

Thermodynamics

- P.K. Nag
- Cengel and Boles

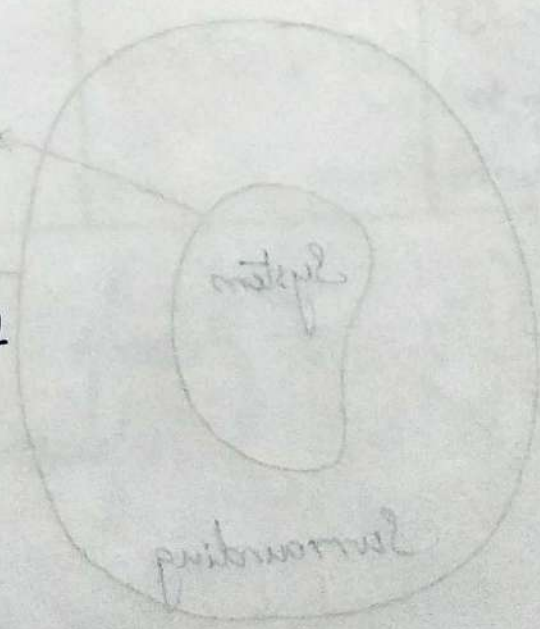
It is the science of energy transfer and its effects on properties.

* Energy transfer may be heat (or) may be work (or) may be both.

The main AIM of thermodynamic study is to Convert dis-organised form of energy (Heat) into organised form of energy (work) in an efficient manner.

Applications:-

- 1) Refrigeration
- 2) Air-conditioning
- 3) Thermal power plant
- 4) I.C. Engines etc.



SYSTEM:-

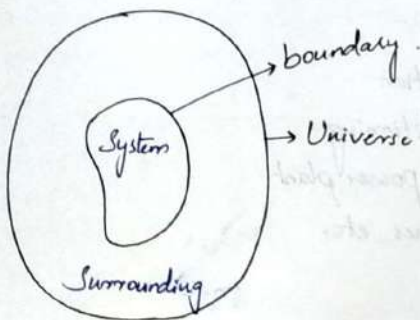
It is a region in space upon which the study is focused i.e., (it is the region in space which we wish to analyse.)

SURROUNDINGS:-

Anything external to system, where the effect of system is felt is known as surroundings.

BOUNDARY:-

The separation between system and surroundings is known as a boundary.



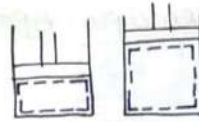
$$\text{Universe} = \text{System} + \text{Surrounding}$$

Note:-

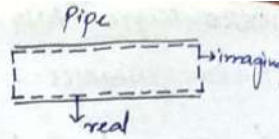
- 1) Boundary can be rigid (or) flexible.
- 2) Boundary can be real (or) imaginary.
- 3) Across the boundary, mass and energy can cross.



rigid.



movable



TYPES OF SYSTEM:-

Type of system	Mass transfer	Energy transfer	Examples
Closed	x	✓	Rigid closed container, Piston cylinder w/o valves
Open	✓	✓	Turbine, Pump, Piston cylinder with valves.
Isolated	x	x	Universe, Hot fluid in a perfectly insulated container

* Isolated system is a special case of closed system in which energy transfer is cut off.

CONTROL VOLUME:-

It is the volume surrounding the device which we wish to analyse across the control volume; both mass and energy can cross the boundary.

* Control volume technique is used in the analysis of open system.

* In closed system; as the analysis is done on fixed mass; also known as control-mass technique.

MICRO-SCOPE AND MACROSCOPIC APPROACH OF THERMODYNAMICS:-

In microscopic approach, the behaviour of individual molecules is taken into consideration. This approach is also known as Statistical thermodynamics. This approach is used at lower densities i.e., at higher altitudes.

In Macroscopic approach, average behaviour of molecules is taken into consideration. This approach is also known as Classical thermodynamics.

THERMODYNAMIC EQUILIBRIUM:-

- (1) Thermal Equilibrium (equality of temp.)
- (2) Mechanical Equilibrium (equality of force/pressure)
- (3) Chemical Equilibrium (equality of chemical potential)

A system is said to be in thermodynamic equilibrium if it satisfies all the three equilibria mentioned above.

PURE SUBSTANCE:-

A substance is said to be a pure substance,

if it is

- (a) homogeneous in chemical composition.
- (b) homogeneous in chemical aggregation (bonding)

NOTE:- A pure substance can exist in a single phase (or) more than one phase.

vapour
water

- (a) ✓
(b) ✓

H ₂ O
water

- (a) ✓
(b) ✗

H ₂ O ₂
water

- (a) ✗
(b) ✗

NOTE:-

Though dry air is a pure substance, moist air is not a pure substance. i.e., water vapour can be separated on condensation.

PROPERTY OF A SYSTEM:-

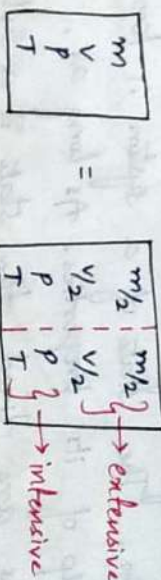
Any measurable characteristic is a property.

Ex:- Pressure, Volume, Temperature, Mass, Density etc.

Properties are of two types:-

- (1) Intensive - independent of size - Pressure, Temp, Density, Viscosity, etc.
- (2) Extensive - dependent of size.

↳ Mass, Volume, all forms of energy



$$m = \frac{m_1 + m_2}{m_3} \Rightarrow m = m_1 + m_2 + m_3$$

NOTE:- Ratio of two extensive properties is an intensive property. Ex:- $\rho = \frac{m}{V}$

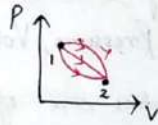
SPECIFIC PROPERTIES:-

These are extensive properties per unit mass.
All specific properties are intensive properties.

Extensive	Intensive
Volume (V) $\longrightarrow$	Sp. vol. (v) = V/m
Internal Energy (U) $\longrightarrow$	Sp. I.E (u) = U/m
Enthalpy (H) $\longrightarrow$	Sp. Enthalpy (h) = H/m
Entropy (S) $\longrightarrow$	Sp. Entropy (s) = S/m

Important points with respect to properties:-

- (1) Properties are point functions.
- (2) Properties are independent of path.
- (3) Properties are exact differentials.
- (4) Properties are macroscopic characteristics.



STATE OF A SYSTEM:-

The Condition of a system is known as State of a system. The state of a system is known with the help of its properties. As long as, the properties are fixed, the state of a system is fixed.

PROCESS:-

Any change of state is known as process.
During a process, properties of system change.

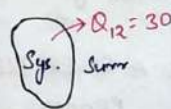
REVERSIBLE AND IRREVERSIBLE PROCESS:-

A process is said to be a reversible process if when reversed, follows the same path without leaving any effect on system and surroundings.

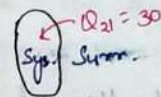
A process which is not reversible process is an irreversible process. Friction is one of the reasons which makes a process irreversible. Most of the practical processes are irreversible.

Reversible processes are discussed in thermodynamics because they are easy to analyse and they are most efficient.

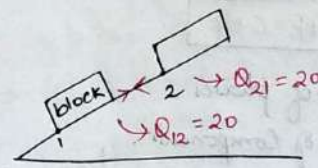
Reversible:-



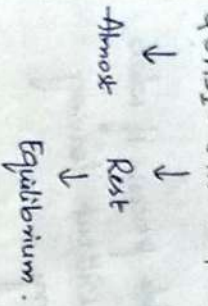
System	Surrounding
$Q_{12} = -30$	$Q_{12} = +30$
$Q_{21} = +30$	$Q_{21} = -30$
<u>0</u>	<u>0</u>



Irreversible:-



QUASI-STATIC PROCESS:-



A process which is carried out with a small gradient (small pressure diff. or small temp. diff.) is known as a Quasi-static process.

On a quasi-static process, every point passes through equilibrium state.

* THOUGH ALL REVERSIBLE PROCESSES ARE QUASI-STATIC, ALL QUASI-STATIC ARE NOT REVERSIBLE.

(some of quasi-static may involve friction).

* FRICTION LESS QUASI-STATIC PROCESS IS A REVERSIBLE PROCESS.

GIBBS-PHASE RULE:

It is an empirical relation. According to this rule;

$$P + F = C + 2$$

where; P = no. of phases

C = no. of component.

F = Degree of freedom of independent intensive properties.



$P = 1, C = 1$

$$1 + F = 1 + 2$$

$$F = 2$$



$P = 2$

$C = 1$

$$2 + F = 1 + 2$$

$$F = 1$$



$P = 3$

$C = 1$

$$3 + F = 1 + 2$$

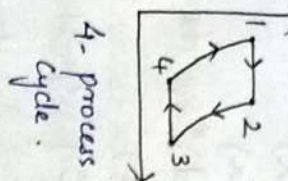
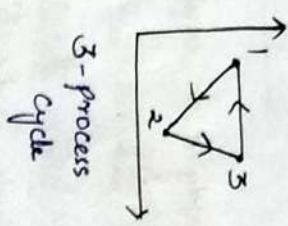
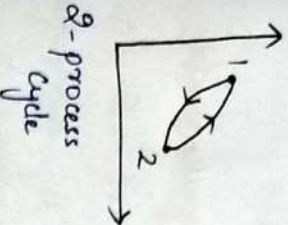
$$F = 0$$

Note:- At triple point; the D.O.F is zero. i.e., at triple point, no intensive property can be varied. It is a unique state.

THERMODYNAMIC CYCLE:-

A system is said to have undergone a thermodynamic cycle; when the initial and final points are same. Minimum no. of processes required for a cycle are two.

Note:- For a thermodynamic cycle, change in property is '0'.



CONCEPT OF CONTINUUM:-

Continuum means Continuous. In Continuum Concept, it is assumed that the matter is spread continuously throughout. For applying Continuum Concept, number of molecules must be large. The Mean free path must be small than the size of system.

Classroom Set:-

1. (d)
2. (a)
3. (b)
4. (a)
5. (b)
6. (c)
7. (a)
8. (a) *weight is also extensive property.
9. (b)
10. (P.R)

11. (a)

12. (d) *Eq. $dz = Mdx + Ndy$ is an exact diff. equation when

13. (c) $\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$

14. (C) optm(d):- $z = \int \frac{dx}{T} - \frac{Vdp}{T}$

$$dz = \frac{dx}{T} - \frac{Vdp}{T} \Rightarrow dz = \left(\frac{1}{T}\right)dx + \left(-\frac{V}{T}\right)dp$$

$$M = \frac{1}{T}, N = -\frac{V}{T}, y = P$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \frac{\partial}{\partial P}\left(\frac{1}{T}\right)_T = 0; \left(\frac{\partial N}{\partial x}\right)_y = \frac{\partial}{\partial T}\left(-\frac{V}{T}\right)_P \Rightarrow \frac{\partial}{\partial T}\left(-\frac{mR}{P}\right)_P = 0$$

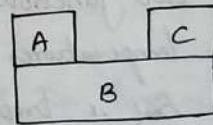
ZEROth LAW OF THERMODYNAMICS:-

(Concept of temperature)

The Concept of temperature was given by Fowler in 1931. This Concept was developed after developing 1st and 2nd law of thermodynamics but to understand 1st and 2nd law; Concept of temperature must be understood. This temperature Concept proceeds 1st and 2nd law and hence "Zero" is assigned to this law.

Statement:-

When body 'A' is in thermal equilibrium with body 'B' and body 'B' is in thermal equilibrium with body 'C' separately, then 'A' and 'C' are in thermal equilibrium.



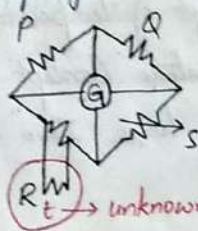
THERMOMETRIC PRINCIPLE:-

In this principle, the property which varies with temperature is calculated first with the help of this property, temperature is subsequently calculated. The property which helps in calculating temperature is known as Thermometric property.

TYPES OF THERMOMETERS:-

1) RESISTANCE THERMOMETER:- (Thermistors)

These thermometers are based on Wheatstone bridge principle. In these thermometers, resistance is thermometric property.

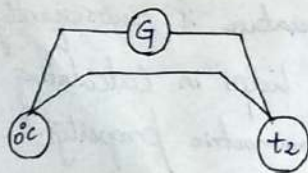


$$\frac{P}{Q} = \frac{R}{S}$$

$$R = R_0 [1 + \alpha t + \beta t^2]$$

2) THERMOCOUPLE:-

These thermometers are based on Seebeck effect. According to Seebeck effect, when two dissimilar metals are joined to form two different junctions and if these two junctions are maintained at different temperatures, emf (or) voltage is generated. This emf is proportional to temp. difference between two junctions. First, emf (or) voltage is calculated. With the help of emf, the unknown temp. is then calculated.



3) CONSTANT-VOLUME GAS THERMOMETER:-

For an ideal gas, when volume is constant, pressure changes with temperature.

$\therefore$ In constant volume gas thermometers, pressure is thermometric property.

4) CONSTANT PRESSURE GAS THERMOMETER:-

In this thermometer, volume is thermometric property.

TEMPERATURE SCALES:-

Temperature scales are arbitrary. [randomly taken]

ICE POINT:-

It is the temperature at which the water is converted into ice at atmospheric pressure.

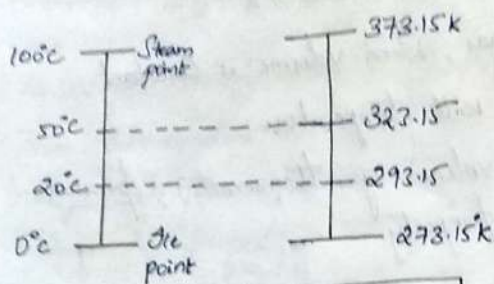
STEAM POINT:-

It is the temperature at which the water is converted into steam at atmospheric pressure.

In Centigrade (or) Celsius scale;

$$\text{Ice point} = 0^\circ\text{C}$$

$$\text{Steam point} = 100^\circ\text{C}$$



$$K = ^\circ C + 273.15$$

* The temp. difference remains same in $^\circ C$ and K.

METHOD USED BEFORE 1954:-

This method is based on two reference temperatures i.e., Ice point ($0^\circ C$) and Steam point ($100^\circ C$).

$$t = at + b$$

$$t_i = ap_i + b$$

$$t_s = ap_s + b$$

$$\Rightarrow 100 = ap_s + b$$

$$0 = ap_i + b$$

$$100 = a(p_s - p_i)$$

$$\Rightarrow a = \frac{100}{p_s - p_i}$$

$$p_s - p_i$$

$$b = -ap_i = -\left(\frac{100}{p_s - p_i}\right) \times p_i$$

$$t = at + b = \left(\frac{100}{p_s - p_i}\right)p - \left(\frac{100}{p_s - p_i}\right) \times p_i$$

$$t = \frac{100}{p_s - p_i} (p - p_i)$$

METHOD USED AFTER 1954:-

This method is based on single fixed temperature i.e., triple point of water. The triple point of water is taken as $0.01^\circ C$ ($273.16 K$)

S.I unit of temperature = Kelvin.

Ideal gas temp. scale and Kelvin scales are same, * So in an Ideal gas equation, Kelvin scale is used.*

$$PV = nRT \Rightarrow P = \frac{nR}{V} \cdot T$$

$$\Rightarrow P = C \cdot T$$

$$P_{tp} = C \cdot T_{tp} \Rightarrow C = \frac{P_{tp}}{T_{tp}}$$

$$P = \frac{P_{tp}}{T_{tp}} \cdot T \Rightarrow T = T_{tp} \left(\frac{P}{P_{tp}} \right) \Rightarrow T = 273.16 \left(\frac{P}{P_{tp}} \right)$$

Note:- Ideal gas thermometer is independent of material of construction because all ideal gases behave in a same manner.

INTERNATIONALLY ACCEPTED DEFINITION OF 1K:-

1 Kelvin is defined as $\frac{1}{273.16}$ times the triple point of water.

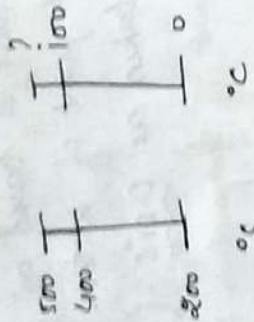
$$1K = \frac{1}{273.16} \times \text{Triple pt. of water.}$$

Classroom Set:-

1. (a)

2. $F = 200 \text{ N}$; $t = 0^\circ \text{C}$

B. $P = 400 \text{ N}$; $t = 100^\circ \text{C}$



$$\frac{500 - 200}{400 - 200} = \frac{^\circ \text{C} - 0}{100 - 0}$$

$$\Rightarrow ^\circ \text{C} = 150 \Rightarrow \text{option (C)}$$

3. (c)

4. (d) * $t = a t^2 + b$; $t = 0^\circ \text{C} \Rightarrow n = 5 \text{ cm}$
 $t = 100^\circ \text{C} \Rightarrow n = 20 \text{ cm}$
 $t = ? \Rightarrow n = 15 \text{ cm}$

5. (d)

$$\Rightarrow 0 = a(25) + b$$

6. (d)

$$100 = 400a + b$$

7. (a)

$$\Rightarrow 100 = 400a + b$$

8. (d)

$$0 = 25a + b$$

9. (c)

$$100 = 375a \Rightarrow a = 0.266$$

$$b = -25a \Rightarrow -6.66$$

$$\therefore t = 0.266(225) - 6.66$$

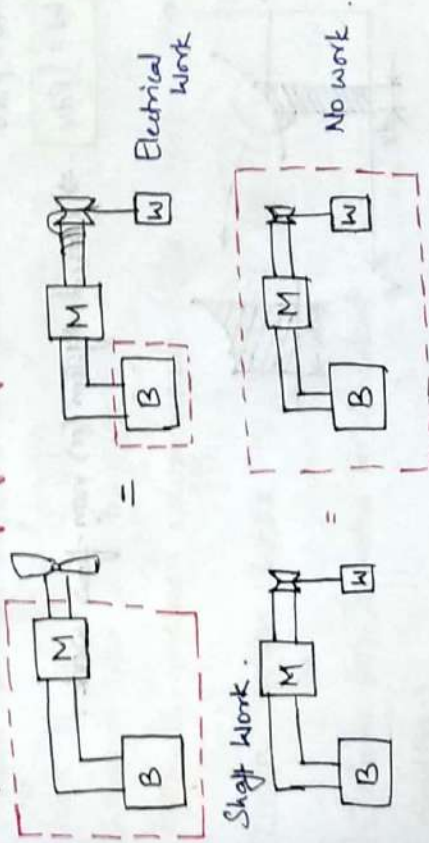
$$t = 53.3^\circ \text{C}$$

* When water is mixed with Sulphuric acid, there is a chemical reaction and because of this, the system is not in thermal equilibrium.

ENERGY INTERACTIONS:- (HEAT AND WORK)

THERMODYNAMIC WORK:-

Work is said to be done by the system when the sole effect on things external to the system can be equated to raising of weights. (Weights may not be actually raised but the effect is equated to raising of weights.)



for Work transfer, it should cross the boundary

* Work transfer is a boundary phenomenon.

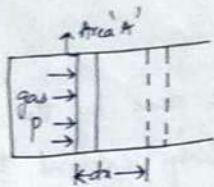
CONVENTION OF WORK TRANSFER:-

Work done by the system is taken as positive (output) and work done on the system is taken as negative (input).

NOTE:- In a closed system, expansion work is +ve. Compression work is -ve.



GENERALISED EQUATION FOR CLOSED SYSTEM WORK:-



$$P = F/A$$

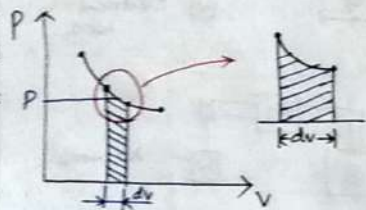
$$F = P \times A$$

$$\text{Work} = F \times x$$

$$dW = P \times A \times dx$$

$$dW = P dv$$

$$W = \int P dv \rightarrow \text{Closed system (or) non-flow work.}$$



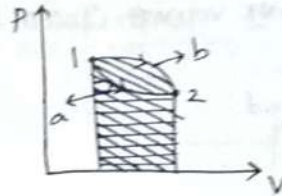
$$\text{Area of strip} = P dv$$

* Area under the curve when projected on volume axis gives Closed System (or) Non-flow work.

Conditions for applying the equation $W = \int P dv$:-

- 1) The system must be a closed system.
- 2) Work should cross the boundary.
- 3) It must be a reversible process.

→ Work



Though the endpoints are same for path 'a' and 'b', work is not same because areas are different as paths are different.

∴ Work transfer depends on path.

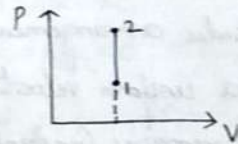
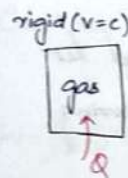
→ Work is a path function.

→ Work is not a property.

→ Work is inexact differential. (dW or δW)

CLOSED SYSTEM WORK IN VARIOUS PROCESSES:-

(1) Ischoric (or) Isometric (or) Constant Volume :-



As Vol = Constant
 $dv = 0$.

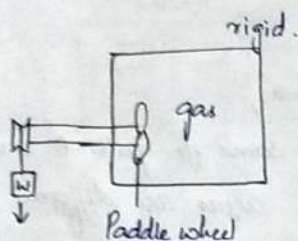
$$W = \int P dv = \int P \cdot 0$$

$$W = 0$$

Constant volume closed system work = 0.

* Rigid Container = Constant volume *

SPECIAL CASE OF CONSTANT VOLUME CLOSED SYSTEM WORK



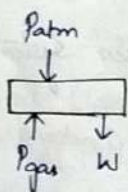
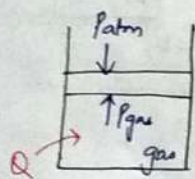
In this case; $PdV=0$ but work is not zero. It is valid for Reversible process whereas paddle wheel work is an irreversible work.

Paddle wheel work is negative; i.e., work is done on the system.

(2) Constant pressure (or) Isobaric (or) Isopiestic:-

Example of Const. pressure process:-

Consider a piston cylinder arrangement and let the piston move at a uniform velocity. During this process, pressure remains constant.



We know;

$$\Sigma F = ma$$

$$\text{As } v=c \Rightarrow a=0$$

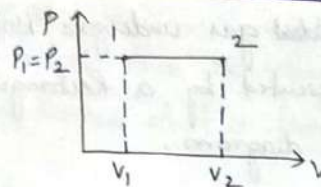
$$\Sigma F = 0$$

$$\Rightarrow P_{atm} + W = P_{gas}$$

$$\Rightarrow P_{gas} \cdot A = P_{atm} \cdot A + W$$

$$\Rightarrow P_{gas} = P_{atm} + \frac{W}{A} = \text{Constant}$$

Work:-

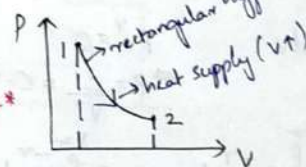
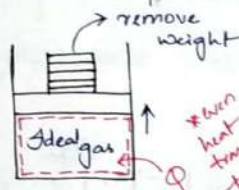


$$W = \text{Area} = P(V_2 - V_1)$$

$$W = \int_{V_1}^{V_2} P dv = P \int_{V_1}^{V_2} dv = P(v) \Big|_{V_1}^{V_2} = P(V_2 - V_1)$$

$$W = P(V_2 - V_1)$$

(3) Constant temperature (or) Isothermal process:- $\rightarrow (T=c)$



$$\text{Ideal gas} \Rightarrow PV = nRT = C$$

$$\Rightarrow P_1 V_1 = P_2 V_2 = C \Rightarrow PV = C \Rightarrow P = \frac{C}{V}$$

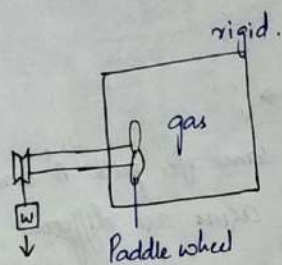
$$W = \int P dv = \int \frac{C}{v} dv = C \int \frac{dv}{v} = C (\ln v) \Big|_{V_1}^{V_2} = C \ln \left(\frac{V_2}{V_1} \right)$$

$$W = nRT \ln \left(\frac{V_2}{V_1} \right) \text{ (or) } P_1 V_1 \ln \left(\frac{V_2}{V_1} \right) \text{ or } P_2 V_2 \ln \left(\frac{V_2}{V_1} \right)$$

Considered Conditions:-

- 1) Closed system
- 2) Reversible
- 3) Ideal gas
- 4) Isothermal.

SPECIAL CASE OF CONSTANT VOLUME CLOSED SYSTEM WORK



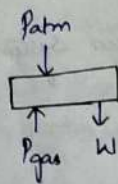
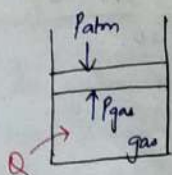
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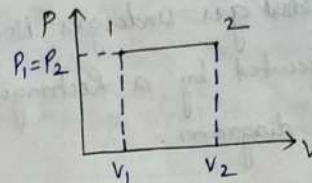
$$\Sigma F = 0$$

$$\Rightarrow P_{atm} + W = P_{gas}$$

$$\Rightarrow P_{gas} \cdot A = P_{atm} \cdot A + W$$

$$\Rightarrow P = P_{atm} + \frac{W}{A} = \text{Constant}$$

Work:-

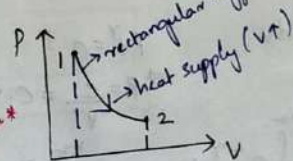
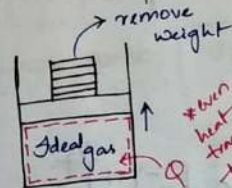


$$W = \text{Area} = P(V_2 - V_1)$$

$$W = \int_{V_1}^{V_2} P dV = P \int_{V_1}^{V_2} dV = P(V_2 - V_1)$$

$$W = P(V_2 - V_1)$$

(3) Constant temperature (or) Isothermal process:-



$$\text{Ideal gas} \Rightarrow PV = mRT = C$$

$$\Rightarrow P_1 V_1 = P_2 V_2 = C \Rightarrow PV = C \Rightarrow P = \frac{C}{V}$$

$$W = \int P dV = \int \frac{C}{V} dV = C \int \frac{dV}{V} = C (\ln V)_{V_1}^{V_2} = C \ln \left(\frac{V_2}{V_1} \right)$$

$$W = mRT \ln \left(\frac{V_2}{V_1} \right) \text{ (or) } P_1 V_1 \ln \left(\frac{V_2}{V_1} \right) \text{ or } P_2 V_2 \ln \left(\frac{V_2}{V_1} \right)$$

Considered Conditions:-

- 1) Closed system
- 2) Reversible
- 3) Ideal gas
- 4) Isothermal.

Sp NOTE:- When an ideal gas process; it is represented by a rectangular Hyperbola on a p-v diagram.

(4) ADIABATIC PROCESS:-

A process in which there is no heat transfer from the system (or) to the system is known as Adiabatic process.

For an Adiabatic process; $PV^\gamma = \text{Constant}$.

where $\gamma = \text{Adiabatic Index} \Rightarrow \gamma > 1$.

or $W = \int_{V_1}^{V_2} P dv$

$$PV^\gamma = C$$

$$\Rightarrow P = \frac{C}{V^\gamma} = C \cdot V^{-\gamma}$$

(2)
$$W = \int_{V_1}^{V_2} C \cdot V^{-\gamma} dv = C \int_{V_1}^{V_2} V^{-\gamma} dv = C \cdot \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2}$$

$$= \frac{C}{-\gamma+1} \left[V_2^{-\gamma+1} - V_1^{-\gamma+1} \right]$$

$$= \frac{C}{-\gamma+1} \left[V_2^{-\gamma} \cdot V_2 - V_1^{-\gamma} \cdot V_1 \right] = \frac{1}{-\gamma+1} \left[\frac{C}{V_2^\gamma} \cdot V_2 - \frac{C}{V_1^\gamma} \cdot V_1 \right]$$

$$= \frac{1}{-\gamma+1} \left[P_2 V_2 - P_1 V_1 \right] = \frac{P_1 V_1 - P_2 V_2}{\gamma-1}$$

$$\therefore W = \frac{P_1 V_1 - P_2 V_2}{\gamma-1}$$

Conditions:-
1) Closed system
2) Reversible
3) Adiabatic
4) Ideal gas

(5) POLYTROPIC PROCESS:-

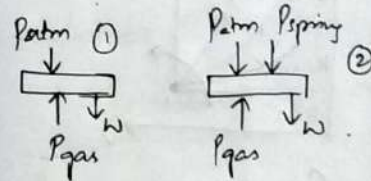
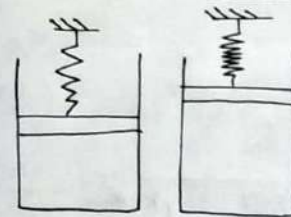
A process in which there are many changes [like temp; volume, pressure etc.] is known as polytropic process.

Example:- Piston-cylinder arrangement with a spring when heated. For a poly-tropic; $PV^n = \text{Constant}$. where 'n' is known as polytropic index.

Generally, $1 < n < \gamma$

In Adiabatic process, there is no heat transfer but in polytropic process; there is a heat transfer.

Polytropic work;
$$W = \frac{P_1 V_1 - P_2 V_2}{n-1}$$



① $\Rightarrow P_{\text{gas}} = P_{\text{atm}} + W$

② $\Rightarrow P_{\text{gas}} = P_{\text{atm}} + W + P_{\text{spring}}$
 $\therefore P_{\text{gas}}$ is increasing.

$\rightarrow P_{\text{gas}}$ is increasing.

$\rightarrow$ Expansion $\Rightarrow$ Vol. is increasing.

$\rightarrow$ Increase in temp $\Rightarrow$

$\rightarrow$

(1) Isochoric: - $PV^k = c$

$$PV^0 = c \Rightarrow k=0$$

(2) Isothermal: -

$$PV = nRT \Rightarrow PV^1 = c \Rightarrow k=1$$

(3) Polytropic: -

$$PV^n = c \Rightarrow k=n$$

(4) Adiabatic: -

$$PV^\gamma = c \Rightarrow k=\gamma$$

(5) Isochoric: -

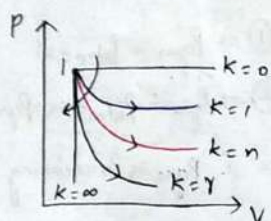
$$PV^k = c$$

$$\Rightarrow (P/V^k)^{1/k} = (c)^{1/k}$$

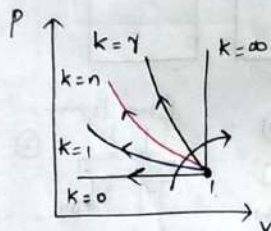
$$\Rightarrow P^{1/k} \cdot V = c$$

$$\Rightarrow P^{1/\infty} \cdot V = c \therefore k=\infty$$

$$\Rightarrow V = c$$



Expansion



Compression

Slope of Isothermal Process on P-V diagram: -



$$\text{Slope} = \frac{dp}{dv} \Rightarrow (dy/dx)$$

$$PV = nRT$$

$$PV = c$$

$$\Rightarrow Pdv + Vdp = 0$$

$$\Rightarrow \frac{dp}{dv} = -P/V$$

Note: - With increase in pressure, volume decreases and hence isothermal slope is negative.

SLOPE OF ADIABATIC PROCESS ON P-V DIAGRAM: -

$$\text{Slope} = \frac{dp}{dv}$$

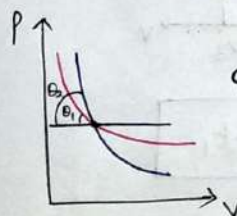
$$PV^\gamma = c$$

$$\Rightarrow P \cdot \gamma V^{\gamma-1} dv + V^\gamma \cdot dp = 0$$

$$\Rightarrow \frac{dp}{dv} = -\frac{\gamma P}{V} = \gamma \left(-P/V \right)$$

$$\therefore \text{Adiabatic Slope} = \gamma (\text{Isothermal Slope}) (\because \gamma > 1)$$

$$\therefore \text{Adiabatic Slope} > \text{Isothermal Slope}$$



$$\theta_2 > \theta_1 \Rightarrow \theta_2 = \text{Adiabatic}$$

$$\theta_1 = \text{Isothermal}$$

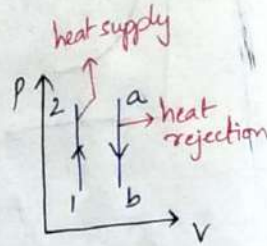
IDEAL GAS EQUATION FOR VARIOUS PROCESSES:-

(1) Constant volume process:-

$$PV = mRT$$

$$\frac{P}{T} = C$$

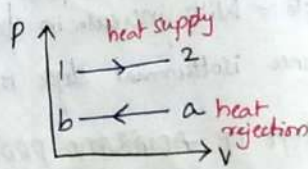
$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \boxed{\frac{P_1}{P_2} = \frac{T_1}{T_2}}$$



(2) Constant Pressure process:-

$$PV = mRT$$

$$\frac{V}{T} = C \Rightarrow \boxed{\frac{V_1}{V_2} = \frac{T_1}{T_2}}$$



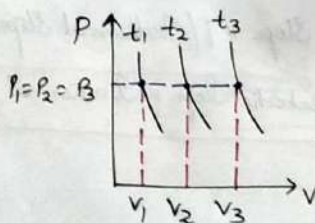
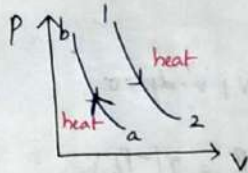
(3) Isothermal process:-

$$PV = mRT$$

$$PV = C$$

$$\Rightarrow P_1 V_1 = P_2 V_2$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{V_2}{V_1}$$



$$\boxed{t_3 > t_2 > t_1}$$

(4) Adiabatic process:-

$$PV = mRT$$

$$PV^\gamma = C$$

$$\Rightarrow \boxed{P_1 V_1^\gamma = P_2 V_2^\gamma} \quad (1)$$

$$P = \frac{mRT}{V}$$

$$\Rightarrow \frac{mRT}{V} \cdot V^\gamma = C$$

$$\Rightarrow TV^{\gamma-1} = C$$

$$\Rightarrow \boxed{T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}} \quad (2)$$

$$(1) = \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{1/\gamma} ; (2) = \frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{1/\gamma-1}$$

$$\Rightarrow \boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}} \quad (3)$$

(5) Polytropic process:-

$$P_1 V_1^n = P_2 V_2^n$$

$$T_1 V_1^{n-1} = T_2 V_2^{n-1}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

* All these above 5 process equations are valid for ideal gases only *

CONVENTIONAL QUESTIONS:-

1) A spherical balloon of 1m dia. Contains a gas at 150 kPa. The gas inside the balloon is heated until the pressure reaches 450 kPa. During the process of heating, the pressure of gas is proportional to cube of dia. of balloon. Find the work done by gas inside the balloon.

Sol:- $P_1 = 150 \text{ kPa}$ $P_2 = 450 \text{ kPa}$
 $D_1 = 1 \text{ m}$ $D_2 = ?$
 $P \propto D^3 \Rightarrow P = kD^3 \Rightarrow k = P/D^3 = P_1/D_1^3 = P_2/D_2^3$
 $\Rightarrow \frac{P_1}{P_2} = \frac{D_1^3}{D_2^3} \Rightarrow \frac{150}{450} = \frac{1^3}{D_2^3}$
 $\Rightarrow D_2^3 = 3 \Rightarrow D_2 = (3)^{1/3} = 1.44 \text{ m}$
 $D_2 = 1.44 \text{ m}$

$$V = \frac{4}{3} \pi \left(\frac{D_2}{2}\right)^3 = \frac{\pi D^3}{6}$$

$$dV = \frac{\pi}{6} [3D^2 dD] = \frac{\pi}{2} D^2 dD$$

$$W = \int kD^3 \cdot \frac{\pi}{2} D^2 dD$$

$$= \frac{k\pi}{2} \int D^5 dD$$

$$= \frac{k\pi}{2} \left[\frac{D^6}{6} \right]_{D_1}^{D_2} = \frac{k\pi}{12} [D_2^6 - D_1^6] = \frac{k\pi}{12} [D_2^6 - D_1^6]$$

$$= \frac{150 \times \pi}{12} [1.44^6 - 1^6] = 310.86 \text{ kJ}$$

(2) An ideal gas is undergoing a process in which P is proportional to $V^{-2/5}$. Calculate Work done by the gas from state 1 to state 2.

State 1	State 2
$P_1 = 10^4 \text{ kPa}$	$P_2 = ?$
$V_1 = 4 \text{ m}^3$	$V_2 = 2 \text{ m}^3$

Sol:- $T \propto V^{-2/5}$

$$T = kV^{-2/5} \Rightarrow TV^{2/5} = k$$

$$PV = mRT$$

$$PV = mR(kV^{-2/5}) \Rightarrow PV \cdot V^{2/5} = C$$

$$\Rightarrow PV^{7/5} = C$$

$PV^{1.4} = C$

$$\Rightarrow P_1 V_1^{1.4} = P_2 V_2^{1.4}$$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{1.4} \Rightarrow P_2 = 10^4 \times \left(\frac{4}{2}\right)^{1.4}$$

$$P_2 = 2.64 \times 10^4 \text{ kPa}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{n-1} = \frac{(10^4 \times 4) - (2.64 \times 10^4 \times 2)}{1.4 - 1}$$

$$= \frac{10^4 \times (-1.28)}{0.4}$$

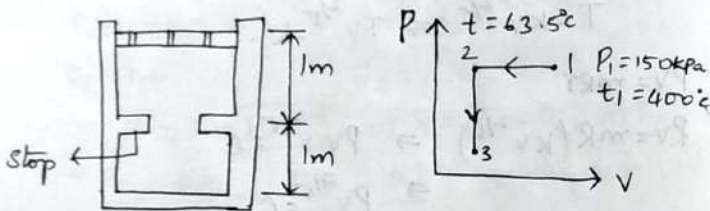
$$= - \frac{12.8 \times 10^4}{4}$$

$$= -32 \times 10^3 = -32 \text{ kJ}$$

W.D on the system.

(2) A piston cylinder arrangement shown in figure initially contains air at 150 kPa and 400°C. The setup is allowed to cool to ambient temperature of 20°C.

- (i) Is the piston resting on stop in the final state? What is the final pressure in the cylinder.
(ii) What is the work done by air per kg during the process.



Sol:- As temp. is reducing, vol. decreases.

$T \propto V$ $\Rightarrow$ Considering here $P=c$

$P_1 = 150 \text{ kPa}$

Constant-vel. moving piston.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow \frac{A L_1}{T_1} = \frac{A L_2}{T_2} \Rightarrow \frac{L_1}{T_1} = \frac{L_2}{T_2}$$

$$\Rightarrow \frac{2}{673} = \frac{L_2}{293} \Rightarrow L_2 = 0.87 \text{ m}$$

(i)

$\therefore$ As the final length of gas is less than 1m, the piston touches the stops.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{A \times 2}{A \times 1} = \frac{673}{T_2}$$

$$T_2 = 336.5 \text{ K} = 63.5^\circ \text{C}$$

(2-3):-

$$\frac{P_2}{P_3} = \frac{T_2}{T_3} \Rightarrow \frac{150}{P_3} = \frac{336.5}{293}$$

$$\Rightarrow P_3 = 130.6 \text{ kPa}$$

(ii) Work done = $W_{1-2} + W_{2-3}$ ($V=c$)

$$= W_{1-2}$$

$$= P(V_2 - V_1) = P_2 V_2 - P_1 V_1$$

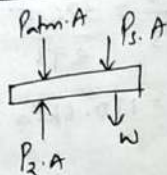
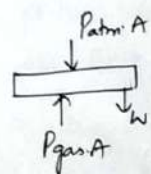
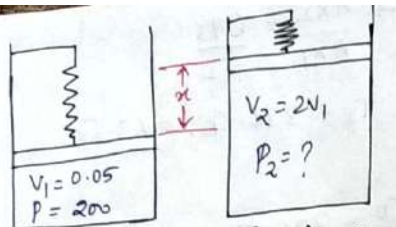
$$= mR(T_2 - T_1)$$

$$= 1 \times 0.287 (336.5 - 673)$$

$$\text{per kg is asked} = -96.57 \text{ kJ}$$

(4) A piston-cylinder assembly contains 0.05 m³ of a gas initially at 200 kPa. At this state, a linear spring which has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Heat is added to the gas causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the c/s area of piston is 0.25 m². Find
(i) Final pressure of gas in cylinder.
(ii) Work done by the gas.

Sol:-



$$\Rightarrow P_1 A = P_{atm} A + W$$

$$\Rightarrow P_1 = P_{atm} + \frac{W}{A} \quad (1)$$

$$P_2 A = P_{atm} A + P_s A + W$$

$$P_2 = P_{atm} + P_s + \frac{W}{A}$$

$$P_2 = P_1 + P_s$$

$$\Rightarrow F_s = kx$$

$$\Rightarrow F_s = 150 \times 0.2$$

$$\Rightarrow P_s = \frac{F_s}{A}$$

$$= \frac{150 \times 0.2}{0.25}$$

$$\Rightarrow P_s = 120 \text{ kPa}$$

$$V_2 - V_1 = A \times x$$

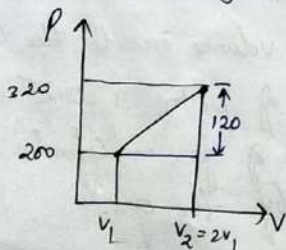
$$2V_1 - V_1 = A \times x$$

$$V_1 = A \times x$$

$$\Rightarrow \frac{0.05}{0.25} = x \Rightarrow 0.2 \text{ m}$$

$$\therefore P_2 = P_1 + P_s = 200 + 120 = 320 \text{ kPa}$$

To find work:- draw P-V diagram:-



$$P_2 = P_1 + P_s$$

$$P_2 = P_1 + \frac{F_s}{A} = P_1 + \frac{k_s x}{A}$$

$$P_2 = P_1 + \frac{k_s \Delta V}{A^2}$$

$$\Delta V = A \times x$$

$$x = \frac{\Delta V}{A}$$

$P_2 \propto \Delta V \Rightarrow$ linear $\Rightarrow$ straight line.

$$W \cdot D = \left(\frac{1}{2} \times 0.05 \times 120 \right) + (200 \times 0.05) = 13 \text{ kJ}$$

$$(or) W = P(V_2 - V_1) = 200 \times 0.05 = 10$$

$$W_s = \frac{1}{2} k x^2 = \frac{1}{2} \times 0.25 \times (0.2)^2 = 3$$

$$W_{tot} = W + W_s = 13 \text{ kJ}$$

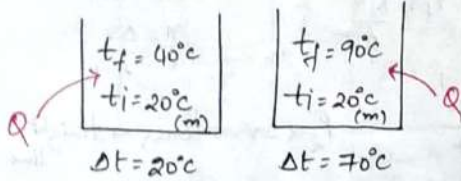
Since it is a constant-pressure process [no spring effect].

In this case, gas is doing two types of work:-

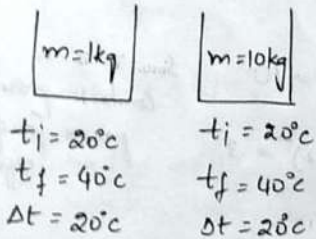
(i) raising the piston.

(ii) Compressing the spring.

HEAT:-



$$Q \propto \Delta t$$



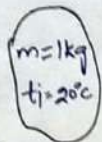
$$Q \propto m$$

$$Q \propto m \Delta t$$

$$Q = mc \Delta t$$

c = specific heat

Consider



$$Q = mc \Delta t$$

$$Q = 1 \times c \times 1$$

$$t_f = 21^\circ\text{C} \quad Q = c$$

Specific Heat:- It is the amount of heat required to raise the temperature of unit mass of substance through unit degree temperature difference.

Specific Heat actually represents heat absorbing capacity.

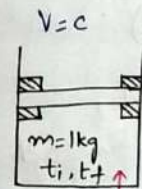
If Specific Heat is more, heat absorbing Capacity will be more.

As the specific heat of water is more than air, water has more heat absorbing Capacity.

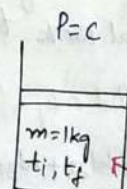
Unit of 'c':- $Q = mc \Delta t$

$$c = \frac{Q}{m \Delta t} = \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}} \quad (\text{or}) \quad \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$c_{\text{water}} = 4.187 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$



$$Q = 100$$



$$Q = 150$$

$$Q_p > Q_v$$

$$\Rightarrow (mc \Delta t)_p > (mc \Delta t)_v \Rightarrow c_p > c_v$$

$$\Rightarrow \gamma = \frac{c_p}{c_v} > 1$$

Specific heat at Constant pressure ' c_p ' > Specific heat at Constant volume ' c_v ' because c_p includes Internal energy and external work where as ' c_v ' includes internal energy only.

Note:-

1) In case of solids and liquids, the expansion work is negligible and hence C_p and C_v are almost same.

FIRST LAW OF THERMODYNAMICS (Conservation of Energy)

For a closed system, undergoing a cycle, net heat transfer (Total heat transfer) is equal to net work transfer (Total work transfer).

$$\Sigma Q = \Sigma W \quad (\text{valid only for a cycle})$$

$$Q_{12} + Q_{23} + Q_{31} = W_{12} + W_{23} + W_{31} \quad \text{any cycle}$$

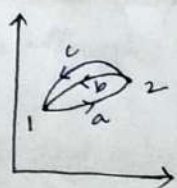
Note:- Heat supply is taken as +ve.

Heat rejection is taken as -ve.

The Equation $\Sigma Q = \Sigma W$ is valid for any cycle. (Rev. or Irrev. Cycle).

* Consequences of 1st law:-

1) Heat transfer is a path function.



$$\Sigma Q = \Sigma W$$

$$Q_{12} + Q_{23} = W_{12} + W_{23}$$

$$dQ_{12} + dQ_{23} = dW_{12} + dW_{23}$$

$$dQ_{23} - dQ_{21} = dW_{23} - dW_{21}$$

We know; that $dW_{2b1} \neq dW_{2c1}$

$$\text{So } \Rightarrow dW_{2b1} - dW_{2c1} \neq 0$$

$$\text{So; } dQ_{2b1} \neq dQ_{2c1}$$

$\Rightarrow$ this proves that heat transfer also depends on path.

Though the end points are same; for paths 'b' and 'c' but the ^{heat} work transfer is not same.

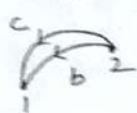
$\therefore$ ^{heat} Work transfer is independent of points and it depends on path. Hence, ^{heat} work transfer is a path function, inexact differential and not a property.

Important points w.r.t Heat and Work:-

- 1) Both are path functions.
- 2) Boundary phenomenon.
- 3) Not a property.
- 4) Inexact differentials.
- 5) Heat and Work are energies in transit. i.e., we observe heat and work only w.r.t to process. i.e., they lose significance once the process is over.

Note:- There is nothing like heat and work, the correct terms are heat transfer and work transfer.

2) Energy is a property.



$$\begin{aligned} dQ_{2b1} - dQ_{2c1} &= dW_{2b1} - dW_{2c1} \\ dQ_{2b1} - dW_{2b1} &= dQ_{2c1} - dW_{2c1} \\ \Rightarrow (dQ - dW)_{2b1} &= (dQ - dW)_{2c1} \end{aligned}$$

Though paths 'b' and 'c' are different, $(dQ - dW)$ is same for paths 'b' and 'c'.

$\therefore dQ - dW$ is independent of path and hence it is a property and this property is known as Energy.

$$(dQ - dW)_{2b1} = (dQ - dW)_{2c1} = dE$$

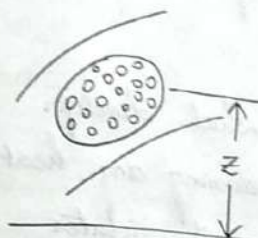
$$(dQ - dW)_{2b1} = dE$$

$$\therefore (dQ = dE + dW)_{2b1}$$

This is the 1st law equation for thermodynamic process for any process.

Note:- The equation $dQ = dE + PdV$ is valid for only Closed System Reversible process because $(dW = PdV)$ is valid only for closed system rev. process.

ENERGY:-



$$E = \underbrace{K.E}_{\text{Macroscopic}} + \underbrace{P.E}_{\text{Microscopic}} + U$$

$$dE = d(K.E + P.E) + dU \quad (\text{generally})$$

$$dE = dU$$

The equation $dQ = dU + dW$ is the first law of thermodynamic equation for any process when K.E and P.E changes are negligible.

INTERNAL ENERGY (U):-

All forms of energy associated with molecules is known as Internal Energy. It is an extensive property and it is generally expressed in KJ. Specific Internal Energy $(u) = \frac{U}{m}$ and it is an intensive property. Generally, expressed in KJ/kg.

3) Energy of an Isolated System is always Constant.

$$dQ = dE + dW$$

In isolated system; $dQ = 0$
 $dW = 0$

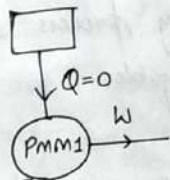
$$0 = dE + 0$$

$$dE = 0$$

$$E_2 - E_1 = 0 = \text{Constant.}$$

4) Perpetual motion machine of first kind (PMM-I) is impossible. (Continuous)

There can be no machine which develops work continuously without consuming any heat. PMM-I is impossible because it violates 1st law of thermodynamics for a cycle.



ENTHALPY (H):-

In thermodynamics, the term " $U+PV$ " appears regularly and for convenience, this term is known as "Enthalpy". It is generally expressed in KJ; Enthalpy is an extensive property.

Specific enthalpy (h) is equal to $\frac{H}{m}$. It is an intensive property. Generally, it is expressed in KJ/kg.

Physical meaning:-

for an ideal gas;

$$\textcircled{1} U = mC_v T \\ du = mC_v dT$$

$$\textcircled{2} H = mC_p T \\ dH = mC_p dT$$

$$\textcircled{3} U = f(T) \text{ only.} \\ \downarrow \\ \text{Joule's law}$$

Joule's law:- Internal energy of an ideal gas is a function of temperature only.

To show that $C_p - C_v = R$ for an ideal gas

$$H = U + PV$$

$$dH = du + d(PV)$$

$$\text{for ideal gas, } PV = mRT$$

$$\Rightarrow mC_p dT = mC_v dT + mRdT$$

$$\Rightarrow C_p = C_v + R$$

$$\Rightarrow \boxed{C_p - C_v = R}$$

$$\text{But } \frac{C_p}{C_v} = \gamma$$

$$C_p = \gamma \cdot C_v$$

$$\Rightarrow C_p - C_v = R \Rightarrow \gamma C_v - C_v = R$$

$$\Rightarrow C_v(\gamma - 1) = R$$

$$\Rightarrow \boxed{C_v = \frac{R}{\gamma - 1}}$$

$$\Rightarrow C_p = \gamma \cdot C_v$$

$$\boxed{C_p = \frac{\gamma R}{\gamma - 1}}$$

for air

$$C_p = 1.005 \text{ KJ/kg} \cdot \text{K}$$

$$C_v = 0.718 \text{ KJ/kg} \cdot \text{K}$$

$$R = 0.287 \text{ KJ/kg} \cdot \text{K}$$

$$\gamma = 1.4$$

HEAT TRANSFER IN VARIOUS PROCESS (CLOSED SYSTEM)

(1) Constant volume process:-

$$\begin{aligned} dQ &= du + dw \\ v = c \Rightarrow dw &= 0 \\ \therefore dQ &= du \end{aligned}$$

Heat transfer in a constant volume closed system process is equal to change in internal energy.

(2) Constant pressure process:-

$$\begin{aligned} dQ &= du + dw \\ dQ_p &= du + d(Pdv) \\ dQ_p &= d(U + Pdv) \\ dQ_p &= dH \end{aligned}$$

Heat transfer in a constant pressure closed system is equal to change in enthalpy.

(3) Isothermal process:-

$$\begin{aligned} dQ &= du + dw \\ U &= f(T) \text{ [ideal gas]} \end{aligned}$$

But in isothermal $\Rightarrow T = c \Rightarrow U = \text{constant}$
 $\Rightarrow du = 0$

$$\therefore dQ = dw$$

In a closed system isothermal process, Heat transfer is equal to work transfer. (Total heat is converted to work) in a "process".

(4) Adiabatic process:-

$$\begin{aligned} dQ &= du + dw; Q = 0 \text{ in adiabatic process.} \\ &[\text{No heat transfer}] \end{aligned}$$

(5) Polytropic process:-

$$\begin{aligned} dQ &= du + dw \\ dQ &= mC_v dT + \frac{P_1 V_1 - P_2 V_2}{n-1} \\ dQ &= mC_v (T_2 - T_1) + \frac{P_1 V_1 - P_2 V_2}{n-1} \\ dQ &= \frac{mR}{\gamma-1} (T_2 - T_1) + \frac{P_1 V_1 - P_2 V_2}{n-1} \\ &= \frac{mRT_2 - mRT_1}{\gamma-1} + \frac{P_1 V_1 - P_2 V_2}{n-1} \\ &= \frac{P_2 V_2 - P_1 V_1}{\gamma-1} + \frac{P_1 V_1 - P_2 V_2}{n-1} \\ &= (P_1 V_1 - P_2 V_2) \left[\frac{1}{n-1} - \frac{1}{\gamma-1} \right] \\ &= \frac{(P_1 V_1 - P_2 V_2)}{(n-1)} \left[\frac{\gamma-1-n+1}{\gamma-1} \right] \\ &= \frac{P_1 V_1 - P_2 V_2}{n-1} \left[\frac{\gamma-n}{\gamma-1} \right] \\ \therefore dQ_{poly} &= \left[\frac{\gamma-n}{\gamma-1} \right] dw_{poly} \end{aligned}$$

HE POLYTROPIC SPECIFIC HEAT:-

$$\begin{aligned}
 (1) \quad Q_{poly} &= \frac{\gamma-n}{\gamma-1} \times \frac{P_1 V_1 - P_2 V_2}{n-1} \\
 &= \frac{\gamma-n}{\gamma-1} \times \frac{1}{n-1} (mRT_1 - mRT_2) \\
 &= \frac{\gamma-n}{\gamma-1} \times \frac{mR}{\gamma-1} (T_1 - T_2) \\
 &= \frac{\gamma-n}{\gamma-1} \times mC_V (T_1 - T_2) = \frac{\gamma-n}{\gamma-1} \times mC_V (-dT) \\
 (2) \quad &= \frac{n-\gamma}{n-1} \times mC_V dT = m \left[\frac{n-\gamma}{n-1} C_V \right] dT
 \end{aligned}$$

$$Q_{poly} = mC_{poly} \cdot dT$$

$$\therefore \boxed{C_{poly} = \frac{n-\gamma}{n-1} C_V}$$

Take $\gamma = 1.4$; $1 < n < \gamma$

$$n = 1.2$$

$$\begin{aligned}
 (3) \quad \Rightarrow C_{poly} &= \frac{\overbrace{1.2-1.4}^{-ve}}{\underbrace{1.2-1}_{+ve}} \times C_V \rightarrow +ve \\
 \therefore \boxed{C_{poly} = -ve.}
 \end{aligned}$$

$$Q_{poly} = \frac{\gamma-n}{\gamma-1} \times W_{poly} = \frac{1.4-1.2}{1.4-1} \times W_{poly}$$

$$Q_{poly} = \frac{0.2}{0.4} \times W_{poly} = \frac{W_{poly}}{2}$$

$$\Rightarrow W_{poly} = 2 \times Q_{poly}$$

For $1 < n < \gamma$; C_{poly} is negative; in such a process though there is a supply of heat, there is a reduction in temp. This is because compared to heat transfer; work transfer is more and this extra work transfer comes from the internal energy of the system. As the internal energy is decreasing, the temp. is also decreasing.

S.No	Process	Ideal gas Equation	Work Transfer	Heat transfer
1.	$V=c$	$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$	0	dU
2.	$P=c$	$\frac{V_1}{V_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$	$P(V_2 - V_1)$	dH
3.	$T=c$	$P_1/P_2 = V_2/V_1$	$mRT \ln(V_2/V_1)$	dW
4.	$PV^\gamma=c$	$T_2/T_1 = (P_2/P_1)^{\frac{\gamma-1}{\gamma}}$	$\frac{P(V_2 - V_1)}{\gamma - 1}$	0
5.	$PV^\gamma=c$	$T_2/T_1 = (P_2/P_1)^{\frac{\gamma-1}{\gamma}}$	$\frac{P(V_2 - V_1)}{\gamma - 1}$	$\left[\frac{\gamma - \gamma}{\gamma - 1} \right] m R \ln \left(\frac{V_2}{V_1} \right)$

To show that $PV^\gamma = \text{constant}$ for an Adiabatic process:-

$$\delta Q = dU + \delta W$$

$$\Rightarrow \delta Q = dU + \delta W$$

$$\delta Q = dU + P dV$$

For an Adiabatic process, $\delta Q = 0$

$$0 = dU + P dV \quad dU = m C_v dT$$

$$\Rightarrow dU = -P dV$$

$$m C_v dT = -P dV \quad \text{--- (1)}$$

$$H = U + PV$$

$$dH = dU + P dV + V dP$$

$$dH = \delta Q + V dP$$

$$\boxed{\delta Q = 0}$$

$$dH = 0 + V dP \Rightarrow dH = V dP$$

for ideal gas; $dH = m C_p dT$

$$m C_p dT = V dP \quad \text{--- (2)}$$

$$\frac{(2)}{(1)} = \frac{m C_p dT}{m C_v dT} = \frac{V dP}{-P dV}$$

$$\Rightarrow \frac{C_p}{C_v} = \frac{V dP}{-P dV} \Rightarrow \gamma = \frac{dP}{P} \left(\frac{-V}{dV} \right)$$

$$\gamma \frac{dV}{V} = -\frac{dP}{P} \Rightarrow \gamma \frac{dV}{V} + \frac{dP}{P} = 0$$

Integrate

$$\gamma \ln V + \ln P = \ln C$$

$$\ln V^\gamma + \ln P = \ln C \Rightarrow \ln P V^\gamma = \ln C$$

$$\Rightarrow \boxed{P V^\gamma = C}$$

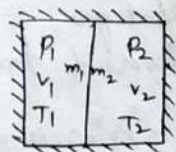
Conditions to apply this equation:-

- i) Adiabatic
- ii) Ideal gas
- iii) Reversible process.

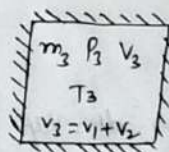
Note:- $P V^\gamma = C$ is valid both for open and closed systems.

1) An insulated rigid pressure vessel is divided into two portions by a partition. The first part of the vessel is occupied by an ideal gas at a pressure P_1 , volume V_1 and temperature T_1 . The other part is occupied by the same ideal gas but at P_2, V_2, T_2 . Suddenly, the partition is removed and two portions mix with each other. Show that, the final pressure P_3 and final temp T_3 are given by $P_3 = \frac{P_1 V_1 + P_2 V_2}{V_1 + V_2}$; $T_3 = \frac{P_1 V_1 + P_2 V_2}{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}}$.

Sol:-



Initial



Final

(i) Mass Conservation:-

$$m_3 = m_1 + m_2$$

$$m = \frac{P V}{R T}$$

$$\Rightarrow \frac{P_3 V_3}{R T_3} = \frac{P_1 V_1}{R T_1} + \frac{P_2 V_2}{R T_2}$$

$$\Rightarrow T_3 = \frac{P_3 V_3}{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}} \quad \text{--- (1)}$$

(ii) Energy Conservation:-

$$dQ = dU + dW$$

$$0 = dU + 0$$

$$dU = 0$$

$$U_f - U_i = 0 \Rightarrow U_f = U_i$$

$$\Rightarrow U_3 = U_1 + U_2$$

$$U = m C_v T$$

$$\Rightarrow m_3 C_v T_3 = m_1 C_v T_1 + m_2 C_v T_2$$

$$\Rightarrow \frac{m_3 R T_3}{\gamma - 1} = \frac{m_1 R T_1}{\gamma - 1} + \frac{m_2 R T_2}{\gamma - 1}$$

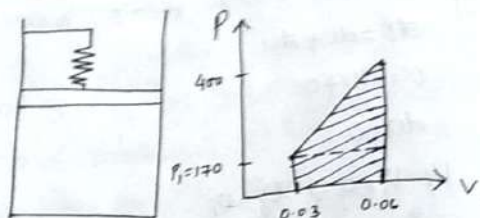
$$\Rightarrow P_3 V_3 = P_1 V_1 + P_2 V_2 \quad \text{--- (2)}$$

$$\Rightarrow \textcircled{2} \text{ in } \textcircled{1} \Rightarrow T_3 = \frac{P_1 V_1 + P_2 V_2}{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}}$$

$$\textcircled{2} \Rightarrow P_3 = \frac{P_1 V_1 + P_2 V_2}{V_3} = \frac{P_1 V_1 + P_2 V_2}{V_1 + V_2}$$

2) A gas is confined to a cylinder by a spring loaded frictionless piston so that the pressure in the fluid is a linear function of volume $P = a + bV$. The internal energy of gas is given by $U = 34 + 3.15PV$ where U is in kJ, P in kPa, V in m^3 . If the gas changes from initial state of $P_1 = 170 \text{ kPa}$, $V_1 = 0.03 \text{ m}^3$ to a final state of $P_2 = 400 \text{ kPa}$, $V_2 = 0.06 \text{ m}^3$. find the magnitude and direction of heat and work.

Sol:-



$$dW = \left(\frac{1}{2} \times 0.03 \times 230 \right) + (0.03 \times 170)$$

$$dW = +8.55 \text{ KJ}$$

→ direction = W.D by the system.

$$dQ = dU + dW$$

$$\rightarrow dU = U_2 - U_1$$

$$U = 34 + 3.15 PV$$

$$U_1 = 34 + 3.15 P_1 V_1$$

$$U_2 = 34 + 3.15 P_2 V_2$$

$$\therefore dU = U_2 - U_1 = 3.15 (P_2 V_2 - P_1 V_1)$$

$$= 3.15 [400 \times 0.06 - 170 \times 0.03]$$

$$dU = 59.53 \text{ KJ}$$

$$dQ = dU + dW = 59.53 + 8.55 = +68.085 \text{ KJ}$$

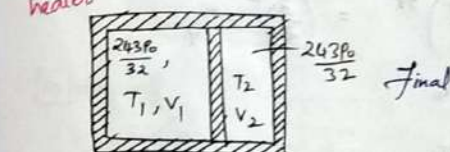
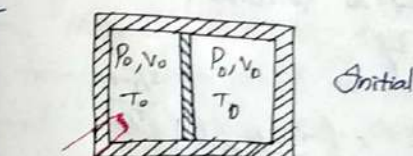
$\therefore$ Heat supplied to the system = direction.

3) A rectangular block as shown in figure has a partition which slides w/o friction along the length of the box. Initially, two chambers of box has 1kg each of ideal gas. ($\gamma = 5/3$) at a pressure P_0 , volume V_0 and temp. T_0 . The chamber

on the left is heated with the help of electric heater. The walls of the box and partition are thermally insulated. The gas in the left chamber expands pushing the partition until the final pressure in both the chambers is $\frac{243P_0}{32}$.

Determine the final temp. of gas in each chamber in terms of T_0 and also calculate work done by the gas in the right chamber.

Sol:-



$$PV = nRT$$

$$m = \frac{PV}{RT}$$

$$\text{mass initial} = \text{mass final}$$

(left) (left)

$$\Rightarrow \frac{P_0 V_0}{RT_0} = \frac{243 P_0 \times V_1}{32 RT_1}$$

$$\Rightarrow V_1 = \frac{32 V_0 T_1}{243 T_0}$$

$$\text{Similarly; } V_2 = \frac{32 V_0 T_2}{243 T_0}$$

$$V_1 + V_2 = 2V_0 \quad ; \quad \frac{32 V_0 T_1}{243 T_0} + \frac{32 V_0 T_2}{243 T_0} = 2V_0$$

$$\Rightarrow \frac{32}{243} (T_1 + T_2) = 2T_0$$

$$\Rightarrow T_1 + T_2 = \frac{243 T_0}{16}$$

As the walls and partition are thermally insulated, there is no heat transfer for the gas in right chamber.

∴ The right chamber gas undergoes adiabatic process. As the partition is frictionless; the process can be treated as reversible.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \frac{T_2}{T_0} = \left(\frac{P_2}{P_0} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{243 P_0}{32 P_0} \times \frac{1}{P_0} \right)^{\frac{5/3-1}{5/3}}$$

$$\Rightarrow \frac{T_2}{T_0} = \left(\frac{243}{32} \right)^{2/5} = \left(\frac{3}{2} \right)^2 = \frac{9}{4}$$

$$\Rightarrow T_2 = 2.25 T_0$$

$$\Rightarrow T_1 = \frac{243 T_0}{16} - 2.25 T_0 = 12.93 T_0$$

$$\boxed{T_1 = 12.93 T_0}$$

$$\text{Work done in right chamber} = \frac{P_i V_i - P_f V_f}{\gamma - 1}$$

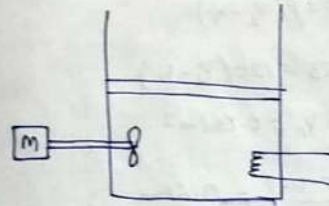
$$W = \frac{nRT_i - nRT_f}{\gamma - 1} = \frac{nR}{\gamma - 1} (T_i - T_f)$$

$$= \frac{1 \times R}{5/3 - 1} (T_0 - 2.25 T_0)$$

$$\boxed{W = -1.875 R T_0}$$

4) A vertical cylinder of c/s area 0.1 m^2 fitted with a leak proof frictionless freely moving piston contains air at a pressure of 120 kPa . The air is mixed by a paddle wheel for 10 min . The shaft of the paddle wheel running at 250 rpm with a torque of 0.5 Nm is driven by a motor during the same period an electrical resistor kept in cylinder and connected to 24 V battery draws a current of 0.45 A to heat air. Find the distance in cm., through which the piston rises given that heat transfer from air to surroundings is 5 kJ ; Internal energy increases by 2 kJ .

Sol:-



$$A = 0.1 \text{ m}^2, T = 0.5 \text{ Nm}; V = 24 \text{ V}, I = 0.45 \text{ A}$$

$$t = 10 \text{ min}$$

$$Q = 5 \text{ kJ}; dU = 2 \text{ kJ}$$

$$N = 250 \text{ rpm}$$

$$dQ = dU + dW$$

$$\Rightarrow -5 = 2 + dW$$

$$dW = -7 \text{ kJ}$$

$$\therefore dW_{\text{paddle}} + W_{\text{elec}} + W_{\text{piston}} = -7$$

$$\text{Paddle work:- } P = \frac{2\pi NT}{60} = \frac{2\pi \times 250 \times 0.5}{60} = 13.08 \text{ W}$$

$$W_{\text{work}} = P \times t = 13.08 \times 60 \times 10 = 7.85 \text{ kJ}$$

Electric work:-

$$P = V \times I \text{ (watt)}$$

$$\text{Work} = P \times t$$

$$= V \times I \times t$$

$$= 24 \times 0.45 \times 10 \times 60 \text{ (J)}$$

$$W_{\text{electric}} = 6.48 \text{ kJ}$$

$$W_{\text{paddle}} + W_{\text{elec}} + W_{\text{piston}} = -7$$

$$\Rightarrow -7.85 - 6.48 + W_{\text{piston}} = -7$$

$$W_{\text{piston}} = 7.33 \text{ kJ}$$

$$\Rightarrow V_2 - V_1 = A \times x$$

$$\Rightarrow x = \frac{V_2 - V_1}{A} \Rightarrow W = P(V_2 - V_1)$$

$$\Rightarrow 7.33 = 120(V_2 - V_1)$$

$$V_2 - V_1 = 0.061 \text{ m}^3$$

$$\Rightarrow x = \frac{0.061}{0.1} = 0.61 \text{ m}$$

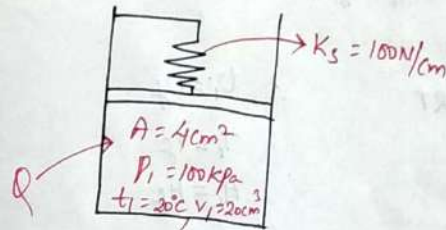
$$x = 61 \text{ cm}$$

5) Air is contained in a piston cylinder assembly as shown in figure with a $\frac{1}{4}$ area of 4 cm^2 and the initial volume is 20 cm^3 . Air is initially at 100 kPa and 20°C . Connected to this massless piston is a spring having a constant of 100 N/cm and the spring is initially undeformed. How much heat must be added to air to increase the

pressure to 300 kPa .

$$\text{Take } C_v = 0.718 \text{ kJ/kgK}, R = 0.287 \text{ kJ/kgK}$$

$$\text{Ans:- } (13.05 \text{ J})$$



Sol:-

FREE EXPANSION:-

The expansion of a gas against vacuum is known as free expansion. For work transfer, there must be resistance; greater the resistance; greater the work. So, the work in free expansion is zero; as there is no resistance.

* Free expansion work = 0 *



$$dQ = dU + dW$$

$$0 = dU + dW$$

$$\Rightarrow U_f - U_i = 0 \Rightarrow U_f = U_i \Rightarrow \text{internal energy is conserved.}$$

FREE EXPANSION OF IDEAL GAS:-

$$U_f = U_i$$

for ideal gas; $U = f(T)$
 $\Rightarrow T_i = T_f$

$$H = U + PV$$

$$H = f(T) + nRT$$

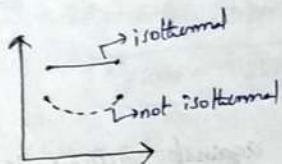
$$H = \phi(T)$$

$$\Rightarrow H_i = H_f$$

$$\therefore U_i = U_f$$

$$T_i = T_f$$

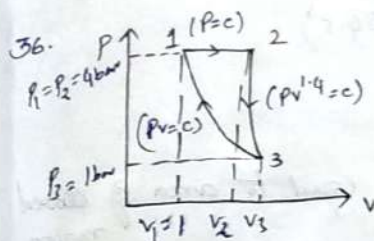
$$H_i = H_f$$



When an ideal gas undergoes free expansion; $T_i = T_f$.
 It does not mean that the process is isothermal because during the free expansion of an ideal gas; initially the temperature decreases and finally when molecules come in contact with walls of the container due to friction, the temperature increases.

$\therefore$ Temperature changes during the process.
 Hence, it is not isothermal.

Classroom Set:-



(3-1):- Isothermal

$$P_1 v_1 = P_2 v_2$$

$$4 \times 1 = 1 \times v_3$$

$$\Rightarrow v_3 = 4$$

(2-3):- Polytropic

$$P_2 v_2^{1.4} = P_3 v_3^{1.4}$$

$$\Rightarrow v_2 = 1.48$$

$$1 \text{ bar} = 100 \text{ kPa}$$

Work done:- $W_{1-2} + W_{2-3} + W_{3-1}$

$$W_{2-3} \Rightarrow \frac{P_2 v_2 - P_3 v_3}{n-1} = \frac{4 \times 1.48 - 1 \times 4}{1.4 - 1} =$$

$$= 4.86 \times 100 = 486 \text{ kJ}$$

$$W_{1-2} = P(v_2 - v_1) = 4(1.48 - 1) = 1.94 \times 100 = 194 \text{ kJ}$$

$$W_{3-1} \Rightarrow P_3 v_3 \ln\left(\frac{v_3}{v_1}\right) = 1 \times 4 \times \ln\left(\frac{4}{1}\right) = 1.38 \times 4 \times 100 = 554.5 \text{ kJ}$$

$$\Rightarrow P_1 v_1 \ln\left(\frac{v_f}{v_i}\right) = P_2 v_2 \ln\left(\frac{v_1}{v_3}\right) = 4 \times 1 \times \ln\left(\frac{1}{4}\right) = -554.5 \text{ kJ}$$

$$\text{Total Work} = 126 \text{ kJ}$$

$$\begin{aligned}
 W_{\text{net}} &= W_{12} + W_{23} + W_{31} \\
 &= 1094 - 4 + 486 + (-159.5) \\
 &= 125.9 \text{ kJ}
 \end{aligned}$$

Observations-

- (1) Net work in a cycle is equal to area of closed region.
- (2) All clockwise cycles on p-v diagrams are positive (work producing cycles).
- (3) All anti-clockwise cycles on p-v diagrams are work absorbing cycles.

1(b)

2(c)

3(c)

Initial = P_0 ; final = P_1

Initial = T_0 ;

$dQ = dW$

$$\begin{aligned}
 P_0 T_0 \ln\left(\frac{V_1}{V_0}\right) \\
 m R T_0 \ln\left(\frac{P_0}{P_1}\right)
 \end{aligned}$$

$$= -RT_0 \ln(P_0/P_1)$$

$$P_0 V_0 = P_1 V_1$$

$$\frac{V_1}{V_0} = \frac{P_0}{P_1}$$

$$3(d) \rightarrow \left(P + \frac{a}{V^2}\right)(V-b) = RT$$

5(b)

$$4(a) \quad P = \frac{RT}{(V-b)} - \frac{a}{V^2}$$

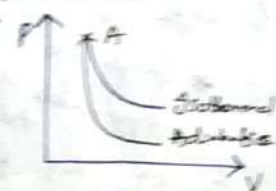
$$\begin{aligned}
 W &= \int P dV = \int \frac{RT}{V-b} dV - \int \frac{a}{V^2} dV \\
 &= RT \ln(V-b) \Big|_{V_1}^{V_2} - a \left[\frac{-2+1}{V} \right]_{V_1}^{V_2} \\
 &= RT \ln \left[\frac{V_2-b}{V_1-b} \right] + a \left[\frac{1}{V_2} - \frac{1}{V_1} \right]
 \end{aligned}$$

7(b)

8(d)

9(c)

Given in nature



10(c)

11(a)

12(a)

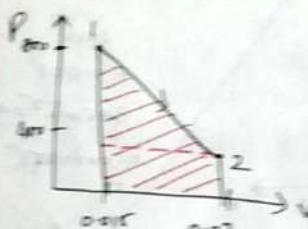
$$P_1 V_1 = P_2 V_2$$

$$\Rightarrow 60 \times 0.015 = P_2 \times 0.03$$

$$P_2 = 400 \text{ kPa}$$

$$\begin{aligned}
 P_1 &= 0.8 \text{ MPa} \\
 &= 0.8 \times 10^6 \text{ Pa} \\
 &= 0.8 \times 10^3 \text{ kPa} \\
 &= 800 \text{ kPa}
 \end{aligned}$$

$$P_2 = ?$$



$$\begin{aligned}
 W.D &= \left(\frac{1}{2} \times (0.015) \times (800 - 400) \right) \\
 &\quad + (400 \times 0.015)
 \end{aligned}$$

$$= 6 + 3$$

$$W.D = 9 \text{ kJ}$$

13(b)

14(a)



$$\begin{aligned}
 P &= 60 \text{ kPa} \\
 V &= 86.4 \text{ m}^3 \\
 t &= 6 \text{ hrs} \\
 P_1 &= 100 \text{ kPa}, T_1 = 273 + 32 \\
 &= 305 \text{ K}
 \end{aligned}$$

$$W_{\text{paddle}} = \frac{200 \text{ kJ}}{60}$$

$$W = \frac{\text{Power}}{\text{Time}}$$

$$\Rightarrow \text{Work} = 60 \times 4 \times 3600 = -864 \text{ kJ}$$

$$dQ = dU + dW$$

$$0 = dU - 864$$

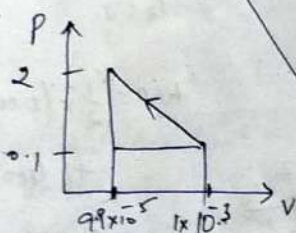
$$\therefore dU = 864 \text{ kJ}$$

$$\begin{aligned}
 m C_v dT &= 864 \\
 \Rightarrow dT &= \frac{864}{m C_v}
 \end{aligned}$$

$$\begin{aligned}
 m &= \frac{P V}{R T} = \frac{100 \times 86.4}{0.287 \times 305} \\
 &= 98.7 \text{ kg}
 \end{aligned}$$

$$\Rightarrow dT = 12.1^\circ \text{C} \Rightarrow \text{heated by } 12.1^\circ \text{C}$$

15.1) $T_1 = 300K$
 $P_1 = 0.1 \text{ MPa}$
 $V_1 = 1 \times 10^{-3} \text{ m}^3$
 $P_2 = 2 \text{ MPa}$
 $P_1 V_1^{1.3} = P_2 V_2^{1.3}$
 $(0.1 \times 10^6) \times (1 \times 10^{-3})^{1.3} = (2 \times 10^6) \times V_2^{1.3}$
 $\Rightarrow 1.25 \times 10^{-4} \times 0.1 \times 10^6 = 2 \times 10^6 \times V_2^{1.3}$
 $= 0.125 \times 10^2 = 2 \times 10^6 \times V_2^{1.3}$
 $\Rightarrow \frac{12.5}{2 \times 10^6} = V_2^{1.3} \Rightarrow V_2 = (6.25 \times 10^{-6})^{1/1.3}$
 $= 9.95 \times 10^{-5} \text{ m}^3$



16.(d) $W.D = \left(\frac{1}{2} \times (10^3 - 9.95 \times 10^5) \times 1.9 \right) + (0.1 \times (10^3 - 9.95 \times 10^5))$

19.(a) $W.D = (1 \times 100) + (1 \times 100) = (8.55 \times 10^4) +$
 $= 200 \text{ kJ}$

20.(a) $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$

$T_2 = 300 \times (3)^{\frac{2.5}{1.5}}$

$T_2 = 300 \times (3)^{\frac{2}{3}}$

(7005×10^3)

15.(c) $P_1 V_1^{1.3} = P_2 V_2^{1.3}$
 $\Rightarrow (0.1 \times 10^3) \times (1 \times 10^{-3})^{1.3} = (2 \times 10^3) \times V_2^{1.3}$
 $\Rightarrow 100 \times (1 \times 10^{-3})^{1.3} = 2000 \times V_2^{1.3}$
 $\Rightarrow \left[\frac{(1 \times 10^{-3})^{1.3}}{20} \right]^{1/1.3} = V_2$
 $\frac{1}{1.3} = 0.769$
 $(20)^{1/1.3} = 10.011$
 $\Rightarrow \frac{1 \times 10^{-3}}{(20)^{1/1.3}} = V_2 \Rightarrow \frac{0.001}{10.011} = V_2 = 9.98 \times 10^{-5}$

$W.D = \left[\frac{1}{2} \times (10^3 - 9.98 \times 10^5) \times (1900) \right] + \left[100 \times (10^3 - 9.98 \times 10^5) \right]$
 $= \left[\frac{1}{2} \times 9 \times 10^4 \times 1900 \right] + \left[100 \times 9 \times 10^4 \right]$
 $= [4.5 \times 10^2] + [9 \times 10^2] = 13.5 \times 10^2 \text{ kJ}$
 $= 13.5 \times 10$
 $= 1350 \text{ J}$

Adiabatic/Polytropic.

$W.D = \frac{P_1 V_1 - P_2 V_2}{n-1} = \frac{100 \times 1 \times 10^{-3} - 2000 \times 9.98 \times 10^{-5}}{0.3}$
 $W = -332 \text{ J}$

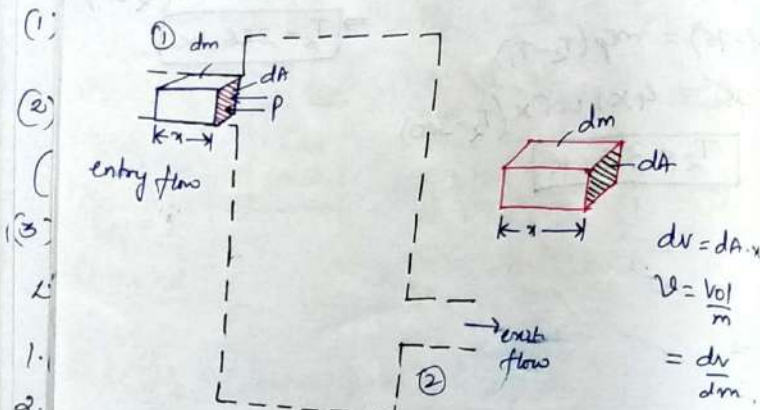
17.(a) $W.D = \frac{1}{2} \times 8 \times 3 = 12$

(b) $W.D = \frac{1}{2} \times 4 \times 4 = 12$

FLOW WORK:-

The work done in causing the fluid element either to enter (or) leave the control volume is known as flow work.

1. Flow work



$$W_{\text{flow}} = \text{force} \times \text{distance}$$

$$= p dA \times dx = p dv$$

$$\frac{W_{\text{flow}}}{\text{mass}} = \frac{p dv}{dm} \Rightarrow \frac{W_{\text{flow}}}{\text{mass}} = p v$$

$$\frac{\text{flow work}}{\text{mass}} = p v$$

$$\Rightarrow \text{flow work} = p v \times \text{mass}$$

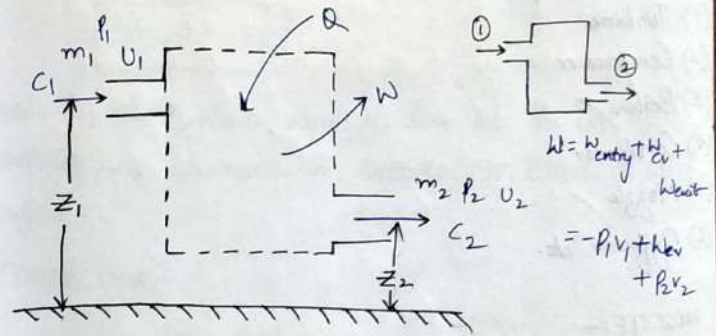
$$\boxed{\text{flow work} = p v}$$

* If the fluid enters at ①; then entry flow work is $-p_1 v_1$; and at ②, the exit flow work is $+p_2 v_2$.

STEADY FLOW ENERGY EQUATION:- (SFE)

A flow is said to be steady, when properties don't change with respect to time. To achieve steady flow conditions, mass entering the control volume = mass leaving the control volume.

Similarly, Energy entering C.V = Energy leaving C.V.



$$E_{\text{entry}} = E_{\text{exit}}$$

$$\frac{1}{2} m_1 C_1^2 + m_1 g z_1 + U_1 + Q = \frac{1}{2} m_2 C_2^2 + m_2 g z_2 + U_2 + W$$

$$m_1 = m_2$$

$$\frac{1}{2} m C_1^2 + m g z_1 + U_1 + Q = \frac{1}{2} m C_2^2 + m g z_2 + U_2 + W$$

$$\frac{1}{2} m C_1^2 + m g z_1 + U_1 + Q = \frac{1}{2} m C_2^2 + m g z_2 + U_2 - p_1 v_1 + W_{\text{c.v}} + p_2 v_2$$

$$\Rightarrow U_1 + p_1 v_1 + \frac{1}{2} m C_1^2 + m g z_1 + Q = U_2 + p_2 v_2 + \frac{1}{2} m C_2^2 + m g z_2 + W_{\text{c.v}}$$

$$\Rightarrow H_1 + \frac{1}{2} m C_1^2 + m g z_1 + Q = H_2 + \frac{1}{2} m C_2^2 + m g z_2 + W_{\text{c.v}}$$

$$\Rightarrow \boxed{h_1 + \frac{C_1^2}{2} + g z_1 + q = h_2 + \frac{C_2^2}{2} + g z_2 + w_{\text{c.v}}}$$

This is known as First Law of Thermodynamics for TURBINE:-
Open system under steady flow conditions.
It is applicable for reversible and irreversible processes.

① Assumption:-

(1) Steady flow.

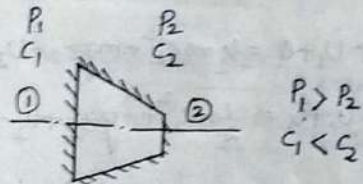
(2) Applications of SFEE:-

- (1) Turbine
- (2) Compressor
- (3) Boiler
- (4) Condenser
- (5) Nozzle
- (6) Diffuser etc.

④ NOZZLE:-

Nozzle is a device which converts pressure into velocity. Diffuser is a device which converts velocity into pressure.

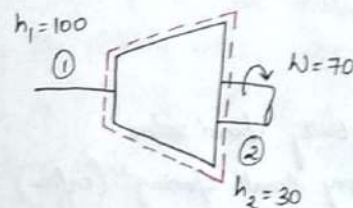
3. SFEE:-



$$h_1 + \frac{c_1^2}{2} + gz_1 + q = h_2 + \frac{c_2^2}{2} + gz_2 + w$$

$$c_1 \ll c_2$$

$$\therefore h_1 = h_2 + \frac{c_2^2}{2}$$



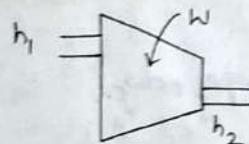
Assumptions:-
(1) Steady flow
(2) Neglect P.E
(3) Neglect K.E
(4) Insulated

$$h_1 + \frac{c_1^2}{2} + gz_1 + q = h_2 + \frac{c_2^2}{2} + gz_2 + w_{c.v}$$

$$w_{c.v} = h_1 - h_2$$

Note:- (1) In turbines, work is done at the cost of enthalpy i.e., decrease in enthalpy is equal to work output.

COMPRESSOR:-



Assumptions:-
(1) Steady flow
(2) Neglect P.E
(3) Neglect K.E
(4) Insulated

$$w = h_1 - h_2$$

$$w = -(h_2 - h_1)$$

$$-w = h_2 - h_1$$

$$\Rightarrow w_{comp} = h_2 - h_1$$

Note:- Increase in enthalpy in a compressor represents work input.

THROTTLING DEVICE:-

Examples of throttling:-

- (1) Flow through a partially opened valve.
- (2) Flow through a very small opening (orifice)
- (3) Flow through a porous plug.

Characteristics of throttling:-

- (1) No work transfer. (Work is lost in overcoming friction)
- (2) No heat transfer. (The length of throttling device is small. So, the time spent by the fluid in the device is less and hence there is no time available for heat transfer.)
- (3) It is an Irreversible process.
- (4) It is isenthalpic process.

$$h_1 + \frac{c_1^2}{2} + g z_1 + q = h_2 + \frac{c_2^2}{2} + g z_2 + dw_{c.v}$$

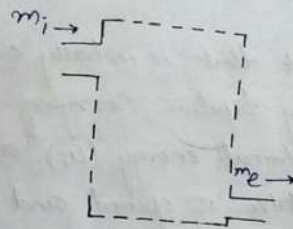
$$\therefore h_1 = h_2$$

UNSTEADY STATE FLOW:- (Variable/Transient)

A flow is said to be unsteady flow when fluid properties change w.r.t time.

Ex:- Filling the tank, emptying the tank.
(discharging)

Let m_i and m_e be the masses entering and leaving the Control volume. Let m_1 and m_2 be the masses in the Control volume under initial and final conditions.



(I) Conservation of Mass:-

$$\dot{m}_i - \dot{m}_e = \left(\frac{dm}{dt} \right)_{c.v} \quad \text{--- (1)}$$

(II) Conservation of Energy:-

$$\left(\frac{dE}{dt} \right)_{c.v} = \dot{E}_i - \dot{E}_e$$

$$= \frac{dE_i}{dt} - \frac{dE_e}{dt}$$

$$E_i = m_i h_i + \frac{1}{2} m_i c_i^2 + m_i g z_i + Q$$

$$E_e = m_e h_e + \frac{1}{2} m_e c_e^2 + m_e g z_e + W_{c.v}$$

$$\Rightarrow \left(\frac{dE}{dt} \right)_{c.v} = \frac{d}{dt} \left(m_i h_i + \frac{1}{2} m_i c_i^2 + m_i g z_i + Q \right) - \frac{d}{dt} \left(m_e h_e + \frac{1}{2} m_e c_e^2 + m_e g z_e + W_{c.v} \right)$$

$\Rightarrow$ Assuming, $d(KE) \& d(PE) = 0$.

$$\Rightarrow E = KE + PE + U \Rightarrow dE = dU$$

$$\Rightarrow \left(\frac{dU}{dt} \right)_{c.v} = \frac{d}{dt} (m_i h_i + Q) - \frac{d}{dt} (m_e h_e + W_{c.v})$$

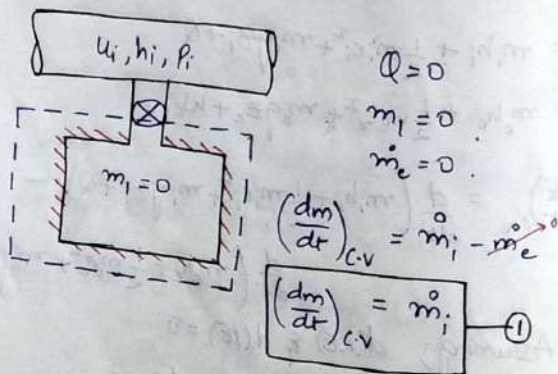
If h_i and h_e does not change with time,

$$\left(\frac{du}{dt}\right)_{c.v} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e + \dot{W}_{c.v} \quad (2)$$

Question:-

1) An insulated storage tank that is initially evacuated is connected to a supply pipeline carrying a fluid at a specific internal energy (u_i) and specific enthalpy (h_i). The valve is opened and fluid flows into the tank from supply pipeline and reaches the pressure same as that of supply pipeline. If the final specific internal energy of fluid in the tank is equal to h_i and hence deduce that if the fluid is an ideal gas, the final temp. T_2 of the gas in the tank is $\gamma T_i \rightarrow T_i$ is supply line temperature.

Sol:-



$$\left(\frac{du}{dt}\right)_{c.v} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e + \dot{W}_{c.v}$$

$$\left(\frac{du}{dt}\right)_{c.v} = \dot{m}_i h_i \quad (2)$$

① in ②

$$\Rightarrow \left(\frac{du}{dt}\right)_{c.v} = \left(\frac{dm}{dt}\right)_{c.v} h_i$$

$$\Rightarrow (du)_{c.v} = (dm)_{c.v} h_i$$

$$\Rightarrow u_2 - u_1 = h_i (m_2 - m_1)$$

$$\Rightarrow m_2 u_2 - m_1 u_1 = h_i m_2 - h_i m_1 \quad [m_1 = 0]$$

$$\Rightarrow m_2 u_2 = h_i m_2$$

$$\Rightarrow u_2 = h_i$$

If ideal gas; $u = C_v T$
 $h = C_p T$

$$\Rightarrow C_v T_2 = C_p T_i$$

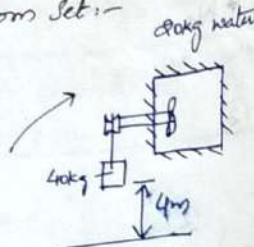
$$\Rightarrow T_2 = \gamma T_i$$

$$m_2 C_v (T_2 - T_i) = m_1 C_p (T_i - T_2)$$

Classroom Set:-

$$\frac{\text{kg} \cdot \text{m}}{\text{s}^2} = \text{J}$$

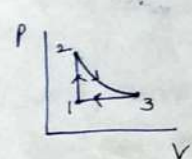
1. (a)
2. (b)
3. (b)
4. (a)
5. (a)
6. (d)
7. (b)
8. (d)



$t_i = 25^\circ\text{C}$
 $Q = 0$
 $W = mgh \times 500$
 $= 40 \times 9.81 \times 4 \times 500$
 $= \frac{10000}{1000}$
 $W = 784.8 \text{ kJ}$

$dQ = dE + dW$
 $dE = dQ - dW$
 property $\rightarrow$ exact.

$dQ = dU + dW$
 $0 = dU - 784.8 \Rightarrow dU = 784.8$
 $\Rightarrow mCdt = 784.8$
 $\Rightarrow 20 \times 4.187 \times (t_2 - 25) = 784.8$
 $\Rightarrow t_2 = 34.4^\circ\text{C}$



(1-2): $V = C$
 $Q = +500 \text{ kJ}$
 $dQ = dU$
 (2-3): $T = C$
 $W = +500 \text{ kJ}$
 $dQ = dW$
 $dQ = 500$
 (3-1): $P = C$
 $W = -200 \text{ kJ}$
 $P(V_2 - V_1) = -200 \text{ kJ}$

(2-3): $P V_2 \ln\left(\frac{V_2}{V_1}\right) = 500 \text{ kJ} \Rightarrow P V_2 - P V_1 = -200 \text{ kJ}$

For cycle; $\Sigma Q = \Sigma W$

$\Rightarrow Q_{12} + Q_{23} + Q_{31} = W_{12} + W_{23} + W_{31}$
 $\Rightarrow 50 + 500 + Q_3 = 0 + 500 - 200$
 $Q_3 = -250 \text{ kJ}$

For process; $dQ = dU + dW$

$-250 = dU - 200$
 $\Rightarrow dU = -50 \text{ kJ}$

9. (d) $\Rightarrow \frac{dn}{dt} = C_p$ $h = u + pv$
 10. (c) $dW = -5000$
 $dQ = -2000$
 $dQ = dU + dW$
 $dU = +3000 \text{ kJ}$

11. (a)
 12. (b) $V_1 = 0.057 \text{ m}^3$; $P_1 = 300 \text{ kPa}$, $T_1 = 300 \text{ K}$
 $t = 10 \text{ hrs}$; $P_2 = 370 \text{ kPa}$, $C_v = 21$; $dU = ?$

$h_1 = h_2$
 $\Rightarrow \frac{U_1}{2} + P_1 V_1 = \frac{U_2}{2} + P_2 V_2$
 $dU = m C_v dT = n C_v dT$
 $\frac{330}{200} = \frac{T_2}{300} = 1 \times 21 \times (30)$
 $\frac{330}{200} = 630 \text{ J/mol}$

13. (c) $\Rightarrow$ free exp. of ideal gas.
 14. (d) $\Rightarrow$ ideal gas $\Rightarrow (T = C)$; Non-ideal gas may increase density.

15. (c) $P = 2 \text{ kW}$; $V = 40 \text{ l}$; $C_p = 4.2$
 $t = 20 \text{ min}$; $dQ = 0$
 $-dU = dW$
 $W = -5000 \text{ J}$
 $Q = +1000 \text{ J}$
 $dQ = dU + dW$
 $1000 = dU - 5000$
 $dU = 6000$
 $m C_p dT = 2400 \Rightarrow 4.2 \times dT$
 $dT = 14.28 \text{ K} = 2400$
 $\frac{\text{kg} \times \text{kJ}}{\text{kgK}} \times \text{K}$
 14.28°C

17. (b)
 18. (b)
 19. (b)
 20. (c)
 (1-2): $Q = 0$
 $W_{12} = -5000 \text{ J}$
 (2-1): $Q_{21} = 1000 \text{ J}$
 $W_{21} = ?$
 $\Sigma Q = \Sigma W$
 $\Rightarrow 1000 = -5000 + W$
 $\Rightarrow W = 6000 \text{ J}$

36()

$$P_1 = 10^3 \text{ kPa}$$

$$T_1 = 350^\circ\text{C} = 350 + 273 = 623 \text{ K}$$

$$P_2 = 10^3 \text{ kPa}; T_2 = ?$$

$$\left(\frac{du}{dt}\right)_{cv} = m_i h_i + Q - m_e h_e - W_{cv}$$

$$du = m_i h_i$$

$$\Rightarrow m C_v dt = m C_p dT$$

37(a)

$$38() \quad \Sigma Q = \Sigma W$$

$$\Rightarrow 180 + Q = 130 + 40$$

$$Q =$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \frac{1}{1} = \frac{350^\circ\text{C}}{T_2}$$

$$\Rightarrow T_2 = 350^\circ\text{C}$$

SECOND LAW OF THERMODYNAMICS:- (directional law)

Need of 2nd law:-

First law of thermodynamics simply says that energy is conserved. It does not give any direction for a particular process. It is the 2nd law; that gives direction for a particular process, through the concept of Entropy.

In order to know the feasibility of a process, Second law of Thermodynamics has to be applied first. Once the direction is known; 1st law is then applied. Hence, 2nd law is known as directional law.

It is observed from experiments that the Complete conversion of heat to work is impossible in a cycle.

$\left[\begin{array}{l} \text{Heat} = \text{Low grade energy} \\ \text{Work} = \text{high grade energy} \end{array} \right]$

Note:- The Complete Conversion of LGE (heat) into HGE (work) may be possible in a process. i.e., when an ideal gas undergoes isothermal process; complete heat is converted into work.

THERMAL ENERGY RESERVOIRS

Source - It is a thermal reservoir which has the capacity to supply heat without undergoing any temp. change.



Sink - It is a thermal reservoir which has the capacity to absorb thermal energy without undergoing any temp. change.

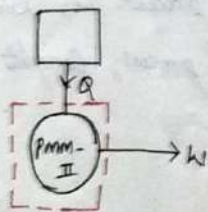
Note - Source and Sink are closed systems with only heat transfer.



STATEMENTS OF SECOND LAW:-

I) Kelvin - Planck Statement:-

It is impossible to develop a device operating on a cycle which produces work continuously while exchanging heat with a single reservoir.



$$\eta = \frac{Q/P}{I/P} = \frac{W}{Q} = 1$$

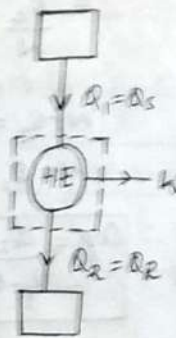
$$\eta = 100\%$$

Note - The efficiency of PMM-II is 100%. As PMM-II is impossible.

$\therefore$ 100% efficiency of any engine is also impossible according to second law.

HEAT ENGINE (work producing)

It is a cyclic operated device producing work. It converts part of heat into work and rejects remaining to sink (or) surroundings.



$$\eta = \frac{Q/P}{I/P} = \frac{W}{Q_1}$$

$$Q_1 = W + Q_2 \Rightarrow W = Q_1 - Q_2$$

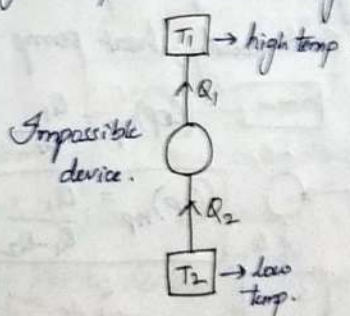
$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_R}{Q_S}$$

* valid for any cycle *

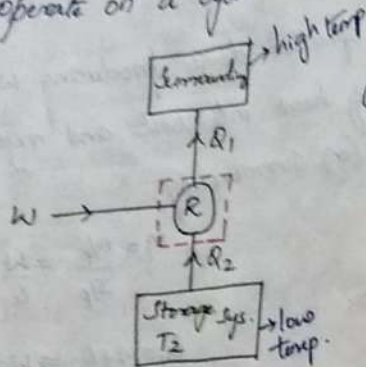
II) Clausius Statement:-

It is impossible to transfer heat from low temp. to high temp. without any external work input.



REFRIGERATOR:-

It is a device which maintains lower temperature compared to surroundings. As, lower temperatures are to be maintained continuously, refrigerator must operate on a cycle.



$$\text{COP} = \frac{\text{desired effect}}{\text{Energy input}}$$

$$= \frac{Q_2}{W}$$

$$W + Q_2 = Q_1$$

$$W = Q_1 - Q_2$$

$$\therefore \text{COP} = \frac{Q_2}{Q_1 - Q_2}$$

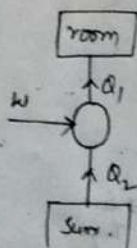
* There is no work of to get efficiency; so COP is introduced *

Valid for any cycle.

HEAT PUMP:-

It is a device which maintains higher temp. compared to surroundings. As high temperature are to be maintained continuously, heat pump must operate on a cycle.

Work absorbing device.



$$(\text{COP})_{\text{HP}} = \frac{Q_1}{W}$$

$$(\text{COP})_{\text{HP}} = \frac{Q_1}{Q_1 - Q_2}$$

Valid for any cycle

Relationship between COP of an H.P and COP of refrigerator operating between same temperature limits.

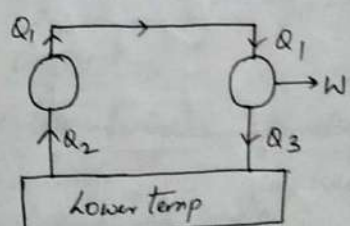
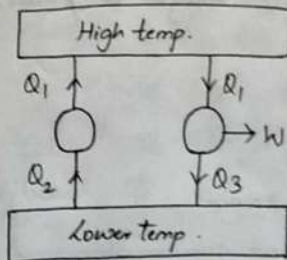
$$(\text{COP})_{\text{HP}} - (\text{COP})_{\text{R}} = \frac{Q_1}{Q_1 - Q_2} - \frac{Q_2}{Q_1 - Q_2}$$

$$= \frac{Q_1 - Q_2}{Q_1 - Q_2} = 1$$

$$(\text{COP})_{\text{H.P}} = 1 + (\text{COP})_{\text{R}}$$

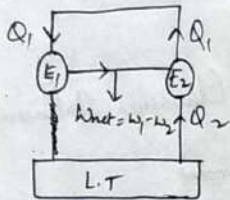
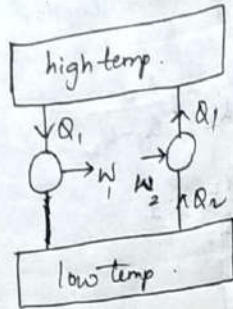
Equivalence of Kelvin-Planck and Clausius Statements:-

Case-I) Violation of Clausius statement:-



Violation of Clausius statement leads to violation of Kelvin-Planck statement. Similarly, violation of Kelvin-Planck leads to Clausius statement. These are called as Parallel laws (statements) of 2nd law.

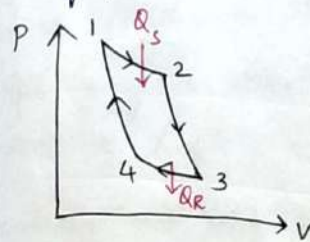
b) Violation of Kelvin-Planck Statement:-



As here, we can see that E_1 is producing work. Initially Q_2 from L.T. sink is producing heat Q_1 , which is enough to produce work.

CARNOT CYCLE:- (Reversible Cycle)

A cycle is said to be a reversible cycle where each and every process in a cycle is reversible.



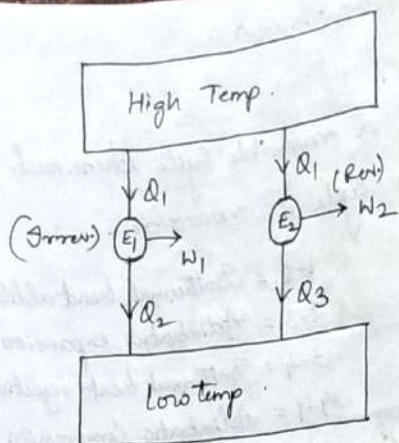
- 1-2 = Isothermal heat addition
- 2-3 = Adiabatic expansion
- 3-4 = Isothermal heat rejection
- 4-1 = Adiabatic Compression

Carnot Cycle consists of two reversible adiabatic and two isothermal processes. Adiabatic process is a fast process and isothermal is a slow process. Hence, such a combination is difficult to achieve. Moreover, in Carnot Cycle each process is a reversible process and hence it is very slow. Therefore, it is difficult to achieve Carnot Cycle in practice.

Though, it is not a practical cycle; but it is used for comparing other practical cycles.

CARNOT'S THEOREM:-

For different cycles, operating between same temperature limits; no cycle has efficiency greater than Carnot Cycle efficiency. [rev. cycle efficiency].

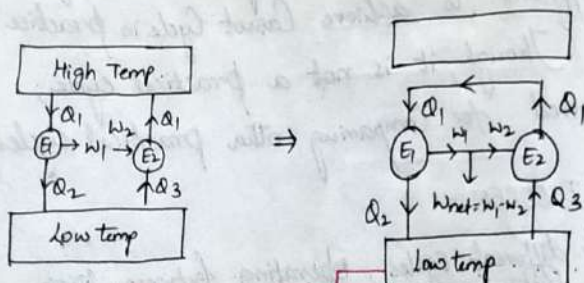


Assume; $\eta_{\text{irrev}} > \eta_{\text{rev}}$

$$\frac{W_1}{Q_1} > \frac{W_2}{Q_1}$$

$$\Rightarrow W_1 > W_2$$

As engine E_2 is a reversible engine; let us reverse the direction.



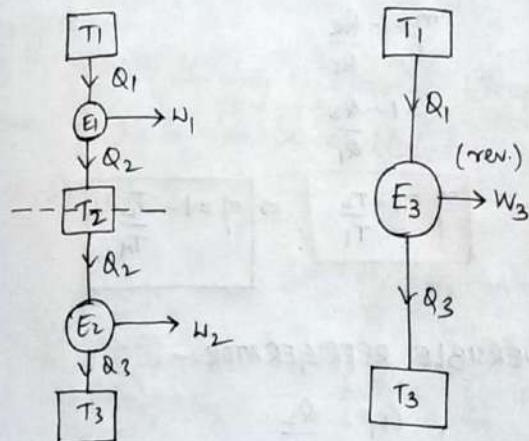
This is a violation of Kelvin-Planck statement. Hence, our assumption of $\eta_{\text{irrev}} > \eta_{\text{rev}}$ is wrong.

$\therefore$ Irreversible cycle efficiency can never be greater than Reversible cycle efficiency operating b/w same temp. limits.

Important points w.r.t Reversible engine :-

- 1) All Rev. engines operating between same temperature limits have same efficiency.
- 2) Efficiency of a reversible engine is independent of working fluid.
- 3) Efficiency of a reversible engine depends only on temperature limits.

Thermodynamic Temperature Scale:-



$$\eta_1 = 1 - \frac{Q_2}{Q_1}$$

$$\eta_1 = f(T_1, T_2) \Rightarrow 1 - \frac{Q_2}{Q_1} = f(T_1, T_2) \Rightarrow \frac{Q_2}{Q_1} = 1 - f(T_1, T_2)$$

$$\frac{Q_1}{Q_2} = \frac{1}{1 - f(T_1, T_2)}$$

$$\Rightarrow \frac{Q_1}{Q_2} = \phi(T_1, T_2)$$

$$\text{Similarly } \Rightarrow \frac{Q_2}{Q_3} = \phi(T_2, T_3) \text{ and } \frac{Q_1}{Q_3} = \phi(T_1, T_3)$$

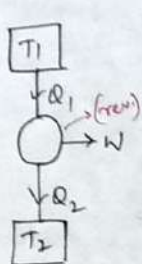
$$\frac{Q_1}{Q_2} = \frac{Q_1/Q_3}{Q_2/Q_3} = \frac{f(T_1, T_3)}{f(T_2, T_3)}$$

$$\frac{Q_1}{Q_2} = \frac{\psi(T_1)}{\psi(T_2)}$$

$$\Rightarrow \boxed{\frac{Q_1}{Q_2} = \frac{T_1}{T_2}} \rightarrow \text{valid ONLY for rev. cycle}$$

T_1, T_2 must be in Kelvin.

EFFICIENCY OF A REVERSIBLE ENGINE:-

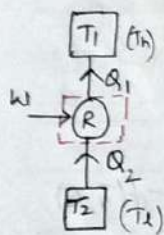


$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1}$$

$$\boxed{\eta = 1 - \frac{T_2}{T_1}} \Rightarrow \boxed{\eta = 1 - \frac{T_L}{T_H}}$$

COP OF A REVERSIBLE REFRIGERATOR:-



$$\text{COP} = \frac{Q_2}{W}$$

$$W = Q_1 - Q_2$$

$$(\text{COP})_R = \frac{Q_2}{Q_1 - Q_2}$$

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} = k \Rightarrow \frac{Q_1}{T_1} = \frac{Q_2}{T_2} = k$$

$$(\text{COP})_R = \frac{KT_2}{KT_1 - KT_2} = \frac{T_2}{T_1 - T_2}$$

$$(\text{COP})_R = \frac{T_L}{T_H - T_L}$$

Note:- COP of Rev. heat pump is $\frac{T_H}{T_H - T_L}$

CLAUSIUS INEQUALITY:-

The cyclic integral of $\frac{dQ}{T}$ is less than or equal to zero.

$$\oint \frac{dQ}{T} \leq 0$$

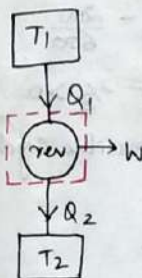
if $\oint \frac{dQ}{T} = 0 \Rightarrow$ reversible cycle

$\oint \frac{dQ}{T} < 0 \Rightarrow$ irreversible cycle.

Significance:-

With the help of Clausius Inequality, the nature of cycle can be established.

CASE-I) Reversible Cycle



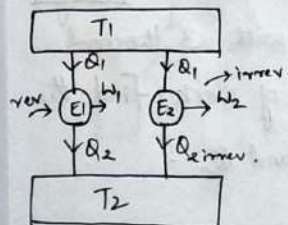
$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\oint \frac{dQ}{T} = \frac{Q_1}{T_1} + \left(\frac{-Q_2}{T_2} \right)$$

$$= 0$$

$\Downarrow$
rev. cycle.

CASE-II) Irreversible Cycle



$$Q_{2\text{irrev}} > Q_2$$

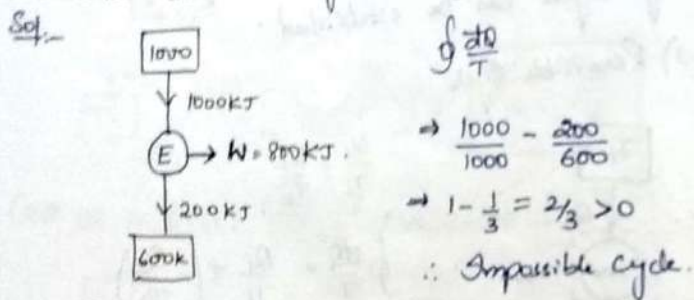
$$\oint_{\text{irrev}} \frac{dQ}{T} = \frac{Q_1}{T_1} + \left(\frac{-Q_{2\text{irrev}}}{T_2} \right)$$

$$\Rightarrow \oint_{\text{irrev}} \frac{dQ}{T} = \frac{Q_2}{T_2} - \frac{Q_{2\text{irrev}}}{T_2}$$

$$\therefore \oint \frac{dQ}{T} = \frac{Q_2 - Q_{2,rev}}{T_2}$$

As $Q_{2,rev} > Q_2 \Rightarrow \oint \frac{dQ}{T} < 0$

1) An inventor has claimed that he has developed an engine which receives 1000 kJ of heat at a temp. of 1000 K and rejects 800 kJ of heat at a temp. of 600 K. Delivering the remaining input as work. Check whether his claim is possible or not using Clausius Inequality. In case, it has to be reversible, determine the amount of heat rejected.



$$\oint \frac{dQ}{T}$$

$$\Rightarrow \frac{1000}{1000} - \frac{800}{600}$$

$$\Rightarrow 1 - \frac{4}{3} = -\frac{1}{3} < 0$$

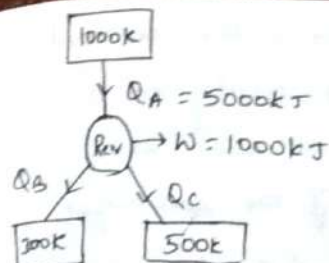
$\therefore$ Impossible cycle.

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \Rightarrow \frac{1000}{1000} \times 600 = Q_2$$

$$\Rightarrow Q_2 = 600 \text{ kJ}$$

2) Fig. shows a system that undergoes a rev. cycle during which it exchanges heat with 3 thermal reservoirs and develops 1000 kJ of work. Find the magnitude and directions of Q_B and Q_C .

Sol:-



$$Q_A = W + Q_B + Q_C$$

$$Q_B + Q_C = 4000 \text{ kJ} \quad \text{--- (1)}$$

$$\oint \frac{dQ}{T} = 0 \Rightarrow \frac{Q_A}{T_A} - \frac{Q_B}{T_B} - \frac{Q_C}{T_C} = 0$$

$$\Rightarrow \frac{Q_A}{T_A} = \frac{Q_B}{T_B} + \frac{Q_C}{T_C}$$

$$\Rightarrow \frac{5000}{1000} = \frac{Q_B}{300} + \frac{Q_C}{500}$$

$$\Rightarrow 5 = \frac{500Q_B + 300Q_C}{15 \times 10^4}$$

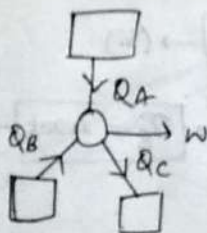
$$\Rightarrow 500Q_B + 300Q_C = 75 \times 10^4$$

$$500Q_B + 500Q_C = 4000 \times 500$$

$$200Q_C = 125 \times 10^4$$

$$Q_C = \frac{12500}{2} = 6250 \text{ kJ}$$

$$Q_B = -2250 \text{ kJ}$$



$$\eta = \frac{W}{Q_s} = \frac{W}{Q_A + Q_B}$$

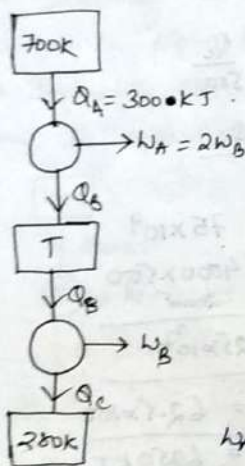
$$= \frac{1000}{7250}$$

$$= 0.13$$

$$\eta = 13\%$$

- 3) Two rev. heat engines A and B are arranged in series. Engine 'A' rejects heat directly to 'B'. Engine 'A' receives 300 kJ of heat at 427°C . While 'B' rejects heat to a sink at 7°C . If the work output of 'A' is 2 times of B. Find (i) Intermediate temp. of A and B (ii) η_A and η_B (iii) Q_R by A i.e. Q absorbed by B (iv) Heat rejected to sink.

Sol:



Rev. cycle

$$\left\{ \begin{array}{l} \frac{Q_A}{T_A} = \frac{Q_B}{T_B} \\ \frac{Q_B}{T_B} = \frac{Q_C}{T_C} \end{array} \right.$$

$$\Rightarrow \frac{Q_A}{T_A} = \frac{Q_C}{T_C}$$

$$\Rightarrow \frac{300}{700} = \frac{Q_C}{280} \Rightarrow Q_C = 120 \text{ kJ} \quad \downarrow \text{(iv)}$$

$$W_A = 2W_B$$

$$\Rightarrow Q_A - Q_B = 2(Q_B - Q_C)$$

$$\Rightarrow Q_A + 2Q_C = 3Q_B$$

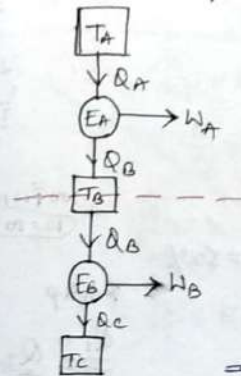
$$[Q_B = 180 \text{ kJ}] \rightarrow \text{(iii)}$$

$$\Rightarrow \frac{Q_A}{T_A} = \frac{Q_B}{T_B} \Rightarrow \frac{300}{700} = \frac{180}{T_B} \Rightarrow T_B = 420 \text{ K} \quad \text{--- (i)}$$

$$\eta_A = 1 - \frac{T_L}{T_H} = 1 - \frac{420}{700} = 40\%$$

$$\eta_B = 1 - \frac{280}{420} = 33.3\%$$

- 4) Two engines A and B are connected in series. Show that the overall efficiency of combined engine is $\eta_0 = \eta_A + \eta_B - \eta_A \eta_B$.



$$\eta_A = \frac{W_A}{Q_A}$$

$$\eta_B = \frac{W_B}{Q_B}$$

$$\eta_0 = \frac{W_A + W_B}{Q_A}$$

$$W_B = \eta_B Q_B$$

$$= \eta_B (Q_A - W_A)$$

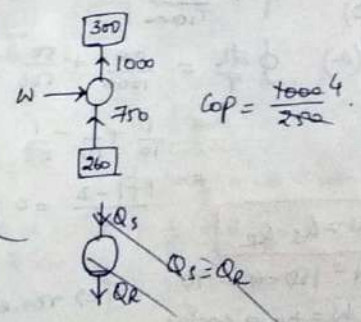
$$\Rightarrow \eta_0 = \frac{W_A}{Q_A} + \frac{W_B}{Q_A} = \eta_A + \eta_B \frac{(Q_A - W_A)}{Q_A}$$

$$\eta_0 = \eta_A + \eta_B - \frac{Q_A}{Q_A} \eta_B = \eta_A + \eta_B - \eta_A \eta_B$$

$$\boxed{\eta_0 = \eta_A + \eta_B - \eta_A \eta_B} \quad \text{--- (Expected Question)}$$

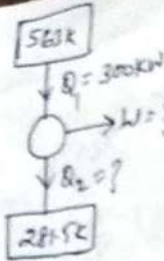
Classroom Set

1. (C)
2. (b)
3. (d)
4. (a)
5. (a) $\rightarrow$ mixture $\rightarrow$ process
6. (c) $\rightarrow$ steam point $\rightarrow$ cycle $\rightarrow$ 7. (c)
7. (c)
8. (C)



24(a)

25(b)



$$\begin{array}{r} 1290 \\ 273 \\ \hline 563 \\ 273 \\ \hline 281.5 \end{array}$$

Rev:-

$$Q_1 = W + Q_2$$

$$\Rightarrow \frac{Q_1}{T_1} = \frac{Q_2}{T_2} \Rightarrow \frac{300kW}{563} = \frac{Q_2}{281.5}$$

(Q-1)

$$\Rightarrow Q_2 = 150kW \Rightarrow \text{Rev.}$$

$\therefore W = 150$ In rev. case, it is $Q_R = 150$
For irrev; it should be more than $(Q_R)_{rev}$
For impossible; it may be less than that

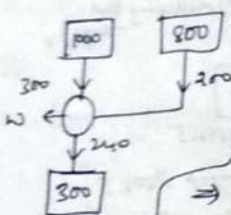
Irrev:-

$$\Rightarrow W = Q_1 - Q_2$$

$$\Rightarrow$$

26.(d)

27.(d)



$$\begin{aligned} \frac{300}{1000} + \frac{200}{800} - \frac{Q_3}{300} &= 0 \\ \Rightarrow \frac{3}{10} + \frac{2}{4} &= \frac{Q_3}{300} \\ \Rightarrow \frac{6+10}{20} &= \frac{Q_3}{300} \\ \Rightarrow \frac{16}{20} &= \frac{Q_3}{300} \\ \Rightarrow Q_3 &= 15 \times 16 = 240 \end{aligned}$$

W =

$$\frac{300}{1000} + \frac{200}{800} - \frac{Q_3}{300} = 0$$

$$\Rightarrow \frac{Q_3}{300} = \frac{3}{10} + \frac{2}{4}$$

$$= 0.3 + 0.5$$

$$= 0.8 \times 300 = 240$$

$$Q_3 = 240$$

$$Q = 165KJ$$

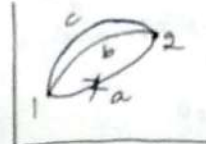
$$W = Q_1 - Q_2$$

$$= 500 - 165$$

$$W = 335KJ$$

ENTROPY

(Q-2) Reversible Cycle:-



Let 1-a-2-b-1 as rev

1-a-2-c-1 as rev

$$\oint \frac{dQ}{T} = 0$$

$$\Rightarrow \left(\frac{dQ}{T} \right)_{1a2} + \left(\frac{dQ}{T} \right)_{2b1} = 0$$

$$\left(\frac{dQ}{T} \right)_{1a2} + \left(\frac{dQ}{T} \right)_{2c1} = 0$$

$$\left(\frac{dQ}{T} \right)_{2b1} - \left(\frac{dQ}{T} \right)_{2c1} = 0$$

$$\Rightarrow \left(\frac{dQ}{T} \right)_{2b1} = \left(\frac{dQ}{T} \right)_{2c1} \Rightarrow \left(\frac{dQ}{T} \right)_{rev,b} = \left(\frac{dQ}{T} \right)_{rev,c}$$

$\Rightarrow$ Doesn't depend on path $\Rightarrow$ property.

Though paths 'b' and 'c' are different but

$\left(\frac{dQ}{T} \right)_{rev}$ is same for paths 'b' and 'c'.

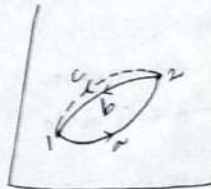
Hence; $\left(\frac{dQ}{T} \right)_{rev}$ must be a property. This property is known as entropy.

$$\therefore \left(\frac{dQ}{T} \right)_{rev,b} = \left(\frac{dQ}{T} \right)_{rev,c} = ds$$

$$\therefore ds = \left(\frac{dQ}{T} \right)_{rev}$$

['ds' $\Rightarrow$ 'd's' written as state is changing from '2' to '1' properties are changing.]

Case-II) Irreversible Cycle:-



1-a-2-b-1 = rev. Cycle
1-a-2-c-1 = irrev. Cycle

$$\oint_{\text{rev}} \frac{dq}{T} = 0$$

$$\Rightarrow \left(\frac{dq}{T} \right)_{1a2} + \left(\frac{dq}{T} \right)_{2b1} = 0$$

$$\Rightarrow \left(\frac{dq}{T} \right)_{1a2} = - \left(\frac{dq}{T} \right)_{2b1} \quad \text{--- (1)}$$

$$\Rightarrow \oint_{\text{irrev}} \frac{dq}{T} < 0$$

$$\Rightarrow \left(\frac{dq}{T} \right)_{1a2} + \left(\frac{dq}{T} \right)_{2c1} < 0$$

$$\Rightarrow \left(-\frac{dq}{T} \right)_{2b1} + \left(\frac{dq}{T} \right)_{2c1} < 0$$

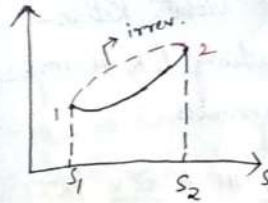
$$\Rightarrow \left(\frac{dq}{T} \right)_{2c1} < \left(\frac{dq}{T} \right)_{2b1}$$

$$\Rightarrow \left(\frac{dq}{T} \right)_{\text{irrev}} < \left(\frac{dq}{T} \right)_{2b1}$$

$$\Rightarrow ds > \left(\frac{dq}{T} \right)_{\text{irrev}} \quad \text{--- (2)}$$

From (1) and (2)

$$ds \geq \left(\frac{dq}{T} \right)$$



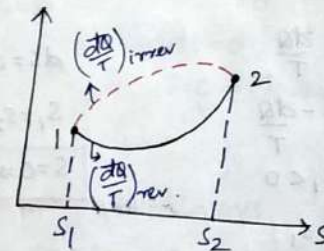
$$dS_{\text{rev}} = S_2 - S_1$$

$$dS_{\text{irrev}} = S_2 - S_1$$

$$dS_{\text{rev}} = dS_{\text{irrev}}$$

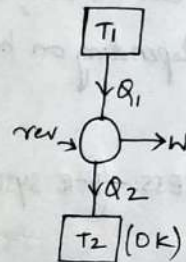
* As long as end points are same, Entropy change will be same, irrespective of nature of process (Rev. or Irrev.)

In order to calculate the entropy change for irrev. process it must be replaced by a rev. process between same end points. As Entropy is a property, it will be same between same end points.



To show that, it is impossible to reach absolute zero according to 2nd law of thermodynamics.

Sol:-



$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0$$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$Q_2 = \frac{Q_1}{T_1} \cdot T_2$$

$$= \frac{Q_1}{T_1} \times 0$$

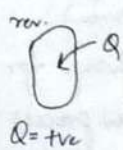
$$Q_2 = 0$$

Violation of Kelvin Planck.

If OK is achieved; then it would violate second law of thermodynamics. Planck statement and hence reaching OK is impossible according to 2nd law of thermodynamics.

ENTROPY CHANGE OF A SYSTEM IN A REV. PROCESS

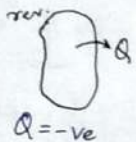
Case-(i) Heat Supply.



$$ds = \frac{dq}{T}$$

$$ds = +\frac{dq}{T} \Rightarrow s_2 - s_1 > 0 \Rightarrow s_2 > s_1 \Rightarrow \boxed{s \uparrow}$$

Case-(ii) Heat Rejection



$$ds = \frac{dq}{T}$$

$$ds = -\frac{dq}{T}$$

$$s_2 - s_1 < 0$$

$$s_2 < s_1$$

$$\boxed{s \downarrow}$$

Case-(iii) Rev. Adiabatic

$$Q = 0$$

$$ds = 0$$

$$s_1 = s_2$$

$$\boxed{s = c \Rightarrow \text{Isentropic}}$$

* In a Rev. process, the system entropy can increase, can decrease or can remain constant depending on heat transfer.*

ENTROPY CHANGE IN IRREV. PROCESS:-(FOR SYSTEM)

Case-(i) Heat Supply

$$ds > \left(\frac{dq}{T}\right)_{\text{irrev}}$$

$$ds = \left(\frac{dq}{T}\right)_{\text{irrev}} + \delta s_{\text{gen}} \quad \delta s_{\text{gen}} \rightarrow +ve$$

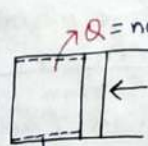
$$\text{For rev. } ds = \left(\frac{dq}{T}\right)_{\text{rev}} + 0 \rightarrow \delta s_{\text{gen}}$$

$\delta s_{\text{gen}} \rightarrow$ Entropy generation (or) Entropy production.

This term appears in Irrev. process. In rev. process;

$$\boxed{\delta s_{\text{gen}} = 0}$$

Irreversible + Non-Adiabatic.



$Q = \text{non-adiabatic}$

$$ds = \frac{dq}{T} + \delta s_{\text{gen}}$$

$$= -3 + 3$$

$$= 0$$

$$ds = 0$$

$$s = c \Rightarrow \text{Isentropic}$$

Though;

A Rev. Adiabatic is ALWAYS Isentropic;

An Isentropic process NEED NOT BE Rev. Adiabatic

Adiabatic process

Rev.

$$ds = \frac{dq}{T}$$

$$ds = 0$$

$$s = c$$

Isentropic

Irrev.

$$ds = \frac{dq}{T} + \delta s_{\text{gen}}$$

$$ds = 0 + \delta s_{\text{gen}}$$

$$ds = +ve$$

Non-isentropic

An Adiabatic process can be isentropic (or) Non-isentropic. If the adiabatic process is reversible; it has to be isentropic. If the Adiabatic process is irreversible, the system increases and it is non-isentropic.

* System Entropy can never decrease in Adiabatic process *

The Entropy of an Irrev. Adiabatic always increases *

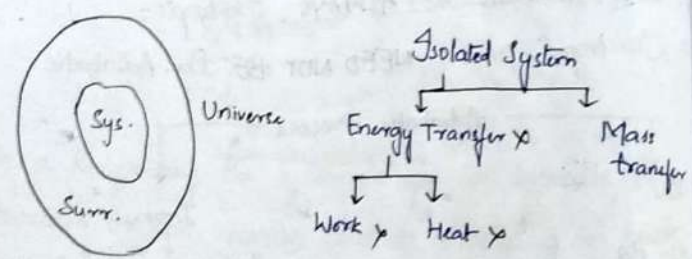
Physical meaning of Entropy:-

$$S = k \ln \Omega$$

$S \propto \ln(\text{randomness})$

Entropy is a measure of disorder (or) randomness of the molecules. Greater the disorder; larger is the entropy and lesser is the efficiency.

Entropy change of Universe:-



$$ds \geq \frac{dq}{T}$$

$$ds \geq \frac{0}{T}$$

$$\Rightarrow ds_{univ} \geq 0$$

$$ds_{univ} = 0 \Rightarrow \text{Reversible}$$

$$ds_{univ} > 0 \Rightarrow \text{Irreversible}$$

$$ds_{univ} < 0 \Rightarrow \text{Impossible for "process"}$$

$$dS_{univ} = dS_{sys} + dS_{surr.}$$

System entropy can increase, can decrease similarly or can remain constant. Similarly surrounding entropy can increase, can decrease or can remain constant but Universe Entropy can never decrease.

This is known as Principle of Increase of Entropy.

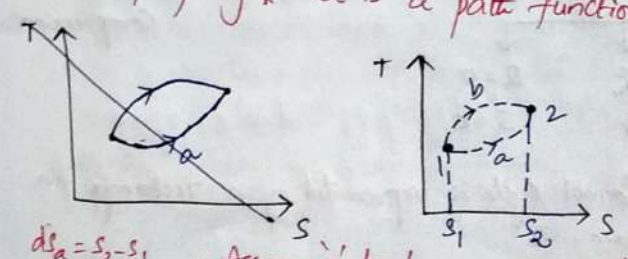
Note:- As Entropy is a measure of randomness, larger the number of molecules; greater is the randomness. Larger is the entropy.

$\therefore$ Entropy depends on mass. $\therefore$ Extensive property.

Entropy generation:-

It is a measure of irreversibilities. Greater the irreversibility; larger is the entropy generation. A process which is more irreversible has more Entropy generation.

* Though Entropy is a property, Entropy generation is not a property *. It is a path function.



$$ds_a = s_2 - s_1$$

$$ds_b = s_2 - s_1$$

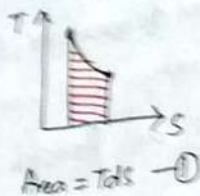
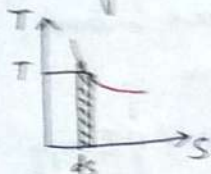
Assume 'a' to be more irreversible

$$ds_a > ds_b \Rightarrow \text{path function.}$$

$$dS_{\text{rev}} = \frac{\delta Q}{T} + dS_{\text{gen}}(\text{rev})$$

$$dS_{\text{rev}} = \left(\frac{\delta Q}{T} \right) + 0$$

T-S Diagram:-



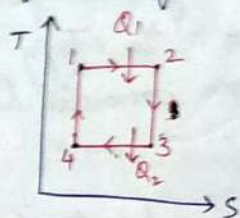
$$\text{Area} = T \Delta S \quad \text{--- (1)}$$

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{rev}} \Rightarrow \delta Q = T dS \quad \text{--- (2)}$$

from (1) and (2) $\Rightarrow \text{Area} = \delta Q_{\text{rev}}$

* Area under curve when projected on Entropy axis gives **REVERSIBLE** heat transfer.

Representation of Carnot Cycle on T-S diagram:-



- (1-2):- Isothermal heat supply
- (2-3):- Adiabatic expansion
- (3-4):- Isothermal heat rejection
- (4-1):- Adiabatic compression.

Rev.

Q_S	Q_R	$Q = 0$
$S \uparrow$	$S \downarrow$	$S = C$

Note:- (1) Carnot cycle is represented by a rectangle on T-S diagram.

(2) A cycle which is clockwise on P-V diagram is always clockwise on T-S diagram.

COMBINED FIRST AND SECOND LAW OF THERMODYNAMICS

$$\delta Q = dU + \delta W \Rightarrow \delta Q = dU + PdV \quad \text{--- (1)}$$

$$dS = \frac{\delta Q}{T}$$

$$\delta Q = T dS \quad \text{--- (2)}$$

$$\Rightarrow T dS = dU + PdV$$

$$\boxed{dS = \frac{1}{T} [dU + PdV]} \rightarrow \text{valid for any process.}$$

* This equation is valid for both rev. and irrev. It connects various properties (point functions) and hence it is applicable for ANY process.

$$H = U + PV$$

$$dH = dU + PdV + VdP$$

$$\delta Q = dU + \delta W$$

$$\delta Q = dU + PdV$$

$$\Rightarrow dH = \delta Q + VdP \quad \text{--- (1)}$$

$$\Rightarrow \delta Q = T dS \quad \text{--- (2)}$$

$$\therefore \boxed{T dS = dH - VdP} \rightarrow \text{valid for any process.}$$

This connects various properties and applicable for ANY process.

Note:- Equations $T dS = dU + PdV$
 $T dS = dH - VdP$ are valid for

- 1) any system [closed & open]
- 2) any process [rev & irrev]
- 3) any working fluid

REPRESENTATION OF CONSTANT VOLUME AND CONSTANT PRESSURE ON T-S DIAGRAM FOR AN IDEAL GAS:-

$$Tds = du + pdv$$

$$(v=c) \Rightarrow pdv = 0$$

$$Tds = du$$

$$\text{Ideal gas} \Rightarrow du = mC_v dT \Rightarrow du = C_v dT$$

$$\Rightarrow Tds = mC_v dT$$

$$\Rightarrow Tds = C_v dT$$

$$\Rightarrow Tds = du$$

$$\Rightarrow Tds = C_v dT$$

$$\Rightarrow \frac{dT}{ds} = \frac{T}{C_v} \left\{ +ve \right.$$

$$Tds = dh - vdp$$

$$(p=c) \Rightarrow dp = 0$$

$$Tds = dh$$

$$dh = mC_p dT \Rightarrow dh = C_p dT$$

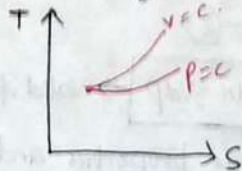
$$Tds = mC_p dT$$

$$\Rightarrow Tds = C_p dT$$

$$\Rightarrow Tds = C_p dT$$

$$\Rightarrow \frac{dT}{ds} = \frac{T}{C_p} \left\{ +ve \right.$$

But $C_p > C_v$, So slope of $(v=c)$ process is more.



* Slope of Constant volume Curve on T-S diagram is greater than slope of Constant pressure Curve *

ENTROPY of solids < Entropy of liq. < Entropy of gases.

Entropy change for an Ideal gas:-

$$Tds = du + pdv$$

$$ds = \frac{du}{T} + \frac{pdv}{T}$$

$$\text{Ideal gas} \rightarrow pv = mRT \rightarrow \frac{p}{T} = \frac{mR}{v}$$

$$\rightarrow du = mC_v dT$$

$$\Rightarrow ds = \frac{mC_v dT}{T} + \frac{mR dv}{v}$$

$$\Rightarrow \int_{s_1}^{s_2} ds = mC_v \int_{T_1}^{T_2} \frac{dT}{T} + mR \int_{v_1}^{v_2} \frac{dv}{v}$$

$$\Rightarrow s_2 - s_1 = mC_v \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{v_2}{v_1}\right)$$

$$\Rightarrow s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right) \rightarrow \text{valid for ideal gas.}$$

$$Tds = dh - vdp$$

$$ds = \frac{dh}{T} - \frac{vdp}{T}$$

$$\text{Ideal gas} \rightarrow pv = mRT \rightarrow \frac{v}{T} = \frac{mR}{p}$$

$$\rightarrow dh = mC_p dT$$

$$\Rightarrow \int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} \frac{mC_p dT}{T} - \int_{p_1}^{p_2} \frac{mR dp}{p}$$

$$\Rightarrow$$

$$s_2 - s_1 = mC_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{p_2}{p_1}\right)$$

$$\Rightarrow s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right) \rightarrow \text{valid for ideal gas only.}$$

$$\begin{aligned}
 \therefore S_2 - S_1 &= \left(\cancel{C_p} - R \right) \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right) \\
 &= C_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right) \\
 &= \cancel{R} \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right) \\
 \text{VWT} \Rightarrow \frac{T_2}{T_1} &= \frac{V_2}{V_1} \\
 &= C_p \ln \left(\frac{T_2}{T_1} \right) + C_p \ln \left(\frac{V_2}{V_1} \right) - C_p \ln \left(\frac{V_2}{V_1} \right) \\
 &= C_p \left[\ln \left(\frac{T_2}{T_1} \right) + \ln \left(\frac{V_2}{V_1} \right) \right] \\
 &= C_p \left[\ln \left(\frac{T_2}{T_1} \cdot \frac{V_2}{V_1} \right) \right]
 \end{aligned}$$

$$\therefore S_2 - S_1 = C_p \ln \left(\frac{V_2}{V_1} \right) + C_v \ln \left(\frac{P_2}{P_1} \right)$$

$$S_2 - S_1 = C_v \ln \left(\frac{P_2}{P_1} \right) + C_p \ln \left(\frac{V_2}{V_1} \right)$$

ENTROPY CHANGE FOR SOLIDS AND LIQUIDS:-

$$Tds = du + pdv$$

Solids
Liquids } Incompressible
 $dv = 0$
 $\Rightarrow (c = c)$

$$\Rightarrow Tds = du + 0$$

$$\Rightarrow Tds = mcdr$$

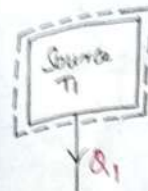
$$ds = \frac{mcdr}{T}$$

$$\Rightarrow \int_{S_i}^{S_f} ds = mc \int_{T_i}^{T_f} \frac{dT}{T}$$

$$S_f - S_i = mc \ln \left(\frac{T_f}{T_i} \right)$$

only for Solids & liquids.

ENTROPY CHANGE FOR A RESERVOIR:-



$$ds = \frac{dQ}{T}$$

$$\Rightarrow ds = -\frac{dQ}{T_1}$$

$$ds = -\frac{Q_1}{T_1}$$



$$ds = +\frac{dQ}{T_2}$$

$$ds = +\frac{Q_2}{T_2}$$

$$ds = \frac{dQ}{T} + dS_{gen}$$

↓ Entropy change
↓ Entropy transfer
↓ Entropy generation

The Entropy change is due to two reasons:-

- 1) Entropy transfer
- 2) Entropy generation

In case of a Rev. process, there is no entropy generation; only entropy transfer takes place.

In case of Adiabatic process; there is no entropy transfer; there is only entropy generation.

$$ds = \frac{dq}{T} + \delta S_{gen}$$

↓ External Interaction (E-I)
↓ Internal Irreversibilities (I-I)

Sys
E-I I-I
Surr = E-I
For Adiabatic; $dQ=0$
E-I=0

$$dS_{univ} \geq 0$$

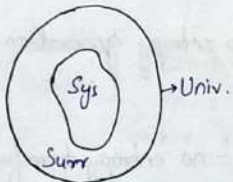
$$\Rightarrow dS_{sys} + dS_{surr} \geq 0 \Rightarrow \text{Adiabatic; } dS_{surr} = 0$$

$$dS_{sys} \geq 0$$

Note:- In an Adiabatic process; as there is no external interaction in the form of heat transfer.

$\therefore$ Surrounding Entropy change is zero

$$dS_{surr} = 0 \Rightarrow \text{Adiabatic}$$



$$ds = \frac{dq}{T} + \delta S_{gen}$$

Isolated System = Universe

E-T & M-T

$$dq = 0$$

$$dS_{univ} = \delta S_{gen}$$

$$\delta S_{gen} = dS_{univ} = dS_{sys} + dS_{surr}$$

ENTROPY CHANGE OF OPEN SYSTEM:-

Entropy is an extensive property which depends on mass. In an open system; as there is a mass transfer, there is entropy change due to mass transfer.

$$\left(\frac{ds}{dt}\right)_{cv} = \frac{dQ}{T} + \delta S_{gen} + \dot{m}_i s_i - \dot{m}_e s_e \quad (\text{open system})$$

$$\text{Steady flow} \Rightarrow \left(\frac{ds}{dt}\right)_{cv} = 0$$

$$\Rightarrow 0 = \frac{dQ}{T} + \delta S_{gen} + \dot{S}_i - \dot{S}_e$$

$$\Rightarrow \dot{S}_e - \dot{S}_i = \frac{dQ}{T} + \delta S_{gen} \quad (\text{closed system})$$

* Air is flowing (steady flow) in an insulated pipe with pressure and temp. at two sections ① and ② are given in the table. Establish the direction of flow of air in a pipe; treating air as an ideal gas.

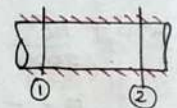
Sol:-

	Sec ①	Sec ②
P	130 kPa	100 kPa
T	50°C	13°C

$$T_1 = 323K$$

$$T_2 = 286K$$

Assume flow from ① to ②



$$dS_{univ} \geq 0$$

$$dS_{sys} + dS_{surr} \geq 0$$

$$dS_{sys} \geq 0 \Rightarrow dS_{air} \geq 0$$

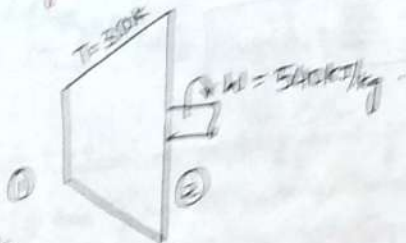
$$dS_{air} = S_2 - S_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$= 1.005 \ln\left(\frac{286}{323}\right) - 0.287 \ln\left(\frac{100}{130}\right) = -0.467 \frac{kJ}{kgK}$$

flow from ② to ①

↑

8) Steam enters a turbine with a pressure of 30 bar, $h_1 = 3230 \text{ kJ/kg}$ and velocity of 160 m/s . Saturated vapor at 100°C ($h_2 = 2676 \text{ kJ/kg}$) exit with a velocity of 100 m/s at a steady state. Turbine develops a $W = 5400 \text{ kJ/kg}$ Heat transfer between turbine and surroundings occurs at an average temp of 350 K . Determine the entropy generation in turbine per kg of steam. Neglect PE. Take $S_f = 6.922 \text{ kJ/kgK}$ and $S_g = 7.361 \text{ kJ/kgK}$.



Steady flow;

$$h_1 + \frac{C_1^2}{2000} + q = h_2 + \frac{C_2^2}{2000} + W_{turb}$$

$$\Rightarrow 3230 + \frac{160^2}{2000} + q = 2676 + \frac{100^2}{2000} + 5400 \text{ (J/kg)}$$

$$q = -217 \text{ kJ/kg}$$

$$ds = \frac{dq}{T} + s_{gen}$$

$$\Rightarrow S_2 - S_1 = -\frac{217}{T} + s_{gen} \quad \text{Turbine is losing heat.}$$

$$\Rightarrow 7.361 - 6.922 = -\frac{(217)}{350} + s_{gen}$$

$$\therefore s_{gen} = 0.491 \text{ kJ/kgK}$$

* Entropy numericals in MS notes *

AVAILABLE ENERGY, AVAILABILITY & IRREVERSIBILITY

Available Energy (A.E.) -

The maximum possible work that can be obtained in a cycle is known as Available Energy.



$$\eta = \frac{W_{net}}{Q_1}$$

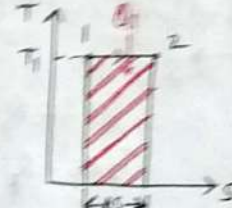
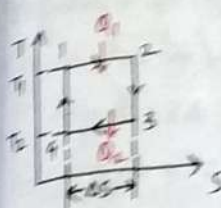
$$\Rightarrow 1 - \frac{T_2}{T_1} = \frac{W_{max}}{Q_1}$$

$$\eta_{max} = \frac{W_{max}}{Q_1}$$

$$\Rightarrow W_{max} = Q_1 \left(1 - \frac{T_2}{T_1} \right)$$

$$\eta_{max} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow A.E. = Q_1 \left(1 - \frac{T_2}{T_1} \right)$$



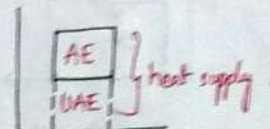
$$Q_1 = T_1 \Delta s$$

$$Q_2 = T_2 \Delta s$$

$$A.E. = Q_1 - T_2 \frac{Q_1}{T_1} = Q_1 - T_2 \Delta s$$

$$\therefore A.E. = Q_1 - T_2 \Delta s$$

Heat rejection (Q_2) = $T_2 \Delta s$. As T_2 is the lowest temp. of heat rejection; $T_2 \Delta s$ is the lowest heat rejection. This lowest heat rejection is called Unavailable Energy (UAE).



$$AE = Q_1 - T_0 \Delta S$$

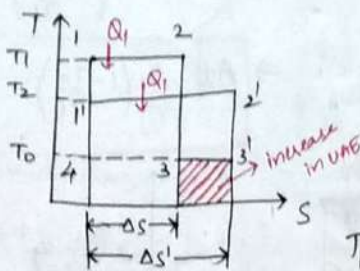
↓
UAE

$$AE = Q_1 - UAE$$

$$Q_1 = AE + UAE$$

Note: - The area below atm. temp. T_0 on T-S diagram represents unavailable energy.

LOSS OF AVAILABLE ENERGY (OR) INCREASE IN 'UAE' WHEN HEAT IS TRANSFERRED THROUGH A FINITE TEMP. DIFFERENCE: -



$$UAE_1 = T_0 \Delta S$$

$$UAE_2 = T_0 \Delta S'$$

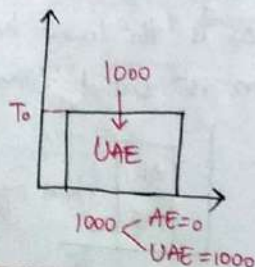
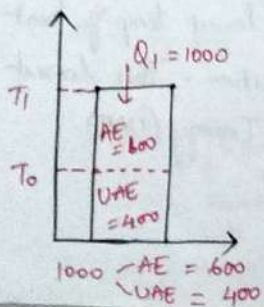
$$\text{Increase in UAE (or) Loss of } = T_0 (\Delta S' - \Delta S)$$

$$T_1 \Delta S = Q_1 \Rightarrow \Delta S = Q_1 / T_1$$

$$T_2 \Delta S' = Q_1 \Rightarrow \Delta S' = Q_1 / T_2$$

$$\text{Loss of AE} = T_0 \left[\frac{Q_1}{T_2} - \frac{Q_1}{T_1} \right] = Q_1 T_0 \left[\frac{T_1 - T_2}{T_1 T_2} \right]$$

$$\therefore \text{Loss of AE} = Q_1 T_0 \left[\frac{T_1 - T_2}{T_1 T_2} \right]$$



* Greater the temp. difference; larger is the loss of A.E. greater is the irreversibility. Hence, to achieve maximum efficiency the temp. difference must be less. If the temp. diff. is less; then the process tends towards Reversible.

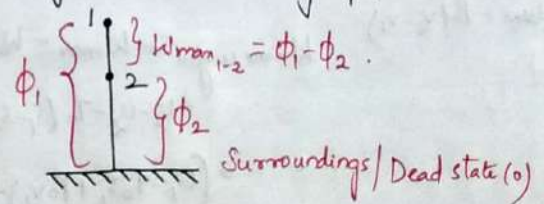
According to 1st law of thermodynamics; thermal energy at lower temperature and equal amount of thermal energy at higher temperature have same meaning.

$\therefore$ First law of thermodynamics is Quantitative law.

According to 2nd law of thermodynamics, thermal energy at higher temp. has greater significance compared to same amount of thermal energy at lower temp. Because, the thermal energy at higher temp. is capable of producing more work. Hence, Second law is known as Qualitative law.

AVAILABILITY (EXERGY) :-

The max. work (useful) that can be obtained in a process when a system comes into equilibrium with surroundings. It is denoted by ' ϕ '.



The reference state in Availability is atm. conditions and these conditions are also known as dead state.

Max. work for closed system:-

Max. work is possible; when the process is reversible.

$$ds = \frac{\delta Q}{T}$$

$$dS_{surr} = \frac{\delta Q_{surr}}{T}$$

$$dS_{surr} = \frac{\delta Q_{surr}}{T_0}$$

$$\delta Q_{surr} = dS_{surr} \cdot T_0 \quad \text{--- (1)}$$

$$\delta Q_{sys} = -\delta Q_{surr} \quad \text{--- (2)}$$

$$\delta Q_{sys} = dU_{sys} + \delta W_{sys}$$

$$\delta W_{sys} = \delta Q_{sys} - dU_{sys} \quad \text{--- (3)}$$

$$W_{max} = -\delta Q_{surr} - dU_{sys} \quad \text{--- (4)}$$

① in ④

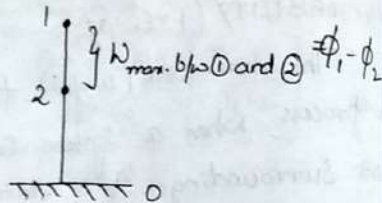
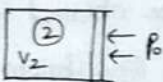
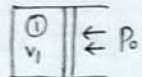
$$\delta W_{sys} = -T_0 dS_{surr} - dU_{sys}$$

$$W_{max} = T_0(S_2 - S_1) - (U_2 - U_1)$$

$$W_{max} = U_1 - U_2 - T_0(S_1 - S_2)$$

Closed system for 1-2 process.

Max. useful work for closed system:-



$$W_{atm} = P_0(v_2 - v_1)$$

$$W_{max, useful} = W_{max} - W_{atm}$$

$$= U_1 - U_2 - T_0(S_1 - S_2) - P_0(v_2 - v_1)$$

$$= (U_1 - T_0 S_1 + P_0 v_1) - (U_2 - T_0 S_2 + P_0 v_2)$$

$$W_{max, useful} = \phi_1 - \phi_2$$

$$\phi_1 = U_1 - T_0 S_1 + P_0 v_1$$

→ availability function for a closed system

Maximum work in open system:-

for closed; $dQ = dU + dW$ --- 1

$$W_{max} = U_1 - U_2 - T_0(S_1 - S_2)$$

for open; SFEE

$$h_1 + \frac{C_1^2}{2} + g z_1 + q = h_2 + \frac{C_2^2}{2} + g z_2 + w$$

Neglect KE and PE

$$\Rightarrow h_1 + q = h_2 + w \Rightarrow q = h_2 - h_1 + w$$

$$\Rightarrow q = dh + w - z$$

from Comparing 1 and 2;

$$W_{max} = h_1 - h_2 - T_0(S_1 - S_2)$$

In open sys; $W_{max} = W_{max, useful}$

$$\Rightarrow W_{max, useful} = (h_1 - T_0 S_1) - (h_2 - T_0 S_2)$$

$$= \phi_1 - \phi_2$$

$$\phi_1 = h_1 - T_0 S_1; \phi_2 = h_2 - T_0 S_2$$

$$\therefore \phi = h - T_0 S \Rightarrow \text{Availability function for open system.}$$

IRREVERSIBILITY:-(I)

$$I = W_{\text{max}} - W_{\text{act}}$$

$$I = T_0 [dS_{\text{sys}} + dS_{\text{surr}}] = T_0 (dS_{\text{univ}})$$

$$I = T_0 (dS_{\text{univ}})$$

Valid for both open and closed

$$dS_{\text{univ}} = dS_{\text{gen}}$$

$$I = T_0 (dS_{\text{gen}})$$

$$I \propto dS_{\text{gen}} \rightarrow \text{Gouy Stodola theorem}$$

HELMHOLTZ FUNCTION (F):-

$$F = U - TS$$

The difference in helmholtz function gives max. work enclosed system.

GIBBS FUNCTION (G):-

$$G = H - TS$$

The difference in Gibbs function gives max. work in open system.

Numericals:-

1. (a)

2. (c)

3. (d)



$$T = 1200\text{K}; T_0 = 300\text{K}$$

$$Q = 3200\text{kW}$$

$$\eta_{\text{max}} = 1 - \frac{T_0}{T} = \frac{W_{\text{max}}}{Q_1}$$

$$\Rightarrow 1 - \frac{300}{1200} = \frac{W_{\text{max}}}{3200}$$

$$\Rightarrow \frac{900}{1200} = \frac{W_{\text{max}}}{3200} \Rightarrow W_{\text{max}} = 2400\text{kW}$$

$$h(C) = (h_1 - T_0 s_1) - (h_2 - T_0 s_2)$$

$$= (400 - 300(1.1)) - (100 - 300(0.7))$$

$$= (400 - 330) - (100 - 210)$$

$$h_1 - T_0 s_1 = 70 - (-110) = 180\text{kJ/kg}$$

5. (C)



$$T_2 = T_0; \text{loss} = \frac{Q_2(T_2 - T_0)}{T_2}$$

$$= \frac{9000 \times (300 - 900)}{900}$$

$$\text{loss} = 6000\text{kJ}$$

6. (a)

7. (C)



$$Q_1 + Q_2 = 1200$$

$$Q_2 = 600\text{J}$$

$$I = T_0 (dS_{\text{univ}})$$

$$dS_{\text{sys}} = -\frac{Q_1}{T_1} = -\frac{1200}{900}; dS_{\text{surr}} = \frac{1600}{300}$$

$$= -4/3; = 2$$

$$dS_{\text{univ}} = -\frac{4}{3} + 2 = \frac{2}{3}$$

$$I = 300 \left(\frac{2}{3} \right) = 200\text{J}$$

$$I = W_{\text{max}} - W_{\text{act}} = 800 - 600 = 200\text{J}$$

8. (a)

$$\text{sphere}; d = 0.1\text{m}; T_1 = 20^\circ\text{C}; T_0 = 20^\circ\text{C}$$

$$\rho = 2700; C = 0.9$$



$$Q = mc\Delta T$$

$$\rho = \frac{m}{V} = m \cdot \frac{1}{V} = 2700 \times \frac{4}{3} \pi \times (0.05)^3 = 1.413\text{kg}$$

$$Q = 1.413 \times 0.9 \times (298 - 473) = -222.5\text{kJ}$$

$$dS_{\text{sys}} = \text{solid}$$

$$= mc \ln \left(\frac{T_2}{T_1} \right)$$

$$= 1.413 \times 0.9 \times \ln \left(\frac{298}{473} \right)$$

$$= -0.58\text{kJ/K}$$

$$dS_{\text{surr}} = \frac{+Q}{T_0}$$

$$= \frac{222.5}{298}$$

$$= 0.74\text{kJ/K}$$

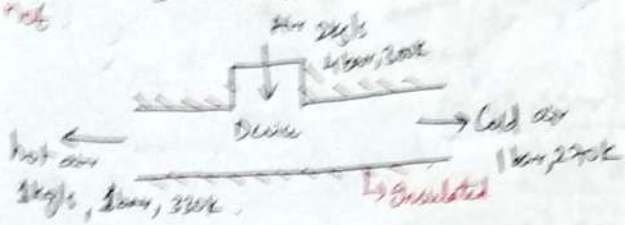
$$dS_{\text{univ}} = 0.166$$

$$I = T_0 (dS_{\text{univ}})$$

$$= 298 \times 0.166$$

$$= 49.66 \approx 47.5\text{kJ}$$

A hypothetical device is supplied with 2 kg/s of air at 4 bar, 300K. Two separate streams of air leave the device as shown in Fig. Each stream is at constant pressure of 1 bar. Mass flow rate is same for both streams. One of the exit streams is said to be at 300K while the other is at 270K. The ambient temp is 300K. Determine whether such a device is possible or not.



$$m = m_1 + m_2$$

$$2 = m_1 + m_2 \Rightarrow m_1 = 1 \text{ kg/s}$$

$$m_2 = 1 \text{ kg/s}$$

$$h_1 = 1.005 \times 300 = 301.5$$

$$h_2 = 1.005 \times 270 =$$

$$\Delta S_{\text{sys}}(1) = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$= 1.005 \ln\left(\frac{300}{300}\right) - 0.287 \ln\left(\frac{1}{4}\right)$$

$$= 0.095 + 0.29 = 0.492 \text{ kJ/kgK} \times 1$$

$$\Delta S_{\text{sys}}(2) = 1.005 \ln\left(\frac{270}{300}\right) - 0.287 \ln\left(\frac{1}{4}\right)$$

$$= 0.291 \times 1$$

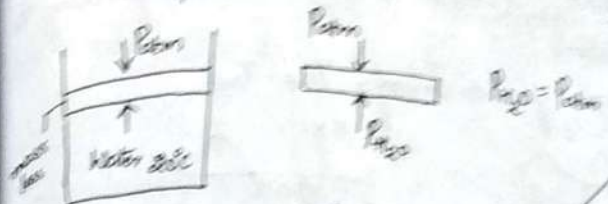
$$\Delta S_{\text{sys}} = (1) + (2) = 0.7846$$

$$\Delta S_{\text{univ}} = \Delta S_{\text{sys}} + 0$$

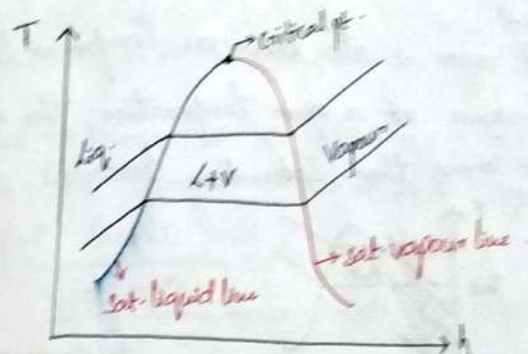
$$> 0.7846 \therefore \Delta S_{\text{univ}} > 0 \Rightarrow \text{Possible device}$$

PROPERTIES OF PURE SUBSTANCES

Water at 20°C → Steam at 20°C
at 1 bar



- (1) Water at 20°C - Water at 100°C
- (2) Water at 100°C - Steam at 100°C
- (3) Steam at 100°C - Steam at 20°C



Critical Point:-

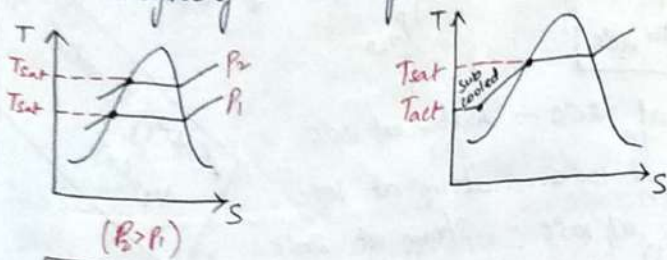
It is the point at which saturated liquid and saturated vapour curves meet. At this point, it is difficult to distinguish between liq. and vapour.

Various Regions:-

① Sub-cooled (or) Undercooled Region:-

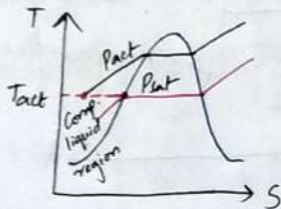
It is the region in which the actual temp. is less than saturation temperature corresponding to that pressure.

$$\text{Degree of Sub-cooling} = T_{\text{sat}} - T_{\text{act}}$$



$$T_{\text{sat}} \propto p$$

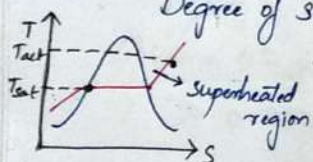
Sub-cooled region is also known as Compressed liquid region because at a given temperature; the actual pressure is more than saturation pressure.



② Super-heated region:-

It is the region in which the actual temp. is greater than saturation temp. corresponding to that pressure.

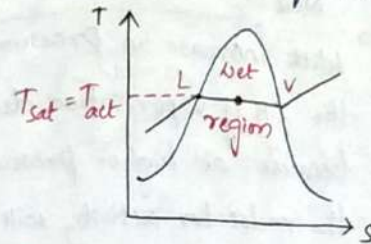
$$\text{Degree of superheating} = T_{\text{act}} - T_{\text{sat}}$$



Super heating and Sub-cooling are Constant pressure processes.

③ Wet Region:-

It is a region in which both liquid and vapour coexist in equilibrium.



$$\text{Sub-cooled} \Rightarrow T < T_{\text{sat}}$$

$$\text{Superheated} \Rightarrow T > T_{\text{sat}}$$

$$\text{Wet region} \Rightarrow T = T_{\text{sat}}$$

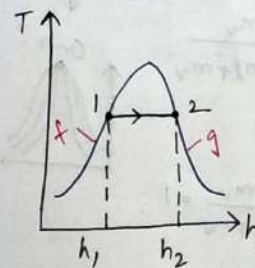
SENSIBLE HEAT:-

The heat transfer associated with temp. difference is called Sensible heat.

$$SH = mc\Delta T$$

LATENT HEAT:-

The heat transfer associated with phase change is known as Latent heat.



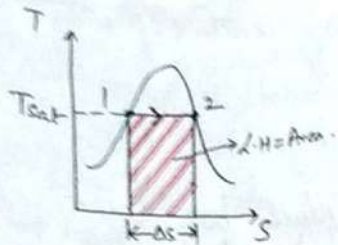
$$\Delta H = Q_{\text{phase change}} = Q_p = dH$$

$$= h_2 - h_1$$

$$= h_g - h_f$$

$$\Delta H = h_{fg}$$

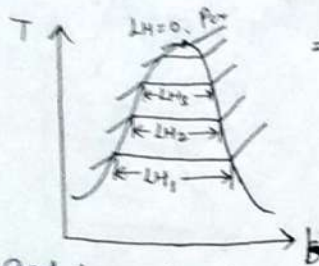
* Phase change is a practical example of rev. process



$$ds = \frac{dQ}{T}$$

$$dQ = T ds$$

$$\boxed{dQ = T_{sat} (ds)}$$



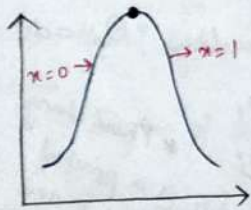
Note:-
 ⇒ With increase in pressure, the L.H of vaporisation decreases because at higher pressures the molecular activity will be high and hence less heat is required for causing phase change.

* At critical point; L.H = 0 i.e., the liquid flashes off suddenly to vapour.

DRYNESS FRACTION/QUALITY OF MIXTURE:-

It is ratio of mass of vapour to the total mass of mixture. The dryness fraction along saturated liq. curve is zero and dryness fraction along saturated vapour curve is one.

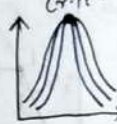
All constant dryness fraction lines Converge at the Critical point.



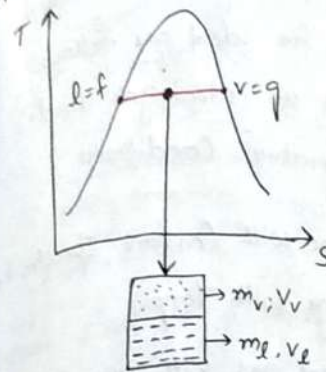
$$x = \frac{m_v}{m_l + m_v}$$

$$x_{liq} = 0$$

$$x_{vap} = \frac{m_v}{0 + m_v} = 1$$



SPECIFIC VOLUME OF MIXTURE:- (v)



$$V = V_l + V_v$$

$$m v = m_l v_l + m_v v_v$$

$$\Rightarrow v = \frac{m_l v_l}{m} + \frac{m_v v_v}{m}$$

$$= \frac{m_l v_l}{m_l + m_v} + \frac{m_v v_v}{m_l + m_v}$$

$$= (1-x) v_l + x v_v$$

$$v = v_l + x(v_v - v_l)$$

$$\Rightarrow v = v_f + x(v_g - v_f) = v_f + x v_{fg}$$

$$\therefore \boxed{v = v_f + x v_{fg}}$$

Extensive

Intensive

1) $V = V_f + V_g$

⇒

$v = v_f + x v_{fg}$

2) $U = U_f + U_g$

⇒

$u = u_f + x u_{fg}$

3) $H = H_f + H_g$

⇒

$h = h_f + x h_{fg}$

4) $S = S_f + S_g$

⇒

$s = s_f + x s_{fg}$

valid only in wet region.

IDEAL GAS:-

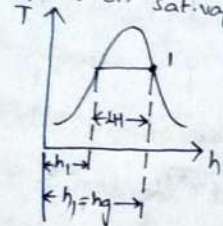
A gas is said to be an Ideal gas when there are no intermolecular forces. An ideal gas follows $PV = mRT$. A gas behaves as an ideal gas under low pressure and high temperature conditions.

Perfect Gas:- It is an ideal gas with constant specific heats (C_p & C_v do not change with temp.)

Semi-perfect gas:- It is an ideal gas with variable specific heats. (C_p & C_v change with temp.)

ENTHALPY AT VARIOUS POINTS:-

① Point on sat. vapour curve:-

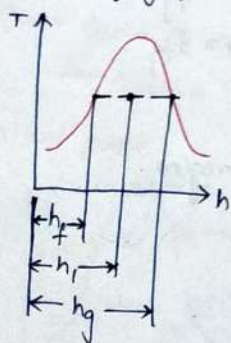


$$h_1 = h_f + L.H$$

$$\Rightarrow h_g = h_f + L.H$$

$$L.H = h_g - h_f$$

② Enthalpy of point at wet region:-

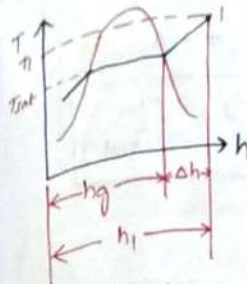


$$h_1 = h_f + x(h_g - h_f)$$

$$h_g - h_f = L.H$$

$$h_1 = h_f + x(L.H)$$

② Point is at superheated region:-



$$h_1 = h_g + \Delta h$$

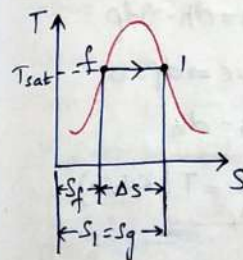
$$= h_g + C_{p,v} \Delta T$$

$$h_1 = h_g + C_{p,vapor} (T_1 - T_{sat})$$

Vapour state is a state which is very close to its liquid state. $\therefore$ Superheated vapour must be treated as an Ideal gas only under low pressure conditions. In case of Vapour Compression Refrigeration Cycle, pressures are low and hence Superheated vapour can be treated as an Ideal Gas. In case of Rankine cycle, pressures are very high and hence Superheated vapour cannot be treated as an Ideal gas.

ENTROPY AT VARIOUS POINTS:-

① Point is on Sat. vapour Curve.



$$S_1 = S_f + \Delta S$$

$$ds = \frac{dq}{T}$$

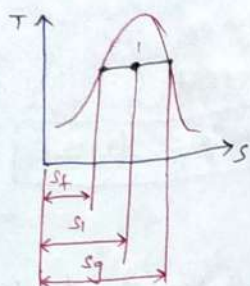
$$\Delta S_{\text{phase change}} = \frac{dq_{\text{phase change}}}{T_{\text{sat}}}$$

$$\Delta S_{\text{phase change}} = \frac{L.H}{T_{\text{sat}}}$$

$$S_1 = S_f + \frac{L.H}{T_{\text{sat}}}$$

$$\Rightarrow S_g = S_f + \frac{L.H}{T_{\text{sat}}} \Rightarrow S_g - S_f = \frac{L.H}{T_{\text{sat}}}$$

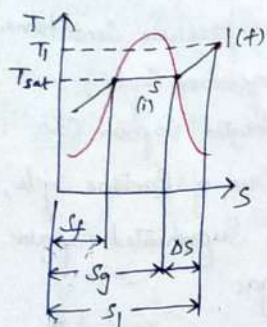
② point at wet region:-



$$s_1 = s_g + x(s_f - s_g)$$

$$s_1 = s_g + x \left(\frac{2H}{T_{sat}} \right)$$

③ point at superheated region:-



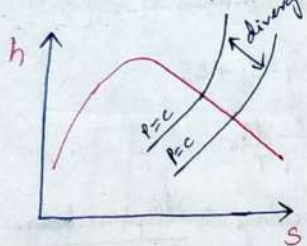
$$s_1 = s_g + \Delta s$$

$$s_1 - s_g = C_p \ln \left(\frac{T_1}{T_{sat}} \right) - R \ln \left(\frac{P_1}{P_i} \right)$$

$$\Delta s = C_{p_{vapour}} \cdot \ln \left(\frac{T_1}{T_{sat}} \right)$$

$$s_1 = s_g + C_{p_{vap}} \cdot \ln \left(\frac{T_1}{T_{sat}} \right)$$

MOLLIER DIAGRAM:-



$$Tds = dh - vdp$$

$$p=c \Rightarrow dp=0$$

$$Tds = dh$$

$$\Rightarrow \frac{dh}{ds} = T \text{ (Kelvin)}$$

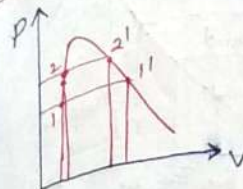
slope of (p=c) lines on h-s diagram

Note:- Constant pressure slope can never be negative

The Constant pressure lines diverge in the superheated vapour with a positive slope.

They diverge because as the temp. increases, slope increases.

P-v diagram



Liquids are incompressible; so the change in volume is negligible.



Solids → liquid = melting
liquid → solid = freezing
liquid → vapour = vaporisation
vapour → liquid = Condensation
solid → vapour = Sublimation

In case of General Substances

S → L = Expands
L → S = Contracts

Water

S(ice) → L(water)

melting → contracts

Water (L) → S(ice) → freezing
expands

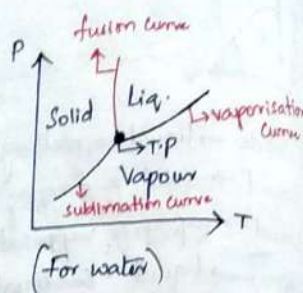
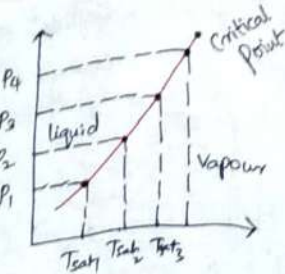
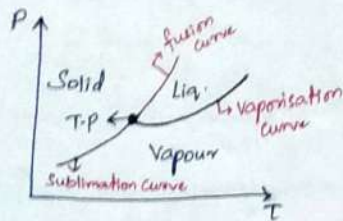
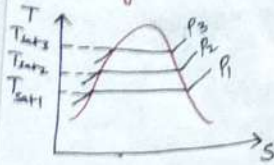
Water

When ice melts, it contracts

When water freezes, it expands.

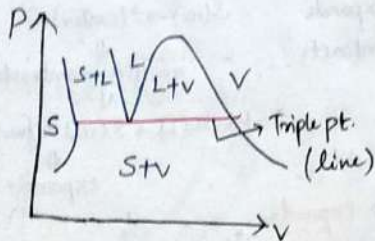
This is due to hydrogen bonding ⇒ the oxygen atom is surrounded by other oxygen atoms tetragonally, with voids (gaps); so, when freezing takes place, voids are filled; in melting it expands.

P-T diagram:-



(For water)

* Substances which expand on freezing [Ex:- Water]; the slope of fusion curve is negative.



$$P + F = C + 2$$

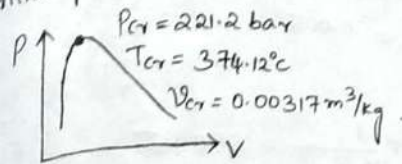
$$3 + F = 1 + 2$$

$$F = 0$$

According to Gibb's Phase Rule; the DOF at triple pt. is 0. This means that at triple point; no intensive properties can be varied. It is a unique state. No intensive property can be varied.

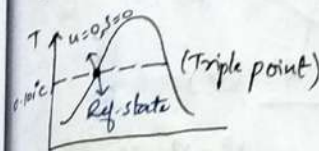
Triple point is a point on P-T diagram and it is a line on P-v diagram.

Critical point data for water:-

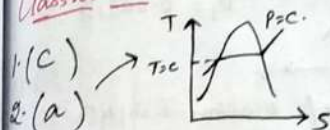


Reference state in Standard Steam tables:-

$$dQ = du + dw$$



Classroom Set:-

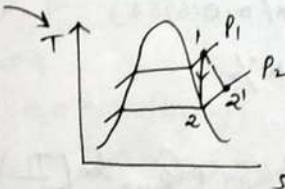


1. (C)

2. (a)

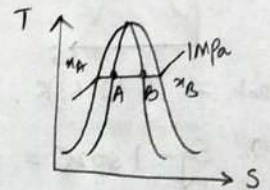
3. (b)

4. (d)



P and T alone are not sufficient. In weight; P and T are not independent. They coincide.

For Adiabatic; $S \uparrow$
 $\Rightarrow$ Superheated steam.



$$5. (C) \quad U_A = U_B; \quad P = 1 \text{ MPa};$$

$$m_A U_A = m_B U_B \quad \text{max} = 1$$

$$\Rightarrow \frac{m_A}{m_B} = \frac{U_B}{U_A} = \frac{U_f + x_B U_{fg}}{U_f + x_A U_{fg}}$$

$$\left(\frac{m_A}{m_B} \right)_{\text{max}} = \frac{U_f + U_{fg}}{U_f} = \frac{U_g}{U_f} = \frac{2583}{161} = 16.04$$

$$U_B = U_f + x(U_g - U_f)$$

$$U_A = U_f + x_A(U_g - U_f)$$

3. THERMODYNAMIC RELATIONS:-

Theorem 1:-

If $dz = Mdx + Ndy$ is an exact differential equation

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

Theorem 2:-

$$f = \phi(x, y, z)$$

$$\left(\frac{\partial x}{\partial y}\right)_f \cdot \left(\frac{\partial y}{\partial z}\right)_f \cdot \left(\frac{\partial z}{\partial x}\right)_f = 1$$

Theorem 3:-

$$z = \phi(x, y)$$

$$\left(\frac{\partial x}{\partial y}\right)_z \cdot \left(\frac{\partial y}{\partial z}\right)_x \cdot \left(\frac{\partial z}{\partial x}\right)_y = 1$$

Maxwell Equations:-

There are four Maxwell's Equations and these are applicable for any substance.

$$Tds = du + pdv$$

$$\Rightarrow du = Tds - pdv$$

$$\Rightarrow dz = Mdx + Ndy$$

$$\Rightarrow \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\Rightarrow \left(\frac{\partial T}{\partial v}\right)_s = \left(\frac{\partial p}{\partial s}\right)_v \quad \text{--- (1)}$$

$$Tds = dh - vdp$$

$$dh = Tds + vdp$$

$$\Rightarrow dz = Mdx + Ndy$$

$$\Rightarrow \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \Rightarrow \left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p \quad \text{--- (2)}$$

$$G = H - TS$$

$$dG = dH - [Tds + SdT]$$

$$dG = dH - Tds - SdT$$

$$Tds = dh - vdp$$

$$dh - Tds = vdp$$

$$\Rightarrow dG = vdp - SdT$$

$$dG = vdp - SdT$$

$$dz = Mdx + Ndy$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \Rightarrow \left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T \quad \text{--- (3)}$$

$$F = U - TS$$

$$dF = dU - Tds - SdT$$

$$dU + pdv = Tds$$

$$dU - Tds = -pdv$$

$$\Rightarrow dF = -pdv - SdT$$

$$\Rightarrow \left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial S}{\partial v}\right)_T \quad \text{--- (4)}$$

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v$$

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial p}{\partial T}\right)_v$$

$$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

$$\left(\frac{\partial T}{\partial v}\right)_s = \left(\frac{\partial p}{\partial s}\right)_v$$

$$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \left(\frac{\partial s}{\partial p}\right)_T$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

Tds Equations:-

1st Tds Equation:- $S = f(T, v)$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

$$Tds = T \left(\frac{\partial s}{\partial T}\right)_v dT + T \left(\frac{\partial s}{\partial v}\right)_T dv$$

$$\frac{dT}{ds} = \frac{T}{C_v} \Rightarrow \left(\frac{dT}{ds}\right)_v = \frac{T}{C_v} \Rightarrow C_v = T \left(\frac{\partial s}{\partial T}\right)_v$$

$$Tds = C_v dT + T \left(\frac{\partial p}{\partial T}\right)_v dv$$

2nd Tds Equation:-

$S = f(T, p)$

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$

$$Tds = T \left(\frac{\partial s}{\partial T}\right)_p dT + T \left(\frac{\partial s}{\partial p}\right)_T dp$$

$$Tds = C_p dT - T \left(\frac{\partial v}{\partial T}\right)_p dp$$

These Tds Equations are applicable for ANY gas. (Ideal or Real). Equating 1st and 2nd Tds Eqn.

$$C_v dT + T \left(\frac{\partial p}{\partial T}\right)_v dv = C_p dT - T \left(\frac{\partial v}{\partial T}\right)_p dp$$

$$\Rightarrow T \left(\frac{\partial v}{\partial T}\right)_p dp + T \left(\frac{\partial p}{\partial T}\right)_v dv = (C_p - C_v) dT$$

$$\Rightarrow dT = \frac{T}{C_p - C_v} \left(\frac{\partial v}{\partial T}\right)_p dp + \frac{T}{(C_p - C_v)} \left(\frac{\partial p}{\partial T}\right)_v dv \quad \text{--- (1)}$$

$$T = f(p, v)$$

$$\Rightarrow dT = \left(\frac{\partial T}{\partial p}\right)_v dp + \left(\frac{\partial T}{\partial v}\right)_p dv \quad \text{--- (2)}$$

Comparing (1) and (2)

$$\Rightarrow \frac{T}{C_p - C_v} \left(\frac{\partial v}{\partial T}\right)_p = \left(\frac{\partial T}{\partial p}\right)_v$$

$$\Rightarrow C_p - C_v = T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial T}\right)_v \quad \text{(valid for any gas)} \quad \text{--- (3)}$$

$$T = f(p, v)$$

From Th:3

$$\Rightarrow \left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1 \Rightarrow -\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p = \left(\frac{\partial p}{\partial T}\right)_v \quad \text{--- (4)}$$

(4) in (3)

$$C_p - C_v = -T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial v} \right)_T \left(\frac{\partial v}{\partial T} \right)_p$$

$$C_p - C_v = -T \left[\left(\frac{\partial v}{\partial T} \right)_p \right]^2 \left(\frac{\partial p}{\partial v} \right)_T \Rightarrow \text{valid for ANY gas.}$$

$$= (-)(+)(+)(-)$$

$$C_p - C_v = +ve \Rightarrow C_p > C_v$$

Note: (1) C_p and C_v are equal at absolute zero (0K)
(2) $\gamma = 1$ at absolute zero (0K)

Coefficient of Volume expansivity: (β)

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p$$

It shows variation of volume with temp. under constant pressure conditions.

Isothermal Compressibility: (K_T)

$$K_T = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T$$

$$C_p - C_v = \frac{TV\beta^2}{K_T} \Rightarrow \text{valid for any gases.}$$

ENERGY EQUATION:-

$$TdS = du + pdv$$

$$\Rightarrow du = Tds - pdv$$

$$\downarrow \text{First Tds Eqn} \Rightarrow Tds = C_v dT + T \left(\frac{\partial p}{\partial T} \right)_v dv$$

$$du = C_v dT + T \left(\frac{\partial p}{\partial T} \right)_v dv - pdv$$

$$du = C_v dT + \left[T \left(\frac{\partial p}{\partial T} \right)_v - p \right] dv \rightarrow \text{Energy Equation.}$$

valid for any gas

Any gas undergoing (v=c) process; $dv=0$

$$\therefore du = C_v dT \Rightarrow \text{valid for ANY gas undergoing Const. vol. process.}$$

Ideal gas:-

$$Pv = mRT \Rightarrow \frac{mR}{v} = \frac{P}{T}$$

$$P = \frac{mR}{v} \cdot T$$

$$\left(\frac{\partial p}{\partial T} \right)_v = \frac{mR}{v} \Rightarrow \left(\frac{\partial p}{\partial T} \right)_v = \frac{P}{T} \Rightarrow T \left(\frac{\partial p}{\partial T} \right)_v - P = 0$$

$$\therefore du = C_v dT \Rightarrow \text{valid for any process.}$$

Note:- The Equation $du = C_v dT$ is valid for an Ideal gas undergoing any process.

The Equation $du = C_v dT$ is valid for a Real gas undergoing constant volume process only.

$$dU = C_v dT + \left[T \left(\frac{\partial p}{\partial T} \right)_v - p \right] dv \quad \text{--- (a)}$$

For an ideal gas; $T \left(\frac{\partial p}{\partial T} \right)_v - p = 0$.

$$\Rightarrow U = f(T, v)$$

$$dU = \left(\frac{\partial U}{\partial T} \right)_v dT + \left(\frac{\partial U}{\partial v} \right)_T dv \quad \text{--- (b)}$$

Comparing (a) and (b)

$$C_v = \left(\frac{\partial U}{\partial T} \right)_v$$

$$\left(\frac{\partial U}{\partial T} \right)_T = T \left(\frac{\partial p}{\partial T} \right)_v - p$$

For an ideal gas; $T \left(\frac{\partial p}{\partial T} \right)_v - p = 0$.

$$\left(\frac{\partial U}{\partial v} \right)_T = 0 \text{ for ideal gas.}$$

This equation shows for an ideal gas; internal energy is independent of volume.

$$U = f(T, p, v)$$

$$T = \phi(U, p, v)$$

From Th: 2

$$\left(\frac{\partial U}{\partial p} \right)_T \cdot \left(\frac{\partial p}{\partial v} \right)_T \cdot \left(\frac{\partial v}{\partial U} \right)_T = 1$$

$$\left(\frac{\partial U}{\partial p} \right)_T \cdot \left(\frac{\partial p}{\partial v} \right)_T = \left(\frac{\partial U}{\partial v} \right)_T \Rightarrow 0$$

$$\Rightarrow \left(\frac{\partial U}{\partial p} \right)_T \cdot \left(\frac{\partial p}{\partial v} \right)_T = 0$$

$$\downarrow 0 \quad \downarrow -ve$$

$$\Rightarrow \left(\frac{\partial U}{\partial p} \right)_T = 0$$

$\Rightarrow$ for ideal gas; internal energy is independent of pressure.

For an ideal gas; internal energy is independent of pressure and volume.

$\therefore$ Internal energy of an ideal gas is a function of Temperature alone. This is known as Joule's Law.

Joule Thomson Coefficient:--
(u)

$$T ds = dh - v dp$$

$$dh = T ds + v dp$$

$$dh = C_p dT - T \left(\frac{\partial v}{\partial T} \right)_p dp + v dp$$

$$dh = C_p dT - \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right] dp \rightarrow \text{valid for any gas.}$$

Ideal gas:--

$$pV = mRT \Rightarrow \frac{mR}{p} = \frac{V}{T}$$

$$V = \frac{mR}{p} \cdot T$$

$$\left(\frac{\partial v}{\partial T} \right)_p = \frac{mR}{p}$$

$$\left(\frac{\partial v}{\partial T} \right)_p = \frac{V}{T} \Rightarrow T \left(\frac{\partial v}{\partial T} \right)_p - v = 0$$

$$dh = C_p dT - [0] dv$$

$$dh = C_p dT$$

1) The above eqn. is valid for ideal gas undergoing any process.

2) The eqn. is valid for a real gas undergoing const. pressure process only.

$$dh = C_p dT - \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right] dp$$

ANY gas undergoing throttling; $dh = 0$.

$$0 = C_p dT - \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right] dp$$

$$\Rightarrow C_p dT = \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right] dp$$

$$\frac{dT}{dp} = \frac{1}{C_p} \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right]$$

$$\Rightarrow \left(\frac{\partial T}{\partial p} \right)_h = \frac{1}{C_p} \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right]$$

$$\mu = \left(\frac{\partial T}{\partial p} \right)_h$$

$$\Rightarrow \mu = \frac{1}{C_p} \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right] \quad \mu = \frac{1}{C_p} \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right]$$

for any gas.

For Ideal gas: $T \left(\frac{\partial v}{\partial T} \right)_p - v = 0$

$$\therefore \mu = \frac{1}{C_p} (0) = \boxed{\mu = 0} \Rightarrow \text{For ideal gas.}$$

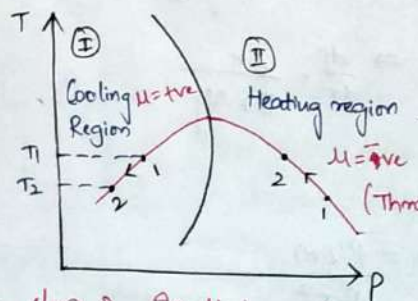
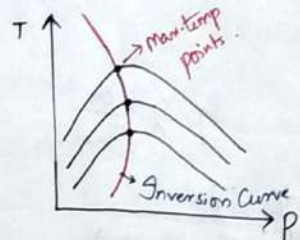
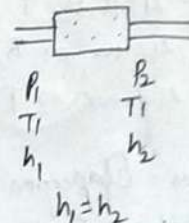
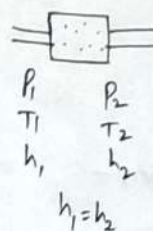
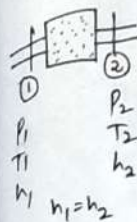
For an Ideal gas, Joule-Thompson Coefficient is zero.

$$\mu_{\text{ideal gas}} = 0.$$

We know that; for an Ideal gas; internal enthalpy is a function of temp. In throttling, as the enthalpy

is Constant; $\therefore$ Temperature = Constant ($dT = 0$). } Ideal gas.

Porous plug



$$\mu = \left(\frac{\partial T}{\partial p} \right)_h \Rightarrow \text{slope.}$$

(Throttling always in decreasing pressure).

- (1) The slope of isenthalpic curves on T-P diagram is equal to Joule-Thompson Co-efficient.
- (2) In Cooling region; Joule-Thompson Co-efficient is +ve. i.e; during throttling; temperature decreases. (used in VCR system).
- (3) In heating region; Joule-Thompson Co-efficient is -ve; during throttling; temperature increases.

(d) During throttling, temp. may increase [cooling region]; or it may decrease [cooling region]; or it may remain constant [ideal gas].

$$\begin{aligned} \Delta H &= 0 \Rightarrow T = \text{const} \\ \Delta H &= +ve \Rightarrow T \downarrow \\ \Delta H &= -ve \Rightarrow T \uparrow \end{aligned}$$

Clausius - Clapeyron Equation:-

From Maxwell Relations;

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

During phase change;

$$S_g - S_f = \frac{LH}{T_{sat}}$$

$$\frac{dP}{dT} = \frac{S_g - S_f}{V_g - V_f} ; \frac{dP}{dT} = \frac{LH}{T_{sat}(V_g - V_f)}$$

$$\frac{dP}{dT} = \frac{LH}{T_{sat}(V_g - V_f)} \Rightarrow \frac{dP}{dT} = \frac{LH}{T_{sat} \cdot V_g}$$

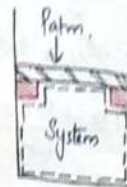
$$V_f \ll V_g$$

$$\frac{dP}{dT} = \frac{P(LH)}{T_{sat} \cdot R \cdot T_{sat}} = \frac{P(LH)}{R \cdot T_{sat}^2}$$

$$\therefore \frac{dP}{dT} = \frac{P(LH)}{R \cdot T_{sat}^2} \Rightarrow \text{Clausius Clapeyron.}$$

Properties of Pure Substances:-

Q.32. (a)



$$m_l = 1 \text{ kg}$$

$$m_v = 0.03 \text{ kg}$$

$$P_1 = 100 \text{ kPa}$$

$$V_2 = 1.5 V_1 ; Q = 1000 \text{ J}$$

$$dS_{sys} = 10 \text{ J/K}$$

$$\Rightarrow x = \frac{m_v}{m_v + m_l} = \frac{0.03}{1.03} = 0.029$$

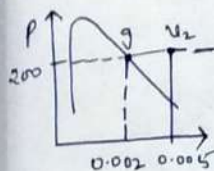
$$\Rightarrow v_1 = v_f + x(v_g - v_f) = 0.003871 \text{ m}^3/\text{kg}$$

$$\Rightarrow V_1 = v_1 \times m = 0.003871 \times 1.03 = 0.003987 \text{ m}^3$$

$$V_2 = 1.5 V_1 = 1.5 \times 0.003987 = 0.00598 \text{ m}^3$$

$[0.003871 = 0.0015 + x(0.002 - 0.0015) \Rightarrow x = \frac{0.0043}{5 \times 10^{-4}} = 8.6]$

$$v_2 = \frac{V_2}{m} = \frac{0.00598}{1.03} = 0.0058 \text{ m}^3/\text{kg}$$



$\therefore$ final state = Superheated state.

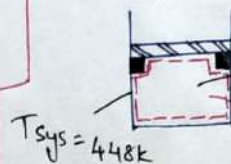
When piston starts moving; it is a constant pressure process. (At 200 kPa)

$$W = P(V_2 - V_1) = 0.39 \approx 0.4 \text{ kJ}$$

32. (d)

33. ()

Entropy generation (Considering only system).



(Not asked in ques.)

$$dS_{sys} = \left(\frac{\delta Q}{T}\right)_{sys} + (\delta S_{gen})_{sys}$$

$$\Rightarrow 10 = \frac{1000}{448} + \delta S_{gen}$$

$$\delta S_{gen} = 7.76 \text{ J/K}$$