$$

$$v_2 - v_1 = 3 \quad \text{--- (II) Solving eqn (I) & (II)}$$

$$v_2 = 3 + v_1$$

$$v_2 = 3 + 6 - 2v_2$$

$$v_2 = 9 - 2v_2$$

$$3v_2 = 9$$

$$3v_2 = 9$$

$$v_2 = 3$$

Putting eqn (II)

$$3 - v_1 = 3$$

$$v_1 = 3 - 3$$

$$v_1 = 0$$

Putting in eqn (I)

$$v_2 - 0 = 3$$

02.3.21

As ball B impact with ball c
from law of conservation of momentum

$$m_2 u_2 + m_3 u_3 = m_2 v_2 + m_3 v_3$$

$$\Rightarrow 4 \times 3 + 8 \times 0.25 = 4 \times v_2 + 8 \times v_3$$

$$\Rightarrow 12 + 2.0 = 4v_2 + 8v_3$$

$$\Rightarrow 14 = 4(v_2 + 2v_3)$$

from collision of elasticity $\Rightarrow v_2 + 2v_3 = 18$ — (3)

elasticity $v_3 - v_2 = e(u_2 - u_3)$

$$v_3 - v_2 = 1(1 - 0.25)$$

$$v_3 - v_2 = 0.75$$
 — (4)

~~$$16 - v_2 - v_2 = 0.75$$

$$16 - 2v_2 = 0.75$$

$$-2v_2 = 0.75 - 16$$

$$-2v_2 = -15.25$$

$$v_2 = 15.25/2$$~~

0.5×5

0.95
 95

$2 \times \frac{1}{2} = 1$

Eq (3) + (4) =

$$v_2 + 2v_3 = 18$$

$$-v_2 + v_3 = 0.75$$

$$3v_3 = 18.75$$

$$v_3 = 18.75/3$$

$$v_3 = 6.25$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$= 4x$$

2.2.1) Direct Impact of a body with a fixed plane

Each fixed plane will not move after impact. So here velocity of approach = velocity of separation

From Newton's law of elastic collision $v_2 = eu$

Here u - initial velocity
 v - final

e = coefficient of restitution

* Here law of momentum is not used as fixed plane has infinite mass.

* The velocity with which the body impinges on the floor, when it falls from a height (H) will be

$$u = \sqrt{2gH}$$

Q A ball is dropped from a height $h_0 = 1m$ on a smooth cloth, knowing that the height of the 1st bounce is $h_1 = 81cm$, determine

i) coefficient of restitution

ii) Expected height as it bounces the second time

$$h_0 = 1m$$

$$h_1 = 81cm$$

Given height from which $h_0 = 1m$

to which ball rises after

$$1^{st} \text{ bounce } = h_1 = 81cm = 0.81m$$

3.3.91

i) velocity with which ball impinges on the floor

$$u = \sqrt{2gh_1}$$

$$= \sqrt{2 \times 9.8 \times 1}$$

$$= \sqrt{19.6} \text{ m/s}$$

velocity with which ball rebound $v = \sqrt{2gh_2}$

$$= \sqrt{2 \times 9.8 \times 0.81}$$

$$= 0.9 \sqrt{19.6} \text{ m/s}$$

Again we know that $v = eu$

$$\Rightarrow e = \frac{v}{u} = \frac{0.9 \sqrt{19.6}}{\sqrt{19.6}} = 0.9$$

$$\therefore e = 0.9$$

ii) Let h_2 be expected height after second bounce.
velocity with which the ball impinges second time
i.e. $u = 0.9 \sqrt{2g}$

velocity with which the ball rebound second time $v = \sqrt{2gh_2}$

$$v = eu$$

$$\Rightarrow \sqrt{2gh_2} = 0.9 \times 0.9 \sqrt{2g}$$

$$\Rightarrow \sqrt{2g} \sqrt{h_2} = 0.81 \sqrt{2g}$$

$$\Rightarrow h_2 = (0.81)^2$$

$$\boxed{h_2 = 0.656 \text{ m}}$$

ENDS OF MECHANICS