<FUNDAMENTALS OF ENGG. MECHANICS>

<CHAPTER-01>

- Mechanics may be defined as the branch of science which deals with the study of effect of forces on material bodies in state of rest or state of motion.
- Applications of laws of mechanics to field of problems / real life problem is termed as Engg. Mechanics.

- Engg. Mechanics
  - Mechanics of solid
    - Mechanics of Rigid body
      - Statics
      - Dynamics
        - Kinetics
        - Kinematics
    - Mechanics of deformable bodies
      - Theory of elasticity
      - Theory of plasticity
  - Mechanics of fluid

Statics

It is that branch of mechanics that deals with the forces and their effect on body at rest.

Dynamics

It is the study of motion of rigid bodies & their relation with the forces causing them.

Kinetics

It deals with the laws of motion of bodies under the action of forces. It gives the relationship between forces & their resulting motions.

Kinematics

It deals with the motion of bodies without considering the causes of motion.

Rigid Body

A rigid body is such that a solid body in which the distance between any two arbitrary points remains constant even if a force is applied to it.

Body

An object is an identifiable collection of matter ~~cont~~ constrained by boundary called as body.

Particle

A particle is defined as a material point or point mass without any dimensions are known as particle. When the size of the body is extremely small compared with its range of motion, it may in certain cases be considered as particle.

Some Important Terms in Mechanics

Mass : Mass is the quantity of matter in a body.

(∵ matter → substance of which physical bodies are composed)

denoted as (M)

Weight : It is force with which a body is attracted towards the center of the earth under its gravitational pull.

$(W) = m \times g$

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Time : Time is the measure of sequence of events. denoted as (t).

Space : Space refers to the geometrical region

length : The linear distance is known as length denoted as (l).

Units & Dimensions

All physical quantities are divided in to two groups.

Fundamental / basic quantities	Derived quantities	
mass	speed	force
length	area	momentum
Time	volume	
	velocity	

Units

Units is defined as the neumerical standard used to measure the qualitative dimensions of a physical quantity.

Types of unit system

M.K.S → meter - kilogram - sec.
C.G.S → centimeter - gram - sec
F.K.S → Foot - kilogram - sec
S.I → International system of units.

Most accepted unit system all over the world.

It consist of 7 base units & 2 supplymentry units & a number of derived units

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Scalars & Vectors

Scalar

A scalar quantity is a physical quantity which needs only magnitude but no direction.

e.g. Mass, area, volume, temperature, energy

Vector

A vector quantity is a physical quantity which needs both magnitude & direction for it's specification.

e.g. velocity, displacement, acceln, force, momentum etc..

It is represented as $\longrightarrow$

2) $v = u + at$

check

$\Rightarrow L^1T^{-1} = L^1T^{-1} + L^1T^{-2}\ T^1$

$\Rightarrow L^1T^{-1} = L^1T^{-1} + L^1T^{-1}$

2) $v^2 - u^2 = 2as$

$[LT^{-1}]^2 - [LT^{-1}]^2 = 2[LT^{-2}][L]$

or $L^2T^{-2} - L^2T^{-2} = 2L^2T^{-2}$

1·2 ~~Chapter 2~~

FORCE

Force is defined as the external agency that changes or tends to change the state of rest or uniform straight line motion of a body ~~of~~ to which it is applied.

– force is a vector quantity, we can specify i.e.
→ it's magnitude
→ it's point of application
→ the direction of it's application.

Unit:

$F = m \times a$ (Newton's 2nd law)

$m \to$ mass of a body
$a \to$ accelⁿ.

In SI unit Newton.

$1N = 1kg \times 1\ m/s^2$

$1N = 1\ kg\,m/s^2$

(C·G·S unit)
$1\ dyne = 0.00001\ N$
or $1\ Dyne = 10^{-5}\ N$
or $1N = 10^5\ dyne$.

Representation

Force is a vector quantity, the sign for a force is $\longrightarrow$ or $\longleftarrow$ (straight line with arrow head)

Characteristics of force

1. Magnitude of force.
2. The direction of force or line of action of force. (i.e. upward, downward, east, west etc)
3. Nature of Force. (either pull or push type)

4. Point of applications
5. physical independence of a force.
6. Transmissibility of forces.
7. Superposition of forces
8. Reaction force generated by the action force.

Effect of a Force

A force may produce one or more of the following changes or effect in a body.

- → It may change the state of the body.
  (If body at rest → motion
  If body is in motion → accelerate)
- → It may change the direction of the motion of a body.
- → It may retard the forces. (de-acceleration)
- → The forces acting on a body, may give rise to internal stresses.
- → May produce turning effect.

Physical Independence of Force

~~If x no. of forces acts on a particle simultaneously then the effect of each force in the group~~

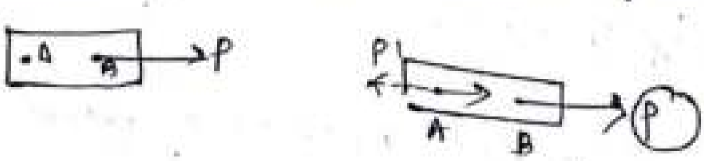
The resultant of a number of forces will have the same effect as produced by all forces acts when acts individually / separately.

or

If a no. of forces are acting simultaneously on a particle, then the resultant of these forces will have the same effect as produced by all forces.

Principle of Superposition

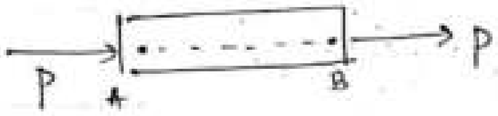
The action of a given system of forces on a rigid body will not be changed if we add or subtract from them another system of forces in equilm.



does not change

Theorem of Transmissibility of force

The point of application of a force may be transmitted along its line of action to another point without changing the effect of the force on any rigid body to which it may be applied.



Action & Reaction

Newton's 3rd Law:

"To every action, there is an equal & opposite reaction"

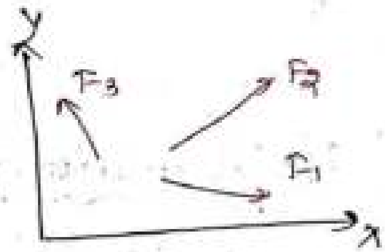
Any force or action causes an equal & opposite force from the support or point of application.

System of Forces

Type of Force

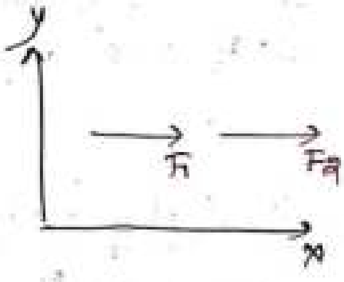
1. coplaner –

whose line of action lies on the same plane. It may be ∥ or non ∥.



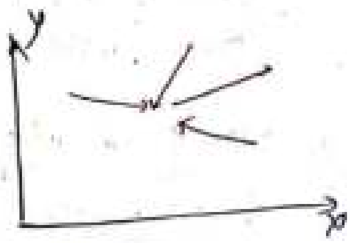
2. Colinnear –

whose line of action lies on the same line



3. concurrent –

The forces which meet at a single point –



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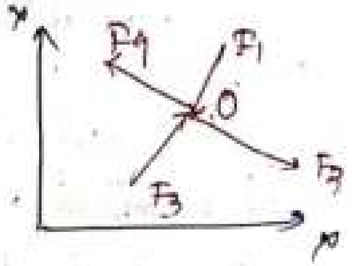
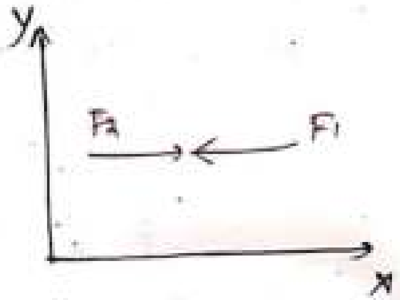
4. Coplanner Cocurrent

The forces which meet at one point & also their line of action lies in one plane.



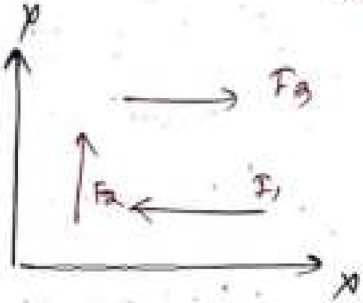
5. collinear concurrent

The forces whose line of action meet at one point.



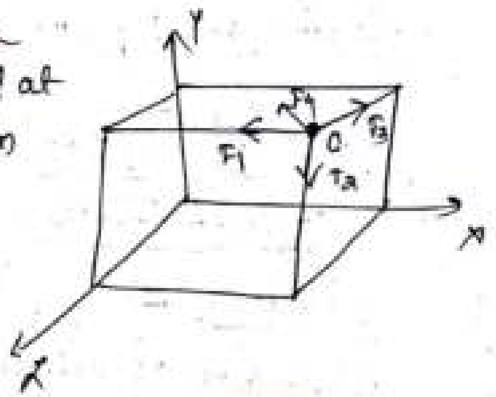
6. Coplanner non concurrent

The forces whose line of action do not meet at a point.



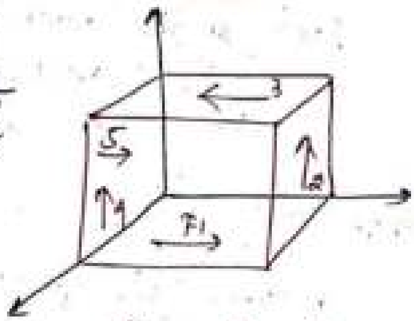
7. Non coplanner concurrent

Whose line of action meet at one point but do not lie in same plane.



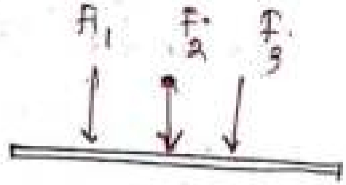
8. No coplanner non concurrent

Whose line of action do not lie on the same plane & they donot meet at one point



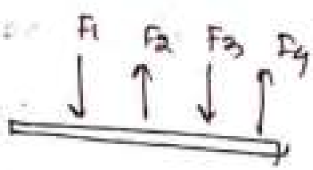
9. Parallel forces

line of action are ∥ to each other in same dire^n.



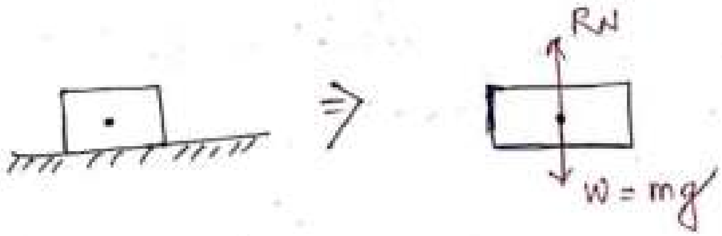
10. Non parallel forces

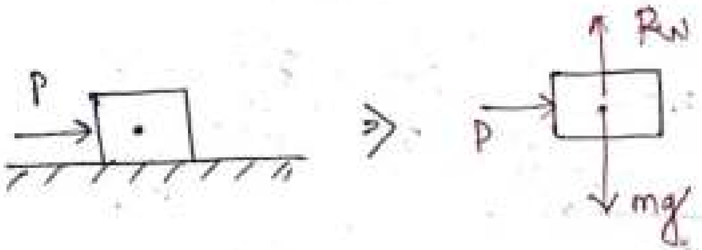
line of action ∥ to each each other but directed in opposite direction.

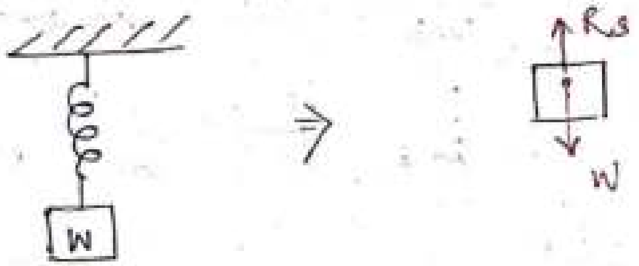


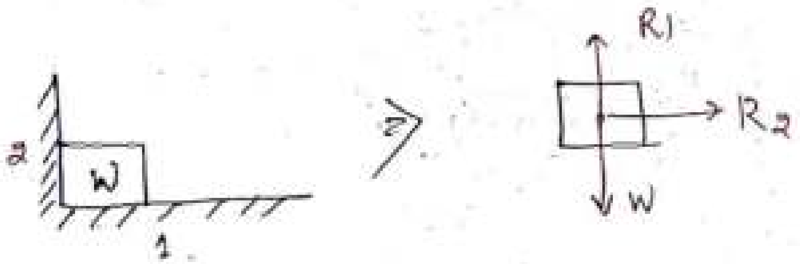
Freebody Diagram

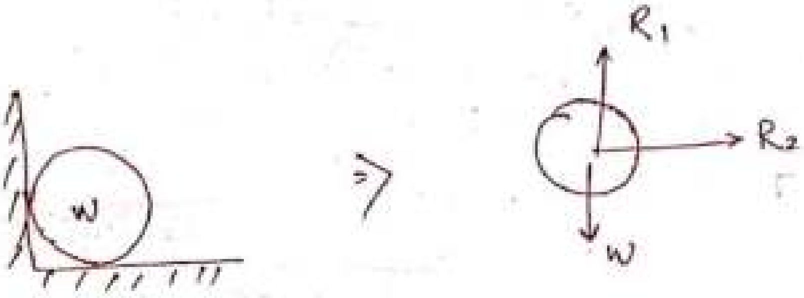
Freebody diagram is a sketch of the isolated body, which shows the external forces on the body & the reaction forces acting on it.

1) 

2) 

3) 

4) 

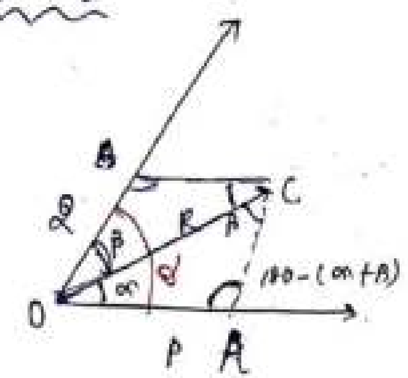
5) 

6) 

1.3 RESOLUTION OF FORCE

- Resolution is the process of splitting of a given force into a set of forces or a number of components without changeing its effect on the body.
- A force is generally resolved along two mutually perpendicular directions.

Case-1



$OA = P$

$OB = Q = AC$

$OC = R$

In ΔAOC $\quad \dfrac{OA}{\sin\beta} = \dfrac{AC}{\sin\alpha} = \dfrac{OC}{\sin\{180-(\alpha+\beta)\}}$

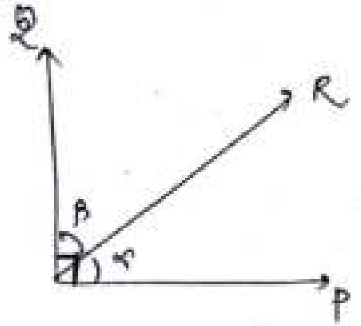
$$\Rightarrow \frac{P}{\sin\beta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin(\alpha+\beta)}$$

$$\boxed{P = R\frac{\sin\beta}{\sin(\alpha+\beta)}}$$

$$\boxed{Q = R\frac{\sin\alpha}{\sin(\alpha+\beta)}}$$

Case-2

*



$\alpha+\beta = 90^\circ$

$\beta = 90^\circ - \alpha$

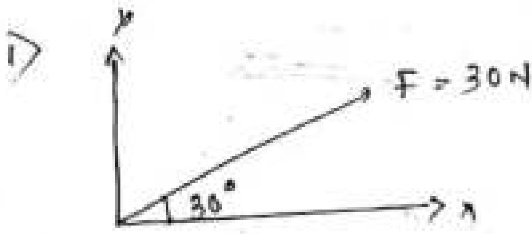
$$P = R\frac{\sin(90-\alpha)}{\sin(90^\circ)} = 1$$

$$\boxed{P = R\cos\alpha}$$

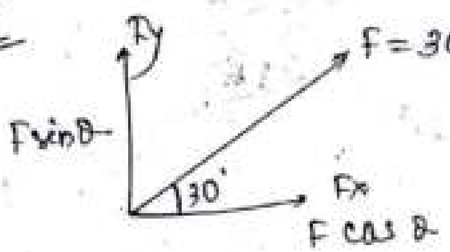
$$Q = R\frac{\sin\alpha}{\sin 90^\circ} \Rightarrow \boxed{Q = R\sin\alpha}$$

Though resolution of force is possible in any given direction, we frequently use resolution with rectangular components, which are always perpendicular to each other.

1)



sol



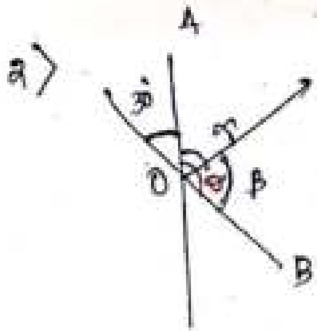
$\theta = 30°$

$F_x = F\cos\theta = 30 \times \cos 30° = 25.98$ Ans

$F_y = F\sin\theta = 30 \times \sin 30° = 15$ Ans

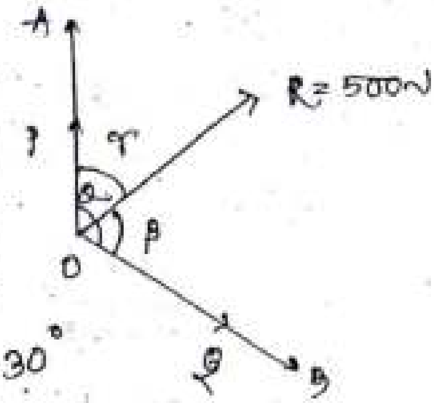
$\boxed{F = F_x + F_y}$

2)



P = 400 N given

Find α & β and OB



$\sin\beta = \frac{P}{R}\sin\theta$

$\theta = 180 - 30°$
$= 150°$

$= \frac{400}{500}\sin 150°$

$= 0.8 \times \sin 150°$

$= 0.4$

$\beta = \sin^{-1} 0.4$
$= 23.57$

$\alpha + \beta = \theta$

$\alpha = \theta - \beta$
$= 150 - 23.57$

$\alpha = 126.42$

3)

1.4 COMPOSITION OF FORCES

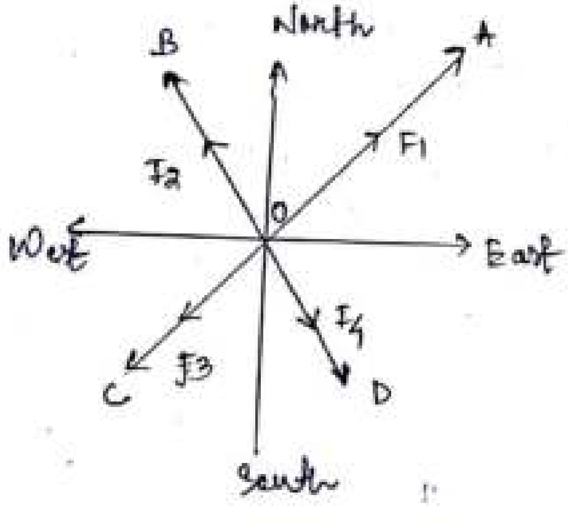
- The process of reduction of a given system of forces to a single force which could replace all other force and produce the same effect as produced by the system of forces is called composition of forces.
- The single force is called as resultant force.

Space diagram

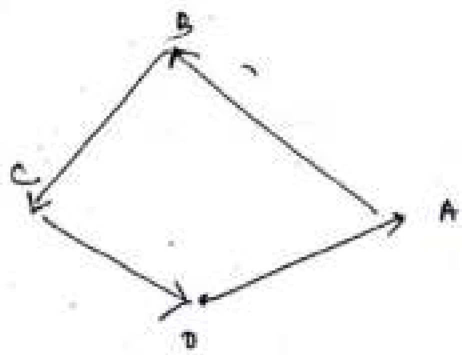
It is a position diagram showing the various forces along with their magnitude, point of application & line of action.

Vector diagram

It is a vector diagram corresponding to a space diagram.



Space diagram



vector diagram

Laws for Composition of Forces

There are three laws for composition of forces & hence determine of resultant of a system of concurrent forces.

They are

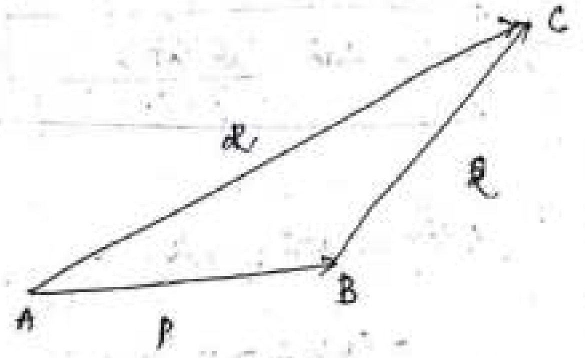
i) Triangle law of composition of forces

ii) parallelogram " " "

iii) polygon " " "

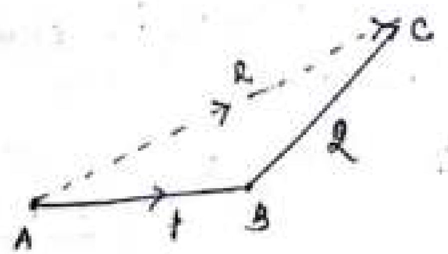
1) Triangle law of composition of forces statement

of the forces, are represented both in magnitude & direction, by the two sides of a triangle taken in the same order, their resultant is resultant (in magnitude and direction) by the third side of the triangle taken in the opposite order.

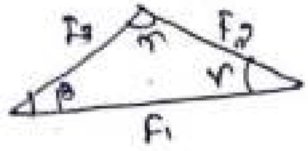


$R = P + Q$

Analytical Methods → consider 2 forces acting on same order.

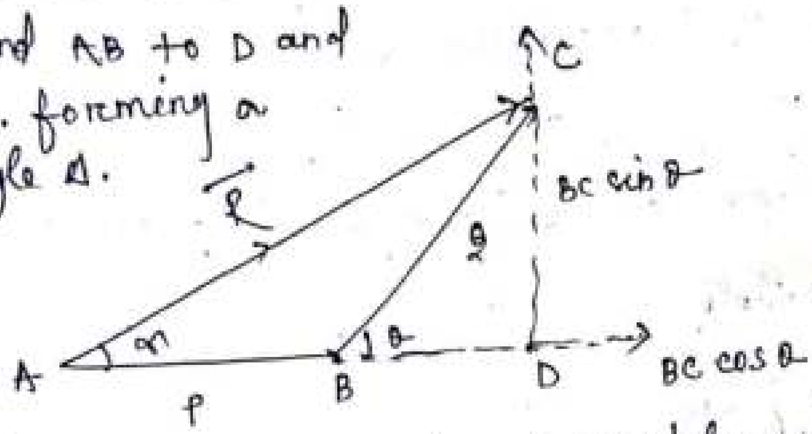


let for sine law



$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

Now extend AB to D and Join CD. forming a right angle Δ.



$\vec{Q}$ makes angle θ with horizontal and
$\vec{R}$ makes angle α " with $\vec{P}$.

$$\tan\alpha = \frac{CD}{AD} \quad (\text{In } \Delta ADC)$$

$$= \frac{CD}{AB+BD} = \frac{BC\sin\theta}{AB+BD} = \frac{BC\sin\theta}{P+BC\cos\theta}$$

$$\Rightarrow \tan\alpha = \frac{BC\sin\theta}{P+BC\cos\theta} = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\Rightarrow \cancel{\tan^{-1}} \Rightarrow \boxed{\alpha = \tan^{-1}\frac{Q\sin\theta}{P+Q\cos\theta}}$$

$||^{ly}$ in ΔADC $\quad AC^2 = AD^2 + CD^2$ (pythagoras theorem)

$$= (AB+BD)^2 + CD^2$$

$$= AB^2 + BD^2 + 2\cdot AB\cdot BD + CD^2$$

$$= AB^2 + BD^2 + CD^2 + 2AB\cdot BD$$

$$= AB^2 + BC^2 + 2AB\cdot BC\cos\theta$$

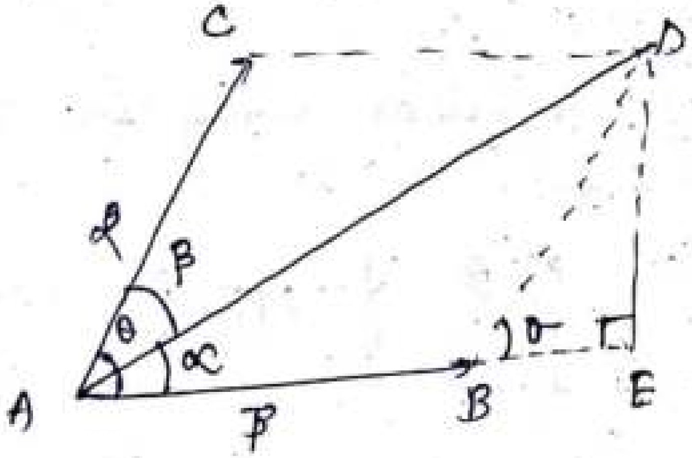
Replacing with vectors.

$$\boxed{R^2 = P^2 + Q^2 + 2P\cdot Q\cos\theta}$$

parallelogram Law of composition of forces

Analytical

Considere 2 forces $\vec{P}$ & $\vec{Q}$ are acting on two sids of a $\parallel^{re}$ gream, at point A, at an agle θ.



AC = BC

AB = CD

Let BD makes an agle θ with the horizental.

extend AB up to E & join DE.

Now in Δ DEA is an rightangled Δ.
Acc. to pythagoras.

$$AD^2 = ED^2 + AE^2$$
$$= (AB + BE)^2 + ED^2$$
$$= AB^2 + BE^2 + 2 \cdot AB \cdot BE + ED^2$$
$$= AB^2 + BE^2 + ED^2 + 2 AB \cdot BE$$
$$= AB^2 + BD^2 + 2 \cdot AB \cdot BD \cos\theta$$

$BE = BD\cos\theta$
$DE = BD\sin\theta$

$$\Rightarrow AD^2 = AB^2 + BD^2 + 2 \cdot AB \cdot BD \cos\theta$$
$$\Rightarrow \boxed{R^2 = P^2 + Q^2 + 2 \cdot P \cdot Q \cos\theta}$$

ΔAED $\tan\alpha = \dfrac{ED}{AE} = \dfrac{BD\sin\theta}{AB + BE} \Rightarrow \dfrac{BD\sin\theta}{AB + BD\cos\theta}$

$$\Rightarrow \boxed{\alpha = \tan^{-1}\frac{Q\sin\theta}{P + Q\cos\theta}}$$

Special cases

1) When 2 forces P & Q acts at very small angle $\theta \approx 0°$ then.

$R^2 = P^2 + Q^2 + 2PQ$ (when the forces acting in ~~of~~ same direction)
$\Rightarrow R = \sqrt{P^2 + Q^2 + 2PQ}$

$R = \sqrt{(P+Q)^2}$
$\Rightarrow R = P + Q$

$R = P - Q$ if $P > Q$
$R = Q - P$ if $Q > P$
opposite direction

$\theta = 0°$ $R^2 = P^2 + Q^2 + 2PQ\cos 0°$. $\cos 0° = 0° = 1$
$\Rightarrow R^2 = (P+Q)^2$
$\Rightarrow R = (P+Q)$

$\theta = 180^\circ$

$$R^2 = P^2 + Q^2 + 2PQ \cos 180^\circ$$

$$= P^2 + Q^2 - 2PQ$$

$$R^2 = (P-Q)^2$$

$$\Rightarrow R = P - Q$$

$\theta = 90^\circ$

$$R = P^2 + Q^2 + 2PQ \cos 90^\circ$$

$$R = \sqrt{P^2 + Q^2}$$

Composition of Forces by Method of Resolution.

When a no. of forces acting at a point can be obtained easily by resolving each of the forces into their respective rectangular components.

$$\Sigma F_x = F_{x_1} + F_{x_2} + F_{x_3} + \cdots$$

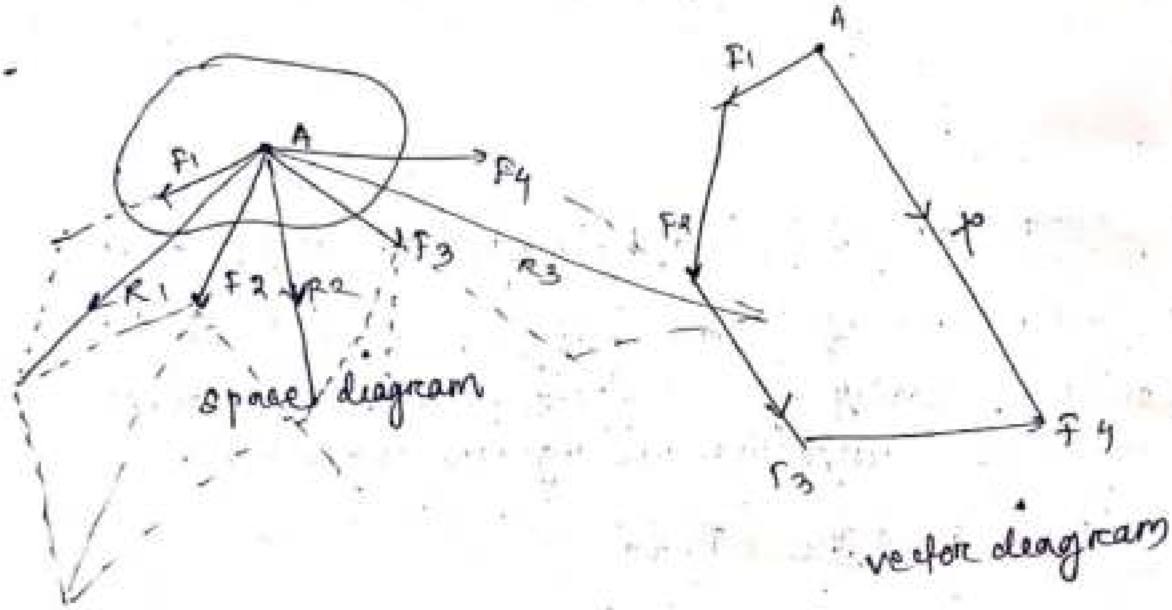
$$\Sigma F_y = F_{y_1} + F_{y_2} + F_{y_3} + \cdots$$

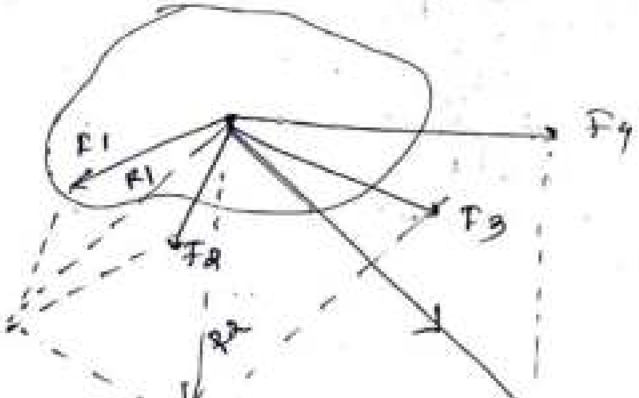
$$R = \sqrt{(F_x)^2 + (\Sigma F_y)^2}$$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

3) Polygon Law of forces

If a no. of forces acting symelteneously at a point are represented by the side of an open polygon, all taken in same order, then their resultant is represented by the closing side of the polygon taken in opposite order.





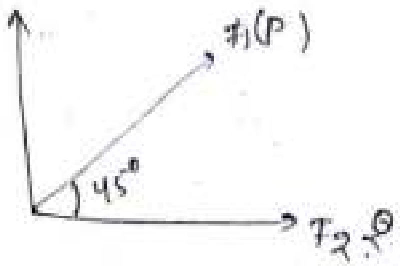
take R_1 by F_1 & F_2

take R_2 by R_1 & F_3

take R_3 by R_2 & F_4

by ||gram methode.

Q) Two forces of 4N & 6N making an angle 45° with each other. Determine resultant force.



$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$= 4^2 + 6^2 + 2\times 24 \times \cos 45°$

$\Rightarrow R^2 = 16 + 36 + 33.94$

$\Rightarrow R = 9.27$ N Ans

$\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta} = \dfrac{6\times\sin 45°}{4 + 6\times\cos 45°}$

$= \dfrac{6\times 0.707}{4 + 4.24}$

$= \dfrac{4.242}{8.24} = 0.51$

$\Rightarrow \alpha = \tan^{-1} 0.51$

$\Rightarrow \alpha = 27.02$

Q) Resultant of two forces acting at right angle is given by 1000 N. The magnitude of both forces is same. Determine the magnitude of the forces & angle of the resultant.

Sol: $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$F_1 = F_2 = F$

$R = \sqrt{F^2 + F^2 + 2F^2\cos 90°}$

$\Rightarrow (1000)^2 = F^2 + F^2 + 2F^2 \cos 90°$

$\Rightarrow 2F^2 = (1000)^2$

$\Rightarrow F = 707.2$ N

$\tan\alpha = \dfrac{F\sin\theta}{F + F\cos\theta} = \dfrac{707\times\sin 90°}{707 + 707\times\cos 90°}$

$= 1$

$\Rightarrow \alpha = \underline{\underline{45°}}$

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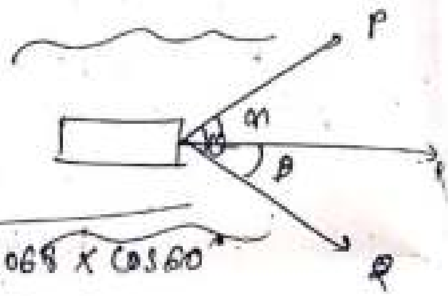
2) A boat is moving uniformly along a canal by two ropes pulling with forces $P = 890\,N$ & $Q = 1068\,N$ acting with an angle $60°$ betⁿ them. Determine the magnitude of resultant pull on the boat & the angle α & β as shown in fig

$P = 890\,N$ $\theta = 60°$

$Q = 1068\,N$



$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\Rightarrow R = \sqrt{(890)^2 + (1068)^2 + 2 \times 890 \times 1068 \times \cos 60°}$

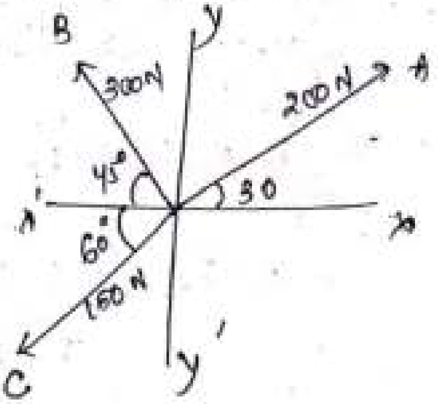
$\Rightarrow R = 1689\,N$

$\alpha = \tan^{-1} \dfrac{Q \sin\theta}{P + Q\cos\theta} = \tan^{-1} 0.65$

$\Rightarrow \alpha = 33°$

$\Rightarrow \beta = 60° - 33° = 27°$

2)



$\Sigma F_x = 200\cos 30° + -300\cos 45° - 150\cos 60°$

$= -113.9\,N$

$\Sigma F_y = 200\sin 30° + 300\cos 45° - 150\cos 60°$

$= 182.2\,N$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 217\,N$

$\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x}$

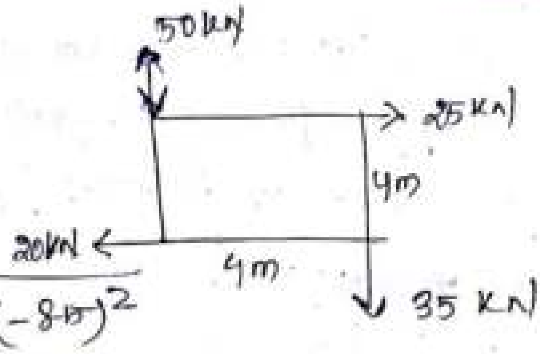
$\Rightarrow \theta \approx 60°$

Q)

$\Sigma H = 25 - 20 = 5 \text{ kN}$

$\Sigma V = -50 + (-35)$

$= -85 \text{ kN}$

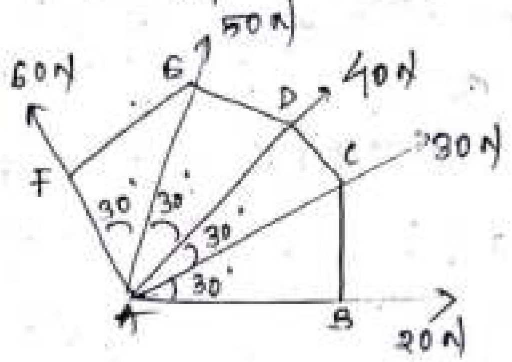


$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{5^2 + (-85)^2}$

$= 85.15 \text{ kN}$

$\tan\theta = \frac{\Sigma V}{\Sigma H} \Rightarrow \theta = \tan^{-1}\frac{\Sigma V}{\Sigma H} = 86.6^\circ$

Q)



$\Sigma H = 20 \cos 0^\circ + 30 \cos 30^\circ$
$+ 40 \cos 60 + 50 \cos 90^\circ$
$+ 60 \cos 120^\circ = 36.0 \text{ N}$

$\Sigma V = 20 \sin 0 + 30 \sin 30 + 40 \sin 60 + 50 \sin 90 + 60 \sin 120^\circ$

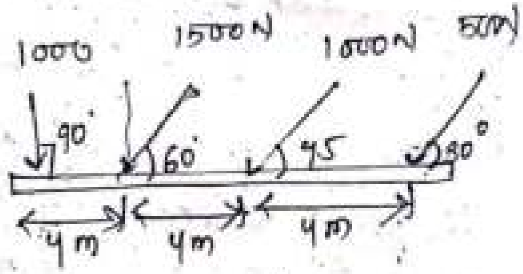
$= 151.6 \text{ N}$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 155.8 \text{ N}$

$\Sigma H = +ve$, $\Sigma V = +ve$ } Resultant lies betn 0 to 90°

$\theta = \tan^{-1}\frac{\Sigma V}{\Sigma H} = 78.6^\circ$ Ans

Q)



$\Sigma H = 1000 + 1500 \cos 60^\circ + 1000 \cos 45^\circ$
$+ 500 \cos 30^\circ$

$= 1890 \text{ N}$

$\Sigma V = 1000 + 1500 \sin 60^\circ + 1000 \sin 45^\circ$
$+ 500 \sin 30^\circ = 3256 \text{ N}$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 3765 \text{ N}$ $\quad \theta = \tan^{-1} \Sigma V / \Sigma H = 59.8^\circ$

θ will betn 0 to 90°

$R \times x = 1000 \times 0 + 1500 \times \sin 60^\circ \times 4 + 1000 \times \sin 45^\circ \times 8$
$+ 500 \sin 30^\circ \times 12$

$x = 4.25 \text{ m}$.

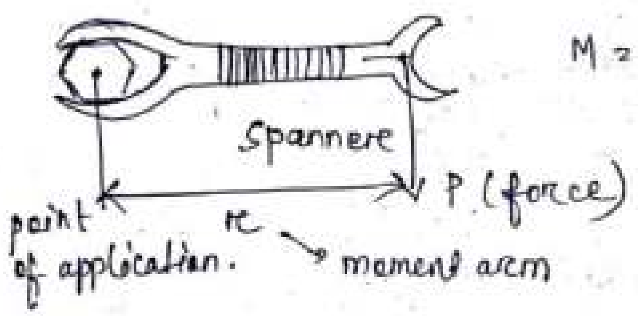
Find the magnitude, dirnn & position of Resultant force.

1.5 MOMENT OF A FORCE & COUPLE

The moment of a force with respect to a point is equal to the product of the magnitude of the force & perpendicular distance from the point to the line of action of the force.

$M = P \times s$ $P \rightarrow$ acting on the body

$s \rightarrow \perp$ distance betⁿ the point, about which the moment is req & the line of action of the force.



$M = P \times r$

Units

$M = F \times r$

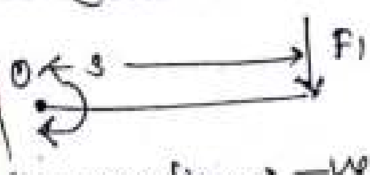
$=$ Newton $\times$ meter

$=$ N/m or KN-m.

Types of Moment

1) clockwise

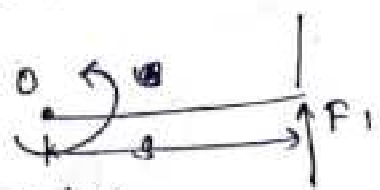
The moment of a force whose effect is to turn or rotate about the moment center in clock direcⁿ.



Sign convention $\rightarrow$ $-ve$

2) Anticlockwise

The moment of a force whose effect is to turn about the moment center in anticlockwise direcⁿ.

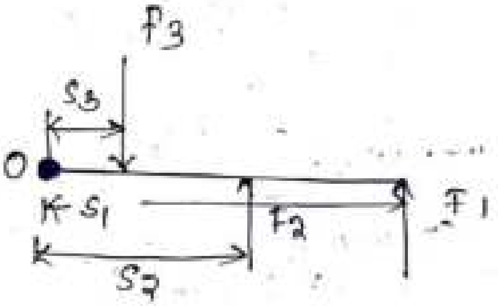


$\rightarrow +ve$

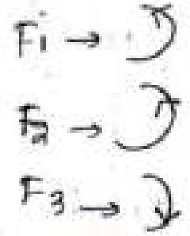
For rotary motion two quantities are always needed i.e. force & moment. These two quantity combined in to a single term known as the (M) moment or torque (τ)

Combinations of Moment

When general forces, in one plane, are involved it is seen that some forces tend to rotate clockwise direc^n & some tend to produce anticlockwise direc^n about the same center.
So net torque or moment of several forces about a common moment center.

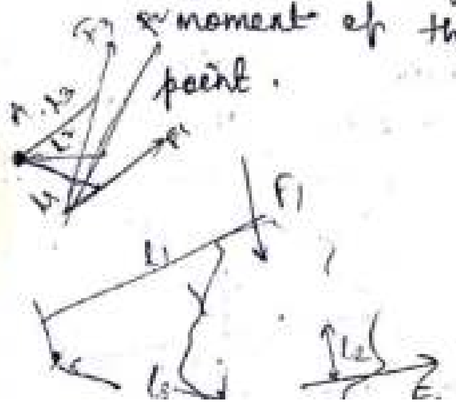


$$\tau = F_1 s_1 + F_2 s_2 - F_3 s_3$$



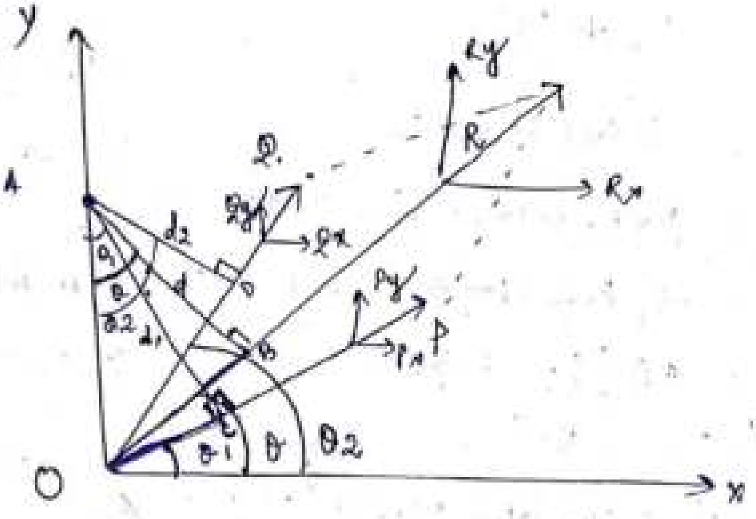
Varignon's Theorem <Law of moments>

If a no. of coplanar forces act simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.



$$\boxed{\sum M^F = M^R}$$

$$R \times x = F_1 l_1 + F_2 l_2 + F_3 l_3$$



Let two forces P & Q are acting at a point O.
Now by taking ||$^{\text{gm}}$ law by projecting P & Q, we get a point, which is the resultant of P & Q.

Let angle of inclination of P with x axis = θ_1
R " " " = θ
Q " " " = θ_2

- Now let a arbitary point A on y axis, at which we will calculate the moment of P & Q.
- Now draw ⊥ line to P, Q & R from point A.

Acc to varignon theorem the moment by P & Q at A is equal to moment by R.

Moment of P at A = $M_A^P = P \times d_1$ —— (1)
" " Q " A = $M_A^Q = Q \times d_2$ —— (2)
" " R " A = $M_A^R = R \times d$
$= R \times AO \cos\theta$

From △ AOB $\cos\theta = \dfrac{d}{AO}$
$\Rightarrow d = AO\cos\theta$

$= AO \times R\cos\theta$
$= AO\, R_x$ —— (3)

Adding equation ① & ②

$$\Sigma M_A^F = M_A^P + M_A^Q$$

$$= P \times d_1 + Q \times d_2$$

$$= P \times AO \cos\theta_1 + Q\, AO \cos\theta_2$$

$$= AO[P\cos\theta_1 + Q\cos\theta_2]$$

$$= AO[P_x + Q_x] \quad \text{--- ④}$$

$$\Sigma M_A^F = AO[R_x] \quad \text{--- ⑤}$$

From ΔAOC

$\cos\theta_1 = \frac{d_1}{AO}$

$\Rightarrow d_1 = AO\cos\theta$

From ΔAOD

$\cos\theta_2 = \frac{d_2}{AO}$

$\Rightarrow AO\cos\theta_2 = d_2$

P_x & Q_x are horizontal component of P & Q.

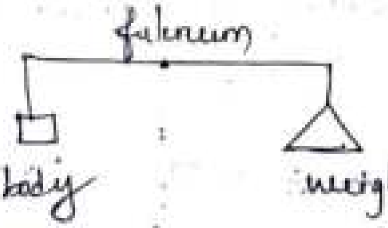
Where $P_x + Q_x = R_x$ all the horizontal components

From equation ③ & ⑤

$$\boxed{\Sigma M_A^F = M_A^R}$$

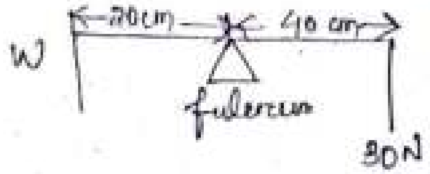
Levers

A lever is a rigid body (bar) (straight, curved or bent) and is hinged at one point (fulcrum). It is free to rotate about the fulcrum. Some common eg. of levers are scissor, weighing balance, tongue etc.



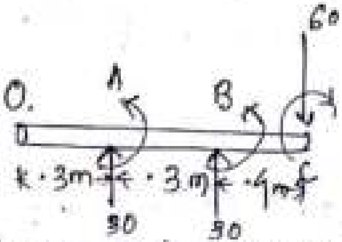
1eg A uniform rectangular bar AB is placed over a fulcrum at a distance of 20cm from end A. A load of 30N is suspended at end B. What is the magnitude of force applied at end A so the bar remains horizontal?

$W \times 20 = 30 \times 40\,cm$



$\Rightarrow W = \frac{30 \times 40}{20} = 60N$ ans

eg-2: A uniform rod OABC of length 1m is hinged at O. A force 30N at A & 30N at B are applied at distances 0.3m & 0.6m from the end O. Another force of 60N is applied at the end C. What is the moment of the forces about O.



Soln: pt A & B produces +ve moment while C produces -ve moment.

So $M = 30N \times .3m + 30N \times .6m - 60 \times 1m$

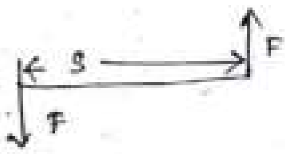
$= -33Nm$ ans

clockwise moment of magnitude 33Nm.

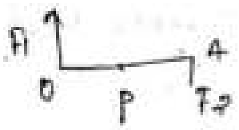
COUPLE

- A pair of equal and unlike parallel forces is known as a couple.

→ The perpendicular distance betn both forces called as arm of couple.



→ The plane to which the forces are acting are called plane of couple.



$F_1 \times OP + F_2 \times AP = 0$

$(\because F_1 = F_2 = F_1)$

Moment of couple

The magnitude of torque is given by $= F_1 \times OP + F_1 \times AP$

$= F_1(OP + AP)$

$= F_1 \times OA$

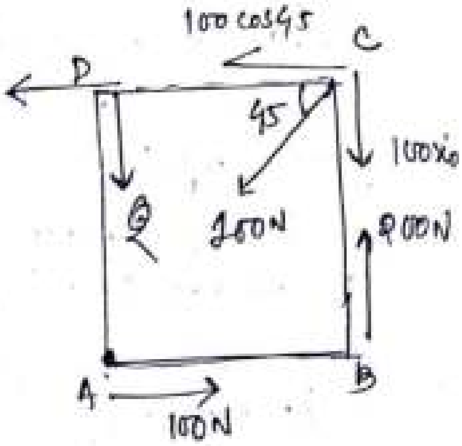
$\tau = F \cdot S$

$=$ Force × perpendicular distance. (Nm)

- It may be +ve or -ve depending upon directn.

(anticlock) (clockwise)

2) A square ABCD has ~~[illegible]~~ forces acting as shown in fig. find values of P & Q. If the system reduces to a couple. Also find the magnitude of the couple. If the side of the square is 1m.



Sol^n each side of square → 1m.

Resolving all horizontal forces.

$100 - 150\cos45' - P = 0$

$\Rightarrow P = 100 - 150\cos45'$

$= 100 - 70.71$

$= 29.28$ N

Resolving all vertical forces

$200 - 150\sin45' - Q = 0$

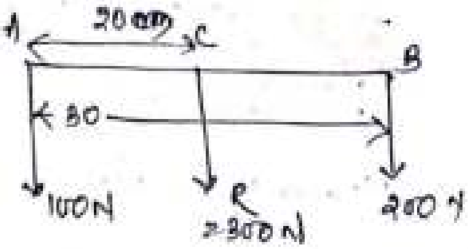
$\Rightarrow Q = 200 - 100\sin45'$

$= 129.3$N

By taking moment about A.

$200 \times 1 +$

2) Two like ∥ᵉ forces of 100N & 200N act as the ends of a rod of 30cm long. Find the magnitude of the resultant & the line of action.



Sol^n Since both the ∥ᵉ forces has have same direc^n. so

$R = F_A + F_B$

$= 300$N

Taking moment at A.

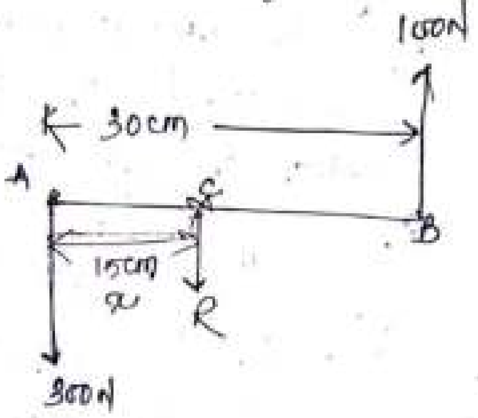
Let R acts at pt C. $200 \times AB \cancel{+0} = 300 \times AC$

$\Rightarrow 200 \times 30 \cancel{+0} = 300 \times AC$

$\Rightarrow AC = \frac{200 \times 30}{} = 20$ cm

Resultant of parallel Forces

Q) Two unlike parallel forces of magnitude 300 N & 100 N are acting in such a way that their line of actions are 30 cm apart. Determine the magnitude & localation of the resultant.



Sol^n Both the forces are in opposite direc^n.

So $R = P - Q$

$= 300 - 100 = 200N$

Let R acts at pt. C.

By taking moment ab A.

$100 \times BA + 0 = R \times AC$

$\Rightarrow 100 \times BA = 200 \times AC$

$\Rightarrow 100 \times 30 = 200 \times AC$

$\Rightarrow AC = 15cm$ Ans

~~$100 \times 30 = 300 \times x$~~

~~$\Rightarrow x = \frac{3000}{300}$~~

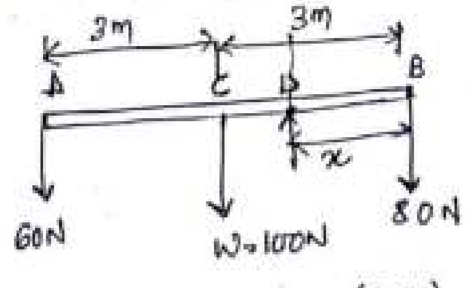
$100 \times (30 - x) = 300 \times x$

$\Rightarrow 3000 - 100x = 300x$

$\Rightarrow 3000 = 200x$

$\Rightarrow x = 15$ ✓

Q) A uniform beam AB of weight 100 N & 6m long has two bodies of weight 60N & 80N suspended from its two ends as shown in fig. Find at what point the beam should be supported so that it may rest horizontally



Sol^n Weight of rod AB = 100N.

Length of the AB = 6m and weights of bodies suspended at A & B are 60N & 80N.

$80 \times x = 100 \times (3 - x) + 60 \times (6 - x)$

Let x = distance between B & D., where the beam should be supported.

The force due to weight of the body, 100N, acts at the mid point of the beam at C.

As the beam is resting horizontally the clockwise -ve moments of the forces above D, should be equal to the anticlockwise moment of the forces.

So
$$80x = 60(6m - x) + 100(3m - x)$$
$$= 360m - 60x + 300 - 100x$$
$$\Rightarrow 240x = 660\,m$$
$$\Rightarrow x = \frac{660}{240} = 2.75m$$
Ans

So, the beam can be balanced at a distance of 2.75m from the 80N force.

<u>properties of a couple</u>

↳ The algebraic sum of the forces constituting the couple is zero.

↳ A couple can not be balanced by a single force.

↳ The algebraic sum of the moment of the force constituting the couple about any point is same and equal to moment of the couple itself.

CHAPTER-02 EQUILIBRIUM OF FORCES

2.1 If a system of forces acting simulteneously on a body produces no change in the state of rest or the state of motion of the body, the system of forces is said to be in equillm.

A system of forces can be in equill^m under two situations.

↳* If the resultant of a number of forces acting at a point is zero.

↳ When the resultant of a system of forces applied on a particle has a non-zero value, then the particle will remain at rest by applying a force equal in magnitude but opposite in dirc^n of the resultant.

Principles of Equilibrium

Two-force principle

When a body is acted upon by two, equal opposite collinear forces, the resultant force is zero. The system of forces is said to be in equilibrium.

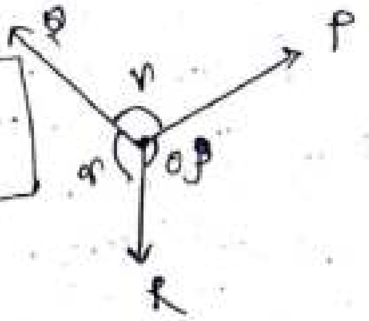
Three Force principle

Three non-parallel forces will be in equilc^n when they lie in one plane, intersect at one point and their free vectors form a closed triangle.

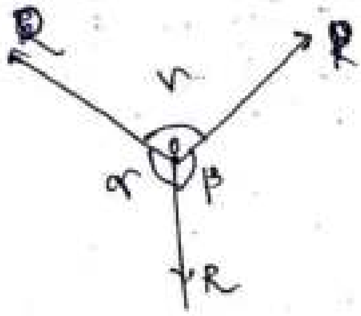
2.2 Lami's Theorem

If three coplanner concurrent forces are acting on a body kept in equilibrium, then each force is proportional to the sine angle between other two forces and the const. of proportionality is the same.

$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma} = k = \text{const.}}$$



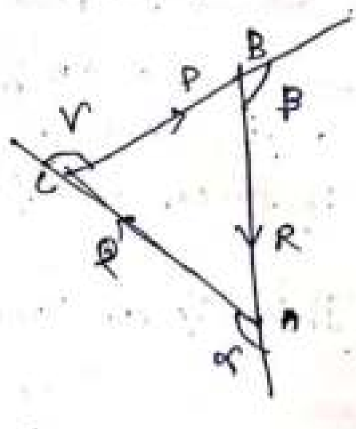
proof



Let force P, Q, R acting at point O.

since P, Q, R are in equilibrium the triangle of forces should be a closed one. (Vector diagram)

Draw a line AB ∥ to force R.
From end A draw a line ∥ to Q.
name it AC. From 'C' draw a line ∥ to P. It will intersect the line AB at B.



$\angle A = \pi - \alpha$

$\angle B = \pi - \beta$

$\angle C = \pi - \gamma$

Applying sine rule to the $\triangle ABC$.

$$\frac{P}{\sin(\pi-\alpha)} = \frac{Q}{\sin(\pi-\beta)} = \frac{R}{\sin(\pi-\gamma)}$$

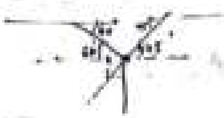
$$\Rightarrow \boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

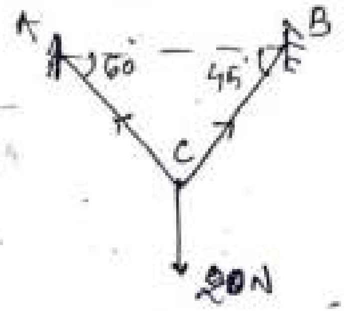
Q) An electric lamp weighing 20N is suspended from a point C supported by 2 wire AC & BC. The point A, B are at same level. AC makes an angle 60° and BC makes 45° to horizontal as shown in fig. Determine the tension in the string AC & BC.

Soln W at C = 20

T_{AC} - tension in AC

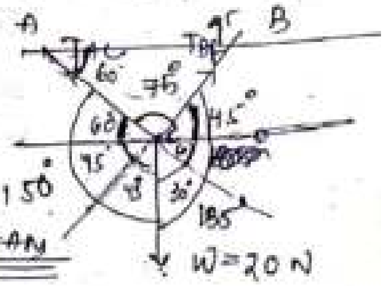
T_{BC} = " " BC.





$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

$$\Rightarrow \frac{20}{\sin 75^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{T_{AC}}{\sin 135^\circ}$$

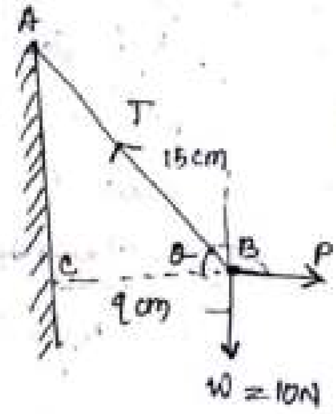


$$T_{AC} = \frac{20 \times \sin 135}{\sin 75} = \frac{14.14}{\sin 75} = \underline{\underline{14.95\ Ans}}$$

$$T_{BC} = \frac{20 \times \sin 150}{\sin 75^\circ} = \frac{10}{\sin 75^\circ} = \frac{10}{.965} = \underline{\underline{10.35}}$$

Q) Body weighing 10N is suspended from a fixed point by a string 15cm long & is kept at rest by a horizontal force P at a distance of 9 cm from the vertical line drawn through the point of suspension. What are the tension of the string & the value of P?

Soln



Let tension T developed in the string AB. The point B is in equil^{m}, under the three forces 10, T_{AB} & P.

Let ∠ABC = θ

Applying Lami's theorem

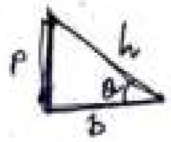
$$\frac{P}{\sin(90+\theta)} = \frac{T_{AB}}{\sin 90} = \frac{10}{\sin(180-\theta)}$$

$$\frac{P}{\cos\theta} = \frac{T}{1} = \frac{10}{\sin\theta}$$

From ΔABC

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow AC^2 = AB^2 - BC^2$$
$$= 15^2 - 9^2$$
$$= 225 - 81$$
$$AC = \sqrt{144}$$
$$= 12 \text{ Ans}$$



$$\sin\theta = \frac{AC}{AB} = \frac{12}{15} = 0.8$$

$$\cos\theta = \frac{BC}{AB} = \frac{9}{15} = 0.6$$

$\sin\theta = p/h$

$\cos\theta = b/h$

$\tan\theta = \frac{p}{b}$

$$\frac{T}{1} = \frac{P}{0.6} = \frac{10}{0.8}$$

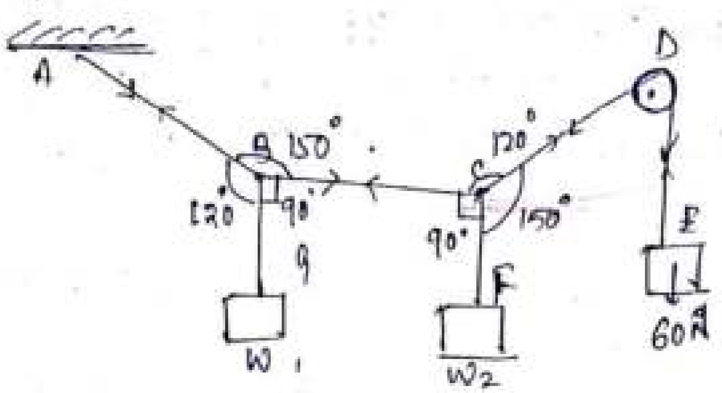
$$\Rightarrow P = \frac{10 \times 0.6}{0.8} = \frac{60}{8} = 7.5N \text{ Ans}$$

$$\Rightarrow T = \frac{10}{0.8} = 12.5 \text{ N Ans}$$

Q) A fine light string ABCDE with one end A fixed, has weights W_1 & W_2 attached to it at B and C. The string passes round a smooth pulley D carrying wt 80N at free end E as shown in fig. If the position of equ^n, BC is horizontal with AB & CD makes an angle 150° & 120° with BC. Find

i) Tension in portion AB, BC, DE.

ii) magnitude of W_1 & W_2



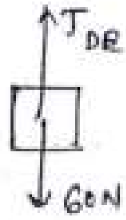
T_{AB} - tension in AB

T_{BC} = 〃 〃 BC

T_{CD} = 〃 〃 CD

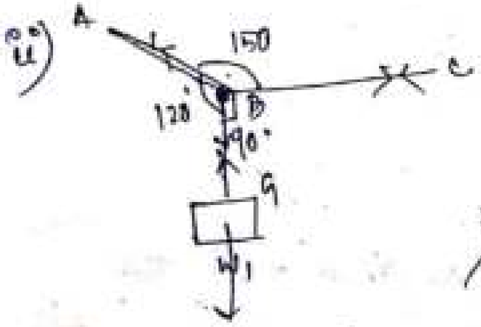
pulley is smooth no friction $T_{CD} = T_{DE}$

$T_{DE} = 60N = T_{CD}$



Apply lamis theorem at C & B.

ii)

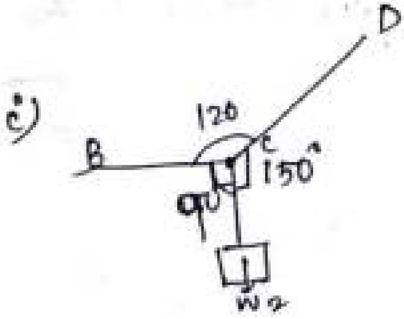


$$\frac{T_{AB}}{\sin 90} = \frac{T_{BC}}{\sin 120} = \frac{W_1}{\sin 150}$$

$$\Rightarrow T_{AB} = \frac{T_{BC} \times \sin 90}{\sin 120} = \frac{30 \times 1}{\sin 120} = 34.63N$$

$$\Rightarrow W_1 = \frac{T_{BC} \times \sin 150}{\sin 120} = \frac{30 \times .5}{.866} = 17.32N$$

i)

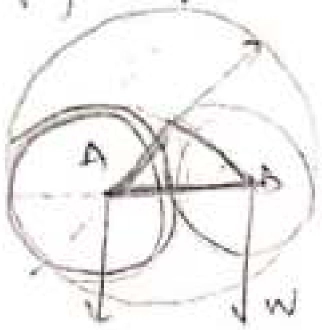


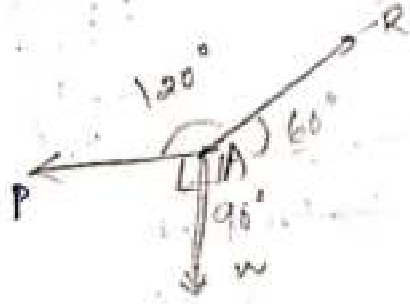
$$\frac{T_{CD}}{\sin 90^\circ} = \frac{T_{BC}}{\sin 150} = \frac{W_2}{\sin 120^\circ}$$

$$\Rightarrow T_{BC} = \frac{T_{CD} \times \sin 150}{\sin 90} = \frac{60 \times .5}{1} = 30N$$

$$\Rightarrow W_2 = \frac{T_{CD} \times \sin 120}{\sin 90^\circ} = 51.96\ N$$

Q) Two equal and heavy spheres of 40 mm radius are in equilm with in a cup of radius 120 mm. Show that the reacn betn the cup & one sphere is double of that betn the two spheres. As shown in the fig





$$\frac{R}{\sin 90^\circ} = \frac{W}{\sin 120} = \frac{P}{\sin 150^\circ}$$

$\Rightarrow R = \frac{W}{\sqrt{3}/2} = \frac{P}{1/2}.$

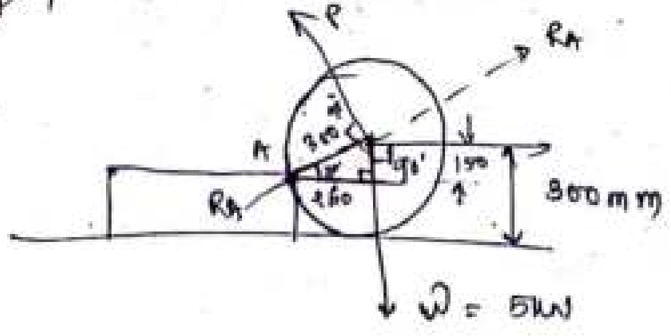
$\Rightarrow R = \frac{P}{1/2}$

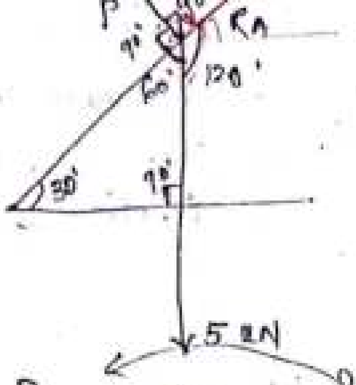
$\Rightarrow R = 2P$ ✓ Ans

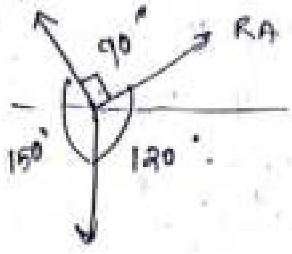
Q) A uniform wheel 600 mm dia weighing 5 kN rest against a rigid rectangular block of 150mm height as shown in the fig. Find the minm force reqr. to turn the wheel over the corner A & find the reacn on the block.

2015 (W)

2016







$$\frac{P}{\sin 120} = \frac{R_A}{\sin 150} = \frac{5000}{\sin 90^\circ}$$

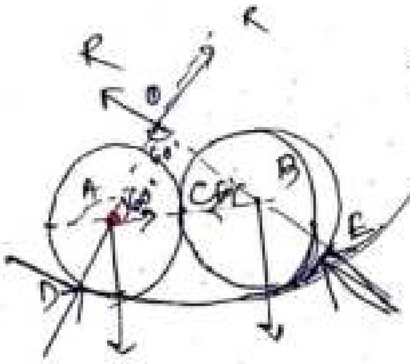
$\Rightarrow$ $P = 4330\,N = 4.33\ kN$

$R_A = 2500\,N$

$= 2.5\ kN$

$\rightarrow$

Two spheres with center A & B, lying in equilm, in cup with center 'O', Let the sphere contact at pt C. and sphere A with cup D & sphere B with cup E.



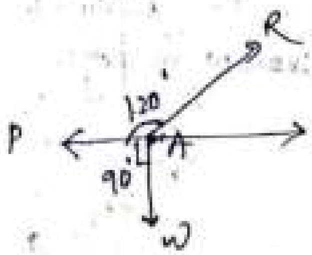
$R \rightarrow$ reaᴺ at D & E

$P \rightarrow$ reaᴺ at C.

from geometry. $OD = 120$ mm $AD = 40$ mm so $AO = 120 - 40 = 80$.

similarly $OB = 80$. $AB = AC + CB = 40 + 40 = 80$

OAB becomes equilateral Δ.



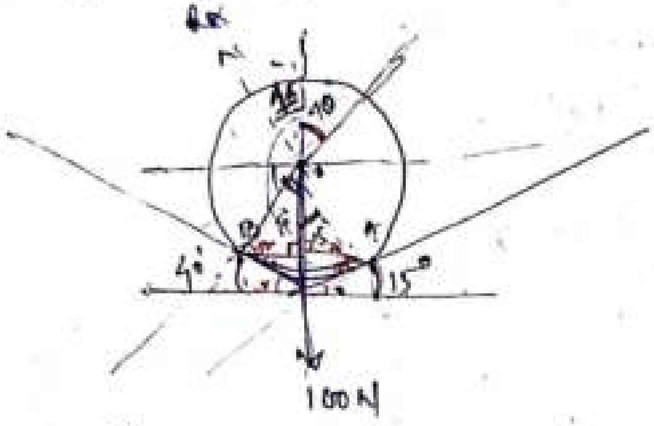
$$\frac{R}{\sin 90} = \frac{W}{\sin 120} = \frac{P}{\sin 150^\circ}$$

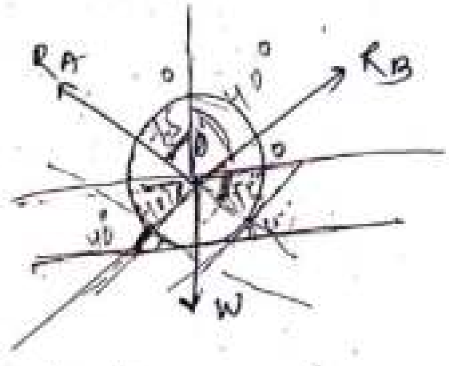
$\Rightarrow$ $R = \frac{W}{\sqrt{3}/2} = \frac{P}{1/2}$

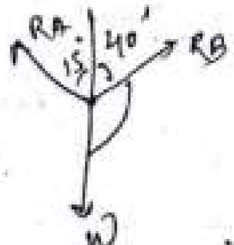
$\Rightarrow$ $R = P/\tfrac{1}{2}$

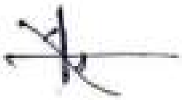
$\Rightarrow$ $R = 2P$

2) A smooth circular cylinder of radius 1.5 meter is laying in triangular groove. One side of which makes 15° angle & other 40° angle with horizontal. Find the reactions at the surface of contact. If there is no friction & the cylinder weighs 100N.









$R_A \rightarrow$ Reacⁿ of A

$R_B \rightarrow$ Reacⁿ of B

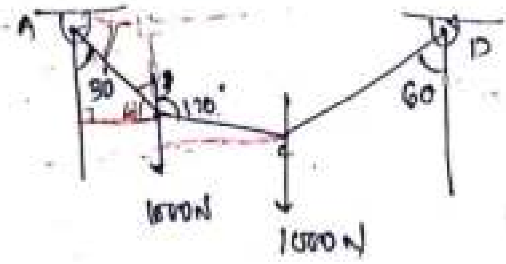
from friction force

$$\frac{R_A}{\sin(180-40)} = \frac{R_B}{\sin(180-15)} = \frac{100}{\sin(15+45)}$$

$R_A = 78.5N$

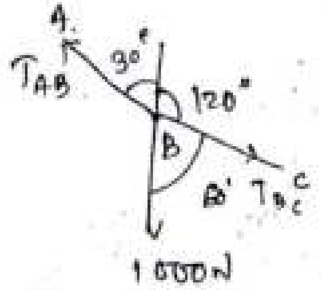
$R_B = 81.6N$

2) A string ABCD attached to fixed points A & D has two equal weights of 1000N attached to B & C. The weight rest with the portions AB & CD inclined angle as shown in fig.



Find the tension in AB, BC & CD

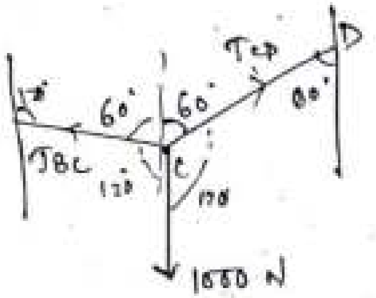
Sol^n Free body diagram.



$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin(180-30)} = \frac{1000}{\sin 150^\circ}$$

$\Rightarrow T_{AB} = 1732$ N

$\Rightarrow T_{BC} = 1000$ N

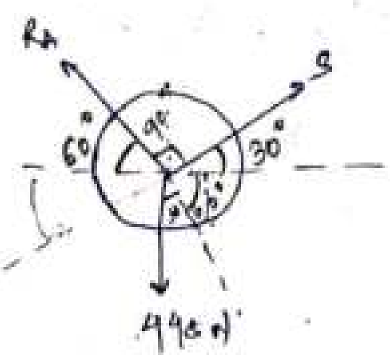


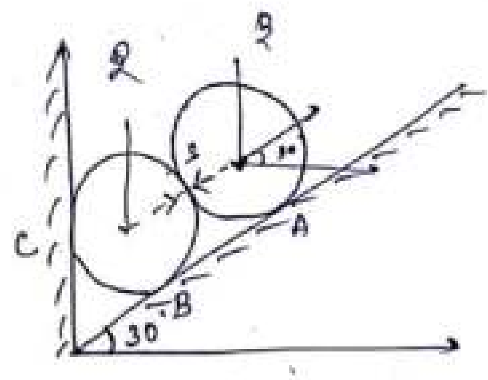
$$\frac{T_{BC}}{\sin 120^\circ} = \frac{T_{CD}}{\sin 120^\circ} = \frac{1000}{\sin 120^\circ}$$

$T_{CD} = 1000$ N Ans

Q) Two identical rollers each of weight $Q = 445$ N are supported by an inclined plane and a vertical wall as shown in the fig. Assuming smooth surface, Find the reactions induced at pt A, B, C

Sol^n



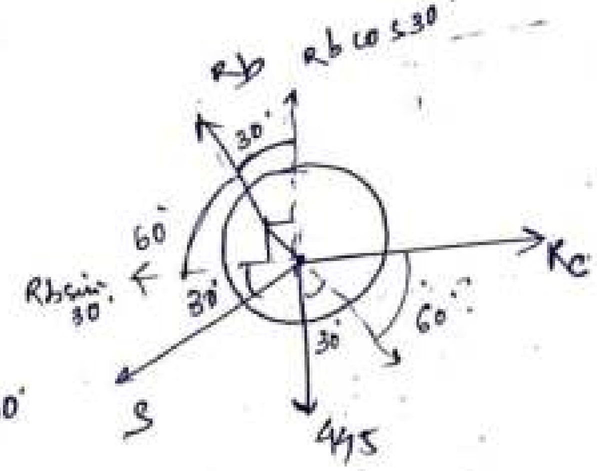


$$\frac{R_A}{\sin 120} = \frac{S}{\sin 150} = \frac{445}{\sin 90^\circ}$$

$\Rightarrow R_A = 385.38$ N $\quad S = 225.5$ N

Resolving vertically

$\Sigma y = 0$



$R_b \cos 30 = 445 + S \sin 30°$

$\Rightarrow \quad R_b = (\quad) N$

Resolving horizontally

$\Sigma x = 0$

$R_b \sin 30° + S \cos 30° = R_c$

$\Rightarrow \quad R_c = (\quad) N$

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CHAPTER → 03 ⟨ FRICTION ⟩

3.1 When a body slides or tends to slide over another surface an opposing force, called as force of friction. It acts tangent to the surface and opposite to the direction the body is moving or tends to move.

- Friction
  - Static
  - Dynamic
    - Sliding
    - Rolling

↳ **Static Friction**

It is experienced by a body when it is at rest or when the body is tend to move.

↳ **Sliding Friction**

It is experienced when a body slide over another body.

↳ **Rolling Friction**

It is experienced when a body rolls over another body.

Limiting Friction

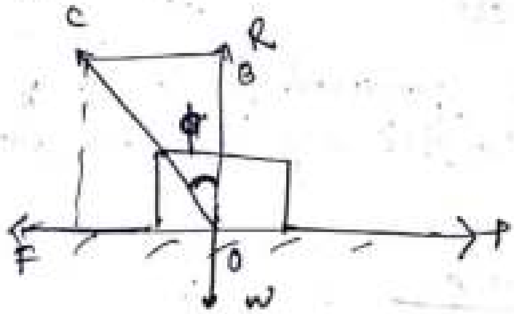
This is the maximum value of frictional force which comes in to play, when a body just begins to slide over another body, known as limiting friction.

If the applied force is less than the limiting friction, the body remains at rest & the friction is called static friction, which may have any value betⁿ zero to limiting friction.

Angle of friction

Angle of friction is the angle, which the resultant of force of limiting friction & normal reaction makes with the normal reaⁿ.

- Let mass m kept on horizontal. pulled by a force P. When the body is just about to slide a limiting (F) friction will act on the opposite side. R be the normal reaⁿ of wt. W.



Let OC is the resultant betⁿ R & F, makes an angle ϕ with R.

$\Delta OBC \quad \tan\phi = \frac{BC}{BO} = \frac{F}{R}$

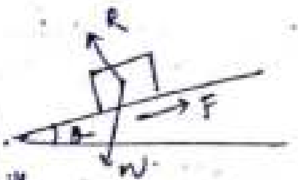
Coefficient of friction

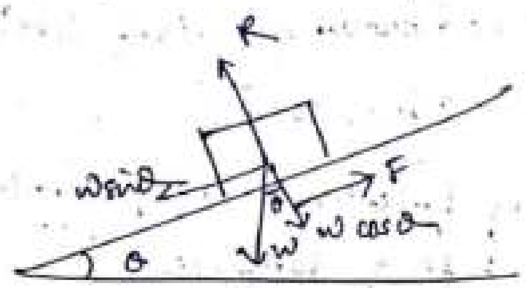
It is the ratio of friction to the normal reaction betⁿ 2 bodies denoted by μ

$$\mu = \frac{F}{R} = \tan\phi \rightarrow \boxed{F = \mu R}$$

Angle of repose

consider the block of weight W resting on an inclined plane which makes an angle θ with horizontal. When θ is very small the block will rest on the plane. If θ increases gradually; a stage is reached at which the block will starts to slide. That angle is called as angle of repose.





$\Sigma V = 0$

$R = W\cos\theta \longrightarrow (1)$

$\Sigma H = 0 \quad F = W\sin\theta \longrightarrow (2)$

$$\frac{\cancel{W}\sin\theta}{\cancel{W}\cos\theta} = \frac{F}{R}$$

$$\Rightarrow \boxed{\tan\theta = \frac{F}{R}}$$

$\therefore \tan\varphi = \tan\theta$

$\Rightarrow \varphi = \theta$

Angle of friction = Angle of repose.

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Laws of friction

↳ Laws of static friction

→ The force of friction always act opposite in the direc^n of applied force.

→ The magnitude of force of friction is exactly equal to the applied forces which tend to move the body.

→ The magnitude of the limiting friction bears a const ratio to normal reaction bet^n the two surface.

$$F/R = const.$$

→ The force of friction is independent of the area of contact bet^n 2 surface.

→ The force of friction depends upon the surface roughness.

→ Laws of Dynamic friction

→ The force of friction always act in a direction opposite in which the body is moving.

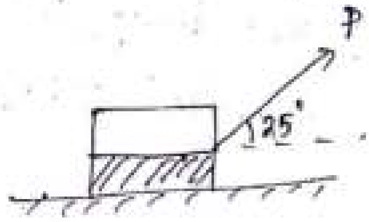
→ For moderate speed the force of friction remains const, but it decreases with increase of the speed.

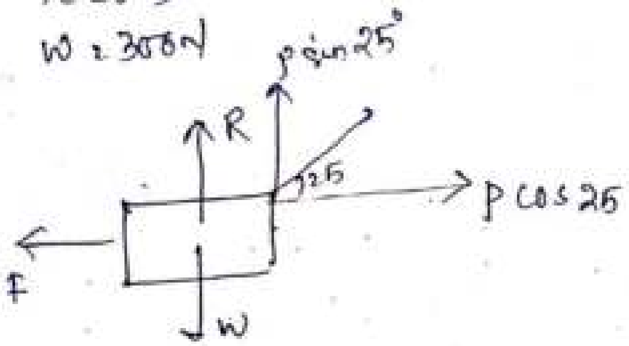
Q) A body of weight 300N is lying on a rough horizontal plane having a co-efficient of friction 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.

Soln

$\mu = 0.3$

$W = 300N$





$\Sigma H = 0 \Rightarrow P\cos 25° = F \Rightarrow F = 0.9063P$

$\Sigma V = 0 \Rightarrow R = W - P\sin 25°$

We know that $F = \mu R$

$\Rightarrow 0.9063P = \mu [W - P \times .4226]$

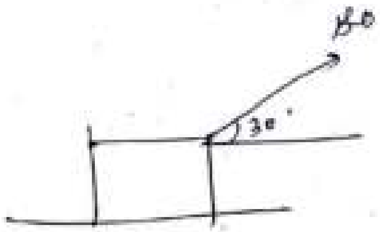
$\Rightarrow 0.9063P = 0.3[300 - .4226P]$

$\Rightarrow 0.9063P = 90 - .1268P$

$\Rightarrow P = 87.1\ N.$ Ans

21/15

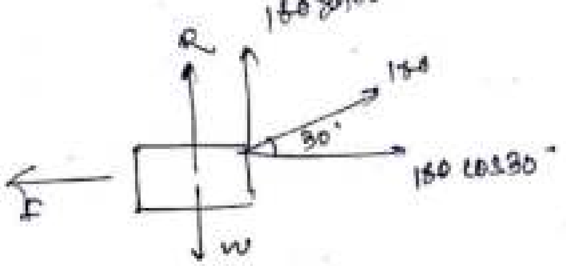
Q) A body resting on a rough horizontal plane required a pull of 180N inclined at 30°, to the plane to move it. It was found that that a push of 220N inclined at 30° to the plane just m[oved] the body. determine the weight of the body and the co-efficient of friction.

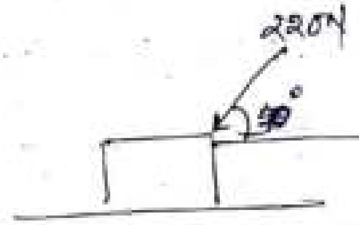


①

Soln

FBD of fig 1





$\Sigma H = 0$

$F_1 = 180 \cos 30°$ N

$\Sigma V = 0$

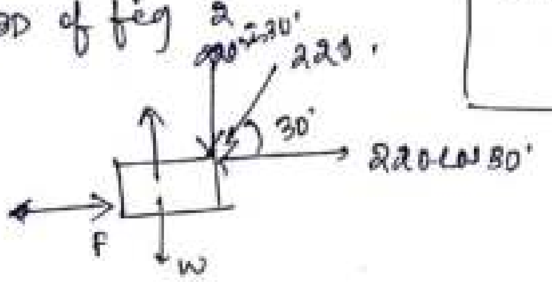
$R = W - 180 \sin 30°$

$\Rightarrow R = W - 90$

$F_1 = \mu R$

$\Rightarrow 155.88 = \mu(W - 90)$ —— ①

FBD of fig 2



$\Sigma H = 0$ $\quad F_2 = 220 \cos 30°$

$\Rightarrow F_2 = 190.52$ N

$\Sigma V = 0$

$R = W + 220 \sin 30°$

$\Rightarrow R = W + 110$

$F = \mu R$

$\Rightarrow 190.52 = \mu(W + 110) - ($

Adding equn ① & ②

Substracting

$$155.88 = \mu W - 90\mu$$
$$-\quad 190.52 = \mu W + 110\mu$$
$$(-) \qquad (-) \qquad (-)$$

$$+34.64 = +200\mu$$

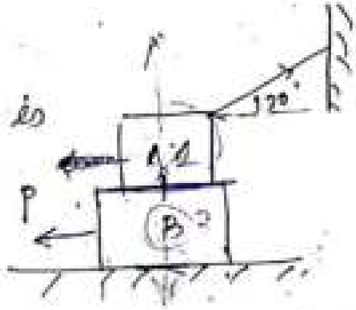
$$\Rightarrow \mu = 0.1732 \quad \underline{\text{Ans}}$$

puting value of μ in eqn (1)

we get $155.88 = 0.1732(W - 90)$

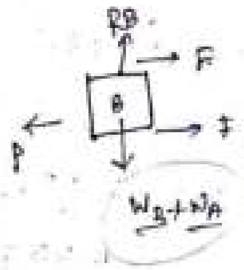
$$W = 991.68\,N$$

2) if co-efficient betn the 2 blocks is 0.3. Find force P req to move the block.

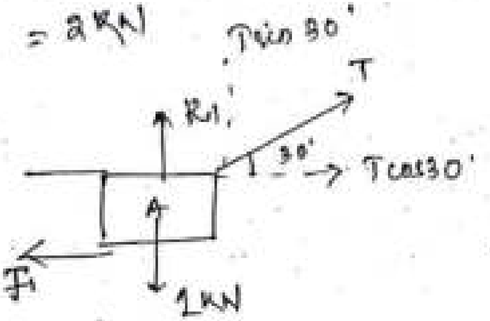


$W_A = 1\,kN$

$W_B = 2\,kN$



soln



(vertically)

$$R_1 + T\sin 30' = 1\,kN$$

$$\Rightarrow T\sin 30' = 1 - R_1 \quad \text{—— (1)}$$

Horizontally

$$T\cos 30° = F_1$$

$$\Rightarrow T\cos 30' = \mu R_1 \quad \text{—— (1)}$$

$$\Rightarrow T\cos 30' = 0.3 R_1$$

Dividing eqn (1) & (2)

$$\frac{T\sin 30'}{T\cos 30'} = \frac{1 - R_1}{0.3\,R_1} \Rightarrow \tan 30° = \frac{1 - R_1}{0.3 R_1}$$

$\Rightarrow 0.5774 = \frac{1-R_1}{0.3R_1}$

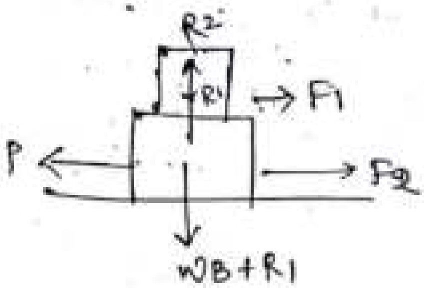
$\Rightarrow 0.5774 \times 0.3R_1 = 1-R_1$

$\Rightarrow 0.173R_1 = 1-R_1$

$\Rightarrow R_1 = 0.85 \text{ kN}$

$F = \mu R_1 = 0.3 \times 0.85$

$= .255 \text{ kN}$



$R_2 = 2 + R_1$

$= 0.85 + 2 = 2.85 \text{ kN}$

$F_2 = \mu R_2$

$= 0.3 \times 2.85 = .855 \text{ kN}$

$P = F_1 + F_2$

$= .255 + .855$

$= 1.11 \text{ kN}$

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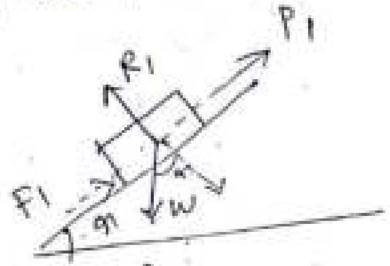
3.2 Equill^m of a body on Rough Inclined plane

Consider a body laying on a rough inclined plane subjected to force P. as shown in fig

1. Minimum force (P_1) which will keep the body in equill^m when it is sliding down word.



(moving downward)

$F_1 = \mu R_1$

Net horizontal force.

$P_1 = W\sin\alpha - F_1$

$\Rightarrow P_1 = W\sin\alpha - \mu R_1$ —①

Net vertical force.

$W\cos\alpha = R_1$ —②

putting value of R_1 in equ ① we get

$P_1 = W\sin\alpha - \mu(W\cos\alpha)$

$= W(\sin\alpha - \mu \times \cos\alpha)$

$= W(\sin\alpha - \tan\phi \times \cos\alpha)$ $(\because \mu = \tan\phi)$

$= W\left(\sin\alpha - \dfrac{\sin\phi}{\cos\phi} \times \cos\alpha\right)$ $\because \left(\tan\phi = \dfrac{\sin\phi}{\cos\phi}\right)$

$\Rightarrow P_1\cos\phi = W(\sin\alpha * \cos\phi - \sin\phi \times \cos\alpha)$

$\Rightarrow P_1\cos\phi = W\sin(\alpha - \phi)$

$\Rightarrow \boxed{P_1 = \dfrac{W\sin\alpha(\alpha - \phi)}{\cos\phi}}$

2. Minimum force (P_1) which will keep the body in equ^m when moving upward.

$P_1 = W\sin\alpha + F_1$ —①

$R_1 = W\cos\alpha$

$\boxed{P_1 = \dfrac{W\sin(\alpha + \phi)}{\cos\phi}}$

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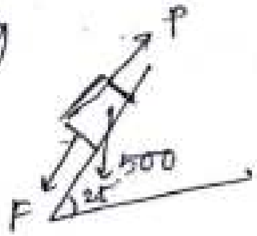
Q1) A body of wt 500 N is lying on a rough plane inclined at an angle of 25°. supported by horizontal force P as shown in fig. Determine P for both upward & downward motion.

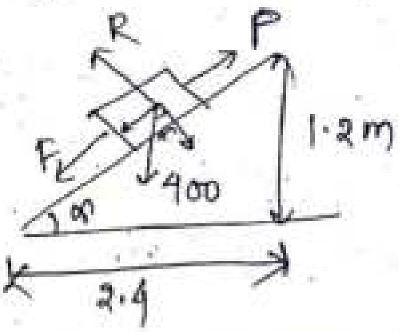


Sol^n

$$P_1 = \frac{W \sin(\alpha - \varphi)}{\cos \varphi} = 46.4 \text{ N}$$

$$P_2 = \frac{W \sin(\alpha + \varphi)}{\cos \varphi} = 376.2 \text{ N}$$

Q2) An inclined plane as shown in fig is used to unload a body of wt 400N. from a height 1.2 m. $\mu = 0.3$. (State weather it is necessary to push the body down the plane or hold it back from sliding down. What minim^m force is req. parallel for this purpose) Find (P) ←



Sol^n $\tan \alpha = \frac{1.2}{2.4} = 0.5$

$\alpha = 26.5°$

& normal reac^n

$R = W \cos \alpha$

$= 400 \times \cos 26.5°$

$= 357.9$ N

$F = \mu R$

$4 \sin \alpha + \mu R = P$

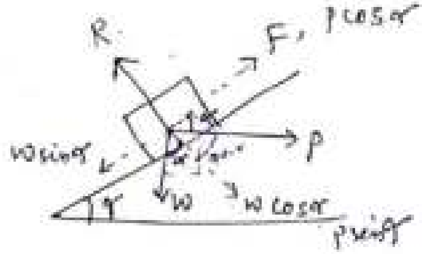
$\Rightarrow P = 400 \times \sin 26.5 + 0.3 \times 357.9$

$=$

Equilibrium of a body on a rough inclined plane subjected to a force acting horizontally

Consider a body lying on a rough inclined plane subjected to a force acting horizontally.

1. Minimum force (P_1) which will keep the body in equi^m, when it is at the point of sliding downward.



$F = \mu R$

$\Sigma H = 0$

$P\cos\alpha + F = W\sin\alpha$

$\Rightarrow P\cos\alpha = W\sin\alpha - F$

$\Rightarrow P\cos\alpha = W\sin\alpha - \mu R$ — ① ($\because F = \mu R$)

$\Sigma V = 0$

$R = W\cos\alpha + P\sin\alpha$ —— ②

putting the value of R in eq^n ①

$P_1\cos\alpha = W\sin\alpha - \mu(W\cos\alpha + P_1\sin\alpha)$

$\Rightarrow P_1\cos\alpha + \mu P_1\sin\alpha = W\sin\alpha - \mu W\cos\alpha$

$\Rightarrow P_1(\cos\alpha + \mu\sin\alpha) = W(\sin\alpha - \mu\cos\alpha)$

put $\mu = \tan\phi$

$$\Rightarrow P_1 = W\frac{(\sin\alpha - \mu\cos\alpha)}{\cos\alpha + \mu\sin\alpha}$$

$$= W\frac{(\sin\alpha - \tan\phi \cdot \cos\alpha)}{(\cos\alpha + \tan\phi \cdot \sin\alpha)}$$

$$= W\frac{\left(\sin\alpha - \frac{\sin\phi}{\cos\phi} \cdot \cos\alpha\right)}{\left(\cos\alpha + \frac{\sin\phi}{\cos\phi} \cdot \sin\alpha\right)}$$

$$= W\left(\frac{\sin\alpha \cdot \cos\phi - \sin\phi \cdot \cos\alpha}{(\cos\alpha \cdot \cos\phi + \sin\phi \cdot \sin\alpha)}\right)$$

$$P_1 = W\frac{\sin(\alpha-\phi)}{\cos(\alpha-\phi)}$$

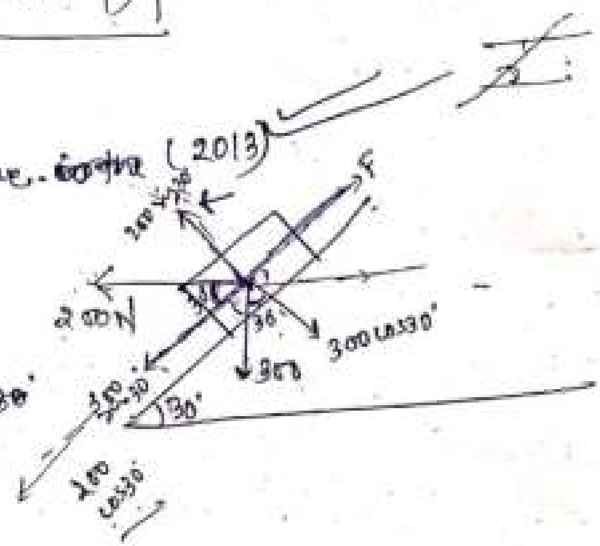
$$\Rightarrow \boxed{P_1 = W\tan(\alpha-\phi)}$$

ii) Maximum force (P_1), when the body is moving upward.

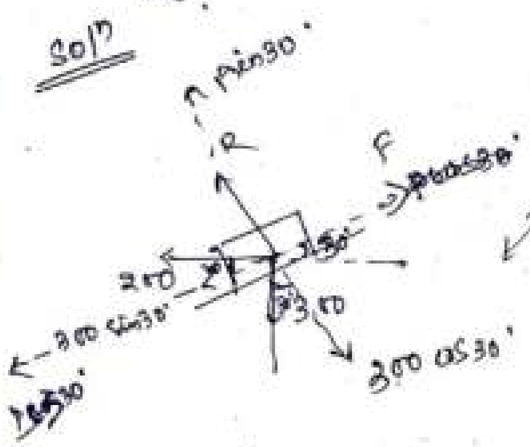
$$P_1 = W\frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)}$$

$$\Rightarrow \boxed{P_1 = W\tan(\alpha+\phi)}$$

Q) Find the total force. (2013)
μ?



Sol^n



$\Sigma V = 0$

$300\cos30° = R + 200\sin30°$

$\Rightarrow R = 300\cos30° - 200\sin30°$

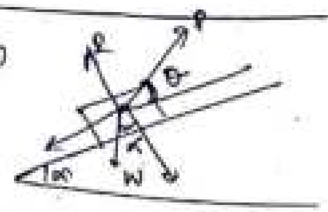
$= (\quad)$

$\Sigma H = 0$

$200\cos30° + 300\sin30° = F$

$\Rightarrow \mu R = 200\cos30° + 300\sin30°$

Minimum force (P_1), keep the body in equilm when sliding downward



$$P_1 = W\frac{\sin(\alpha+\phi)}{\cos(\theta+\phi)}$$

$$P_{min} = P_2 = \frac{W\sin(\alpha+\phi)}{\cos(\theta-\phi)}$$

$$P_1 = W\frac{\sin(\alpha-\phi)}{\cos(\alpha-\phi)}$$

$$\Rightarrow \boxed{P_1 = W\tan(\alpha-\phi)}$$

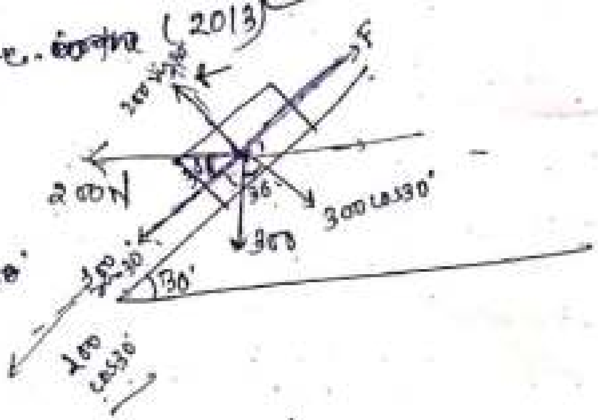
ii) Maximum Force force (P_1), when the body is moving upward.

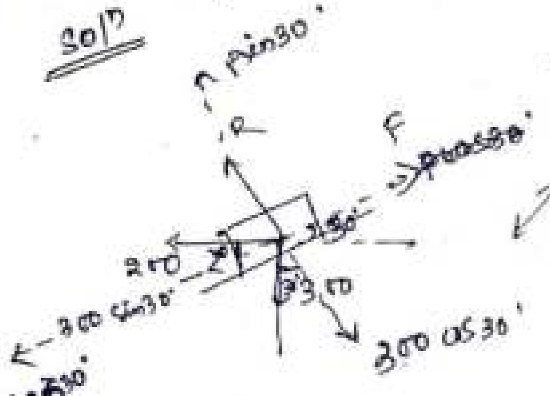
$$P_1 = W\frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)}$$

$$\Rightarrow \boxed{P_1 = W\tan(\alpha+\phi)}$$

2) Find the total force (2013) μ?

Soln





$\Sigma V = 0$

$300\cos 30° = R + 200\sin 30°$

$\Rightarrow R = 300\cos 30° - 200\sin 30°$

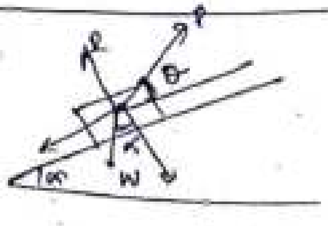
$= (\quad)$

$\Sigma H = 0$

$200\cos 30° + 300\sin 30° = F$

$\Rightarrow \mu R = 200\cos 30° + 300\sin 30°$

Minimum force (P_1), keep the body in equilm when sliding downward

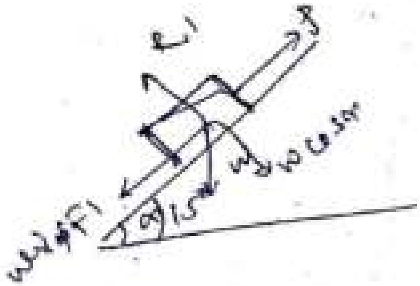


$$P_1 = W\frac{\sin(\alpha-\phi)}{\cos(\theta+\phi)}$$

$$P_{max} = P_2 = \frac{W\sin(\alpha+\phi)}{\cos(\theta-\phi)}$$

2) An effort of 200N is required just to move certain body up an inclined plane at an angle 15° the forces acting ∥ to plane. If angle of inclin^n is 20°, then the effort req is found to be 230N. Find weight of the body & μ.

Given: $P_1 = 200N$ $P_2 = 230N$
$\alpha = 15°$ $\alpha = 20°$



$\Sigma F_H = 0$

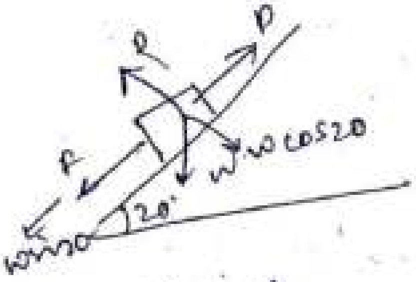
$F_1 + W\sin\alpha_1 = 200$

$\Rightarrow \mu R_1 + W\sin 15 = 200$

$\Rightarrow \mu W\cos\alpha + W\sin 15 = 200$

$\Rightarrow W(\mu\cos\alpha + \sin\alpha) = 200$ —(1)

$\Sigma F_V = 0$
$R_1 = W\cos\alpha$



$\Sigma F_H = 0$

$P = W\sin 20 + F$

$\Rightarrow \mu R + W\sin 20 = 230$

$\Rightarrow \mu W\cos 20 + W\sin 20 = 230$

$\Rightarrow W(\mu\cos 20 + \sin 20) = 230$ —(2)

$\Sigma F_V = 0$
$R_2 = W\cos 20$

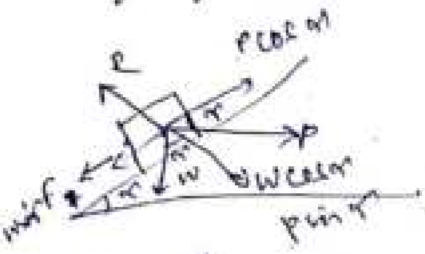
$$\frac{eq^n (2)}{eq^n (1)} = \frac{\mu\cos 20 + \sin 20}{\mu\cos 15 + \sin 15} = \frac{230}{200}$$

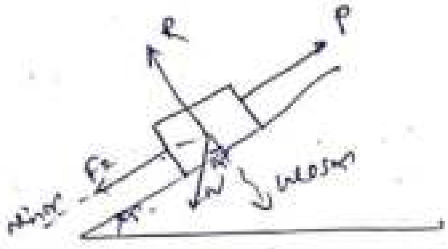
$\Rightarrow \mu = 0.259$

$eq^n (1) \rightarrow W(.259 \times \cos 15 + \sin 15) = 200$

$\Rightarrow W = 392\,N$ Ans

Q) A load of 1.5kN resting on an inclined rough plane, can be moved up the plane by a force of 2kN applied horizontally & by a force of 1.25 kN applied ∥ to the plane. Find angle of inclination & μ





$P = w\tan(\alpha+\varphi)$

$\Rightarrow 2 = 1.5\tan(\alpha+\varphi)$

$\Rightarrow \alpha+\varphi = 53.1^\circ$

$P = w\,\dfrac{\sin(\alpha+\varphi)}{\cos\varphi}$

$\Rightarrow 1.25 = 1.5\,\dfrac{\sin(53.1)}{\cos\varphi}$

$\Rightarrow \cos\varphi = .96$

$\Rightarrow \varphi = 16.3^\circ$

$\alpha = 53.1 - 16.3^\circ$

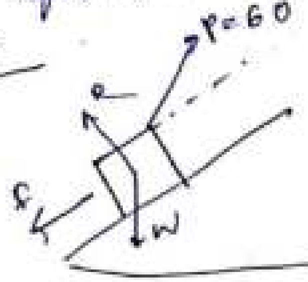
$= 36.8^\circ$

$\mu = \tan\varphi = \tan \times 16.3^\circ$

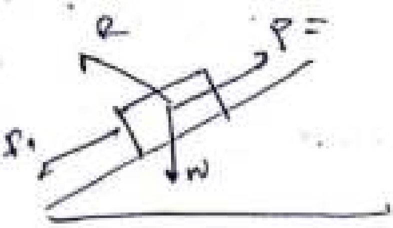
$= .292$

Q) Find the force req. to move a load 300N upon rough plane the force being ∥ to the plane. The inclination of the plane is such that when the same load is kept on a perfectly smooth plane inclined at same angle, a force 60N applied at an inclination of 30° to the plane, keep the same load in equilm. $\mu = 0.3$.

Sol)



Smooth



Rough.

For smooth $\mu = 0 \therefore \varphi = 0$

$P = W\,\dfrac{\sin(\alpha+\varphi)}{\cos(\theta-\varphi)} \Rightarrow 60 = \dfrac{300\sin\alpha}{\cos 30^\circ} \Rightarrow \alpha = 10^\circ$

For Rough

$P = W\,\dfrac{\sin(\alpha+\varphi)}{\cos\varphi} \Rightarrow P = 140.7N$

$\mu = 0.3$

$\tan\varphi = 0.3$

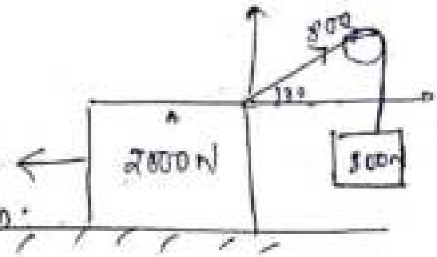
$\varphi = \tan^{-1}0.3$

$= 16.7$

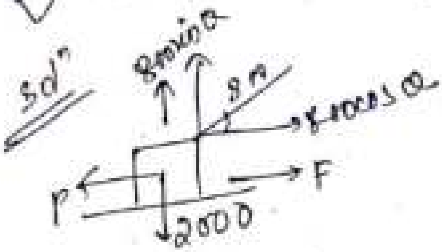
Q) $\mu = 0.35$

2014 Determine value of P.

Consider the pulley is frictionless.



Sol^n



$P = F + 800 \cos 30° \Rightarrow P = \mu R_n + 800 \cos 30°$

$2000 = R_n + 800 \sin 30°$

$\Rightarrow R_n = 2000 - 800 \sin 30°$

$\Rightarrow$ putting value of R_n.

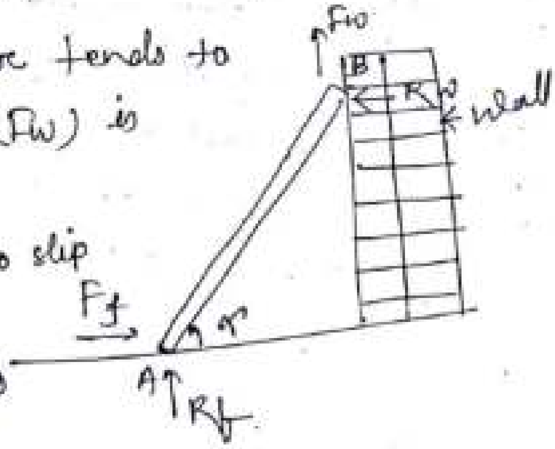
$P = \mu \times (2000 - 800 \sin 30°) + 800 \cos 30°$

$= (1252.82)$ ✓

Application of friction

3.3 LADER FRICTION

A ladder is a device for climbing on walls.



- As upper end of the ladder tends to slip downward, friction (Fw) is upward.

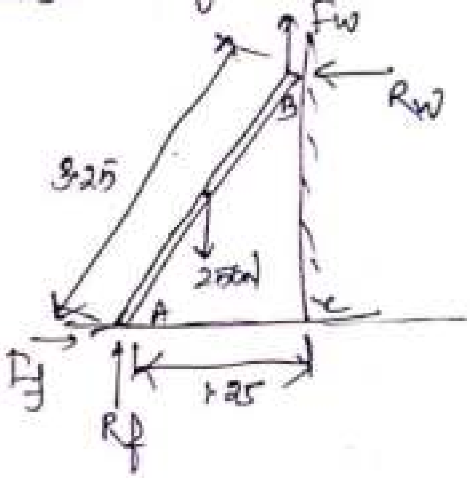
→ As the lower end tries to slip away from wall i.e. friction (Ff) is towards the wall.

- Since the system is in equil^m, therefore the algebraic sum of horizontal & vertical components of the forces must also be equal to zero.

2014 9) A uniform ladder of length 3.25m and weighing 250 N placed against a smooth vertical wall. It's lower end 1.15 m from the wall. The co-efficient of friction betn ladder & floor is 0.3. Determine frictional force acting on ladder at point of contact betn ladder & floor.

Soln $\Sigma V = 0$

$R_f = 250\,N$



from geometry

$BC^2 = \sqrt{AB^2 - AC^2}$

$= 30m$

Taking moment about 0.

$R_f \times 1.25 - 250 \times \left(\frac{1.25}{2}\right) = F_f \times 3$

$\Rightarrow R_f = 521.\,N$

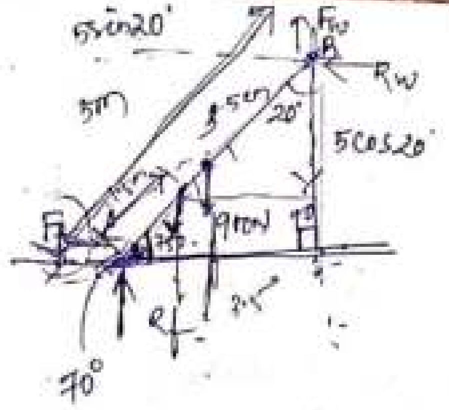
2015 2) A ladder 5 meter long rest on a horizontal ground and leans against a smooth vertical wall at an angle 70° with horizontal. The weight of ladder is 900N and acts at it's middle. The ladder is at the point of sliding, when a man weighing 750N stands on the ladder 1.5m from bottom. calulate μ.

Sol $l = 5m$

$\alpha = 70^\circ$

$W_1 = 900N$

$W_2 = 750N$



$Rf = 900 + 750 = 1650N$

$Ff = \mu_f \times Rf = \mu_f \times 1650 N$

Taking moment about B

$R_f \times 5\cos 70 - 900 \times 2.5 \cos 70 - 750 \times 3.5 \cos 70 = F_f \times 5 \sin 70$

$R_f \times 5 \sin 20^\circ - 900 \times 2.5 \sin 20 - 750 \times 3.5 \sin 20^\circ = F_f \times 5 \cos 20^\circ$

put the value of F_f

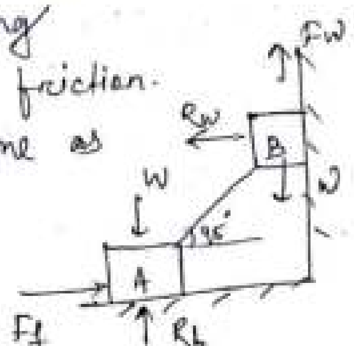
$R_f \times 5 \sin 20 - 900 \times 2.5 \sin 20 - 750 \times 3.5 \sin 20 = \mu_f \times 1650 \times 5 \cos 20^\circ$

$\Rightarrow 1650 \times 5 \sin 20^\circ = (\mu_f \times 1650 \times 5 \cos 20^\circ) + 975$

$= 4533 \mu_f + 975$

$\Rightarrow \mu_f = 0.15$ Ans

2) Two identical blocks of weight W are supported by a rod inclined at 45° with horizontal, as shown in fig. If both the blocks are limiting equilibrium, Find the coefficient of friction (μ). assuming it to be same as floor as well as at wall.



Sol^n Resolving forces vertically.

$$F_w + R_f = 2W$$

$$\Rightarrow \mu R_w + R_f = 2W \quad \text{——} (1)$$

Now resolving the forces horizontally.

$$R_w = F_f$$

$$\Rightarrow R_w = \mu R_f \quad \text{——} (2)$$

Substituting R_w in equ^n (1).

$$\mu(\mu R_f) + R_f = 2W$$

$$\Rightarrow \mu^2 R_f + R_f = 2W$$

$$\Rightarrow R_f = \frac{2W}{(1+\mu^2)} \quad \text{——} (3)$$

putting value of R_f in equ^n (2)

$$R_w = \mu \times \frac{2W}{\mu^2+1}$$

Taking moment of the forces about block A

$$R_w \times l \cos 45^\circ + F_w \times l \cos 45^\circ = W \times l \cos 45^\circ.$$

$$R_w + F_w = W$$

$$\Rightarrow R_w + \mu R_w = W$$

$$\Rightarrow R_w (1+\mu) = W$$

putting value of R_w $\frac{\mu \times 2W}{\mu^2+1}(1+\mu) = W$

$$\Rightarrow 2\mu(1+\mu) = \mu^2+1$$

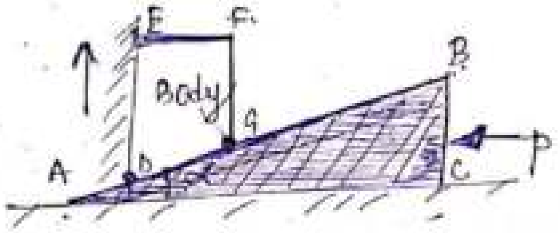
$$\Rightarrow 2\mu + 2\mu^2 = \mu^2+1$$

$$\Rightarrow \mu^2 + 2\mu - 1 = 0$$

$$\mu = \frac{-2 \pm \sqrt{(2)^2+4}}{2} = 0.414 \ \underline{\text{Ans}}$$

WEDGE FRICTION

A wedge is usually, of a triangular in cross-section & is, generally, used for slight adjustments in the position of a body i.e for tightening fits or keys for shafts. Sometimes, a wedge is also used for lifting heavy weight. It is made of up wood or metal.



Wedge ABC, used to lift the body DEFG.

↑ effect
← P (effort)

W = weight of the body DEFG

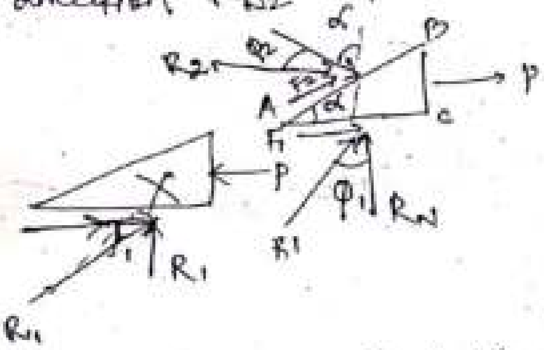
P = Force req. to lift the body

μ = co-efficient of friction = $\tan\varphi$

W_{wedge} → Not considered.

due to horizontal movement we get vertical.

When force P is applied in, the body will lift in upward direction

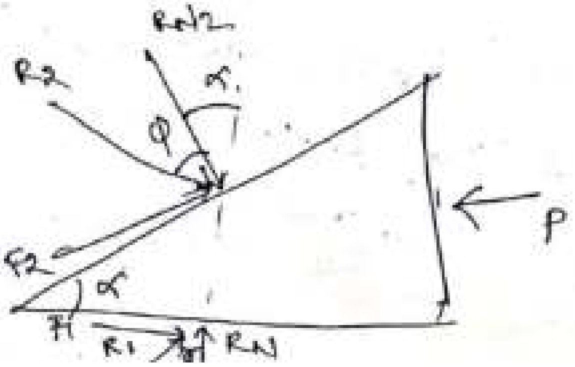


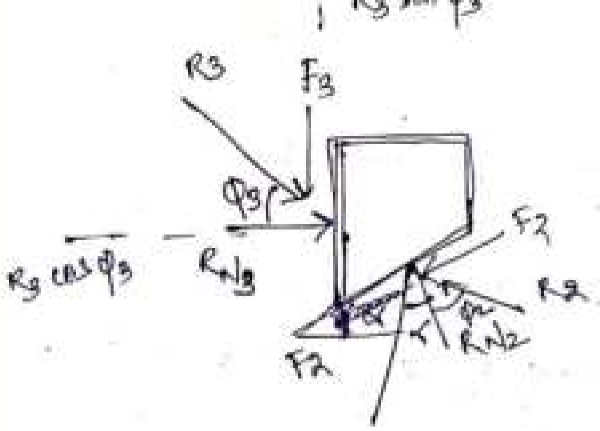
R_1 → resultant of frictional force & normal reacⁿ betⁿ floor & wedge. F_1 & R_{N1}

φ_1 & φ_2 → angle of friction.

R_{N2} → normal reacⁿ at AG, & frictional force F_2.

The resultant of both is R_2. making an angle φ_2.

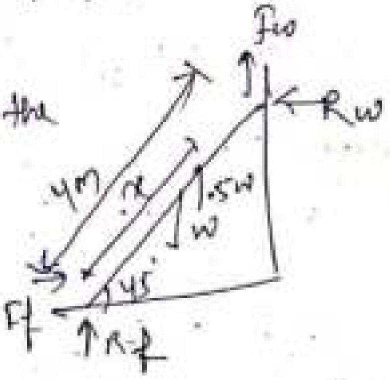




2) A uniform ladder of 4m length rests against a vertical wall with which it makes an angle of 45°. The co-effi of friction betn ladder & wall 0.4 & that betn ladder & floor 0.5. If a man whose weight is one-half of that ladder ascends it, how high it will be when the ladder slips?

Soln $x \to$ distance betn A & the man.

weight of man $= \frac{W}{2}$
$= .5W$



$F_f = \mu_f R_f = 0.5 R_f$

$F_w = \mu_w R_w = 0.4 R_w$

$R_w = F_f = 0.5 R_f$

$R_f = 2R_w$

Resolving vertically $R_f + F_w = W + 0.5W$

$\Rightarrow 2R_w + 0.4 R_w = 1.5W$

$\Rightarrow R_w = \frac{1.5W}{2.4} = 0.625W$

Scanned by CamScanner

$F_W = .4 \times .625W$
$= 0.25W$

Taking moment about A.

$(W \times 2\cos 45^\circ + .5W \times x\cos 45^\circ)$
$= R_W \times 4\sin 45^\circ + F_W \times 4\cos 45^\circ$

put value of R_W & F_W

$x = 3.0\,m$.

CHAPTER → 04 Centre of Gravity

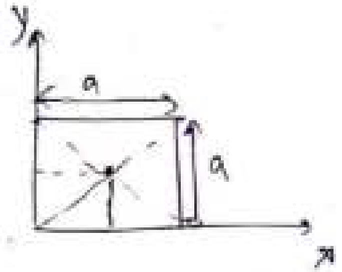
Centre of gravity can be defined as a point through which the whole weight of the body acts, irrespect of it's position. It may be noted that every body has one and only one centre of gravity.

4.1 Centroid

The plane figures like triangle, rectangle, circle etc have only area, but no mass. The centre of area of such fig is known as centroid.

Centroid of basic geometrical figures

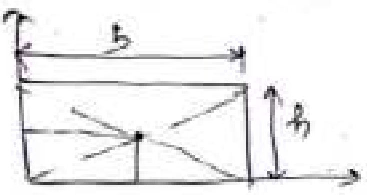
1)



$\bar{x} = a/2$

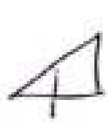
$\bar{y} = a/2$

2)



$\bar{x} = b/2$

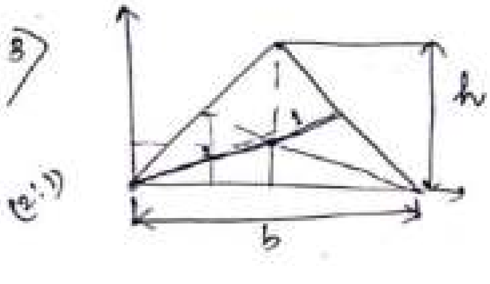
$\bar{y} = h/2$



$x = \frac{2b}{3}$

$y = h/3$

3)



$\bar{x} = b/2$

$\bar{y} = h/3$

median devided into 2:1 ratio.

draw a line FQ

BFQ ≅ DABD.

$\frac{AQ}{D} = \frac{AF}{AC}$

DQ BF ≅ D DBG

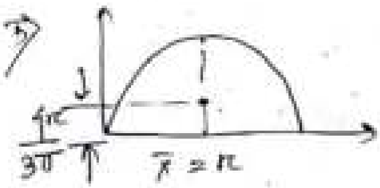
$FG/GB = \frac{QD}{DB} = \frac{1}{2}$

$QD:DB = 1:2$

1) 

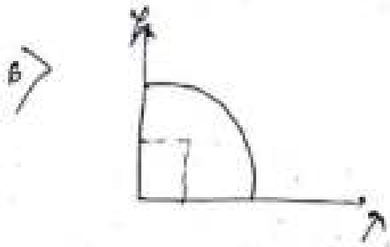
$\bar{x} = r$

$\bar{y} = r$

2) 

$\bar{x} = r$

$\bar{y} = 4r/3\pi$

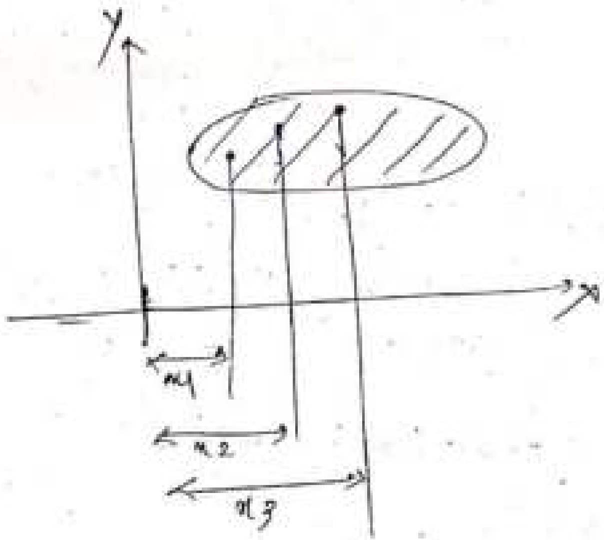
3) 

$\bar{x} = 4r/3\pi$

$\bar{y} = 4r/3\pi$

Where $\bar{x}$ & $\bar{y}$ is the co-ordinates of centroid given

<u>Centre of gravity by Moments</u>



Consider a body of mass M whose centre of gravity is required to be found out. Let it is divided into small masses $m_1, m_2, m_3 \ldots$ & the co-ordinates are (x_1, y_1) (x_2, y_2) & (x_3, y_3)

$$M\bar{x} = m_1x_1 + m_2x_2 + m_3x_3 \ldots$$

$$\bar{x} = \frac{\Sigma mx}{M}$$

$$\bar{y} = \frac{\Sigma my}{M}$$

$$M = m_1 + m_2 + m_3 + \cdots$$

Axis of Reference

The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference, called as axis of reference. from where $\bar{x}$ & $\bar{y}$ is calculated.

Centre of gravity of plane figure

The plane geometrical sections such as T, I, L Sections only have area but no mass. For these the centroid & centre of gravity is same.

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}$$

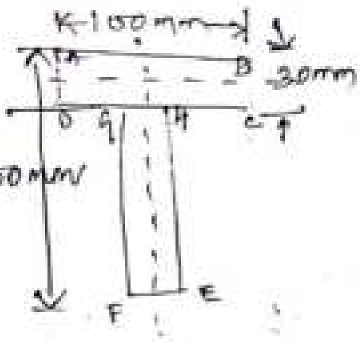
$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}$$

Centre of gravity of Symmetrical Sections

- If the given section is symmetrical about x-x axis then we have to find $\bar{x}$.

- If it is symmetrical to y-y axis then we have to find $\bar{x}$ & $\bar{y}$.

2) Find the centre of gravity of 100mm x 150mm x 30mm of T-section.



Sol^n: This section of is symmetrical about Y-Y axis.

Split the section in 2 section.

ABCD ; EFGH

For rectangle ABCD.

$a_1 = 100 \times 30 = 3000 mm^2$

$y_1 = (150 - \frac{30}{2}) = 135 mm$

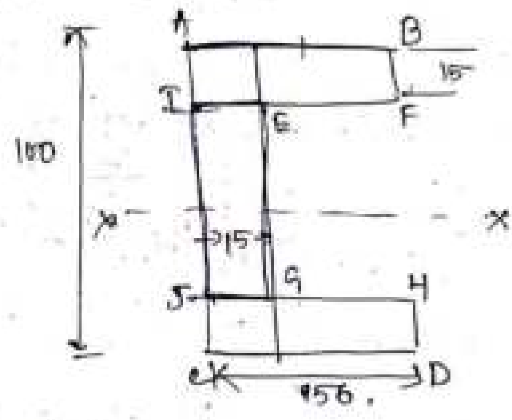
rectangle EFGH $a_2 = (150-30) \times 30 = 120 \times 30$
$= 3600 mm^2$

$y_2 = 120/2 = 60 mm.$

$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2} = \frac{3000 \times 135 + 3600 \times 60}{3000+3600}$

$= 94.1 mm.$

3) Symmetrical about X-X axis.



1) Rectangle ABIF.

$a_1 = 15 \times 50 = 750 mm^2$

$x_1 = 50/2 = 25 mm$

2) Rectangle CDHJ

$a_2 = 50 \times 15 = 750 mm^2$

$x_2 = 50/2 = 25 mm.$

3) Rectangle IEJG

$a_3 = \cancel{(50-30)}\ 15 \times (100-30)$

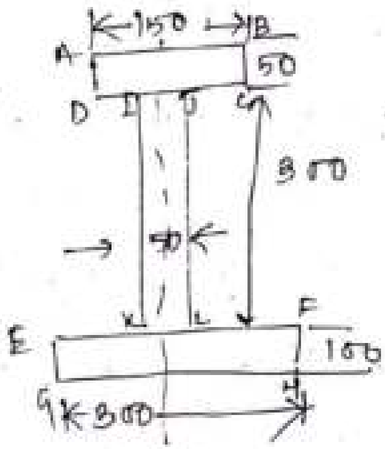
$= 1050 mm^2$

$x_3 = 15/2 = 7.5 mm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{750 \times 25 + 750 \times 25 + (1050 \times 7.5)}{750 + 1050 + 750}$$

$$= 17.8mm$$

Q) 

$a_1 = 150 \times 50$

$y_1 = 100 + 300 + \frac{50}{2}$

$= 400 + 25 = 425mm$

$a_2 = 300 \times 100$

$y_2 = 100/2 = 50mm$

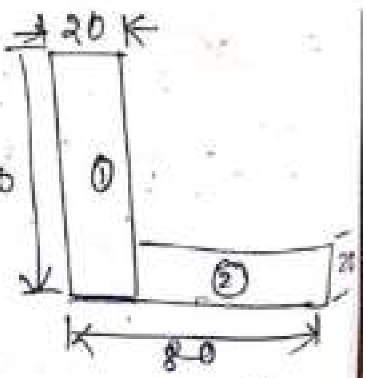
$a_3 = 300 \times 50$

$y_3 = 100 + \frac{300}{2} = 250mm$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

Center of gravity of unsymmetrical section

Q) Find C.G of the given L section



Rectangle ①

1 $a_1 = 20 \times 100 = 2000mm^2$

$y_1 = 100/2 = 50mm$

$x_1 = 20/2 = 10mm$

Rectangle ② $a_2 = \cancel{80} \overset{(80-20)}{} \times 20 = 1200mm^2$

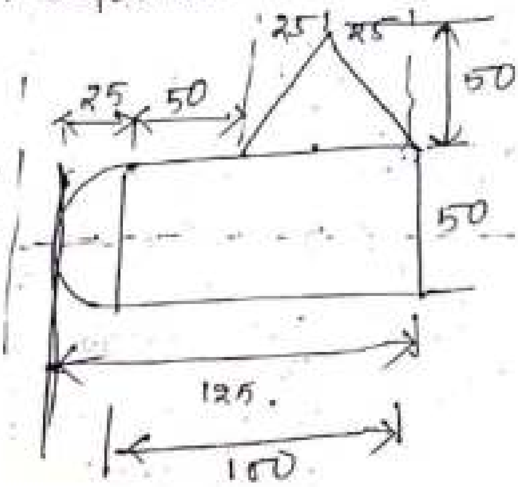
$y_2 = 20/2 = 10mm$ $x_2 = 20 + \frac{(80-20)}{2}$

$= 50 mm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = 25mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = 35mm$$

2) A uniform lamina is shown in fig. Determine the C.G of the lamina.



a) for the rectangle:

$$a_1 = 100 \times 50 = 5000 \, mm^2$$

$$x_1 = 25 + 100/2 = 75mm$$

$$y_1 = 50/2 = 25mm$$

for semicircle:

$$a_2 = \frac{\pi r^2}{2} = \pi/2 \, (25)^2 = 982mm^2$$

$$x_2 = 25 - 4r/3\pi = 14.4 \, mm$$

$$y_2 = 50/2 = 25mm$$

for Δ.

$$a_3 = \frac{1}{2} \times b \times h = \frac{1}{2} \, 50 \times 50 = 1250mm^2$$

$$x_3 = 25 + 50 + 25 = 100mm$$

$$y_3 = 50 + 50/3 = 66.7mm$$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = 71.1 \, mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = 32.2mm$$

4.2 MOMENT OF INERTIA

Moment of force = F × ⊥ distance. (1st moment of force.

of F × ⊥ distance × ⊥ distance (2nd moment of force) or moment, moment of force)

M.M.O.F / Second moment of force.

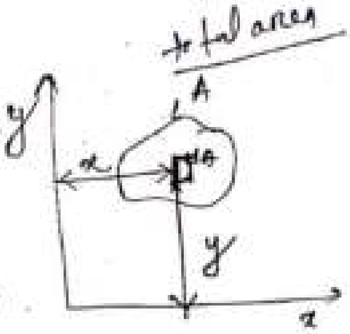
Sometime Area & mass can be found out by above methode.

→ also known as Moment of inertia.

$\begin{cases} M.M.O.A \\ M.M.O.M \end{cases}$

$I_{yy} = \Sigma dA \cdot x$ (M.I about yy)

$= \Sigma dA \cdot x \cdot x$



$\boxed{I_{yy} = \Sigma dA \cdot x^2}$ — MI about yy axis

$\boxed{I_{yy} = \int dA \cdot x^2}$

$\boxed{I_{xx} = \int dA \cdot y^2}$ — M.I about xx axis

$\boxed{\text{Moment of inertia} = \text{Force} \times (\text{perpendicular distance})^2}$

$\boxed{\text{unit} = N\,m^2}$

Moment of inertia of a rectangular section.

consider a rectangular section ABCD.

b → width of the section

d → depth of the section

Consider a small strip PQ of thickness dy ∥ to X-X axis at a distance y from the centre axis.

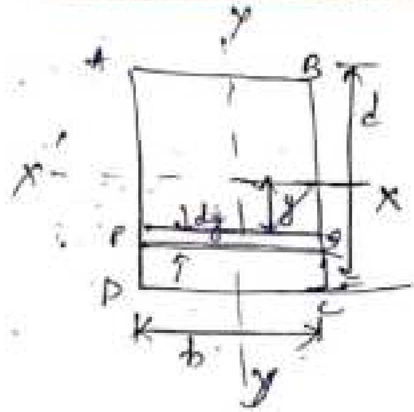
Area of small strip = $dA = b \times dy$

M.O.I of strip about $x-x$ axis

$= \text{Area} \times y^2$

$= dA \cdot y^2$

$= b \times dy \cdot y^2$



$$I_{x-x} = \int_{-d/2}^{d/2} dA \cdot y^2$$

$$= \int_{-d/2}^{d/2} b \cdot dy \cdot y^2$$

$$= b \int_{-d/2}^{d/2} y^2 \cdot dy = b \left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= b \left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right]$$

$$= b \left[\frac{d^3/8 - (-d^3/8)}{3}\right]$$

$$= b \left[\frac{\cancel{2}d^3}{\cancel{8}4 \times 3}\right]$$

$$I_{xx} = bd^3/12.$$

for hollow 

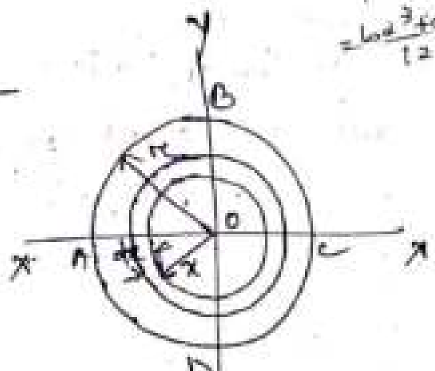
$$I_{xx} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$$

$$= db^3/12 - d_1 b_1^3/12$$

IIIy similarly $I_{yy} = \dfrac{db^3}{12}$.

$I_p = \dfrac{bd^3}{12} + \dfrac{db^3}{12} = \dfrac{bd^3 + db^3}{12}$

M.I of a circular section



- Consider a circle ABCD with centre O.
- Consider a ring of radius x and thickness dx.

area of the ring $da = 2\pi x \cdot dx$

M.O.I about xx axis = area × distance^2

or yy axis $= 2\pi x \cdot dx \times x^2$

$= 2\pi x^3 \, dx.$

Now M.I about the central axis let it be I_{xx}.

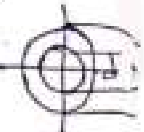
$I_{zz} = \int_0^r 2\pi r^3 . \, dx = 2\pi \int_0^r x^3 \, dx$

$= 2\pi \left[\frac{x^4}{4}\right]_0^r$

$= \frac{\pi}{2} \times r^4 = \frac{\pi}{32} d^4 \quad (r = d/2)$

$\therefore I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{\pi}{64} d^4$

for hollow



$I_{xx} = \frac{\pi}{64}(D^4 - d^4) = I_{yy}$

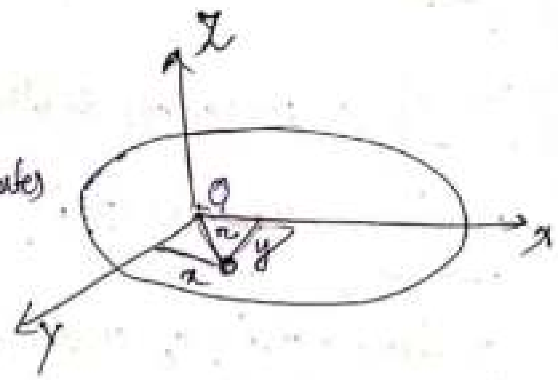
Theorem of perpendicular Axis

It states that if I_{xx} & I_{yy} be the moment of inertia of a plane section about 2 perpendicular axis meeting at O, The moment of inertia I_{zz} about the zz axis perpendicular to the plane and passing through intersection of x-x & y-y is given by

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Proof

consider a lamina (p) of area da having coordinates x & y along ox & oy axis as shown in fig.



consider a plane oz ⊥ to ox & oy. Let r be the distance of lamina p from zz axis. op = r

from geometry $r^2 = x^2 + y^2$

M.I about XX $\quad I_{xx} = da \cdot y^2$

YY $\quad I_{yy} = da \cdot x^2$

$I_{zz} = da \cdot r^2$

$= da(x^2 + y^2)$

$= da\, x^2 + da \cdot y^2$

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Theorem of parallel axis

It states that if the M.I of a plane area about an axis through it's centre of gravity is denoted by I_G, then moment of inertia of the area about any other axis AB, parallel to the 1st, and at a distance h from the C.G is given by

$$\boxed{I_{AB} = I_G + ah^2}$$

$I_{AB} \rightarrow$ M.I of the area about axis AB.

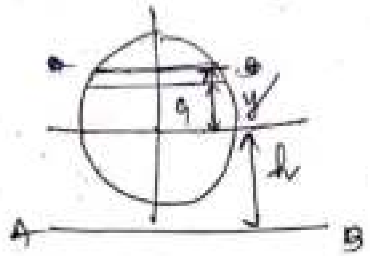
$I_G \rightarrow$ M.I . . . about C.G

$a \rightarrow$ area of section

$h \rightarrow$ distance betn C.G & secn AB.

proof

consider a strip of a circle, where M.I required to be found out



let δa = area of strip

y = distance of strip from. C.G.

h = distance of C.G from axis AB

M.I of whole section about an axis passing through

C.G $= \delta a \cdot y^2$

$I_G = \Sigma \delta a \cdot y^2$ M.I of whole secn passing through C.G.

M.I of section about AB

$$I_{AB} = \sum \delta a (h+y)^2$$

$$= \sum \delta a (h^2+y^2+2hy)$$

$$= \left(\sum h^2 \delta a\right) + \left(\sum y^2 . \delta a\right) + \left(\sum \cancel{2hy}\, \delta a\right)^{0}$$

$$\boxed{I_{AB} = ah^2 + I_G}$$

$\sum h^2 \delta a = ah^2$ sum of moments

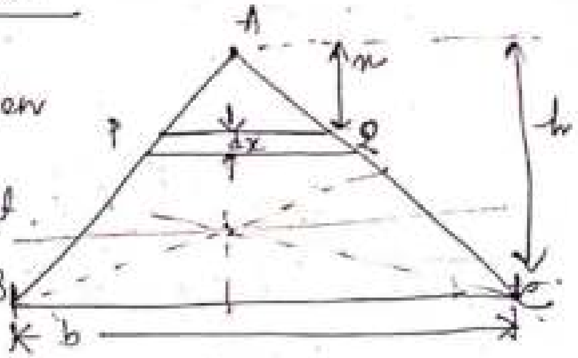
$\sum y^2 \delta a = I_G$

M.I of a triangular section

Consider a triangular section ABC whose ~~and~~ M.I is required to be found out.

b → base

h → height



(BC = base = b)

Consider a small secⁿ PQ of thickness dx at x distance from vertex A.

for △APQ, △ABC

$$\frac{PQ}{BC} = \frac{x}{h}$$

$$\Rightarrow PQ = \frac{BC.x}{h} = \frac{b.x}{h}$$

Small area of ~~da~~ PQ $= \frac{b.x}{h} \times dx$

M.I of strip about BC $= \text{Area} \times (\text{distance})^2$

$$= \frac{bx}{h}.dx \times (h-x)^2$$

$$= \frac{bx}{h}(h-x)^2.\, dx$$

M.I of whole ~~section~~ △ can be found out by integrating the above from 0 to h

$$I_{BC} = \int_0^h \frac{bx}{h}(h-x)^2\,dx$$

$$= \frac{b}{h}\int_0^h x\cdot(h^2+x^2-2hx)\,dx$$

$$= \frac{b}{h}\int_0^h (xh^2+x^3-2hx^2)\,dx$$

$$= \frac{b}{h}\left[\frac{x^2h^2}{2}+\frac{x^4}{4}-\frac{2hx^3}{3}\right]_0^h$$

$$= \frac{b}{h}\left[\frac{h^4}{2}+\frac{h^4}{4}-\frac{2h^4}{3}\right] = \frac{b}{h}\left[\frac{2h^4+h^4}{4}-\frac{2h^4}{3}\right]$$

$$= \cancel{\frac{b}{h}\left[\frac{h^4}{2}-\frac{2h^4}{3}\right]}$$

$$= \cancel{\frac{b}{h}\left[\frac{3h^4-2h^4}{3}\right] = \frac{b}{h}\left[\frac{h^4}{3}\right] = \frac{bh^3}{3}}$$

$$= \frac{b}{h}\left[\frac{3h^4}{4}-\frac{2h^4}{3}\right] = \frac{b}{h}\left[\frac{9h^4-8h^4}{12}\right] = \frac{bh^3}{12}$$

M.I. of triangular section through axis of the centre of gravity, parallel to x-axis

$$I_G = I_{BC} + ad^2 \qquad d = h/3$$

$$= \frac{bh^3}{12} - \frac{bh}{2}\times\left(\frac{h}{3}\right)^2$$

$$I_{BC} = I_G + ah^2$$

$$\boxed{I_G = \frac{bh^3}{36}}$$

Moment of Inertia of a composite Section.

Steps

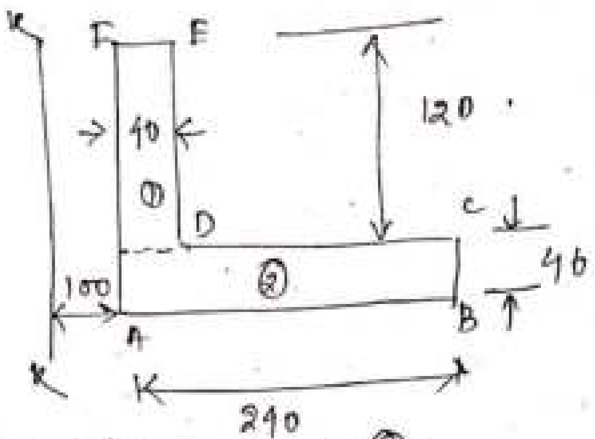
↳ 1st split up the given section into plane areas.

↳ Find M.I of these areas about their respective C.G.

↳ Apply parallel axis theorem.

↳ Obtain the M.I.

(2) Find M.I about axis KK



Split up the secⁿ into ① & ②.

for secⁿ ①. I_{G1} = M.I about c.G about the axis K-K.

$$I_{G1} = \frac{db^3}{12} = \frac{120 \times 40^3}{12} = 640 \times 10^3 \ mm^4$$

$$h_1 = 100 + \frac{40}{2} = 120 \ mm$$ (distance betⁿ c.G of secⁿ ① & axis K-K)

M.I of secⁿ ① axis K-K.

$$I_{KK} = I_{G1} + a_1 h_1^2$$
$$= [(640 \times 10^3) + (120 \times 40) \times (120)^2]$$
$$= 69.76 \times 10^6 \ mm^4$$

Similarly M.I of section ② above it's c.G & parallel to axis K-K.

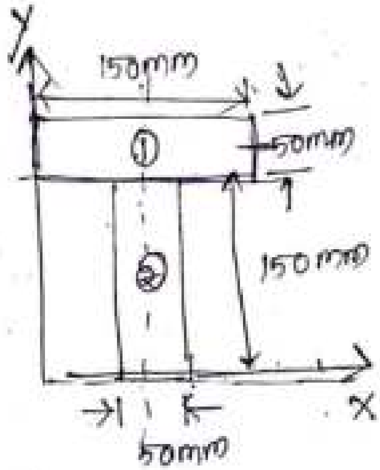
$$I_{G2} = \frac{db^3}{12} = 46.08 \times 10^6 \ mm^4$$

$$h_2 = 100 + \frac{240}{2} = 220 \ mm$$

$$I_{G2} + a_2 h_2^2$$
$$= [(46.08 \times 10^6) + (240 \times 40) \times (220)^2]$$
$$= 510.72 \times 10^6 \ mm^4$$

$$I_{KK} = 69.76 \times 10^6 + 510.72 \times 10^6$$
$$= 580.48 \times 10^6 \ mm^4$$

Q) Find the M.I of a T-section with as 150 mm × 50 mm and web 150 mm × 50 mm about x-x & y-y axis through the centre of gravity of the section.



Soln Rectangle ①

$a_1 = 150 \times 50 = 7500\,mm^2$

$y_1 = 150 + \frac{50}{2} = 175mm$

Rectangle ②

$a_2 = 150 \times 50 = 7500\,mm^2$

$y_2 = \frac{150}{2} = 75\,mm$

$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500+7500} = 125mm$

M.I of ① about X-X axis

$I_{G1} = \frac{db^3}{12} = \frac{150 \times 50^3}{12} = 1.5625 \times 10^6\,mm^4$

$h_1 = \overbrace{150 + \frac{50}{2}}^{y_1} - \underbrace{125}_{\bar{y}} = 50mm$

y → distance from C.G.

M.I about x-x axis $I_{G1} + a_1h_1^2$

$= 1.5625 \times 10^6 + 7500 \times (50)^2$

$= 20.3125 \times 10^6\,mm^4$

Similarly M.I of ② about X-X axis

$I_{G2} = \frac{bd^3}{12} = \frac{50 \times (150)^3}{12} = 14.06 \times 10^6\,mm^4$

$h_2 = 125 - \frac{150}{2} = 50\,mm$

M.I about XX axis $I_{G2} + a_2h_2^2$

$= 14.06 \times 10^6 + 7500 \times 50^2$

$= 32.8125 \times 10^6\,mm^4$

$I_{xx} = 20.3125 \times 10^6 + 32.8125 \times 10^6$

$= 53.125 \times 10^6\,mm^4$ Ans

Moment about yy axis ...

$$I_{G_1} = \frac{db^3}{12} = \frac{50 \times 150^3}{12} = 14.0625 \times 10^6 \, mm^4$$

$$I_{G_2} = \frac{db^3}{12} = \frac{150 \times 50^3}{12} = 1.5625 \times 10^6 \, mm^4$$

From y axis the distance is zero.

M.I about Y-Y axis ①.

$$I_{G_1} + \cancel{a_1 h_1^2} \approx 0 = 14.0625 \times 10^6 \, mm^4$$

M.I about Y-Y axis ②

$$I_{G_2} + \cancel{a_1 h_1^2} \approx 0 = 1.5625 \times 10^6 \, mm^4$$

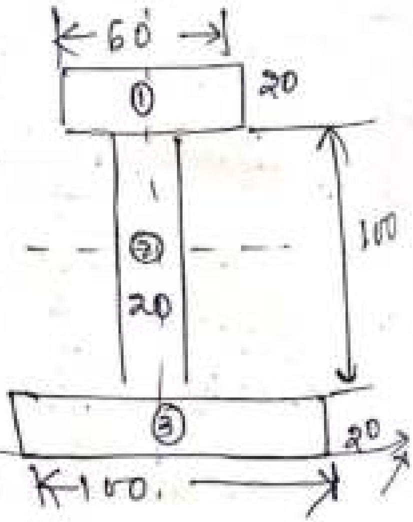
$$I_{yy} = 14.0625 \times 10^6 + 1.5625 \times 10^6$$

$$= 15.625 \times 10^6 \, mm^4 \quad \underline{\text{Ans}}$$

~~2019~~ Q)

Find the M.I of the given section about horizontal axis passing through C.G. Find M.I about X-X axis

~~Soln~~ This secn is symmetric about y axis. C.G. pass



Rect ①. $a_1 = 60 \times 20 = 1200 \, mm^2$

$x_1 = 60/2 = 30$

$y_1 = 120 + \frac{20}{2} = 130 \, mm$

② $a_2 = 100 \times 20 = 2000$

$y_2 = 20 + \frac{100}{2} = 70 \, mm$

③ $a_3 = 100 \times 20 = 2000$

$y_3 = 20/2 = 10 \, mm$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = 60.8 \, mm$$

Scanned by CamScanner

$I_{G_1} = \frac{bd^3}{12} = \frac{60 \times 20^3}{12} = 40 \times 10^3 \, mm^4$

$h_1 = y_1 - \bar{y} = 130 - 60.8 = 69.2 mm$

M.I of rectangle (1) about X-X

$I_{G_1} + a_1 h_1^2 = 40 \times 10^3 + [1200 \times (69.2)^2]$

$= 5786 \times 10^3 mm^4$

for (2)

$I_{G_2} = \frac{bd^3}{12} = \frac{20 \times 100^3}{12} = 1666.7 \times 10^3 mm^4$

$h_2 = y_2 - \bar{y} = 70 - 60.8 = 9.2 mm$

$I_{xx(2)} = I_{G_2} + a_2 h_2^2 = 1836 \times 10^3 mm^4$

for (3) $I_{G_3} = \frac{100 \times 20^3}{12} = 66.7 \times 10^3 mm^4$

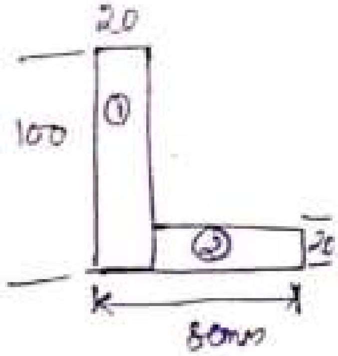
$h_3 = \bar{y} - y_3 = 60.8 - 10 = 50.8 mm.$

$I_{xx(3)} = I_{G_3} + a_3 h_3^2 = 5228 \times 10^3 mm^4$

$I_{xx} = (5786 \times 10^3) + (1836 \times 10^3) + (5229 \times 10^3)$

$= 12850 \times 10^3 mm^4$

(2018 S) Find the M.I about the centroidal X-X & Y-Y axis of the angle section.



Sol^n This section is not symmetrical about x or y axis.

Rectangle (1)

$a_1 = 100 \times 20 = 2000\ mm^2$

$y_1 = 100/2 = 50\ mm$

(2) $a_2 = 80 \times 20 = 1600\ mm^2$

$y_2 = \frac{20}{2} = 10\ mm$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{2000 \times 50 + 1600 \times 10}{2000 + 1600} = 35mm$

M.I of ① about X-X axis.

$I_{G1} = \frac{bd^3}{12} = \frac{20 \times (100)^3}{12} = 1.667 \times 10^6\ mm^4$

$h_1 = y_1 - \bar{y} = 50 - 35 = 15mm$

$I_{xx(1)} = I_{G1} + a_1 h_1^2 = 1.667 \times 10^6 + 2000 \times (15)^2$

$= 2.117 \times 10^6\ mm^4$

M.I of ② about X-X-axis

$I_{G2} = \frac{bd^3}{12} = \frac{60 \times 20^3}{12} = 0.04 \times 10^6\ mm^4$

$h_2 = \bar{y} - \bar{y}_2 = 35 - 10 = 25mm$

$I_{xx(2)} = I_{G1} + a_2 h_2^2 = 0.79 \times 10^6\ mm^4$

A

$I_{x-x} = I_{xx(1)} + I_{xx(2)} = 2.407 \times 10^6 \ mm^4$

M.I about y axis

$x_1 = 20/2 = 10 \ mm$

$x_2 = 20 + 60/2 = 50 \ mm$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = 25 mm.$$

M.I of ① about y-y axis

$$I_{G_1} = \frac{db^3}{12} = \frac{100 \times 20^3}{12} = 0.06 \times 10^6 \ mm^4$$

$h_1 = \bar{x} - x_1 = 25 - 10 = 15 mm$

$I_{yy(1)} = I_{G_1} + a_1 h_1^2 = 0.06 \times 10^6 + 2000 \times 15^2$

$= 0.517 \times 10^6 \ mm^4$

M.I of ② y-y

$$I_{G_2} = \frac{db^3}{12} = \frac{20 \times 80^3}{12} = 0.36 \times 10^6 \ mm^4$$

$h_2 = x_2 - \bar{x} = 50 - 25 = 25 mm$

$I_{yy(2)} = I_{G_2} + a_2 h_2^2 = 1.11 \times 10^6 \ mm^4$

$I_{yy} = I_{yy(1)} + I_{yy(2)}$

$= 1.627 \times 10^6 \ mm^4$

CHAPTER – 05 Principle of Lifting Machines.

5.1 ↳ Machine :- It is an assembly of interconnected components arranged to transmit or modify force in order to perform useful work.

↳ Simple machine :- It is defined as a machine which helps to do some work at some point when effort of force is applied to it.

↳ Compound machine :- It can be defined as a device which consist of no. of simple machine which enable us to do some work at a faster speed with less effort as compare to simple machine.

↳ Lifting Machine :- The machine which are use to lift heavily load are called lifting machine. In a lifting machine a force or load (W) applied at one point by means of another force called effort (P) applied at another point.

1) Mechanical Advantage (M.A)

$$M.A = \frac{\text{Weight load lifted}}{\text{effort applied}} = \frac{W}{P}$$

$$M.A = \frac{W}{P}$$

2) Velocity Ratio (V.R)

$$V.R = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{y}{x}$$

9) Input :- It can be defined as work done on the machine. It is measured by the product of effort applied & the distance covered by the effort.

$i/p = P \times y$ or effort $\times$ effort distance.

5) output :- It is defined as the work done by the machine. It is the product of load lifted & distance covered by the load.

output $= W \times x$ Load $\times$ load distance.

Efficiency (η) / Relation betn η, M.A, V.R

Ratio of $\dfrac{\text{work done by the machine.}}{\text{work done on the m/c}}$

$$= \frac{W \times x}{P \times y} = \frac{W}{P} \times \frac{x}{y}$$

$$= \frac{W}{P} \times \frac{1}{y/x} = \frac{M.A}{1} \times \frac{1}{V.R}$$

$$\boxed{\eta = \frac{M.A}{V.R}} < 1.$$

Ideal Machine

$$\eta = \frac{M.A}{V.R} = 100\%$$

i.e $\boxed{o/p = i/p.}$

Q) In a certain weight lifting m/c a weight of 1 kN is lifted by an effort of 25 N. While wt. moves by 100mm, the point of application of effort moves by 8 m. Find MA, VR & η.

Sol^n

$W = 1 kN$

$P = 25 N$

$x = 100 mm = .1 m$

$y = 8$

$MA = W/P = 40$

$VR = y/x = 80$

$\eta = MA/VR = 0.5 = 50\%$

2) Effort $= 50$ N (P)

Load (W) $= 500$ N

Effort distance $= (y) = 55$ cm $= .55$ m

Load distance $= (x) = 5$ cm $= 0.05$ m

$VR = y/x = \frac{.55}{.05} = 11$

$M \cdot A = \frac{500}{50} = 10$

$\eta = \frac{10}{11} = 0.9\% = 90\%$

3) $V.R = 50$ Determine W & P = 60

$\eta = 70\%$

$VR = y/x$ $\eta = \frac{MA}{VR}$

$\Rightarrow .70 = \frac{MA}{50}$

$MA = \frac{W}{P}$

$\Rightarrow W = 2100$ N. $\Rightarrow MA = 35$

Reversibility of a Machine

Sometimes, a machine is also capable of doing some work in the reversed direction, after effort is removed. Such a m/c is called a reversible m/c & known as reversibility of a machine.

Condition for Reversibility of a m/c

W → load lifted by the m/c

P → effort req to lift the load

y → distance moved by effort

x → distance moved by load.

$i/p = p \times y$

$o/p = w \times x$

We know that m/c friction = $i/p - o/p$

$= p \times y - w \times x$

If the m/c is reversible then the o/p of the machine should be more than friction.

$w \times x > p \times y - w \times x$

$\Rightarrow 2w \times x > p \times y$

$\Rightarrow \frac{w \times x}{p \times y} > \frac{1}{2}$

$\Rightarrow \frac{w/p}{y/x} > 1/2$

$\frac{M.A}{V.A} > 1/2$

$\frac{MA}{VR} > 50\%$

$\eta > 50\%$

So the condition is if the machine is reversible the efficiency is more than 50%.

Self locking m/c

Some time a machine is not capable of doing any work when the effort is removed. Such machine is called as self locking machine. Here the efficiency should not be more than 50%.

Law of Machine.

Law of machine may be defined as the relationship between effort applied & load lifted.

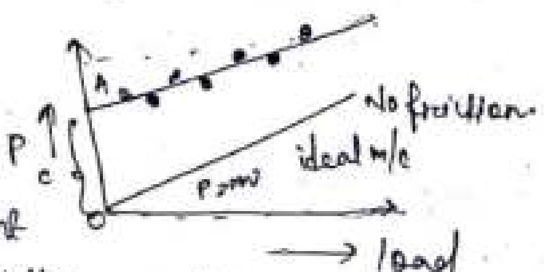
Mathmetically it is $p = mW + C$

$p \rightarrow$ effort

$w \rightarrow$ Load lifted

(slope) $m \rightarrow$ const

$C \rightarrow$ Another const. represent m/c friction.



amount of friction need to overcome by the machine.

(Q) What load can be lifted by an effort of 120N, if the velo. ratio is 18 & $\eta = 60\%$. Determine the law of the machine, if it is observed that an effort of 200N is req. to lift a load of 2600N & find the effort req. to run the m/c at a load of 3.5kN.

Sol^n $V.R = y/x = 18$ $P = 120$

$\eta = .6$

$$\frac{W/P}{V/R} = .6 \Rightarrow \frac{W}{P} = V.R \times .6$$
$$= 18 \times .6$$
$$= 10.8$$

$$\Rightarrow W = 120 \times 10.8$$
$$= 1296\,N$$

Laws of m/c $P = 200$
$W = 2600$

$P = mW + c$

$120 = m \times 1296 + c$ —— ①
$200 = m \times 2600 + c$ —— ②

$-80 = -m\,1304$

$\Rightarrow m = 0.061$

put the value of m in equ^n ②

$\cancel{120 = 0.061 \times 1296 + C}$ $200 = 0.061 \times 2600 + c$

$\cancel{\Rightarrow C = 1.15}$ $\Rightarrow C = 44$

Now effort req. to lift a load of $3.5kN = 3.5 \times 10^3 N$

$P = .061 \times 3.5 \times 10^3 + 44$

$P = 257\,N$ Ans

Q) In a lifting m/c an effort of 40N raised a load of 1kN. If efficiency of the m/c is 0.5. what is its velocity ratio? If on this m/c an effort of 74 N raised a load of 2kN. what is new efficiency? what will be the effort req. to raise a load of 5kN.

Sol) $P = 40N$ $\quad \eta = 0.5$

$W = 1kN = 1000N$ $\quad P = 74N$ $\quad W = 2kN = 2000N$.

velocity ratio when effi is 0.5

$M.A = \frac{W}{P} = \frac{1000}{40} = 25$

$\eta = \frac{M.A}{V.R} = \frac{25}{V.R} \Rightarrow V.R = \frac{25}{0.5} = 50$

Effi when P is 74 & W = 2000N

$M.A = \frac{W}{P} = \frac{2000}{74} = 27$

$\eta = \frac{M.A}{V.R} = \frac{27}{50} = 54\ \%$

effort req. to raise a load of 5kN or 5000N

$P = mW + C$

$40 = m \times 1000 + \cancel{C}$

$74 = m \times 2000 + \cancel{C}$

$\Rightarrow 34 = 1000\ m$

$\Rightarrow m = 0.034$

value of C.

$40 = m \times 1000 + C$

$\Rightarrow 40 = 0.034 \times 1000 + C$

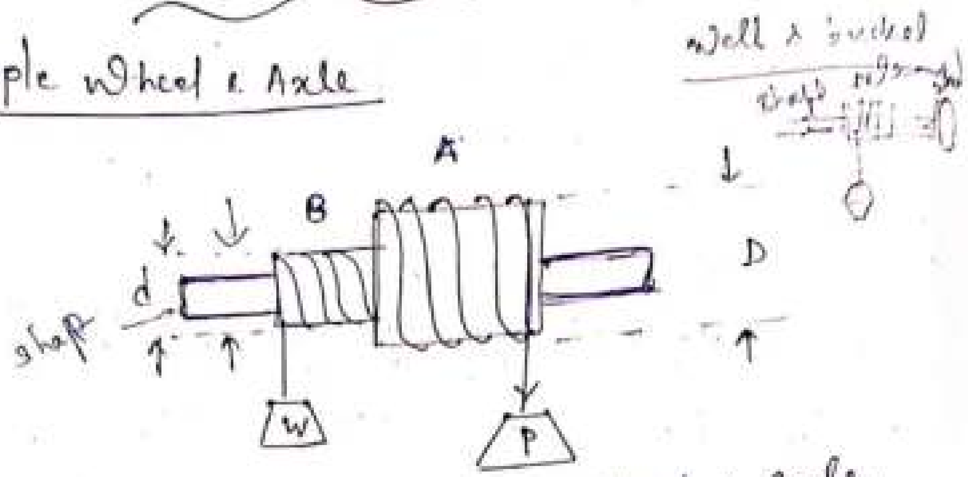
$\Rightarrow C = 6$

$P = 0.034\ W + 6$

$\Rightarrow P = 0.034 \times 5000 + 6 = \underline{176\ N}$

5.2 Simple Lifting Machine

Simple Wheel & Axle



The above is the fig of simple wheel & Axle.

- The Wheel A & axle B are keyed to the same shaft. The shaft is mounted on ball bearing, to reduce the frictional resistance minimum.
- A string is wound round the axle B, which carries the load to be lifted. A second string is wound round the wheel A in the opposite direc^n to that of the string on B.

D → Dia of effort wheel W → load lifted
d → " " " load axle P → Effort applied

- one end of the string is fixed to the wheel, while the other is free & the effort is applied to this end.
- Since the two strings are wound in opposite directions, therefore a downward motion of the effort (P) will raise the load (W)

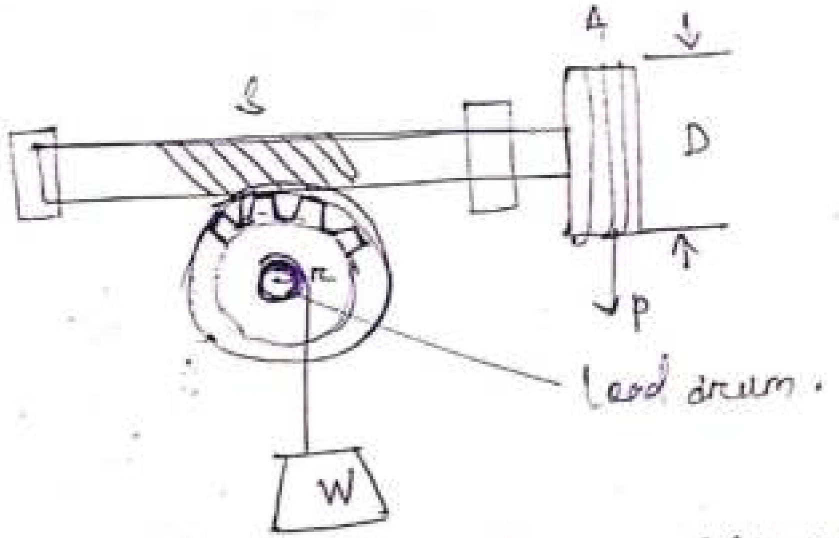
$$M.A = \frac{W}{P}$$

Distance/Displacement by the wheel $= \pi D$
" " " Axle $= \pi d$

$$V.R = \frac{\pi D}{\pi d} \Rightarrow V.R = \frac{D}{d}$$

$$\eta = \frac{M.A}{V.R}$$

Worm & Worm wheel



↳ It consist of a square threaded screw. S (known as worm) & a toothed wheel (known as worm wheel) geared to each other..

↳ A wheel A is attached to the worm, over which passes a rope as shown in fig.

D → Dia of effort wheel

r → radius of the load drum.

W → load

P → Effort applied

T → No. of teeth on the worm wheel.

$$M.A = \frac{W}{P}$$

~~Displace~~ Distance moved by wheel = πD

" " " Load drum = $\frac{2\pi r}{T}$

$$V.R = \frac{\pi D}{2\pi r/T} = \frac{DT}{2r}$$

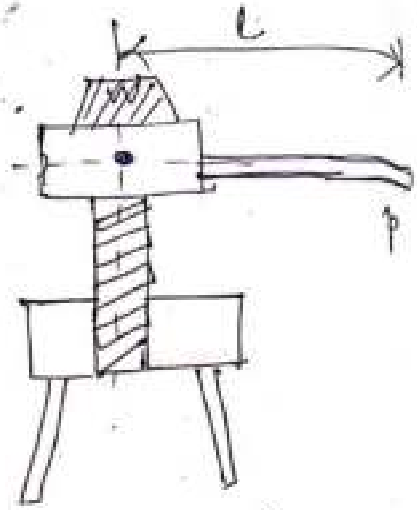
$$\eta = \frac{M.A}{V.R}$$

if the there is thread of n no.

then $V.R = \frac{DT}{n \times 2 \times r}$

Simple Screw Jack

It consist of a screw, fitted in a nut, which forms the body of the Jack. The principle, on which a screw works, is similar to that of an inclied plane.



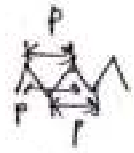
↳ The fig shows a simple screw Jack.

↳ $L \rightarrow$ length of effort arm

$P \rightarrow$ effort

$W \rightarrow$ load

$p \rightarrow$ pitch of the screw



The distance moved by the effort in one revolution $= 2\pi l$

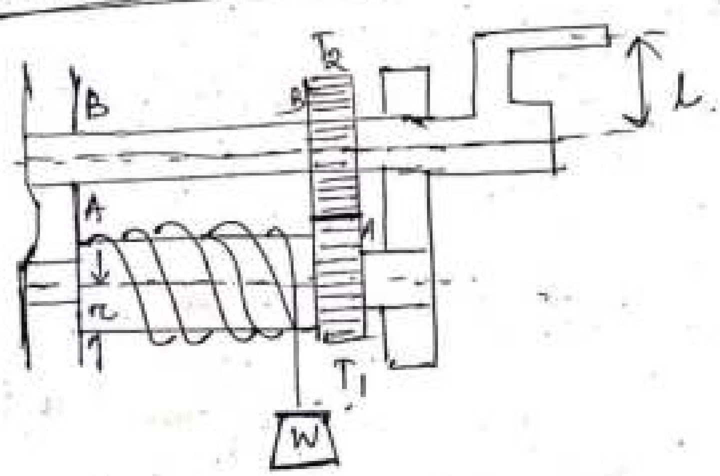
Distance moved by the load $= p$

$$V \cdot R = \frac{2\pi l}{p}$$

$$M \cdot A = \frac{W}{P}$$

$$\boxed{\eta = \frac{M \cdot A}{V \cdot R}}$$

Single purchase Crab winch



In a single purchase crab winch, a rope is fixed to the drum & is wound a few turns around it.

The free end of the rope carries a load W.

↳ A toothed wheel A is rigidly mounted on the load drum

↳ Another toothed wheel B called pinion is geared with wheel A.

$T_1 \rightarrow$ no. of teeth in wheel / gear A.

$T_2 \rightarrow$ " " " " / " B.

$l \rightarrow$ length of handle

$r \rightarrow$ radius of load drum

$W \rightarrow$ Load

$P \rightarrow$ effort.

Distance moved by the effort in one revolution of handle

$= 2\pi l$

no. of revolⁿ made by pinion B = 1

" " " " A $= \frac{T_2}{T_1}$

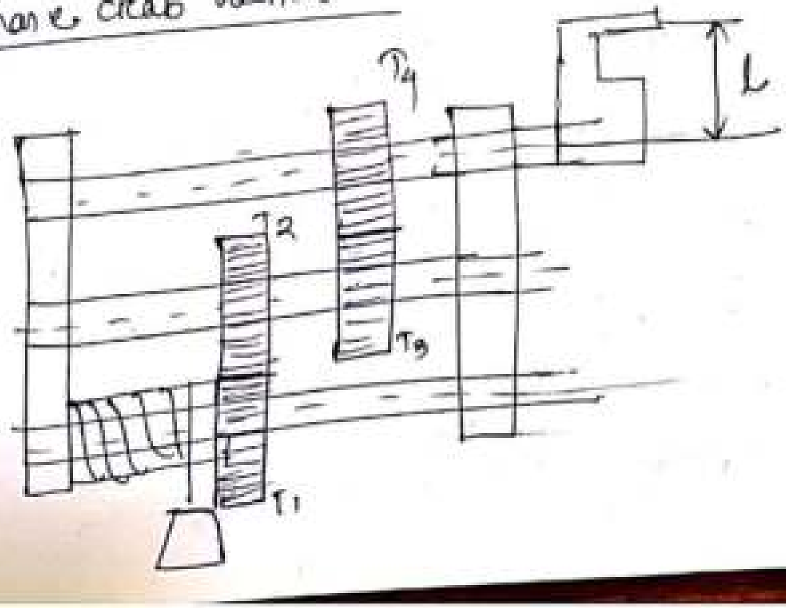
" " " " load drum $= T_2/T_1$

distance moved by load $= 2\pi r \times T_2/T_1$

$$V.R = \frac{2\pi l}{2\pi r \times T_2/T_1} = \frac{T_1 \times l}{T_2 \times r}$$

$$M.A = \frac{W}{P} \qquad \boxed{\eta = \frac{M.A}{V.R}}$$

Double purchase crab winch



It is the improved version of single purchase crab winch. Here there are 2 spur wheel & 2 pinion.

T_1 meshed with T_2 (pinion)

T_3 " " T_4 (pinion)

l = length of the handle.

T_1 & T_3 = No. of teeth in spur wheels

T_2 & T_4 = " " " pinion "

r = radius of drum

W = load

P = effort

Distance moved by effort in one revolution of handle

$= 2\pi l$

No. of revoln made by pinion 4 = 1

" " " " spur 3 = T_4/T_3

" " " " pinion 2 = T_4/T_3

" " " " spur 1 = $\frac{T_2}{T_1} \times \frac{T_4}{T_3}$

Distance moved by load = $2\pi r \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}$

$$V.R = \frac{2\pi l}{2\pi r\,(T_2/T_1)\,(T_3/T_4)} = \frac{l}{r}\left(T_1/T_2 \times T_4/T_3\right)$$

$M.A = W/P$

$$\boxed{\eta = \frac{M.A}{V.R}}$$

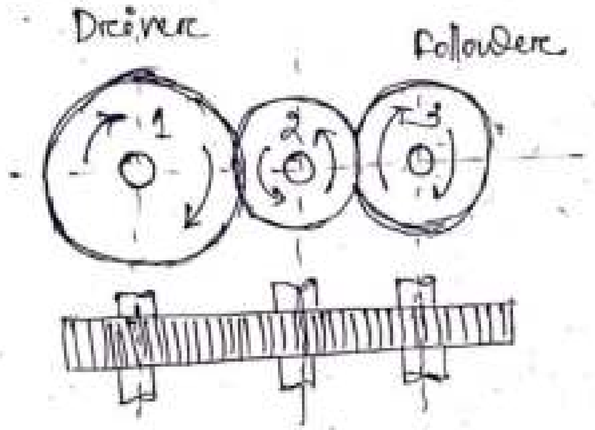
Gears

A gear may be defined as a pulley or wheel having projections on its rim known as teeth. It is also used for power transmission.

Gear train

Sometime two or more gears are made to mesh with each other, so as to operate as a single system, to transmit power from one shaft to another. Such a combination is called gear train or train of wheels.

1) simple gear train



The above fig shows a simple gear train.

$N_1 \rightarrow$ speed of driver

$T_1 \rightarrow$ No. of teeth on the driver

$N_2 \rightarrow$ speed of Intermediate

$T_2 \rightarrow$ No. of teeth on "

$N_3 \rightarrow$ Speed of follower

$T_3 \rightarrow$ No. of teeth " "

Velocity ratio of gear 1 & 2 $\quad \dfrac{N_2}{N_1} = \dfrac{T_1}{T_2}$ ——— (1)

Velocity ratio of gear 2 & 3 $\quad \dfrac{N_3}{N_2} = \dfrac{T_2}{T_3}$ ——— (2)

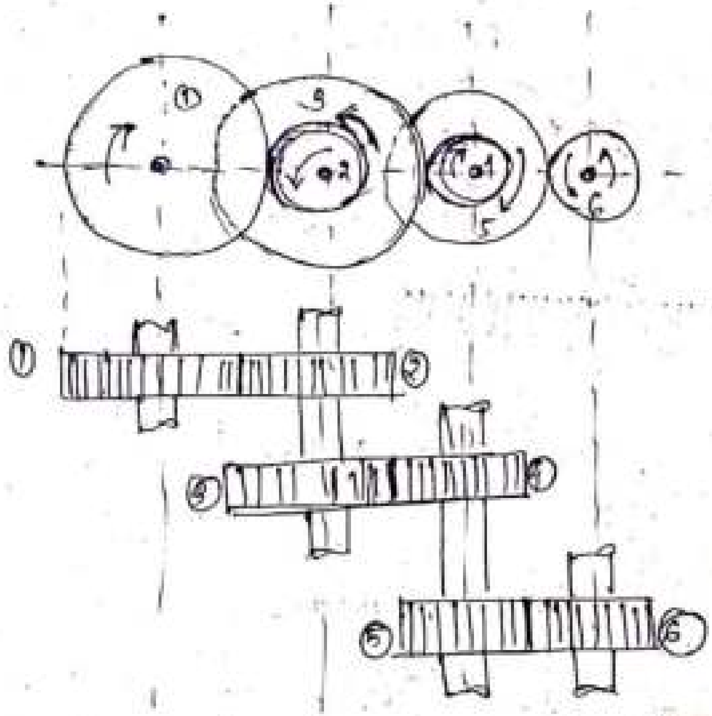
Multiplying eqn ① & ②

$$\frac{N_3}{N_2} \times \frac{N_2}{N_1} = \frac{T_2}{T_3} \times \frac{T_1}{T_2}$$

$$\Rightarrow \boxed{\frac{N_3}{N_1} = \frac{T_1}{T_3}}$$

Compound gear train

When more then one gear is mounted on same shaft it is known as compound gear train.



$N_1, N_2, N_3, N_4, N_5, N_6 \longrightarrow$ Speed of respective wheel

$T_1, T_2, T_3, T_4, T_5, T_6 \longrightarrow$ no. of teeth " "

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \quad (\text{for } 1 \ \& \ 2)$$

$$\frac{N_4}{N_3} = \frac{T_3}{T_4} \quad (\text{" } 3 \ \& \ 4)$$

$$\frac{N_6}{N_5} = \frac{T_5}{T_6} \quad (\text{" } 5 \ \& \ 6)$$

Multiplying ① ② ③.

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\Rightarrow \boxed{\frac{N_6}{N_1} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}} \quad \left\{ \begin{array}{l} N_2 = N_3 \\ N_4 = N_5 \end{array} \right\}$$

5.3 Derrik

A derrik is a lifting device. These are used to lift heavy loads. Normally used in building construction, port etc, marine sector,

↳ These are also known as stationary cranes.

↳ The most basic type of derrick is controlled by 3 or 4 lines connected to the top of the mast/column which allow it to move in lateral directⁿ & up&down motion.

↳ Normally the height of a derrick is 265 ft (80m).

*A Shaft is a live member & axle is a dead member. Shaft is used to transmit power from one mechanical member to another, while axle is used to only support the load / transmit motion.

CHAPTER-06 | DYNAMICS

6.1 **Dynamics** :- It is the study of motion of rigid body and their relation with the forces causing them.

The entire system of dynamics is based on 3 laws of motion. Also known as newton law's of motion.

Newton's 1st law

It states that " Every body continues in its state of rest or of uniform motion in a straight line, unless it is acted up on by some external force.
It is also called as law of inertia.

↳ A body at rest has a tendency to remain at rest called inertia of rest.

↳ A body in uniform motion in a straight line has a tendency to preserve its motion. known as inertia of motion. *I

Newton's 2nd Law

" The rate of change of momentum is directly proportional to the impressed force and takes place in the same dirn in which the force acts".

m = mass of a body
u = initial velo. of the body
v = final velo of the body
a = const. acclⁿ
t = time in seconds req. to change the velo u to v.
F = Force req. to change velo from u to v in t sec.

Initial momentum = mu
final " = mv

Rate of change of momentum = $\frac{mv - mu}{t} = \frac{m(v-u)}{t}$
$= ma$
$\left(\because \frac{v-u}{t} = a\right)$

Acc to 2nd law $F \propto ma$

$\Rightarrow F = Kma$

$K \rightarrow$ const.

For convenience, the unit of force adopted in such that it produces unit accel^n in unit mass.

$F = ma$ = mass x accel^n

In S.I system unit of force is Newton → N.
A Newton may be defined as the force which acting upon a mass of 1 kg, produces an accel^n of $1 m/s^2$ in the direc^n of which it acts.

_Also known as law of dynamics.

If accel^n is due to gravity $a = 9.8 \ m/s^2$ = 1 kg.wt

$F = ma$
$\Rightarrow F = 9.8$ ~~kg~~ N
~~= 9.8~~
= 1 kg.wt
= 1 kg.F

(1 kg.wt = 9.8 N)
[1 kg.F = 9.8 N)

Q) body has 50 kg mass on earth. Find a where $g = 9.8 m/s^2$
b) on moon $g = 1.7 m/s^2$
c) " sun. $g = 270 m/s^2$

$F_1 = 50 \times 9.8$

$F_2 = 50 \times 1.7$

$F_3 = 50 \times 270$

Kinetic equations

1) $v = u + at$

2) $s = ut + \frac{1}{2}at^2$

3) $v^2 = u^2 + 2as$

2) A particle of mass 50gm starts from rest & moves under the influence of a const force. It acquires a speed of 6m/s after 12s.

i) Find force on the particle

ii) Find speed at $t = 16s$

iii) Find distance covered by the particle in 1st 10s.

iv) Find " " " " during time interval 5 to 15s.

Soln

m = mass of the particle = 50g = 0.05 kg

$u = 0$

$v = 6$ m/s

$t = 12$. i) $v = u + at$

$F = ma$ $\Rightarrow 6 = 0 + a \times 12$

$\Rightarrow a = 6/12 = 0.5$ m/s^2

i) $F = m \times a$

$= .05 \times 0.5 = 0.025$ N

ii) $v = u + at$

$= 0 + (0.5 \times 16) = 8.0$ m/s

iii) $s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 0.5 \times 10^2 = 25$m

iv) $s_2 - s_1 = \left(ut_2 + \frac{1}{2}at_2^2\right) - \left(ut_1 + \frac{1}{2}at_1^2\right)$

$= (\quad)$

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Newton 3rd law of Motion

To every action there is an equal & opposite reacn.

Momentum :- It is the product of mass with velocity.

m x v

Force :- Any external agent which produce or tends to produce, destroys or tends to destroy the motion of any body. known as Force. unit N.

F = m x a

Inertia :- The property which offers resistance to change state of rest or motion is known as inertia.

Newton 3rd law for recoil of gun

When bullet is fired from a gun, the opposite reaction of the bullet is known as recoil of gun.

$M \rightarrow$ Mass of gun.

$m \rightarrow$ Mass of bullet.

$V \rightarrow$ velo. of gun

$v \rightarrow$ velo of bullet after being fired.

momentum ~~before~~ of the gun $= MV$

" " , bullet $= mv$

$$\boxed{MV = mv}$$

Law of conservation of Momentum.

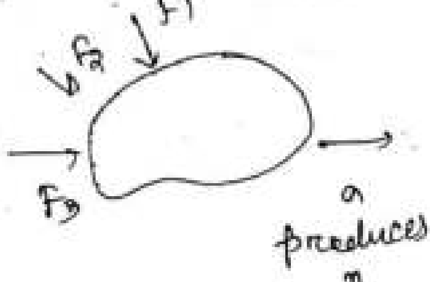
D'alembert's Principle

A system of forces acting on a body in motion is in dynamic equilm with inertia force of the body.

Inertia $\rightarrow$ Resist motion

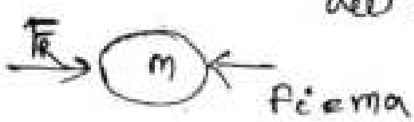
$\rightarrow$ resist to be at rest

The resultant of F_1, F_2, F_3 let R.



Let a mass m.

Resultant to bring the body at rest, we have to apply a force ~~was~~ in opposite direction whose value is equal to ma.



~~known as inertia force~~

~~whose value is~~

known as inertia force, to bring the body in static equl^n.

$$\Sigma F = 0$$

$$F_R - ma = 0$$

$$\Rightarrow F_R = ma \quad \Rightarrow \boxed{F_i = ma}$$

$-ma \rightarrow$ inertia force $= F_i$, Also known as reversed force.

6.2 Work, power, Energy

Work

When force acts on a body, the body undergoes a displacement, work is said to be done on the body by the force.

$$W = F \cdot S$$

unit

$$W = F \cdot S$$

$$= N\text{-}m = 1 \text{ Joule} \quad (SI)$$

$$1 \text{ erg} = CGS = 1 \text{ dyne} = 10^{-7} \text{ Joule}$$

power

It is the rate of doing work.

unit = Watt = J/s = N-m/s

Energy

It is the capacity to do work.
It exists in many forms, mechanical, electrical, Chemical, heat, light etc.

unit

Same as work = Joule

Mechanical Energy — Kinetic = $\frac{1}{2}mv^2$
— potential = mgh

Kinetic Energy

Energy possed by a body, by virtue of its mass & velocity.

P.E

Energy possed by a body, by virtue of its position.

Q) A truck of mass 15 tonnes travelling at 1.6 m/s. impact with a spring

Law of conservation of Energy

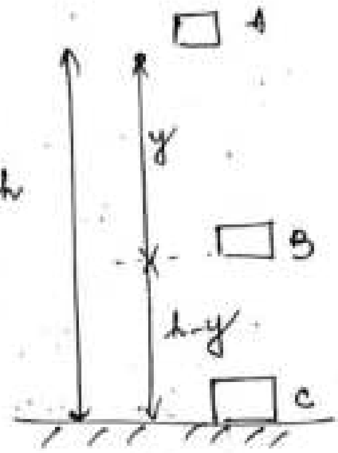
It states that " Energy can neither be created nor destroyed. though it can transformed from one form to another form.

Transformation of Energy

Consider a body of mass m which is released from rest from height h above the ground.

m = mass of the body

h = height



Energy at A

Since at A body has 0 velocity

$KE = 0$

$PE = mgh$

total energy = $PE + KE = mgh$

Energy at B

The body travelled y distance from A to B. so

$v = \sqrt{2gy}$

KE at B $= \frac{mv^2}{2} = \frac{m \times (\sqrt{2gy})^2}{2} = mgy$

$PE = mg(h-y) = mgh - mgy$

total energy $= KE + PE = \cancel{mgy} + mgh \cancel{- mgy}$

$= mgh$

Energy at C

At C body has fallen a height h.

$v = \sqrt{2gh}$

$KE = \frac{mv^2}{2} = \frac{m(\sqrt{2gh})^2}{2} = mgh$

$PE = 0$

total energy $= KE + PE = mgh$

Q) A 100gm ball is released from rest from the top of 20m high building. Find the change in P.E & K.E when it is at a height of 10 from the ground.

Soln

$m = 100gm = 0.1kg$

$h_1 = 20m$

$P.E_1 = mgh_1 =$

$K.E_1 = 0$

Impulse → When a const. force F acts on a body for a time interval t. known as Impulse.

$\boxed{I = F \times t}$ unit N-s

Linear momentum -

Law of conservation of linear momentum

Acc to newton's 2nd law, the net external force acting on a body is equal to rate of change of linear momentum / momentum.

This leads to the law of conservation of linear momentum for a body.

which states that the linear momentum of a body remains const. if the external force on a body is zero.

6.3 Collision of Elastic Bodies

When two bodies strikes with each other with certain velocity it is known as collision.

↳ If one body is in rest and even if another body strikes to it (wall or floor) also known as collision.

↳ Let any ball strikes to the floor, it rises certain height or rebounded.

↳ This property of bodies by virtue of which, they rebounded after impact is called elasticity.

↳ But if a body does not rebound at all, after impact called as inelastic collision.

Phenomenon of collision

- The bodies, immediately after collision, come momentarily to rest.
- The two bodies tend to compress each other, so long as they are compressed to the maxi^{m} value called as time of compression. (t_c)
- The process of regaining of original shape from the deformed shape of the bodies called restitution. Time taken for that called as time of restitution (t_r)

Time of collision = Time of compression + Time of restitution

Law of conservation of Momentum

It states that the total momentum of two bodies remains const. after their collision.

$$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

m_1 = mass of 1st body

m_2 = " " 2nd body

u_1, u_2 = initial velocity of mass m_1 & m_2 respectively

v_1, v_2 = final " " " m_1 & m_2 "

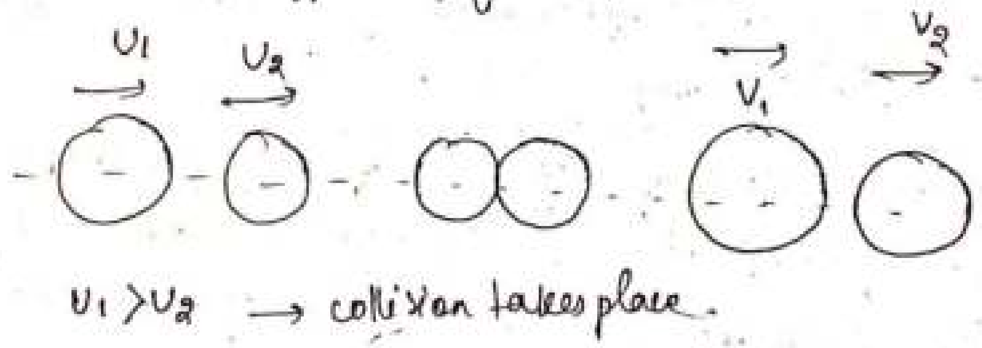
Newtons Law of collision of elastic bodies

It states when two moving bodies collide with each other, their velo. of separation bears a const ratio to their velo. of approach.

$$(v_2 - v_1) = e\,(u_1 - u_2)$$

$$e = \frac{u_1 - u_2}{v_2 - v_1}$$

e = co-efficient of restitution.



$u_1 > u_2 \rightarrow$ collision takes place.

$v_2 > v_1 \rightarrow$ separation takes place.

Types of collision

→ Direct collision

→ Indirect "

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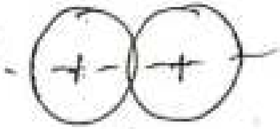
Direct collision

The line of impact of the two colliding bodies, is in the line Joining the centers of the 2 bodies, known as point of contact or point of collision.

$\vec{m_1}$ $\vec{m_2}$



$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$.

The value of e is in betn 0 to 1

if e = 0 collision is inelastic

e = 1 " " elastic.

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Q) A ball of mass 2 kg moving with a velocity 2m/sec hit another ball of mass 4 kg at rest, after impact the 1st ball comes to rest. Cal. velo. of the 2nd ball after impact & coeffi of restitution.

u_1

$m_1 = 2kg$ $u_1 = 2 m/s$

$m_2 = 4 kg$ $v_1 =$

$v_2 =$

$u_2 =$

$e =$

$v_1 = 0$ (comes to rest m_1)

$u_2 = 0$ at rest (m_2)

$m_1 u_1 + m_2 \cancel{u_2}^{0} = m_1 \cancel{v_1}^{0} + m_2 v_2$

$\Rightarrow 2 \times 2 = 4 \times v_2$

$\Rightarrow v_2 = 1 m/s$

$(v_2 - v_1) = e(u_1 - u_2)$

$\Rightarrow e = \frac{1-0}{2-0} = \frac{1}{2} = 0.5$ Ans

Q) 2015, 2018

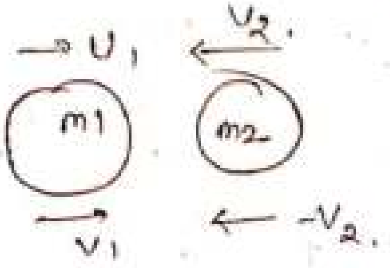
Two balls of masses 2kg & 3kg are moving with velo 2m/s & 3m/s towards each other. If e = 0.5. find velocity of the two balls after collision.

Sol)

$m_1 = 2$ $u_1 = 2$

$m_2 = 3$ $u_2 = 3$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$



$\Rightarrow \frac{1}{2} = \frac{v_2 - v_1}{\cancel{2-(-3)}} = \frac{-v_2 - v_1}{2-(-3)}$

$\Rightarrow \cancel{v_2 - v_1 = \ldots \frac{1}{2}}$

$\Rightarrow \frac{1}{2} = \frac{-v_2 - v_1}{5}$

$\Rightarrow -v_2 - v_1 = 5/2$ ——— (2)

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\Rightarrow 2 \times 2 + 3(-3) = 2v_1 + (-3v_2)$

$\Rightarrow 2v_1 - 3v_2 = -5$ ——— (1)

multiply 2 in eq (2) (2 × 2)

$$\begin{aligned} \cancel{2v_1} - 3v_2 &= -5 \\ -\cancel{2v_1} - 2v_2 &= {}_{+}5 \\ \hline -5v_2 &= 0 \\ v_2 &= 0 \text{ m/s} \end{aligned}$$

$\Rightarrow v_2 + v_1 = -5/2$

$\Rightarrow v_2 = -5/2 - v_1$

put the value at equ^n (2)

$2v_1 - 3(-5/2 - v_1) = -5$

$\Rightarrow 2v_1 + 15/2 + 3v_1 = -5$

$\Rightarrow v_1 = -2.5$ m/s

Now $v_1 = -5/2 = -2.5$ m/s

$v_2 = 0$

2) A ball is dropped from a height of 10m on a smooth floor and it rebound to a height of 5m. Determine the coefficient of restitution between the ball & the floor & also determine the expected height of the 2nd rebound.

$u \rightarrow$ velo before impact

$v \rightarrow$ " after "

$h \rightarrow$ hight before = 10m

$h_1 \rightarrow$ " after 1st rebound 5m

$h_2 \rightarrow$ " " 2nd , ?

$U = \sqrt{2gh}$ (when the body is at a height 10m $V=0$)

$V = \sqrt{2gh_1}$

$e = \frac{V}{U} = \frac{\sqrt{2gh_1}}{\sqrt{2gh}}$

$\Rightarrow e = \sqrt{h_1/h}$

$\Rightarrow e = 0.7$

similarly

$e = \sqrt{h_2/h_1}$

$\Rightarrow (0.7)^2 = \frac{h_2}{5}$

$\Rightarrow \underline{\underline{h_2 = 2.5m}}$